

Title: Unsupervised detection of quantum phases and their order parameters from projective measurements

Speakers:

Collection: Machine Learning for Quantum Many-Body Systems

Date: June 14, 2023 - 2:00 PM

URL: <https://pirsa.org/23060038>

Abstract: Recently, machine learning has become a powerful tool for detecting quantum phases. While the sole information about the presence of transition is valuable, the lack of interpretability and knowledge on the detected order parameter prevents this tool from becoming a customary element of a physicist's toolbox. Here, we report designing a special convolutional neural network with adaptive kernels, which allows for fully interpretable and unsupervised detection of local order parameters out of spin configurations measured in arbitrary bases. With the proposed architecture, we detect relevant and simplest order parameters for the one-dimensional transverse-field Ising model from any combination of projective measurements in the x, y, or z basis. Moreover, we successfully tackle the bilinear-biquadratic spin-1 model with a nontrivial nematic order. We also consider extending the proposed approach to detecting topological order parameters. This work can lead to integrating machine learning methods with quantum simulators studying new exotic phases of matter.

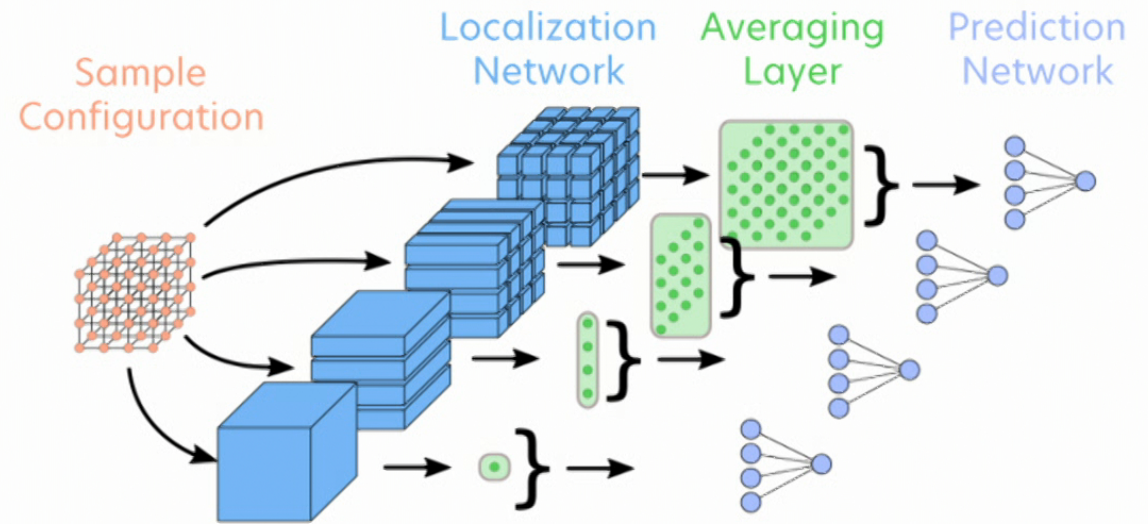
Unsupervised detection of quantum phases and their order parameters from projective measurements



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ML for QMBP at Perimeter Institute
June 14, 2023

Interpretation Net

S. Wetzel *et al.*



Receptive Field Size	Train Loss	Validation Loss
28×28	$6.1588e - 04$	0.0232
1×2	$1.2559e-04$	$1.2105e-07$
1×1	0.2015	0.1886
baseline	0.6931	0.6931

} separate trainings

Interpretation Net

S. Wetzel *et al.*

1x1

$$f(s_i) = f_0 + f_1 s_i + f_2 \underbrace{s_i^2}_1 + f_3 \underbrace{s_i^3}_{s_i} + \dots$$

$$F\left(\frac{1}{N} \sum_i f(s_i)\right)$$

1x2

$$f(s_i, s_j) = f_{0,0} + f_{1,0} s_i + f_{0,1} s_j + f_{2,0} s_i^2 + f_{1,1} s_i s_j + f_{0,2} s_j^2 + \dots$$

$$F\left(\frac{1}{N} \sum_{\langle i,j \rangle_T} s_i s_j\right)$$

Receptive Field Size	Train Loss	Validation Loss
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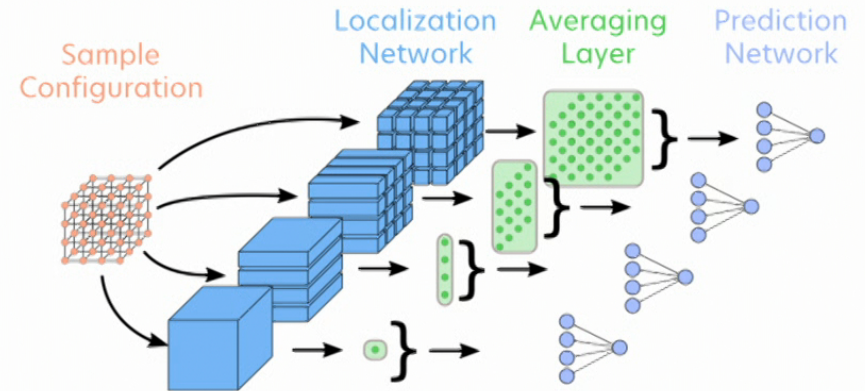


Phys. Rev. B 96, 184410 (2017)

Interpretation Net

S. Wetzel *et al.*

- It requires multiple training runs
The more complex correlators you consider, the larger number of training runs.
- Multi-class problems are challenging to implement
What if one phase is driven by one-body and another by two-body correlators?

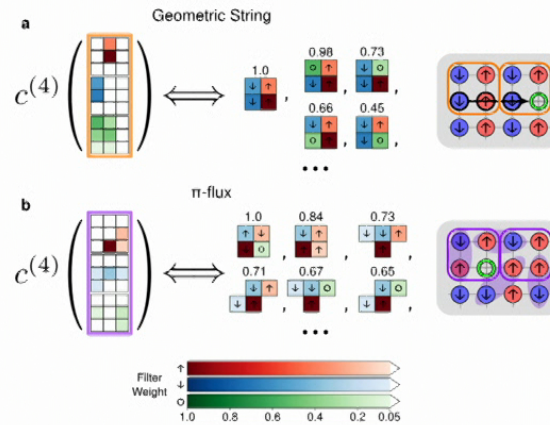
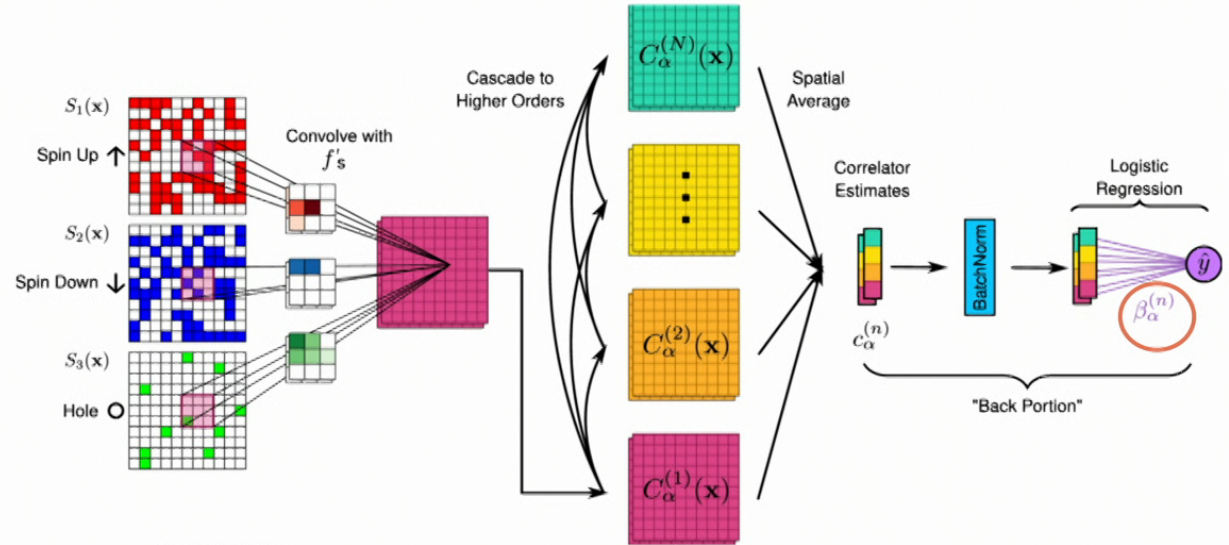
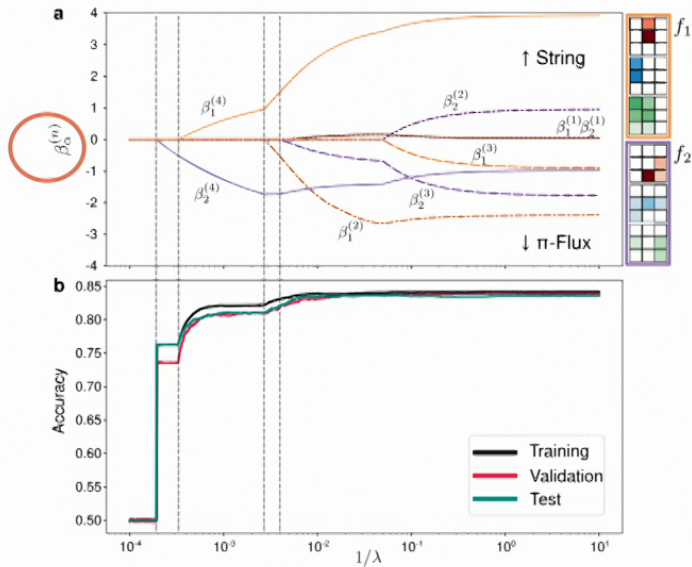


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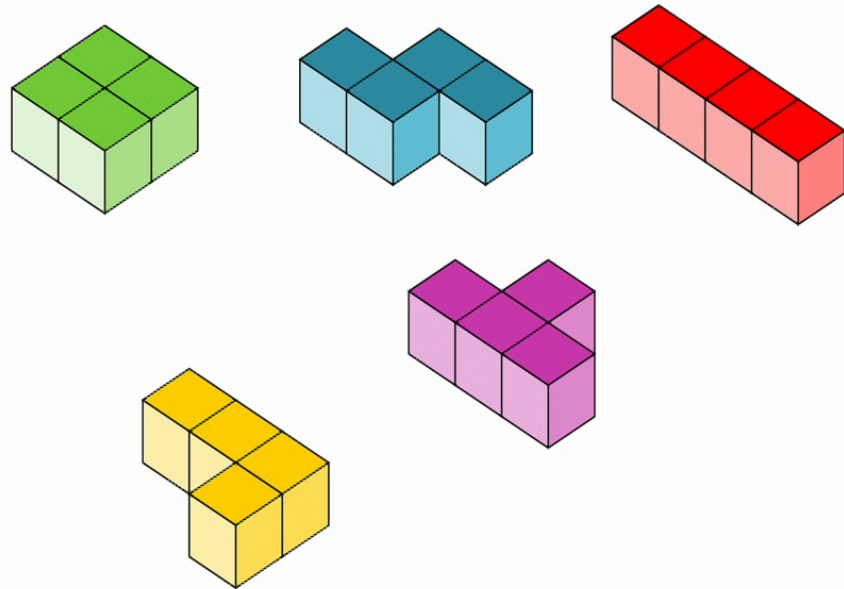
} separate trainings

Correlator CNN

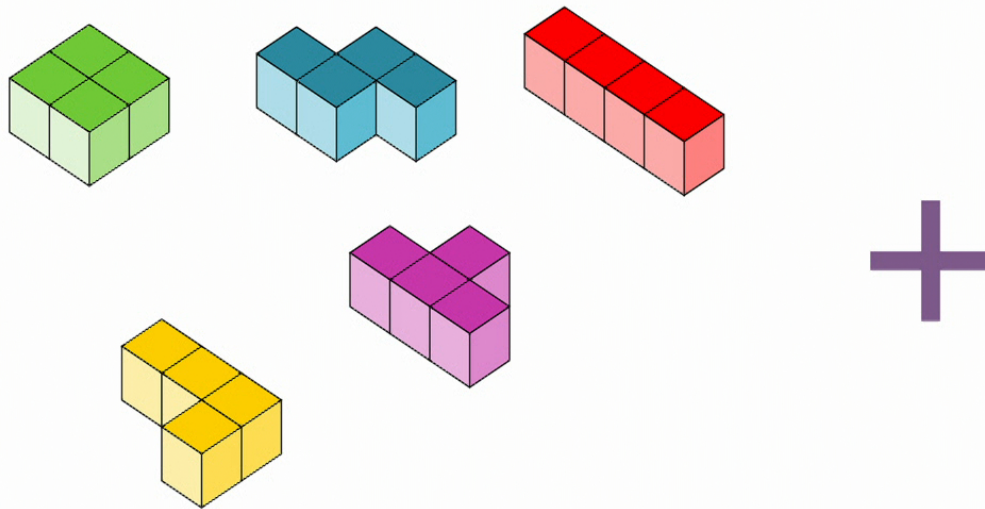
C. Miles, A. Bohrdt,
E.-A. Kim *et al.*



Tetris ~~Adaptive CNN~~



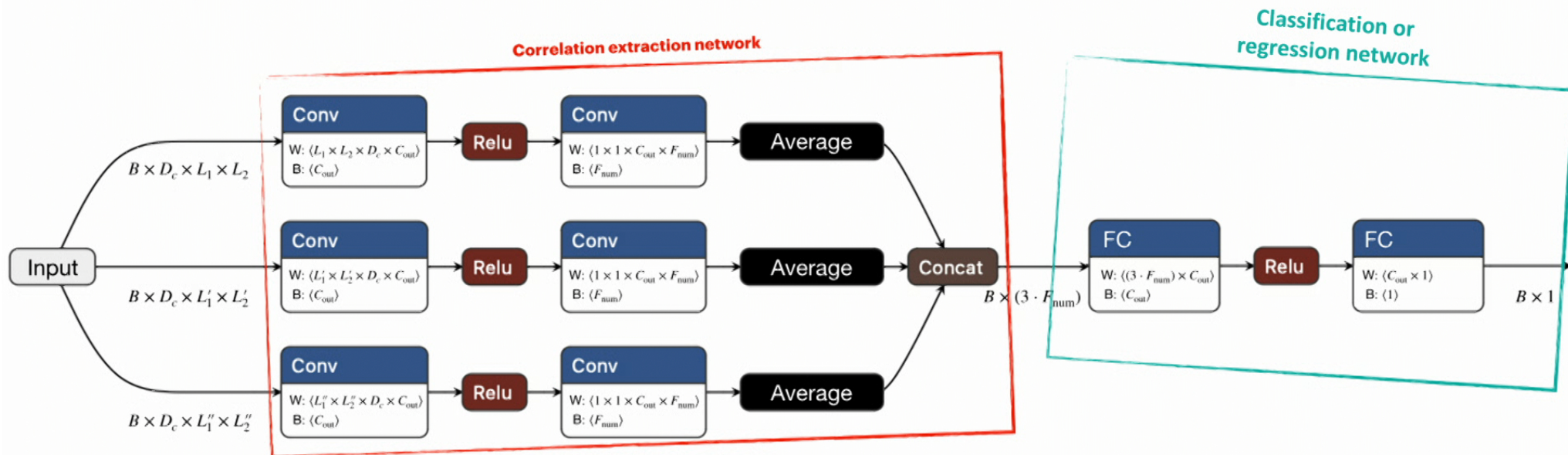
Tetris ~~Adaptive CNN~~



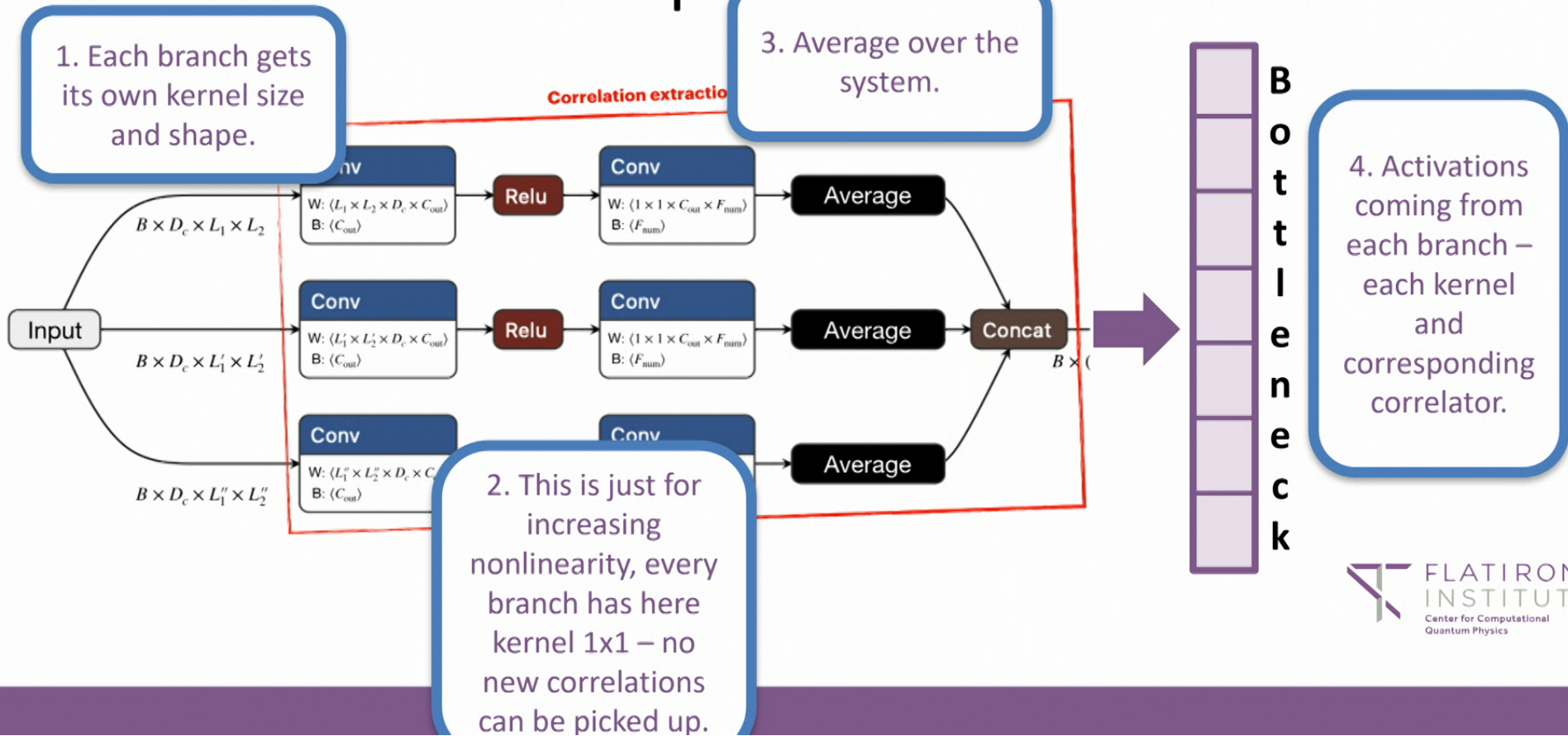
**Loss function
promoting the use of:**

1. simple kernels
(correlators)
2. the smallest number
of kernels (correlators)

Adaptive CNN for order parameter detection

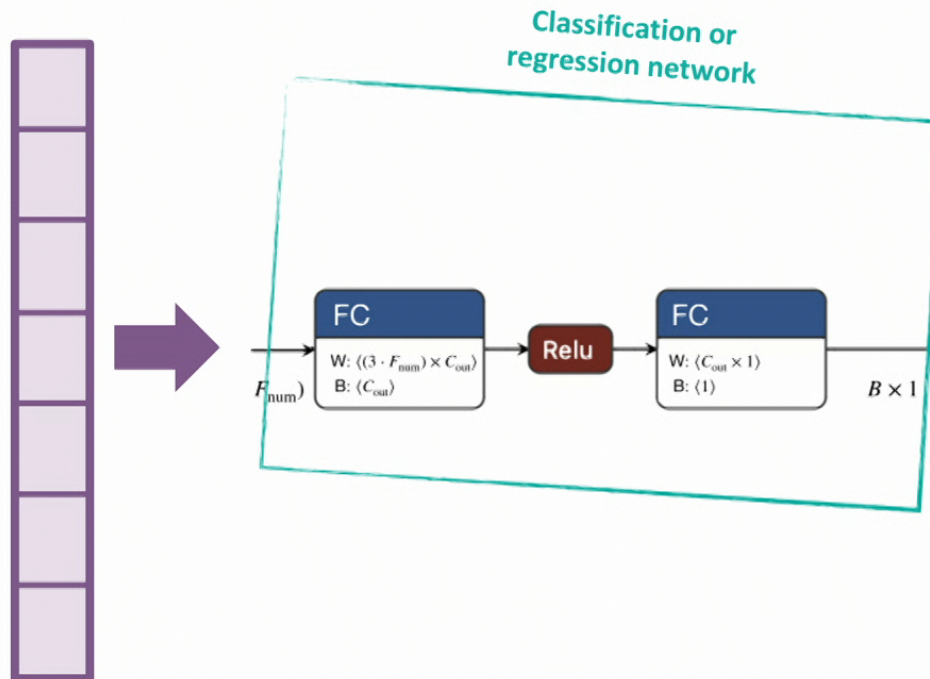


Adaptive CNN for order parameter detection

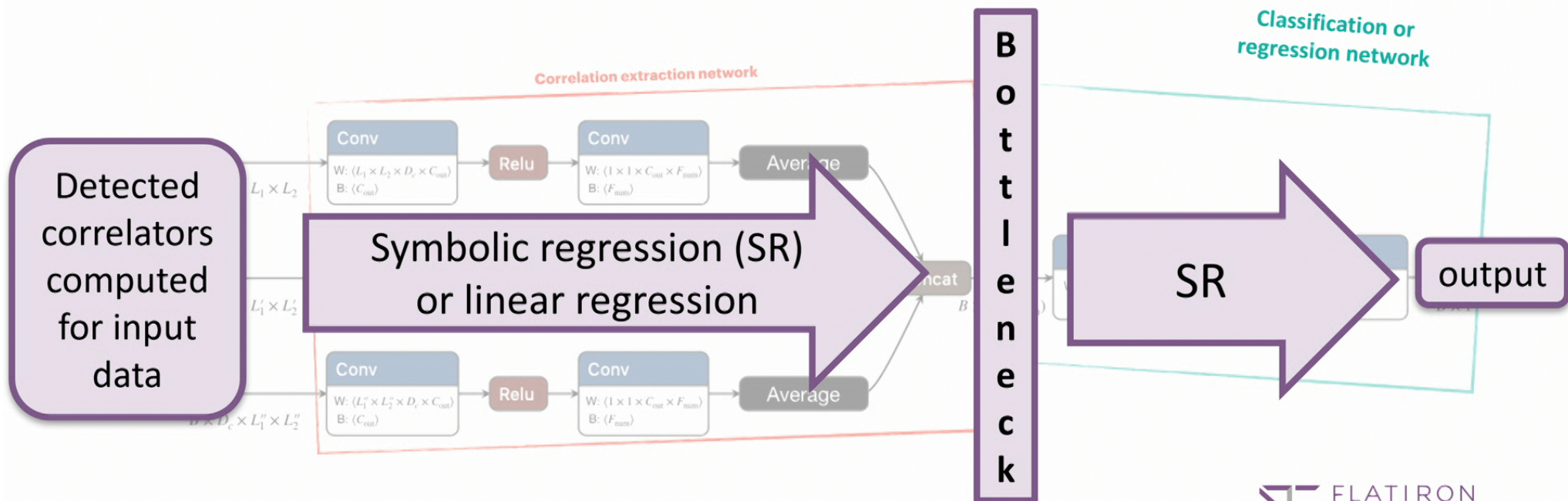


Adaptive CNN for order parameter detection

4. Activations coming from each branch – each kernel and corresponding correlator.



Adaptive CNN for order parameter detection



Outline



State-of-the-art of order
parameter detection



Adaptive CNN



Preliminary results

1D transverse-field Ising

$$\hat{H} = -J \left(\sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z + g \sum_j \hat{S}_j^x \right)$$

Two setups:

1. supervised classification: training data far from the transition
2. unsupervised analysis with regression (task: predict g)

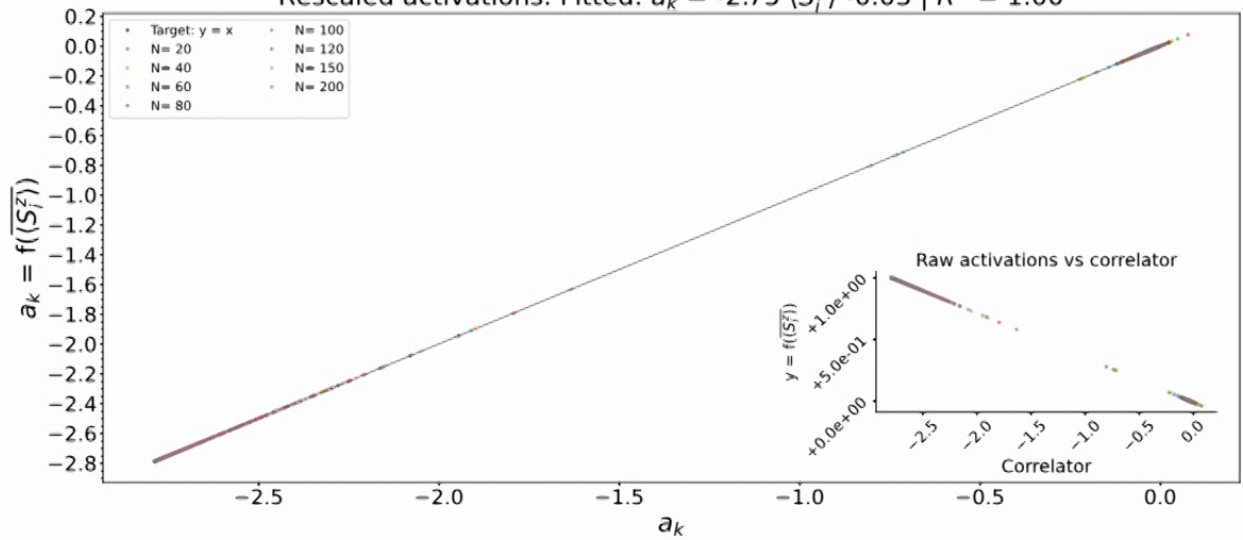
DMRG using ITensors.jl library in Julia

1D transverse-field Ising

Data in z basis

Kernel 0 activations vs correlators

Rescaled activations. Fitted: $a_k = -2.75 \overline{\langle S_i^z \rangle} - 0.03 \mid R^2 = 1.00$



Detected correlators computed for input data

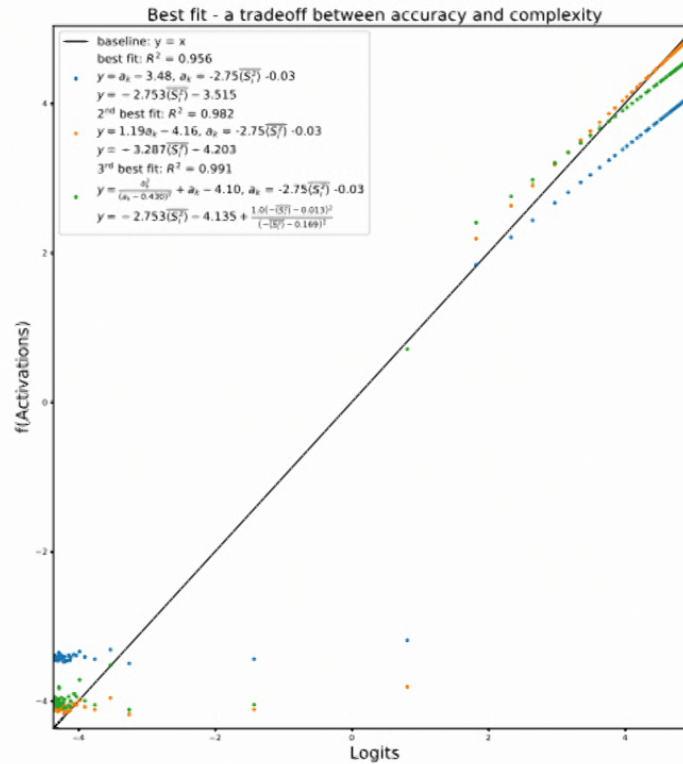
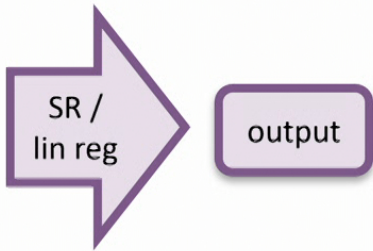
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1D transverse-field Ising

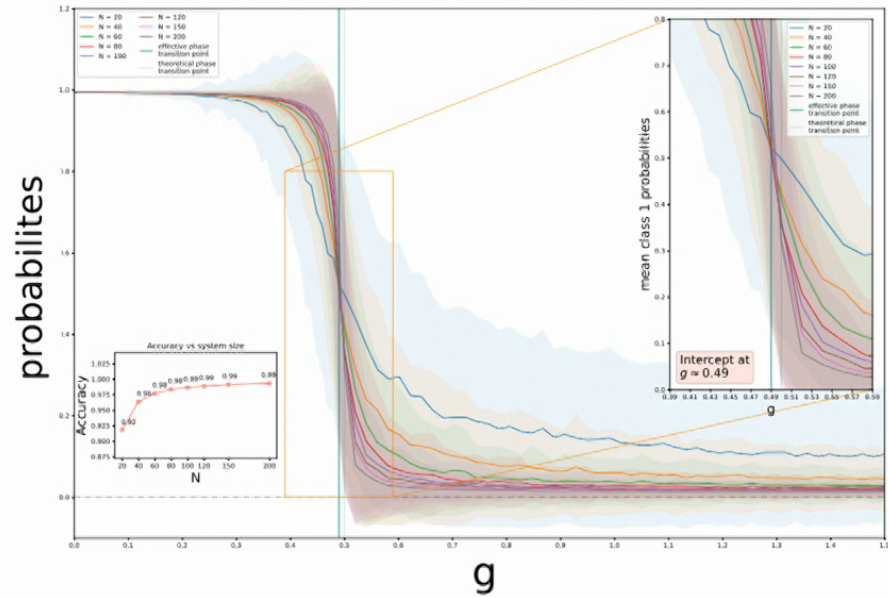
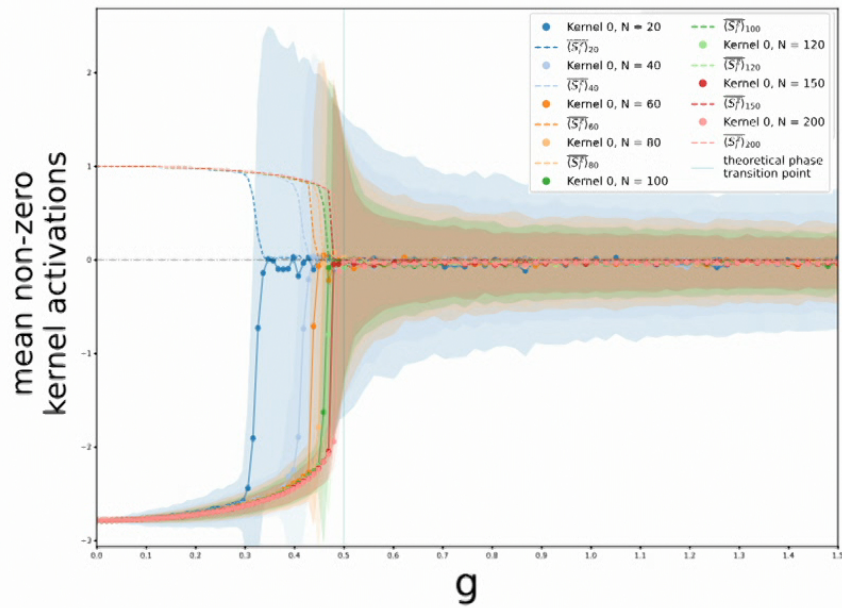
Data in z basis

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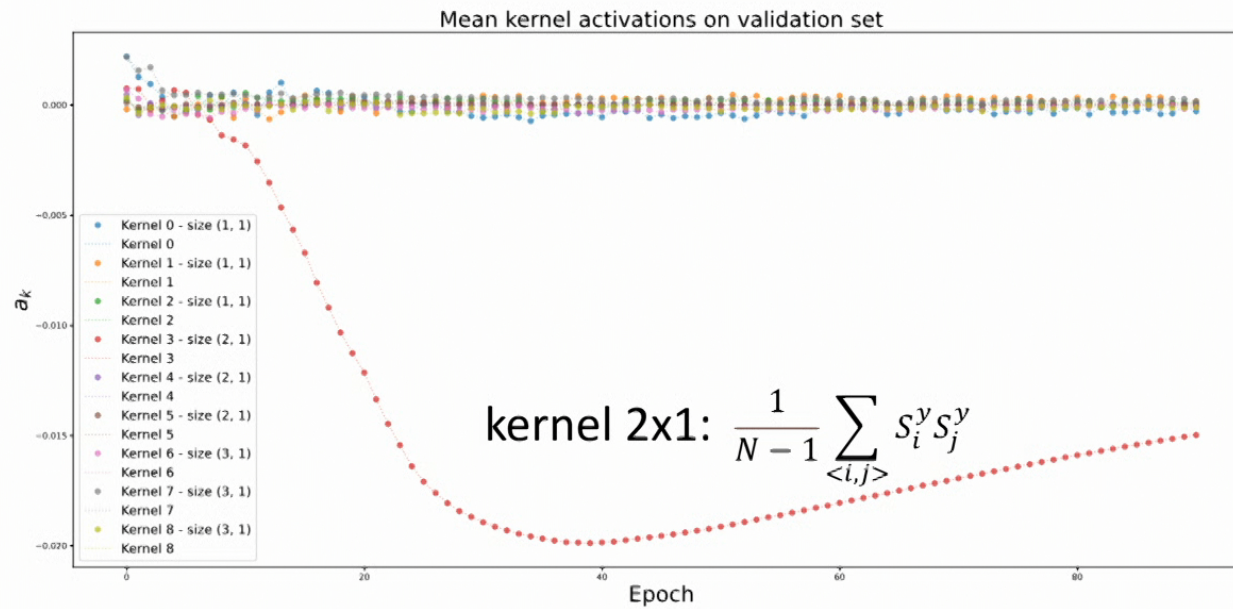
1D transverse-field Ising

Data in z basis



1D transverse-field Ising

Data in y basis

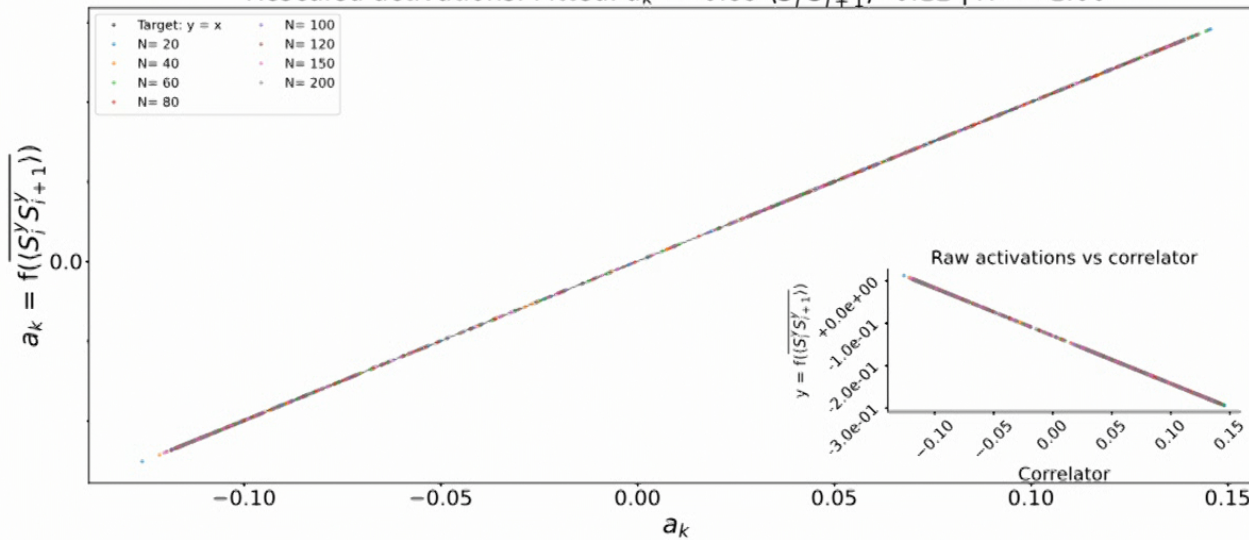


1D transverse-field Ising

Data in y basis

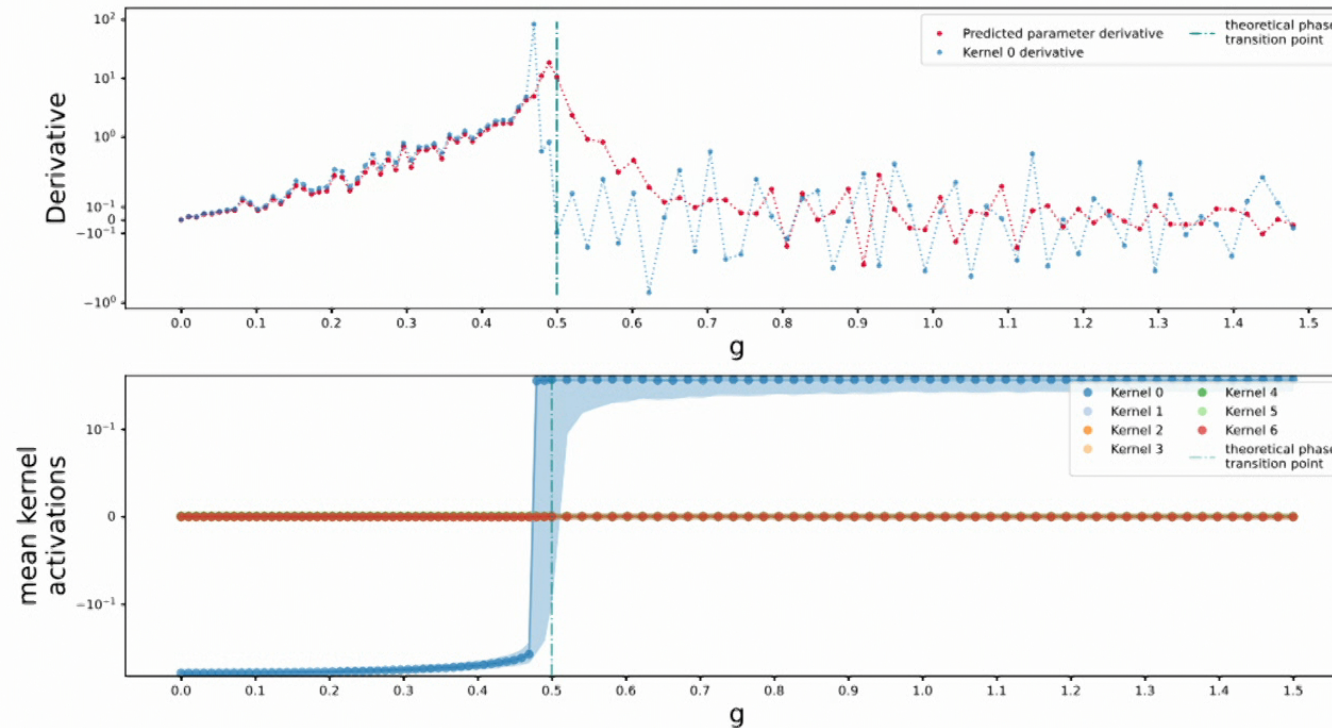
Kernel 3 activations vs correlators

Rescaled activations. Fitted: $a_k = -0.89 \overline{\langle S_i^y S_{i+1}^y \rangle} - 0.12 \mid R^2 = 1.00$



How to make it unsupervised?

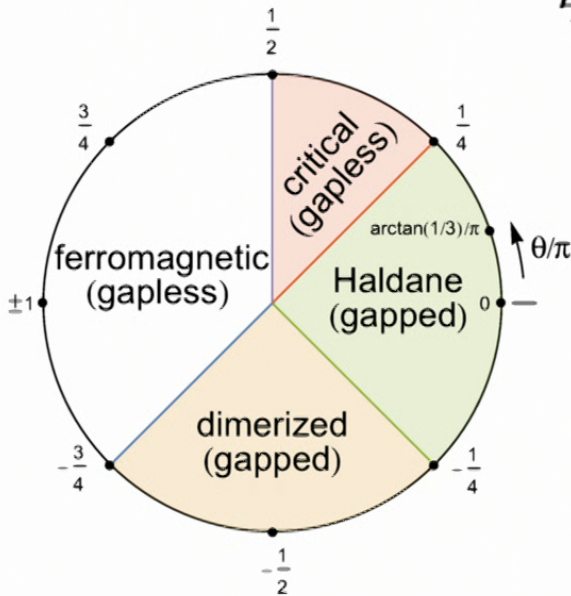
Prediction-based method by Eliška *et al!*



$$y = -1.176\overline{(S_i^2)} + 0.426$$

1D bilinear-biquadratic **spin-1** Heisenberg model

$$\hat{H} = \sum_{i=1}^N \cos(\theta) (\vec{S}_i \otimes \vec{S}_{i+1}) + \sin(\theta) (\vec{S}_i \otimes \vec{S}_{i+1})^2$$



We need to modify our Taylor expansion (spin-1!)

$$(S_i^d)^p = \begin{cases} (S_i^d)^2 & \text{if } p \text{ is even} \\ S_i^d & \text{if } p \text{ is odd} \end{cases} \implies f(S_i^z, S_{i+1}^z) = C_1 + C_2 S_i^z + C_3 S_{i+1}^z + C_4 S_i^z S_{i+1}^z + C_5 (S_i^z)^2 + C_6 (S_{i+1}^z)^2 + C_7 (S_i^z)^2 S_{i+1}^z + C_8 S_i^z (S_{i+1}^z)^2 + C_9 (S_i^z S_{i+1}^z)^2$$

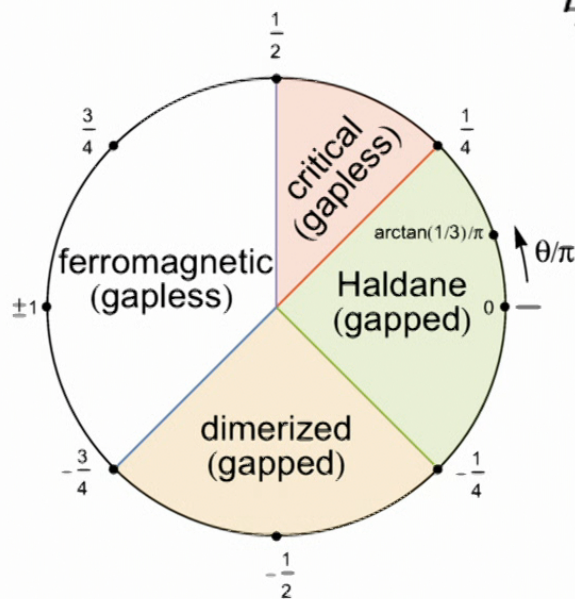
$(S_i^d)^2 \in \{0,1\}$
 $S_i^d \in \{-1,0,1\}$

M. V. Rakov & M. Weyrauch (2022)
Phys. Rev. B 105, 024424

DMRG using ITensors.jl library in Julia

1D bilinear-biquadratic **spin-1** Heisenberg model

$$\hat{H} = \sum_{i=1}^N \cos(\theta) (\vec{S}_i \otimes \vec{S}_{i+1}) + \sin(\theta) (\vec{S}_i \otimes \vec{S}_{i+1})^2$$



M. V. Rakov & M. Weyrauch (2022)
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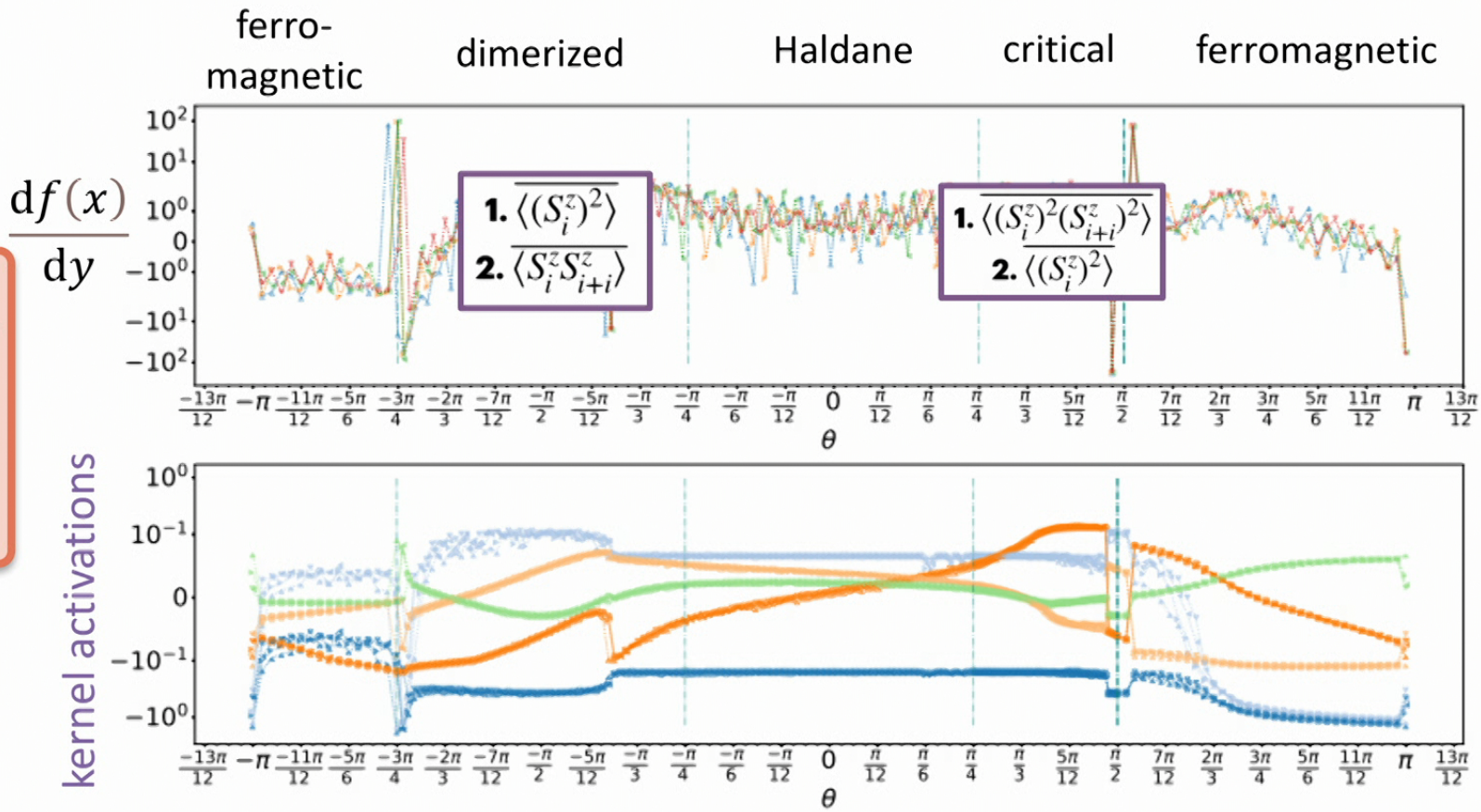
Preliminary!!

- Because our DMRG-generated data is suboptimal
- Because we need to regularize the CNN more

DMRG using ITensors.jl library in Julia

1D bilinear-biquadratic spin-1 Heisenberg model

Preliminary!!



various N

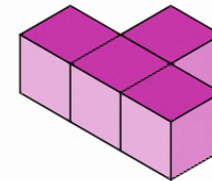
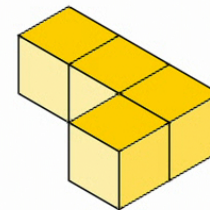
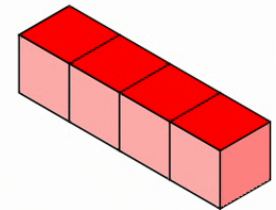
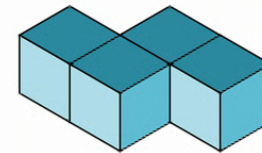
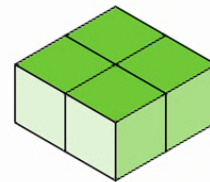
$1 \times 1, 1 \times 1$
 $2 \times 1, 2 \times 1$
 $2 \times 1 +$
 dilation



Läuchli et al. (2006) Phys. Rev. B 74, 144426

Extensions

1. Design special kernels to detect string-type order parameters
2. Apply to 2D: symmetric and non-symmetric gapped convolutions
3. Apply to 2D with topological order (Ising gauge theory-ready!)
4. Combine with Siamese NNs
5. Adaptive convolution kernels,
arXiv:2009.06385



Take-home message

Force your NN to speak the language you know!

Beware! Limits expressivity!