Title: Near Term Distributed Quantum Computation using Optimal Auxiliary Encoding

Speakers:

Collection: Machine Learning for Quantum Many-Body Systems

Date: June 12, 2023 - 2:00 PM

URL: https://pirsa.org/23060030

Abstract: ZOOM: https://pitp.zoom.us/j/94595394881?pwd=OUZSSXpzYlhFcGlIRm81Y3VaYVpCQT09

NEAR TERM DISTRIBUTED QUANTUM COMPUTATION USING OPTIMAL AUXILIARY ENCODING

Abigail McClain Gomez, Taylor Patti,
Anima Anandkumar, and Susanne Yelin

6/12/2023

INGREDIENTS

- Distributed Quantum Computing with Limited Information Transfer
- Mean-Field Corrections
- Auxiliary Qubits
- Optimal Encoding

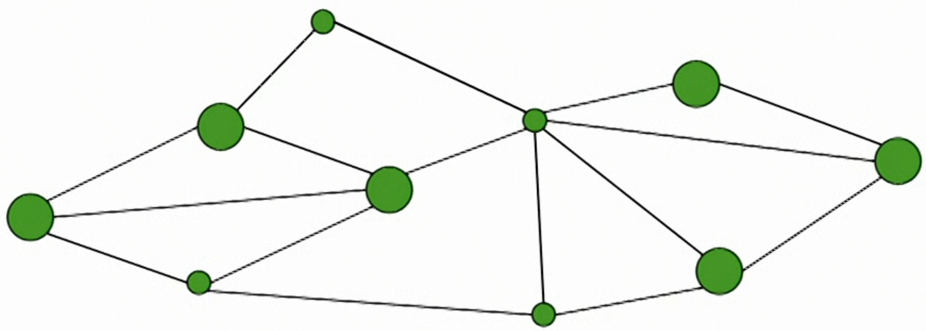
FRAGMENTED TIME EVOLUTION

- Classical Link Results
- Quantum Link Results
- Role of Auxiliary Encoding

FRAGMENT-INITIALIZED VQE

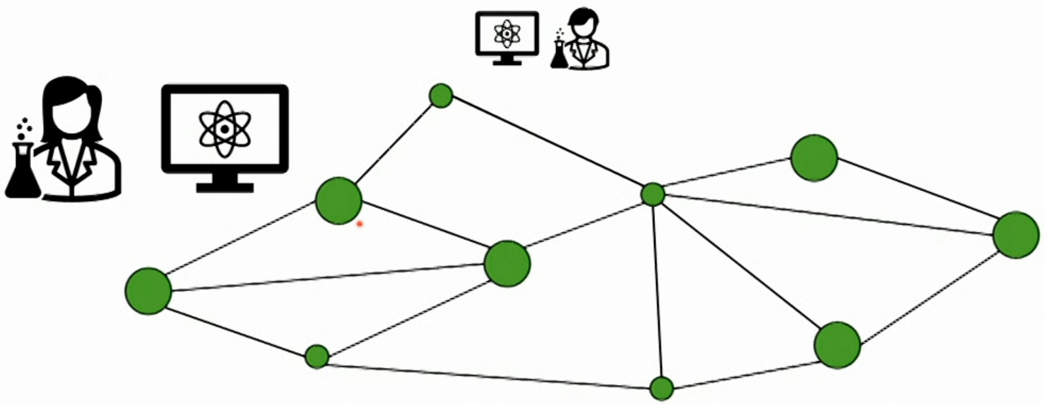
- Initialization Scheme
- Comparison to Random Initialization

DISTRIBUTED QUANTUM COMPUTING



Each **node** is a quantum simulator.

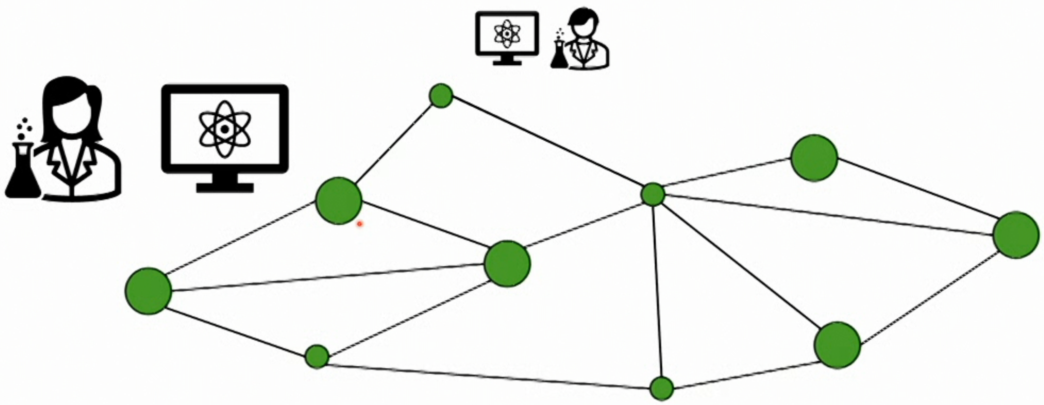
DISTRIBUTED QUANTUM COMPUTING



Each **node** is a quantum simulator.

The nodes are in communication with each other, forming a **network**.

DISTRIBUTED QUANTUM COMPUTING

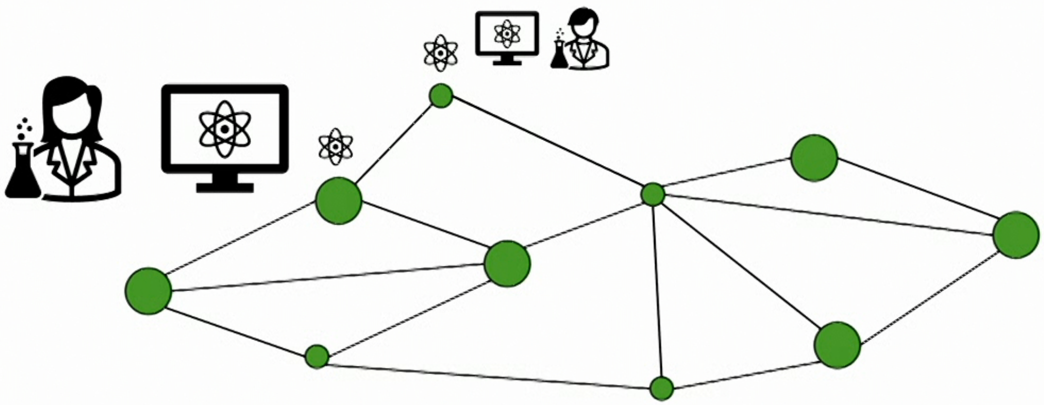


Each **node** is a quantum simulator.

The nodes are in communication with each other, forming a **network**.

Through cooperation, the allocated resources can address a single quantum computing task.

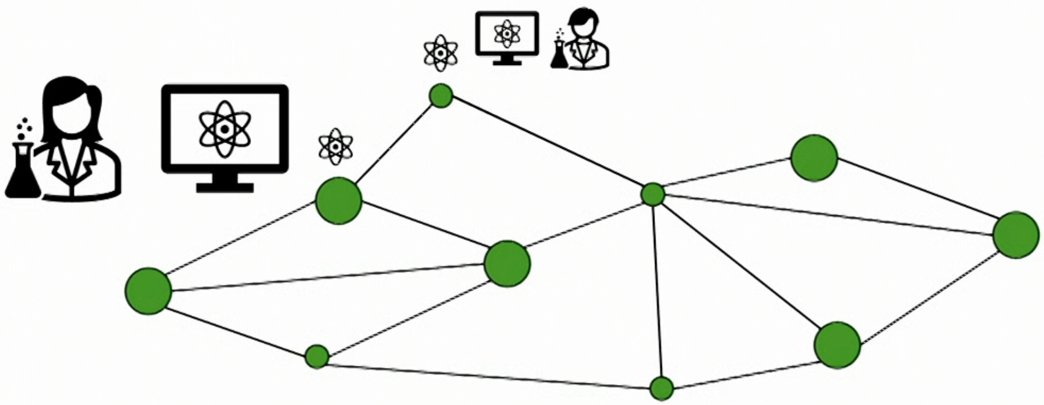
DISTRIBUTED QUANTUM COMPUTING



Non-local operations are required for the distributed simulators to behave as a cooperative network.

In the near-term, non-local operations are **costly**!

DISTRIBUTED QUANTUM COMPUTING



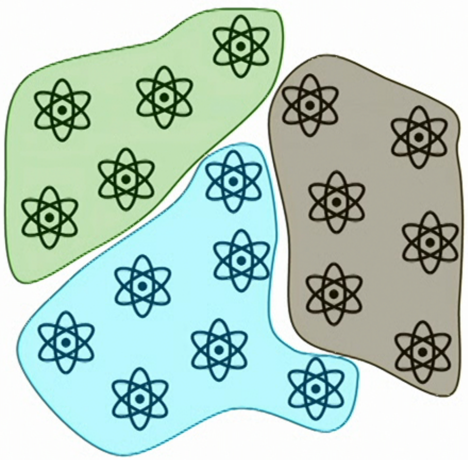
Non-local operations are required for the distributed simulators to behave as a cooperative network.

In the near-term, non-local operations are **costly**!

Motivating Question:

To what extent is distributed quantum computation viable when limited information is passed between simulators?

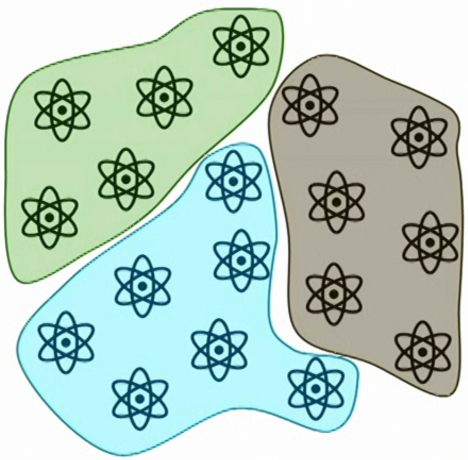
A FRAGMENTED SYSTEM



Motivating Question:

To what extent is distributed quantum computation viable when **limited information** is passed between simulators?

A FRAGMENTED SYSTEM



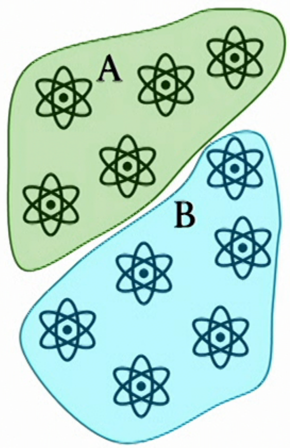
Motivating Question:

To what extent is distributed quantum computation viable when limited information is passed between simulators?

Classical Link – classical channel between distributed simulators for limited classical information transfer

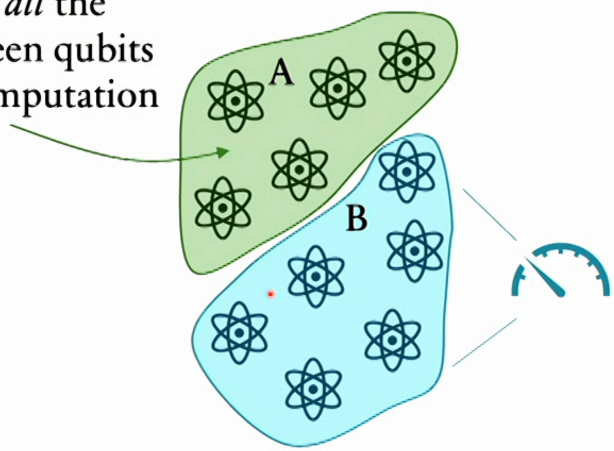
Quantum Link – quantum channel between distributed simulators for limited quantum + classical information transfer

PASSING INFORMATION ACROSS THE INTERFACE



PASSING INFORMATION ACROSS THE INTERFACE

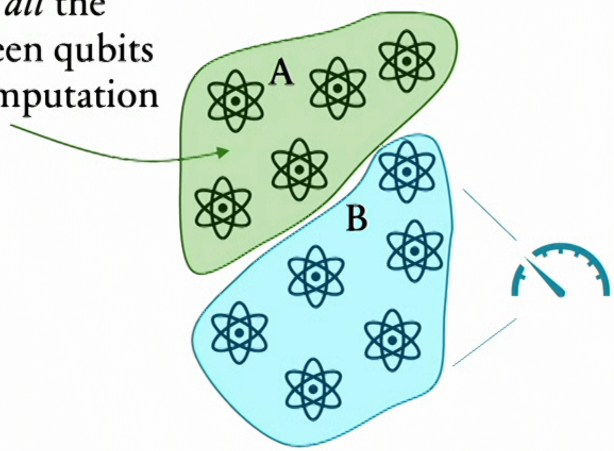
Within a fragment, *all* the **interactions** V_{ij} between qubits are included in the computation



Use measurement-informed ***mean-field corrections*** to approximate the presence of environment qubits

PASSING INFORMATION ACROSS THE INTERFACE

Within a fragment, *all* the **interactions** V_{ij} between qubits are included in the computation

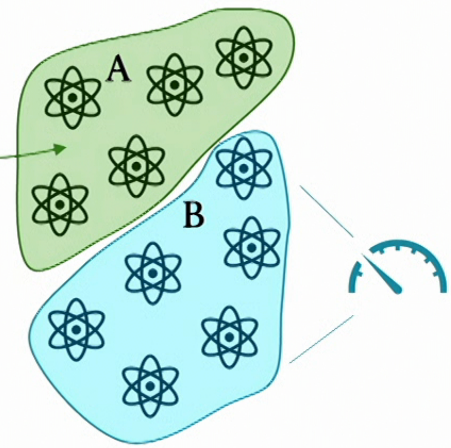


Use measurement-informed ***mean-field corrections*** to approximate the presence of environment qubits

Use ***extra qubits*** ("auxiliary qubits") to facilitate the computation through additional interactions.

PASSING INFORMATION ACROSS THE INTERFACE

Within a fragment, *all* the **interactions** V_{ij} between qubits are included in the computation



Use measurement-informed ***mean-field corrections*** to approximate the presence of environment qubits

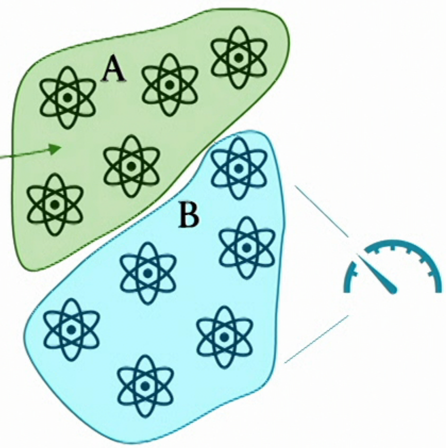
Use ***extra qubits*** ("auxiliary qubits") to facilitate the computation through additional interactions.

We are going to focus on spin model Hamiltonians:

$$H = -\sum_{j<i} J_{ij}\hat{S}_z^i\hat{S}_z^j - h\sum_{i=1}^{N}\hat{S}_x^i$$

PASSING INFORMATION ACROSS THE INTERFACE

Within a fragment, *all* the **interactions** V_{ij} between qubits are included in the computation



Use measurement-informed ***mean-field corrections*** to approximate the presence of environment qubits

Use ***extra qubits*** ("auxiliary qubits") to facilitate the computation through additional interactions.

We are going to focus on spin model Hamiltonians:

$$H = -\sum_{j<i} J_{ij} \hat{S}_z^i \hat{S}_z^j - h \sum_{i=1}^{N} \hat{S}_x^i$$

$$H_{f=A} = -\sum_{j<i\in A} J_{ij} \hat{S}_z^i \hat{S}_z^j - h \sum_{i=1}^{N_A} \hat{S}_x^i + V_{corr}(B)$$

MEAN-FIELD FRAGMENT CORRECTION

To mimic the presence of the environment qubits, use **mean-field corrections**

Quantum Ising-Like Model: $$H = -\sum_{j<i} J_{ij}\hat{S}_z^i\hat{S}_z^j - h\sum_i \hat{S}_x^i$$

Mean-Field Approximation: $$H_{MF} = -\sum_{j<i} J_{ij}\left(\hat{S}_z^i\left\langle\hat{S}_z^j\right\rangle + \hat{S}_z^j\langle\hat{S}_z^i\rangle\right) - h\sum_i \hat{S}_x^i$$

MEAN-FIELD FRAGMENT CORRECTION

To mimic the presence of the environment qubits, use **mean-field corrections**

Quantum Ising-Like Model: $$H = -\sum_{j<i} J_{ij} \hat{S}_z^i \hat{S}_z^j - h \sum_i \hat{S}_x^i$$

Mean-Field Approximation: $$H_{MF} = -\sum_{j<i} J_{ij} \left(\hat{S}_z^i \left\langle \hat{S}_z^j \right\rangle + \hat{S}_z^j \langle \hat{S}_z^i \rangle \right) - h \sum_i \hat{S}_x^i$$

Fragment Hamiltonian w/ Mean-Field Correction:

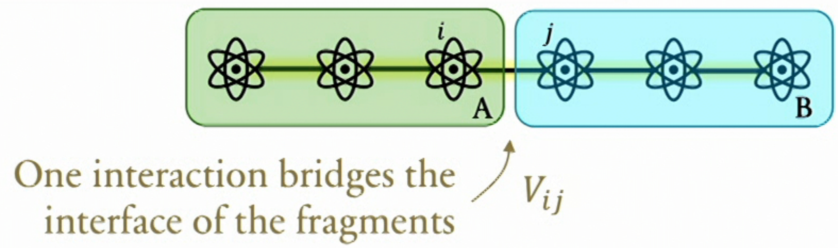
$$H_{f=A} = -\sum_{j<i\in A} J_{ij} \hat{S}_z^i \hat{S}_z^j - h \sum_{i\in A} \hat{S}_x^i - \sum_{i\in A} b_i \left(\left\{ \left\langle \hat{S}_z^j \right\rangle \forall j \in B \right\} \right) \hat{S}_z^i$$

$$b_i \left(\left\{ \left\langle \hat{S}_z^j \right\rangle \forall j \in B \right\} \right) = \sum_{j\in B} J_{ij} \left\langle \hat{S}_z^j \right\rangle$$

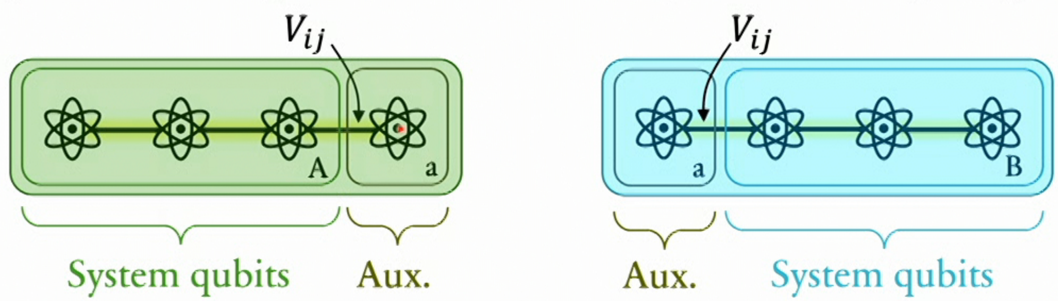
AUXILIARY QUBITS

To enable entanglement beyond a single fragment, incorporate extra "auxiliary" qubits that can mediate additional interactions with the system qubits.

Nearest Neighbor interactions:



Have the auxiliary qubit in each fragment interact with the system according to V_{ij}:



OPTIMAL AUXILIARY ENCODING

Which interactions are "most important"?

Select auxiliaries to minimize the short time simulation error due to fragmentation

$$\epsilon = 1 - \left|\langle U_f^\dagger(dt) U(dt)\rangle\right|^2 = var(H - H_f)dt^2 + \mathcal{O}(dt^4)$$

Rule: encode the environment qubit that has the largest contribution to $var(H - H_f)$.

$$H - H_f = -\sum_{\langle i,j\rangle \in I} J_{ij}\hat{S}_z^i\hat{S}_z^j$$

OPTIMAL AUXILIARY ENCODING

Which interactions are "most important"?

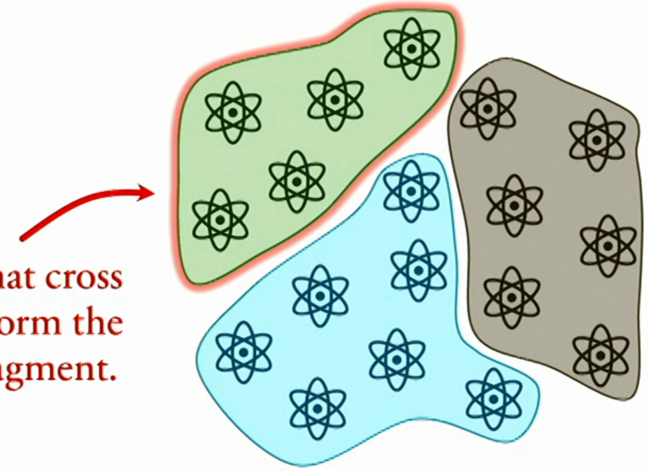
Select auxiliaries to minimize the short time simulation error due to fragmentation

$$\epsilon = 1 - \left|\langle U_f^\dagger(dt)U(dt)\rangle\right|^2 = var\left(H - H_f\right)dt^2 + \mathcal{O}(dt^4)$$

Rule: encode the environment qubit that has the largest contribution to $var\left(H - H_f\right)$.

$$H - H_f = -\sum_{\langle i,j\rangle \in I} J_{ij}\hat{S}_z^i\hat{S}_z^j$$

Any interactions that cross the red boundary form the interface of this fragment.



INGREDIENTS

- Distributed Quantum Computing with Limited Information Transfer
- Mean-Field Corrections
- Auxiliary Qubits
- Optimal Encoding

FRAGMENTED TIME EVOLUTION

- Classical Link Results
- Quantum Link Results
- Role of Auxiliary Encoding

FRAGMENT-INITIALIZED VQE

- Initialization Scheme
- Comparison to Random Initialization

AUXILIARY QUBITS: CLASSICAL VS. QUANTUM LINK

$$\epsilon = 1 - \left|\langle U_f^\dagger(dt)U(dt)\rangle\right|^2 = var\left(H - H_f\right)dt^2 + \mathcal{O}(dt^4)$$

Rule: encode the environment qubit(s) that has the largest contribution to $var\left(H - H_f\right)$.

Classical Links: Estimate the error contribution of each environment qubit after one time step to select the auxiliary encoding – this choice will be **fixed** throughout the simulation.

The auxiliary qubits are *extra* qubits included in each fragment!

AUXILIARY QUBITS: CLASSICAL VS. QUANTUM LINK

$$\epsilon = 1 - \left|\langle U_f^\dagger(dt)U(dt)\rangle\right|^2 = var\left(H - H_f\right)dt^2 + \mathcal{O}(dt^4)$$

Rule: encode the environment qubit(s) that has the largest contribution to $var\left(H - H_f\right)$.

Classical Links: Estimate the error contribution of each environment qubit after one time step to select the auxiliary encoding – this choice will be **fixed** throughout the simulation.

The auxiliary qubits are *extra* qubits included in each fragment!

Quantum Links: Use **nonlocal operations** to selectively shuttle/teleport only a handful of qubits, and **actively adapt** this choice as the variance measurements change over time.

AUXILIARY QUBITS: CLASSICAL VS. QUANTUM LINK

$$\epsilon = 1 - \left|\langle U_f^\dagger(dt)U(dt)\rangle\right|^2 = var\left(H - H_f\right)dt^2 + \mathcal{O}(dt^4)$$

Rule: encode the environment qubit(s) that has the largest contribution to $var\left(H - H_f\right)$.

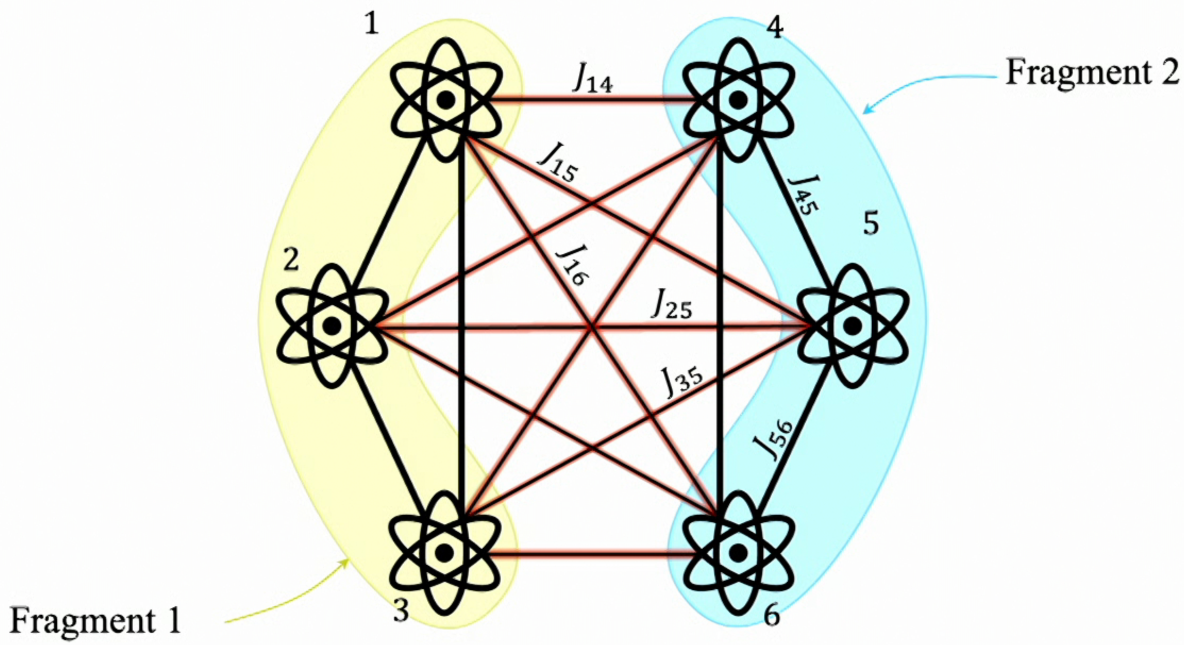
Classical Links: Estimate the error contribution of each environment qubit after one time step to select the auxiliary encoding – this choice will be **fixed** throughout the simulation.

The auxiliary qubits are *extra* qubits included in each fragment!

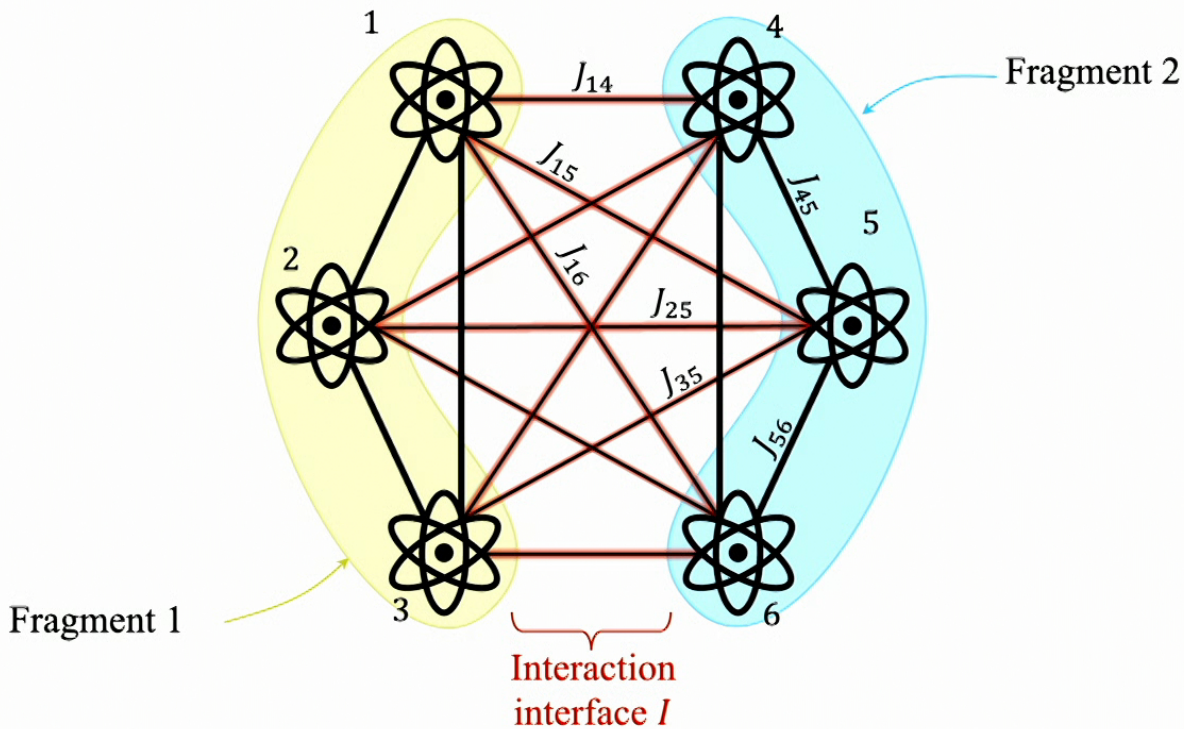
Quantum Links: Use **nonlocal operations** to selectively shuttle/teleport only a handful of qubits, and **actively adapt** this choice as the variance measurements change over time.

The information held by auxiliary qubits is *shared* between fragments!

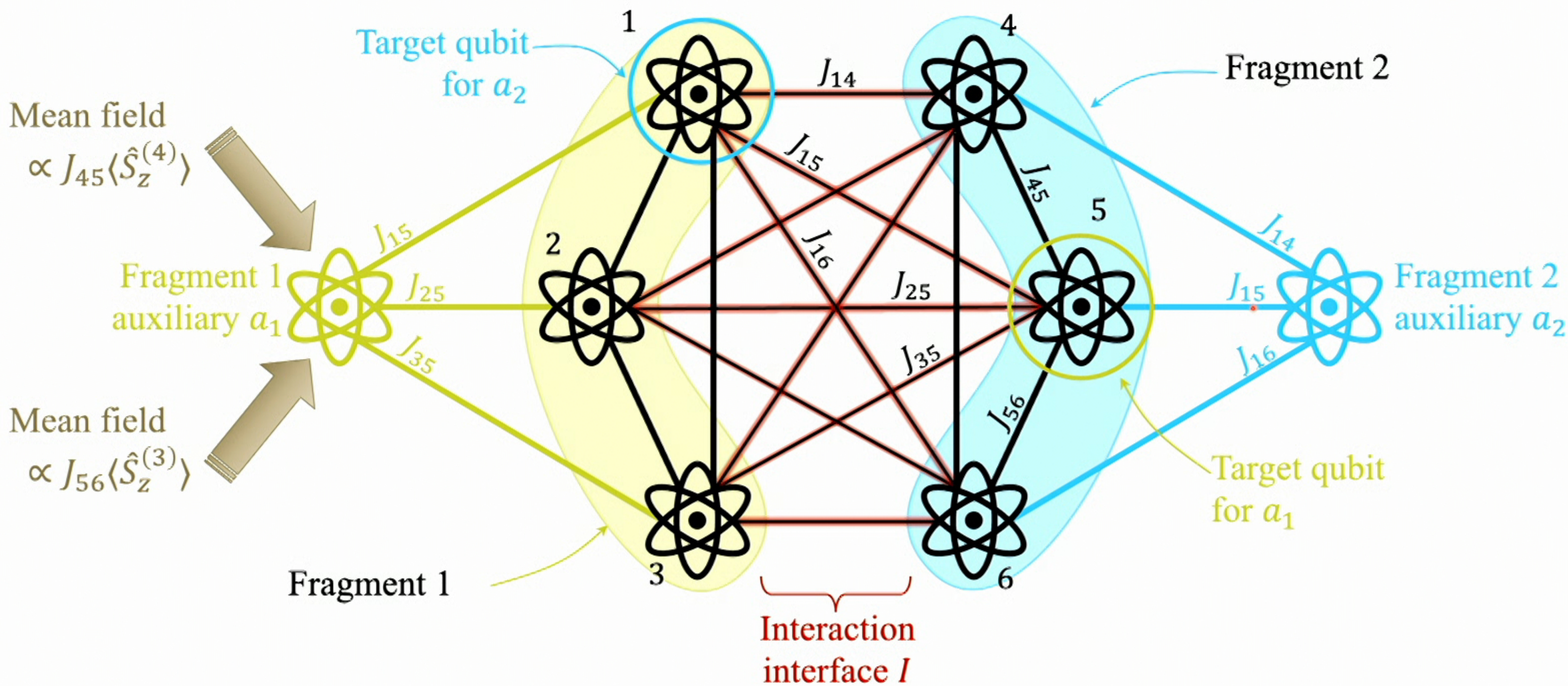
CLASSICAL LINK: AUX. QUBITS + MF CORRECTIONS



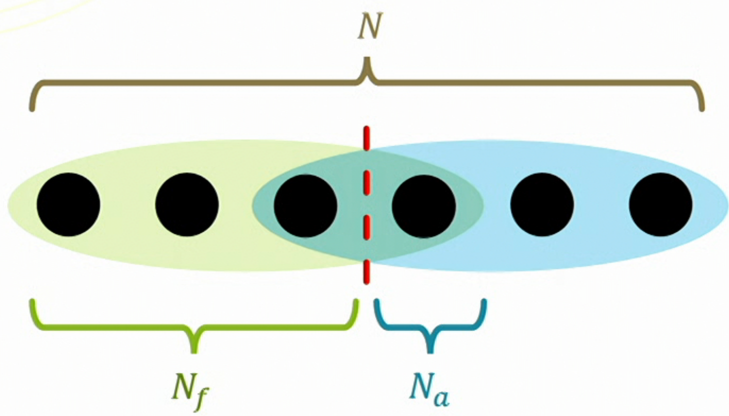
CLASSICAL LINK: AUX. QUBITS + MF CORRECTIONS



CLASSICAL LINK: AUX. QUBITS + MF CORRECTIONS



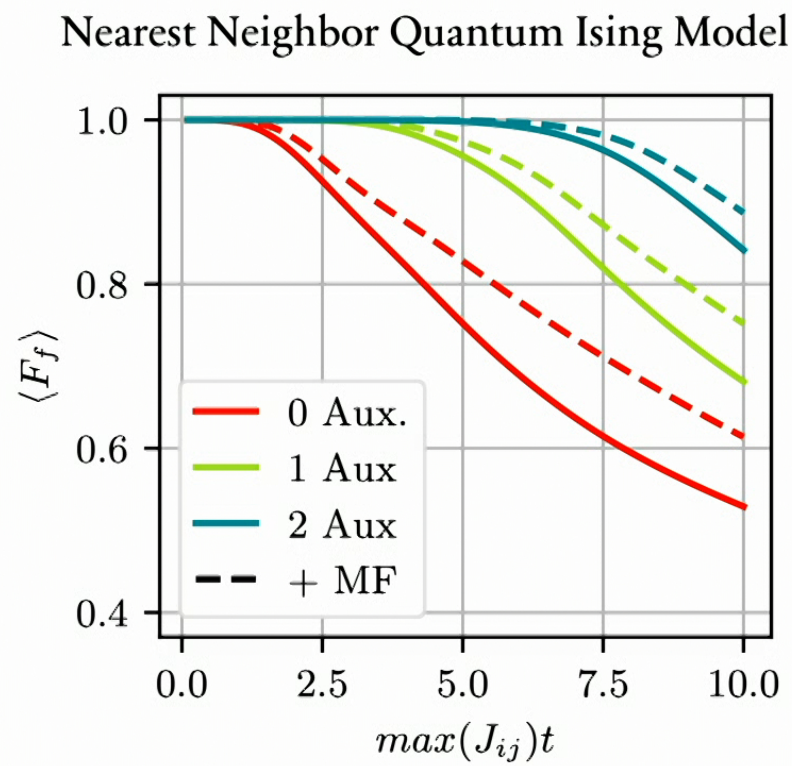
CLASSICAL LINK: TIME SIMULATION RESULTS



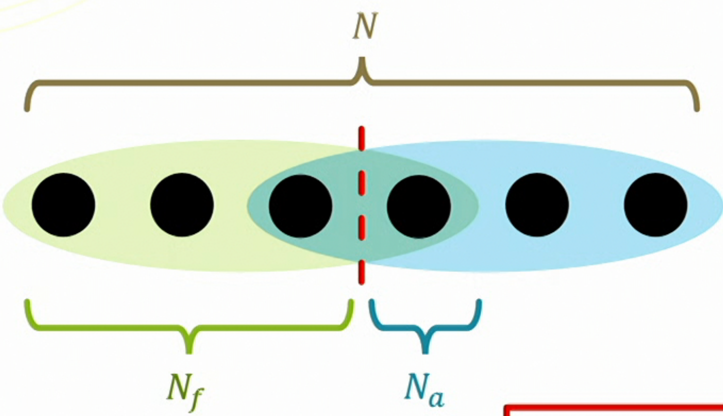
$N = 12$

$N_f = 6$

$N_a = 0, 1, 2$



CLASSICAL LINK: TIME SIMULATION RESULTS



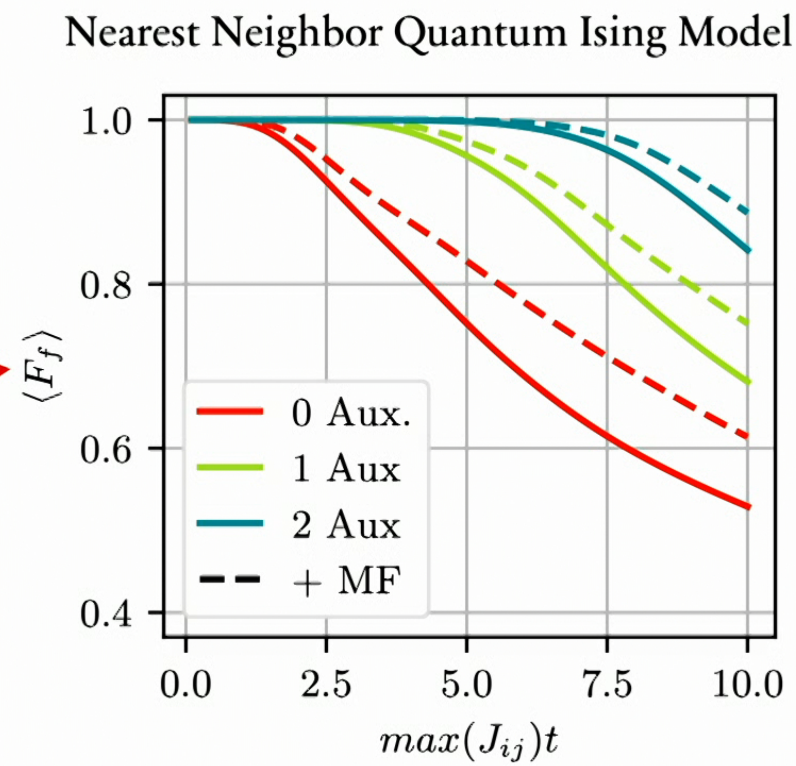
$N = 12$

$N_f = 6$

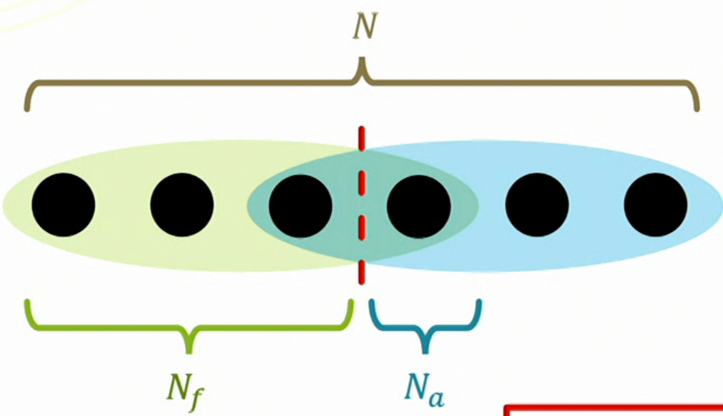
$N_a = 0, 1, 2$

Fragment Fidelity:

$$F_f = \left(Tr\sqrt{\sqrt{\rho_f}\rho_f^{ex}\sqrt{\rho_f}}\right)^2$$



CLASSICAL LINK: TIME SIMULATION RESULTS



$N = 12$

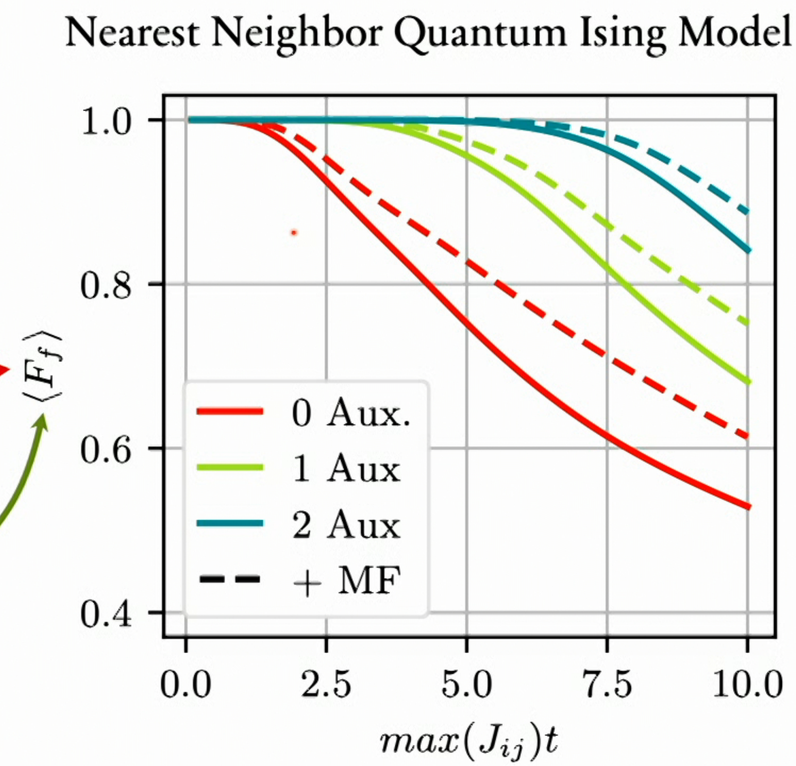
$N_f = 6$

$N_a = 0, 1, 2$

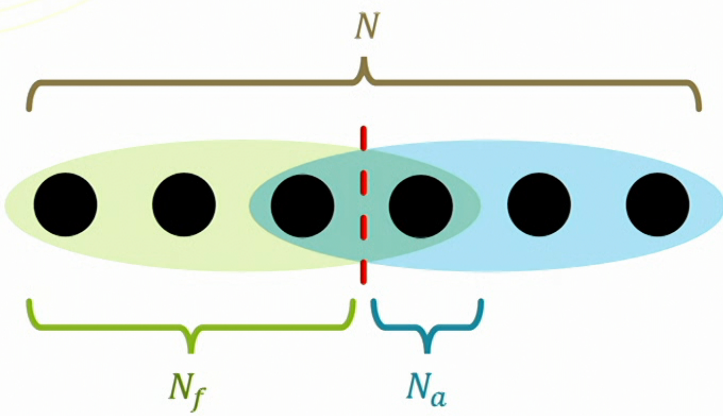
Fragment Fidelity:

$$F_f = \left(Tr\sqrt{\sqrt{\rho_f}\rho_f^{ex}\sqrt{\rho_f}}\right)^2$$

Averaged over transverse field h



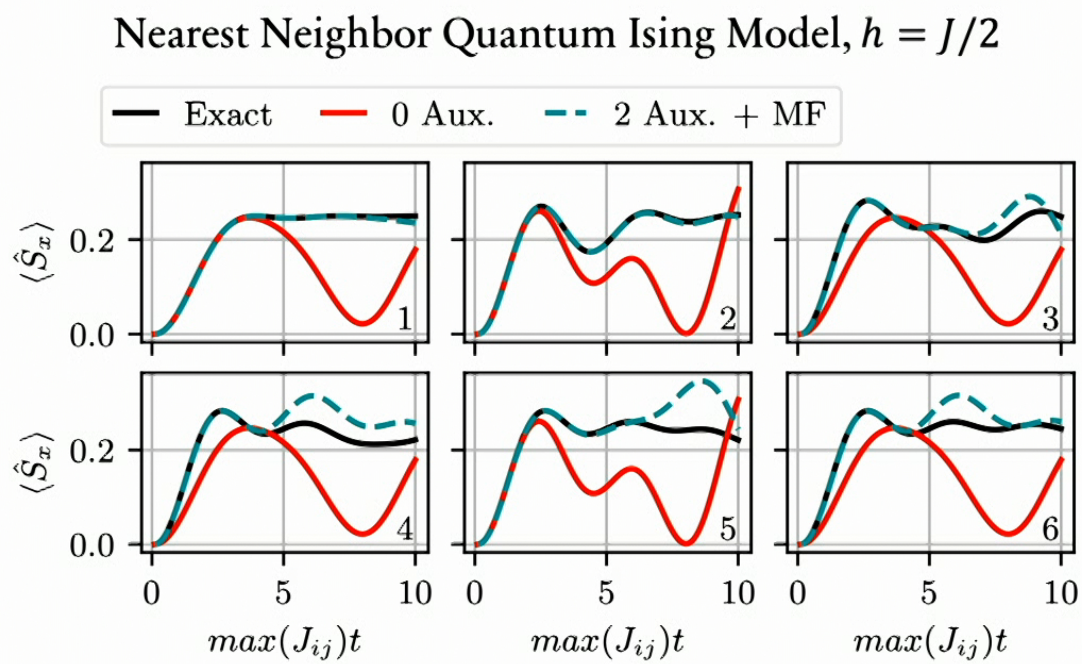
CLASSICAL LINK: TIME SIMULATION RESULTS



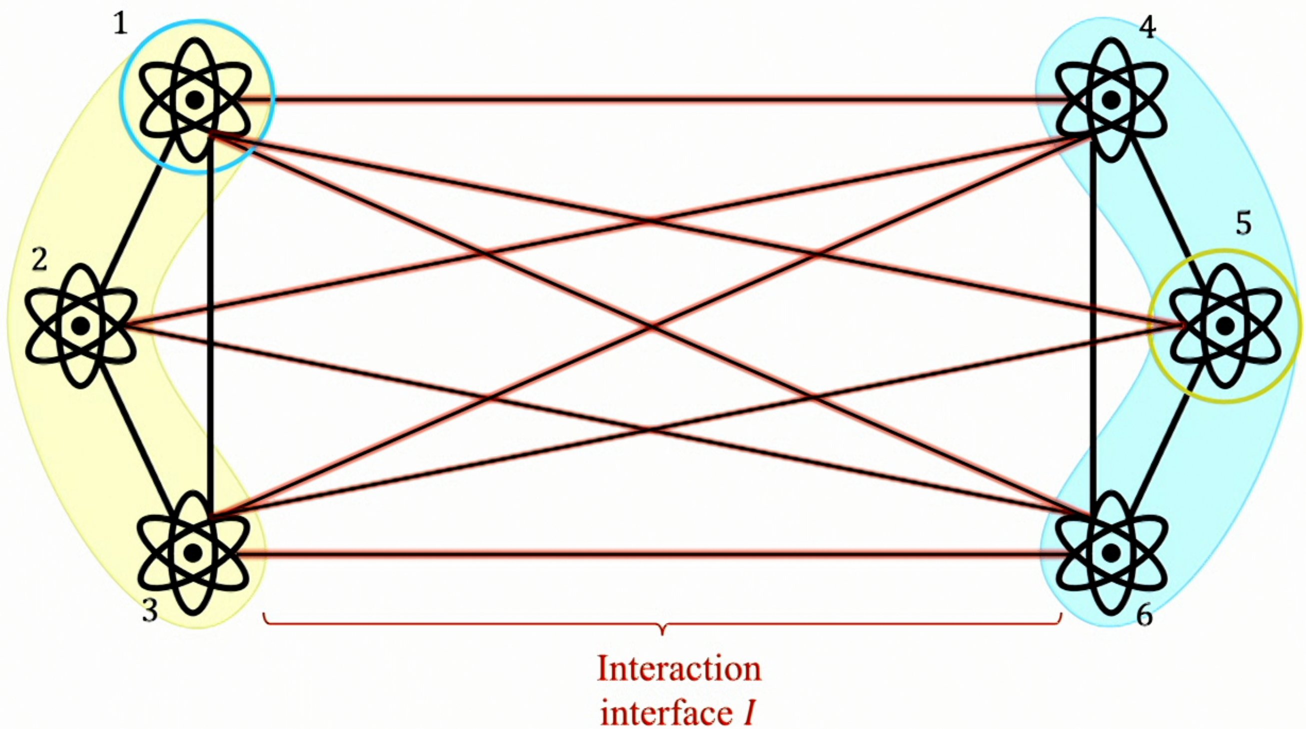
$N = 12$

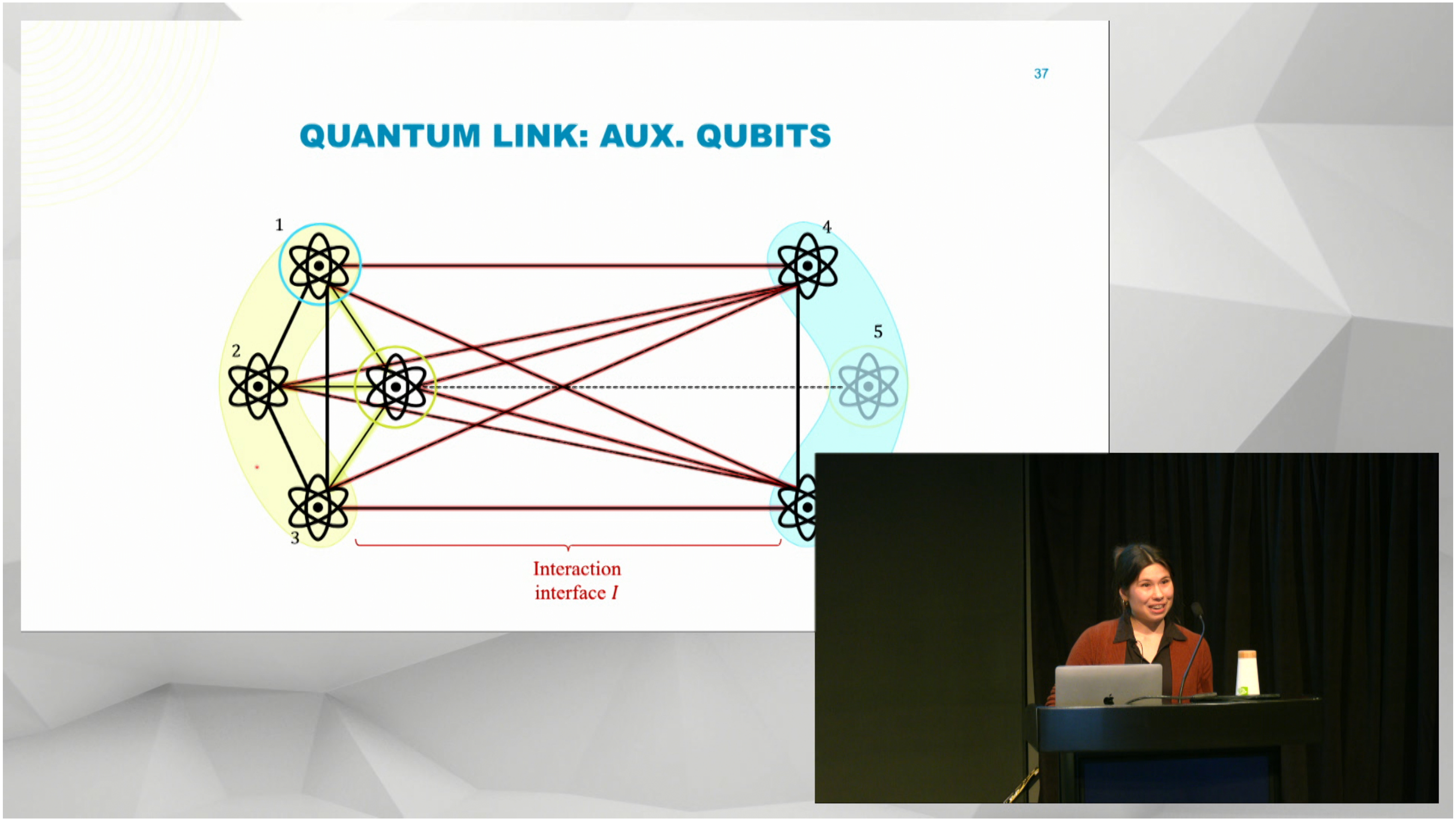
$N_f = 3$

$N_a = 0, 2$

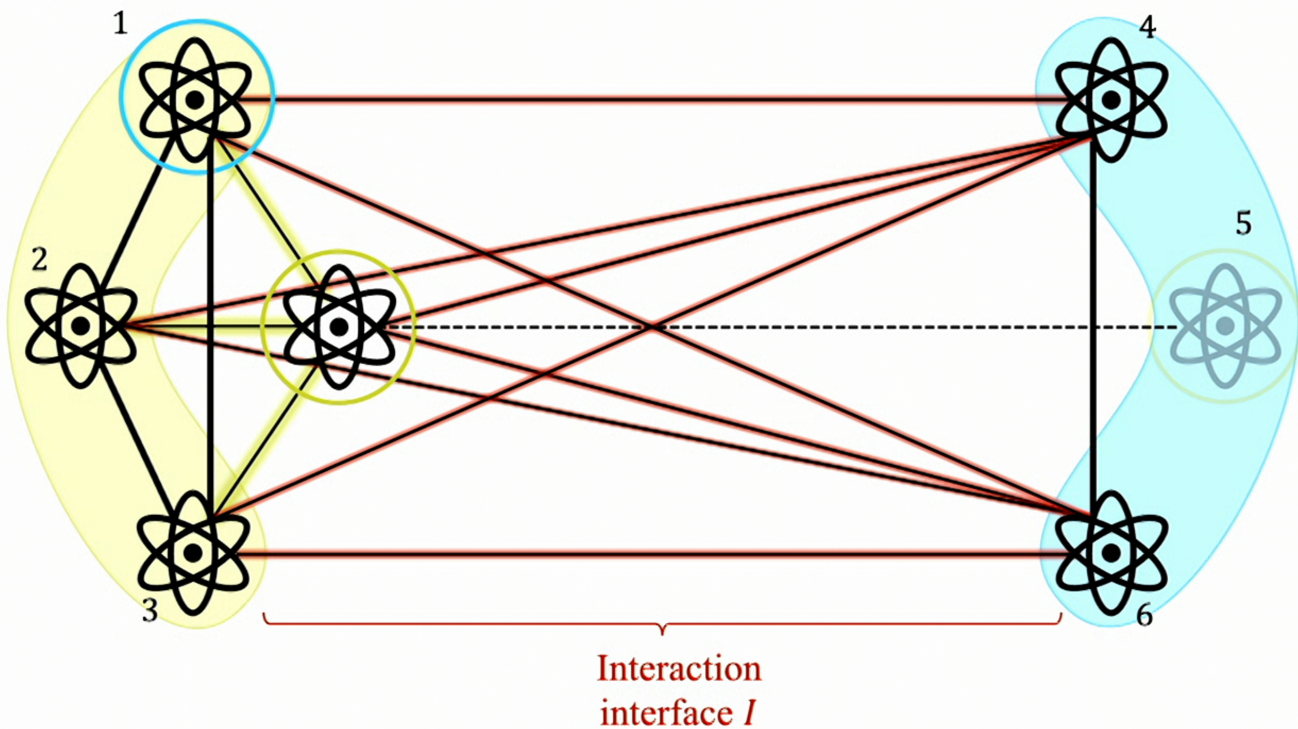


QUANTUM LINK: AUX. QUBITS

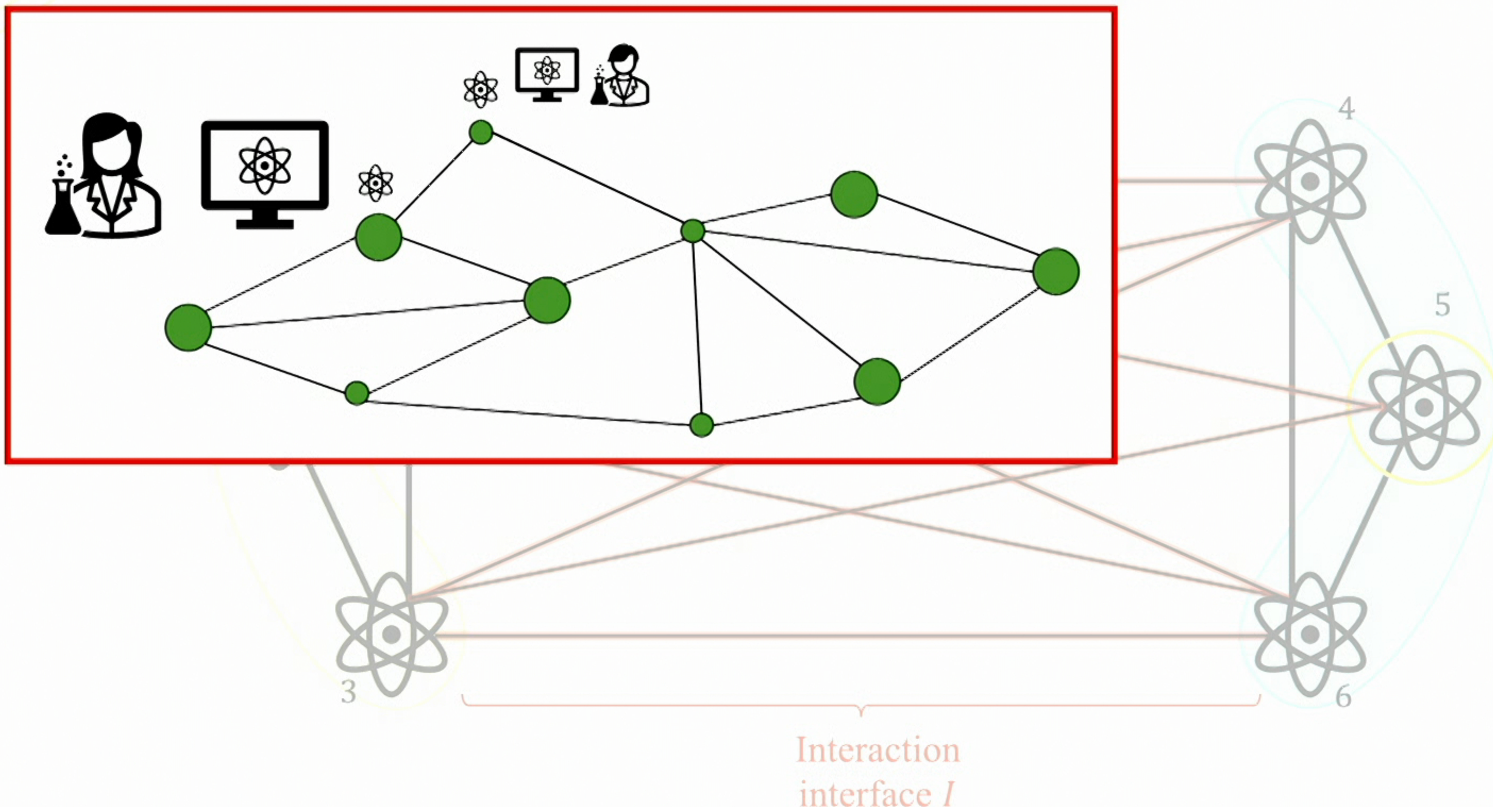



37
QUANTUM LINK: AUX. QUBITS
1
2
3
4
5
Interaction
interface I

QUANTUM LINK: AUX. QUBITS



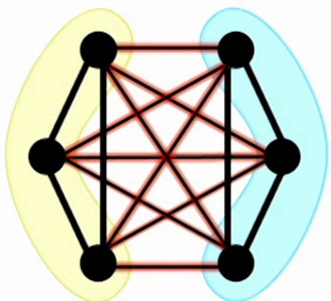
QUANTUM LINK: AUX. QUBITS



QUANTUM LINK: TIME EVOLUTION RESULTS

All-to-all Ising-like model

$$J_{ij} = \begin{cases} +1\,, & p = 0.25 \\ -1\,, & p = 0.25 \\ 0\,, & p = 0.5 \end{cases}$$



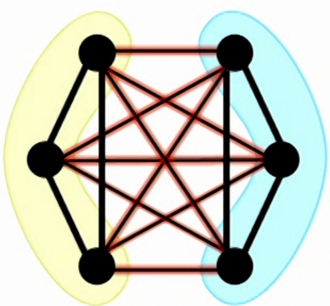
$\boldsymbol{N_f = 3}$

$\boldsymbol{N_a = 2}$

QUANTUM LINK: TIME EVOLUTION RESULTS

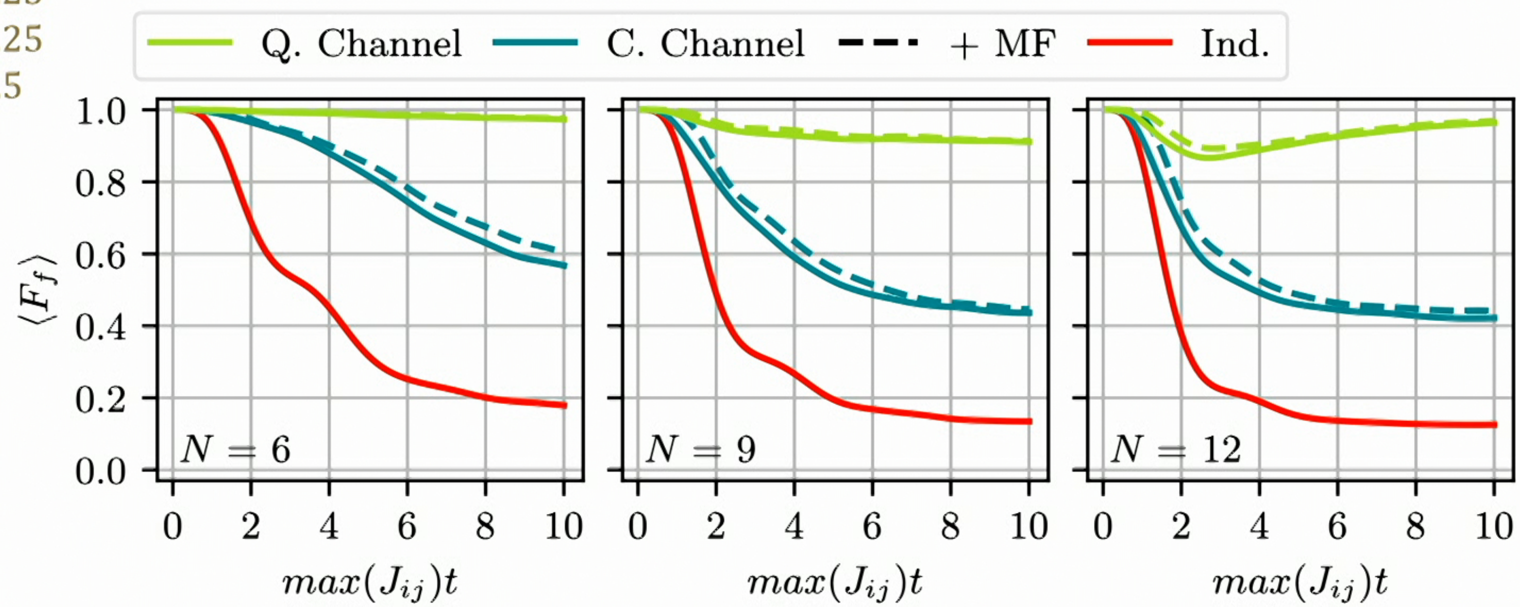
All-to-all Ising-like model

$$J_{ij} = \begin{cases} +1\,, & p = 0.25 \\ -1\,, & p = 0.25 \\ 0\,, & p = 0.5 \end{cases}$$



$N_f = 3$

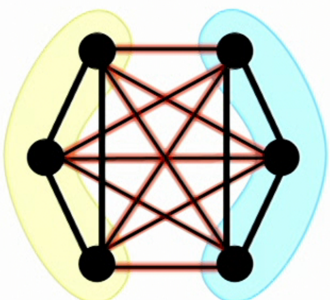
$N_a = 2$



THE ROLE OF OPTIMAL AUXILIARY ENCODING

All-to-all Ising-like model

$J_{ij} = \{\pm 1.0, \pm 0.9, \ldots, \pm 0.1\}$



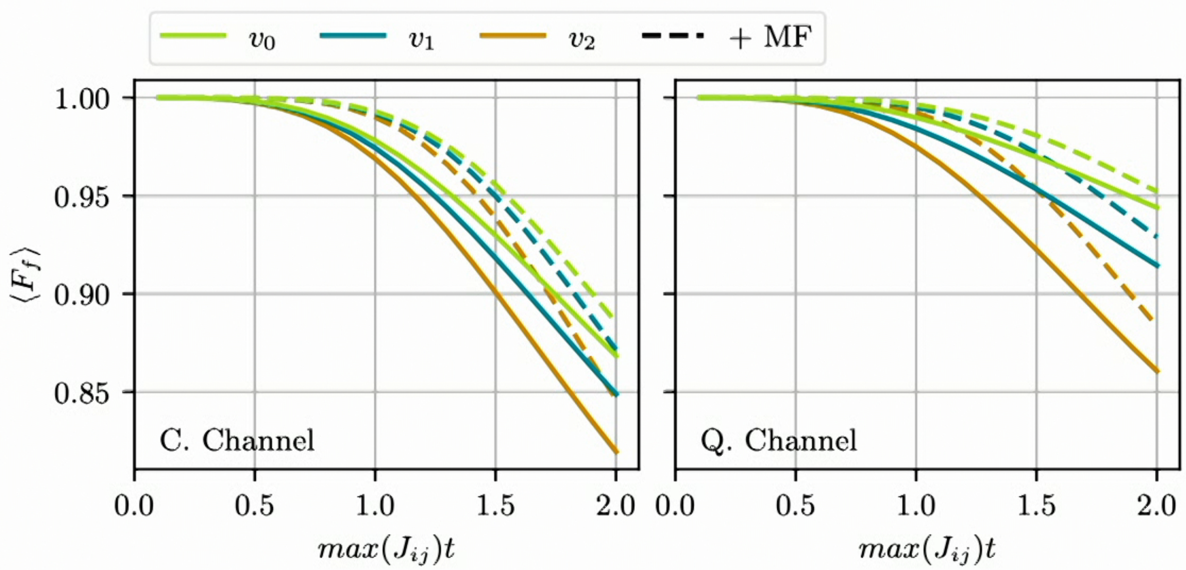
$N = 6$

$N_f = 3$

$N_a = 1$

$$\epsilon = 1 - \left|\langle U_f^\dagger(dt) U(dt)\rangle\right|^2 \approx var(H - H_f)dt^2$$

v: contribution to variance $(v_0 \geq v_1 \geq v_2)$



INGREDIENTS

- Distributed Quantum Computing with Limited Information Transfer
- Mean-Field Corrections
- Auxiliary Qubits
- Optimal Encoding

FRAGMENTED TIME EVOLUTION

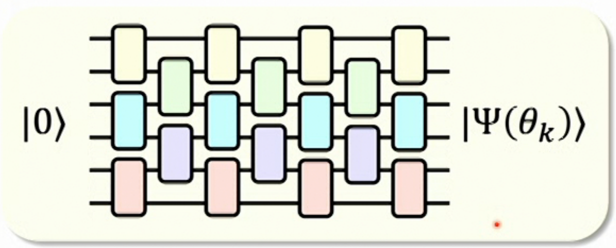
- Classical Link Results
- Quantum Link Results
- Role of Auxiliary Encoding

FRAGMENT-INITIALIZED VQE

- Initialization Scheme
- Comparison to Random Initialization

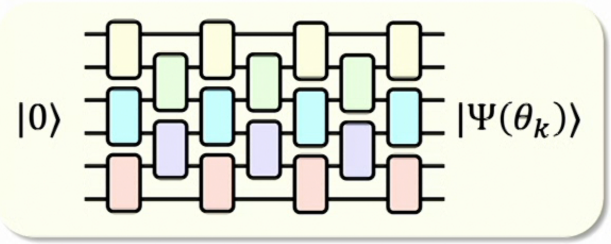
VARIATIONAL QUANTUM EIGENSOLVER

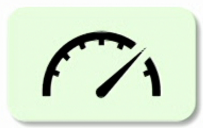
VQE is a hybrid algorithm designed to find the ground state of a Hamiltonian.



VARIATIONAL QUANTUM EIGENSOLVER

VQE is a hybrid algorithm designed to find the ground state of a Hamiltonian.





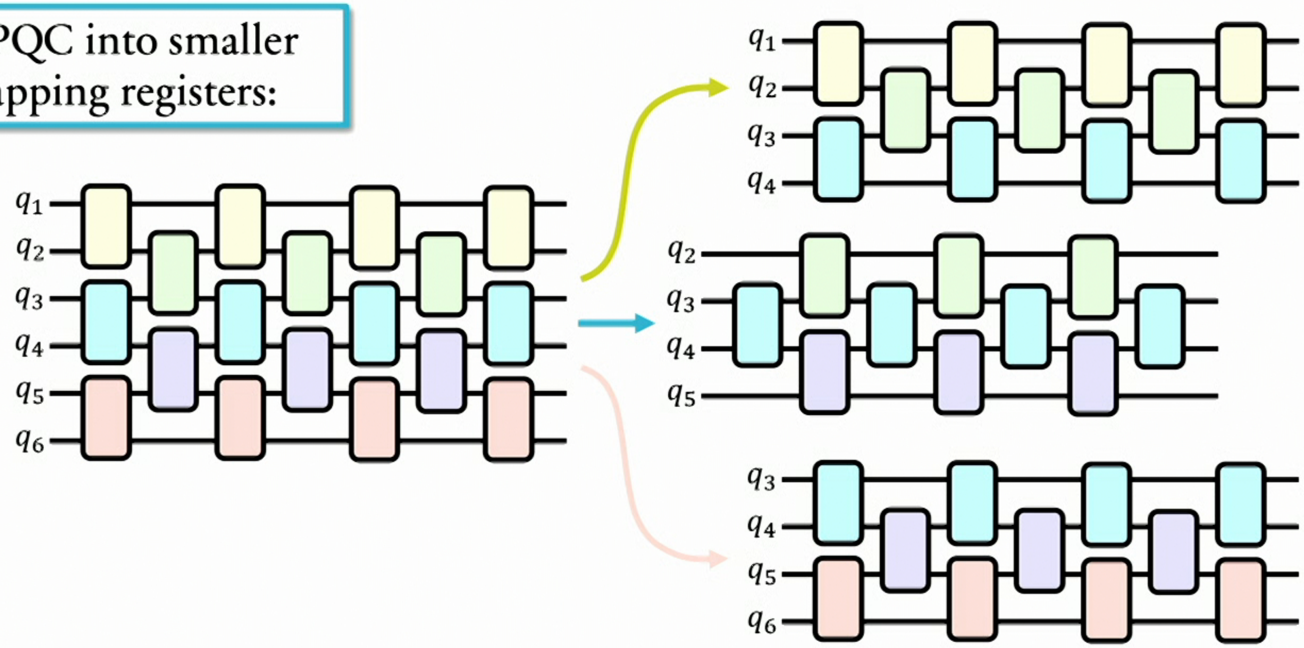
$\frac{\partial \mathcal{L}}{\partial \theta}$

$\mathcal{L} = \langle \Psi(\theta) | H | \Psi(\theta) \rangle \rightarrow E_0$

Classical Optimization Algorithm

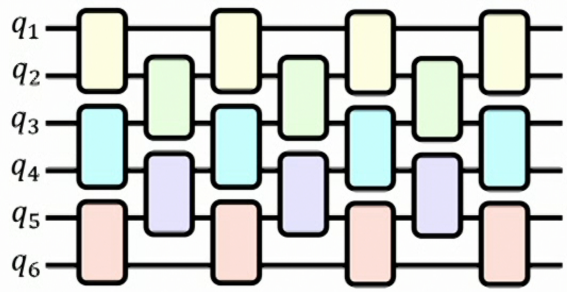
FRAGMENT-INITIALIZED VQE

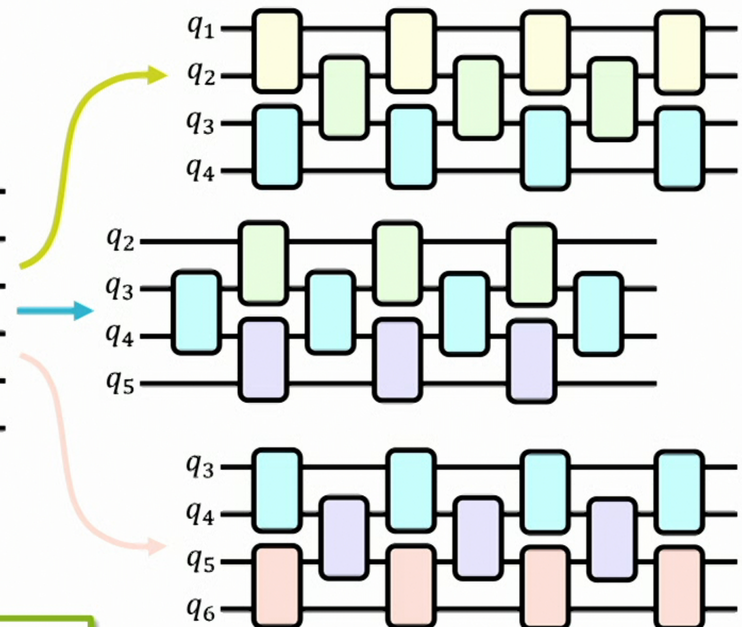
Fragment the full PQC into smaller circuits with overlapping registers:



FRAGMENT-INITIALIZED VQE

Fragment the full PQC into smaller circuits with overlapping registers:

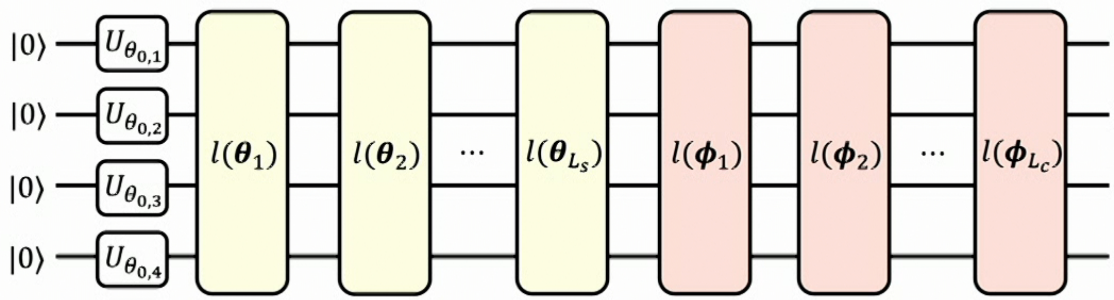


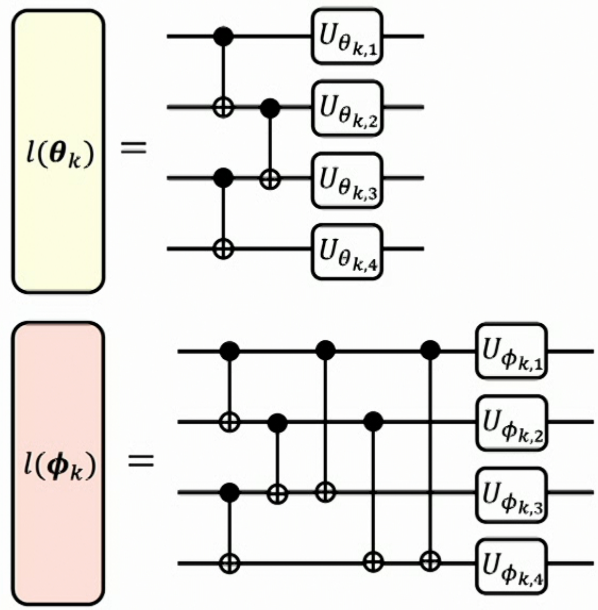


Loss function for a fragmented PQC:

$$\langle H_f \rangle = \left\langle - \sum_{\langle i,j \rangle \in f} J_{ij} \hat{S}_z^i \hat{S}_z^j - \sum_{i \in f} h_i \hat{S}_x^i - \sum_{i \in f} \sum_{j \in E} b_i \hat{S}_z^i \langle \hat{S}_z^j \rangle \right\rangle$$

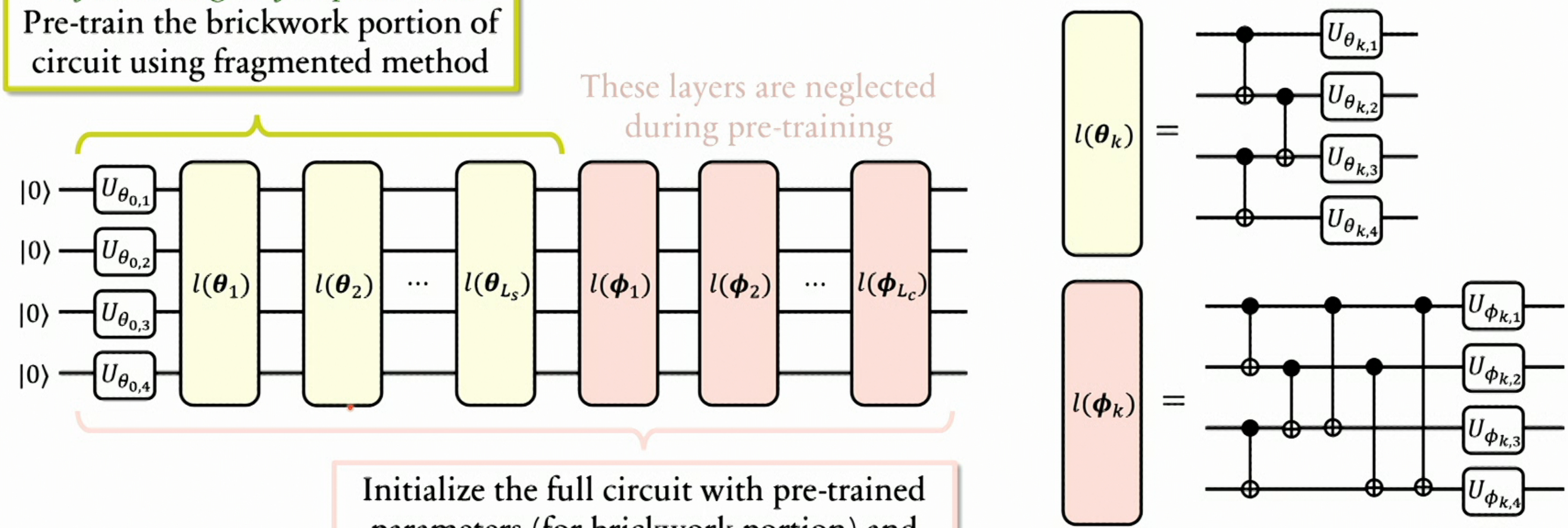
FRAGMENT-INITIALIZED VQE





FRAGMENT-INITIALIZED VQE

Before solving the full problem…
Pre-train the brickwork portion of circuit using fragmented method



Initialize the full circuit with pre-trained parameters (for brickwork portion) and small random values for remaining layers

FRAGMENT-INITIALIZED VQE

PSEUDO CODE:

1. Randomly initialize fragmented PQCs
2. Initialize mean field measurements to be zero
3. In loop (until parameters converge):

   For each fragmented circuit:

   i. Initialize any overlapping parameters with the current values from neighboring circuit.

   ii. Update the circuit's parameters using mean-field modified loss.

   iii. Update mean field measurements of the circuit's system qubits.

FRAGMENT-INITIALIZED VQE

PSEUDO CODE:

1. Randomly initialize fragmented PQCs
2. Initialize mean field measurements to be zero
3. In loop (until parameters converge):

   For each fragmented circuit:

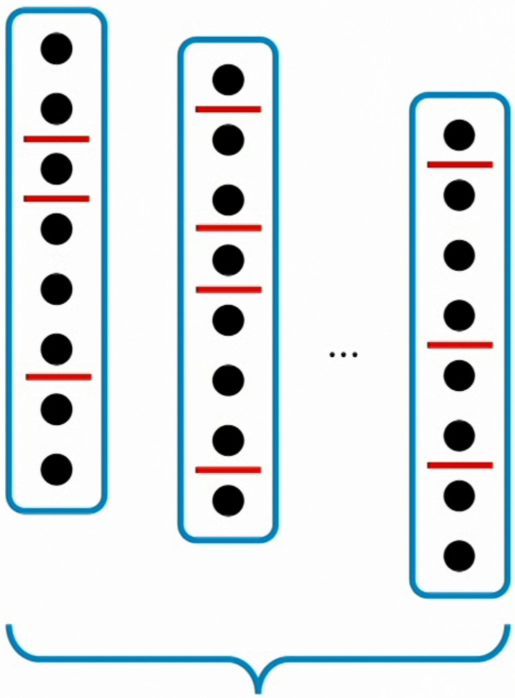
   i. Initialize any overlapping parameters with the current values from neighboring circuit.

   ii. Update the circuit's parameters using mean-field modified loss.

   iii. Update mean field measurements of the circuit's system qubits.

   One optimization iteration complete.

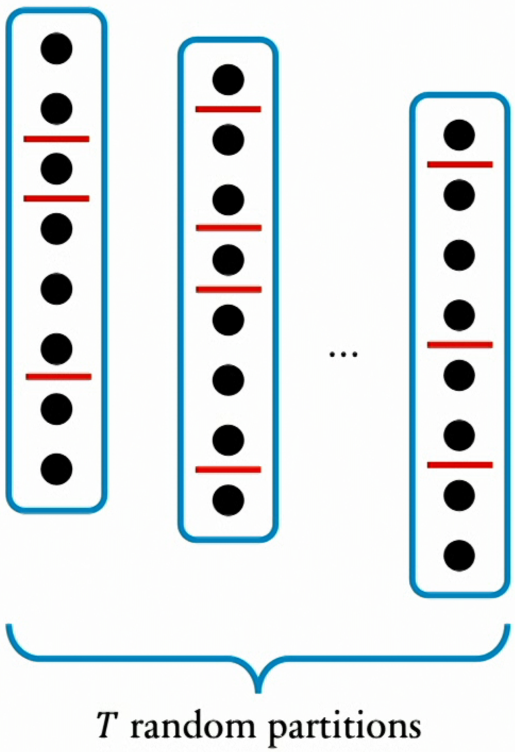
BATCHED PRE-TRAINING



T random partitions

1. Generate T random partitions (with some maximum fragment size and fixed number of auxiliary qubits
   - Set maximum fragment size M and number of auxiliary qubits N_a to remain **classically** tractable
2. Train each set of partitioned circuits
   - Use **classical** resources to do this in parallel
3. Estimate the loss for each set of optimal parameters
   - Use **quantum** resources
4. Select set of parameters with minimum loss to perform final optimization

BATCHED PRE-TRAINING



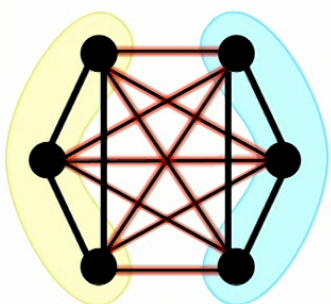
1. Generate T random partitions (with some maximum fragment size and fixed number of auxiliary qubits
   - Set maximum fragment size M and number of auxiliary qubits N_a to remain **classically** tractable
2. Train each set of partitioned circuits
   - Use **classical** resources to do this in parallel
3. Estimate the loss for each set of optimal parameters
   - Use **quantum** resources
4. Select set of parameters with minimum loss to perform final optimization
   - Use **quantum** and **classical** resources

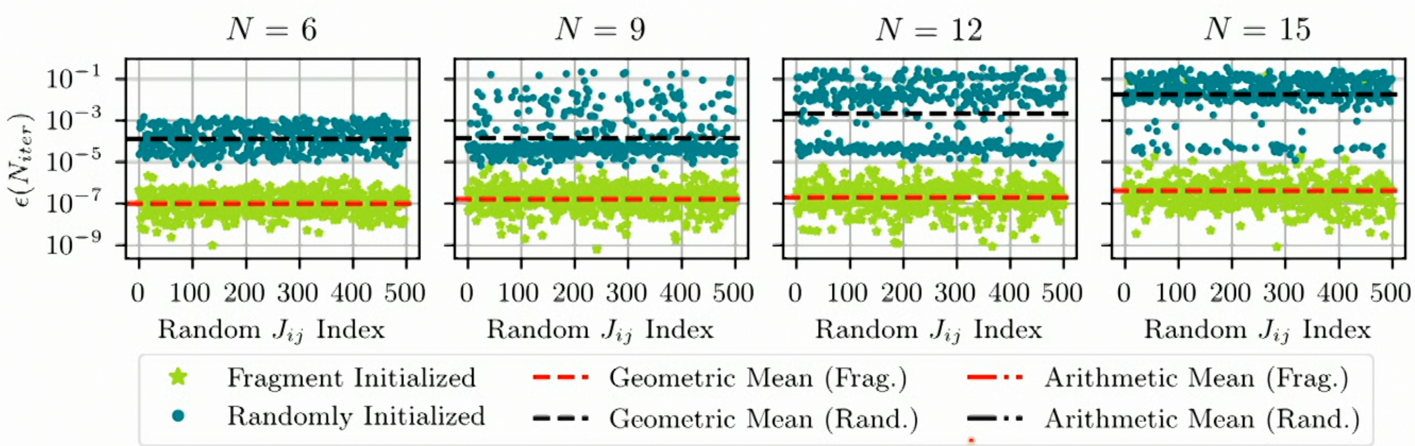
FRAGMENT-INITIALIZED VQE

All-to-all Ising-like model

$h = 0$

$$J_{ij} = \begin{cases} +1, & p = 0.25 \\ -1, & p = 0.25 \\ 0, & p = 0.5 \end{cases}$$



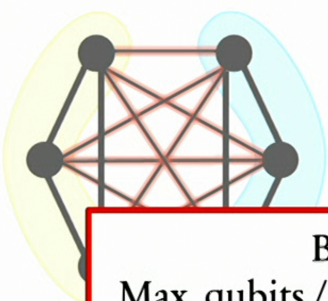


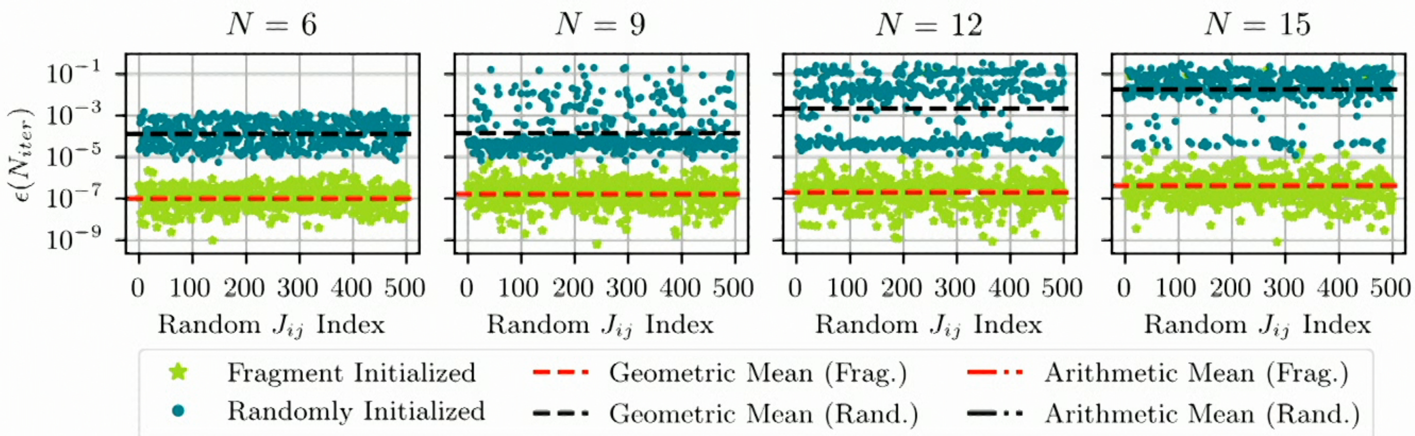
FRAGMENT-INITIALIZED VQE

All-to-all Ising-like model

$h = 0$

$$J_{ij} = \begin{cases} +1, & p = 0.25 \\ -1, & p = 0.25 \\ 0, & p = 0.5 \end{cases}$$





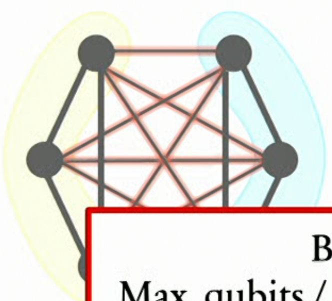
Batch size: $\boldsymbol{T} = \mathbf{10}$
Max. qubits / fragment: $\boldsymbol{M} = \mathbf{3}$
Auxiliary qubits: $\boldsymbol{N_a} = \mathbf{2}$

FRAGMENT-INITIALIZED VQE

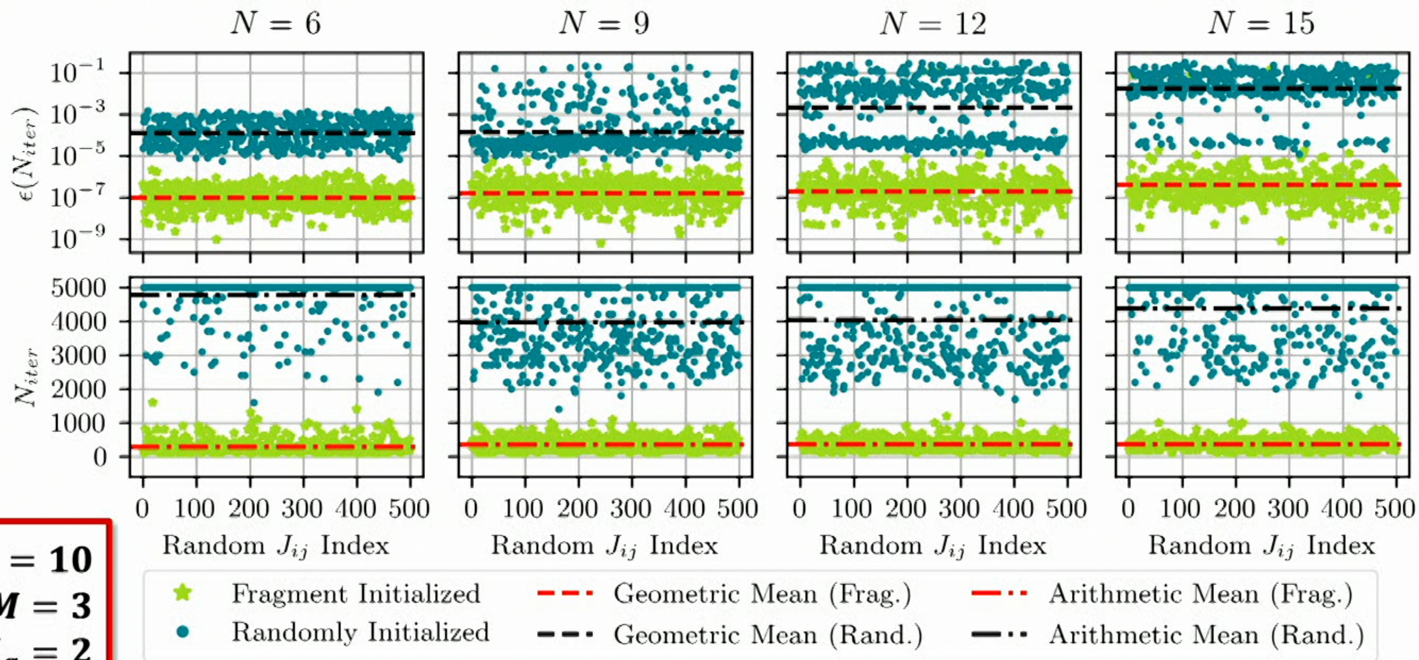
All-to-all Ising-like model

$h = 0$

$$J_{ij} = \begin{cases} +1, & p = 0.25 \\ -1, & p = 0.25 \\ 0, & p = 0.5 \end{cases}$$



Batch size: $\boldsymbol{T} = \mathbf{10}$
Max. qubits / fragment: $\boldsymbol{M} = \mathbf{3}$
Auxiliary qubits: $\boldsymbol{N_a} = \mathbf{2}$

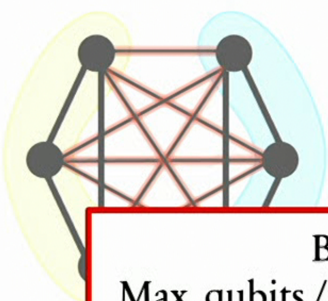


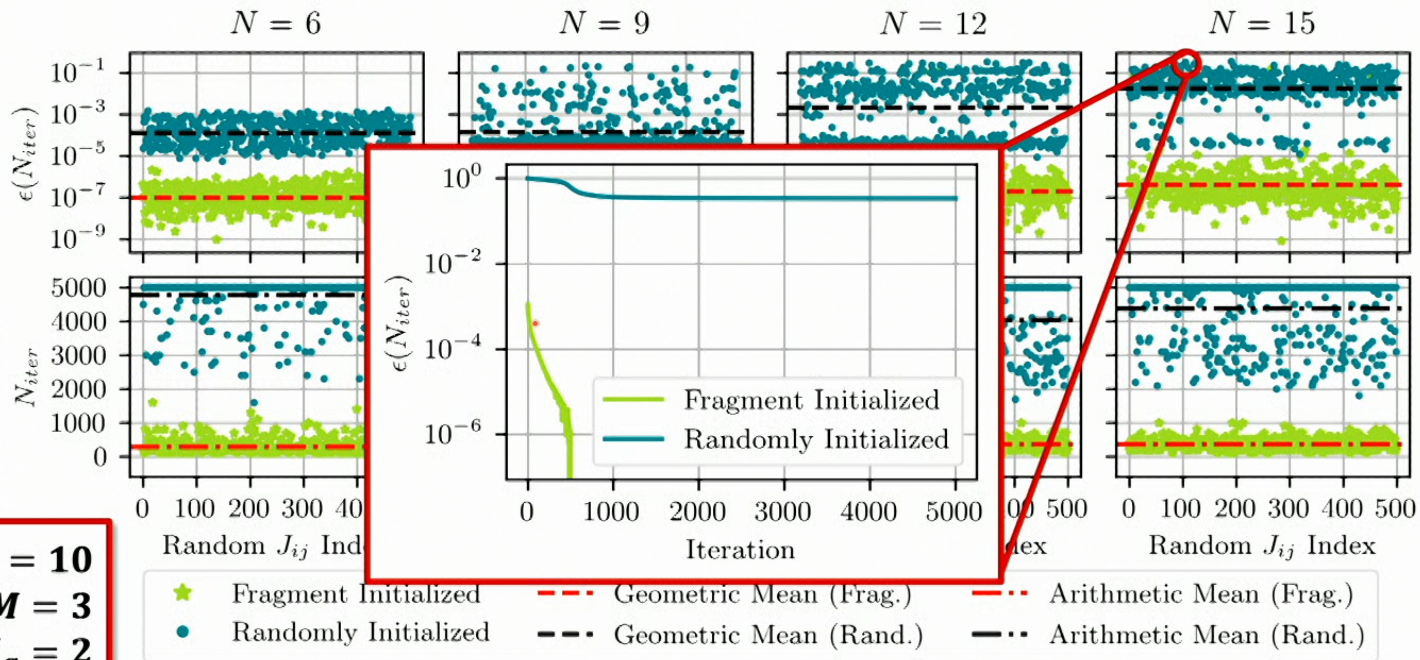
FRAGMENT-INITIALIZED VQE

All-to-all Ising-like model

$h = 0$

$$J_{ij} = \begin{cases} +1\,, & p = 0.25 \\ -1\,, & p = 0.25 \\ 0\,, & p = 0.5 \end{cases}$$

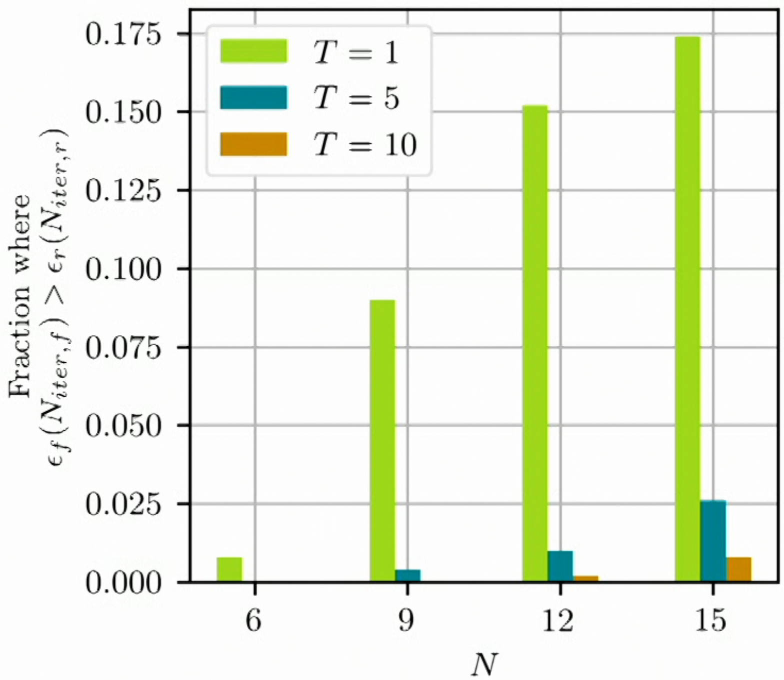




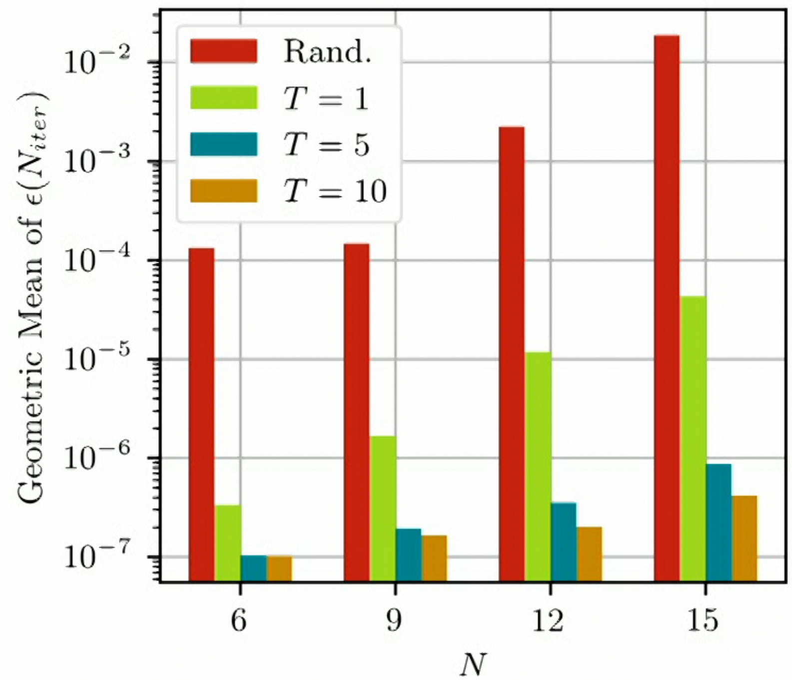
Batch size: $\boldsymbol{T} = \mathbf{10}$
Max. qubits / fragment: $\boldsymbol{M} = \mathbf{3}$
Auxiliary qubits: $\boldsymbol{N_a} = \mathbf{2}$

DEPENDENCE ON BATCH SIZE T

Case by case comparison to random initialization:

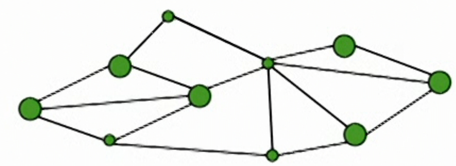


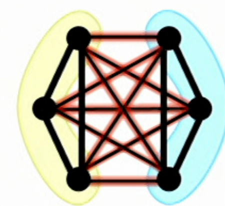
Performance on average:



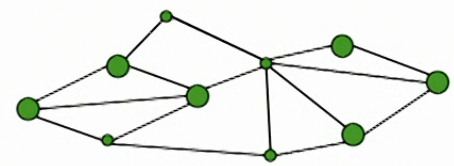
IN SUMMARY…

- Separate quantum simulators can be linked via limited communication channels

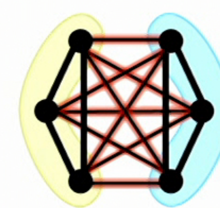




IN SUMMARY…



- Separate quantum simulators can be linked via limited communication channels
- Performance is steadily increased with mean-field corrective terms and auxiliary qubits



- Allowing quantum information transfer – even if limited – can further improve performance
- Using the same fragmented framework, a PQC can be pre-trained using a piece-wise approach that is classically implemented