Title: Standard Model

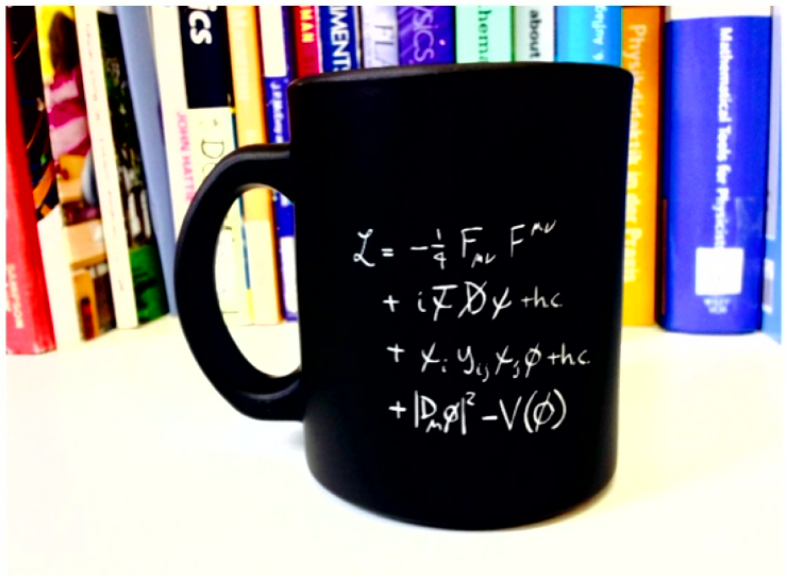
Speakers: Brian Shuve

Collection: TRISEP 2023

Date: June 19, 2023 - 9:00 AM

URL: https://pirsa.org/23060053

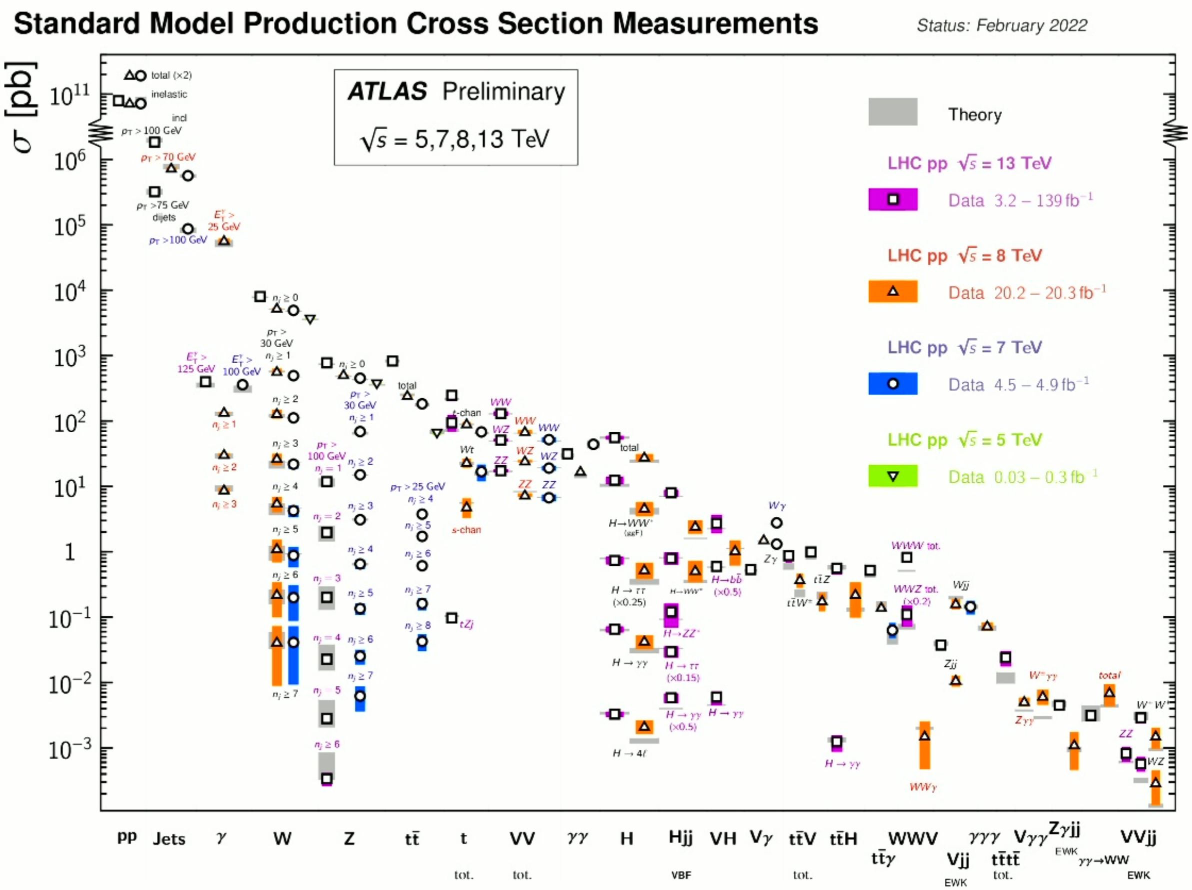
Standard Model Theory



Standard Model to a Theorist

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\, W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\theta\,\alpha_{\mathrm{s}}}{4\pi}\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu} \\
& + (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \frac{1}{2}\lambda\left(\phi^\dagger\phi\right)^2 \\
& + \sum_{f=1}^{3}\left(\bar{\ell}^f_L i\not{D}\ell^f_L + \bar{\ell}^f_R i\not{D}\ell^f_R + \bar{q}^f_L i\not{D}q^f_L + \bar{d}^f_R i\not{D}d^f_R + \bar{u}^f_R i\not{D}u^f_R\right) \\
& - \sum_{f=1}^{3} y^f_\ell\left(\bar{\ell}^f_L\phi\ell^f_R + \bar{\ell}^f_R\phi^\dagger\ell^f_L\right) \\
& - \sum_{f,g=1}^{3}\left(y^{fg}_d\bar{q}^f_L\phi d^g_R + (y^{fg}_d)^*\bar{d}^g_R\phi^\dagger q^f_L + y^{fg}_u\bar{q}^f_L\tilde{\phi}u^g_R + (y^{fg}_u)^*\bar{u}^g_R\tilde{\phi}^\dagger q^f_L\right),
\end{aligned}
$$

Standard Model to an Experimentalist



Standard Model Summary

Credit: Wikipedia

Parameters of the Standard Model [hide]

#	Symbol	Description	Renormalization scheme (point)	Value
1	m_e	Electron mass		0.511 MeV
2	m_μ	Muon mass		105.7 MeV
3	m_τ	Tau mass		1.78 GeV
4	m_u	Up quark mass	$\mu_{\overline{MS}}$ = 2 GeV	1.9 MeV
5	m_d	Down quark mass	$\mu_{\overline{MS}}$ = 2 GeV	4.4 MeV
6	m_s	Strange quark mass	$\mu_{\overline{MS}}$ = 2 GeV	87 MeV
7	m_c	Charm quark mass	$\mu_{\overline{MS}} = m_c$	1.32 GeV
8	m_b	Bottom quark mass	$\mu_{\overline{MS}} = m_b$	4.24 GeV
9	m_t	Top quark mass	On shell scheme	173.5 GeV
10	θ_{12}	CKM 12-mixing angle		13.1°
11	θ_{23}	CKM 23-mixing angle		2.4°
12	θ_{13}	CKM 13-mixing angle		0.2°
13	δ	CKM CP violation Phase		0.995
14	g_1 or g'	U(1) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.357
15	g_2 or g	SU(2) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.652
16	g_3 or g_s	SU(3) gauge coupling	$\mu_{\overline{MS}} = m_Z$	1.221
17	θ_{QCD}	QCD vacuum angle		~0
18	v	Higgs vacuum expectation value		246 GeV
19	m_H	Higgs mass		125.09 ±0.24 GeV

$$\theta_{12}^{\rm PMNS} = 33.82^\circ \qquad \theta_{13}^{\rm PMNS} = 8.6^\circ$$
$$\theta_{23}^{\rm PMNS} = 48.6^\circ \qquad \delta^{\rm PMNS} = 108^\circ - 404^\circ$$

Standard Model: Why?

- Some 'why' questions don't have answers (…yet?)
  - Electron mass, strength of gauge couplings, number of fermion generations, …
- Other 'why' questions have answers rooted in empirical observations and physical constraints on the theory
  - Must haves: causality (locality + Lorentz invariance), unitarity
  - Consequences include charge conservation/gauge invariance, Higgs mechanism, structure of interactions among SM fields

Plan for the Week

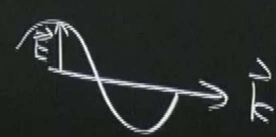
- **Monday:** massive spin-1 particles & Higgs mechanism
- **Tuesday:** the electroweak theory
- **Wednesday:** tests of the electroweak theory, connections to quantum chromodynamics
- **Thursday:** neutrinos and flavour

Lecture 1: Higgs Mechanism

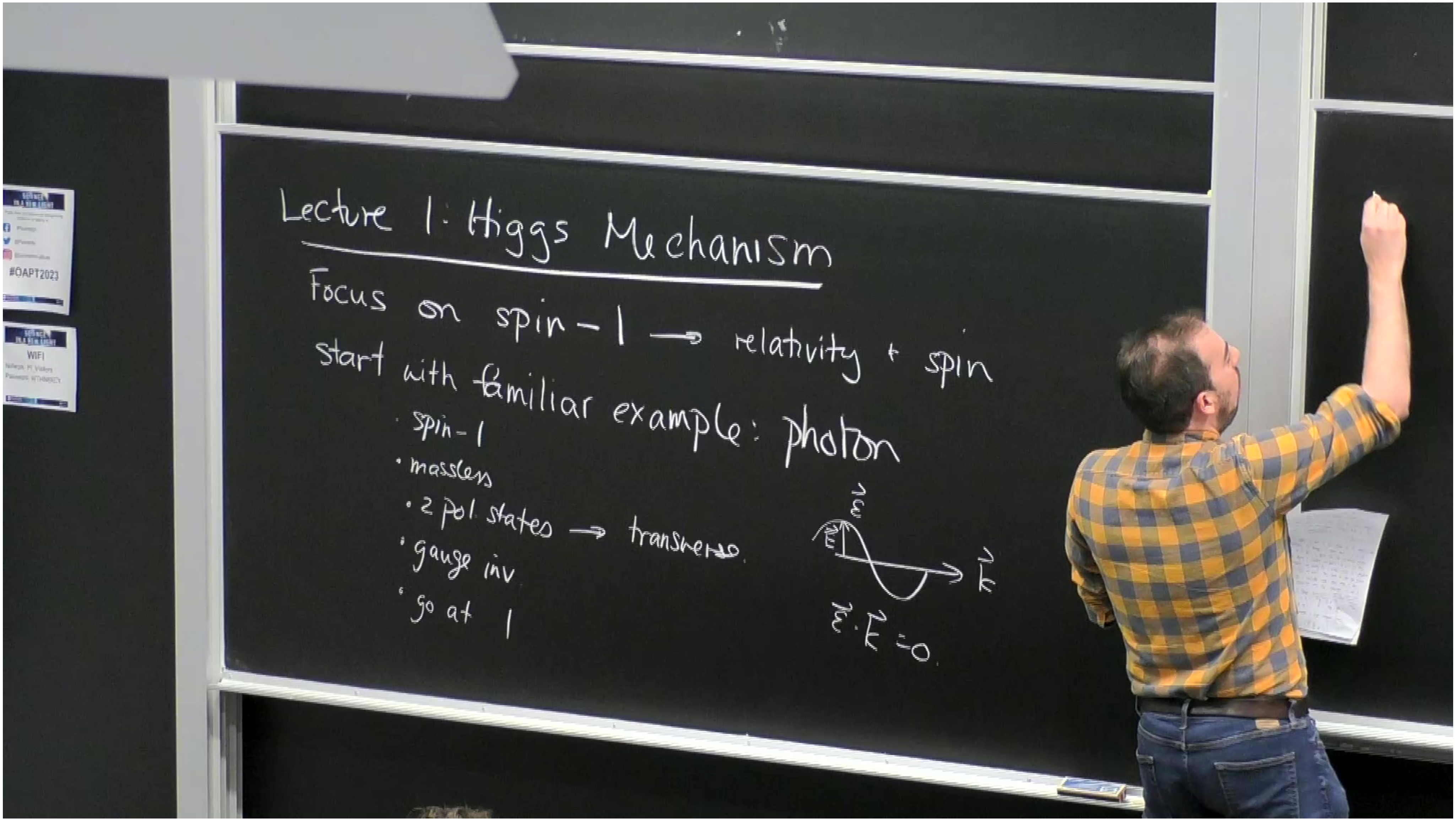
Focus on spin-1 ⟹ relativity + spin

Start with familiar example: photon

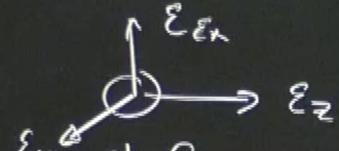
- spin-1
- massless
- 2 pol. states ⟶ transverse



- gauge inv
- go at |


Lecture 1: Higgs Mechanism
Focus on spin-1 → relativity + spin
Start with familiar example: photon
· spin-1
· massless
· 2 pol. states → transverse
· gauge inv
· go at 1
$\vec{\varepsilon}$
$\vec{k}$
$\vec{\varepsilon} \cdot \vec{k} = 0$
#OAPT2023
WIFI

Now, $m \neq 0$

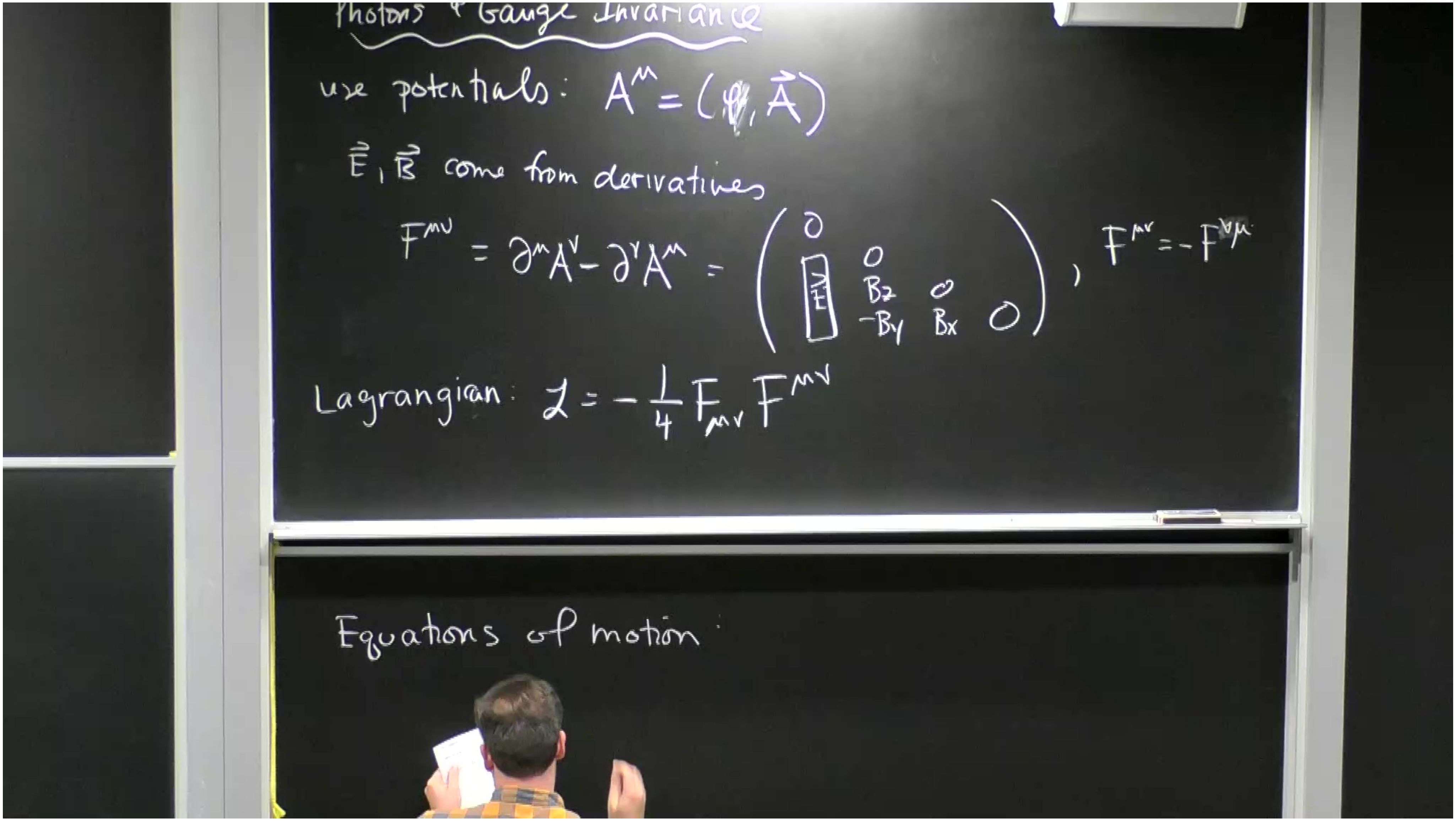


rest frame.

3 polarizations!

<u>conflict</u>: expect $v \sim c$ limit of $m \neq 0$ theory should turn into $m \to 0$ theory

If you take $m \neq 0$ spin-1 particle, $E \gg M$ limit, things go wrong!


Photons & Gauge Invariance
use potentials: $A^\mu = (\phi, \vec{A})$
$\vec{E}, \vec{B}$ come from derivatives
$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & & & \\ \boxed{\vec{E}} & 0 & & \\ & B_z & 0 & \\ & -B_y & B_x & 0 \end{pmatrix}$, $F^{\mu\nu} = -F^{\nu\mu}$
Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
Equations of motion:

Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Equations of motion:

$$\partial_\mu F^{\mu\nu} = \boxed{J^\nu} = (\rho, \vec{J})$$

wrong!

Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Equations of motion:

$$\partial_\mu F^{\mu\nu} = \boxed{J^\nu} = (\rho, \vec{J})$$

$$\boxed{\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = J^\nu}$$

$$\frac{\partial^2 A^\nu}{\partial t^2} - \nabla^2 A^\nu$$

wrong!

Gauge invariance: $A^\mu \rightarrow A^\mu + \partial^\mu \Lambda(x)$

$$F^{\mu\nu} \rightarrow \left[\partial^\mu\left(A^\nu + \cancel{\partial^\nu \Lambda}\right) - \partial^\nu\left(A^\mu + \cancel{\partial^\mu \Lambda}\right)\right]$$

$$\rightarrow F^{\mu\nu}.$$

e.o.m. is invariant ($\mathcal{L}$ is invariant) under gauge trans.

Now going to consider $J=0$ (free propagation)

Choose gauge: Coulomb gauge

$$\vec{\nabla}\cdot\vec{A} = 0$$

combine Coulomb gauge & $\vec{J}=0$.

$$\partial_\mu A^\mu = 0. \quad \text{(Lorenz gauge)}$$

eq. of motion: $\boxed{\partial_\mu \partial^\mu A^\nu = 0}$

solutions: $A^\mu = \varepsilon^\mu(k)\, e^{-ik\cdot x}$

$\xrightarrow{\text{gauge cond.}}$ $\vec{k}\cdot\vec{\varepsilon} = 0$

$\varepsilon^0 = 0$

Choose gauge: Coulomb gauge

$$\vec{\nabla}\cdot\vec{A} = 0$$

combine Coulomb gauge & $\vec{J} = 0$.

$$\partial_\mu A^\mu = 0. \qquad \text{(Lorenz gauge)}$$

eq. of motion: $\boxed{\partial_\mu \partial^\mu A^\nu = 0}$

solutions: $A^\mu = \underbrace{\varepsilon^\mu(k)}_{\text{pol. vector}} e^{-ik\cdot x}$

gauge cond.

$\vec{k}\cdot\vec{\varepsilon} = 0$

$\varepsilon^0 = 0$

For photon momentum $k^\mu = (k, 0, 0, k)$

$$\varepsilon_x = (0, 1, 0, 0)$$

$$\varepsilon_y = (0, 0, 1, 0)$$

In this theory, charge is conserved.

$$\partial_\mu F^{\mu\nu} = J^\nu.$$

$$0 = \partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu J^\nu.$$

For photon momentum $k^\mu = (k, 0, 0, k)$

$$\varepsilon_x = (0, 1, 0, 0)$$

$$\varepsilon_y = (0, 0, 1, 0)$$

In this theory, charge is conserved.

$$\partial_\mu F^{\mu\nu} = J^\nu.$$

$$0 = \partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu J^\nu.$$

$$\partial_\mu A^\mu = \partial_0 A^0 - \vec{\nabla}\cdot\vec{A}$$

☆ gauge inv. $\longleftrightarrow$ 2 pols

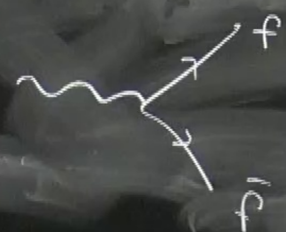
$\longleftrightarrow$ current cons $\longleftrightarrow$ masslessness

$\cdot\vec{A}$

So far all classical.

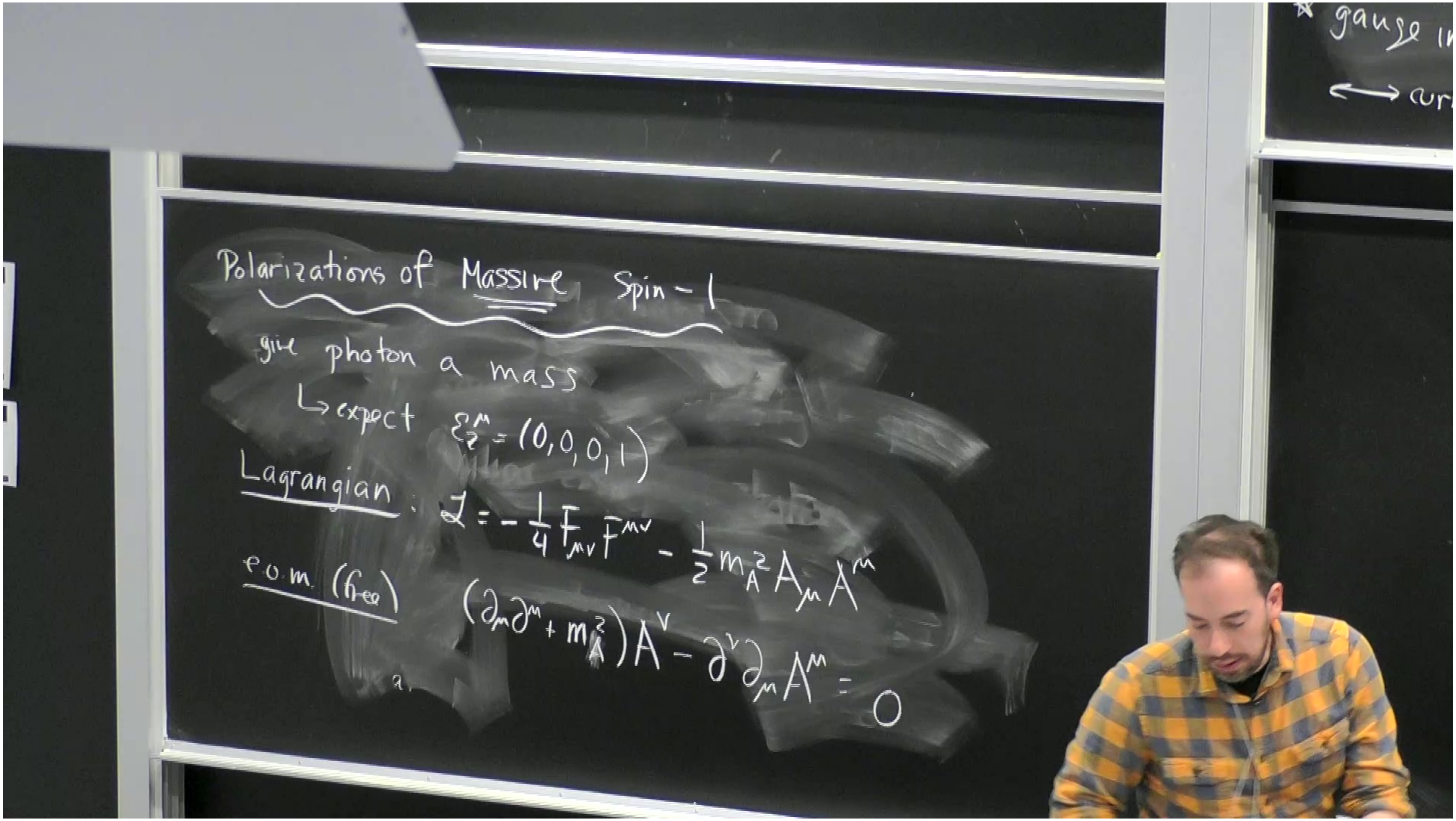
QFT: $A^\mu(x) = \sum_{s,\vec{k}} \Big[\varepsilon_s(\vec{k})\, a_s(\vec{k})\, e^{-ikx} + \varepsilon_s^*(\vec{k})\, a_s^\dagger(\vec{k})\, e^{ikx} \Big]$

ε show up in quantum amplitudes:



$$M \sim \varepsilon(p_\gamma)_\mu\, \bar{u}(p_f)\, \Gamma^\mu\, v(p_{\bar{f}})$$

$$\sim \varepsilon(p_\gamma)_\mu\, M^\mu .$$


gauge in
⟷ cur
Polarizations of Massive Spin-1
give photon a mass
↳ expect $\varepsilon_z^\mu = (0,0,0,1)$
Lagrangian: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu$
e.o.m. (free) $(\partial_\mu\partial^\mu + m_A^2)A^\nu - \partial^\nu\partial_\mu A^\mu = 0$

$\longleftrightarrow$ current cons $\longleftrightarrow$ masslessness

Take ∂_ν of both sides:

$$(\cancel{\partial_\mu \partial^\mu} + m_A^2)\,\partial_\nu A^\nu - \cancel{\partial_\nu \partial^\nu \partial_\mu A^\mu} = 0$$

$$\boxed{\partial_\nu A^\nu = 0}$$

sub into e.o.m: $(\partial_\mu \partial^\mu + m_A^2) A^\nu = 0$

$A^\mu = \varepsilon^\mu e^{-ikx}$ $\rightarrow$ no longer true that $\varepsilon^0 = 0$

$\longleftrightarrow$ current cons $\longleftrightarrow$ masslessness)

Take ∂_ν of both sides:

$$(\cancel{\partial_\mu\partial^\mu} + m_A^2)\partial_\nu A^\nu - \cancel{\partial_\nu\partial^\nu\partial_\mu A^\mu} = 0$$

$$\boxed{\partial_\nu A^\nu = 0}$$

sub into e.o.m: $(\partial_\mu\partial^\mu + m_A^2)A^\nu = 0$

$A^\mu = \varepsilon^\mu e^{-ikx}$ $\rightarrow$ no longer true that $\varepsilon^0 = 0$, but 3 pols.

start in photon rest frame: $k^\mu = (M, 0, 0, 0)$

$$\varepsilon_L^\mu = (0, 0, 0, 1)$$

longitudinal pols.

Now, do Lorentz trans. along $\hat{z}$: $\vec{k}' = k\hat{z}$

$$\varepsilon_L'^\mu = \frac{1}{M_A}(|\vec{k}|, 0, 0, E) \qquad \frac{E}{M_A} = \gamma$$

start in photon rest frame: $k^\mu = (M, 0, 0, 0)$

$$\varepsilon_L^\mu = (0, 0, 0, 1)$$

longitudinal pols.

Now, do Lorentz trans. along $\hat{z}$: $\vec{k}' = k\hat{z}$

$$\varepsilon_L'^\mu = \frac{1}{M_A}\left(|\vec{k}|, 0, 0, E\right) \qquad \frac{E}{M_A} = \gamma$$

Amplitudes $\sim E/M_A \to \infty$

start in photon rest frame: $k^\mu = (M, 0, 0, 0)$

$$\varepsilon_L^\mu = (0, 0, 0, 1)$$

longitudinal pols.

Now, do Lorentz trans. along $\hat{z}$: $\vec{k}' = k\hat{z}$

$$\varepsilon_L'^\mu = \frac{1}{M_A}(|\vec{k}|, 0, 0, E) \qquad \frac{E}{M_A} = \gamma.$$

Amplitudes $\sim E/M_A \rightarrow \infty$ ☹ violate unitarity in $E \gg M_A$ limit!

☆ **cannot** have arbitrarily coupled long. polarizations! in high-energy limit

can we fix this? can I devise $\epsilon_\mu M^{\mu\ldots}$ so that longitudinal polarizations decouple?

$\longleftrightarrow$ cur

Take

Sub in

A^μ

* cannot have arbitrarily coupled long. polarizations! in high-energy limit

can we fix this? can I devise $\varepsilon_\mu M^\mu$ so that longitudinal polarizations decouple?

note that $E \gg M_A$. $\varepsilon^\mu_L \approx \underbrace{\frac{(E,0,0,|\vec{k}|)}{M_A}}_{\frac{k^\mu}{M_A}} - \frac{1}{2}\left(\frac{M_A}{|\vec{k}|},0,0,-\frac{M_A}{|\vec{k}|}\right) + O(M_A^2)$

Take ∂_ν of

$(\partial_\mu \partial$

sub into e.o

$A^\mu = \varepsilon^\mu e$

If theory were invariant under

$$\varepsilon_L^\mu \to \varepsilon_L^\mu - \frac{k^\mu}{m_A}, \text{ then OK}$$

This is exactly what gauge invariance does!

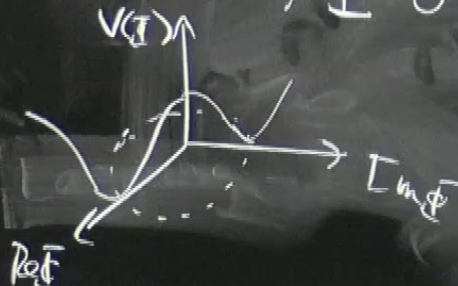
$$\Lambda = -\frac{i}{M_A} e^{-ikx}, \quad A^\mu \to A^\mu + \partial^\mu \Lambda$$

The Higgs Mechanism: Abelian Theory

Add Higgs field $\Phi(x)$ (2 d.o.f)

For starters, let's consider turning off gauge interactions

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi), \quad \text{invariant under } \Phi \to e^{i\alpha}\Phi$$



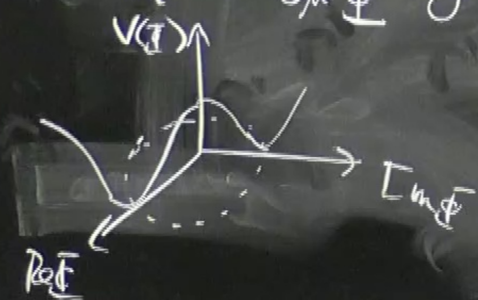
$$V(\Phi) = -\mu^2 |\Phi|^2 + \frac{\lambda}{4} |\Phi|^4$$

The Higgs Mechanism: Abelian Theory

Add Higgs field $\Phi(x)$ (2 d.o.f)

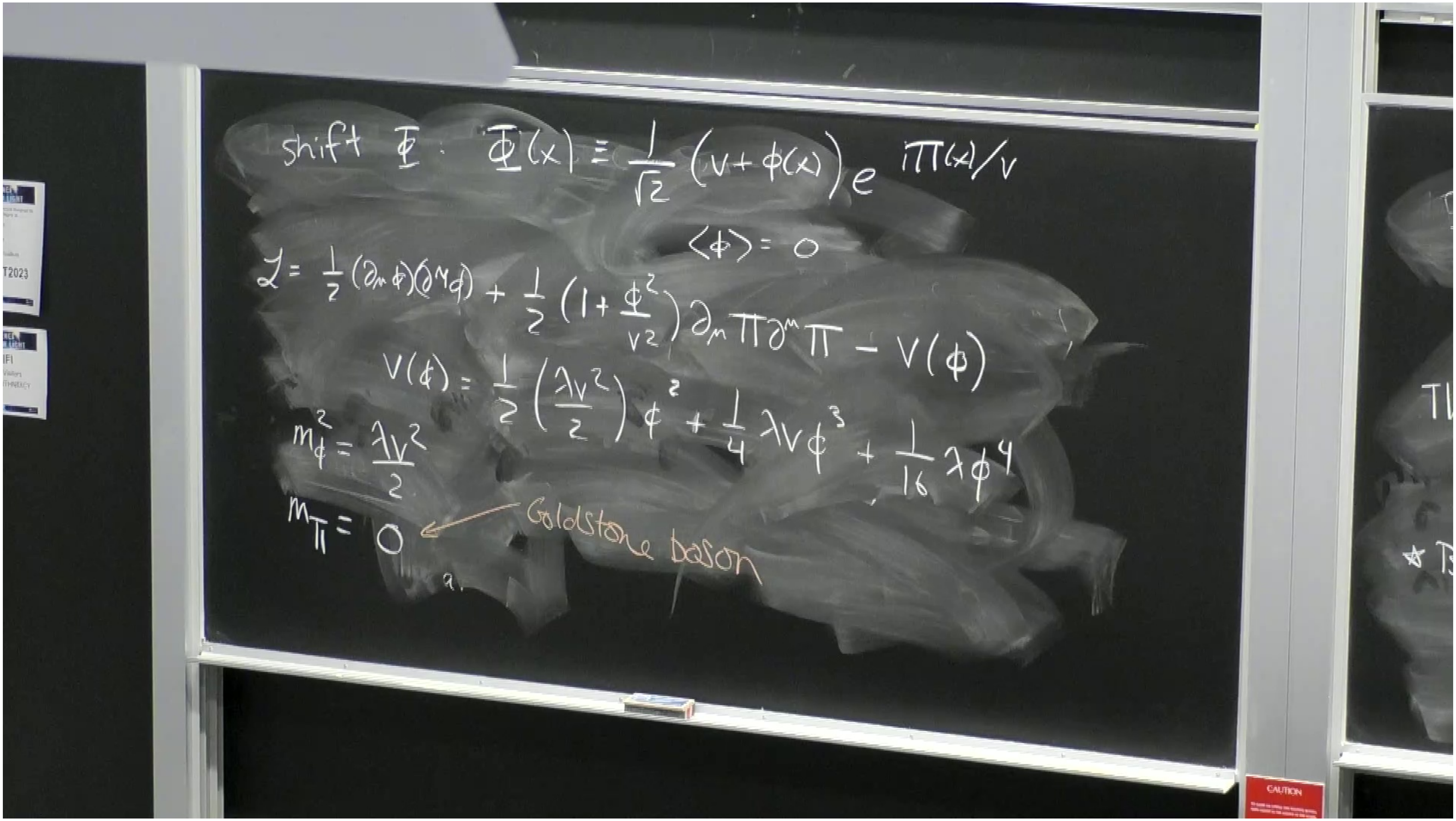
For starters, let's consider turning off gauge interactions

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi), \quad \text{invariant under } \Phi \to e^{i\alpha}\Phi$$



$$V(\Phi) = -\mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4$$

$$\rightarrow \text{min. } \langle\Phi\rangle = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$


shift Φ: $\Phi(x) \equiv \frac{1}{\sqrt{2}}(v+\phi(x))e^{i\pi(x)/v}$
$\langle\phi\rangle = 0$
$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \frac{1}{2}\left(1+\frac{\phi^2}{v^2}\right)\partial_\mu\pi\partial^\mu\pi - V(\phi)$
$V(\phi) = \frac{1}{2}\left(\frac{\lambda v^2}{2}\right)\phi^2 + \frac{1}{4}\lambda v\phi^3 + \frac{1}{16}\lambda\phi^4$
$m_\phi^2 = \frac{\lambda v^2}{2}$
$m_\pi = 0$
Goldstone boson

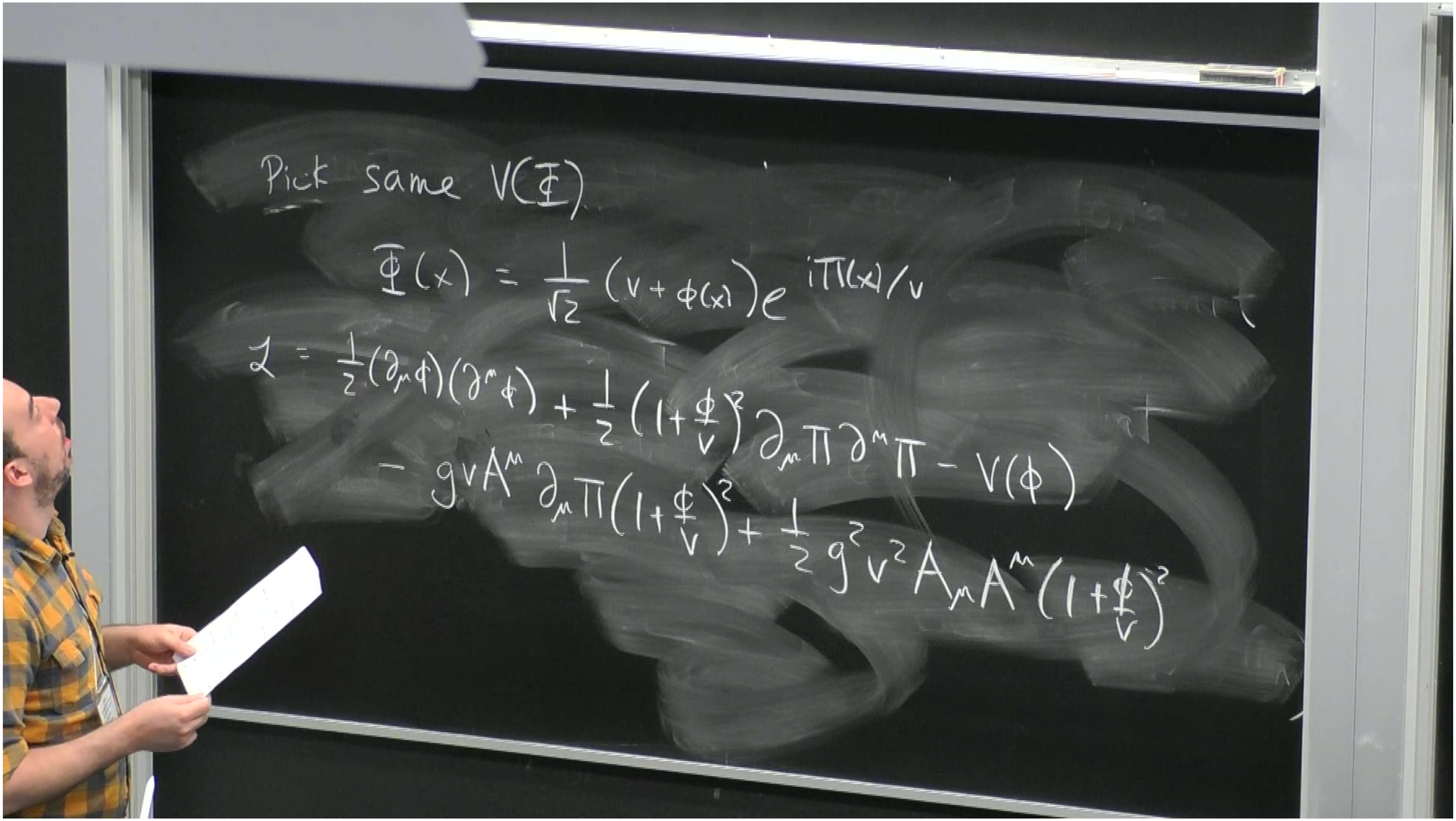
$$D_\mu \Phi = (\partial_\mu - igA_\mu)\Phi$$

$$\xrightarrow{g.t.} (\partial_\mu - igA_\mu - ig\partial_\mu\Lambda)\left[e^{ig\Lambda(x)}\Phi(x)\right]$$

$$= e^{ig\Lambda(x)}(\partial_\mu - igA_\mu)\Phi$$

$$(D_\mu\Phi)(D_\mu\Phi)^* \xrightarrow{g.t.} (D_\mu\Phi)(D^\mu\Phi)^* e^{ig\Lambda}e^{-ig\Lambda}$$

A_μ

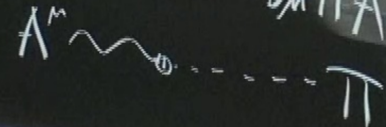

Pick same $V(\Phi)$
$\Phi(x) = \frac{1}{\sqrt{2}}(v + \phi(x))e^{i\pi(x)/v}$
$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\left(1+\frac{\phi}{v}\right)^2 \partial_\mu \pi \partial^\mu \pi - V(\phi)$
$- g v A^\mu \partial_\mu \pi \left(1+\frac{\phi}{v}\right)^2 + \frac{1}{2} g^2 v^2 A_\mu A^\mu \left(1+\frac{\phi}{v}\right)^2$

Pick same $V(\Phi)$

$$\Phi(x) = \frac{1}{\sqrt{2}}\left(v + \phi(x)\right) e^{i\pi(x)/v}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\left(1+\frac{\phi}{v}\right)^2 \partial_\mu \pi \, \partial^\mu \pi - V(\phi)$$

$$- \underbrace{g v A^\mu \partial_\mu \pi}_{\text{constant} \cdot \partial_\mu \pi A^\mu} \left(1+\frac{\phi}{v}\right)^2 + \frac{1}{2} g^2 v^2 A_\mu A^\mu \left(1+\frac{\phi}{v}\right)^2$$

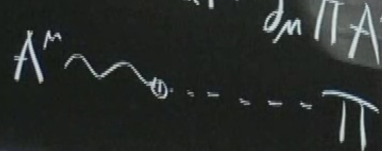


Pick same $V(\Phi)$

$$\Phi(x) = \frac{1}{\sqrt{2}}\left(v + \phi(x)\right) e^{i\pi(x)/v}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\left(1+\frac{\phi}{v}\right)^2 \partial_\mu \pi\, \partial^\mu \pi - V(\phi)$$

$$- \underbrace{g v A^\mu \partial_\mu \pi}_{\text{constant}\cdot\, \partial_\mu \pi A^\mu} \left(1+\frac{\phi}{v}\right)^2 + \underbrace{\frac{1}{2} g^2 v^2 A_\mu A^\mu}_{\text{constant } A_\mu A^\mu} \left(1+\frac{\phi}{v}\right)^2$$



mass for photon. $\boxed{m_A^2 = g^2 v^2}$