

Title: Standard Model

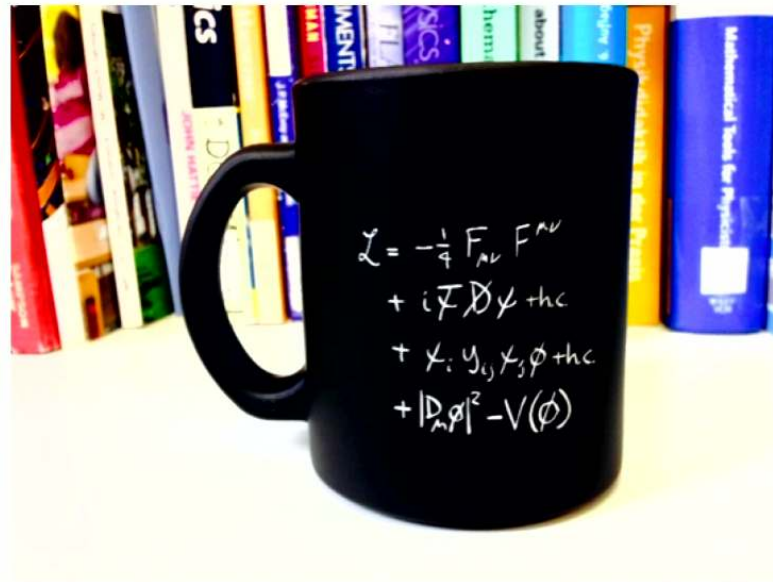
Speakers: Brian Shuve

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Standard Model Theory

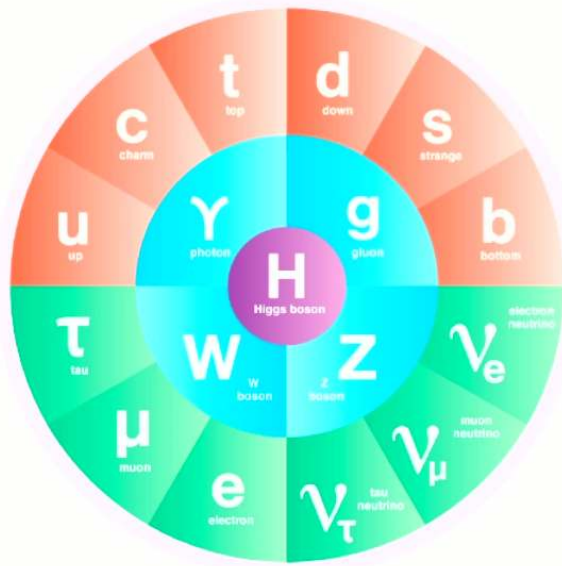


Standard Model to a Theorist

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{2}\text{Tr} G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}\text{Tr} W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\theta\alpha_s}{4\pi}\text{Tr} G_{\mu\nu}\tilde{G}^{\mu\nu} \\
 & + (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \frac{1}{2}\lambda(\phi^\dagger\phi)^2 \\
 & + \sum_{f=1}^3 \left(\bar{\ell}_L^f i\not{D}\ell_L^f + \bar{\ell}_R^f i\not{D}\ell_R^f + \bar{q}_L^f i\not{D}q_L^f + \bar{d}_R^f i\not{D}d_R^f + \bar{u}_R^f i\not{D}u_R^f \right) \\
 & - \sum_{f=1}^3 y_\ell^f \left(\bar{\ell}_L^f\phi\ell_R^f + \bar{\ell}_R^f\phi^\dagger\ell_L^f \right) \\
 & - \sum_{f,g=1}^3 \left(y_d^{fg}\bar{q}_L^f\phi d_R^g + (y_d^{fg})^*\bar{d}_R^g\phi^\dagger q_L^f + y_u^{fg}\bar{q}_L^f\tilde{\phi}u_R^g + (y_u^{fg})^*\bar{u}_R^g\tilde{\phi}^\dagger q_L^f \right),
 \end{aligned}$$

Standard Model Summary

Credit: Wikipedia



Parameters of the Standard Model

[hide]

#	Symbol	Description	Renormalization scheme (point)	Value
1	m_e	Electron mass		0.511 MeV
2	m_μ	Muon mass		105.7 MeV
3	m_τ	Tau mass		1.78 GeV
4	m_u	Up quark mass	$\overline{MS} = 2 \text{ GeV}$	1.9 MeV
5	m_d	Down quark mass	$\overline{MS} = 2 \text{ GeV}$	4.4 MeV
6	m_s	Strange quark mass	$\overline{MS} = 2 \text{ GeV}$	87 MeV
7	m_c	Charm quark mass	$\overline{MS} = m_c$	1.32 GeV
8	m_b	Bottom quark mass	$\overline{MS} = m_b$	4.24 GeV
9	m_t	Top quark mass	On shell scheme	173.5 GeV
10	θ_{12}	CKM 12-mixing angle		13.1°
11	θ_{23}	CKM 23-mixing angle		2.4°
12	θ_{13}	CKM 13-mixing angle		0.2°
13	δ	CKM CP violation Phase		0.995
14	g_1 or g'	U(1) gauge coupling	$\overline{MS} = m_Z$	0.357
15	g_2 or g	SU(2) gauge coupling	$\overline{MS} = m_Z$	0.652
16	g_3 or g_s	SU(3) gauge coupling	$\overline{MS} = m_Z$	1.221
17	θ_{QCD}	QCD vacuum angle		~ 0
18	v	Higgs vacuum expectation value		246 GeV
19	m_H	Higgs mass		$125.09 \pm 0.24 \text{ GeV}$

$$\theta_{12}^{\text{PMNS}} = 33.82^\circ$$

$$\theta_{13}^{\text{PMNS}} = 8.6^\circ$$

$$\theta_{23}^{\text{PMNS}} = 48.6^\circ$$

$$\delta^{\text{PMNS}} = 108^\circ - 404^\circ$$

Standard Model: Why?

- Some 'why' questions don't have answers (...yet?)
 - Electron mass, strength of gauge couplings, number of fermion generations, ...
- Other 'why' questions have answers rooted in empirical observations and physical constraints on the theory
 - Must haves: causality (locality + Lorentz invariance), unitarity
 - Consequences include charge conservation/gauge invariance, Higgs mechanism, structure of interactions among SM fields

Plan for the Week

- **Monday:** massive spin-1 particles & Higgs mechanism
- **Tuesday:** the electroweak theory
- **Wednesday:** tests of the electroweak theory, connections to quantum chromodynamics
- **Thursday:** neutrinos and flavour

Lecture 1: Higgs Mechanism

Focus on spin-1 \rightarrow relativity + spin

start with familiar example: photon

- spin-1
- massless
- 2 pol states \rightarrow transverse
- gauge inv
- go at 1

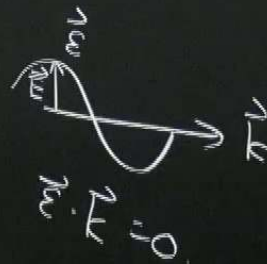


Lecture 1: Higgs Mechanism

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Now, $m \neq 0$



3 polarizations!

conflict: expect $v \sim c$ limit of $m \neq 0$ theory should
turn into $m \rightarrow 0$ theory
If you take $m \neq 0$ spin-1 particle, $E \rightarrow M$ limit, things go wrong!

Photons & Gauge Invariance

use potentials: $A^M = (\phi, \vec{A})$

\vec{E}, \vec{B} come from derivatives

$$F^{M\nu} = \partial^M A^\nu - \partial^\nu A^M = \begin{pmatrix} 0 & & & \\ \boxed{\vec{E}} & 0 & & \\ & B_z & 0 & \\ & -B_y & B_x & 0 \end{pmatrix}, \quad F^{\mu\nu} = -F^{\nu\mu}$$

Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{M\nu} F^{M\nu}$

Equations of motion

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Equations of motion:

$$\partial_\mu F^{\mu\nu} = \boxed{J^\nu} = (\rho, \vec{J})$$

wrong!

CAUTION

Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Equations of motion:

$$\partial_\mu F^{\mu\nu} = \boxed{J^\nu} = (\rho, \vec{J})$$

$$\boxed{\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = J^\nu}$$

$$\frac{\partial^2 A^\nu}{\partial t^2} - \nabla^2 A^\nu$$

wrong!

CAUTION

Gauge invariance: $A^\mu \rightarrow A^\mu + \partial^\mu \Lambda(x)$

$$F^{\mu\nu} \rightarrow \left[\partial^\mu (A^\nu + \partial^\nu \Lambda) - \partial^\nu (A^\mu + \partial^\mu \Lambda) \right]$$
$$\rightarrow F^{\mu\nu}$$

e.o.m. is invariant (\mathcal{L} is invariant) under gauge trans.
~~Now going to consider~~ $J=0$ (free propagation)

Choose gauge: coulomb gauge

$$\vec{\nabla} \cdot \vec{A} = 0$$

combine Coulomb gauge $\vec{J} = 0$.

$$\partial_m A^m = 0.$$

(Lorenz gauge)

eq. of motion: $\partial_m \partial^m A^\nu = 0$

Solutions: $A^m = \epsilon^m(k) e^{-ik \cdot x}$

gauge
cond.

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$$\epsilon^0 = 0$$

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gauge
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$$\vec{E} \cdot \vec{\epsilon} = 0$$

$$\epsilon^0 = 0$$

For photon momentum $k^M = (k, 0, 0, k)$

$$\epsilon_x = (0, 1, 0, 0)$$

$$\epsilon_y = (0, 0, 1, 0)$$

In this theory, charge is conserved.

$$\partial_m F^{mv} = J^v$$

$$0 = \partial_\nu \partial_m F^{mv} = \partial_\nu J^v$$

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★ gauge inv. \leftrightarrow 2 pols

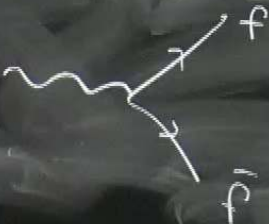
\leftrightarrow current cons \leftrightarrow masslessness

$$\partial_m A^m = \partial_0 A^0 - \vec{\nabla} \cdot \vec{A}$$

So far all classical

$$\text{QFT: } A^\mu(x) = \sum_{s, \vec{k}} \left[\epsilon_s(\vec{k}) a_s(\vec{k}) e^{-ikx} + \epsilon_s^*(\vec{k}) a_s^\dagger(\vec{k}) e^{ikx} \right]$$

ϵ show up in quantum amplitudes:



$$M \sim \epsilon(p_\epsilon)_\mu \bar{u}(p_f) \Gamma^\mu v(p_{\bar{f}})$$

$$\sim \epsilon(p_\epsilon)_\mu M^\mu$$

Polarizations of Massive Spin-1

give photon a mass

↳ expect $\xi^m = (0, 0, 0, 1)$

Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_A^2 A_\mu A^\mu$$

e.o.m. (free)

$$(\partial_\mu \partial^\mu + m_A^2) A^\nu - \partial^\nu \partial_\mu A^\mu = 0$$

* gauge in
↔ cur

\leftrightarrow current cons \leftrightarrow masslessness

Take ∂_ν of both sides:

$$(\cancel{\partial_\mu \partial^\mu} + m_A^2) \partial_\nu A^\nu - \partial_\nu \cancel{\partial^\mu} \partial_\mu A^\mu = 0$$

$$\boxed{\partial_\nu A^\nu = 0}$$

sub into e.o.m:

$$(\partial_\mu \partial^\mu + m_A^2) A^\nu = 0$$

$$A^\mu = \epsilon^\mu e^{-ikx}$$

\rightarrow no longer true that $\epsilon^0 = 0$

← current cons ↔ masslessness

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$$(\partial_\mu \partial^\mu + m_A^2) A^\nu = 0$$

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→ no longer true that $\epsilon^0 = 0$, but 3 pols.

start in photon rest frame: $k^\mu = (M, 0, 0, 0)$

$$\rightarrow \epsilon_L^\mu = (0, 0, 0, 1)$$

longitudinal
pols.

now, do Lorentz trans. along \hat{z} : $\vec{k}' = k \hat{z}$

$$\epsilon_L'^\mu = \frac{1}{M_A} (|\vec{k}|, 0, 0, E) \quad \frac{E}{M_A} = \gamma$$

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Amplitudes $\sim E/M_A \rightarrow \infty$  violate unitarity in $E \Rightarrow M_e$ limit!

* cannot have arbitrarily coupled long
polarizations! in high-energy limit
can we fix this? can I devise $E_{\mu} M^{\mu}$ so that
longitudinal polarizations decouple?

* cannot have arbitrarily coupled long polarizations! in high-energy limit

can we fix this? can I devise $\epsilon_{\mu\nu}$ so that longitudinal polarizations decouple?

note that $E \Rightarrow M_A$.

$$\epsilon_{\mu\nu}^M \approx \underbrace{\frac{(E_0, 0, 0, |\vec{E}|)}{M_A}}_{\frac{k}{M_A}} - \frac{1}{2} \left(\frac{M_A}{|\vec{E}|}, 0, 0, -\frac{M_A}{|\vec{E}|} \right) + o(M_A^2)$$

Take ∂_ν of

$(\partial_\mu \partial_\nu)$

sub into e.o.m

$$A^\mu = \epsilon^{\mu\nu} \partial_\nu \phi$$

If theory were invariant under

$$A^M \rightarrow A^M - \frac{k^M}{m_A} \text{ then OK}$$

This is exactly what gauge invariance does!

$$\Delta = -\frac{c}{M_A} e^{-ikx}, \quad A^M \rightarrow A^M + \partial^M \Delta$$

The Higgs Mechanism: Abelian Theory

Add Higgs field $\Phi(x)$ (2 d.o.f)

For starters, let's consider turning off gauge interactions

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi), \text{ invariant under } \Phi \rightarrow e^{i\alpha} \Phi$$



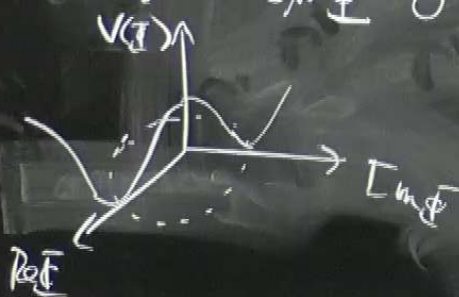
$$V(\Phi) = -\mu^2 |\Phi|^2 + \frac{\lambda}{4} |\Phi|^4$$

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For starters, let's consider turning off gauge interactions

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$$V(\Phi) = -\mu^2 |\Phi|^2 + \frac{\lambda}{4} |\Phi|^4,$$

$$\rightarrow \text{min. } \langle \Phi \rangle = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$

shift Φ : $\Phi(x) = \frac{1}{\sqrt{2}} (v + \phi(x)) e^{i\pi(x)/v}$

$$\langle \phi \rangle = 0$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) + \frac{1}{2} \left(1 + \frac{\phi^2}{v^2}\right) \partial_\mu \pi \partial^\mu \pi - V(\phi)$$

$$V(\phi) = \frac{1}{2} \left(\frac{\lambda v^2}{2}\right) \phi^2 + \frac{1}{4} \lambda v \phi^3 + \frac{1}{16} \lambda \phi^4$$

$$m_\phi^2 = \frac{\lambda v^2}{2}$$

$$m_\pi = 0$$

Goldstone boson

$$D_\mu \bar{\Phi} = (\partial_\mu - ig A_\mu) \bar{\Phi}$$

$$\xrightarrow{g.t} (\partial_\mu - ig A_\mu - ig \partial_\mu \Lambda) [e^{ig\Lambda(x)} \bar{\Phi}(x)]$$

$$= e^{ig\Lambda(x)} (\partial_\mu - ig A_\mu) \bar{\Phi}$$

$$(D_\mu \bar{\Phi})(D_\mu \bar{\Phi})^* \xrightarrow{g.t.} (D_\mu \bar{\Phi})(D_\mu \bar{\Phi})^* e^{ig\Lambda} e^{-ig\Lambda}$$

A_n

Pick same $V(\Phi)$

$$\Phi(x) = \frac{1}{\sqrt{2}} (v + \phi(x)) e^{i\pi(x)/v}$$

$$\mathcal{L} = \frac{1}{2} (\partial_m \phi) (\partial^m \phi) + \frac{1}{2} \left(1 + \frac{\phi}{v}\right)^2 \partial_m \pi \partial^m \pi - V(\phi) \\ - g v A^m \partial_m \pi \left(1 + \frac{\phi}{v}\right)^2 + \frac{1}{2} g^2 v^2 A_m A^m \left(1 + \frac{\phi}{v}\right)^2$$

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constant $\partial_\mu \pi A^\mu$
 $A^\mu \rightsquigarrow \dots \pi$

Pick same $V(\Phi)$

$$\Phi(x) = \frac{1}{\sqrt{2}} (v + \phi(x)) e^{i\pi(x)/v}$$

$$\mathcal{L} = \frac{1}{2} (\partial_n \phi) (\partial^n \phi) + \frac{1}{2} \left(1 + \frac{\phi}{v}\right) \partial_n \pi \partial^n \pi - V(\phi) - g v A^m \partial_n \pi \left(1 + \frac{\phi}{v}\right)^2 + \frac{1}{2} g^2 v^2 A_m A^m \left(1 + \frac{\phi}{v}\right)^2$$

constant $\partial_n \pi A^m$
 $A^m \rightsquigarrow \dots \pi$

constant $A_m A^m$
 \rightarrow mass for photon

$$m_A^2 = g^2 v^2$$