

Title: Data-centric learning of Quantum Many-body States with Classical Machines

Speakers: Eun-Ah Kim

Collection: Machine Learning for Quantum Many-Body Systems

Date: June 13, 2023 - 1:30 PM

URL: <https://pirsa.org/23060049>

# DATA-CENTRIC LEARNING OF QUANTUM MANY-BODY STATES WITH CLASSICAL MACHINES

EUN-AH KIM

CORNELL UNIVERSITY, HARVARD RADCLIFFE INSTITUTE, EWHA WOMANS UNIVERSITY

PERIMETER, 6/13/2023





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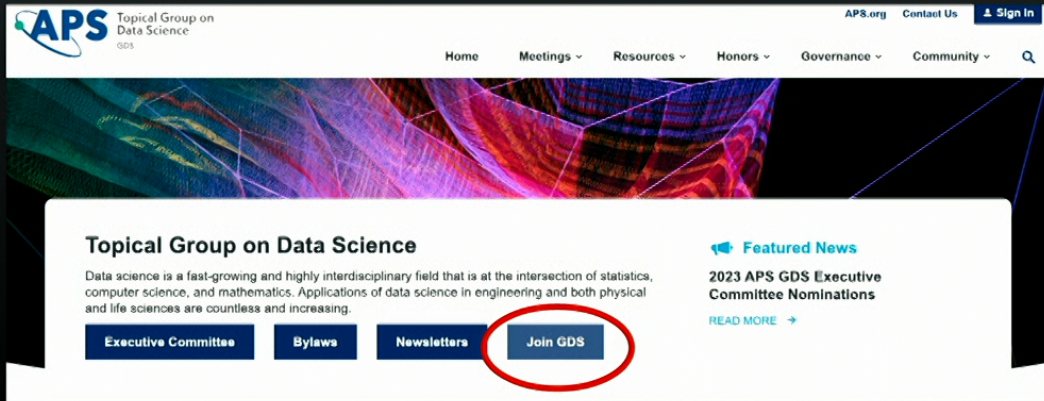
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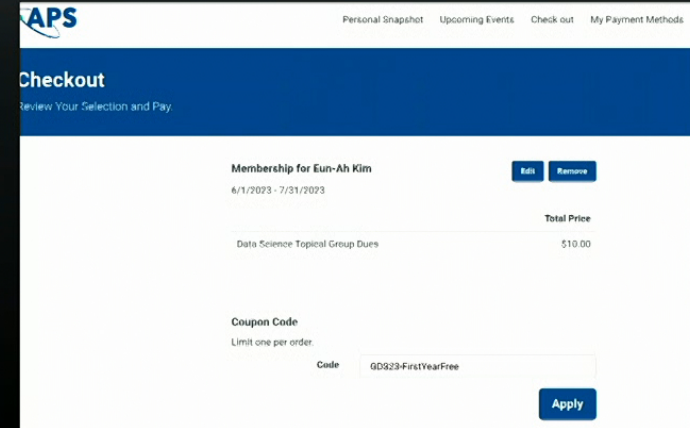
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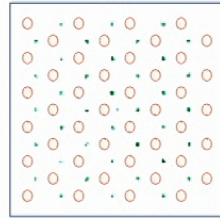
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# Quantum Simulators

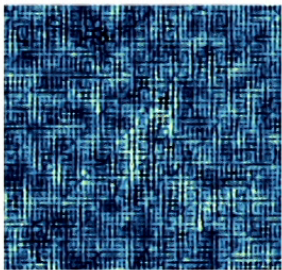
Analog



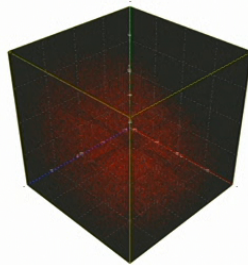
Digital



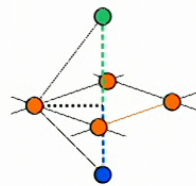
## Complex Data



Scanning Probe

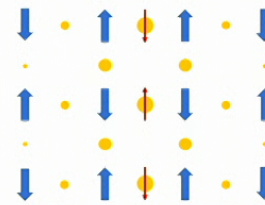


Bulk Probe

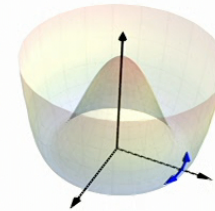


Materials Search

## Theoretical Insight



Order



Fluctuations



Entanglement

# X-ray diffraction in 1913

## *The Reflection of X-rays by Crystals.*

By W. H. BRAGG, M.A., F.R.S., Cavendish Professor of Physics in the University of Leeds; and W. L. BRAGG, B.A., Trinity College, Cambridge.

(Received April 7,—Read April 17, 1913.)

Proceedings of Royal Society A, 01, July 1913

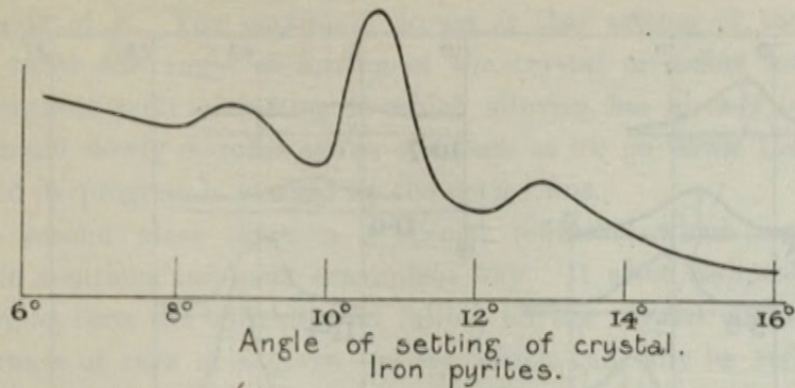


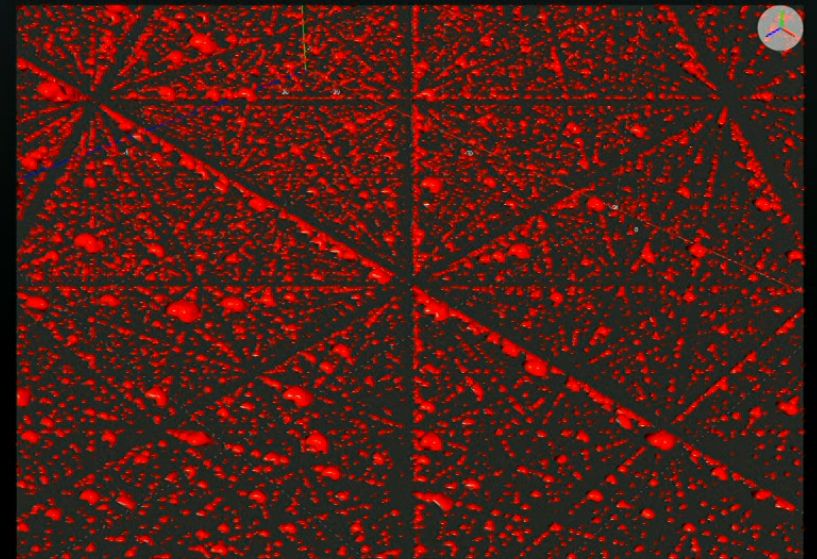
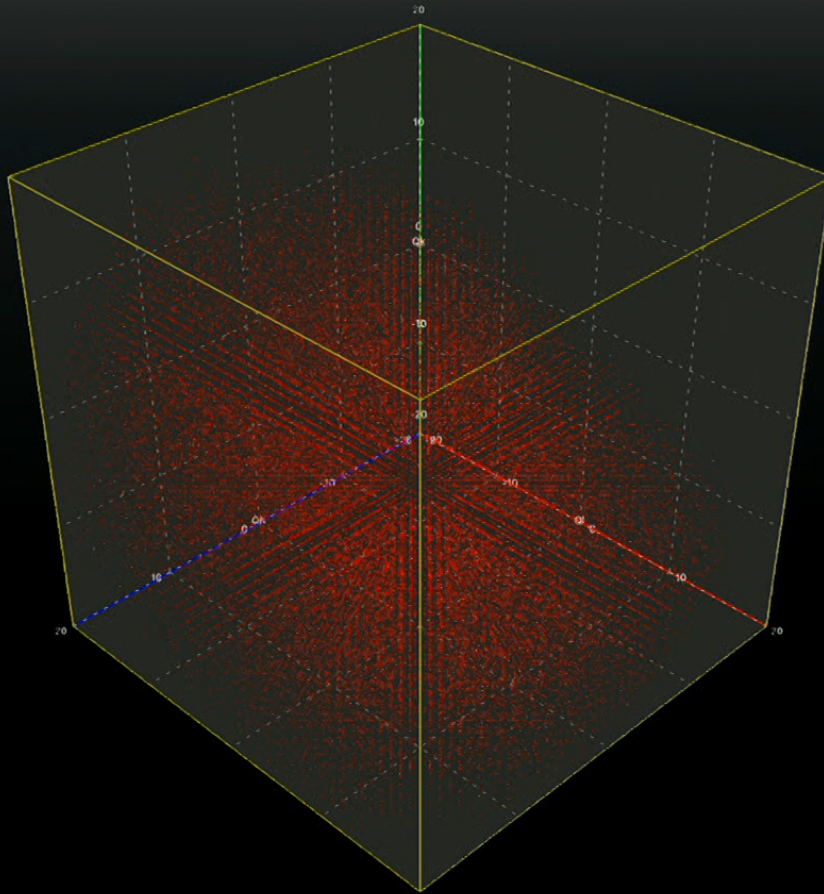
FIG. 2.—Reflection from face (100) of iron pyrites, at varying angles of incidence. Abscissa—Angle of incidence of rays on crystal face; Ordinate—Strength of reflected beam, arbitrary scale.



$$n\lambda = 2d \sin \theta$$

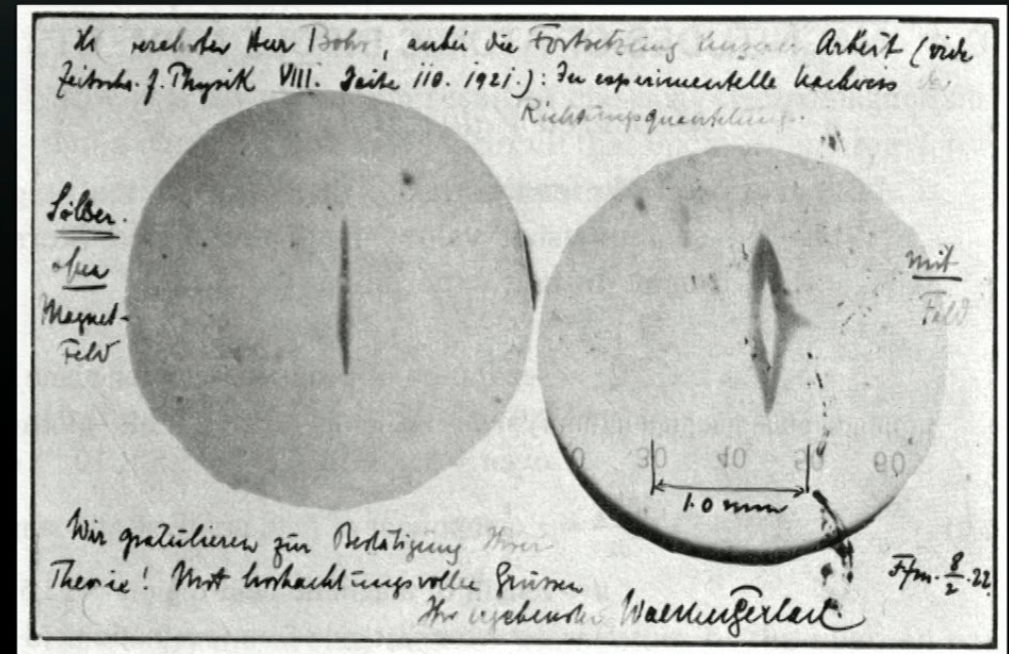
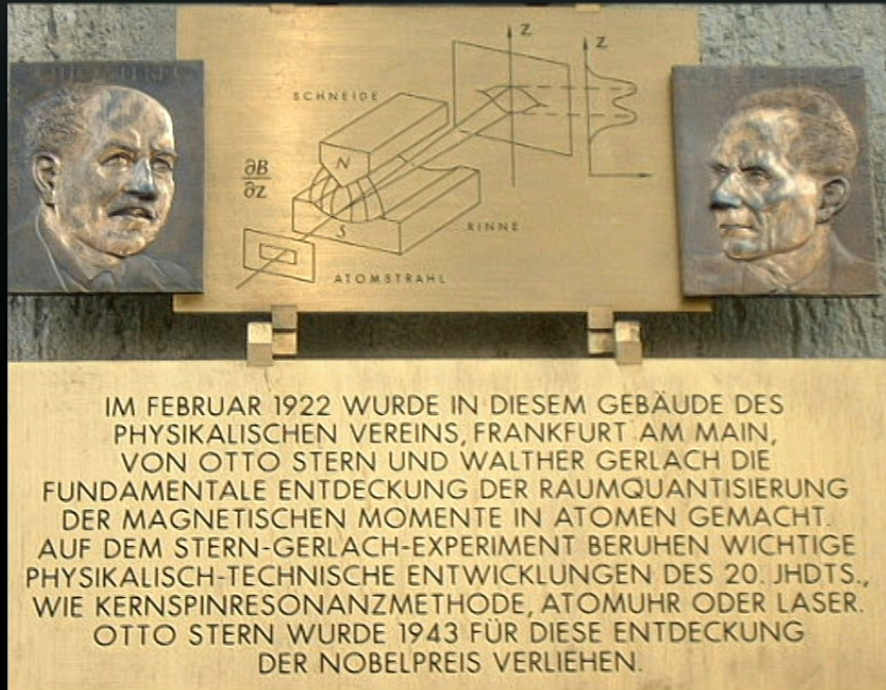


# X-ray diffraction Today





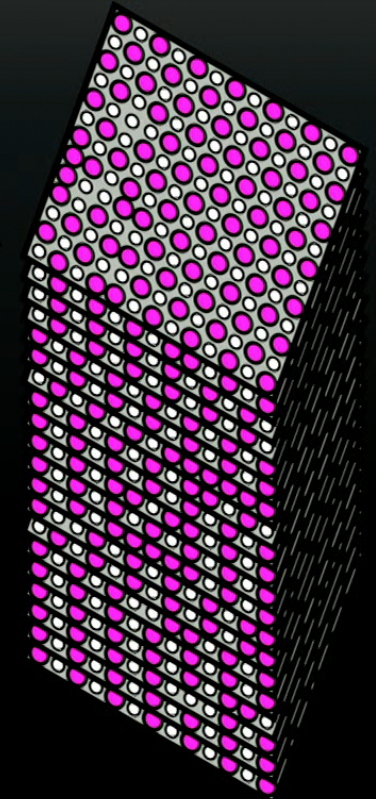
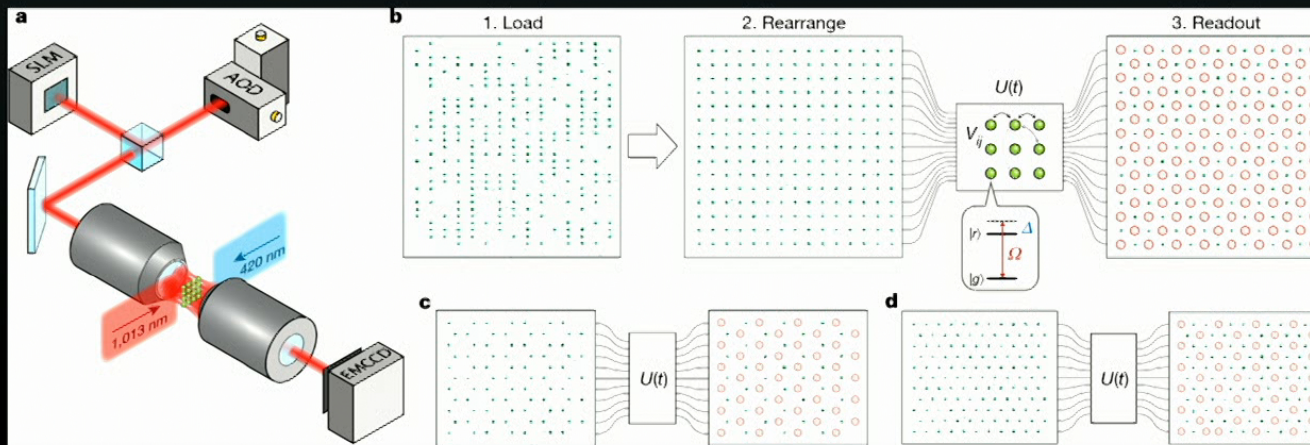
# Projective Measurements in 1922





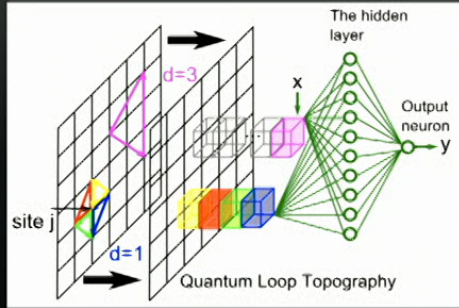
# Projective Measurements Today

## Quantum Simulators

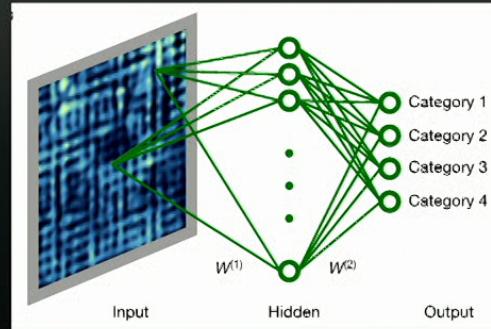


Taken from Ebadi. et al., *Nature* **595** (227), 2021

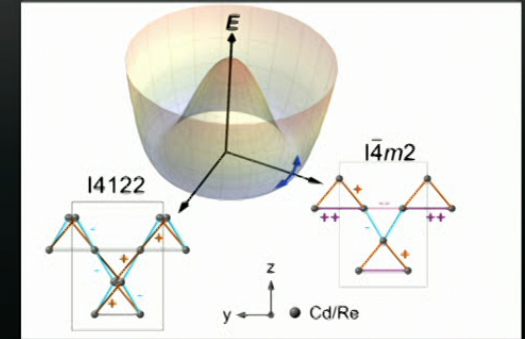
# Learning Quantum many-body States



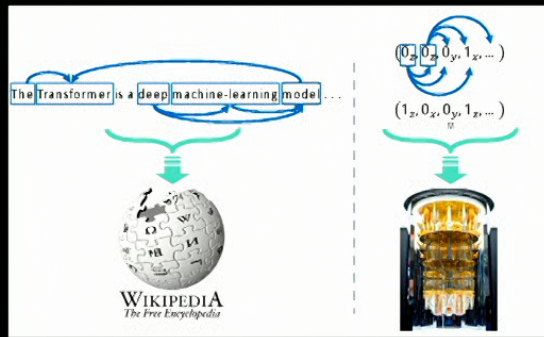
Quantum loop topography  
Zhang & Kim, PRL 118, 216401 (2017)



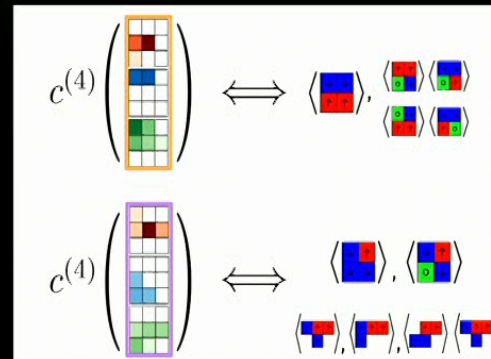
STS on high  $T_c$  cuprate BSSCO  
Testing Charge Order Hypothesis  
Zhang et al, Nature 570, 484 (2019)



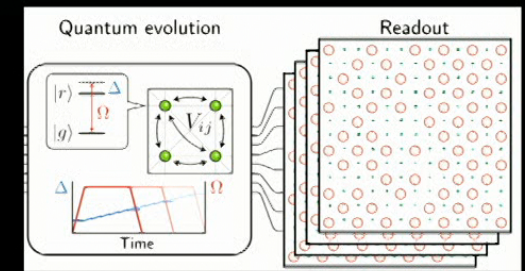
X-ray-Temperature Clustering  
Venderley et al, PNAS (2022)



Attention-based Quantum Tomography  
Cha et al, MLST (2021)



QGM on Fermi Hubbard model  
Miles et al, Nat. Comm. 12, 3905 (2021)



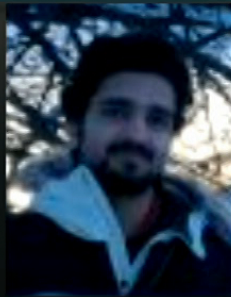
Quantum Simulator Snapshots  
Miles et al, PRR 5, 013026 (2023)



# X-TEC (XRD Temperature Clustering)



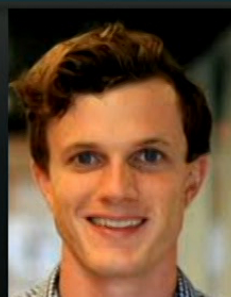
J. Venderley



K. Mallayya



M. Matty



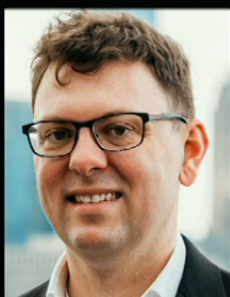
G. Pleiss  
(Cornell, CS)



M. Krogstad  
(ANL)



K. Winberger  
(Cornell, CS)



A. Wilson  
(NYU)



J. Ruff  
(CHESS)



M. Norman  
(ANL)



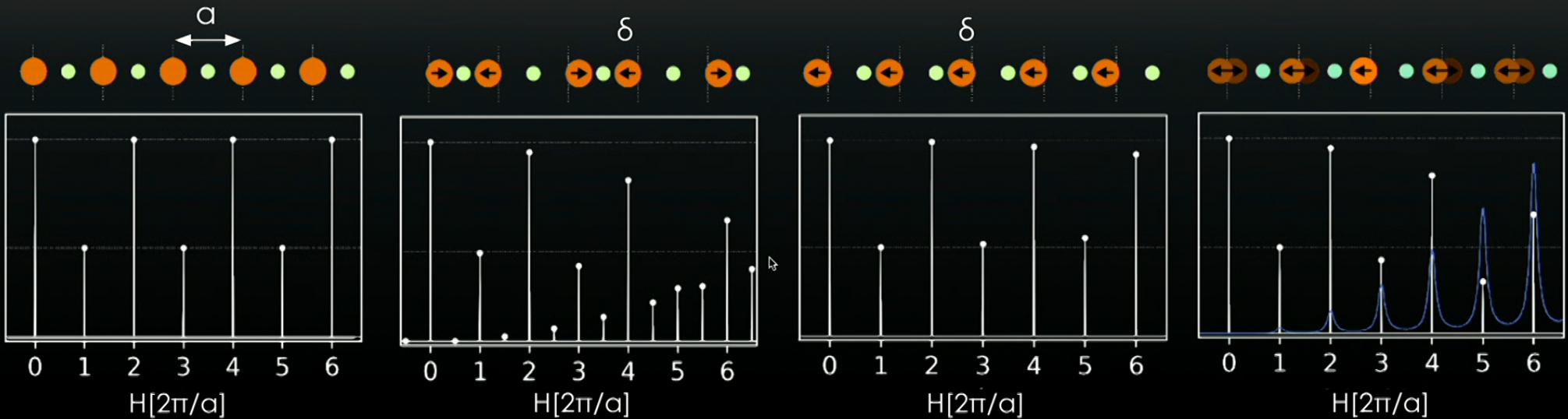
R. Osborn  
(ANL)



S. Rosenkranz  
(ANL)

J. Venderley, EAK et al, PNAS 119, e2109665119 (2022)

# Orders and fluctuations as seen in XRD



Fully Symmetric  
(undistorted)

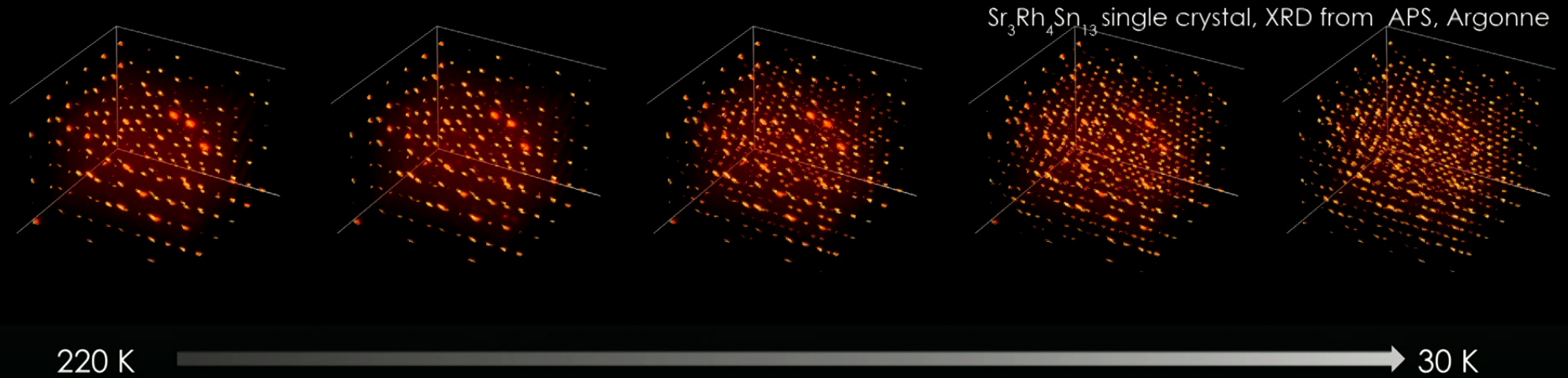
CDW with period  $2a$ :  
superlattice peaks

IUC order:  
Form factors of Bragg  
peaks.

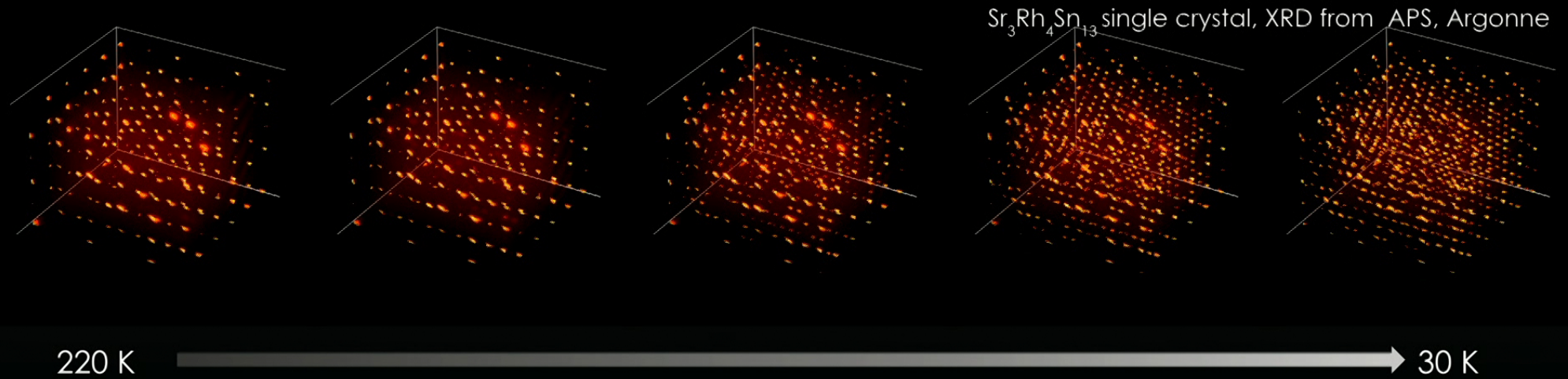
Fluctuations



# Temperature evolution of XRD



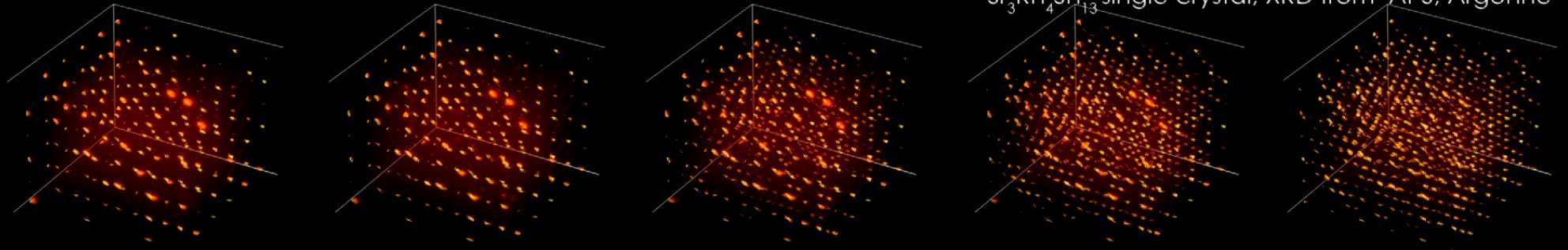
# Temperature evolution of XRD



**X-ray Temperature Clustering  
(X-TEC):**

# Temperature evolution of XRD

$\text{Sr}_3\text{Rh}_4\text{Sn}_{13}$  single crystal, XRD from APS, Argonne

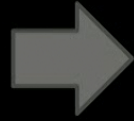


220 K

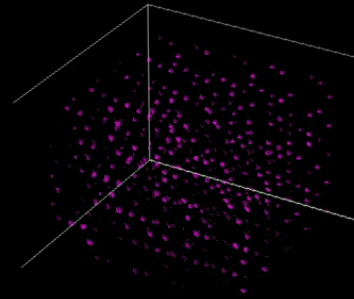
30 K



**X-ray Temperature Clustering (X-TEC):**

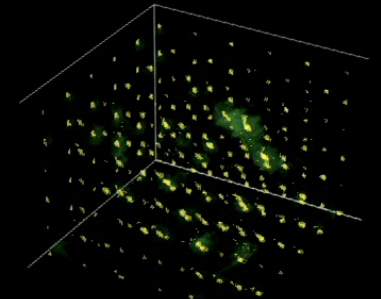


**Charge Density Wave**



+

**Bragg and diffuse**





# Learn the sorting criteria for emergence?

---

$$F = E - TS$$

# Learn the sorting criteria for emergence?

---

$$F = E - TS$$

- Temperature series ( $T_d$  - points) for each  $\vec{q}_i$



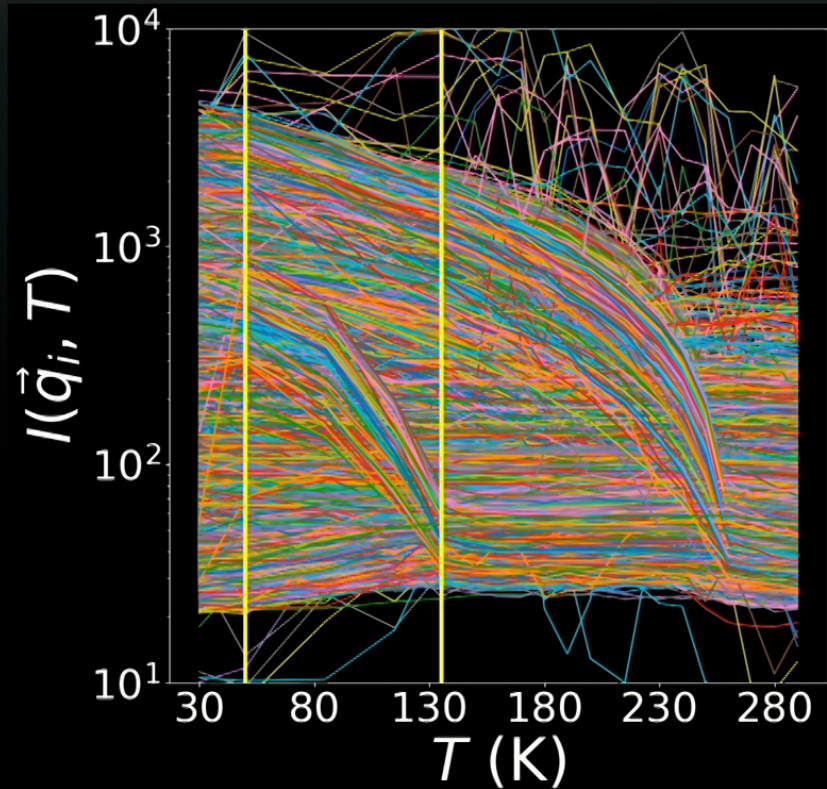
# Learn the sorting criteria for emergence?

---

$$F = E - TS$$

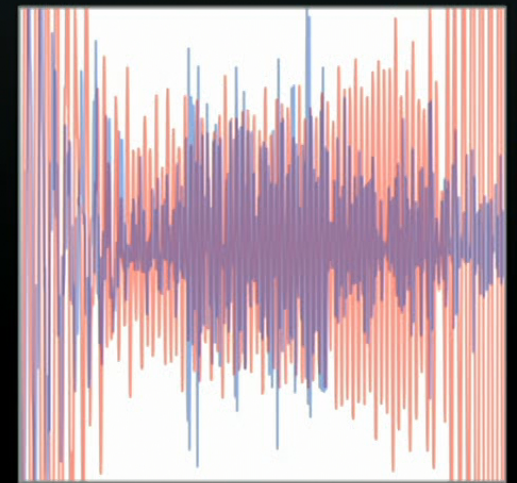
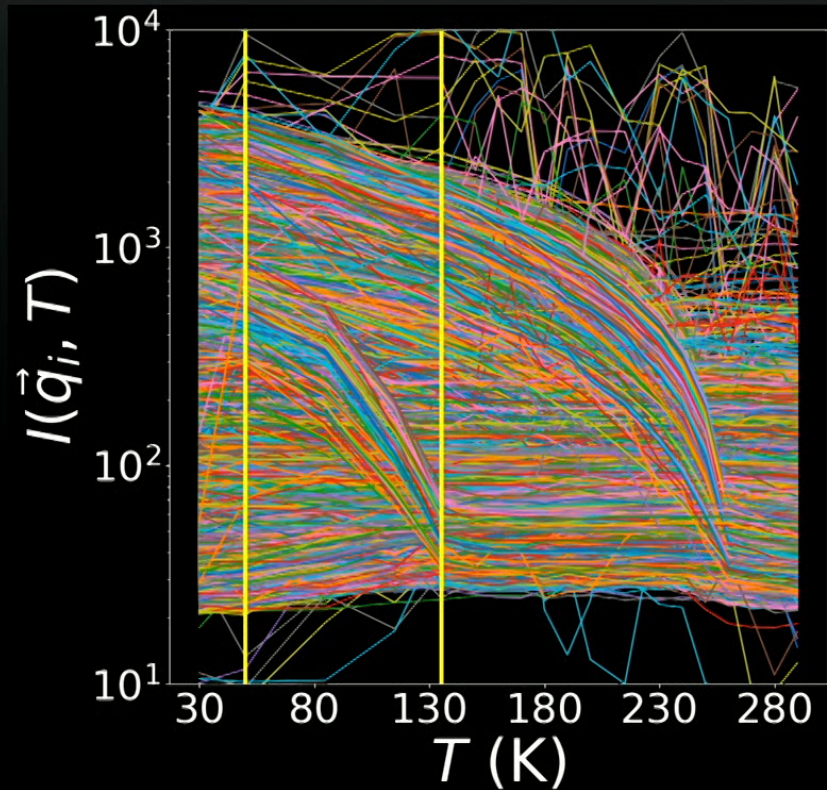
- Temperature series ( $T_d$  - points) for each  $\vec{q}_i$
- A point in  $T_d$  -dim space
- Expect different clusters of T-series among the population of  $\vec{q}_i$

# Raw T-Series and Speaker Verification

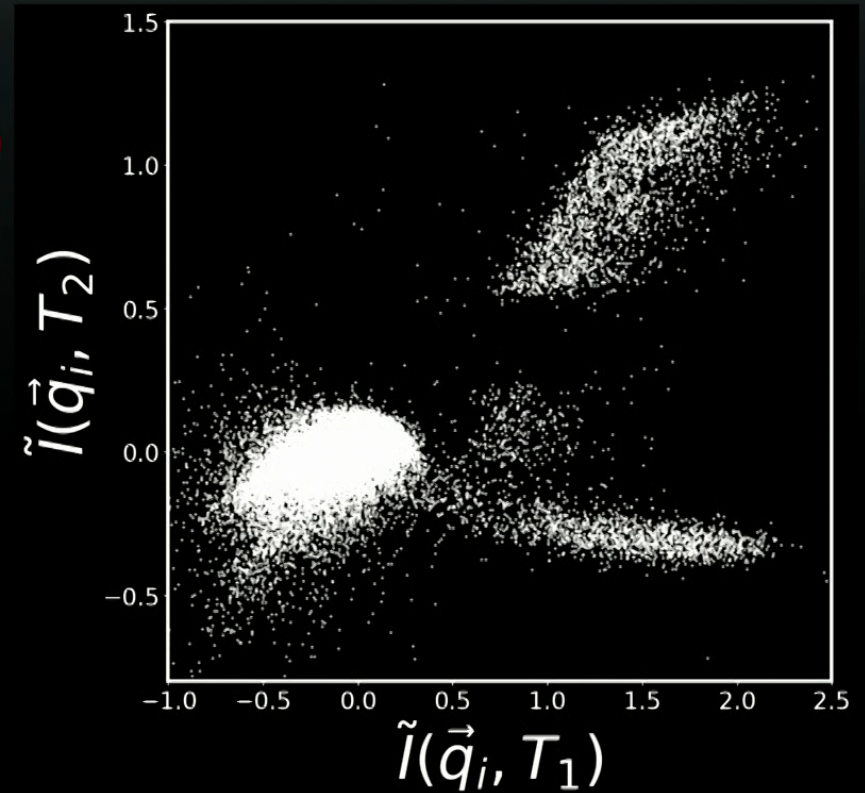
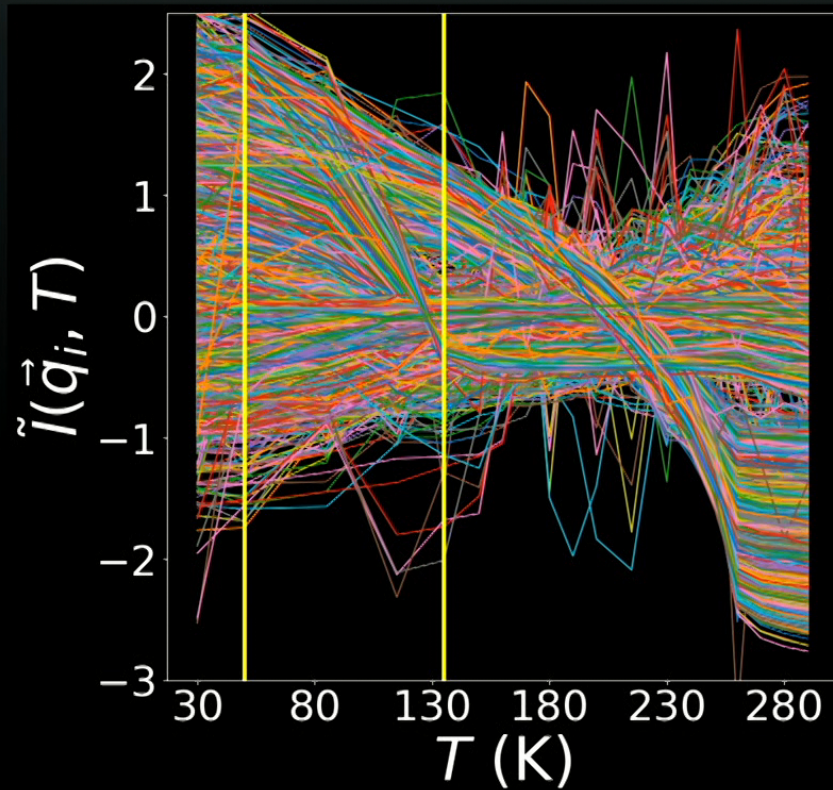




# Raw T-Series and Speaker Verification



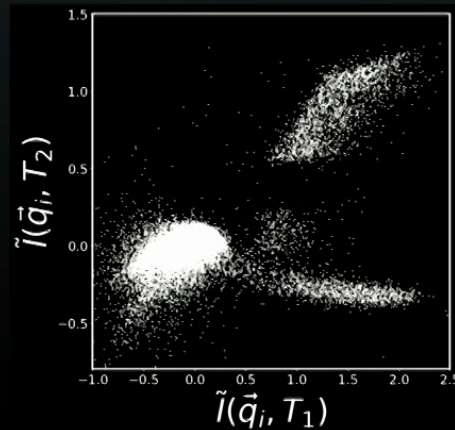
# T-series as a point in high-dim space





# Clustering based on Probabilistic Modeling

d-temperature series  $\leftrightarrow$  A point in d-dimension



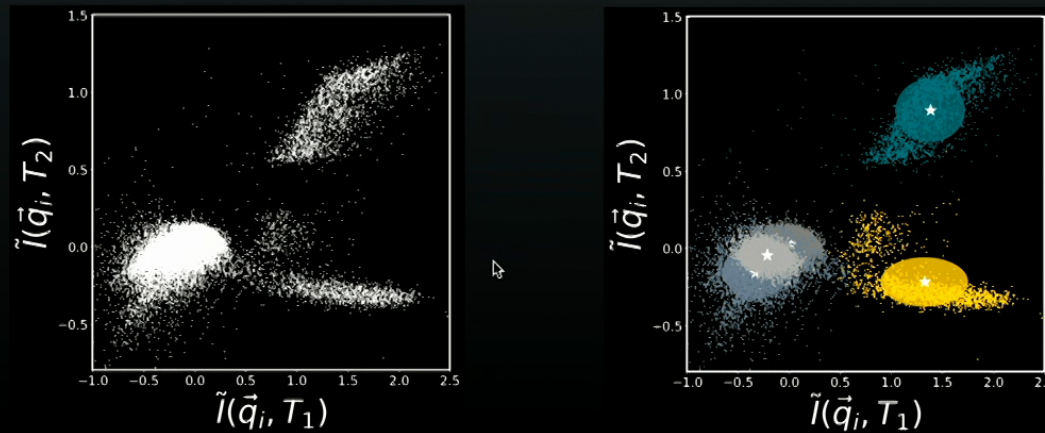
- Gaussian Mixture Model

$$p(I) = \sum_{k=1}^K \pi_k \mathcal{N}(I | m_k, s_k)$$



# Clustering based on Probabilistic Modeling

d-temperature series  $\leftrightarrow$  A point in d-dimension

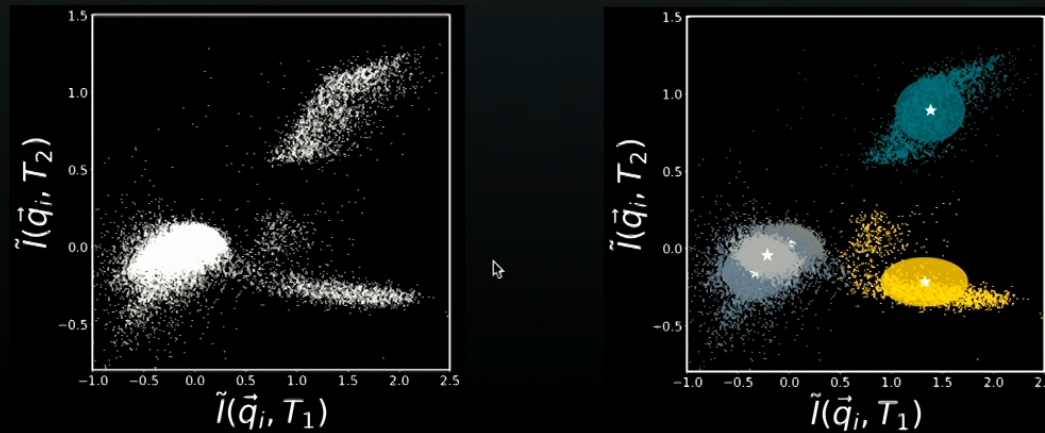


- Gaussian Mixture Model

$$p(I) = \sum_{k=1}^K \pi_k \mathcal{N}(I | m_k, s_k)$$

# Clustering based on Probabilistic Modeling

d-temperature series  $\leftrightarrow$  A point in d-dimension



- Gaussian Mixture Model

$$p(I) = \sum_{k=1}^K \pi_k \mathcal{N}(I | m_k, s_k)$$

# Learning the model and clustering

- Find the maximum likelihood solution for  $\{I\} = \{I(\vec{q}_i)\}$

$$\log p(\{\tilde{\mathbf{I}}(\vec{q}_i)\}|\pi, \mathbf{m}, \mathbf{s}) = \sum_{\vec{q}_i} \log \left[ \sum_{k=1}^K \pi_k \mathcal{N}(\tilde{\mathbf{I}}(\vec{q}_i)|\mathbf{m}_k, \mathbf{s}_k) \right]$$

- Cluster assignment for each data point  $\vec{q}_i$

$$w_i^k = \frac{\pi_k \mathcal{N}(\tilde{\mathbf{I}}(\vec{q}_i)|\mathbf{m}_k, \mathbf{s}_k)}{\sum_k \pi_k \mathcal{N}(\tilde{\mathbf{I}}(\vec{q}_i)|\mathbf{m}_k, \mathbf{s}_k)}$$



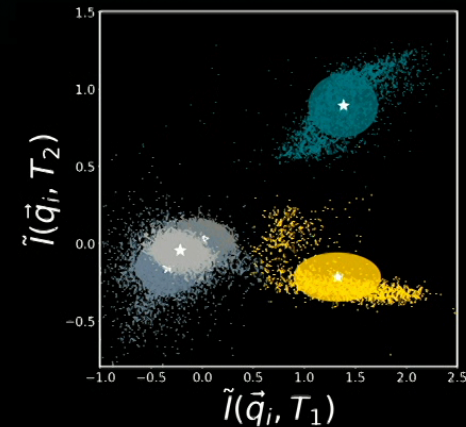
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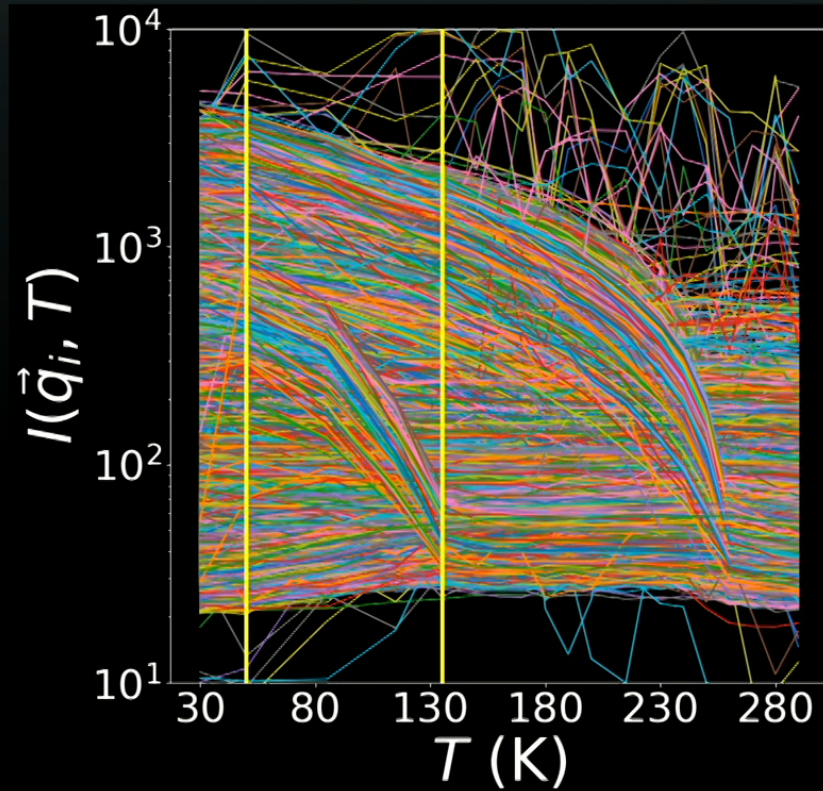
- Cluster assignment for each data point  $\vec{q}_i$

$$w_i^k = \frac{\pi_k \mathcal{N}(\tilde{\mathbf{I}}(\vec{q}_i)|\mathbf{m}_k, \mathbf{s}_k)}{\sum_k \pi_k \mathcal{N}(\tilde{\mathbf{I}}(\vec{q}_i)|\mathbf{m}_k, \mathbf{s}_k)}$$

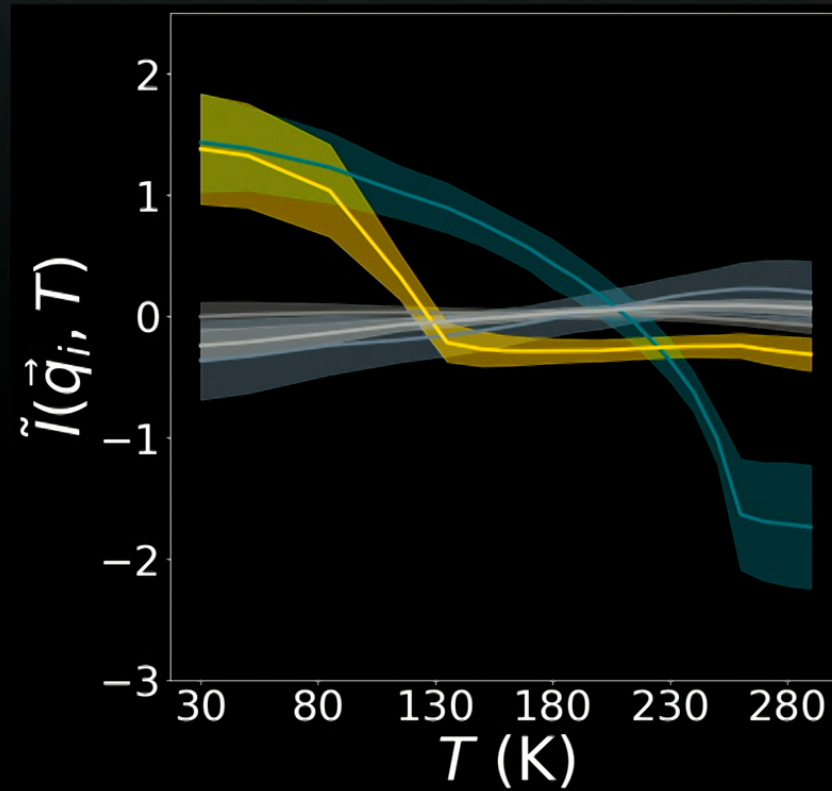




# Discovery of order parameters

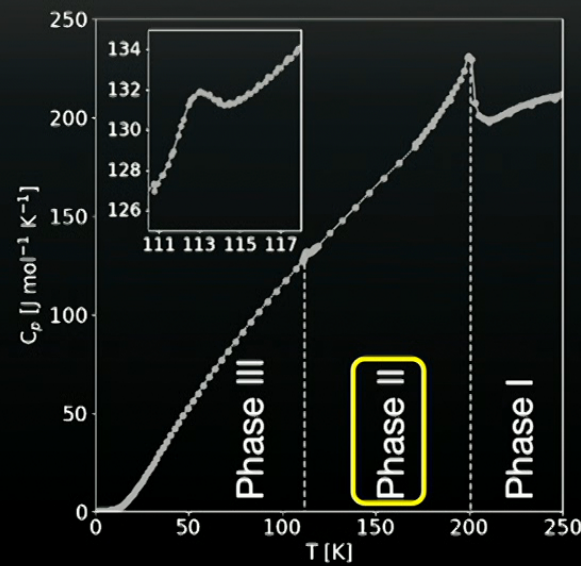
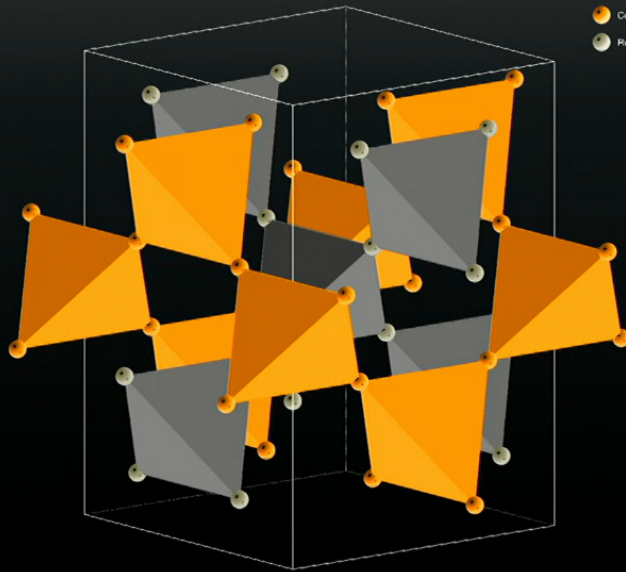


# Discovery of order parameters



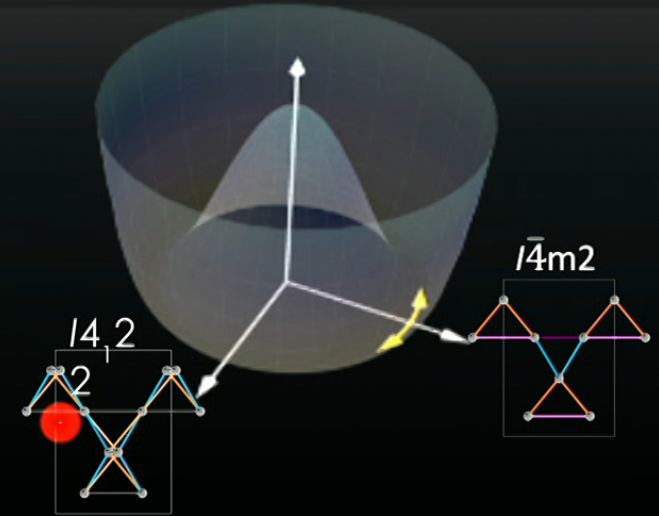
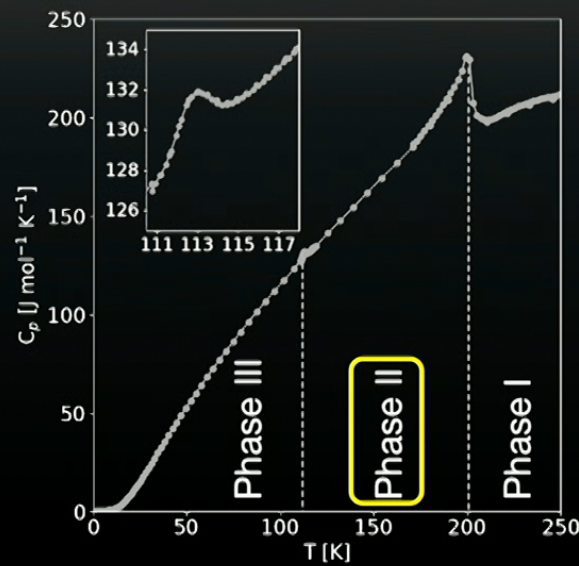
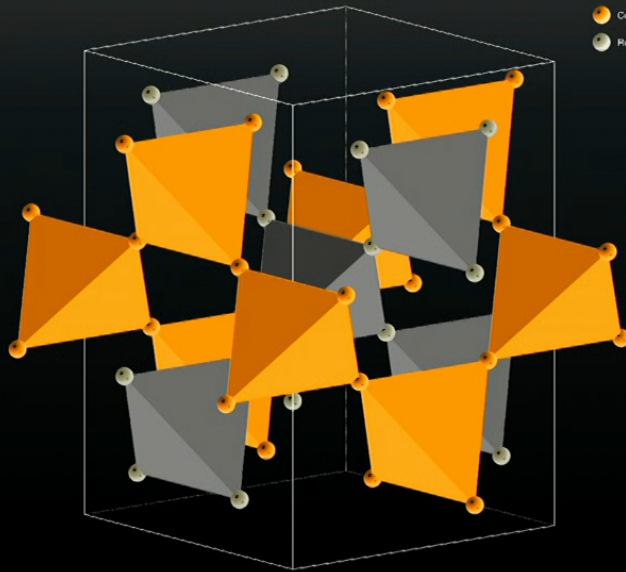


# Pyrochlore $\text{Cd}_2\text{Re}_2\text{O}_7$



- Large signal in  $C_p$  vs small displacements, 5d oxide.
- Ordering of Phase II :  
2 dim  $E_u$  vs 1 dim  $T_{2u}$

# Pyrochlore $\text{Cd}_2\text{Re}_2\text{O}_7$

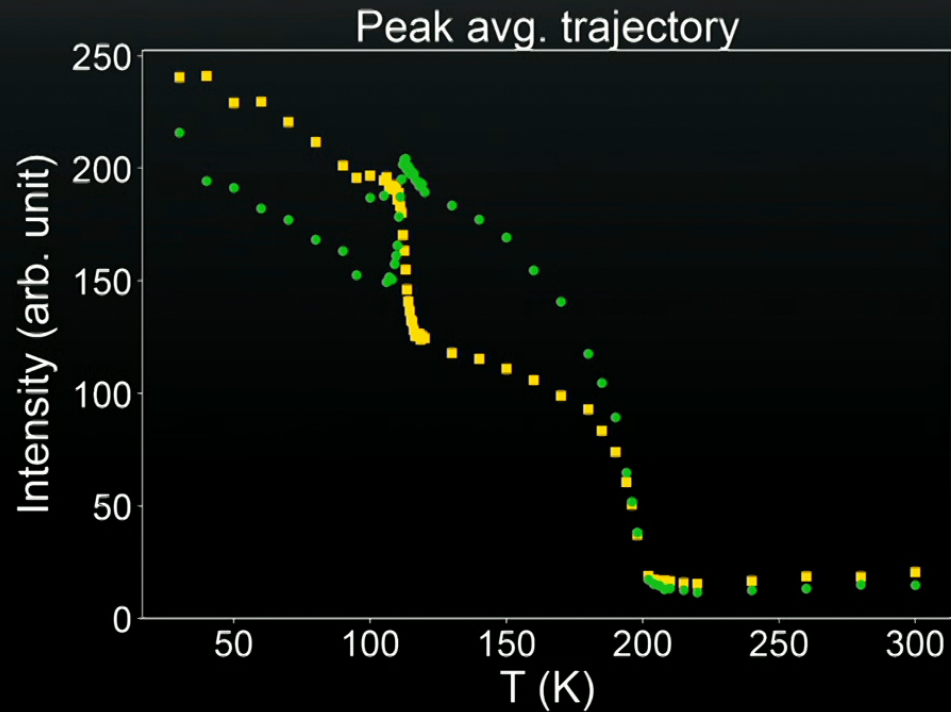


- Large signal in  $C_p$  vs **small displacements, 5d oxide.**
- Ordering of Phase II : **2 dim  $E_u$  vs 1 dim  $T_{2u}$**



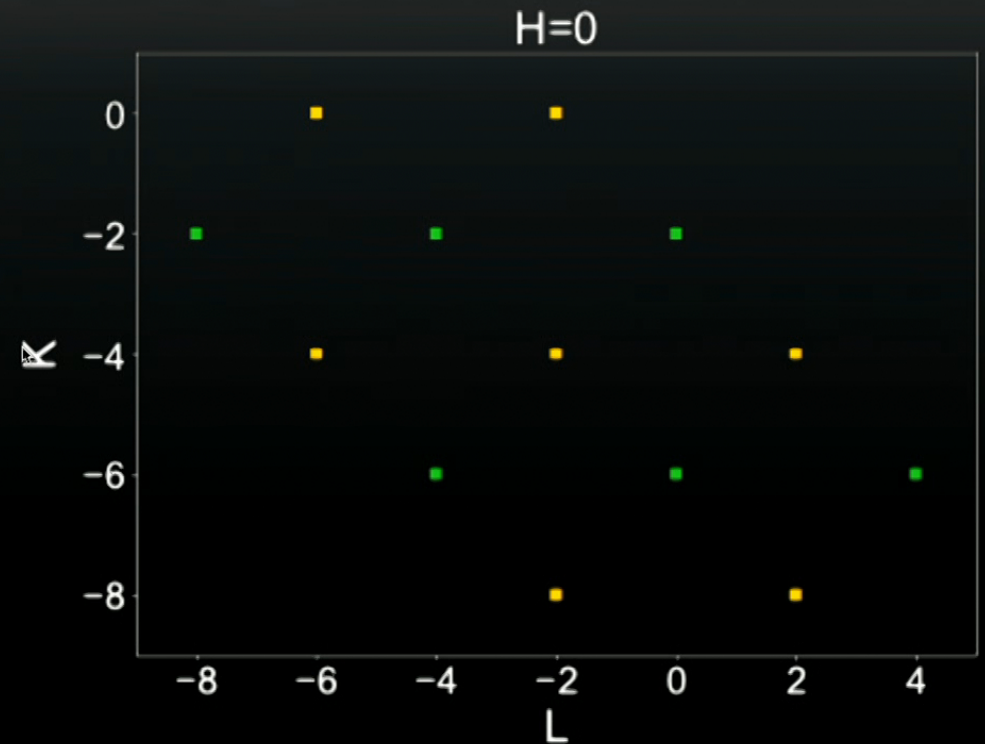
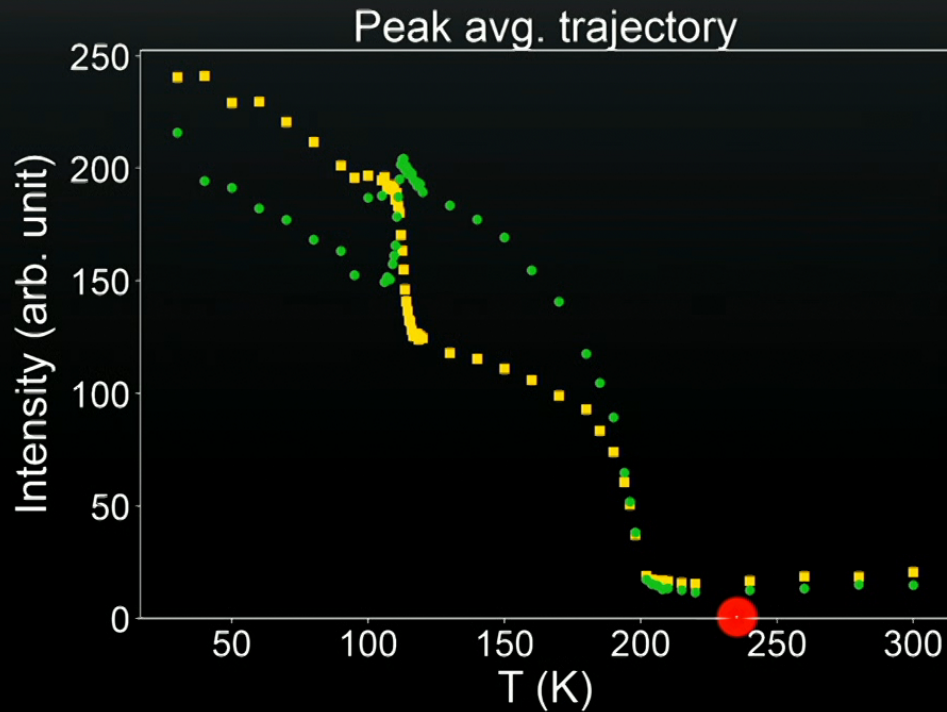
8 TB in ~10 min

# Discovery I: Re displacement



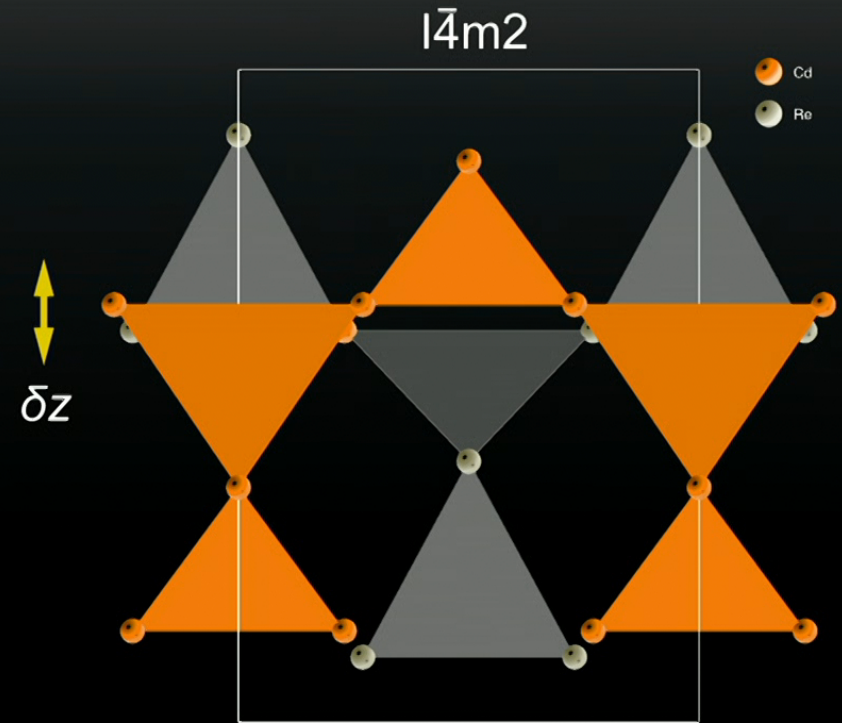
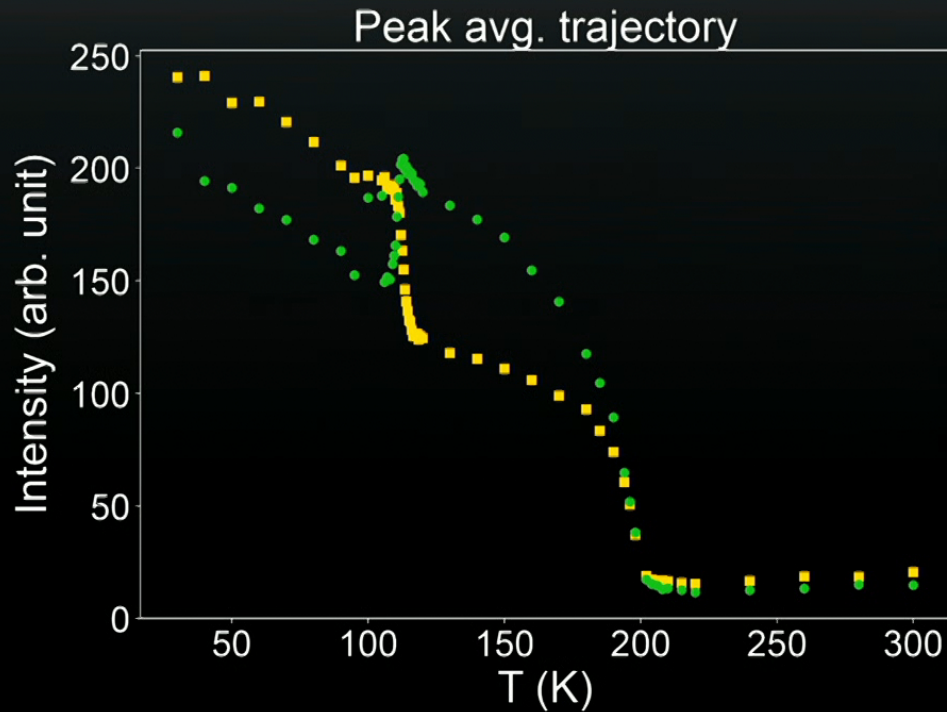


# Discovery I: Re displacement



$(4n_1, 4n_2, 4n_3 + 2)$  vs  $(4n_1 + 2, 4n_2, 4n_3)$  or  $(4n_1, 4n_2 + 2, 4n_3)$

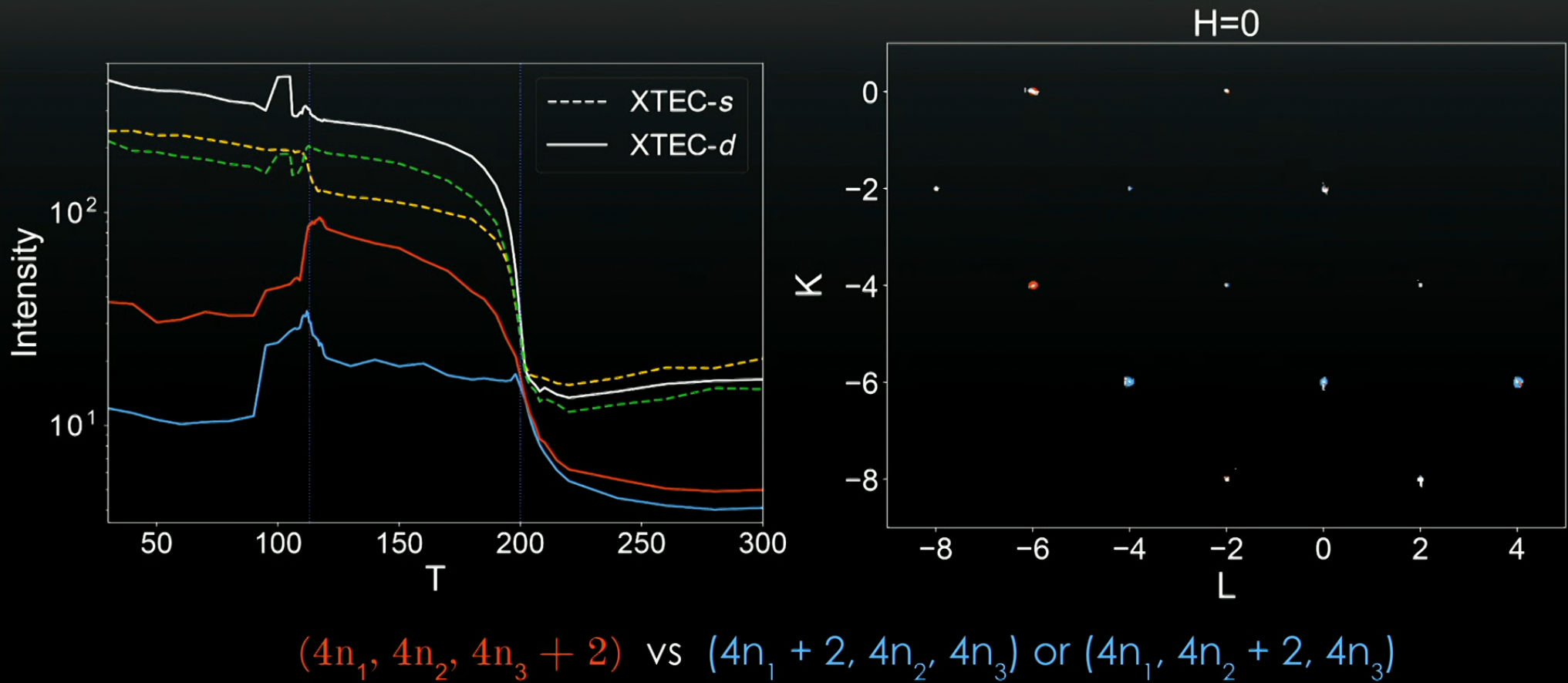
# Discovery I: Re displacement



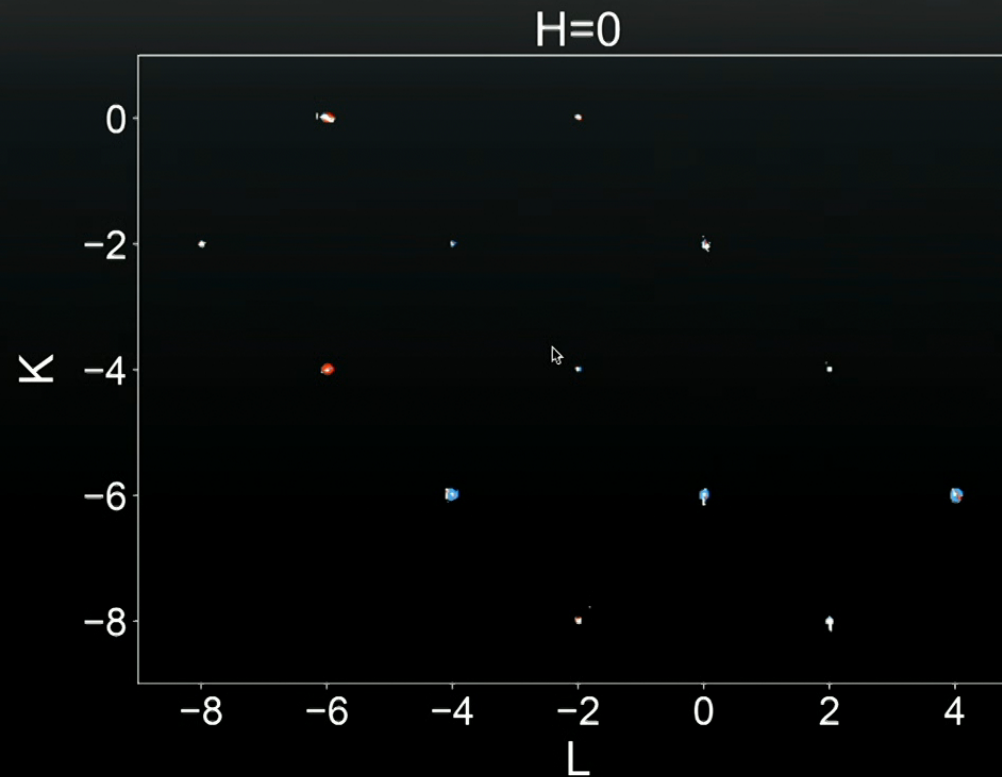
$$(4n_1, 4n_2, 4n_3 + 2) \text{ vs } (4n_1 + 2, 4n_2, 4n_3) \text{ or } (4n_1, 4n_2 + 2, 4n_3)$$



# Discovery II: Goldstone mode



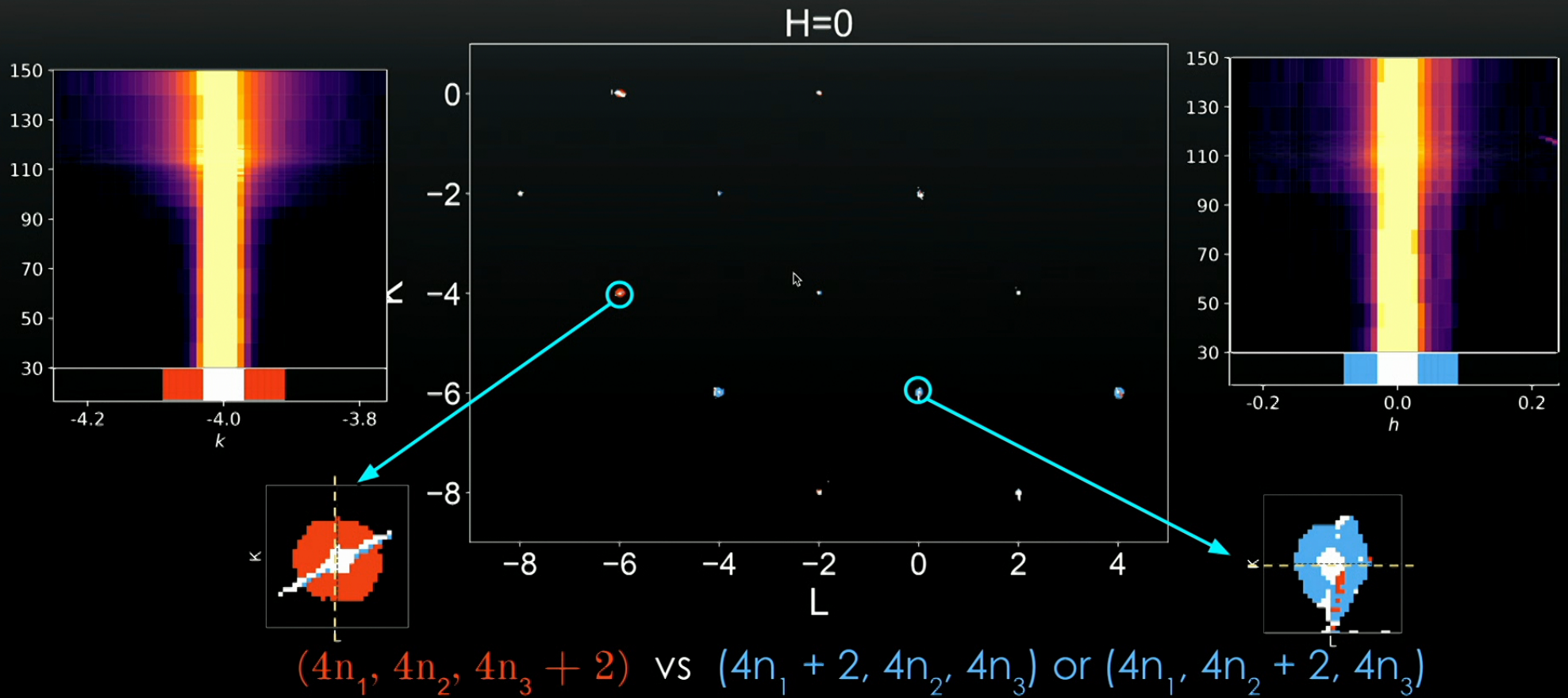
# Discovery II: Goldstone mode



$(4n_1, 4n_2, 4n_3 + 2)$  vs  $(4n_1 + 2, 4n_2, 4n_3)$  or  $(4n_1, 4n_2 + 2, 4n_3)$

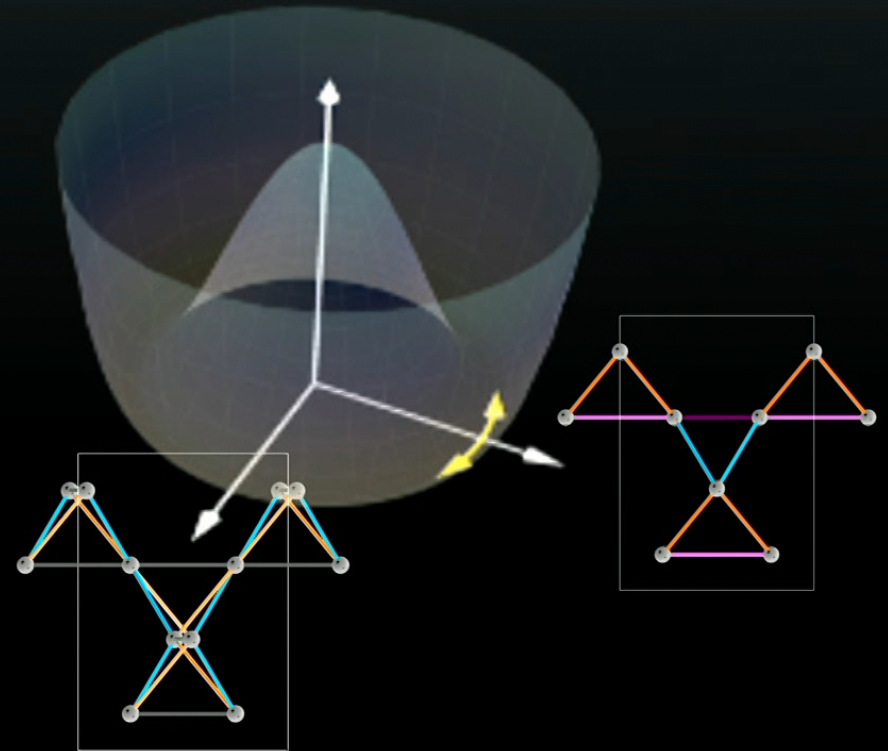
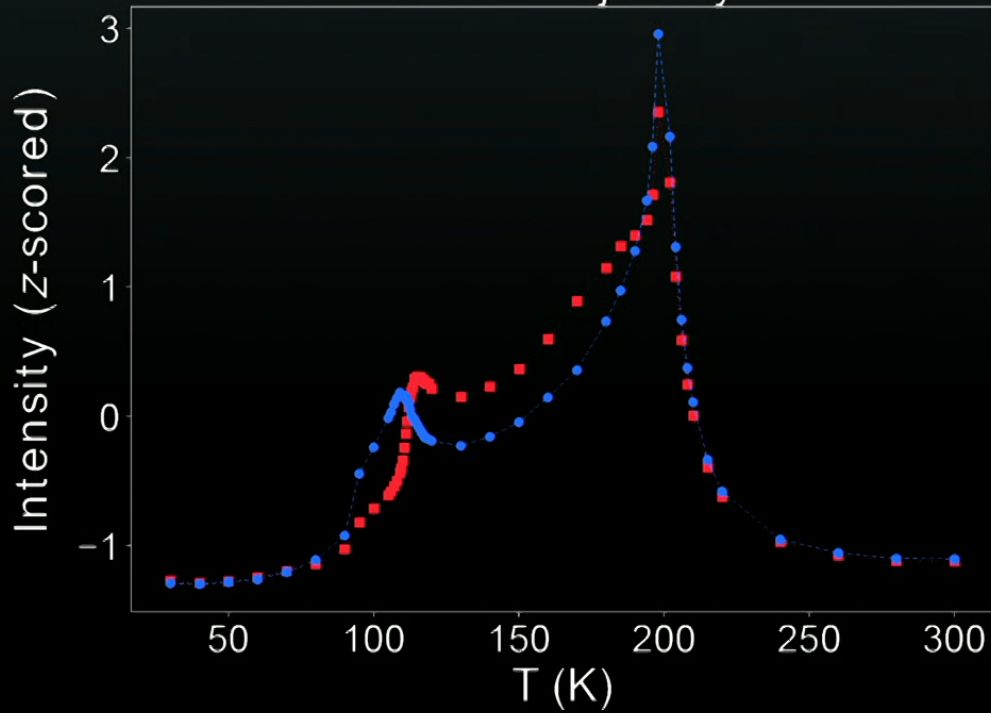


# Discovery II: Goldstone mode



# Discovery II: Goldstone mode

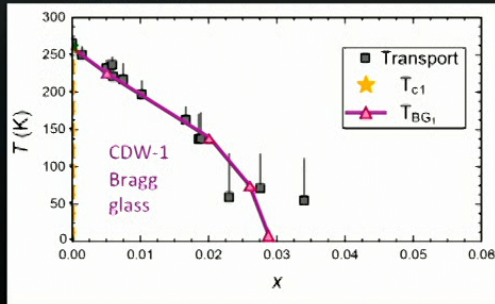
Diffuse trajectory





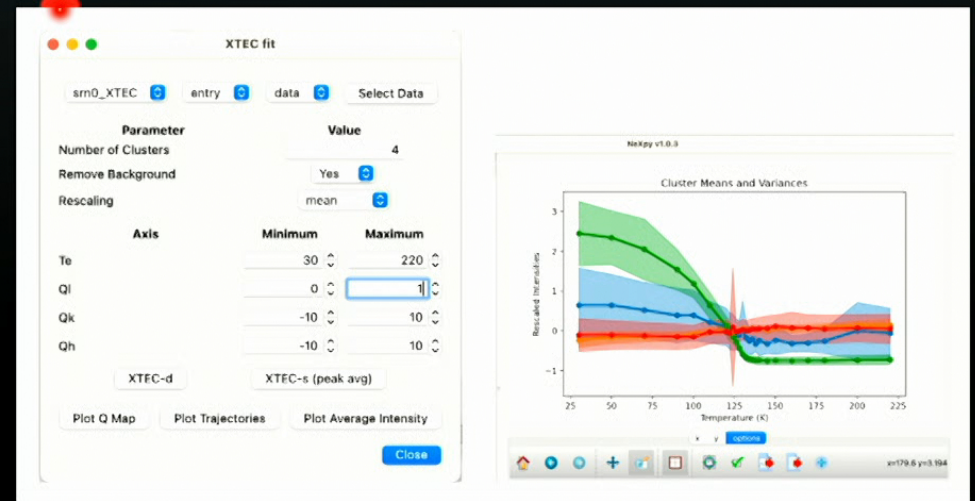
# Versatility

- Bragg glass in Pd-ETe3

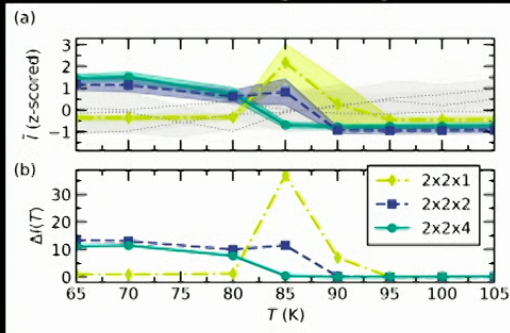
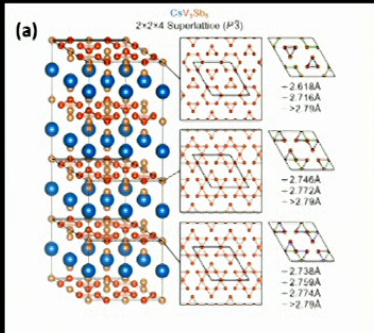


Mallayya, EAK et al (arXiv:2207.14795)

- Plug-in for NeXPy for Advanced Photon Source and CHESS



- Kagome metal  $CsV_3Sb_5$



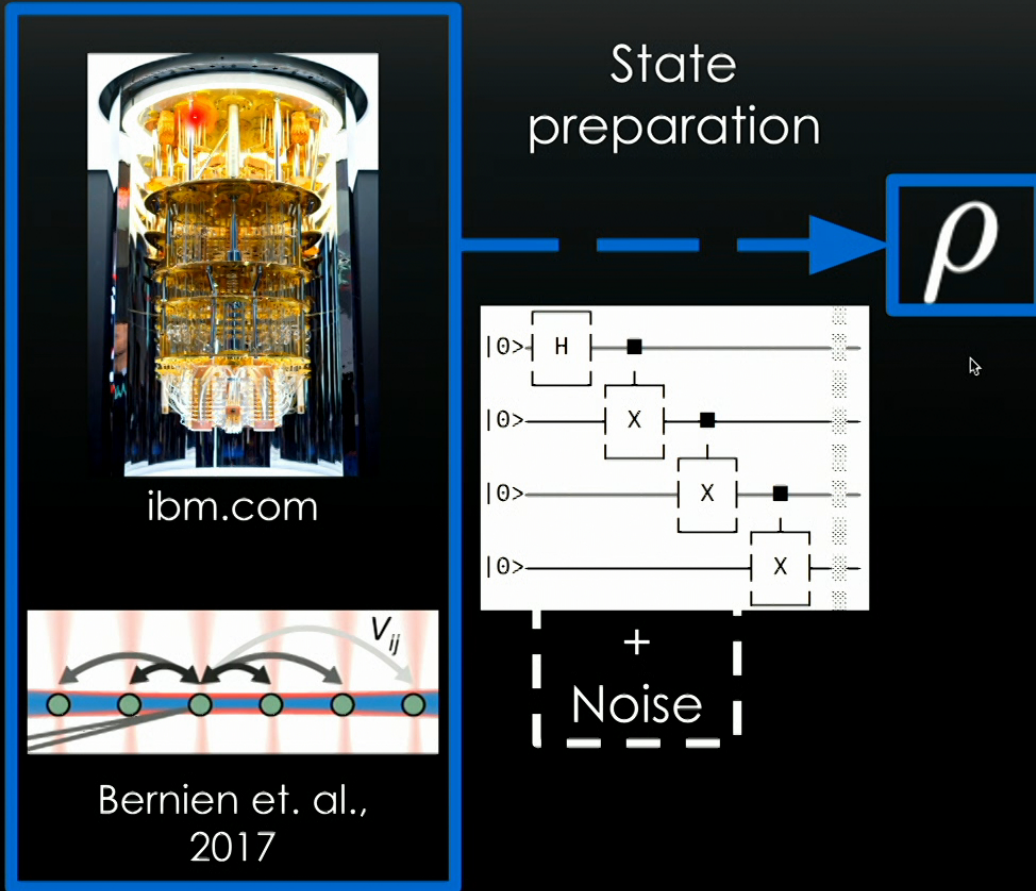
Kautzsch, EAK, Wilson et al, PRM (2023)

- Time-resolved XRD
- 4D STEM data

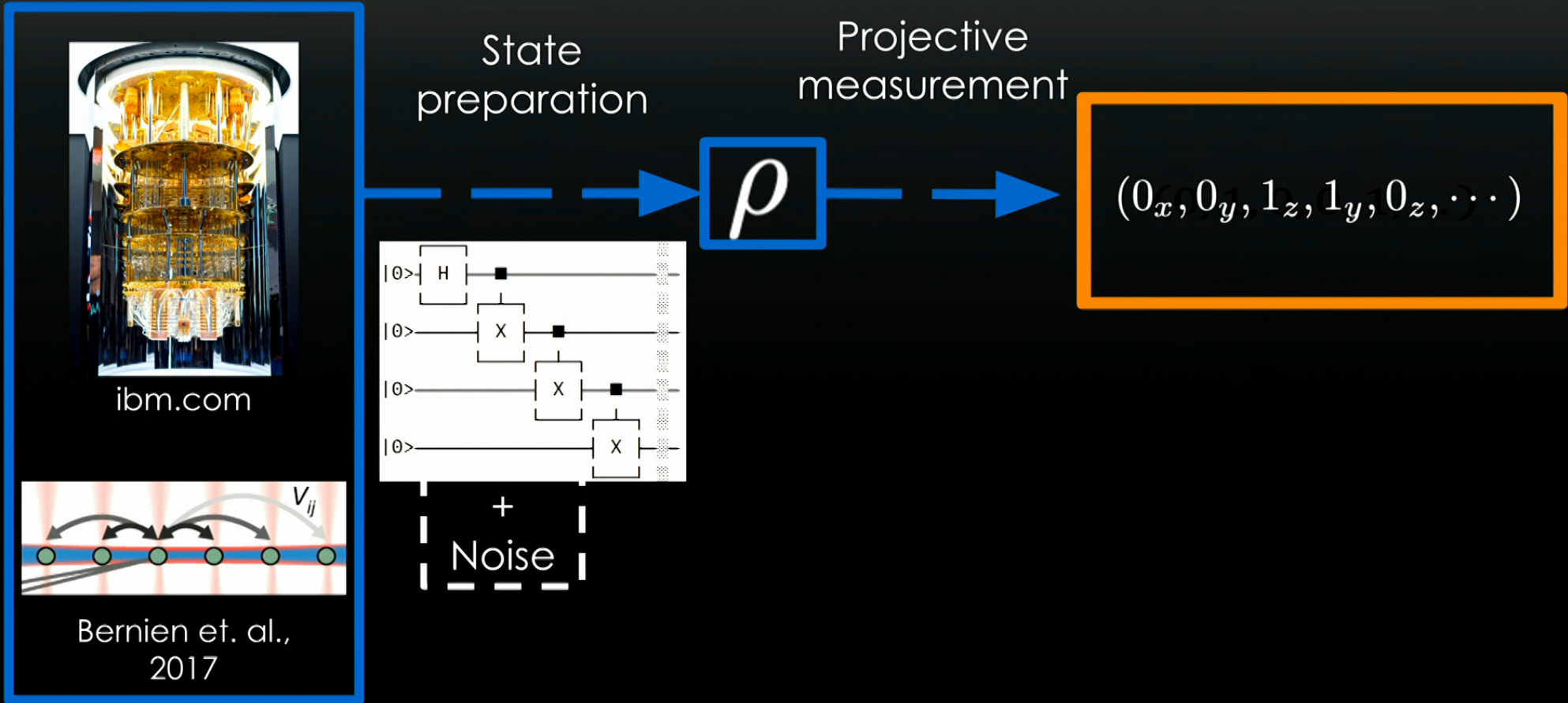
# “Decoding” Quantum Simulator Data



# Projective measurements



# Projective measurements





# Challenge of Tomography

- $N_q$ -QUBIT SYSTEM  $\leftrightarrow \sim 4^{N_q}$  COMPLEX #'S
- $N_q = 10 \sim 10^6$  #'S  $\sim 1$  MB
- $N_q = 20 \sim 10^{12}$  #'S  $\sim 1$  TB
- $N_q = 53 \sim 10^{30}$  #'S  $\sim 1$  QUINTILLION TB



Library of Congress  $\sim 20$  TB  
Source: loc.gov

- MAXIMUM LIKELIHOOD ESTIMATION (MLE):  $N_q < 10$



# Attention-based quantum tomography

MLST 3, 01LT01 (2021)



Peter Cha



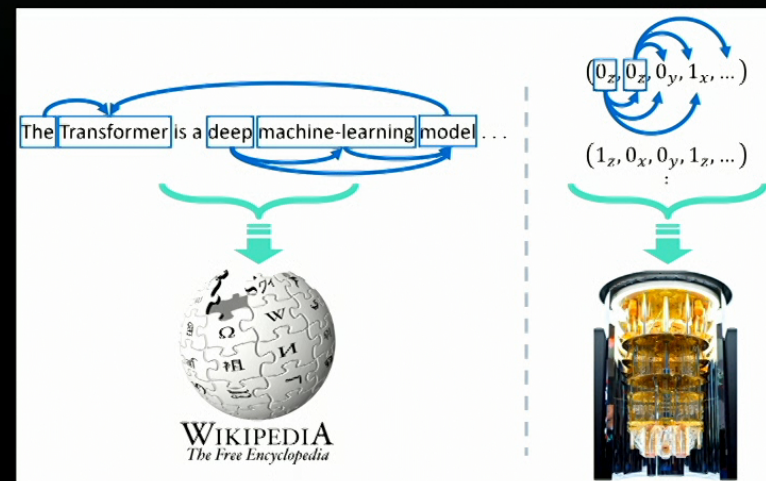
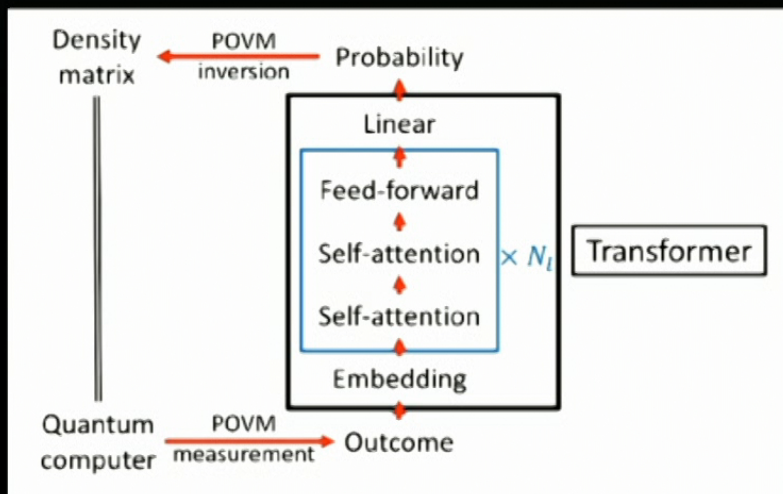
Felix Wu  
(Cornell, CS)



Paul Ginsparg  
(Cornell, CIS & Physics)



Peter McMahon  
(Cornell, AEP)



# Self-attention

---

He lives in the white house.

X1 X2 X3 X4 X5 X6



# Self-attention

He lives in the white house.

x1 x2 x3 x4 x5 x6

$$S_{ij} = \langle QX_i, KX_j \rangle$$

$$A_{ij} = \frac{\exp(S_{ij})}{\sum_{ik} \exp(S_{ik})}$$

$$\hat{X}_i = \sum_j A_{ij} V_j$$

- Q,K,V are to be learned.



# Self-attention

He lives in the white house.

x1 x2 x3 x4 x5 x6

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$$\hat{X}_i = \sum_j A_{ij} V_j$$

- Q,K,V are to be learned.
- High, non-local correlations are used.

# Attention-based Quantum Tomography (AQT)



Density matrix

POVM inversion

Probability

Transformer

Linear

Self-attention

Embedding

Quantum Hardware

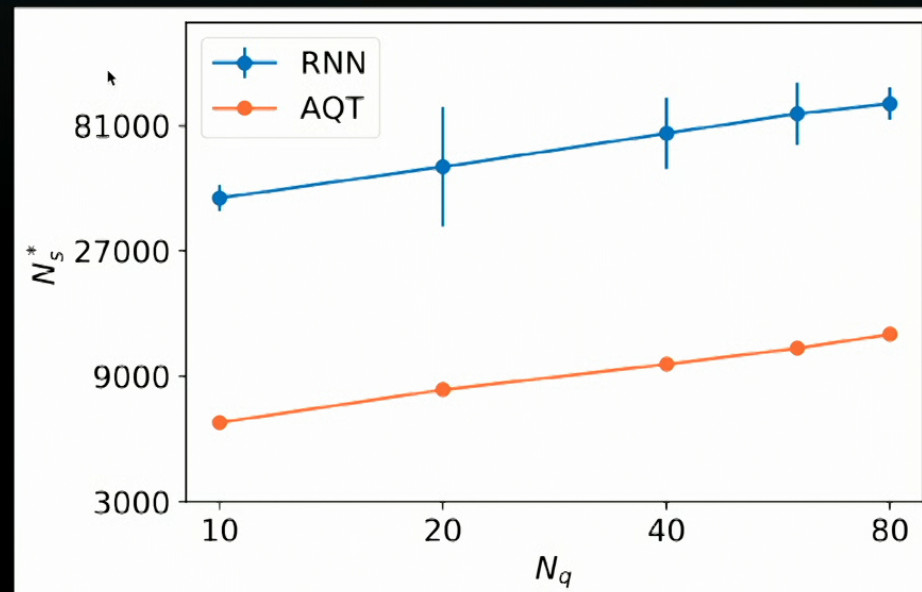
Positive Operator-Valued Measurements (POVM)

Outcome

# Best among ML based tomographies

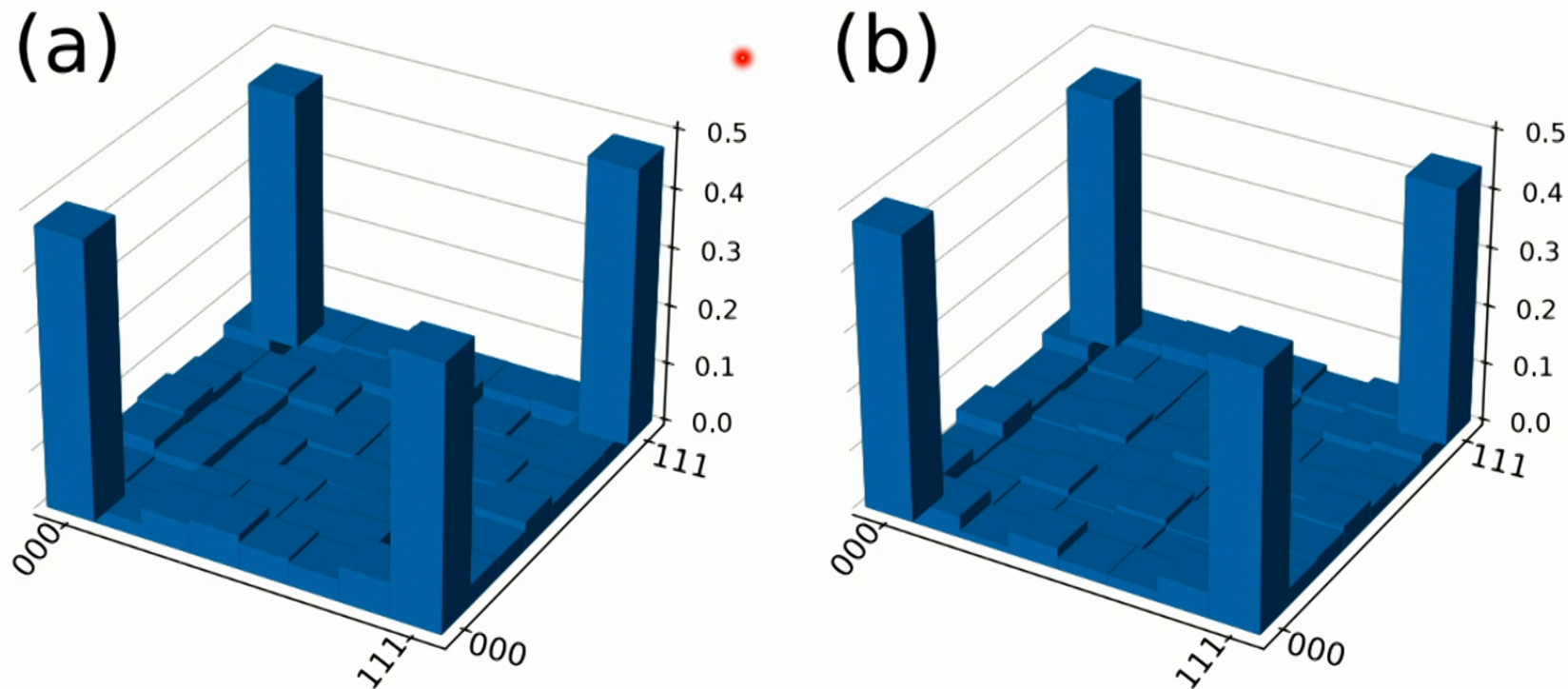
$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$$

- Classical Fidelity  $F_c(p_0, p_1) = \sum_{\vec{a}} \sqrt{p_0(\vec{a})p_1(\vec{a})}$
- Minimum sample size for  $F_c=0.99$





# Comparable to MLE tomography



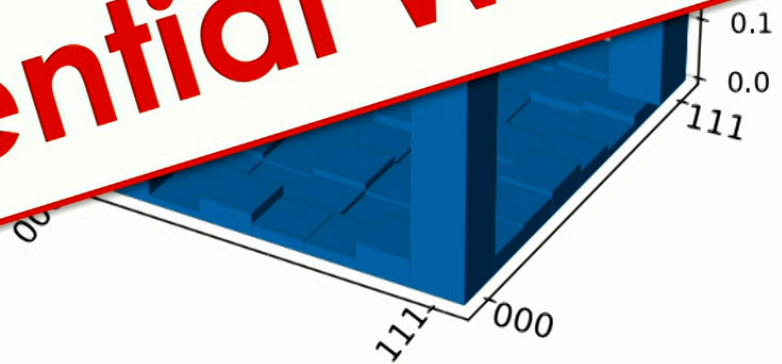
**Figure 3.** Benchmarking AQT (a) to MLE tomography offered by IBM's Qiskit library (b) for a noisy 3-qubit GHZ state data generated on the IBMQ\_OURENSE quantum computer. Each bar represents the absolute value of a density matrix (DM) element.

# Comparable to MLE tomography

(a)



(b)



**No Escaping  
the Exponential Wall!**

Comparing AQT (a) to MLE tomography offered by IBM's Qiskit library (b) for a noisy 3-qubit GHZ state data on the IBMQ\_OURENSE quantum computer. Each bar represents the absolute value of a density matrix (DM) element.



Take 2. Phase recognition  
from single instances:  
Ordered Phases  
(classical)



# MACHINE LEARNING DISCOVERY OF NEW PHASES IN PROGRAMMABLE RYDBERG QUANTUM SIMULATOR

## SNAPSHOTS

PRR 5, 013026 (2023)



Cole Miles



K. Winberger  
(Cornell, CS)



R. Samajdar  
(Harvard)



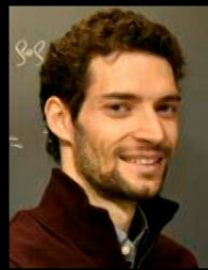
S. Sachdev  
(Harvard)



S. Ebadi  
(Harvard)



T. Wang  
(Harvard)



H. Pichler  
(Innsbruck)



M. Greiner  
(Harvard)

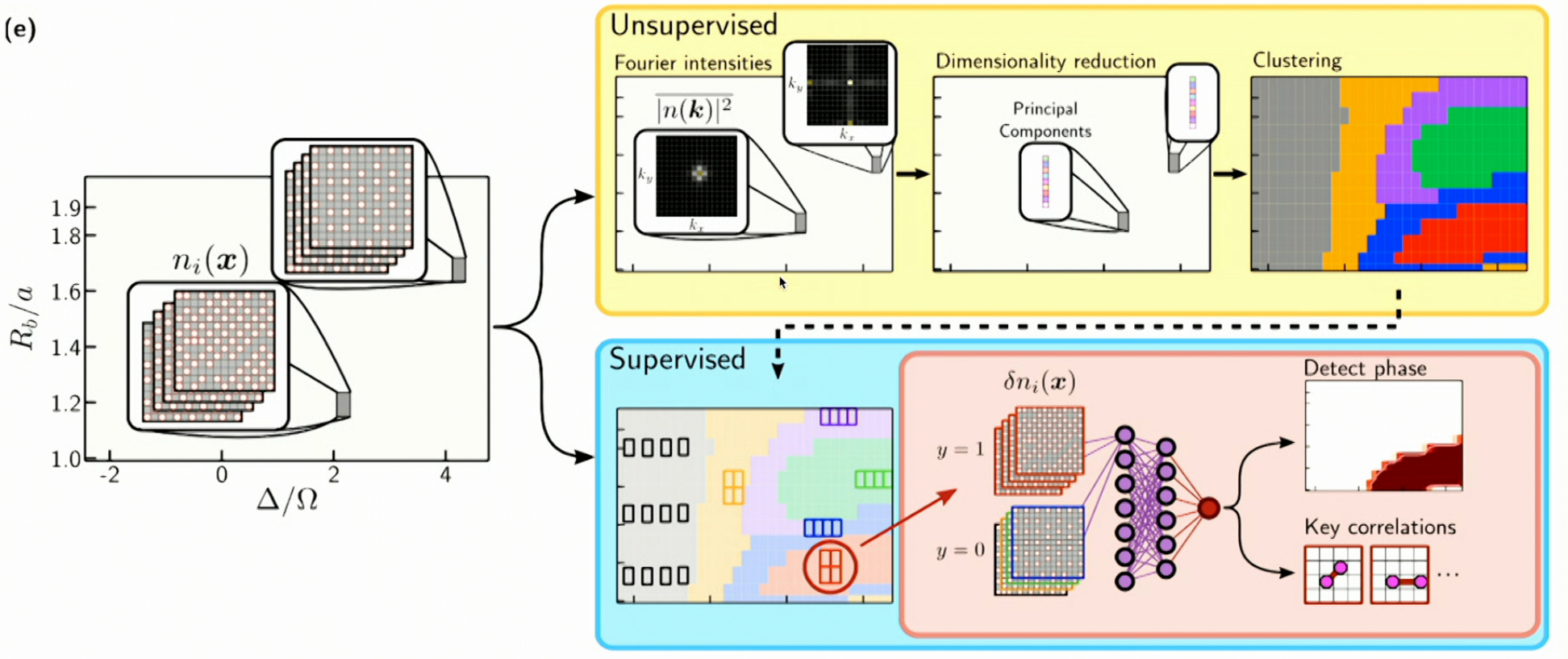


M. Lukin  
(Harvard)



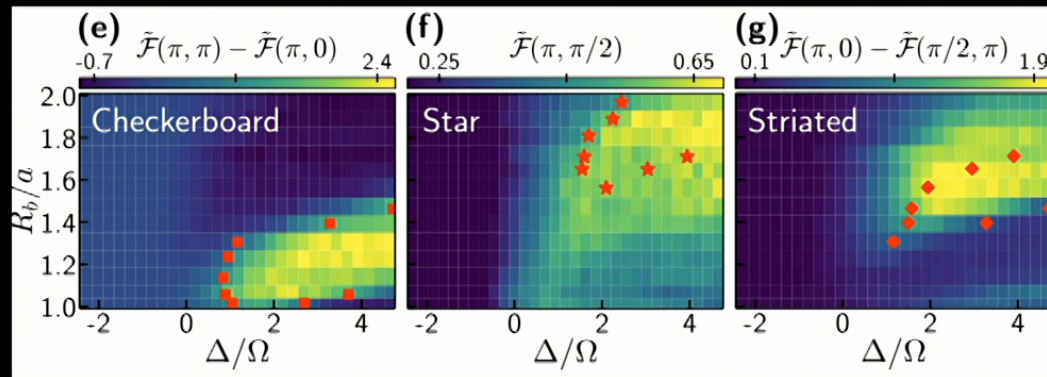
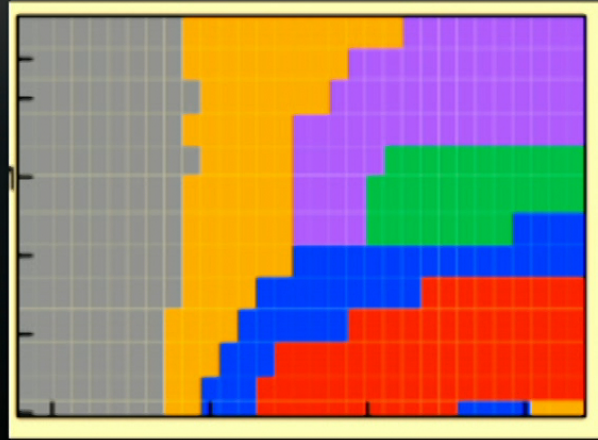
# Hybrid-CCNN

(e)

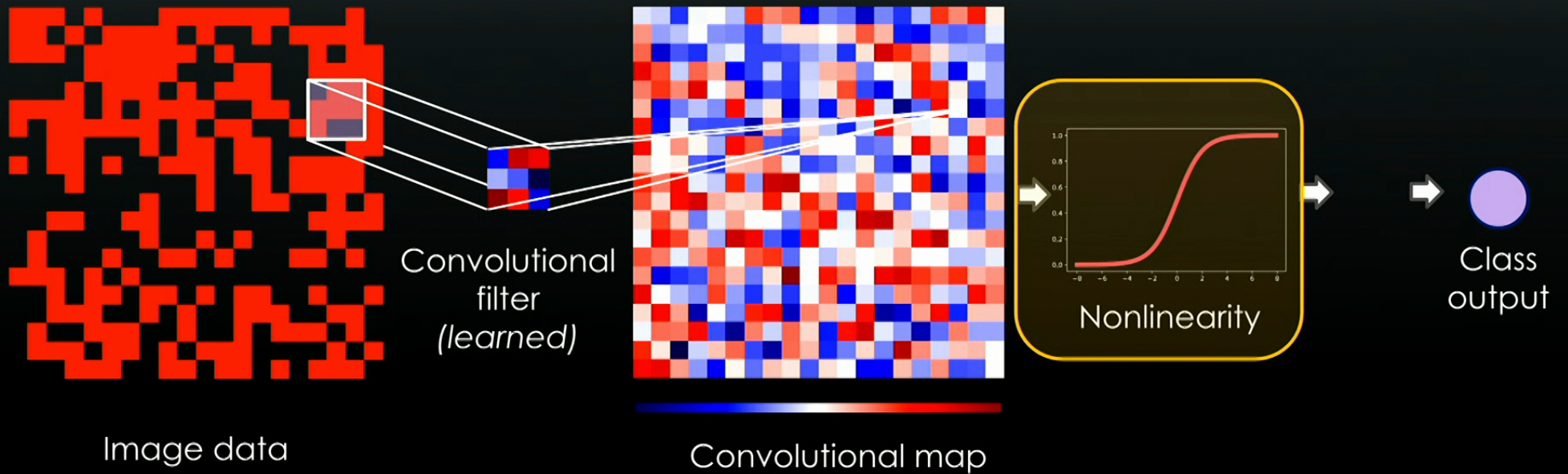




# Identification of Phases

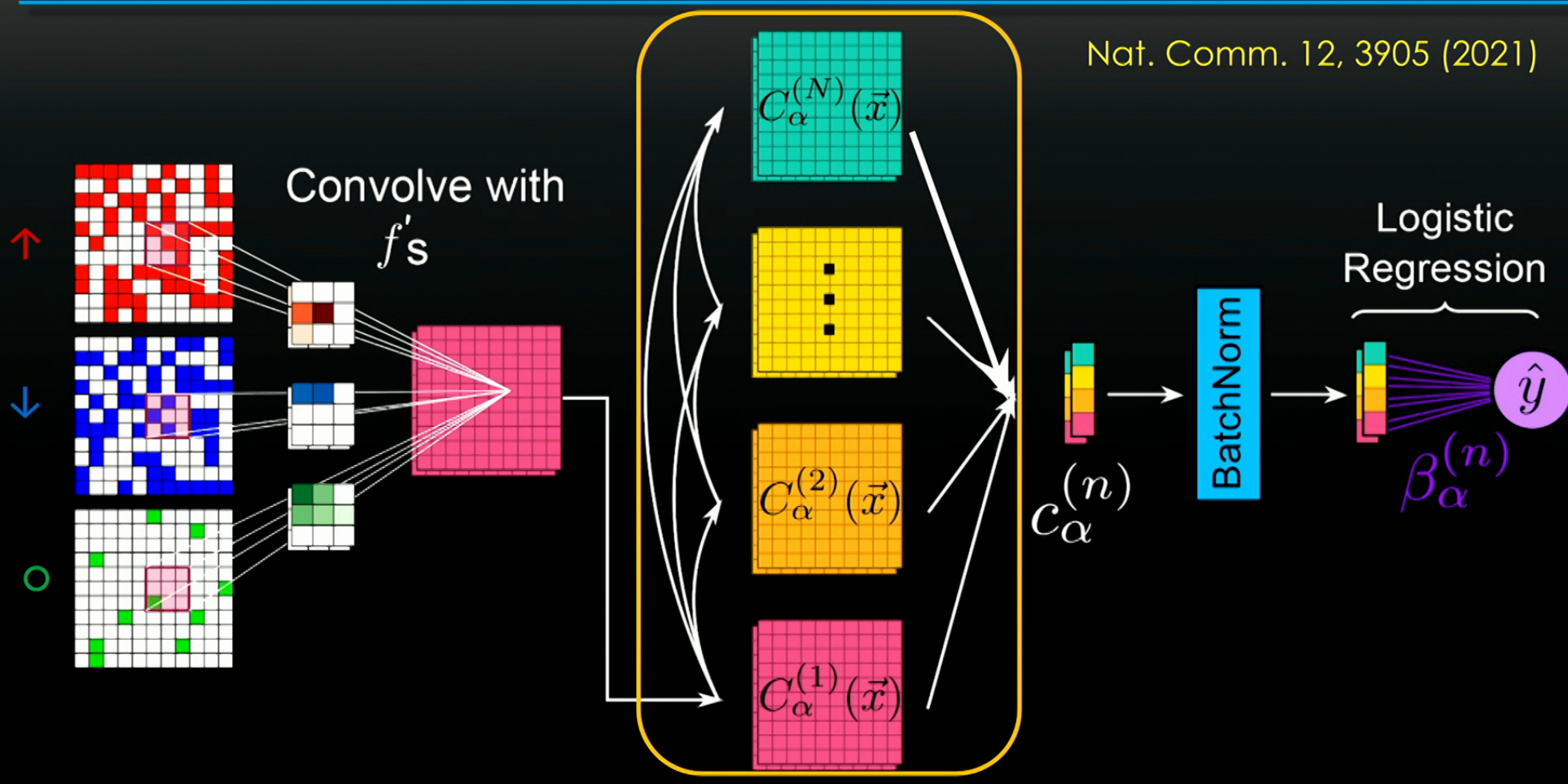


# Convolutional Neural Networks (CNN)





# Correlation CNN (CCNN)



# Regularization Path Analysis (RPA)

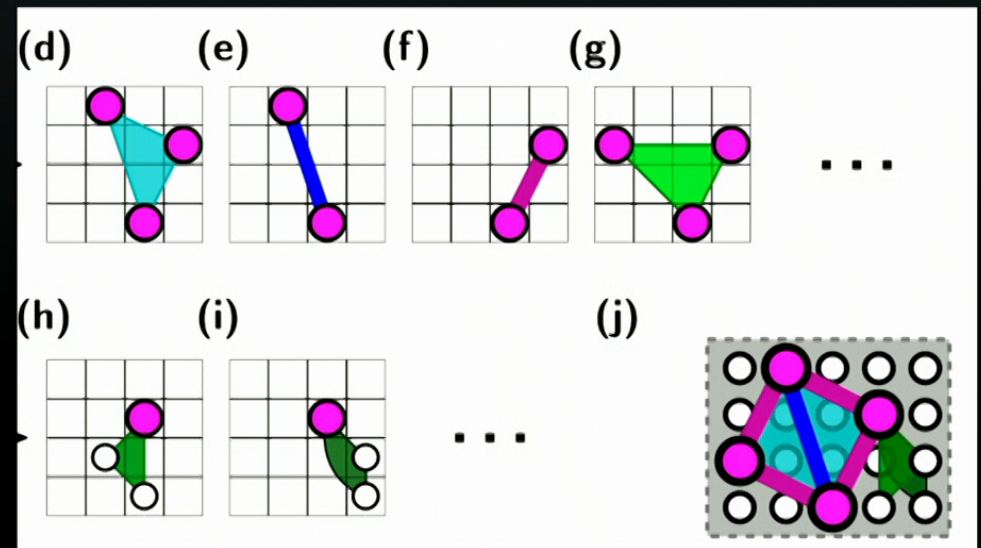
## 1. Fix learned filters



## 2. Retrain the final layer under new loss

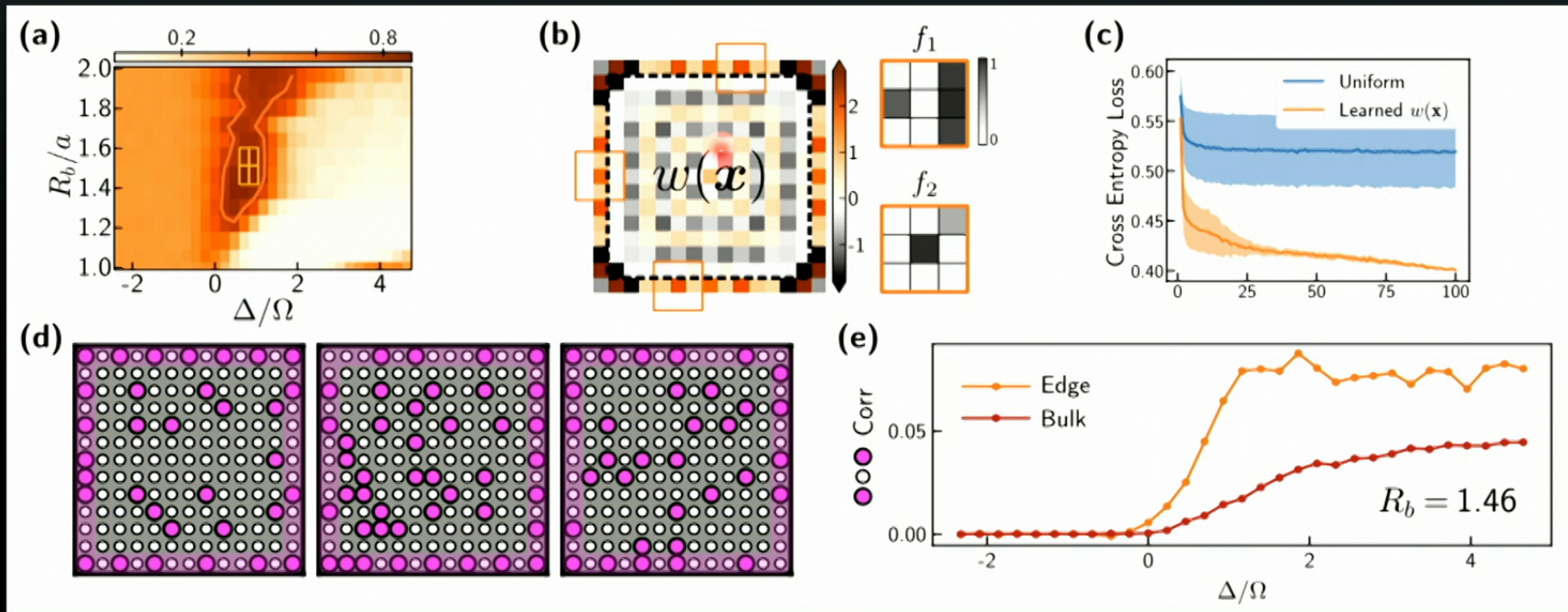
$$\mathcal{L} = \frac{1}{N} \sum_i (-y_i \log \hat{y}_i - (1 - y_i) \log(1 - \hat{y}_i)) + \gamma \sum_{\alpha, a} |f_{\alpha}(\mathbf{a})|,$$

## 3. Uncover signature motifs

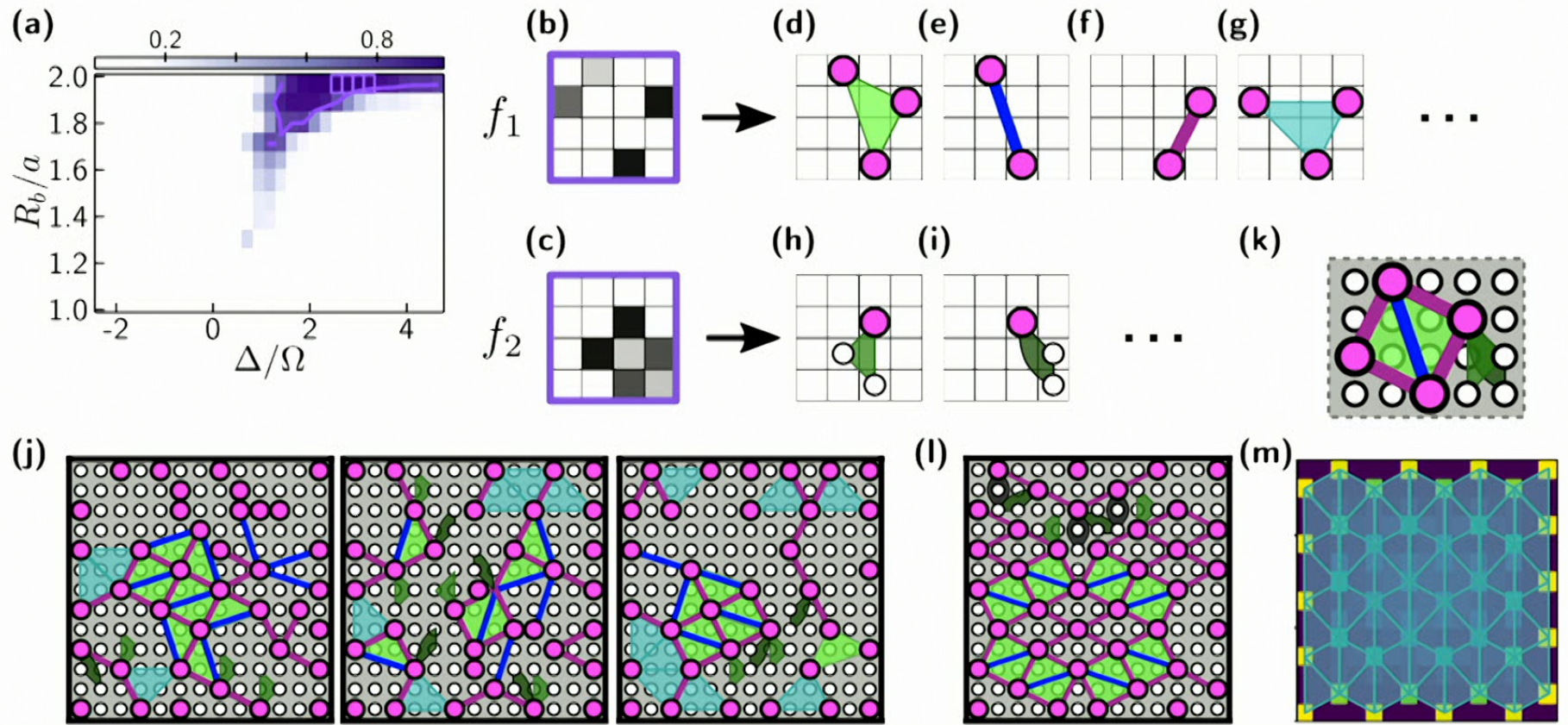




# I: Edge-ordering



# II. Staggered/Rhombic + Nematic





# Machine learning feature discovery of spinon Fermi surface



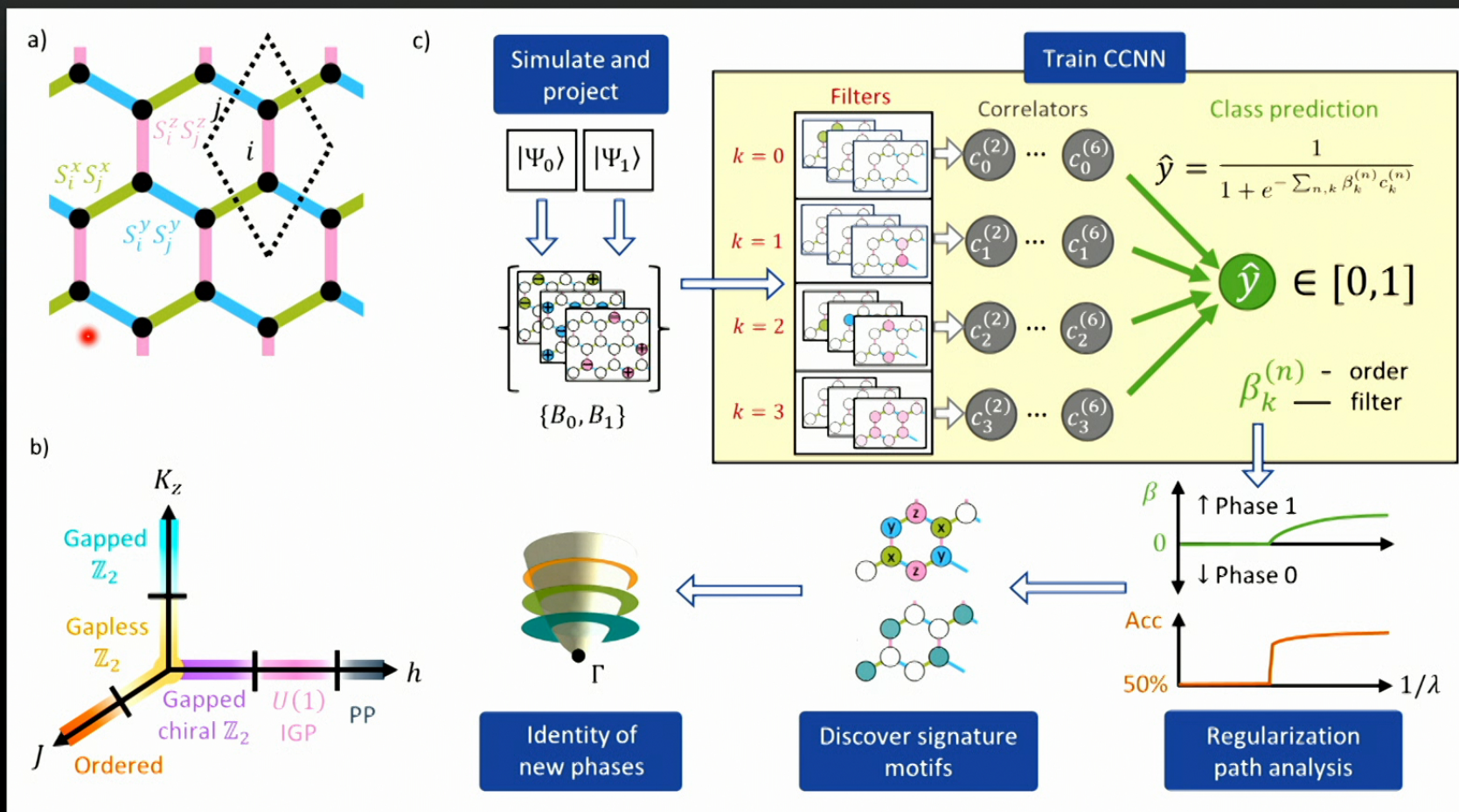
Kevin Zhang



<https://engage.spa.org/gds/home>

Kevin Zhang, Feng Shi, Yuri Lensky, Nandini Trivedi, Eun-Ah Kim (arXiv:2306.03143)

# Discovering features of quantum state



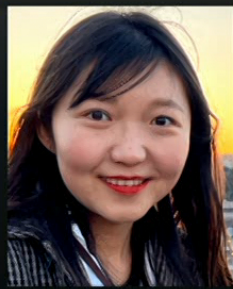


# Take 3. Learn quantum complexity from a collection of bitstrings

# Attention to Quantum Complexity



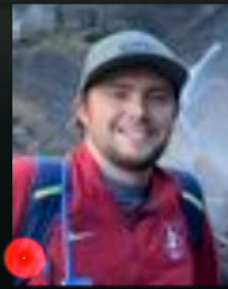
H. Kim



C. Wan  
(Cornell, CS)



K. Varma



J. Hoke  
(Stanford)



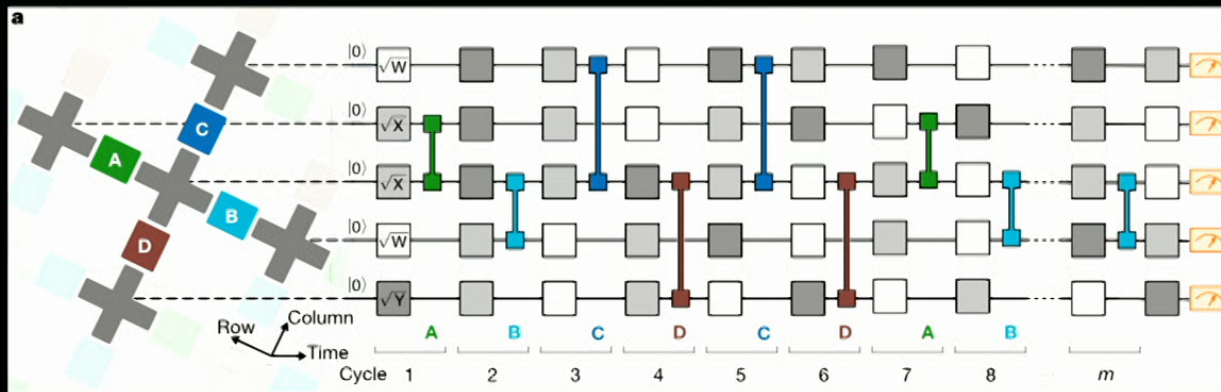
Yuri Lensky



P. Roushan  
(Google)



K. Winberger  
(Cornell, CS)

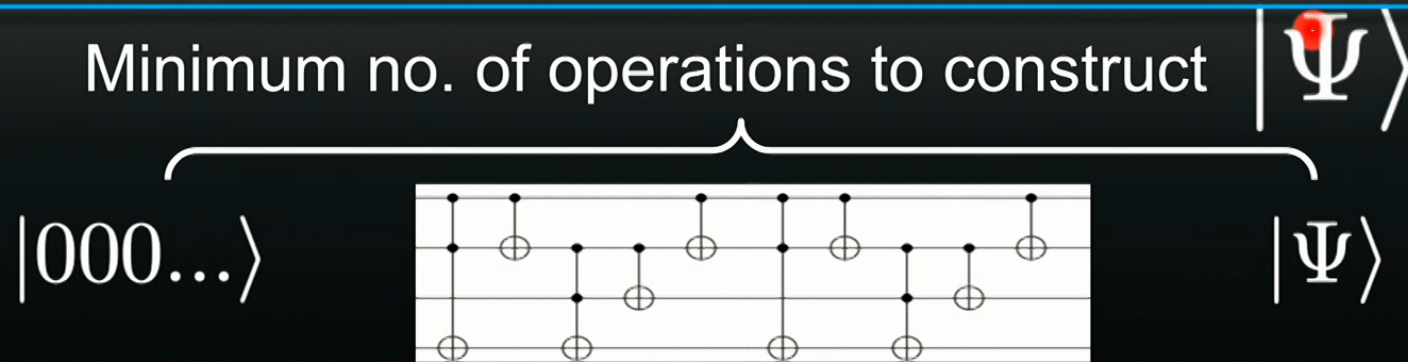


H.Kim, EAK et al, in preparation  
(2023)

Quantum supremacy using a  
programmable  
superconducting processor  
Arute et al, Nature 574, 505  
(2019)



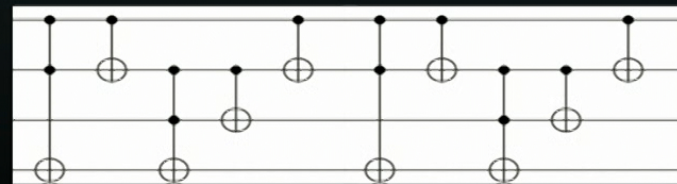
# Complexity of Random Circuits



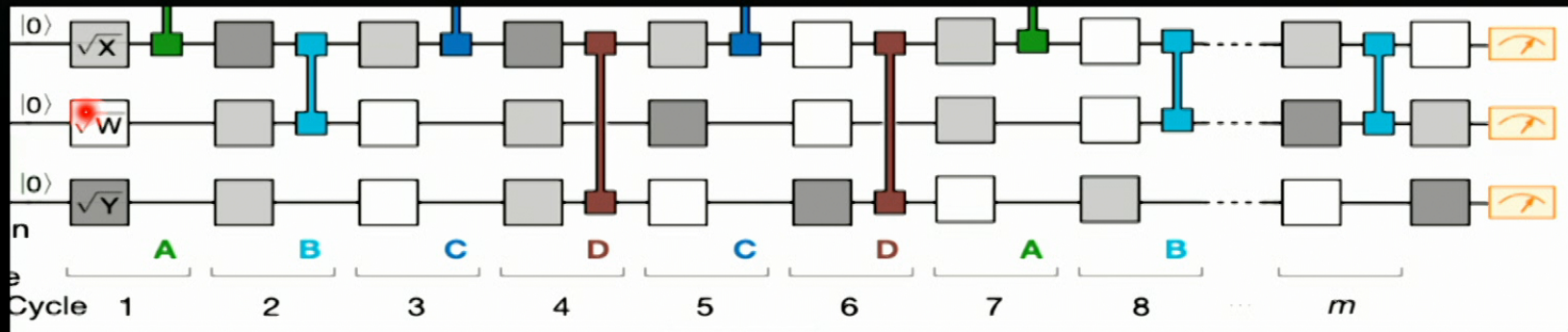
# Complexity of Random Circuits

Minimum no. of operations to construct  $|\Psi\rangle$

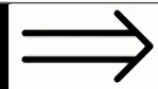
$|000\dots\rangle$



$|\Psi\rangle$



Increasing depth

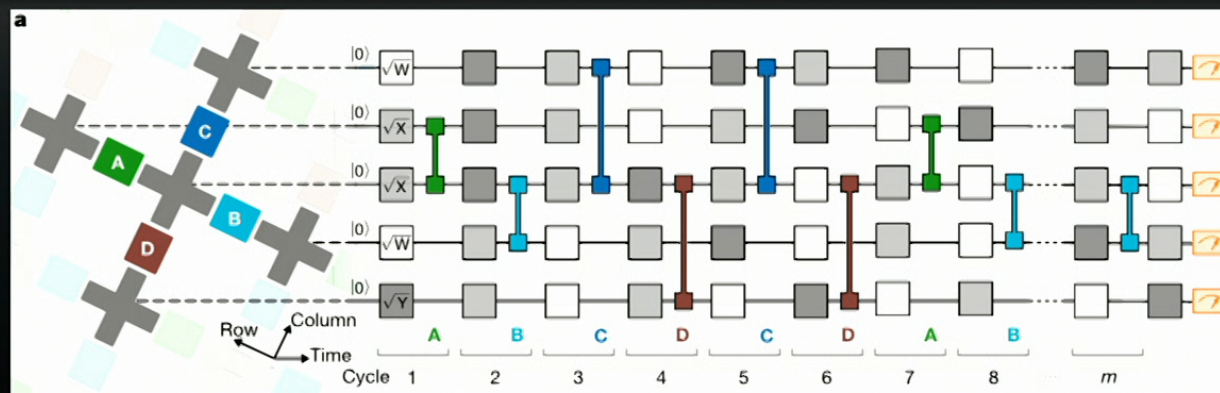


Increasing Complexity

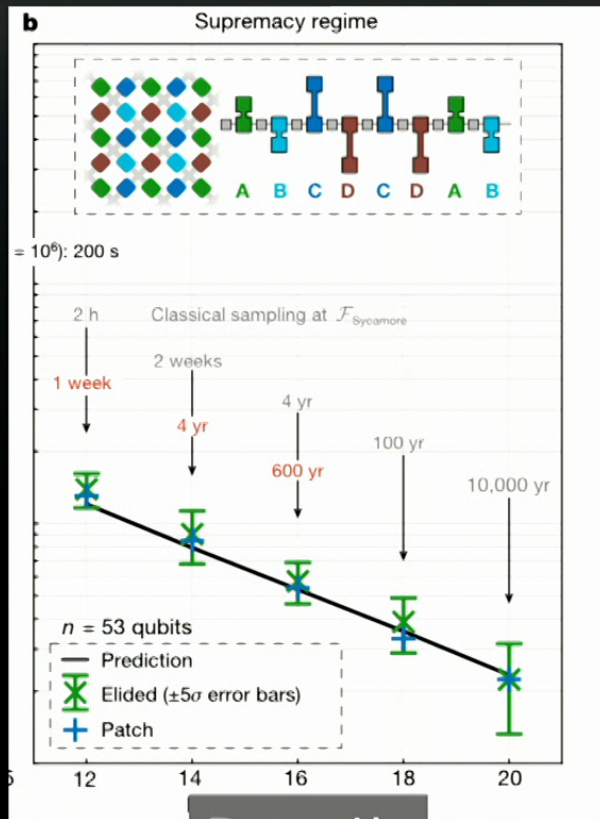
Arute, F., Arya, K., Babbush, R. et al. *Nature* 574, 505–510 (2019)



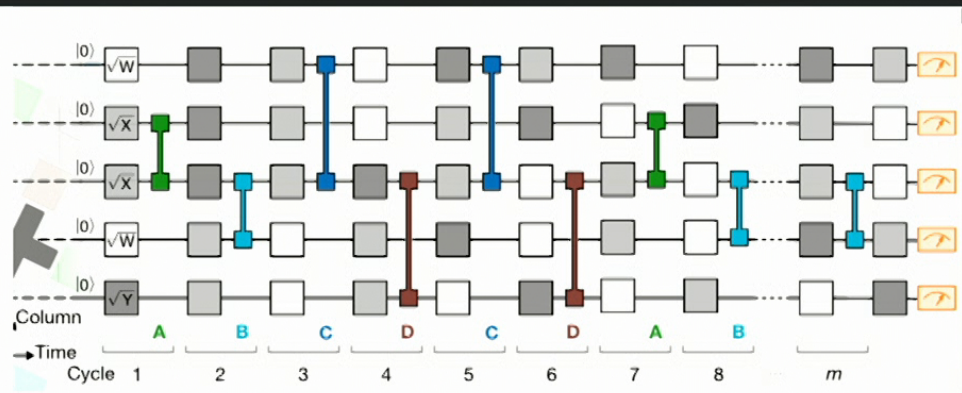
# Quantum Supremacy?



# Quantum Supremacy?



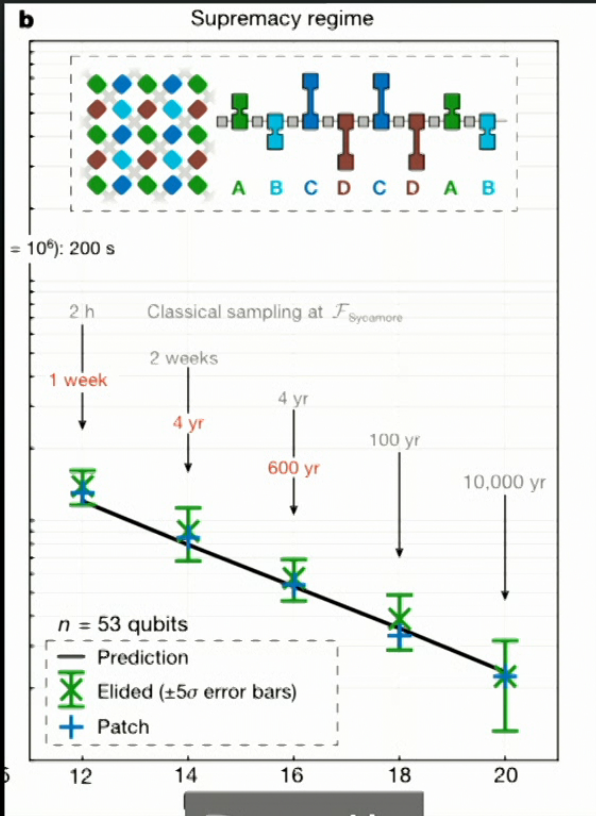
Depth



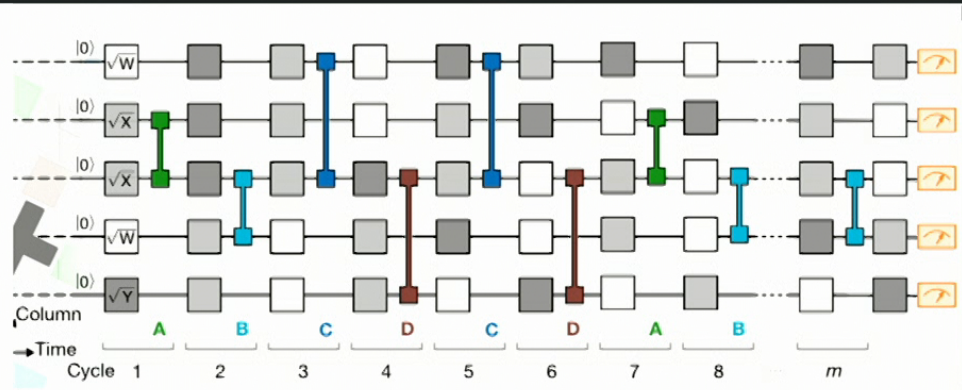


# Quantum Supremacy?

$\mathcal{F}_{\text{XEB}}$



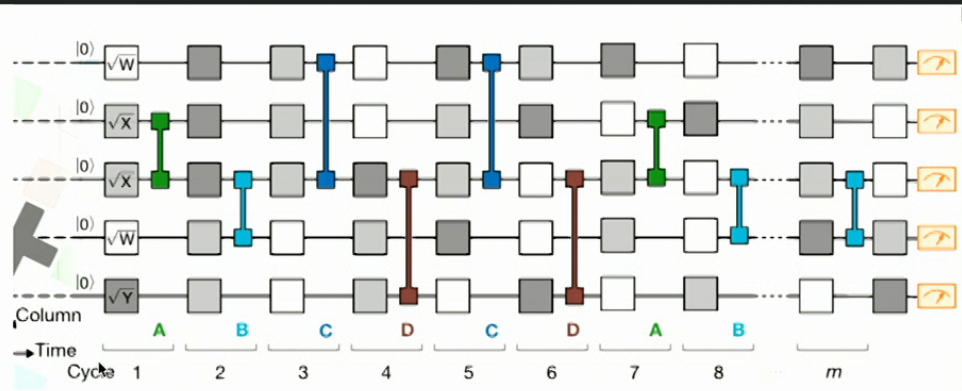
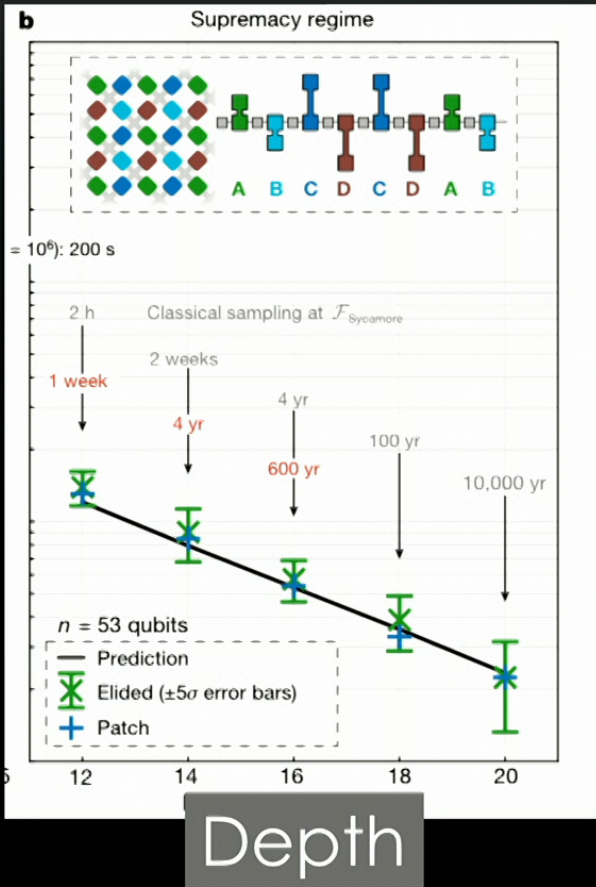
Depth



$$\mathcal{F}_{\text{XEB}} = 2^n \langle P(x_i) \rangle_i - 1$$

# Quantum Supremacy?

$\mathcal{F}_{\text{XEB}}$

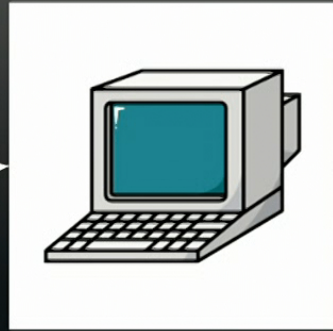
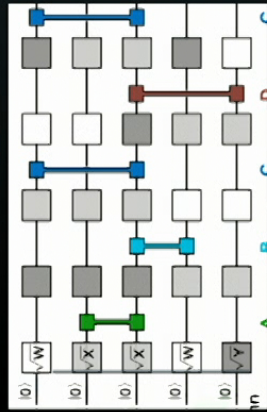


$$\mathcal{F}_{\text{XEB}} = 2^n \langle P(x_i) \rangle_i - 1$$

Complex :

$$\mathcal{F}_{\text{XEB}} \rightarrow 1$$

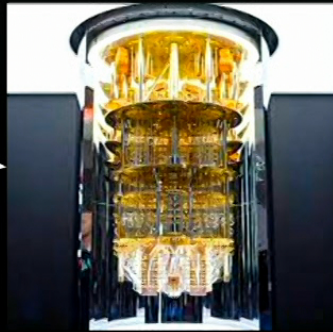




Classical Simulation

$$|\Psi\rangle$$

Full wavefunction



Quantum Processor

0	1	0	0	1
1	0	1	0	1
1	1	0	0	0
0	1	1	0	1
1	0	1	0	0
0	0	1	0	1
1	0	1	0	0

Z-basis measurements

# k-design

---

- K-design random circuit: match perfect infinite-depth up to k-moments

$$U^{\otimes k} \otimes (U^\dagger)^{\otimes k}$$



# k-design

---

- K-design random circuit: match perfect infinite-depth up to k-moments

$$U^{\otimes k} \otimes (U^\dagger)^{\otimes k}$$

- K is expected to grow with depth and system size.

# Self-attention

He lives in the white house.

X1 X2 X3 X4 X5 X6

$$S_{ij} = \langle QX_i, KX_j \rangle$$

$$A_{ij} = \frac{\exp(S_{ij})}{\sum_{ik} \exp(S_{ik})}$$

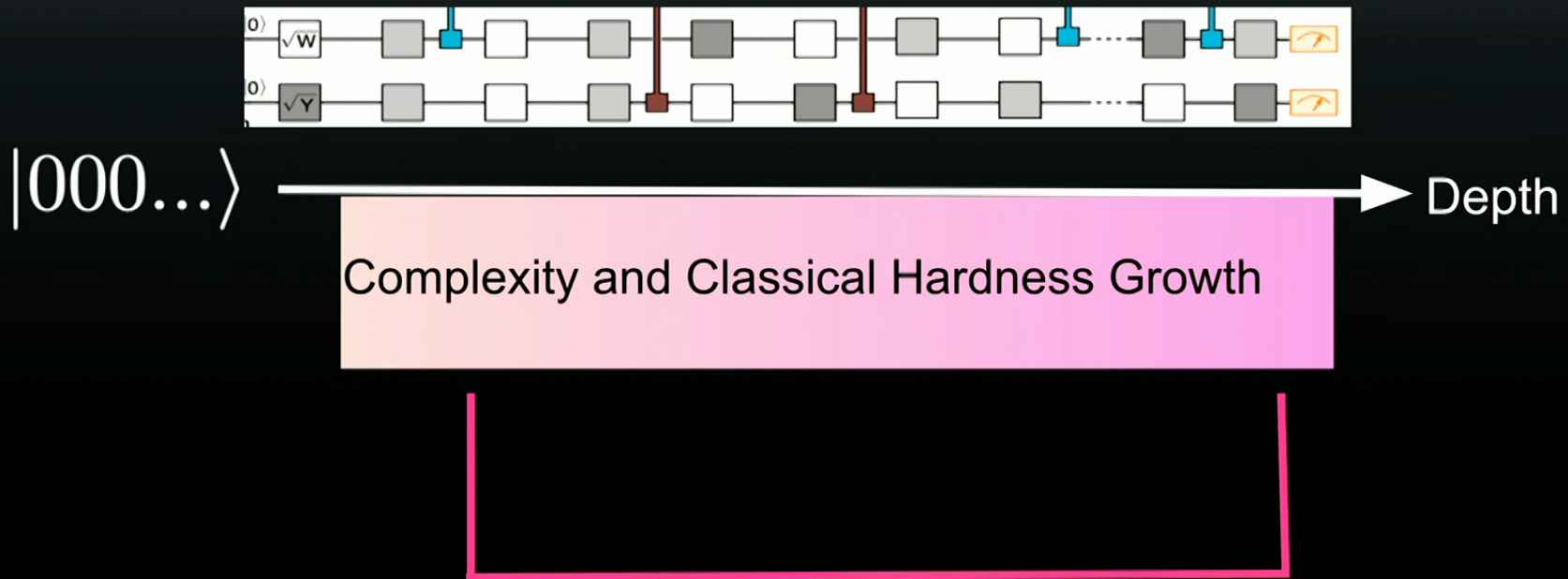
$$\hat{X}_i = \sum_j A_{ij} V_j$$

- Q,K,V are to be learned.

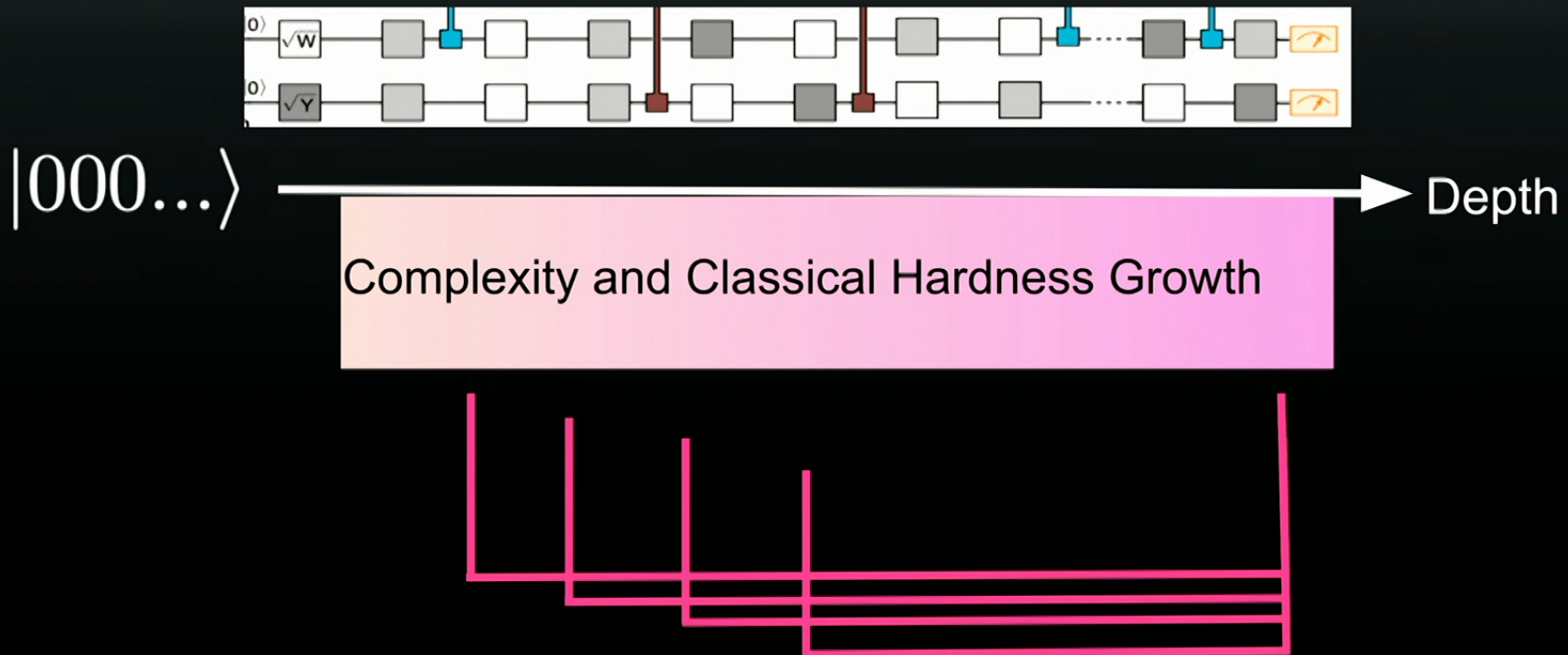
- High, non-local correlation are used.



# Learn the evolution of data



# Learn the evolution of data





# Learn the evolution of data

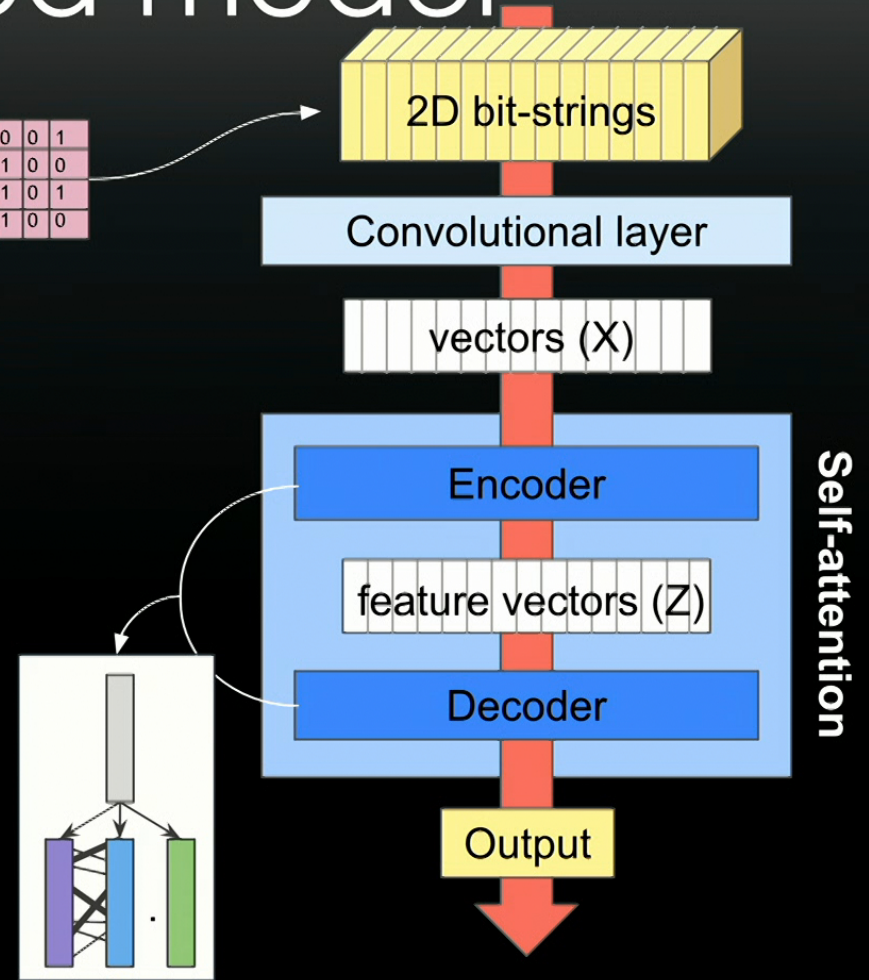


# Attention based model

INPUT: 2D BIT-STRINGS

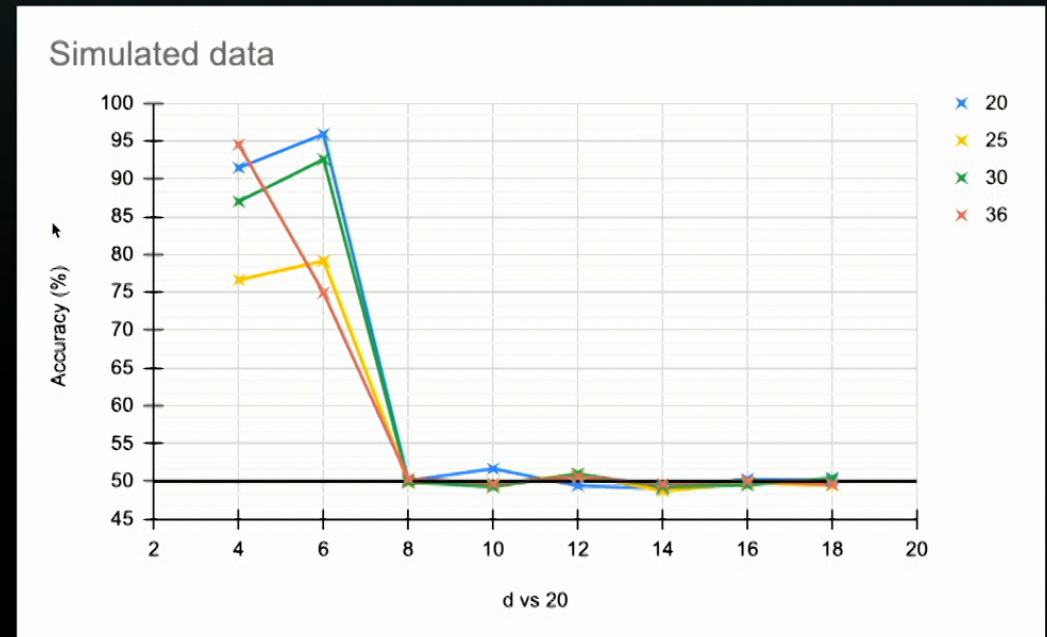
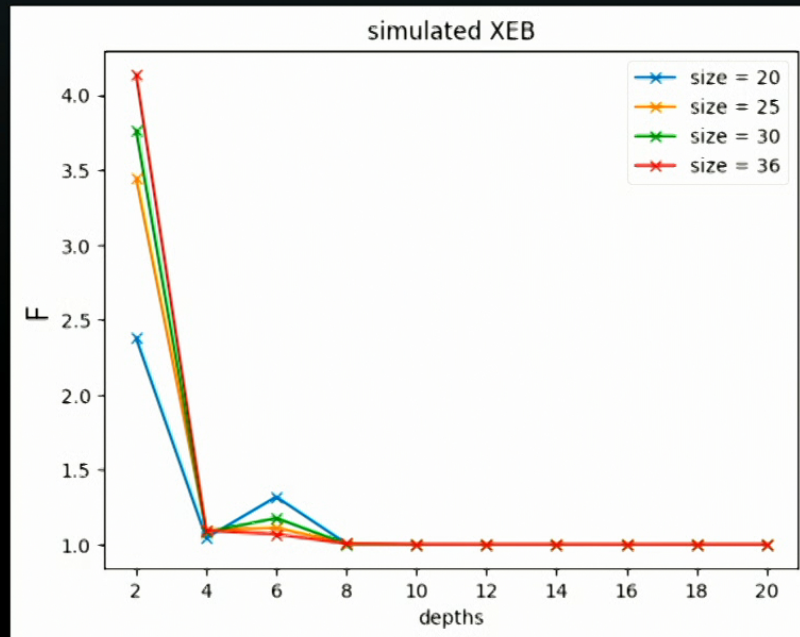
- ONE CONVOLUTIONAL LAYER
- ATTENTION BLOCKS
- OUTPUT: BINARY DEPTH CLASSIFICATION

0	1	0	0	1
1	0	1	0	0
0	0	1	0	1
1	0	1	0	0





# Preliminary results



H.Kim, EAK et al, in preparation (2023)

# Data-centric learning of Quantum Many-body States With Classical Machines

- X-TEC: Clustering in Reciprocal space
  - Comprehensive analysis enabling discoveries
  - Time-resolved X-ray and more...
- Supervised ML: Decoding Quantum Simulator Data
  - Order parameters and correlations from single instances
  - Learning complexity from a collection of instances





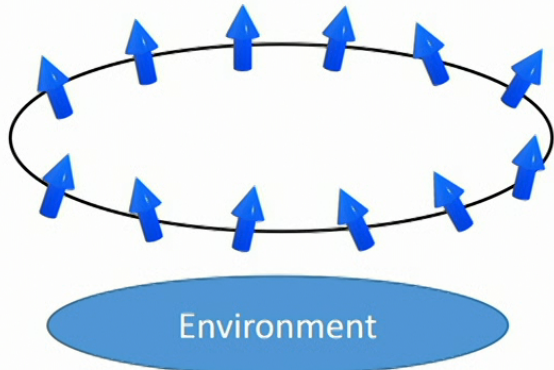
Dorothy Vaughan and West Area Computers becoming first programmers of IBM 7090 in 1961,  
From 'Hidden Figures', 20<sup>th</sup> Century Fox (2016)

# Dynamics of Open Quantum Systems

Why simulate Open Quantum Systems?

..to discover new physics:

A lack of computational tools prevents the exploration of new interesting physics:



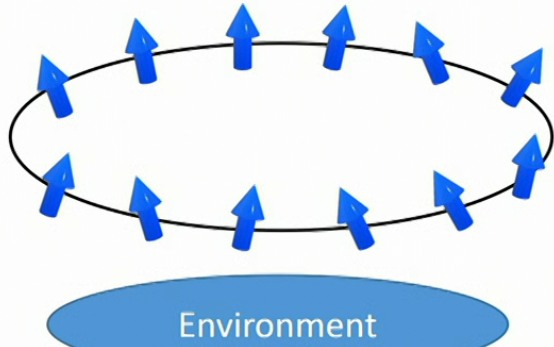


# Dynamics of Open Quantum Systems

Why simulate Open Quantum Systems?

..to discover new physics:

A lack of computational tools prevents the exploration of new interesting physics:



..as a benchmarking tool:

Quantum simulators are sensitive to outside noise – require tools to benchmark these devices:

