Title: Cosmology

Speakers:

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String Cosmology is the attempt to understand the cosmology of our Universe from string theory perspective.

In the early days one approach consisted of looking at the low energy string actions and using duality symmetries to generate cosmologies which involved collapsing universes bouncing into FWRL type scenarios - known as pre big bang scenarios

These mainly involved evolving fields in flat potentials, and suffered from the fact that the quantum loop corrections (g_s) and corrections from integrating out heavy string modes (α') were not really under control.

Running alongside this was an approach towards considering the stabilisation of the key light fields associated with the string dynamics, as these determine the fundamental constants and involved non-perturbative contributions. It is hard !

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In this lecture we will concentrate on the latter, but hopefully touch on the former, as I can't help but think there is still something relevant in it !

Some PBB related reviews:

M. Gasperini & G. Veneziano —Astropart. Phys. 1 (1993) 317-339;

Nuovo Cim. C 38 (2016) 5, 160

J.Lidsey, D. Wands & EJC — Phys. Rept 337 (2000) 343-492

Reviews concentrating on string cosmology stabilising moduli, Inflation and Dark Energy:

M. Cicoli, J. Conlon, A. Maharana, S. Parameswaran & F. Quevedo

e-print: 2303.04819

C. Burgess & L. McAllister — CQG 28 (2011) 204002

L. McAllister & E. Silverstein GRG 40 (2008) 565-605

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String Theory exists in ten spacetime dimensions

This is crucial as we only perceive four in nature.

For consistency of the theory with observation, we need to deal with those extra spatial dimensions, typically curling them up into different small shapes.

Moduli fields are key - control the shape and size of the extra dimensions.

It means they set the magnitude and couplings of the 4D effective field theory.

Provide good candidates for inflation, also can acquire non-trivial time dependence in post inflationary string cosmology.

Today - all moduli must be either stuck in their potential minima or at best slowly varying - how do we guarantee this can happen ?

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Moduli and String Compactification

Starting from 10D theory, the different fields have to be decomposed into their components in the 4 non-compact dimensions and the extra compact dimensions.

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & g_{\mu n} \\ g_{n\nu} & g_{mn} \end{pmatrix} \qquad \mu, \nu = 1, \cdots, 4\,; \qquad m, n = 1, \cdots, 6$$

For example - the 10D graviton g_{MN} splits into a 4D graviton $g_{\mu\nu}$, a set of scalars g_{mn} (the moduli fields) and also vector fields $g_{\mu n}$.

Similar decompositions occur with higher form ATF B_{MN}, C_{MNP} etc…

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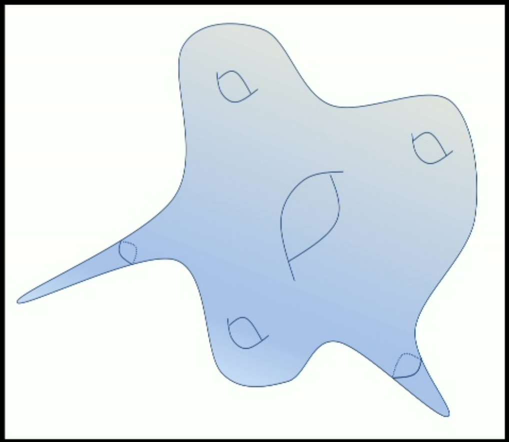
Most compactifications preserve N=1 SUSY

Favoured internal spaces are Calabi Yau manifolds.

Possess SU(3) holonomy, are Ricci flat, they are complex Kähler manifolds, hence the metric can be written as a second derivative of a Kähler potential

$$K(z_i, \bar{z}_j): \quad g_{ij} = \partial_i \partial_j K$$

These manifolds tend not have isometries - so far no known analytic metrics for compact CY manifolds. Make use of their topological structure such as non-trivial homological cycles (Hodge numbers)



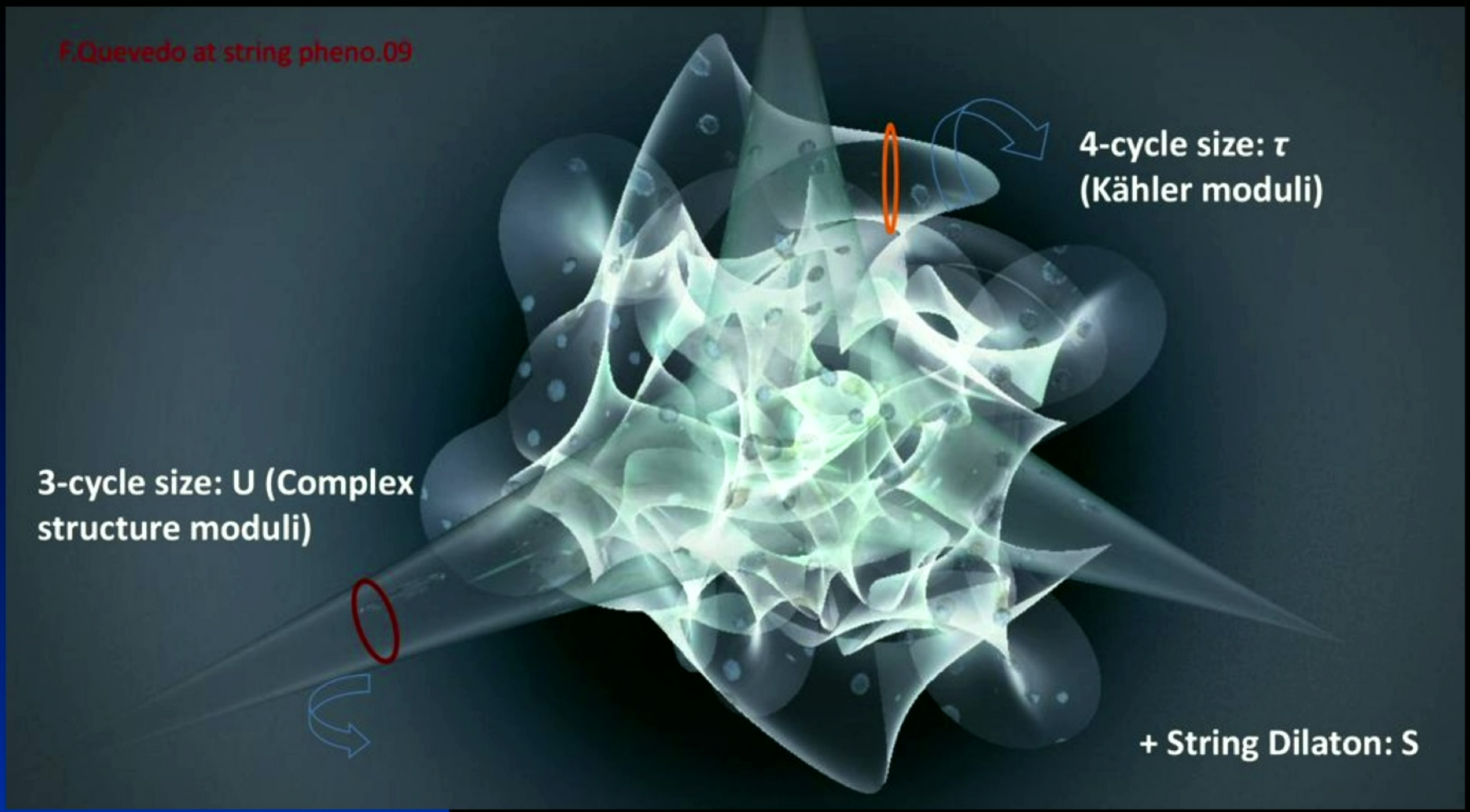
[Recent photo of CY manifold - from Cicoli et all 2023]

Simplest CY is a 2-Torus - gives 2 geometric moduli

$$T = \sqrt{g} + iB_{12}, \qquad U = \frac{\sqrt{g}}{g_{22}} + i\,\frac{g_{12}}{g_{22}},$$

Re T (Kähler modulus) - size of torus

Re U (complex structure modulus) — shape



[Credit - Tetsutaro Higaki]

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Massless boson spectrum of the five string theories and of 11 D SUGR

Theory	Dimension	Supercharges	Massless Bosons
Heterotic $E_8 \times E_8$	10	16	g_{MN}, B_{MN}, ϕ A_M^{ij}
Heterotic $SO(32)$	10	16	g_{MN}, B_{MN}, ϕ A_M^{ij}
Type I $SO(32)$	10	16	g_{MN}, ϕ, A_M^{ij} C_{MN}
Type IIA	10	32	g_{MN}, B_{MN}, ϕ C_M, C_{MNP}
Type IIB	10	32	g_{MN}, B_{MN}, ϕ C_0, C_{MN}, C_{MNPQ}
M-Theory	11	32	g_{MN}, C_{MNP}

[Cicoli et al 2023]

α' – Inverse string tension determines characteristic string length scale:

$$l_s \equiv \sqrt{\hbar c \alpha'}$$

Effective D-dim Planck length dep on dilaton and inv string tension:

$$l_{\mathrm{Pl}}^{(D)} \equiv e^{\varphi/(D-2)} \sqrt{\hbar c \alpha'}$$

Gauge coupling strength :

$$\alpha_{\mathrm{gauge}} \sim g_s^2 \equiv e^{\varphi} = \left(\frac{l_{\mathrm{Pl}}}{l_s}\right)^{D-2}$$

$$e^{\varphi} << 1 -- \text{weak coupling}$$

Dynamics of dilaton → profound differences from case of Einstein gravity with fixed Planck length

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Effective bosonic actions

$\alpha' \to 0$ Massive modes decouple, only massless sector remains

Under constraint, quantum corrections do not break conformal invariance, obtain effective actions for massless bosonic sector.

Ex 1: **Type IIB Superstring** – consistency requires 10 spt dimensions:

$$S_{\rm IIB} = \frac{1}{16\pi\alpha'^4}\left\{\int d^{10}x\sqrt{|g_{10}|}\left[e^{-\Phi}\left(R_{10} + (\nabla\Phi)^2 - \frac{1}{12}(H_3^{(1)})^2\right)\right.\right.$$
$$\left.\left. -\frac{1}{2}(\nabla\chi)^2 - \frac{1}{12}(H_3^{(2)} + \chi H_3^{(1)})^2 - \frac{1}{240}(F_5)^2\right] + \int A_4 \wedge H_3^{(2)} \wedge H_3^{(1)}\right\}$$

R_{10} --- Ricci scalar in 10D: $g_{10} = \det g_{MN}$:

Φ 10D dilaton determines string coupling

Note dilaton-graviton sector interpreted as 10D Brans-Dicke theory where coupling w=-1.

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Moduli are the way extra dimensions manifest in 4D - fundamental.

Gauge singlet scalars, typically with gravitational strength interactions.

In simplest SUSY cases, potential is flat and moduli massless — would be ruled out from fifth force constraints !

Require moduli stabilisation mechanisms to lift the potential, give the moduli mass and allow for viable phenomenological models.

This is key - it determines the cosmology - no free dimensionless parameters!

1. Moduli are uncharged under std model gauge gps — `no quick decay modes'
2. Couplings and interactions of moduli come with $(M_{Pl})^{-1}$ - interact weakly, hard to produce, but then live for a long time without thermalising.
3. As increase VEV of the fields, head towards decompactification limit.
4. Moduli often carry shift symmetries - (often axions in gauge kinetic functions). This means for moduli stabilisation, a potential for the modulus can not be generated perturbatively - requires non-pert like brane instanton or gaugino condensation. In weakly coupled theory, non-pert effects are small, hence moduli are light.

Moduli stabilisation - details are model dependent.

Have seen at the low energy regime, moduli are massless with flat potentials - we need to lift them. A few possible contributors include :

- Fluxes - p form flux threads a p-cycle in the internal manifold. In terms of overall radius of compactification, R, contributes to the lifting of the flatness of the radial direction as $V(R) \propto R^{-6-2p}$

- Localised objects - space filling D branes and orientifold planes - for a p-dim localised object of tension T_p we find $V(R) \propto T_p R^{p-15}$

- Order by order α' and loop (g_S) corrections - lifts the flat potential. Leading α' correction in type IIB leads to $V(R) \propto \frac{1}{R^{18}}$

- Non-perturbative effects such as gaugino condensates play a key role in stabilising flat directions associated with axionic shift symmetries

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- SUSY breaking leads to (low energy) loop corrections to V(R)

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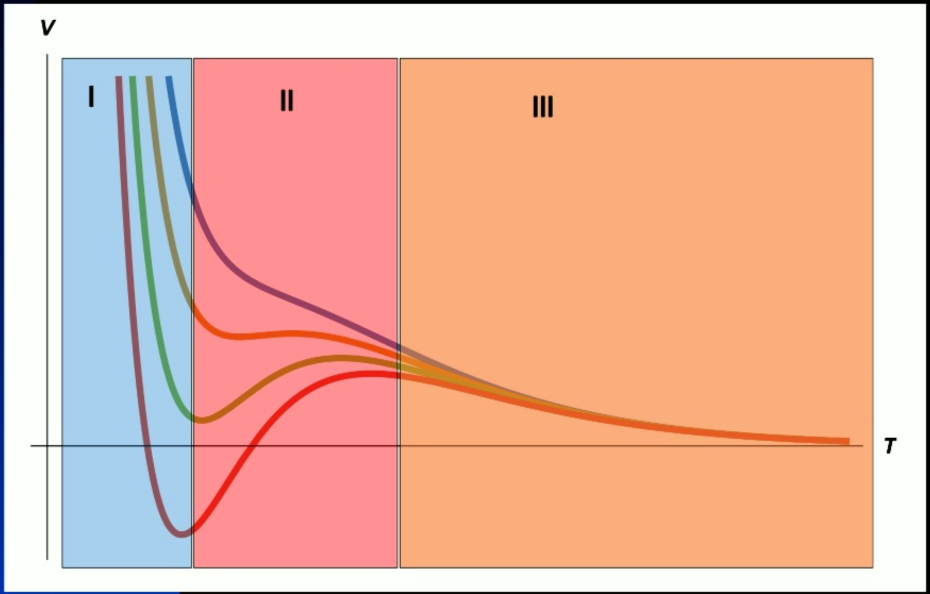
Dine-Seiberg problem

Parameters for the loop and α' expansion are actually moduli VEVs (dilaton and vol of compactification). To leading order they are flat directions, stabilisation requires tradeoff between sub leading terms - so should include all terms ! Expansion can only be trusted in runaway region — the 10D free theory

Briefly: Trust runaway region III, min will likely occur in untrustworthy small vol/strong coupling regime I, reliable min should be in II - will require help of integer fluxes, ranks of gauge gps to ensure that can happen



[Cicoli et al 2023]

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Actual details of stabilisation constructions pretty complicated. In Type IIB, for specific flux choices and localised sources, can obtain 10D solution which incorporates back reaction of fluxes.

These fluxes stabilise the dilaton and complex structure moduli.

Basic idea: start with orientifold CY, 3-form fluxes F_3 and H_3 satisfy Dirac quantisation conditions as they thread 3-cycles of the CY.

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2A(y)}\tilde{g}_{mn}(y)dy^m dy^n,$$

10D Einstein frame metric has `warp factor' — $e^{-2A(y)}$ which is sourced by a 3 form flux (G_3) and localised D3-branes and O3 planes where

$$G_3 = F_3 - iSH_3, \text{ where } S = g_s^{-1} - iC_0 \text{ is the axiodilaton}$$

Eqns of motion force G_3 to be imaginary self dual which in turn fixes the value of the complex structure modulus and the dilaton. Kähler moduli remain unfixed.

Stabilisation of Kähler moduli

Obtain 4D effective action description describing low energy fluctuations about these bgds (once KK modes are integrated out). Have complex structure moduli U, axio-dilaton S and Kähler moduli T=τ + i θ.

$$K = K_{kah} + K_{dil} + K_{cs}$$

F-term SUGR scalar potential for a superpotential $W(\Phi_I)$ and Kahler potential $K(\Phi_I, \bar{\Phi}_{\bar{J}})$

$$V_F = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \overline{W} - 3|W|^2 \right)$$

where

$$D_I \equiv \partial_I + (\partial_I K) W$$

Shift symmetries of axion part of Kähler moduli forbid their appearance in Superpotential to all order in pert theory. But the moduli represent gauge couplings for matter on D7-branes, then non-pert effects like gaugino condensation on a D7-brane can generate a superpotential for them. At tree level we have

$$K_{\mathrm{kah}} = -2 \ln \mathcal{V} \qquad \mathcal{V} = \ell_S^{-6} \int_X \sqrt{g_{(6)}}\, d^6 y$$

Volume of internal space

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With these corrections obtain:

$$W = W_{\rm flux}(S,U) + W_{\rm np}(S,U,T)$$

and

$$K_{\rm kah} = -2\ln\mathcal{V} + K_{\rm p}$$

They lead to a potential for the Kähler moduli which generates a min for the moduli stabilising them.

Two major scenarios:

•KKLT construction and Large Volume Scenario (LVS)

KKLT uses fact VEV of $W_{\rm flux}$ can be tuned to be small, hence act as a small parameter allowing terms from $W_{\rm np}$ to compete —> AdS min which is supersymmetric.

LVS uses fact $K_{\rm p}$ has contribution from an internal volume dependent α' correction. Competes with non-pert correction on a small 4-cycle —> AdS non-supersymmetric min where at the min

$$\mathcal{V} = e^{1/g_s} \gg 1$$

is exponentially large in string units

KKLT construction

Turning on fluxes in type IIB generates pot for dilaton and complex structure moduli but leaves Kähler moduli flat. Integrate out dilaton and complex structure moduli leaving low energy action for Kähler moduli. For case of single modulus:

$$W_{\rm np} = A(U,S)\, e^{-aT}$$

arising from Euclidean 3-branes or gaugino condensations on wrapped D7-branes

Full super potential $$W = W_0 + A\, e^{-aT}$$

Key requirement fluxes tuned so $|W_0| \ll 1$

Using tree-level Kähler potential: $$K = -3\log(T + \bar{T})$$ we obtain:

$$V = \frac{|aA|^2}{6\tau} e^{-2a\tau} + \frac{a|A|^2}{2\tau^2} e^{-2a\tau} + \frac{a\,\mathrm{Re}(AW_0^* e^{-i\theta})}{2\tau^2} e^{-a\tau}$$

Adjusting the phase of the axion to its min, obtain a susy min $D_T W=0$ with

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$$\tau \sim \frac{1}{a} \ln |W_0|^{-1} > 1$$

Can see why require $|W_0| \ll 1$ to ensure τ large enough to suppress pert corrections to Kähler pot

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LVS construction

Same start as KKLT, int out complex structure and axio-dilaton moduli. But LVS requires at least two moduli: CY has `Swiss cheese' structure, overall volume with subsequent moduli describing `holes'

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$$

τ_b - vol of big 4-cycle, τ_s - vol of a hole in it.

Leading α' correction to Kähler potential leads to:
where $\xi \propto \chi(X)$ - Euler number of the CY:

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}\right)$$

Full super potential inc non-pert contribution on the small cycle:

$$W = W_0 + A_s\, e^{-a_s T_s}$$

In limit $\tau_b >> \tau_s$, after fixing axionic partner of τ_b at its min, scalar potential is:

$$V = \frac{4}{3}\frac{a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{s\mathcal{V}} - \frac{2a_s A_s |W_0| \tau_s\, e^{-a_s\tau_s}}{s\mathcal{V}^2} + \frac{3\sqrt{s}\,\xi\,|W_0|^2}{8\mathcal{V}^3}$$

06/23/2008 Min at:

$$\langle\mathcal{V}\rangle \simeq \frac{3\sqrt{\langle\tau_s\rangle}\,|W_0|}{4a_s A_s}\, e^{a_s\langle\tau_s\rangle} \quad \text{and} \quad \langle\tau_s\rangle \simeq \frac{1}{g_s}\left(\frac{\xi}{2}\right)^{2/3}$$

Exp large in size of small 4 cycle

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Exp large in size of small 4 cycle

Note - •small value of the dilaton helps guarantee EFT under control. For $g_s \lesssim 0.1$, have both τ_b and τ_s much larger than string scale -> SUGR approx is good.

•Can construct models for natural values of $|W_0| \sim O(1\text{-}10)$

•Clean separation between string, KK and moduli scales in large vol limit.

$$M_{KK} \propto \frac{M_{Pl}}{\mathcal{V}^{2/3}}, \quad M_S \propto \frac{M_{Pl}}{\mathcal{V}^{1/2}}, \quad M_{cs} \propto \frac{M_{Pl}}{\mathcal{V}} \longrightarrow M_{moduli} \ll M_{KK} \ll M_S$$

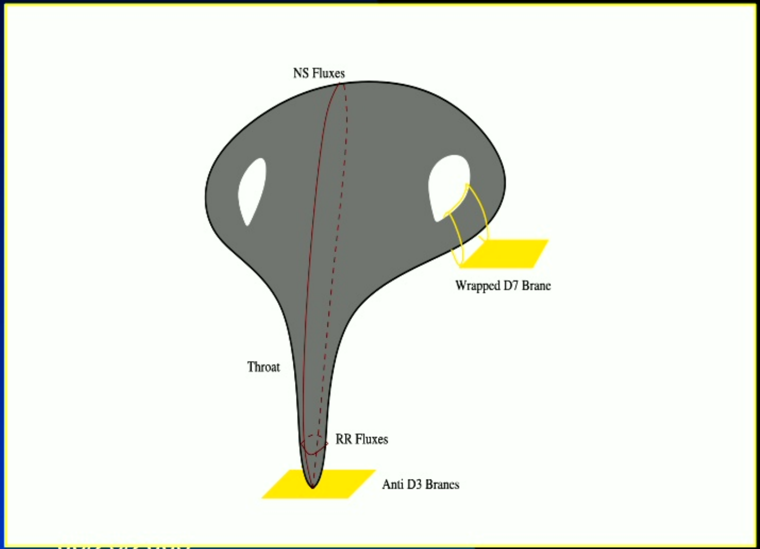
Moduli eff action is therefore a good description of low energy dynamics including addressing Dine Seiberg problem by trade off between K_p and W_{np}.

De Sitter ?

Vacua obtained in KKLT and LVS so far are AdS. A number of approaches taken to obtaining dS vacua by incorporating additional affects as part of the low energy effective action.

•Add anti-D3 branes - experience a potential in the imaginary self-dual bgd, driving them to the bottom of a warped throat within the compactification. Makes a positive definite contribution to the potential in terms of the warp factor at the bottom of the throat $\exp(A_0)$ - lifts to deS vacua.

$$V_{\overline{D3}} \sim \frac{e^{4A_0}}{(T+\overline{T})^2}$$



Typical CY in KKLT and LVS. D7 branes wrapped 4-cycles host gauge gps that provide non-pert effects in super potential. Non-trivial fluxes lead to 3-cycles corresponding to long throats, giving rise to warped factors in the metric. May host anti-D3 branes at their tip to provide dS uplift

[Cicoli et al 2023]

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String Inflation

Having stabilised the majority of the moduli, it opens up the possibility of a final moduli dominating the energy density as it evolves towards it's minima. This can lead to a period of cosmological inflation.

There are many models out there, and for a review see [Cicoli 2023] but I will concentrate on the two we have been discussing - KKLT and LVS

What do we require?

- Having stabilised the moduli, we dont want any runaway directions to appear for them during inflation, as this would destabilise the inflationary dynamics.
- Ideally the same $V(\phi)$ would offer a late time description inc reheating and DE domination maybe through de Sitter or Quintessence.
- Controlled low energy EFT - hence mass hierarchy throughout inflation

$$M_{\rm inf} < H_{\rm inf} < M_{\rm KK} < M_s < M_{\rm Pl}$$

- Controlled low energy EFT - wrt quantum corrections, so size of all cycles fixed above string scale with α' and g_s expansions under control
- CY embedding - the same one should allow for the realisation of a Std Model like sector with non-abelian gauge gp, chiral matter, viable reheating all after inflation !! This is very demanding !

Kahler Moduli Inflation [Conlon and Quevedo 2006]

Recall they consider large volume scenarios within a class of Type IIB flux compactifications on a CY orientifold.

Internal volume of CY

$$\mathcal{V} = \frac{\alpha}{2\sqrt{2}}\left[(T_1+\bar{T}_1)^{\frac{3}{2}} - \sum_{i=2}^{n}\lambda_i (T_i+\bar{T}_i)^{\frac{3}{2}}\right] = \alpha\left(\tau_1^{3/2} - \sum_{i=2}^{n}\lambda_i\tau_i^{3/2}\right)$$

Complex Kahler moduli $T_i = \tau_i + i\theta_i$

τ_i – volume of internal four cycles in CY

θ_i – axionic partners

Full scalar potential for moduli fields

$$V = \sum_{\substack{i,j=2\\ i<j}}^{n} \frac{A_iA_j\cos(a_i\theta_i - a_j\theta_j)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} e^{-(a_i\tau_i+a_j\tau_j)}\left(32(2\mathcal{V}+\xi)(a_i\tau_i + a_j\tau_j + 2a_ia_j\tau_i\tau_j) + 24\xi\right)$$
$$+ \frac{12W_0^2\xi}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \sum_{i=2}^{n}\left[\frac{12e^{-2a_i\tau_i}\xi A_i^2}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \frac{16(a_iA_i)^2\sqrt{\tau_i}e^{-2a_i\tau_i}}{3\alpha\lambda_i(2\mathcal{V}+\xi)}\right.$$
$$\left. + \frac{32e^{-2a_i\tau_i}a_iA_i^2\tau_i(1+a_i\tau_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} + \frac{8W_0A_ie^{-a_i\tau_i}\cos(a_i\theta_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)}\left(\frac{3\xi}{2\mathcal{V}+\xi} + 4a_i\tau_i\right)\right] + V_{uplift}\,.$$

Slow roll inflation supported when: $\mathcal{V} \gg 1$ implies $\tau_1 \gg \tau_i, i = 2..n$

Idea: displace just one moduli from its minimum, keeping the others fixed and show consistent slow roll inflation can be obtained with that moduli evolving back to its minima

Displace τ_2 with parameter $\rho \ll 1$ where

$$\rho \equiv \frac{\lambda_2}{a_2^{3/2}} : \sum_{i=2}^{n} \frac{\lambda_i}{a_i^{3/2}}$$

$$V_{LARGE} = \frac{BW_0^2}{\mathcal{V}^3} - \frac{4W_0 a_2 A_2 \tau_2 e^{-a_2\tau_2}}{\mathcal{V}^2}$$

Note axions assumed fixed in their minima

Intriguing results obtained for 50-60 efolds:

$$\eta \simeq -\frac{1}{N_e}, \qquad \epsilon < 10^{-12},$$

$$0.960 < n_s < 0.967, \quad 0 < |r| < 10^{-10}$$

$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6.$$

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Planck 2015 : $n_s = 0.968 \pm 0.006$; $r < 0.09$

Testing the initial assumptions - let the fields evolve [Blanco-Pillado et al 2009]

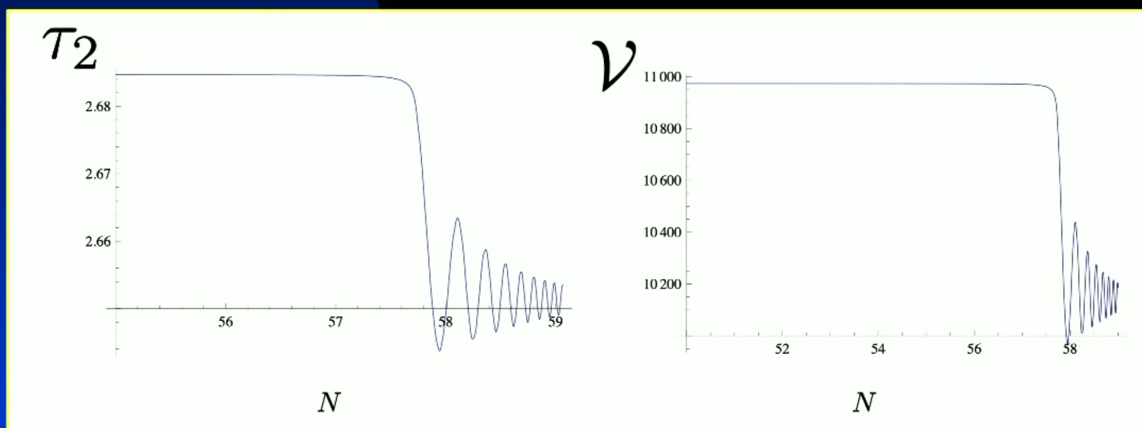
Numerically solve the full equations. The question is what happens if we allow the moduli to evolve so that they all have to find their minima. Do we find the kind of evolution that Conlon and Quevedo assumed in their analytic model ?

$\mathrm{Ex}: \ \rho \sim 0.99$

Global min: $\tau_1^f = 2555.95, \quad \tau_2^f = 4.7752, \quad \tau_3^f = 2.6512, \quad \mathcal{V}^f = 10143.94363$

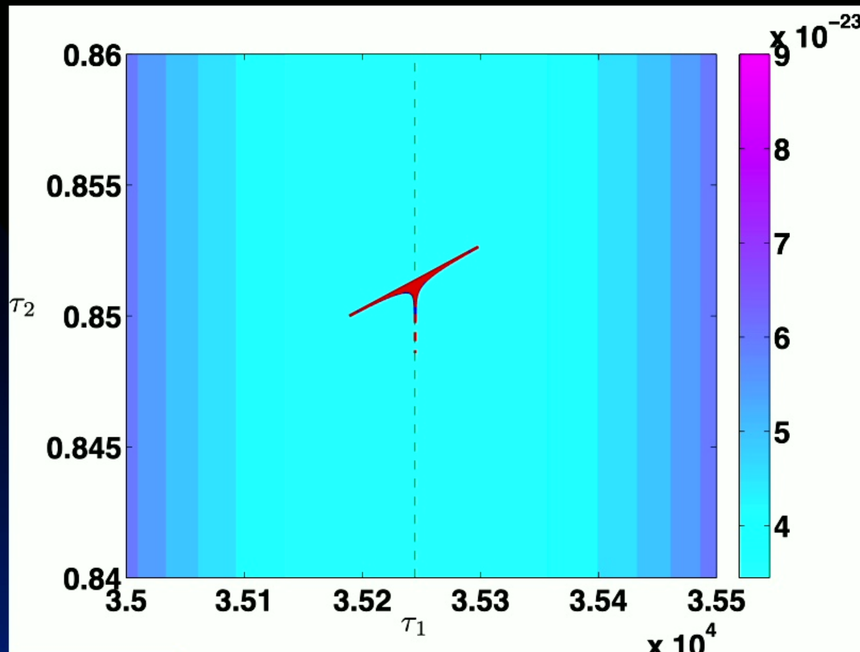
Displace: $\tau_2^i = 78.7752067$

New min: $\tau_1^i = 2781.185086997, \quad \tau_3^i = 2.684717126, \quad \mathcal{V}^i = 10973.9$



$n_s = 0.960$

Basin of attraction [Blanco-Pillado et al 2009]



Original model: Dashed line - trajectory which maintains const vol at fixed τ_3

$$\text{Fix } \tau_3 \text{ allow } \tau_1, \tau_2, (\theta_2 \text{ red line) to evolve}$$

The model works for broader range of parameters and sizes of moduli fields than might be expected. Two potential issues:

$$\text{gravitino mass generally too large}$$

$$\delta\theta_3 \ll 1 \text{ to avoid runaway decompactifiction}$$

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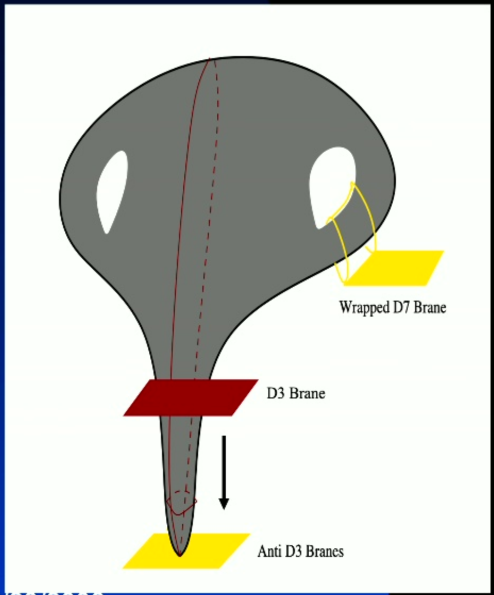
KKLT - pictorially at least

$D3 - \overline{D3}$ Inflation

Coulomb attraction gives rise to scalar potential for brane separation [Burgess et al 01]

In context of warping, this can lead to a successful slow roll due the throat of the manifold, as the branes move towards each other [Burgess et al 2003]

Predicts: $N_e \simeq 52, \quad n_s \simeq 0.968$ and $r \simeq 7 \times 10^{-8}$



Warped brane-anti brane inflation. KKLT plus moving D3 brane attracted to anti-brane at the tip of a warped throat.

However, potential issue - moduli are fixed non-perturbatively, so extra contributions to slow roll parameter η of order O(1) - the resurfacing of the η problem.

06/23/2008 [Cicoli et al 2023]

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Have some predictions from different models [Cicoli et al 2023]

String model	n_s	r
Fibre Inflation	0.967	0.007
Blow-up Inflation	0.961	10^{-10}
Poly-instanton Inflation	0.958	10^{-5}
Aligned Natural Inflation	0.960	0.098
N-Flation	0.960	0.13
Axion Monodromy	0.971	0.083
D7 Fluxbrane Inflation	0.981	5×10^{-6}
Wilson line Inflation	0.971	10^{-8}
D3-$\overline{\text{D3}}$ Inflation	0.968	10^{-7}
Inflection Point Inflation	0.923	10^{-6}
D3-D7 Inflation	0.981	10^{-6}
Racetrack Inflation	0.942	10^{-8}
Volume Inflation	0.965	10^{-9}
DBI Inflation	0.923	10^{-7}

As you can see there are many - some close to being or already ruled out !

After inflation !

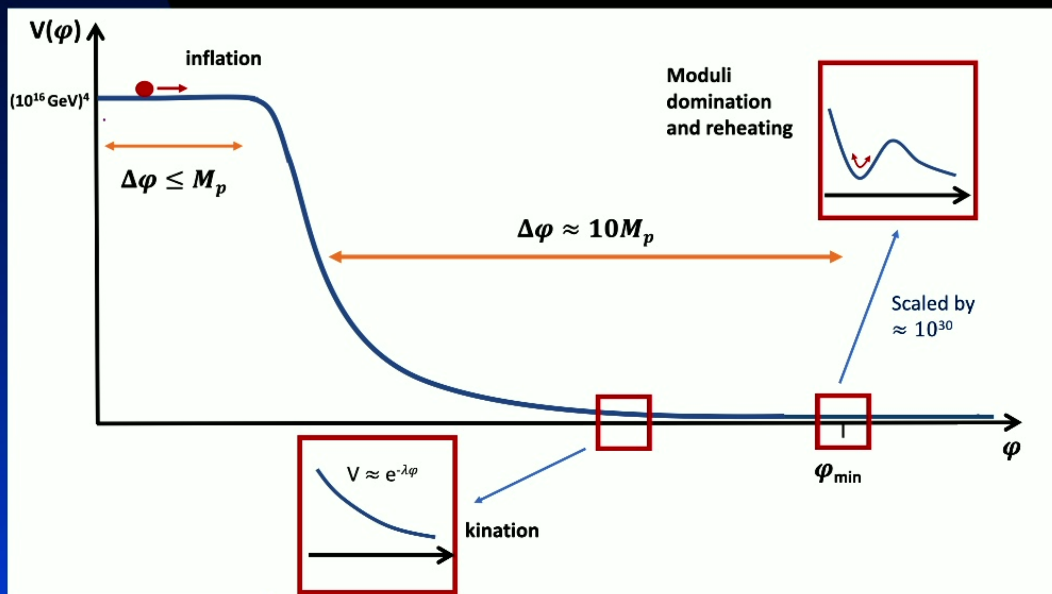
The bit between the end of inflation and the thermal HBB.

Potentially new features could emerge which would modify the standard picture.

For example, large field displacements between end of inflation and final vacuum

No necessary relationship between inflaton field and field responsible for reheating. In fact in D3-anti D3 brane case, inflaton disappears.

Long Kination and moduli dominated epoch leading to moduli driven reheating



Cicoli et al 2023

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Will concentrate on one important element of this - the overshoot problem [Brustein and Steinhardt 93] !

The barrier that has to eventually trap the moduli field can be 20 or more orders of magnitude smaller than the energy scale during inflation. The field should simply shoot straight past and decompactify spacetime !

In cosmology as in many areas of physics we often deal with systems that are inherently described through a series of coupled non-linear differential equations.

By determining the late time behaviour of some combination of the variables, we often see that they may approach some form of attractor solution.

From the stability of these attractor solutions we can learn about the system.

Moreover the phase plane description of the system is often highly intuitive enabling easy analysis and understanding of the system.

Examples inc the relative energy densities in scalar fields compared to the bgd rad and matter densities, as well as the relative energy density in cosmic strings.

Enter Tracker solutions:

Wetterich (88) Peebles and Ratra (88), EJC, Liddle and Wands

Scalar field: $$\phi : \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi); \; p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM:
$$\dot{H} = -\frac{\kappa^2}{2}(\dot{\phi}^2 + \gamma\rho_{\rm b})$$
$$\dot{\rho}_b = -3\gamma\rho_b$$
$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

+ constraint:
$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_{\rm b})$$

Intro new variables x and y:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}; \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}; \quad \lambda \equiv \frac{-1}{\kappa V}\frac{dV}{d\phi}; \quad \Gamma - 1 \equiv \frac{d}{d\phi}\left(\frac{1}{\kappa\lambda}\right)$$

Eff eqn of state: $\gamma_\phi = \frac{2\dot{\phi}^2}{2V+\dot{\phi}^2}$; $\Omega_\phi = \frac{\kappa^2\rho_\phi}{3H^2} = x^2 + y^2$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda\sqrt{\tfrac{3}{2}}y^2 + \tfrac{3}{2}x[2x^2 + \gamma(1 - x^2 - y^2)]$$

$$y' = -\lambda\sqrt{\tfrac{3}{2}}xy + \tfrac{3}{2}y[2x^2 + \gamma(1 - x^2 - y^2)]$$

$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)$$

$$\frac{\kappa^2\rho_b}{3H^2} + x^2 + y^2 = 1$$

where $x' \equiv \frac{d}{d(\ln a)}$

Note: $0 \le \gamma_\phi \le 2$; $0 \le \Omega_\phi \le 1$

Scaling solutions: (x`=y`=0)

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall\lambda,\gamma$	$SP: 0<\gamma$ $SN: \gamma=0$	0	Undefined
2a	1	0	$\forall\lambda,\gamma$	$UN: \lambda<\sqrt{6}$ $SP: \lambda>\sqrt{6}$	1	2
2b	-1	0	$\forall\lambda,\gamma$	$UN: \lambda>-\sqrt{6}$ $SP: \lambda<-\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1-\frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2\le 6$	$SP: 3\gamma<\lambda^2<6$ $SN: \lambda^2<3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2}\frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2\ge 3\gamma$	$SN: 3\gamma<\lambda^2<\frac{24\gamma^2}{9\gamma-2}$ $SS: \lambda^2>\frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

$$V = V_0 e^{-\lambda\kappa\phi}$$

Late time attractor is scalar field dominated

$\lambda^2 \le 6$

Field mimics background fluid.

$\lambda^2 \ge 3\gamma$

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EJC, Liddle and Wands

$$V = V_0 e^{-\lambda\kappa\phi}$$

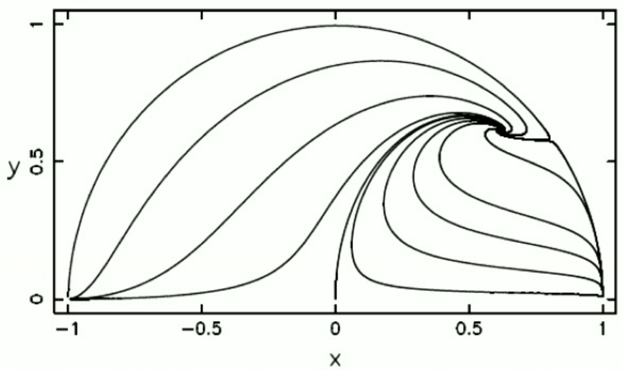


FIG. 3. The phase plane for $\gamma = 1$, $\lambda = 2$. The scalar field dominated solution is a saddle point at $x = \sqrt{2/3}$, $y = \sqrt{1/3}$, and the late-time attractor is the scaling solution with $x = y = \sqrt{3/8}$.

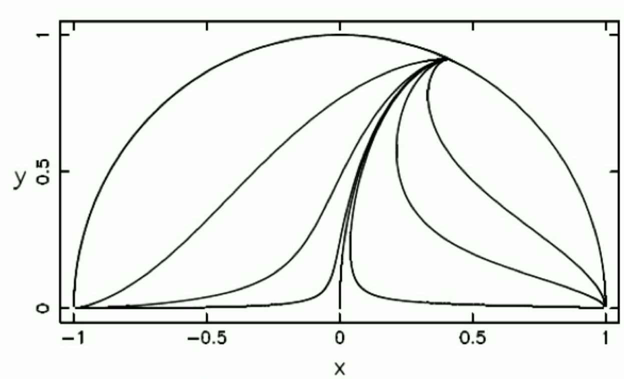


FIG. 2. The phase plane for $\gamma = 1$, $\lambda = 1$. The late-time attractor is the scalar field dominated solution with $x = \sqrt{1/6}$, $y = \sqrt{5/6}$.

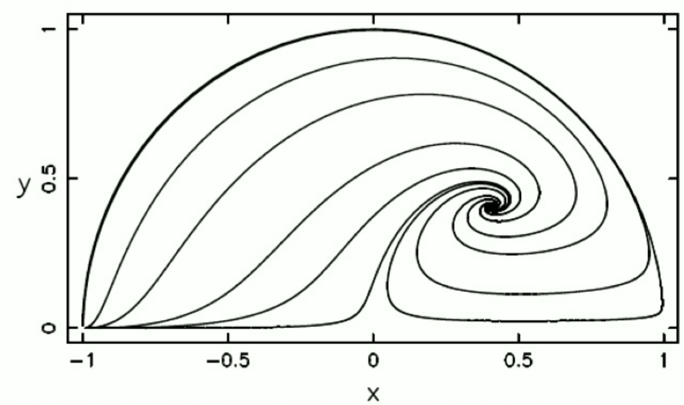
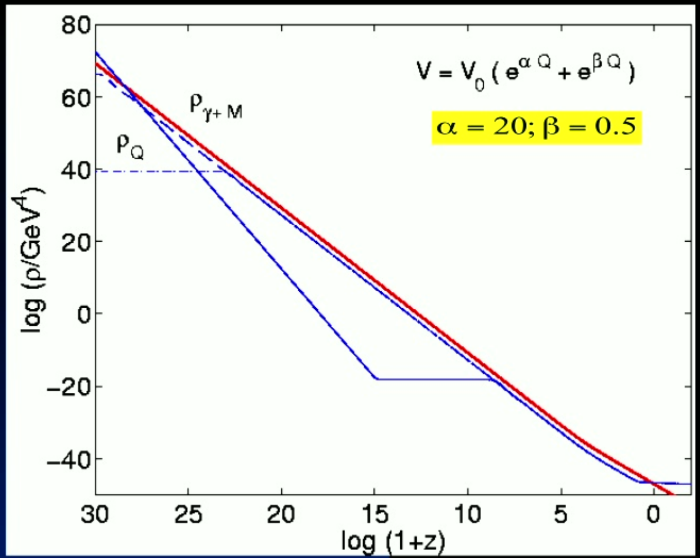
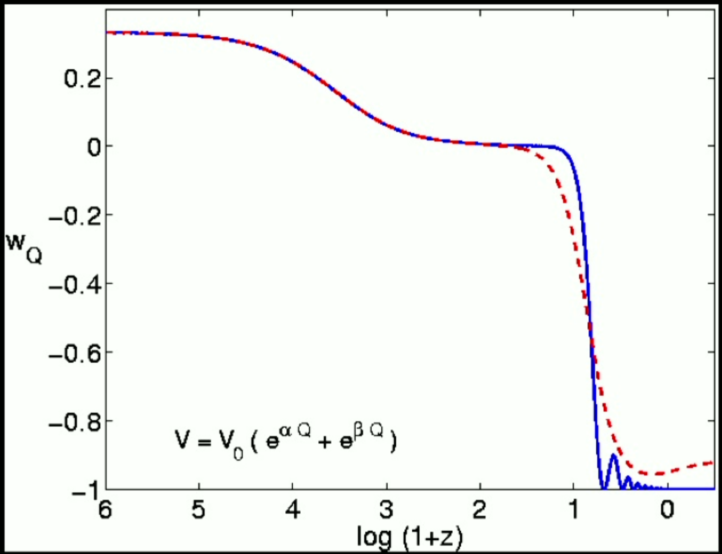


FIG. 4. The phase plane for $\gamma = 1$, $\lambda = 3$. The late-time attractor is the scaling solution with $x = y = \sqrt{1/6}$.

1. Scaling solutions in Dark Energy - Quintessence





Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$

Fifth force !

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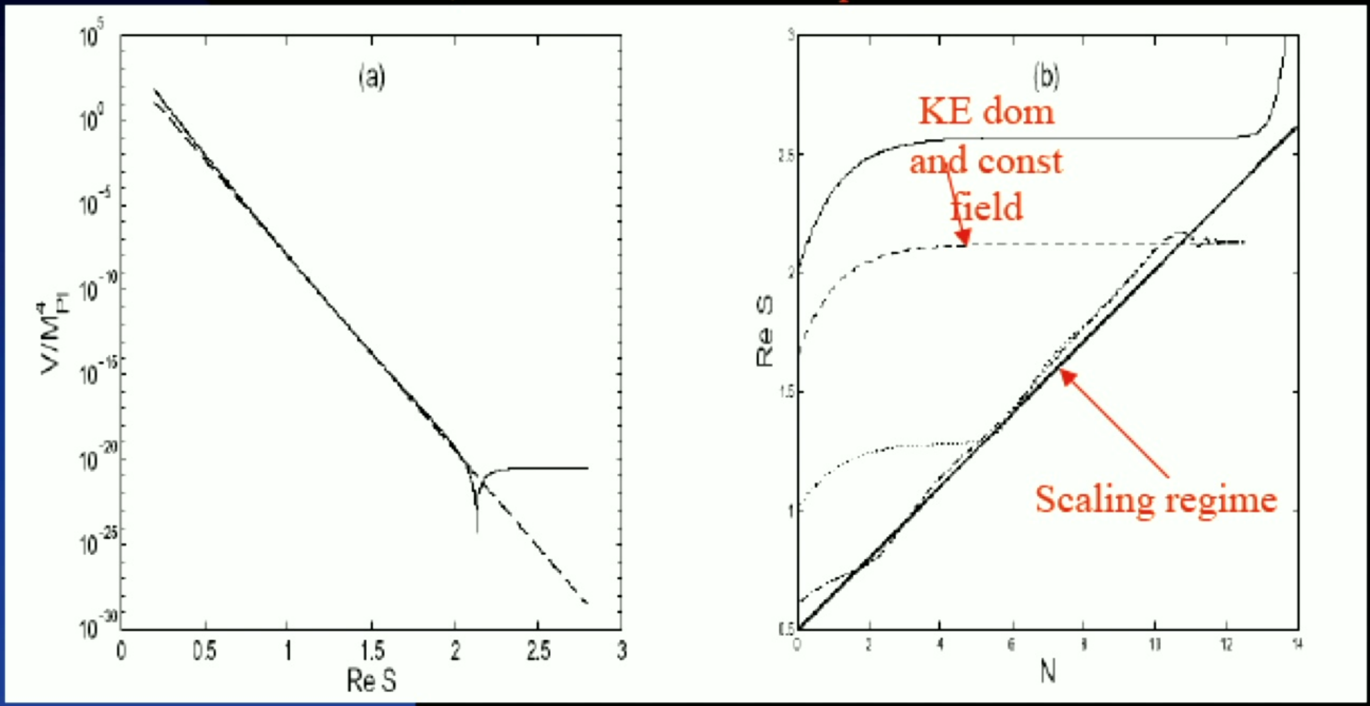
2. **Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.**

Barreiro, de Carlos and EC : hep/th-9805005

Brustein, Alwis and Martins : hep-th/0408160



Two condensate model with V~e^{-aReS} as approach minima

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Barreiro et al : hep-th/0506045

3. Stabilising volume moduli ($\sigma=\sigma_r+\sigma_i$) in KKLT [Kachru et al 2003]

$$\ddot{\sigma_r} + 3H\dot{\sigma_r} - \frac{1}{\sigma_r}(\dot{\sigma_r}^2 - \dot{\sigma_i}^2) + \frac{2\sigma_r^2}{3}\partial_{\sigma_r}V = 0$$

$$\ddot{\sigma_i} + 3H\dot{\sigma_i} - \frac{2}{\sigma_r}\dot{\sigma_r}\dot{\sigma_i} + \frac{2\sigma_r^2}{3}\partial_{\sigma_i}V = 0$$

$$\dot{\rho_b} + 3H\gamma\rho_b = 0$$

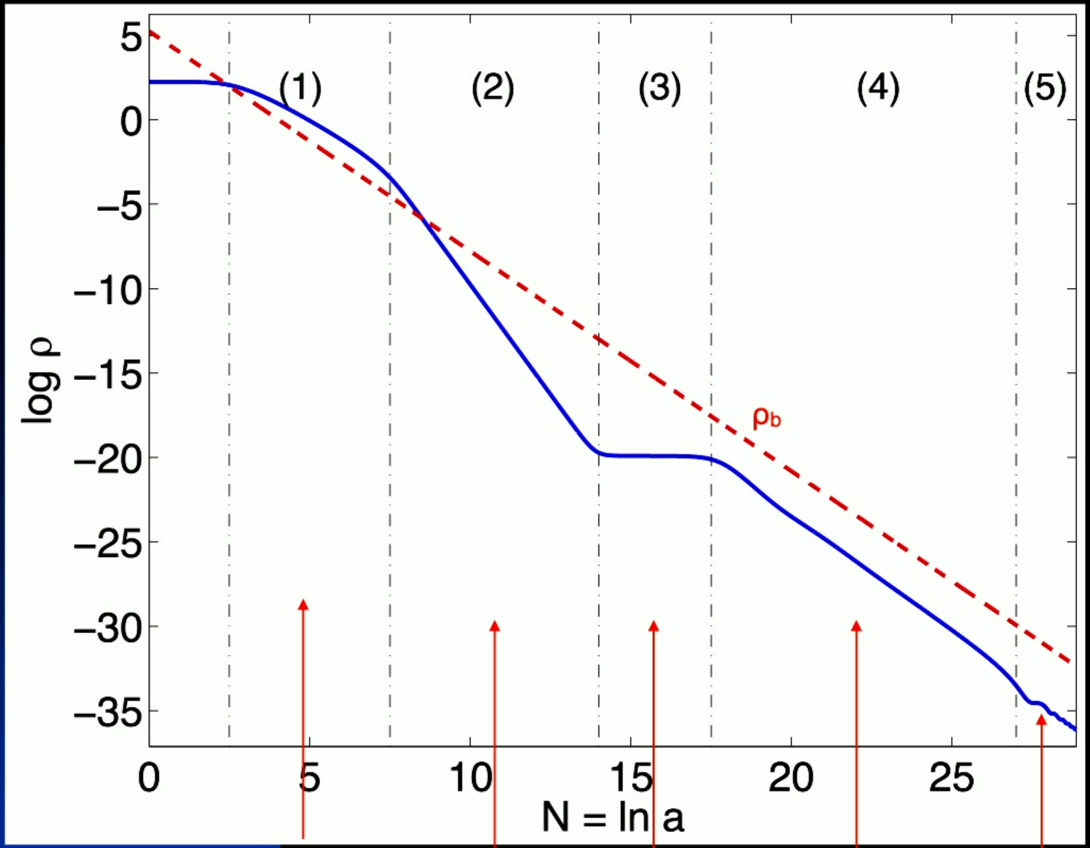
$$3H^2 = \frac{3}{4\sigma_r^2}(\dot{\sigma_r}^2 + \dot{\sigma_i}^2) + V + \rho_b$$

$$V = \frac{\alpha A \mathrm{e}^{-\alpha\sigma_r}}{2\sigma_r^2}\left[A\left(1 + \frac{\alpha\sigma_r}{3}\right)\mathrm{e}^{-\alpha\sigma_r} + W_0\cos(\alpha\sigma_i)\right] + \frac{C}{\sigma_r^3}\ .$$

[including contribution from D term to uplift the potential to de Sitter]

[for discussion on validity of D term addition see also Burgess et al 2003; Achucarro et al 2006]

Evolution of energy density of $\phi \propto \ln\sigma_r$ in KKLT and Kallosh Linde type potentials



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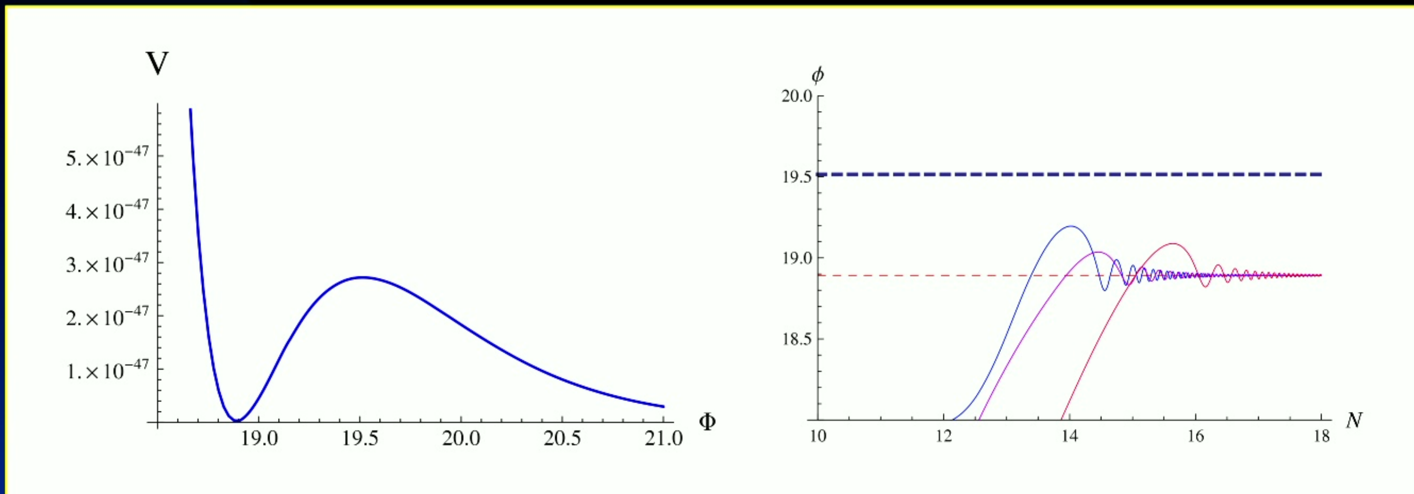
[Brustein et al 2004; Barreiro et al 2005]

4. Large volume modulus inflation - high scale inflation & low scale SUSY co-existing [Conlon et al 2008]

$$V = V_0\left((1-\epsilon\,\Phi^{3/2})e^{-\sqrt{27/2}\Phi} + C\,e^{-10\Phi/\sqrt{6}} + De^{-11\Phi/\sqrt{6}} + \delta e^{-\sqrt{6}\Phi}\right)$$

Toy example - but general features



Steep potential after inflation would normally have runaway solutions but presence of radiation leads to additional Hubble Friction which leads to attractor behaviour and field settles in its minimum.

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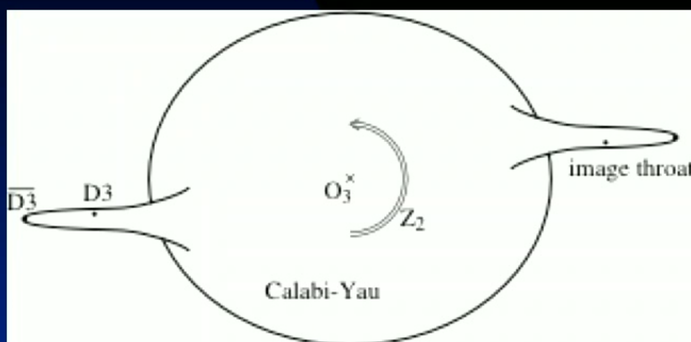
Strings in KLMT © model -- an example.

[Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi 03]

IIB string theory on CY manifold, orientifolded by Z_2 sym with isolated fixed points, become O3 planes. Warped metric:

$$ds^2 = e^{2A(x_\perp)}\eta_{\mu\nu}dx^\mu dx^\nu + ds_\perp^2 \; .$$



Inflaton: sep of D3 and anti D3 in throat.

Annihilation in region of large grav redshift,

$$min\{e^{A(x_\perp)}\} = e^{A_0} \ll 1$$

Redshift in throat important. Inflation scale and string tension, as measured by a 10 dim inertial observer, are set by string physics -- close to the four-dimensional Planck scale. Corresponding energy scales as measured by a 4 dim obs are suppressed by a factor of e^{A_0}

03/16/2012

Many approaches have been taken to String Cosmology that we have not covered here.

They are driven by the need to address a number of questions including the origin of inflation, the nature of primordial density fluctuations and the resolution of the initial singularity. A few types include:

1. Pre Big Bang Cosmology (Veneziano & Gasperini 91) -- low energy string action, has singular collapsing phase in low curvature regimes but require higher order curvatures to avoid the singularity and bounce -- not well controlled.

2. Axion inspired cosmology (Kim et al 04). Example of large field inflation. Use PQ symmetry to protect the inflaton as it evolves from super-Planckian distances. Models based on N-flation (collection of many axions) or axion monodromy (McAllister et al 08). Linear potential and non-perturbative oscillating corrections leads to modulations in the power spectrum and to interesting non-gaussian features in CMB.

3. Eternal inflation models -- example of the string landscape in action (Susskind, Linde ...). Lots of issues over how to properly define the measure in such a landscape and to make predictions of what we expect to see in the CMB.

4. Ekpyrotic and Cyclic models (Khoury et al 01, Steinhardt and Turok 01). Returning to idea of PBB, replacing HBB singularity with prior-contracting phase. Once again have to control the higher curvature effects as enter bounce regime, and there is a lot to be done to understand the propagation of perturbations through the bounce. Not clear how it is embedded in string theory.

5. String gas cosmology (Brandenberger and Vafa 89). The intercommuting of a gas of strings on T^9 dynamically favours the emergence of 3 space dimensions. Still require the true equations that describe the background of the gas of strings.

6. Wonderful fun objects like oscillons that can form as we begin reheating, and are string model specific.

7. Uniques GW signatures of cosmic superstrings and oscillons — with lots of luck maybe detectable by LISA and even LIGO for strings.

I think it is fair to say that all of the models proposed have technical issues concerning the detailed predictive powers they have and in justifying the assumptions made in formulating them in the first place.

Summary

1.We have seen the approaches taken to moduli stabilisation and cosmology in string theory, in particular KKLT and LVS. They both provide exciting observational features which we could search for, although there are of course issues.

2. Radiation in the early universe can be the moduli's friend guiding it to its minima and stopping it running away and decompactifying everything - scaling solutions.

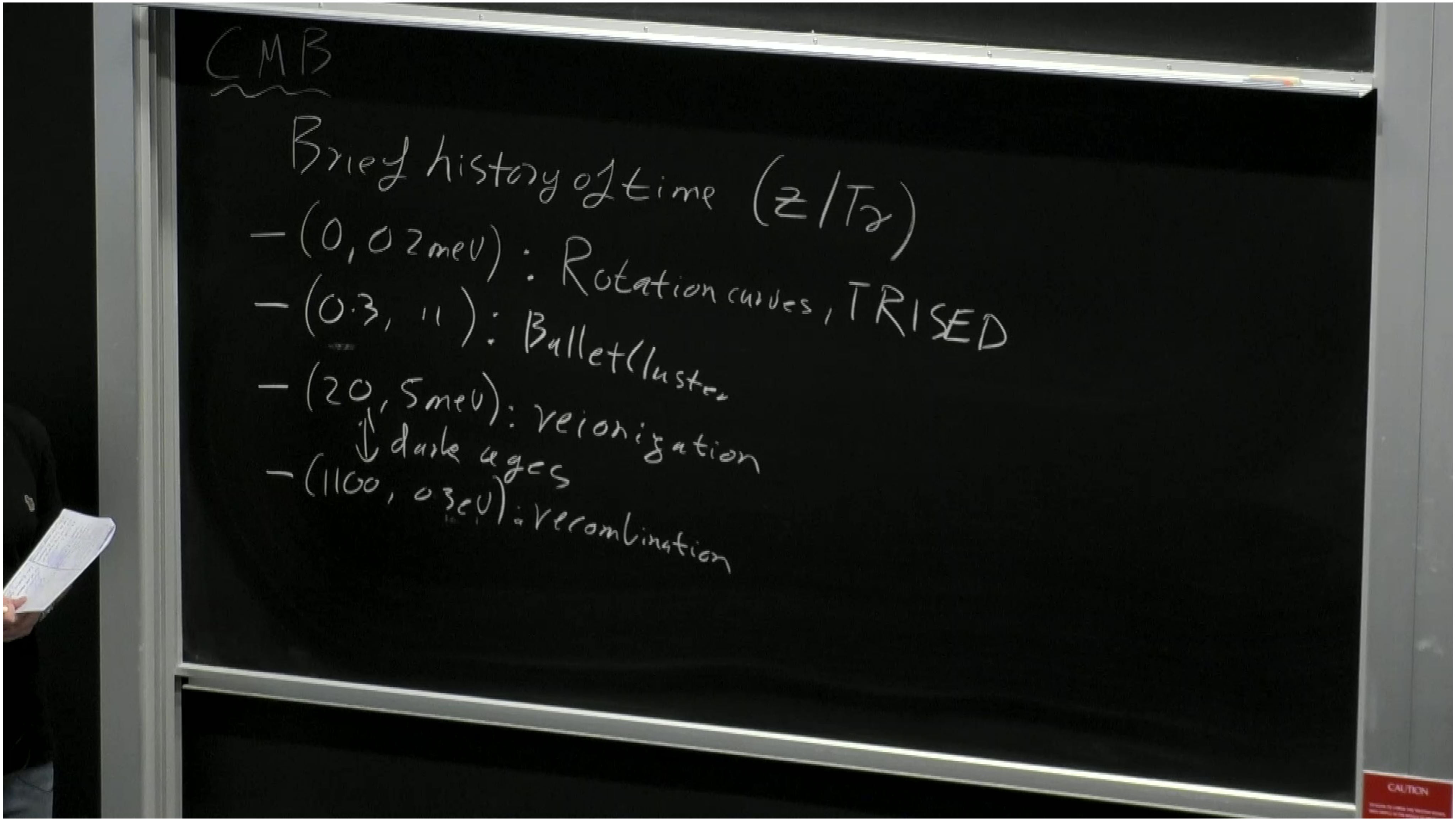
3. Large Volume Inflation appears robust to allowing many moduli fields to evolve - maybe issues over the light axions which can't evolve too far way from their minima

4. Beginning now to get CMB constraints on the cosmic superstring parameters through the B mode Power spectrum, although much more to do through the bispectrum.

5. Future constraints will be enhanced through GW signatures and Pulsar bounds.

It's an exciting field of research - come and join it !

Thank you for listening and all the best to you all !


CMB
Brief history of time (z/Tγ)
— (0, 0.2 meV): Rotation curves, TRISED
— (0.3, 11): Bullet Cluster
— (20, 5 meV): reionization
↕ dark ages
— (1100, 0.3 eV): recombination

— (3400 / 0.8 eV): matter–radiation equality

— (2×10^{9} / 5 MeV): BBN begins

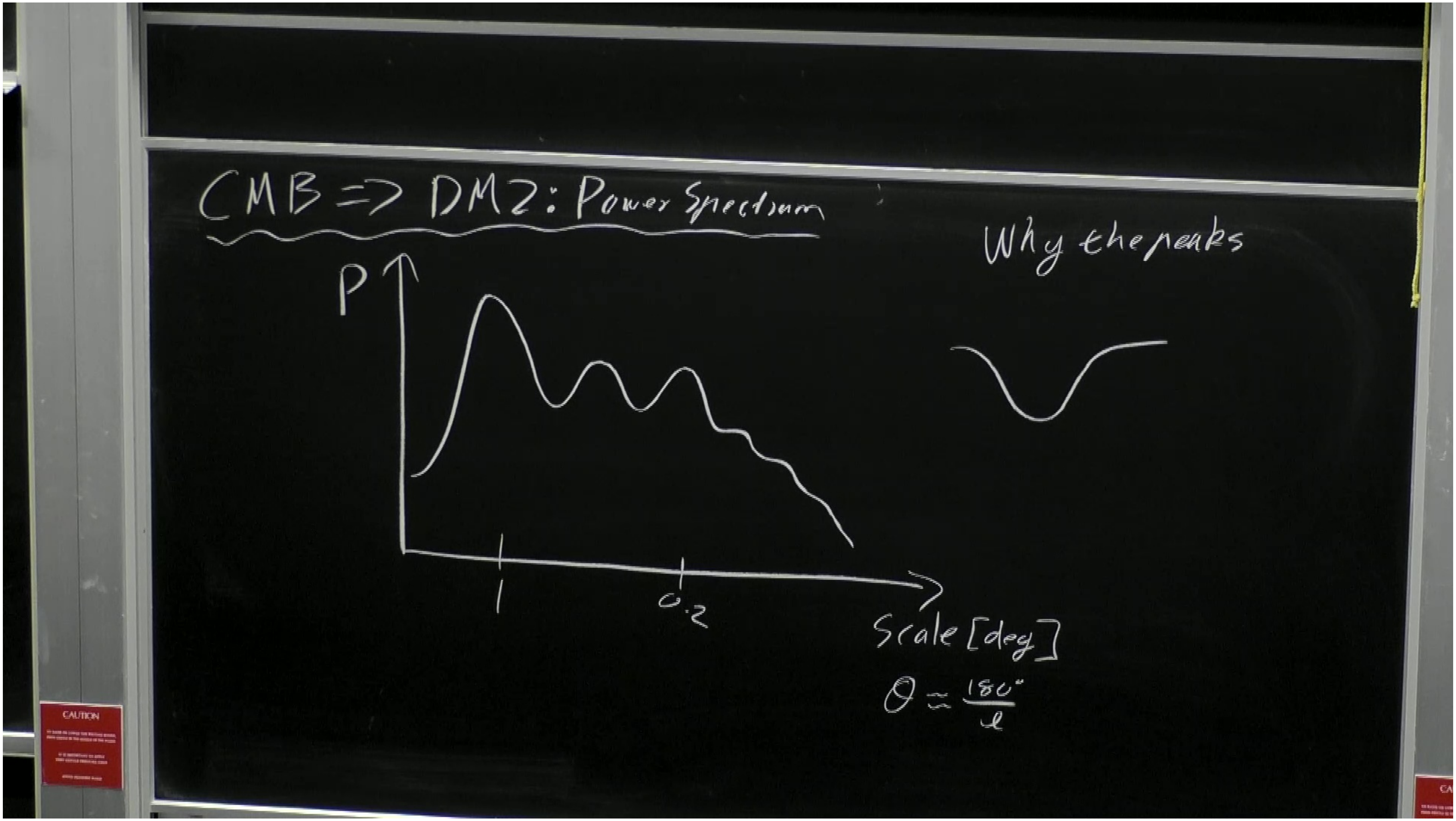
— ($T_\gamma > 5$ MeV): ???

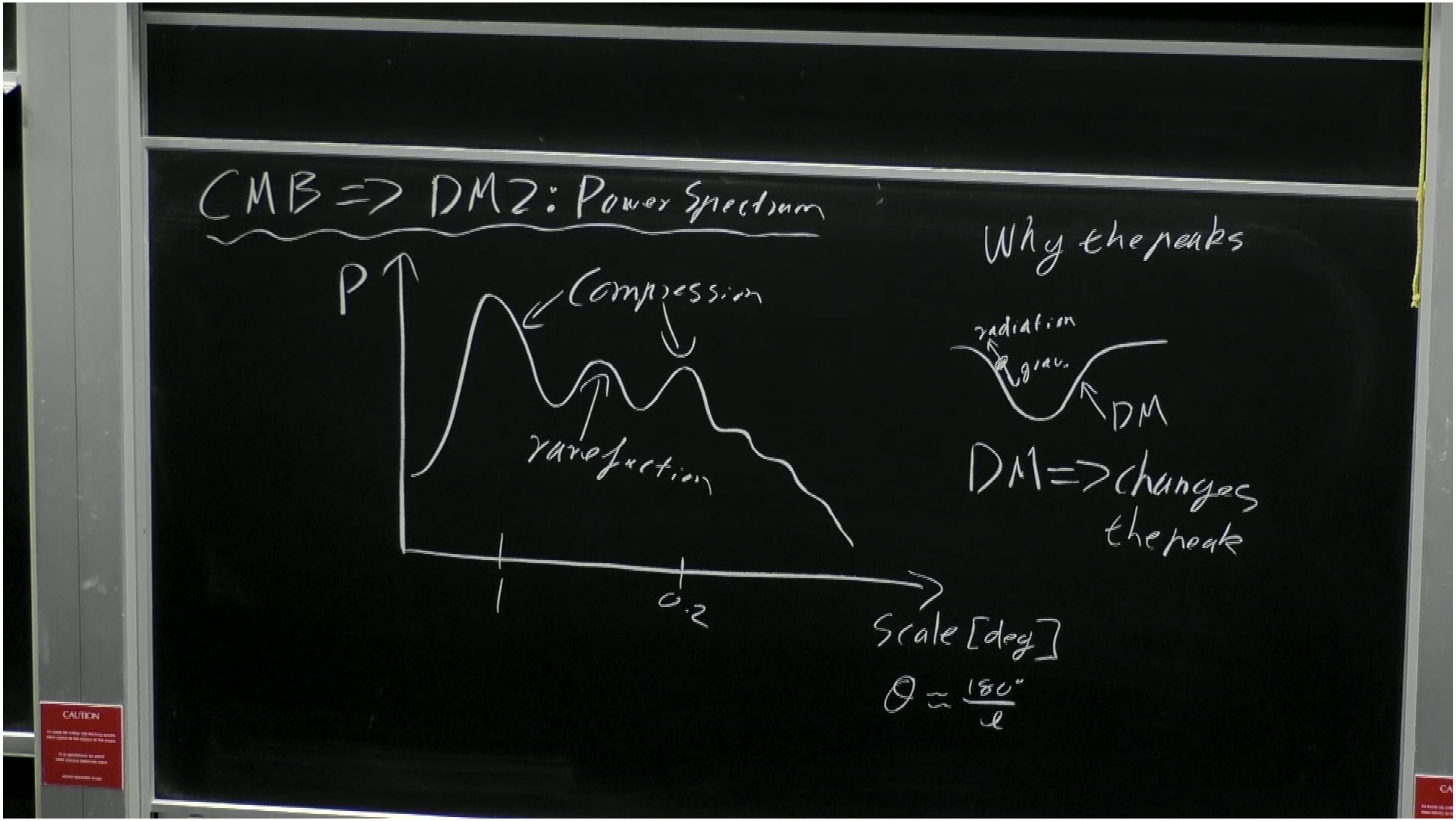
CMB ⇒ DM ⊥ : LSS

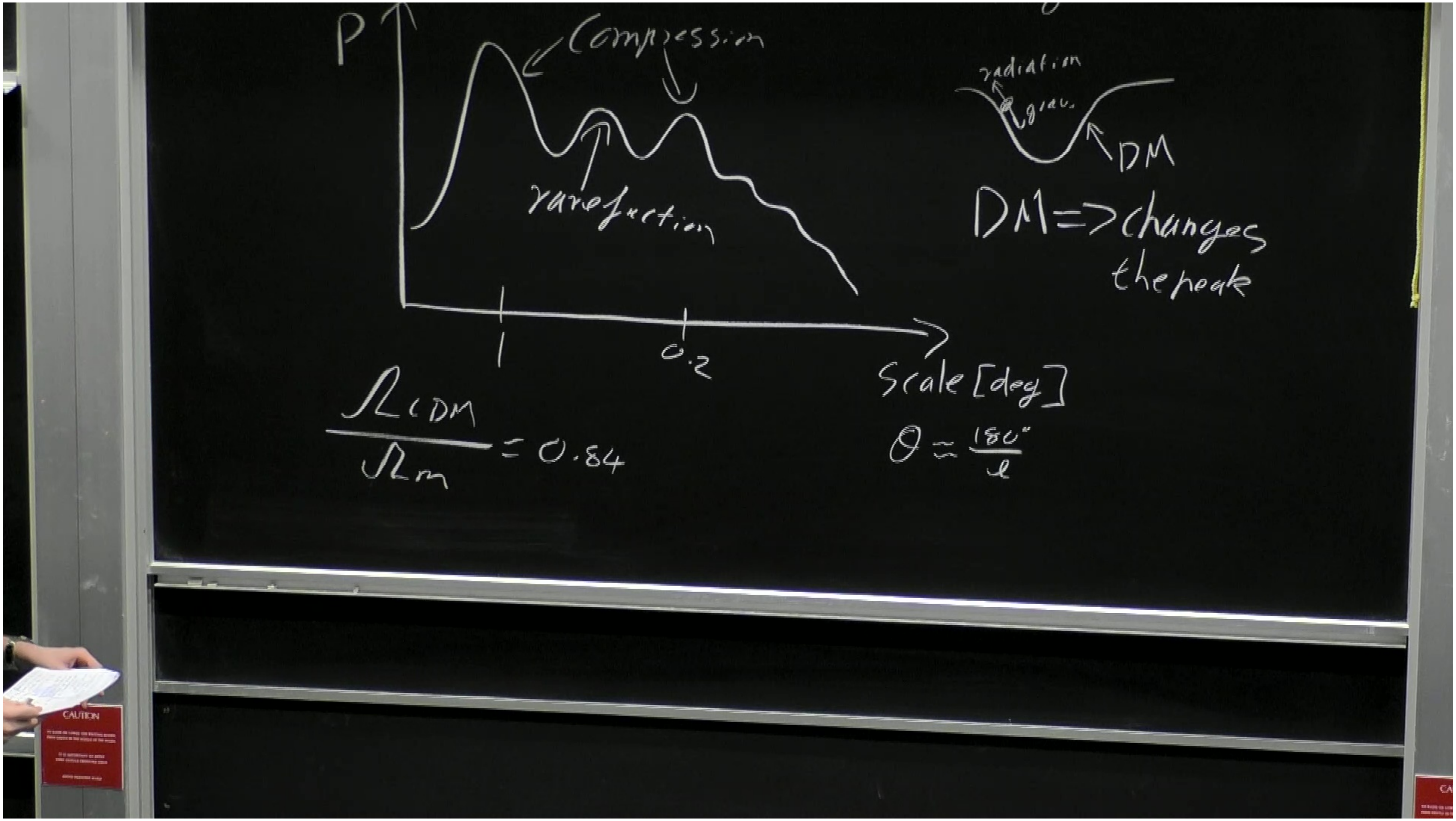
$\Delta T/\bar{T}$ in CMB : $\Delta\rho_b/\bar{\rho}_b = \delta_b \sim 10^{-5}$

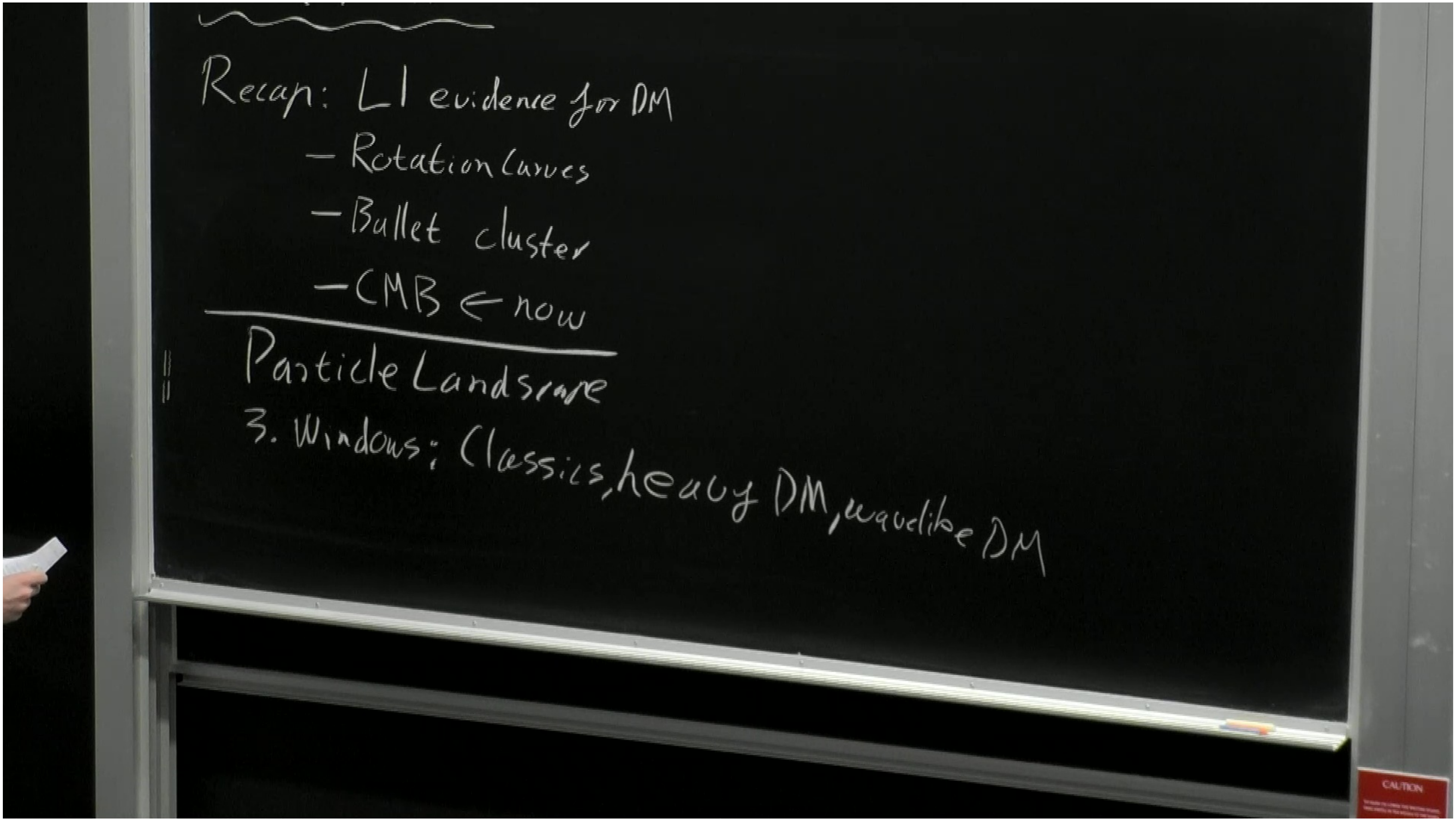
In mat. dom. era $\Rightarrow \delta_b \propto a$

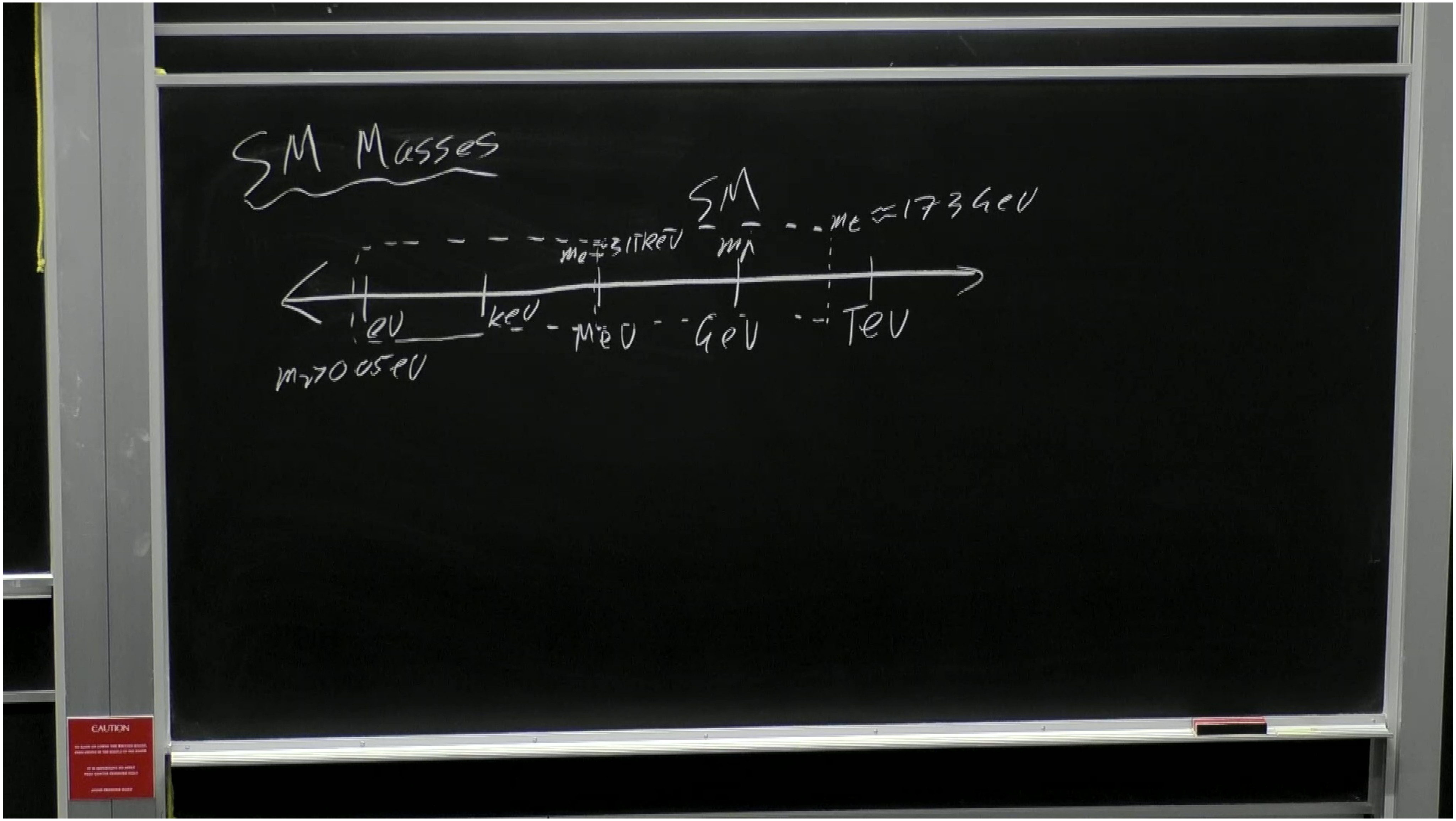
$\delta_b^0 \simeq 10^{-5}\times 1100 \sim 10^{-2} \ll 1$

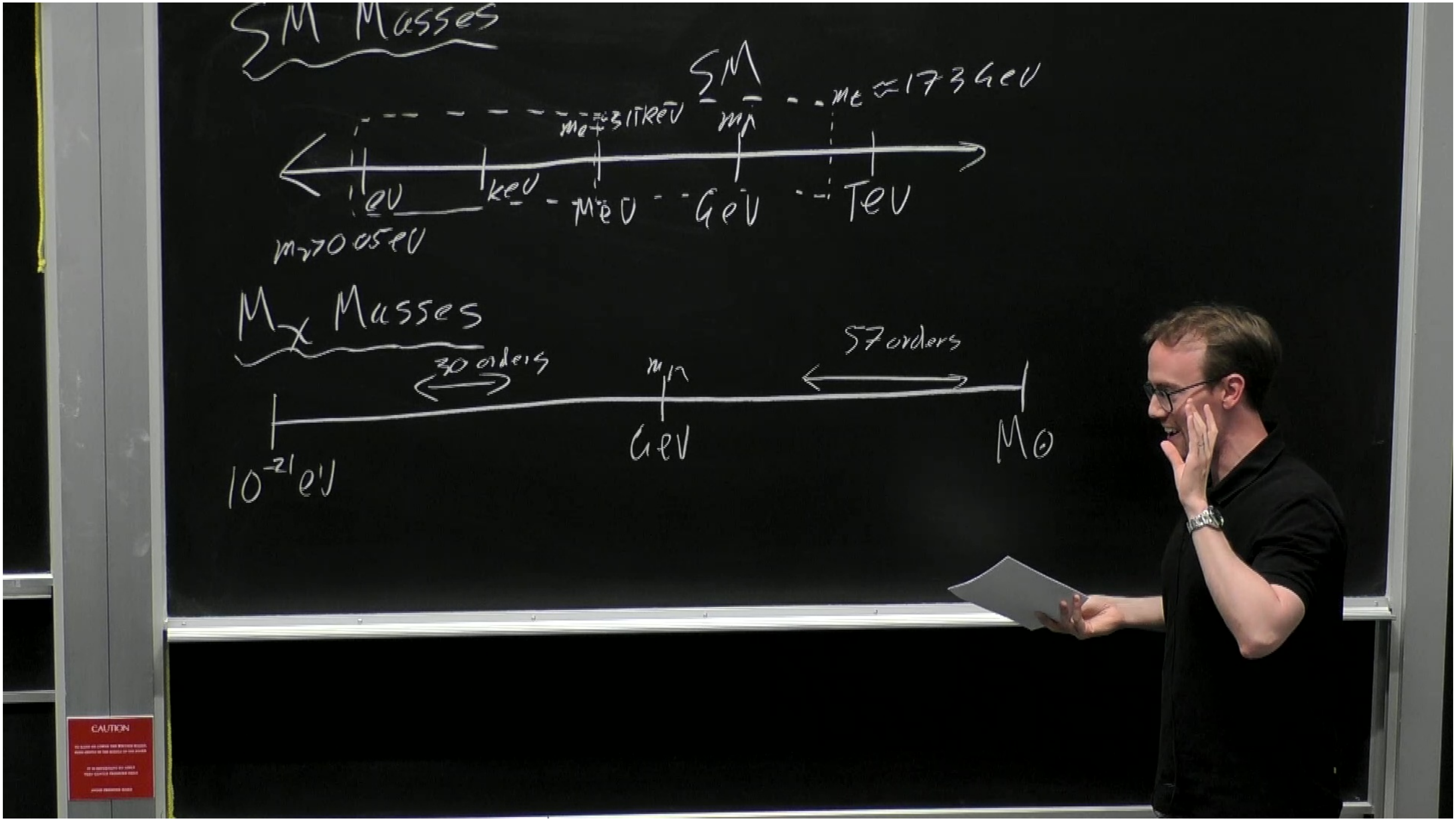

CMB => DMZ: Power Spectrum
Why the peaks
P
1
0.2
Scale [deg]
θ ≈ 180°/ℓ
CAUTION


CMB => DM2: Power Spectrum
P
Compression
rarefaction
1
0.2
Scale [deg]
$\theta \simeq \frac{180°}{\ell}$
Why the peaks
radiation
grav.
DM
DM => changes the peak


P
Compression
rarefaction
radiation
grav.
DM
DM => changes the peak
1
0.2
Scale [deg]
ΩCDM / Ωm = 0.84
θ ≈ 180° / ℓ
CAUTION

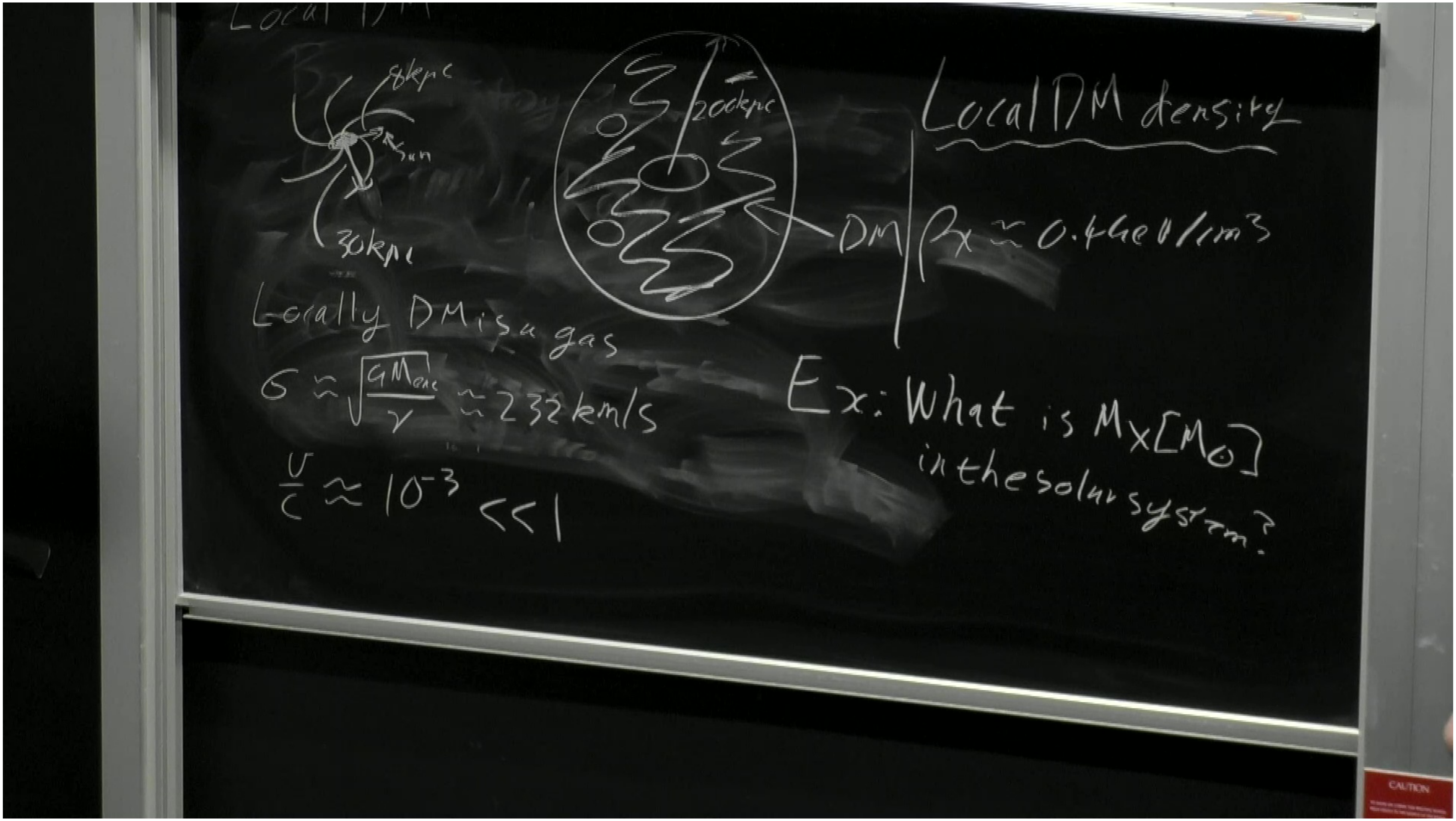

Recap: L1 evidence for DM
— Rotation Curves
— Bullet cluster
— CMB ← now
Particle Landscape
3. Windows: Classics, heavy DM, wavelike DM

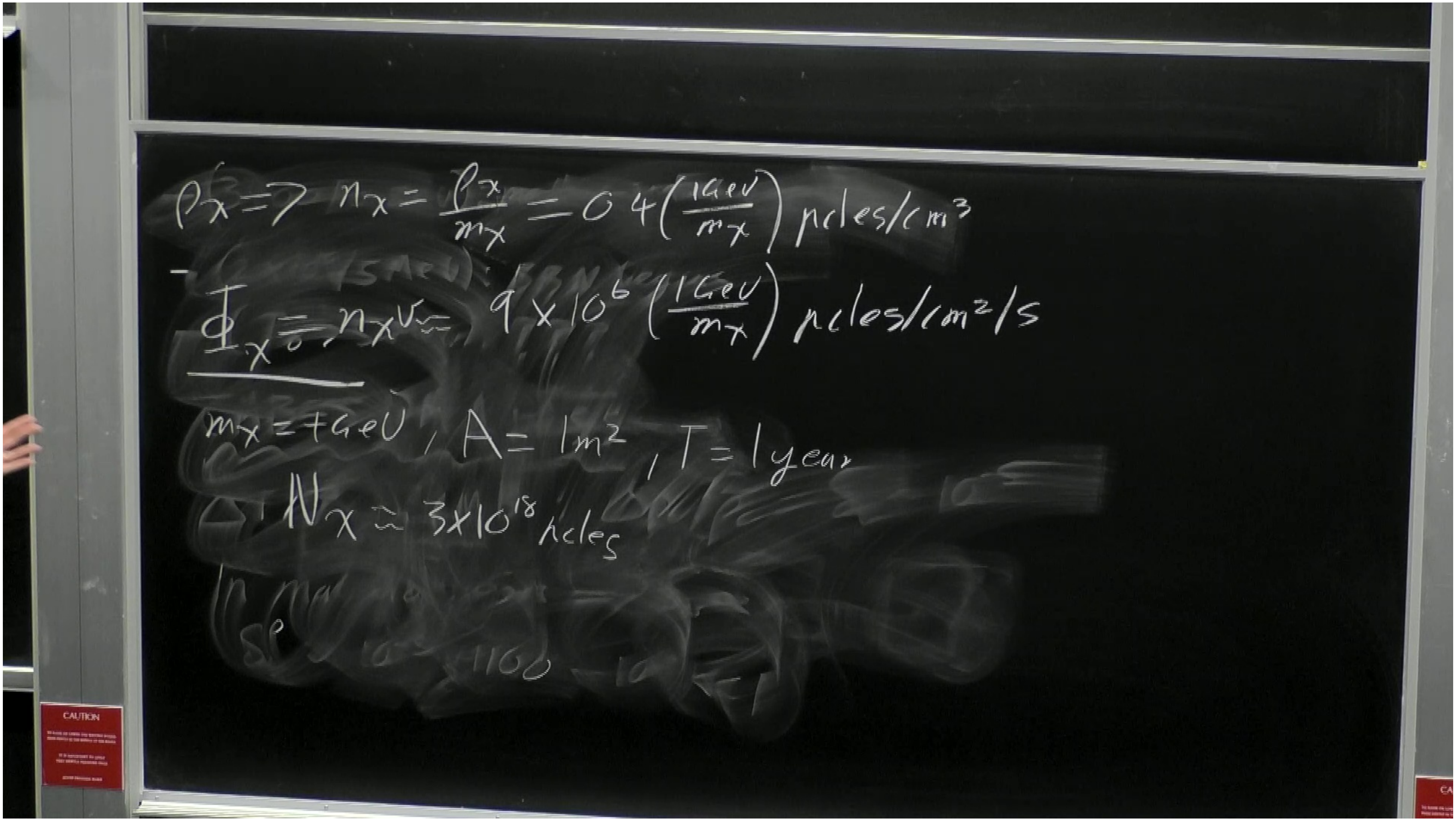

SM Masses
SM
m_t ≃ 173 GeV
m_e ≃ 511 keV
eV
keV
MeV
GeV
TeV
m_ν ≳ 0.05 eV

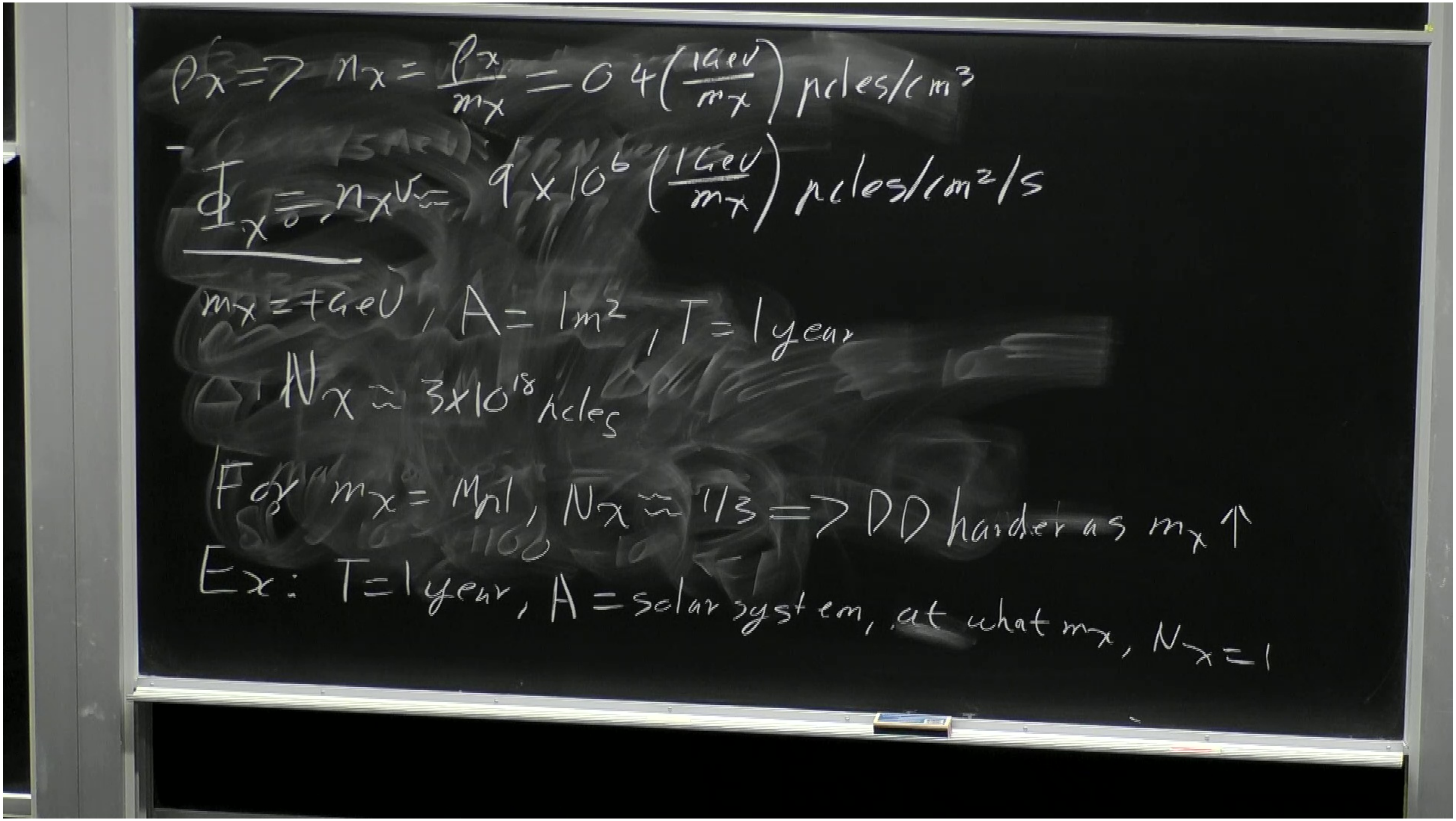

SM Masses
SM
eV
keV
MeV
GeV
TeV
Mχ Masses
30 orders
57 orders
GeV
M⊙

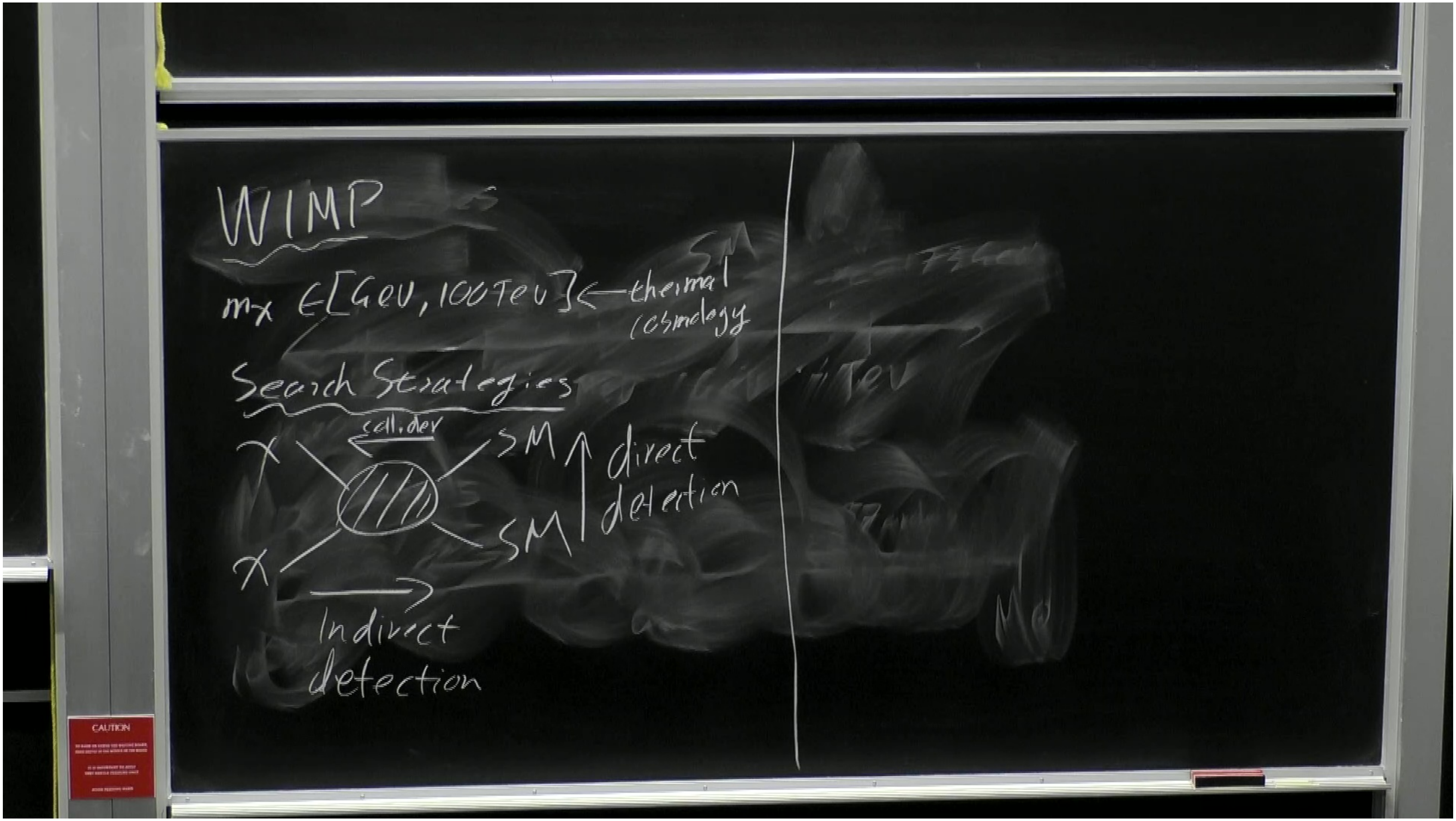
Units $M_\odot \simeq \frac{M_{pl}^3}{m_p^2} \simeq \frac{(10^{19}\,GeV)^3}{(1\,GeV)^2} \simeq 10^{57}\,GeV$

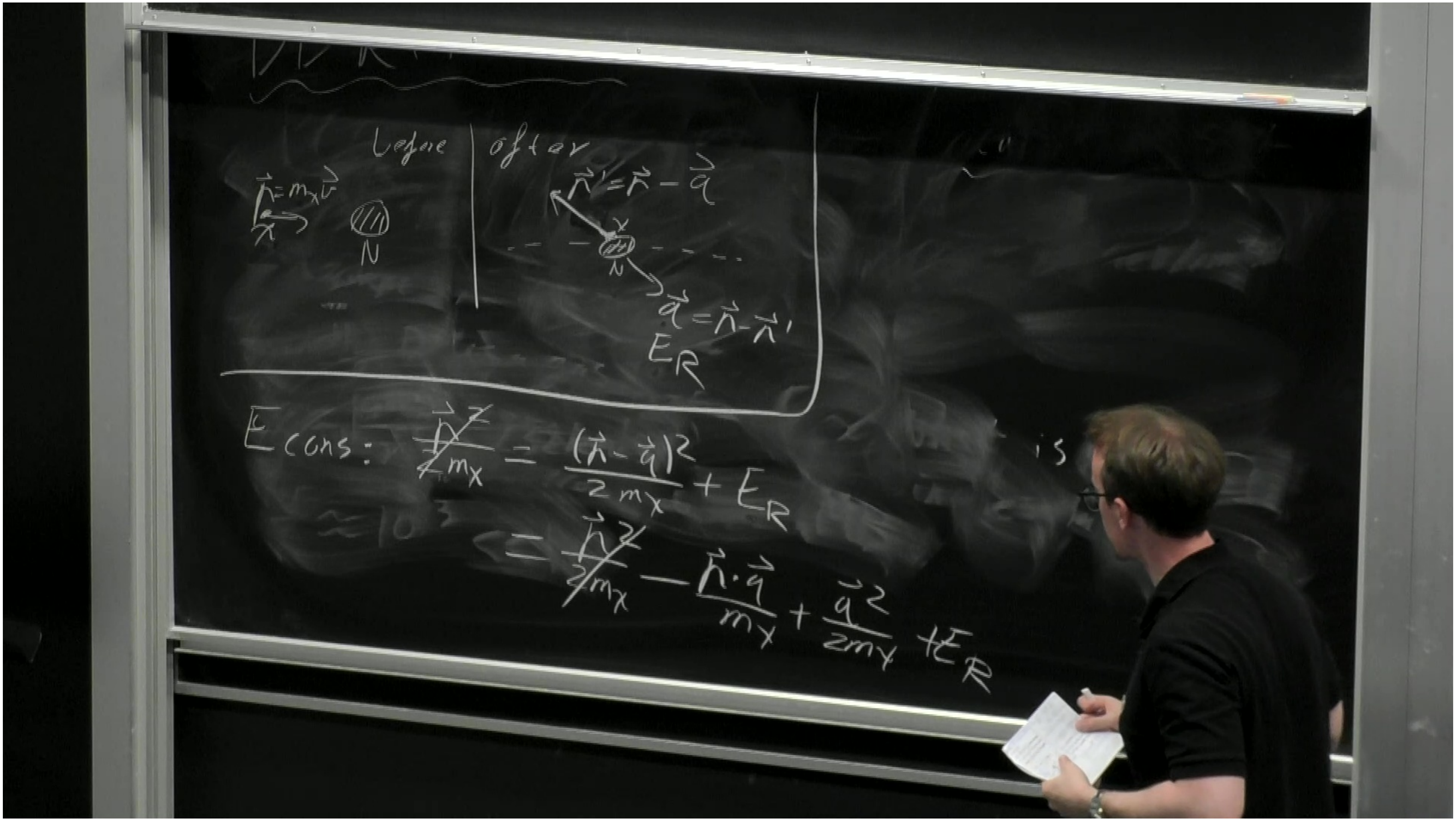
Ex: apply virial thm to a star. What is the # of particles when $P_{gas} = P_{rad}$? See Carr + Rees 1979

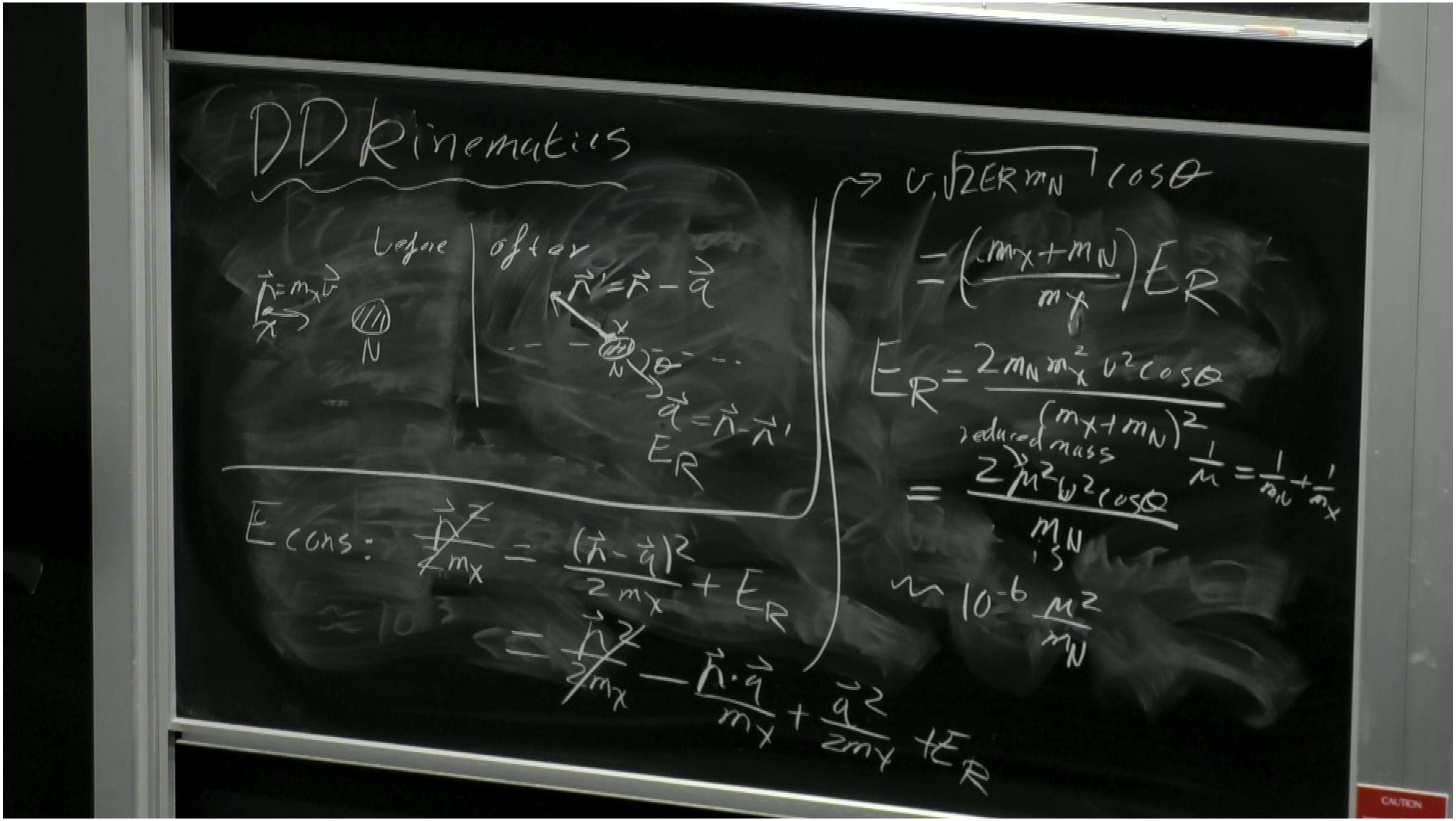

Local DM
8kpc
Sun
30kpc
200kpc
Local DM density
DM
ρχ ≃ 0.4 GeV/cm³
Locally DM is a gas
σ ≃ √(GM_enc / r) ≃ 232 km/s
v/c ≃ 10⁻³ << 1
Ex: What is Mχ[M⊙]
in the solar system?

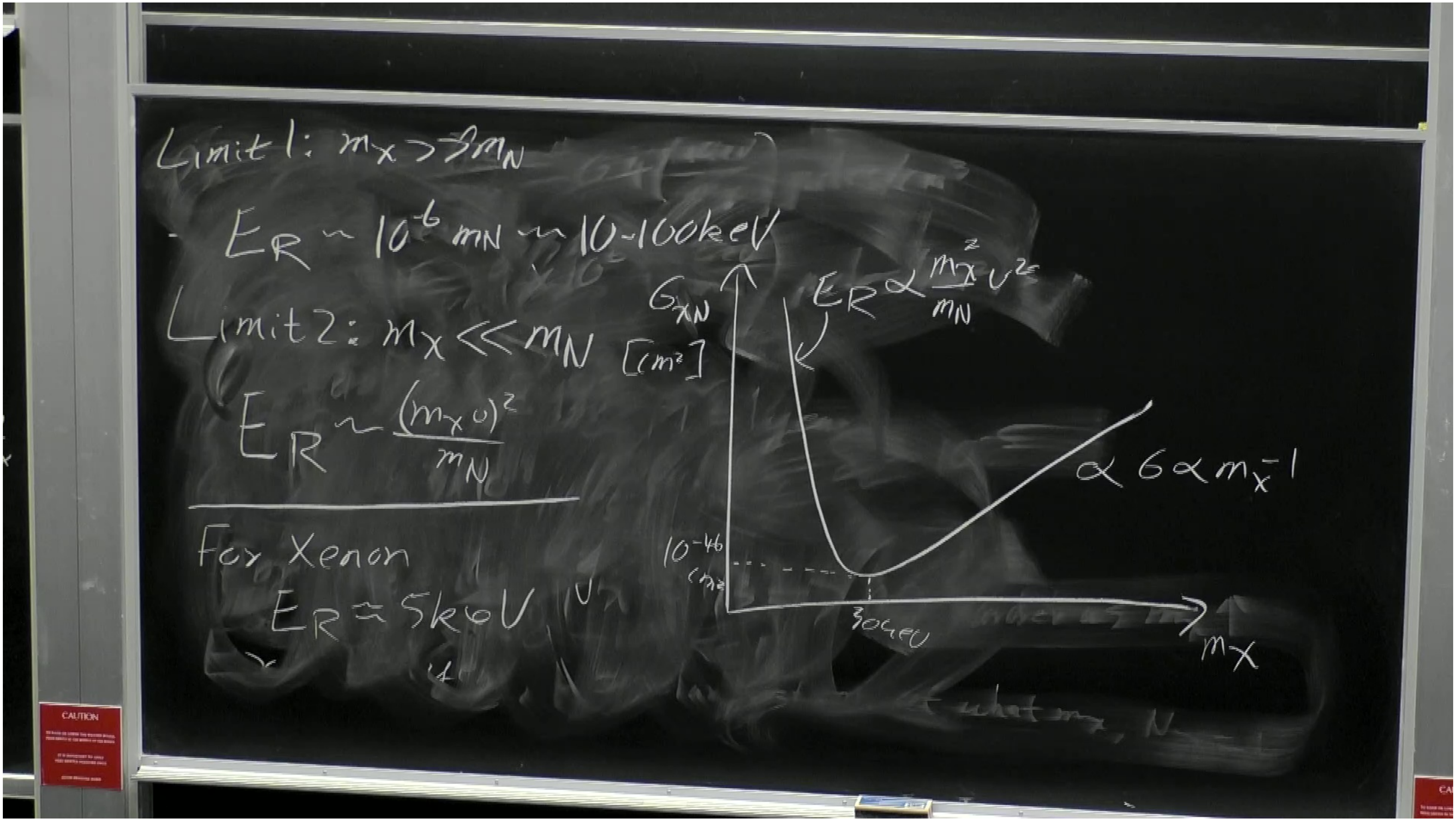

ρχ ⇒ nχ = ρχ/mχ = 0.4 (1GeV/mχ) pcles/cm³
Φχ = nχ v ≃ 9×10⁶ (1GeV/mχ) pcles/cm²/s
mχ = 1GeV , A = 1m² , T = 1 year
Nχ ≃ 3×10¹⁸ pcles

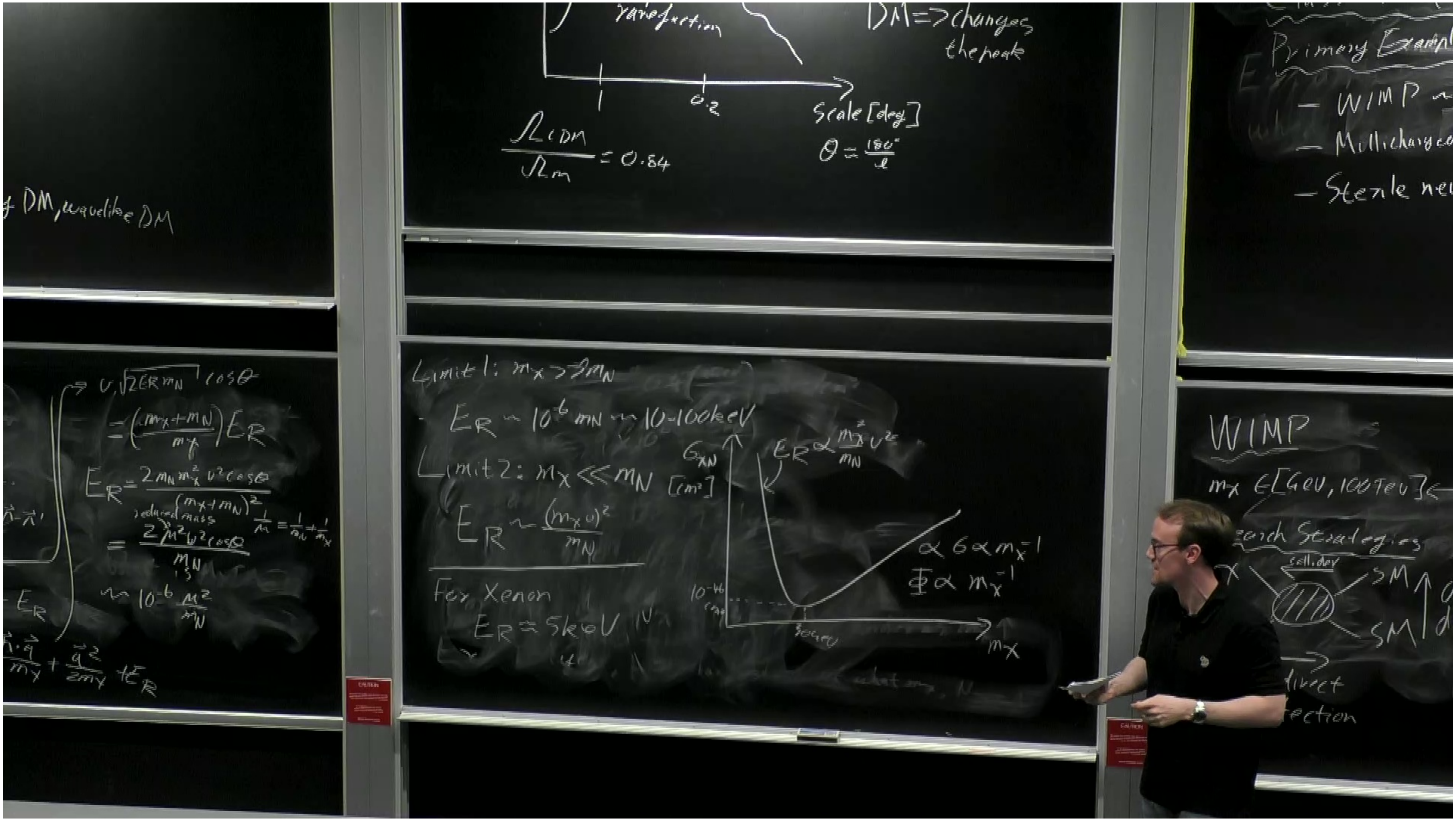

ρχ => nχ = ρχ/mχ = 0.4(1GeV/mχ) pcles/cm³
Φχ ≈ nχv ≈ 9×10⁶ (1GeV/mχ) pcles/cm²/s
mχ = 1GeV, A = 1m², T = 1year
Nχ ≈ 3×10¹⁸ pcles
For mχ = Mpl, Nχ ≈ 1/3 => DD harder as mχ ↑
Ex: T=1year, A = solar system, at what mχ, Nχ=1

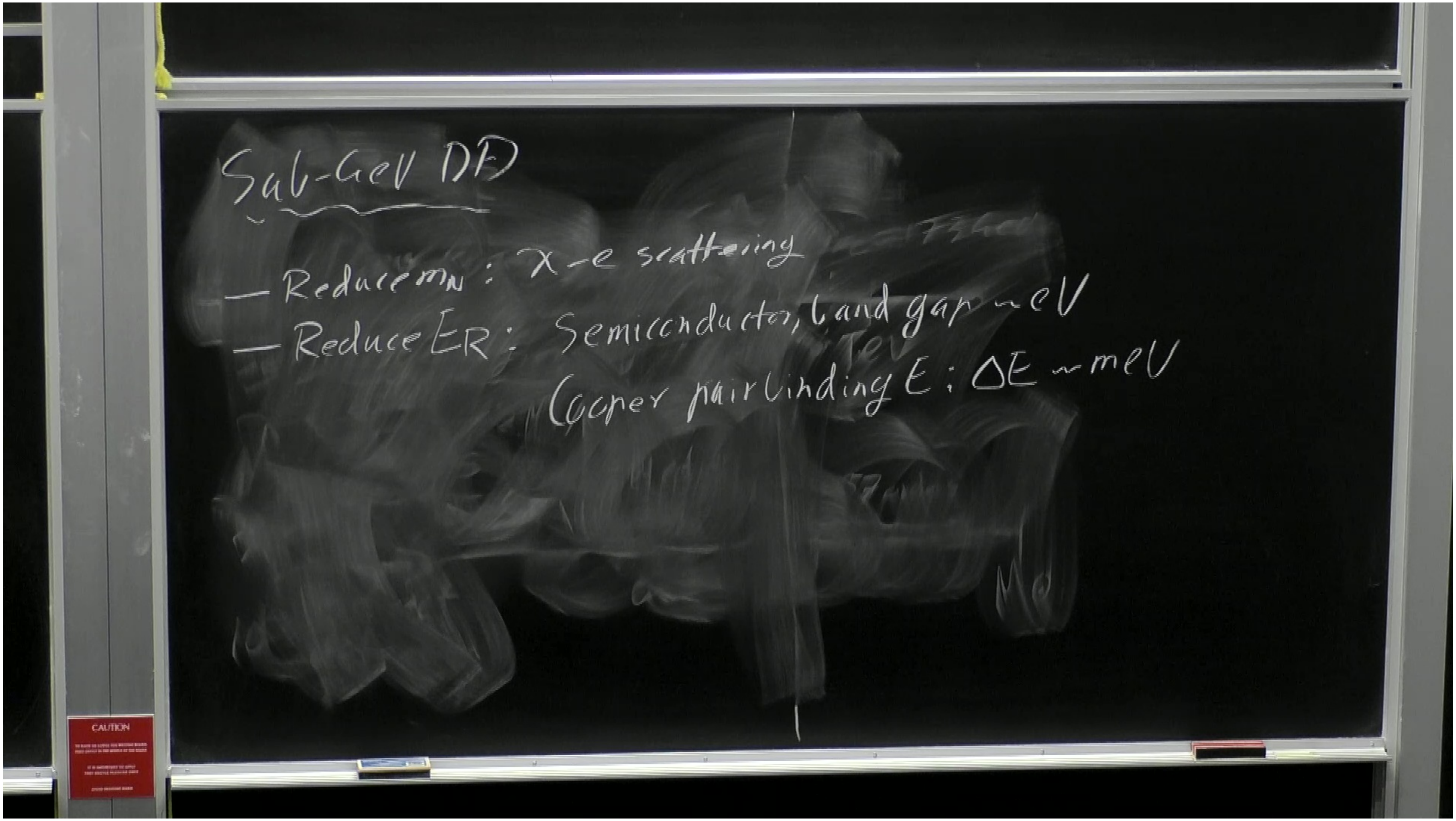

WIMP
mχ ∈ [GeV, 100TeV] ← thermal cosmology
Search Strategies
collider
χ
χ
SM
SM
direct detection
Indirect detection


before
after
$\vec{p} = m_\chi \vec{v}$
χ
N
$\vec{p}' = \vec{p} - \vec{q}$
$\vec{q} = \vec{p} - \vec{p}'$
E_R
E cons: $\frac{\vec{p}^2}{2m_\chi} = \frac{(\vec{p}-\vec{q})^2}{2m_\chi} + E_R$
$= \frac{\vec{p}^2}{2m_\chi} - \frac{\vec{p}\cdot\vec{q}}{m_\chi} + \frac{\vec{q}^2}{2m_\chi} + E_R$
is


DD Kinematics
before | after
p = mχ v
N
p' = p − q
χ
N
θ
q = p − p'
E_R
E cons: p²/2mχ = (p − q)²/2mχ + E_R
= p²/2mχ − p·q/mχ + q²/2mχ + E_R
→ v √(2E_R m_N) cos θ
= ((mχ + m_N)/mχ) E_R
E_R = 2 m_N mχ² v² cos θ / (mχ + m_N)²
reduced mass
= 2 μ² v² cos θ / m_N
1/μ = 1/m_N + 1/mχ
~ 10⁻⁶ μ²/m_N


Limit 1: mχ >> mN
ER ~ 10^-6 mN ~ 10-100 keV
Limit 2: mχ << mN
ER ~ (mχ v)^2 / mN
For Xenon
ER ≃ 5 keV
σχN [cm²]
ER ∝ mχ² v² / mN
∝ σ ∝ mχ^-1
10^-46 cm²
30 GeV
mχ


DM=>changes the peak
Scale [deg]
Limit 1: mχ >> mN
Limit 2: mχ << mN
For Xenon
WIMP
mχ ∈ [GeV, 100TeV]
Search Strategies
direct
detection
Primary Example
- WIMP
- Sterile neu


Sub-GeV DM
— Reduce mN : χ-e scattering
— Reduce ER : Semiconductor, band gap ~ eV
Cooper pair binding E : ΔE ~ meV