Title: Gravitational Waves Experiments

Speakers: Jess McIver

Collection: TRISEP 2023

Date: June 23, 2023 - 9:00 AM

URL: https://pirsa.org/23060061

Gravitational wave experiments

Hello!

I'm Jess McIver (she/her) from UBC; CRC in GW astrophysics; LIGO Scientific Collaboration Deputy Spokesperson; Aspirationally mediocre euphonium player.

I adore hiking, sailing, trees, and generally being outside.

Please feel free to reach out to me anytime: mcIver@phas.ubc.ca

Native-land.ca

One way to support the Musqueam community:

The Aboriginal Housing Management Association (AHMA) provides a spectrum of culturally safe housing including affordable housing units, homeless shelters, transition homes, supportive housing, and assisted living facilities. Many of AHMA's members also offer support services and more. AHMA members make up over one-third of Indigenous housing providers in Canada.

If you like, you can endorse the **AHMA's Provincial Urban Rural and Northern Indigenous Housing Strategy**: https://www.ahma-bc.org/how-to-support

Let's move!

What topics are you most interested in learning about during our 3 lectures?

Definitions: gravitational wave vs. gravity wave

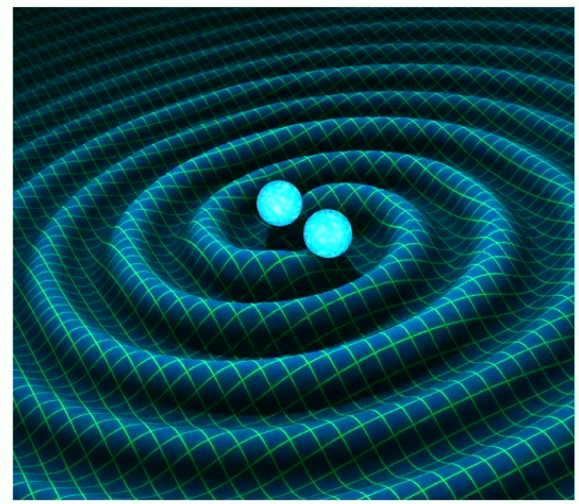
Definitions: gravitational wave vs. gravity wave

Gravitational waves: ripples in the fabric of spacetime

Gravity waves: gravity is the restoring force (e.g. water surface waves, atmospheric waves)

National Weather Service

UWaterloo



R.Hurt/Caltech

Gravitational wave experiments

We will (loosely) follow 'Discovering gravitational waves with Advanced LIGO'
McIver & Shoemaker, Contemporary Physics (2021)

Other references:
- Sathyaprakesh and Schutz's Living Review in Relativity
- Cole Miller's notes on gravitational radiation

You may want to grab paper/pen/pencil

7

Our approach

What are GWs? (GW emission, GW sources, many approximations)

Amplitude, energy, luminosity, characteristic emission for compact binaries

Analysis: searching for and characterizing GW sources

Fourier transforms, searches for modelled and un-modelled GW sources

GW detectors

GW detector sources of noise; design for different frequency regimes

GW results

What have we learned so far? What else could we learn?

The big picture: future GW detectors, multi-messenger astronomy

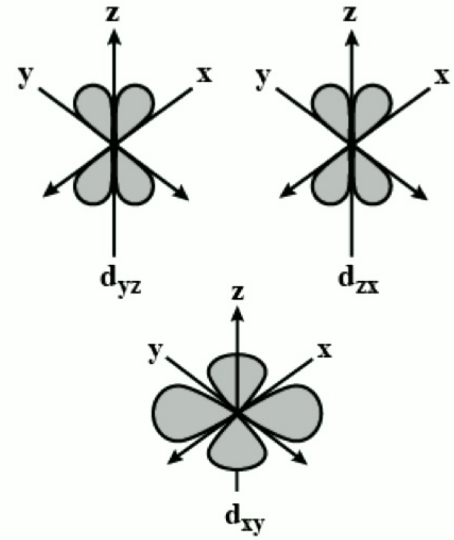
GW emission

To estimate useful properties of gravitational wave emission, we will need to solve Einstein's field equations when spacetime is not flat.

GR happens, yielding:

$$h_{ij} = \frac{2G}{c^4 R}\frac{d^2 I_{ij}}{dt^2}$$

Where I_{ij} is a tensor describing the **quadrupole moment** of the system.



Why is the quadrupole moment the 1st contributing moment?

	Monopole moment	Dipole moment	Quadrupole moment
If mass-energy density is $\rho(\vec{r})$:	$\int \rho(\vec{r}) d^3 r$	$\int \rho(\vec{r}) \vec{r} d^3 r$	$I_{ij} = \int \rho(\vec{r})\ r_i r_j\ d^3 r$
	Total mass-energy	Center of mass-energy	Changes over time (for systems with asymmetry)

Quick calculation…

We saw that $h_{ij} = \dfrac{2}{R}\dfrac{G}{c^4}\dfrac{d^2 I_{ij}}{dt^2}$.

If a source is 5 billion lightyears away, what order of magnitude must $d^2 I_{ij}/dt^2$ be to induce a local spacetime strain on the order of 10^{-21} ?

Hint: $c = 3 \times 10^8$ m/s $G = 6.67 \times 10^{-11}$ m^3 / kg s^2 5 billion lightyears $\approx 5 \times 10^{25}$ m

Bonus for early birds: What are the units of $d^2 I_{ij}/dt^2$?

GW emission

To estimate useful properties of gravitational wave emission, we will need to solve Einstein's field equations when spacetime is not flat.

GR happens, yielding:

$$h_{ij} = \frac{2G}{c^4R}\frac{d^2I_{ij}}{dt^2}$$

Where I_{ij} is a tensor describing the **quadrupole moment** of the system.

Why is the quadrupole moment the 1st contributing moment?

If mass-energy density is $\rho(\vec{r})$:

Monopole moment

$$\int \rho(\vec{r}) d^3r$$

Total mass-energy

Dipole moment

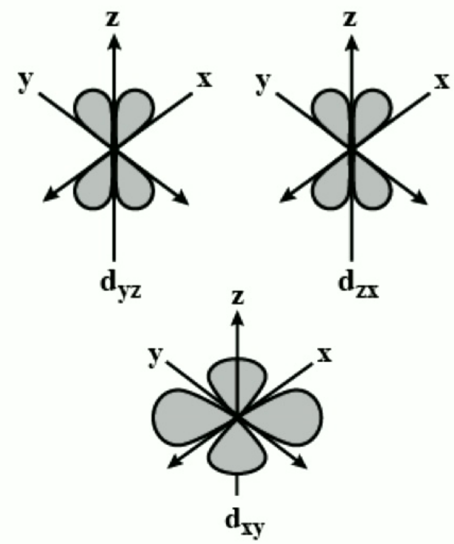
$$\int \rho(\vec{r})\vec{r} d^3r$$

Center of mass-energy

Quadrupole moment

$$I_{ij} = \int \rho(\vec{r})\ r_i r_j\ d^3r$$

Changes over time (for systems with asymmetry)



Quick calculation…

We saw that $h_{ij} = \frac{2}{R}\frac{G}{c^4}\frac{d^2 I_{ij}}{dt^2}$.

If a source is 5 billion lightyears away, what order of magnitude must $d^2 I_{ij}/dt^2$ be to induce a local spacetime strain on the order of 10^{-21} ?

Hint: $c = 3 \times 10^8$ m/s $G = 6.67 \times 10^{-11}$ m^3 / kg s^2 5 billion lightyears $\approx 5 \times 10^{25}$ m

Bonus for early birds: What are the units of $d^2 I_{ij}/dt^2$?

Thought exercise…

Given the definition of I_{ij} (i.e. the spherical asymmetry condition), and your calculation result, **what astrophysical sources do you think could feasibly generate observable gravitational waves** (where "observable" means strains on the order of 10^{-21}) ?

What if the source is galactic (e.g. $R \sim 10^{20}$ m) ?

Thought exercise…

Given the definition of I_{ij} (i.e. the spherical asymmetry condition), and your calculation result, **what astrophysical sources do you think could feasibly generate observable gravitational waves** (where "observable" means strains on the order of 10^{-21}) ?

What if the source is galactic (e.g. $R \sim 10^{20}$ m) ?

Challenge follow-up: is currently possible for humans to generate observable GWs?

Thought exercise…

If you have a very good model for $\frac{d^2 I_{ij}}{dt^2}$ can you infer the distance, R, from the measured strain amplitude, h_{ij} ? Why or why not?

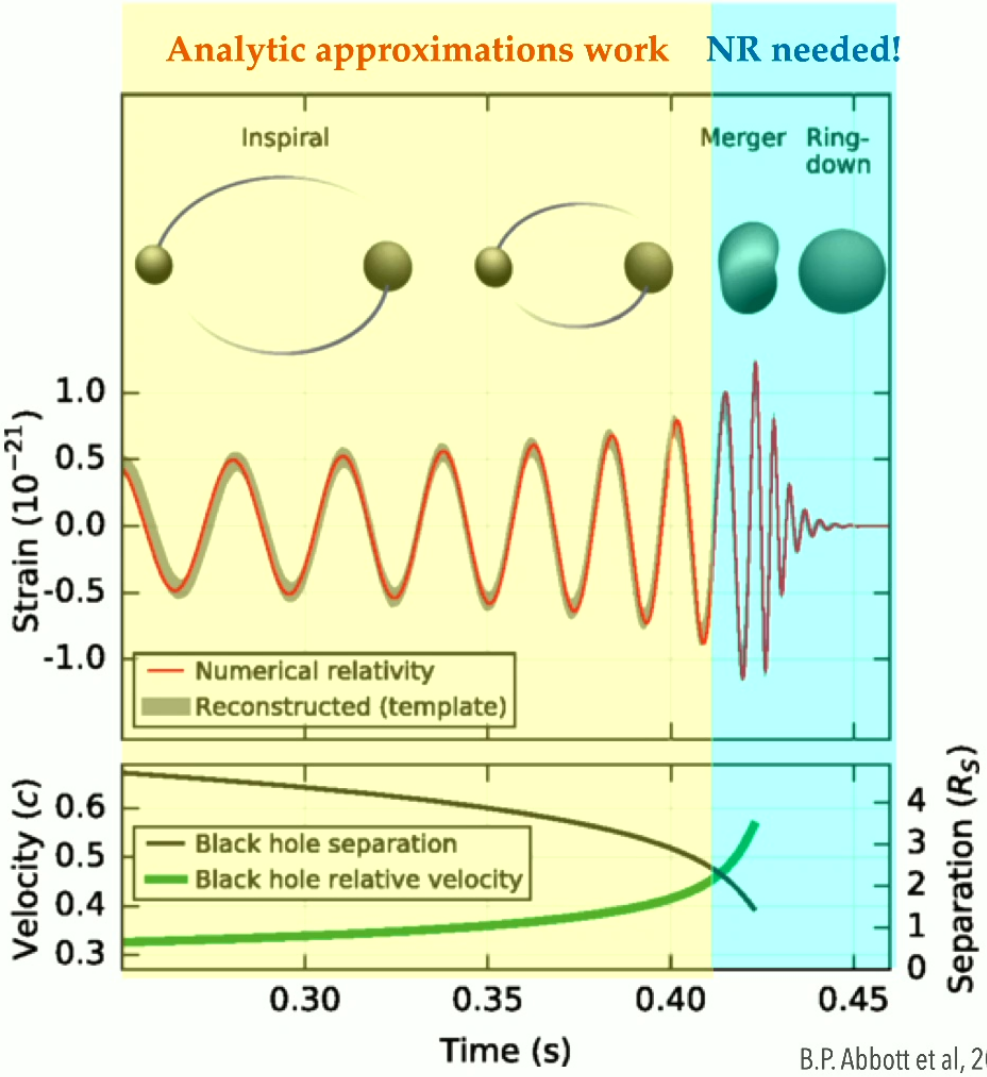
Compact binary coalescences

In the far-field limit (the "inspiral" phase) , the frequency evolution is given (to leading order, via Post-Newtonian approximations) by:

$$\dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f^{11/3}$$

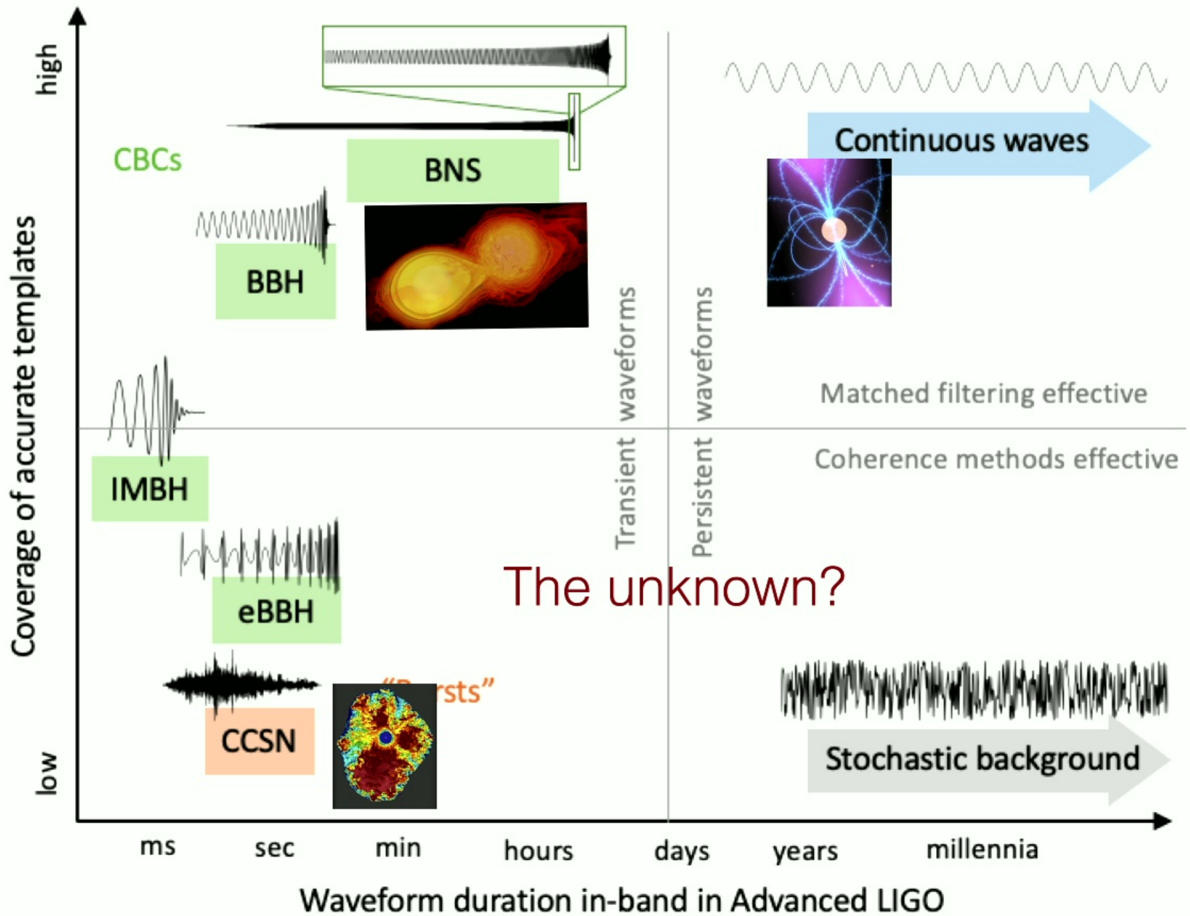
Where M_c is the **chirp mass**.

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$



B.P. Abbott et al, 2016

What might we detect with current ground-based detectors?



McIver and Shoemaker, 2021

Review: CBCs: amplitude, energy, and luminosity

For CBCs, we can derive a (remarkably!) simple estimate of the GW strain amplitude:

$$|h| \approx \frac{r_{S_1} r_{S_2}}{Rr}$$

Where $r_S = \dfrac{2GM}{c^2}$

R is the radius of the orbit

r is the distance to the CBC source

We can also estimate the luminosity emitted by CBCs as:

$$L_{CBC} = \frac{4}{5}\frac{c^5}{G}\left(\frac{2GM_{tot}}{c^2 R}\right)^5$$

Near merger, BH binaries will emit ~5% of their total rest mass energy in GWs within a fraction of a second.

How much energy is this for a black hole binary with $M_{tot} = 152\ M_\odot$?

Quick calculation…

Recall the approximation: $L_{CBC} = \frac{4}{5}\frac{c^5}{G}\left(\frac{2GM_{tot}}{c^2R}\right)^5$

Approximate the GW luminosity of a system comprised of a 70 $M_\odot$ black hole and 50 $M_\odot$ black hole orbiting with 10 r_s separation (where r_s is the Schwarzschild radius of one of the BHs).

Compare your answer with the characteristic luminosity of the following sources:

- SN: $\sim 10^{36}$ W
- AGN: $\sim 10^{37}$ W
- The total visible light of all the galaxies in the observable Universe: $\sim 10^{49}$ W