

Title: Standard Model

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I'm Canadian, but living in LA has its perks...



(view from campus this year)



- 900 students (all undergraduates)
- 20-25 physics majors per class, diverse student body (gender parity among physics majors)
- research is important, but more teaching/mentoring than at research-intensive university

R1 (where most of you are)

- Regular-track faculty and teaching-track faculty

Research Scientist/National Lab Staff Scientist

R2/R3 universities

- Still graduate doctoral and master's students, more teaching responsibilities than R1

Primarily Undergraduate Institution (PUI)

- Almost exclusively undergrads (maybe some master's students), some have more research emphasis than others

Minority Serving Institution (MSI)

- Historically Black Colleges & Universities (HBCUs); Hispanic Serving Institutions (HSIs); Tribal Colleges & Universities and Indigenous-serving non-tribal universities; Asian-American & Pacific-Islander Serving Institutions

Community Colleges

Recap: • need to be careful with long. pol.
of gauge fields

U(1) Higgs mechanism

$$\Phi(x) \rightarrow e^{ig\Lambda(x)}\Phi(x), \quad A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\Lambda(x)$$

$$\mathcal{L} = |\partial_\mu\Phi|^2 - V(\Phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu\Phi = \partial_\mu\Phi - igA_\mu\Phi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\Phi) \text{ min. at } \Phi = v$$

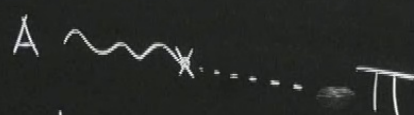
$$V(\Phi) \text{ min. at } \Phi = \frac{v}{\sqrt{2}}, \quad \Phi(x) = \frac{1}{\sqrt{2}} (v + \phi(x)) e^{i\pi(x)/v}$$

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - V(\phi) - g v A^\mu \partial_\mu \pi \left(1 + \frac{\phi}{v}\right)^2 + \frac{1}{2} g^2 v^2 A_\mu A^\mu \left(1 + \frac{\phi}{v}\right)^2$$

$$m_A^2 = g^2 v^2$$

$$m_\phi^2 = \frac{\lambda v^2}{4}$$

π gets eaten by A



unitary gauge: $\Phi(x) \rightarrow \bar{\Phi}(x) e^{-i\pi(x)/v}$
 $\pi(x) \rightarrow 0$

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π gets eaten by A

$A \sim \dots \pi$

unitary gauge

$$\Phi(x) \rightarrow \Phi(x) e^{-i\pi(x)/v}$$

$$\pi(x) \rightarrow 0$$

Higgs accounted

$v=0$ limit

$\Phi, \Phi^\dagger$ 2 dof

A_μ^T 2 dof

$\uparrow$ what physics looks
like $E \gg v$

$v \neq 0$ (broken phase)

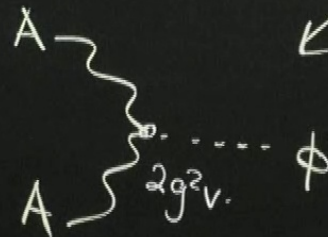
ϕ 1 dof

A_μ^T, A_μ^L 3 dof

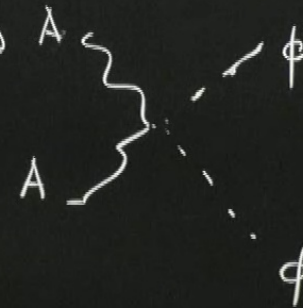


left with remnant ϕ (Higgs boson)

$$\mathcal{L} \supset \frac{1}{2} g^2 v^2 A_\mu A^\mu \left(1 + \boxed{\frac{2\phi}{v}} + \boxed{\frac{\phi^2}{v^2}} \right)$$



crucial
term



Stückelberg limit

Do we actually need a Higgs to give $U(1)$ gauge boson a mass?

$$m_\phi = \frac{\sqrt{\lambda}}{2} v$$

$$m_A = gv$$

$$\frac{m_\phi}{m_A} = \frac{\sqrt{\lambda}}{2g}$$

→ if I fix $m_A = gv$, I can make $m_\phi \gg m_A$ by taking $g \rightarrow 0$, $v = \frac{m_A}{g} \rightarrow \infty$

Stückelberg limit

Do we actually need a Higgs to give $U(1)$ gauge boson a mass?

$$m_\phi = \frac{\sqrt{\lambda}}{2} v$$

$$m_A = gv$$

$$\frac{m_\phi}{m_A} = \frac{\sqrt{\lambda}}{2g}$$

→ if I fix $M_A = gv$, I can make $m_\phi \gg m_A$
by taking $g \rightarrow 0$, $v = \frac{m_A}{g} \rightarrow \infty$

→ then $m_\phi \rightarrow \infty$

Stückelberg limit allows $U(1)$ gauge boson to have mass w/o Higgs if it couples to a conserved current

↳ this requires allowing g to be whatever I want

Fermion masses

massless: helicity (proj. of spin on dir. of motion)
(Weyl fermions) is Lorentz invariant

→ called chirality

RH and LH massless are different particles



RH



LH

antiparticles ψ_L χ_R

ψ_L^+ RH

χ_R^+ LH



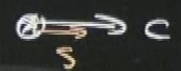
Fermion masses

massless: helicity (proj. of spin on dir. of motion)
(Weyl fermions) is Lorentz invariant

→ called chirality

There is always a LH & RH chiral basis, $m=0$ coincides with helicity

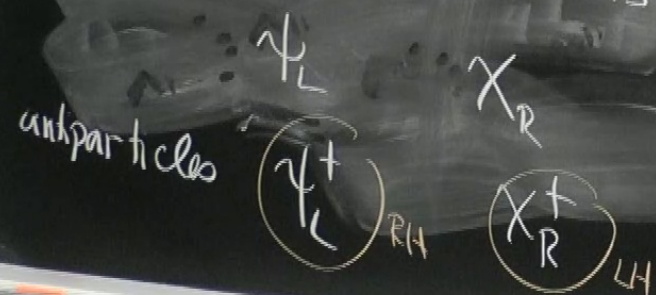
RH and LH massless are different particles



RH



LH



$m \neq 0$: 2 ways

① Dirac masses

take 2 unrelated Weyl fermions

$$\Psi = \begin{pmatrix} e_L^- \\ e_R^- \end{pmatrix}$$

↑
superposition of LH & RH e^-

$$\bar{\Psi} = \begin{pmatrix} (e_L^-)^\dagger & (e_R^-)^\dagger \end{pmatrix}$$

$$\bar{\Psi} = \Psi^\dagger \gamma^0 = (e_R^+ \ e_L^+)$$

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -M \bar{\Psi} \Psi \\ &= -M (e_R^+ e_L + e_L^+ e_R) \end{aligned}$$

② Majorana Masses

single Weyl field ψ_L LH
 $\psi_L^\dagger$ RH

$$\mathcal{L} = -M (\psi_L \psi_L + \psi_L^\dagger \psi_L^\dagger)$$

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_L^\dagger \end{pmatrix}$$

* cannot be used for charged states

Fermion masses from Higgs

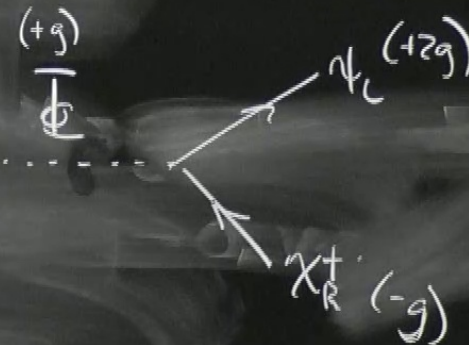
- massive photon, Φ (g)
- add 2 Weyl fermions, ψ_L^{2+} and χ_R^{1+} (2g, 1g)

Fermion masses from Higgs

- massive photon, $\Phi (g)$
- add 2 Weyl fermions, ψ_L^{2+} and $\chi_R^{1+} (2g, 1g)$

No mass possible at Lagrangian level w/o violating charge conservation

But: $\mathcal{L} = -y \Phi \psi_L^+ \chi_R + \text{h.c.}$



Turn Higgs field on in unitary gauge

$$\Phi = \frac{1}{\sqrt{2}}(v + \phi)$$

$$\mathcal{L} = -\frac{Y}{\sqrt{2}}(v + \phi)\psi_L^\dagger \chi_R + \text{h.c.} \quad \checkmark \quad \text{Dirac mass!}$$

Turn Higgs field on in unitary gauge

$$\Phi = \frac{1}{\sqrt{2}}(v + \phi)$$

$$\mathcal{L} = -\frac{y}{\sqrt{2}}(v + \phi)\bar{\psi}_L \chi_R + \text{h.c.} \quad \checkmark \quad \text{Dirac mass!}$$

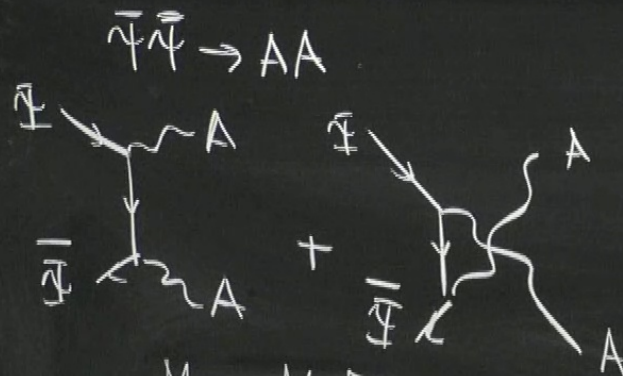
$$\bar{\Psi} = \begin{pmatrix} \bar{\psi}_L & \bar{\chi}_R \end{pmatrix}$$

$$J^\mu = \bar{\Psi} \gamma^\mu \not{D} \Psi \quad \text{is not conserved in absence of Higgs}$$

$$+ \bar{\Psi} \gamma^\mu \not{D} \Psi$$



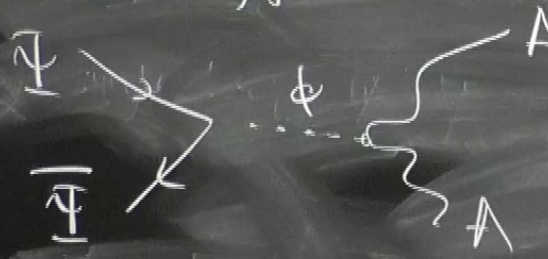
Unitarity of Theory



at high energies,
 $\epsilon_L^M = \frac{1}{m_A}(\vec{k}, 0, 0, k)$

$M \sim \frac{M_F \sqrt{s}}{m_A^2} \rightarrow \text{grows with energy!}$
 $\uparrow$
 matrix element

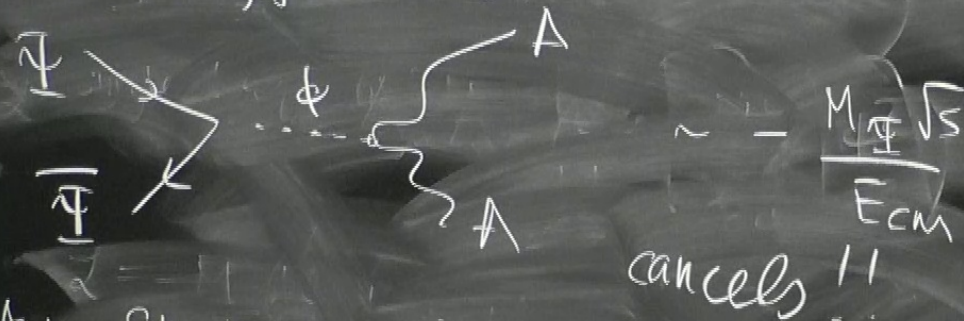
with Higgs field



$$\sim - \frac{M_{\Psi} \sqrt{s}}{E_{cm}}$$

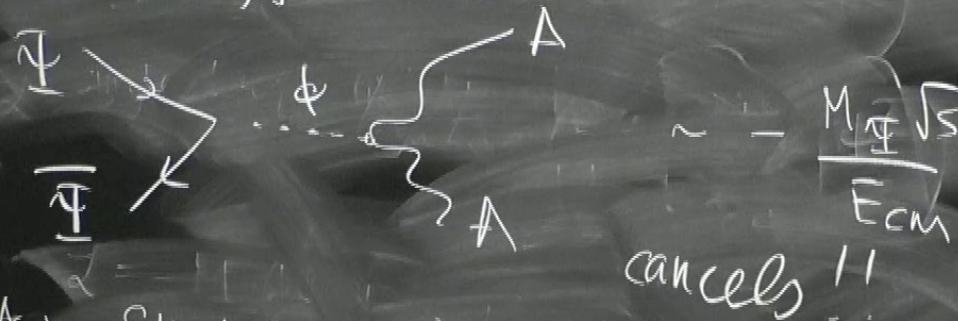
cancels !!

with Higgs' field



* no Stückelberg limit (violation of charge cons., relation between Φ , χ_L , χ_R charges)

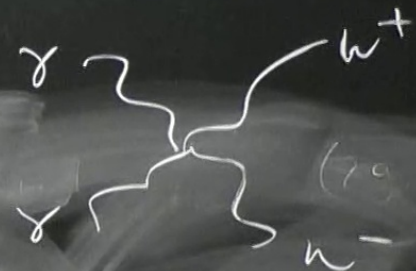
with Higgs field



* no Stückelberg limit (violation of charge cons., relation between Φ, χ_L, χ_R charges)
 → if I want to self-consistently take S.C. $M_\Phi \rightarrow \infty$

Non-Abelian Gauge Theory

gauge boson self-interactions



→ need non-Abelian gauge invariance

→ gauge boson self-interactions, "minimal" charge

$$\underline{SU(2)}: \Phi(x)_\pm \rightarrow e^{ig\Lambda^a(x)(T_R^a)_{\pm\pm}} \Phi(x)_\pm$$

$\uparrow$ multiple gauge params $\uparrow$ generators
 or benchmark

$$\Psi = \begin{pmatrix} \psi_L \\ \chi_R \end{pmatrix}$$

$$J^\mu = 2g \bar{\Psi} \gamma^\mu T^a \Psi$$

is not conserved in absence of Higgs

$$\Phi \approx \Phi + ig \Lambda^a T_R^a \Phi$$

infinitesimal
mass Λ

generators obey $[T_R^a, T_R^b] = i f^{abc} T_R^c$

Looks like $[J_x, J_y] = i J_z$. Many representations:

① Doublet representation (2)

$$T_F^i = \frac{1}{2} \sigma^i$$

CAUTION

$$\tilde{\Phi} \approx \Phi + ig \Lambda^a T_R^a \Phi$$

infinitesimal general rep

generators obey $[T_R^a, T_R^b] = i f^{abc} T_R^c$

Looks like $[J_x, J_y] = i J_z$. Many representations:

① Doublet representation (2)

$$T_R^i = \frac{1}{2} \sigma^i$$

fundamental
rep

② analogy with spin-1
triplet, 3 states

CAUTION

DO NOT TOUCH THE BOARD
OR THE BOARD OR THE BOARD
OR THE BOARD OR THE BOARD
OR THE BOARD OR THE BOARD

Special rep: adjoint rep.

$$(T_A^a)^{bc} = -i \epsilon^{abc}.$$

→ gauge fields transform in adjoint rep → 3 3×3 matrices

Define $D_\mu \Phi = \partial_\mu \Phi - ig A_\mu \Phi$

$\uparrow$ $\uparrow$ $\uparrow$
 matrices

Define $U = e^{ig\Lambda^a T_R^a}$, $\Phi \xrightarrow{g.t.} U\Phi$, $A_\mu \xrightarrow{g.t.} A'_\mu$

want: $D_\mu \Phi \rightarrow U(D_\mu \Phi)$

get: $A'_\mu = UA_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$

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$\uparrow$ $\uparrow$ $\uparrow$
 matrices

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infinitesimal: $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda^a T_R^a + ig \alpha [T_R^a A_\mu]$

$$A^\mu = A^\mu_a T^a_R$$

$$A^\mu \rightarrow A^\mu_a T^a_R + \partial_\mu \Lambda^a T^a_R + ig A^\mu_a A^\nu_b [T^a_R, T^b_R]$$

$$A^\mu_a = A^\mu_a + \partial_\mu \Lambda^a + ig \Lambda^b (T^b_A)^{ac} A^\mu_c$$

$$A^\mu \equiv A^{\mu a} T_R^a$$

$$A_\mu \rightarrow A_\mu^a T_R^a + \partial_\mu \Lambda^a T_R^a + ig A_\mu^a A_\nu^b [T_R^a, T_R^b]$$

$$A_\mu^a = A_\mu^a + \partial_\mu \Lambda^a + ig \Lambda^b (T_A^b)^{ac} A_\mu^c.$$

Field strengths

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^a A_\nu^b$$

$$A^\mu = A^{\mu a} T_R^a$$

$$A_\mu \rightarrow A_\mu^a T_R^a + \partial_\mu \Lambda^a T_R^a + ig A_\mu^a A_\nu^b [T_R^a, T_R^b]$$

$$A_\mu^a = A_\mu^a + \partial_\mu \Lambda^a + ig \Lambda^b (T_A^b)^{ac} A_\mu^c$$

Field strengths

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^a A_\nu^b$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}$$

