

Title: Theory Modelling for the Analogue Unruh Effect

Speakers:

Collection: Quantum Simulators of Fundamental Physics

Date: June 09, 2023 - 11:15 AM

URL: <https://pirsa.org/23060026>



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Theory Modelling for the Analogue Unruh Effect

Bridging from theory to experiment

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Circular motion analogue Unruh effect in a $2 + 1$ thermal bath: Robbing from the rich and giving to the poor

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arXiv[2303.12690]

Third sound detectors in accelerated motion

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Cisco Gooding,¹ Grégoire Ithier,² Xavier Rojas,² Jorma Louko,¹ and Silke Weinfurtner^{1,3}

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Non-Equilibrium Systems, University of Nottingham, Nottingham NG7 2RD, UK

arXiv[2302.12023]



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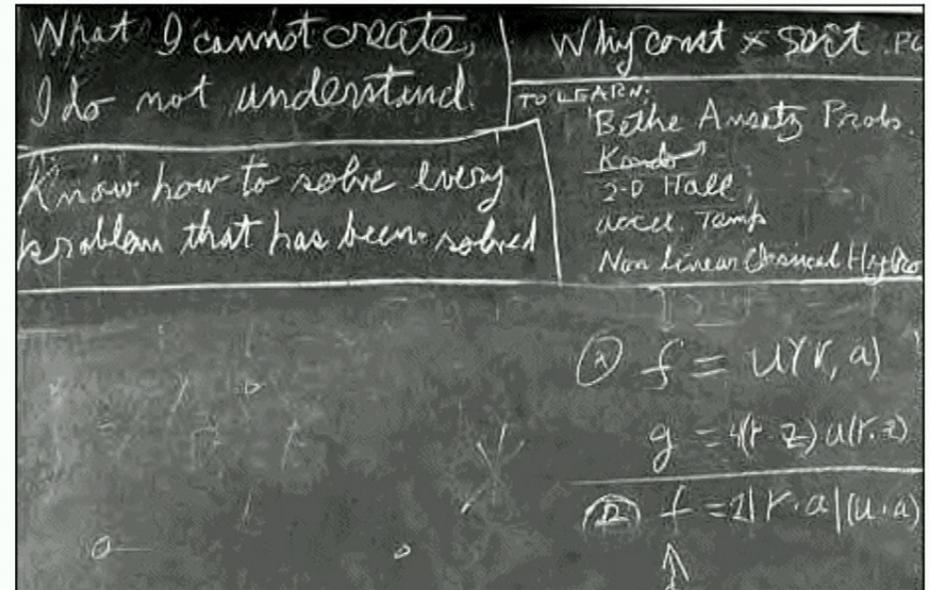


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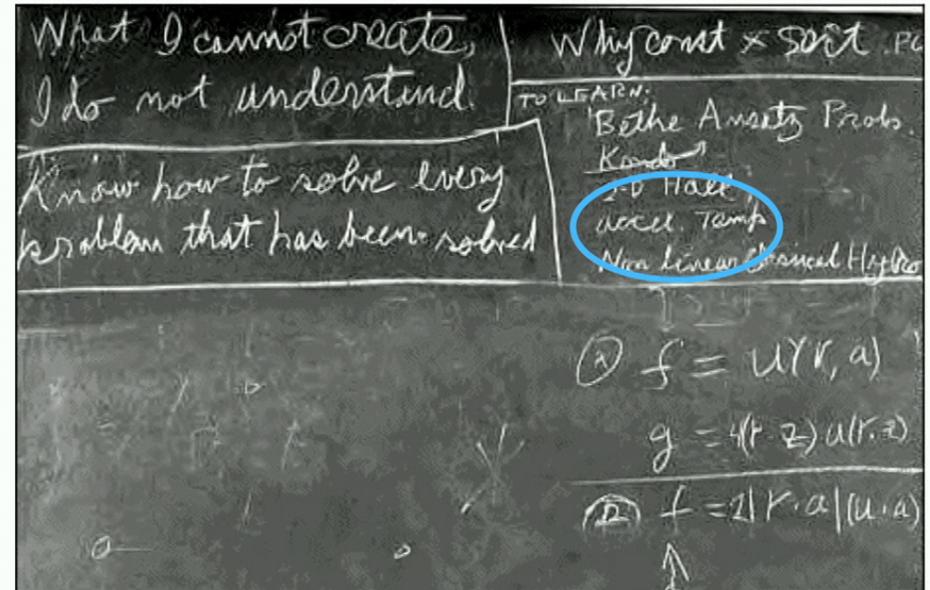
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A primer on the Unruh effect



A primer on the Unruh effect



Feynman's blackboard

- Quantum scalar field $\hat{\phi}(\mathbf{x})$ in Minkowski spacetime $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$
- Couple to a detector $H_{int} = \lambda\chi(\tau)\hat{\phi}(\mathbf{x})\hat{\mu}(\tau)$, the *Unruh-DeWitt detector*

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- λ (small) coupling constant
- $\chi(\tau)$ *switching function*
- $\hat{\mu}(\tau)$ *monopole moment operator* for two level detector with energy gap E

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- Detector excitation probability $\mathcal{P}(E) = \lambda^2 |\langle E | \hat{\mu}(0) | 0 \rangle|^2 \int_{\mathbb{R}} d\tau' \int_{\mathbb{R}} d\tau'' e^{-iE(\tau' - \tau'')} \langle \hat{\phi}(\tau') \hat{\phi}(\tau'') \rangle$

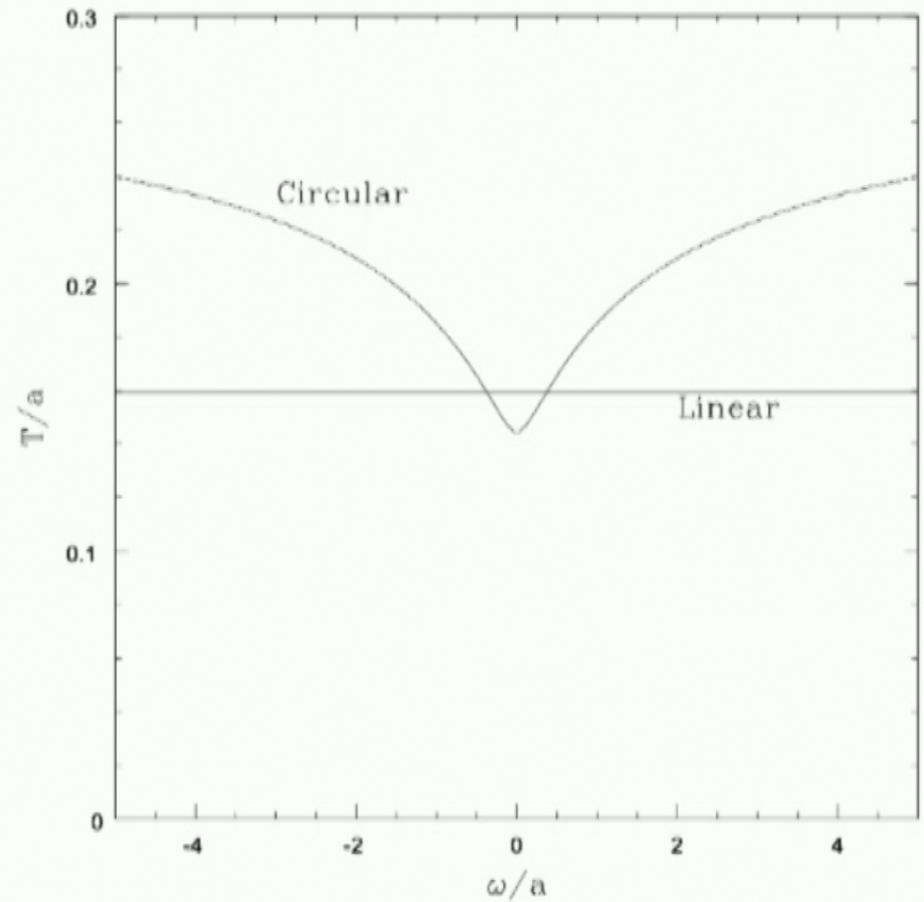
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- Excitation and de-excitation probabilities obey *Einstein's detailed balance condition*:

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- Accelerated trajectory $x^\mu(\tau)$
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$$\mathcal{P}(E) e^{E/T} = \mathcal{P}(-E)$$

The Unruh Effect: $T = \frac{a}{2\pi}$

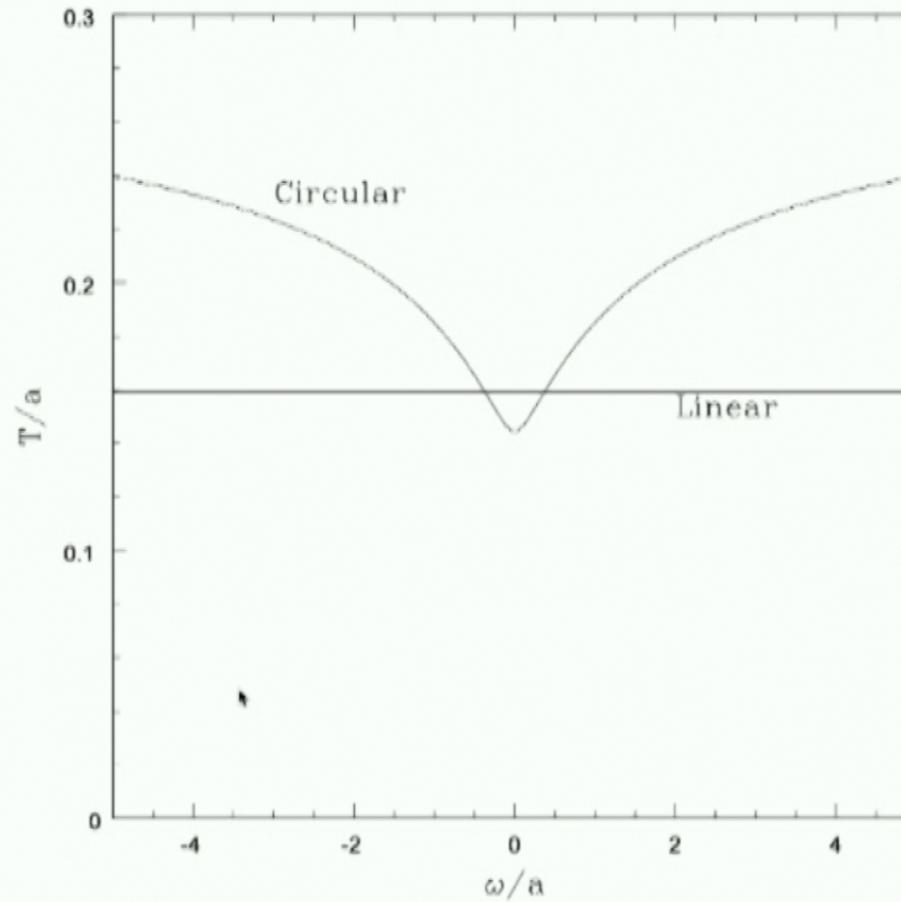
A primer on the circular motion Unruh effect



W. G. Unruh, *Acceleration Radiation for Orbiting Electrons*, 1998

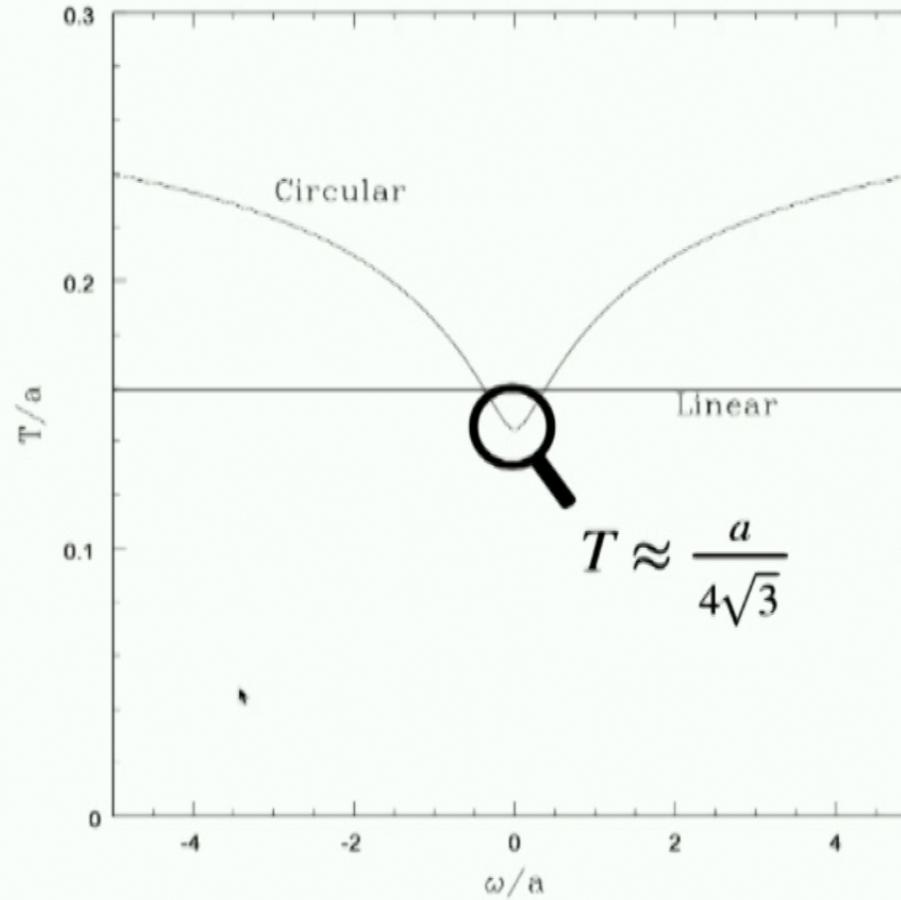
Effective temperature

$$T(\omega) = \frac{\omega}{\ln\left(\frac{\mathcal{P}(-\omega)}{\mathcal{P}(\omega)}\right)}$$

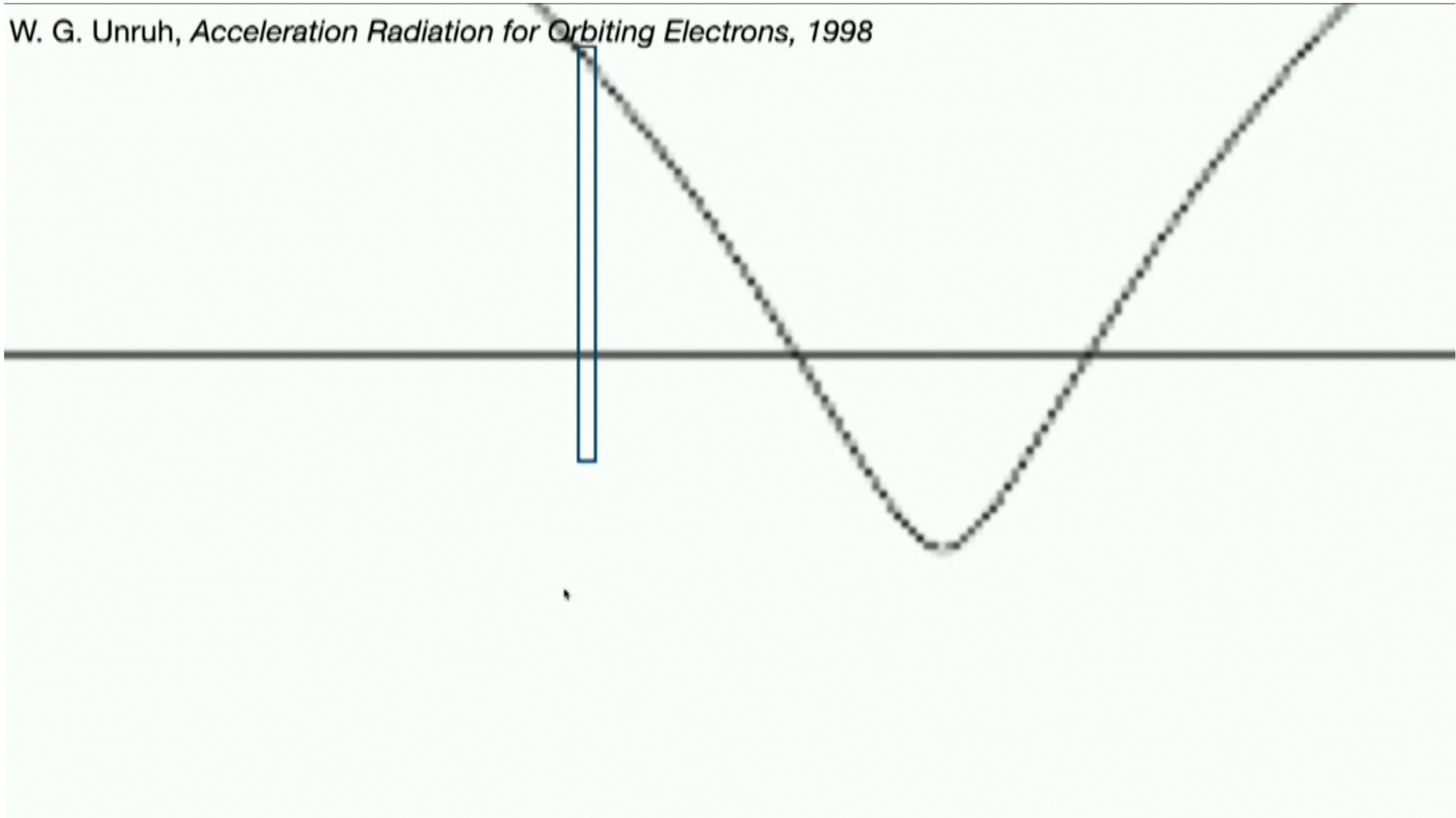


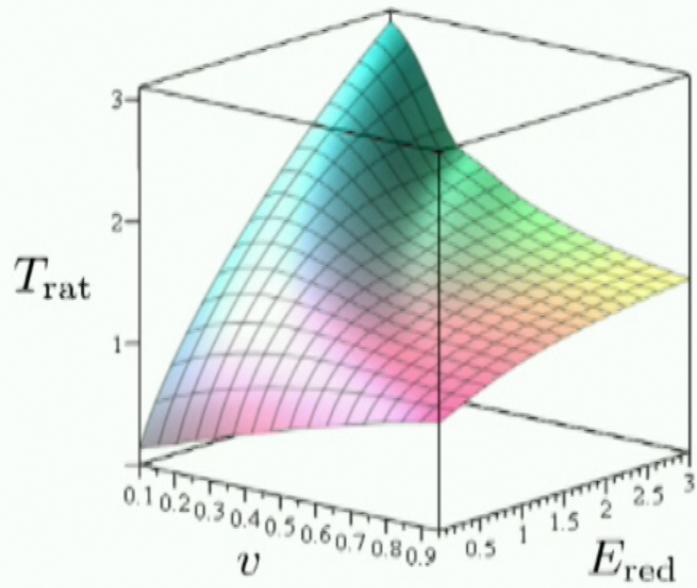
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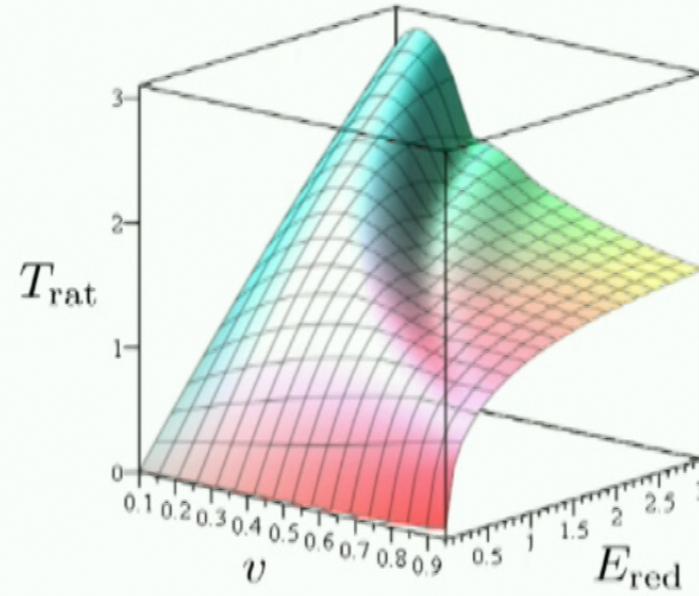


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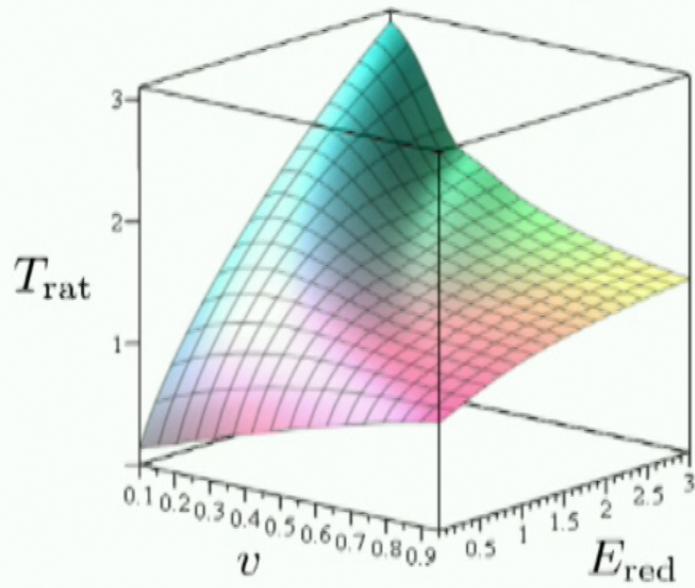


3 + 1 dimensions

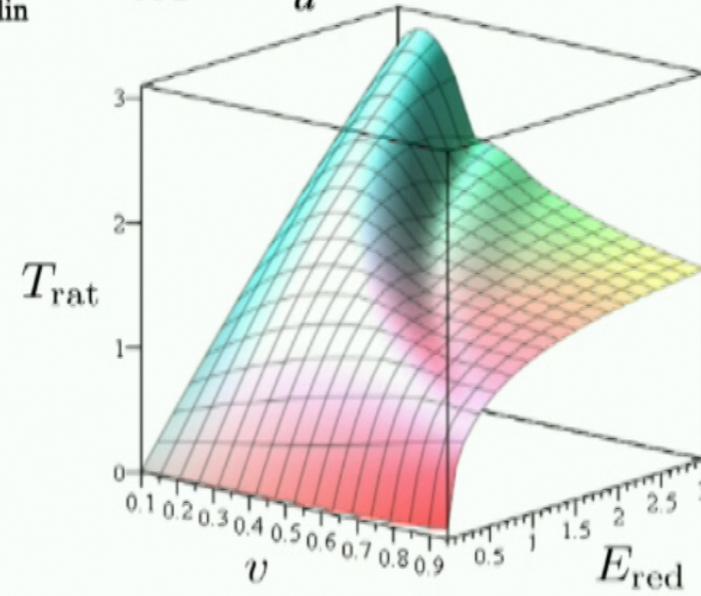


2 + 1 dimensions

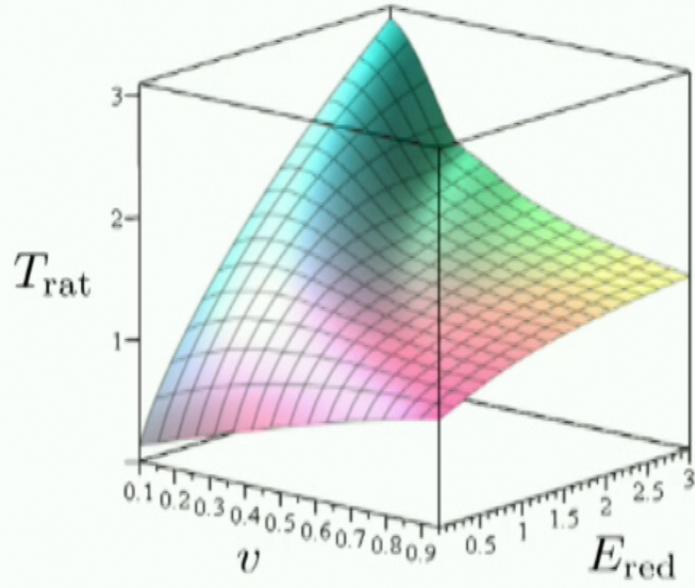
$$T_{\text{rat}} := \frac{T_{\text{circ}}}{T_{\text{lin}}} \quad E_{\text{red}} := \frac{E}{a}$$



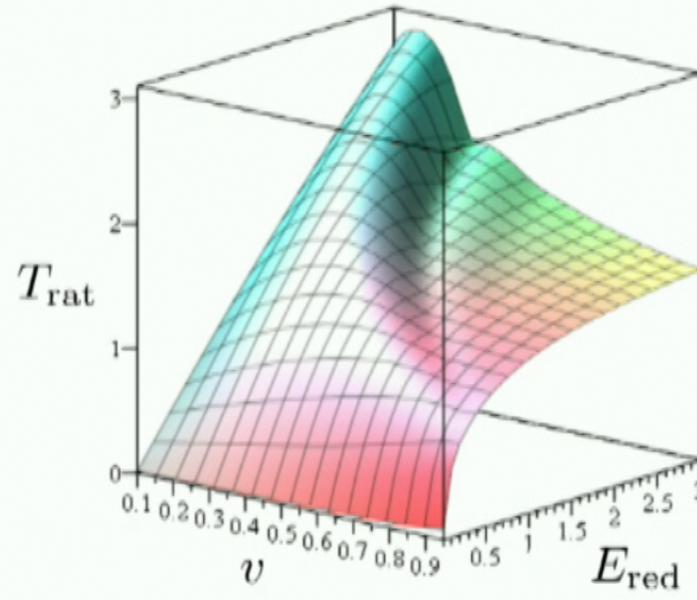
3 + 1 dimensions



2 + 1 dimensions

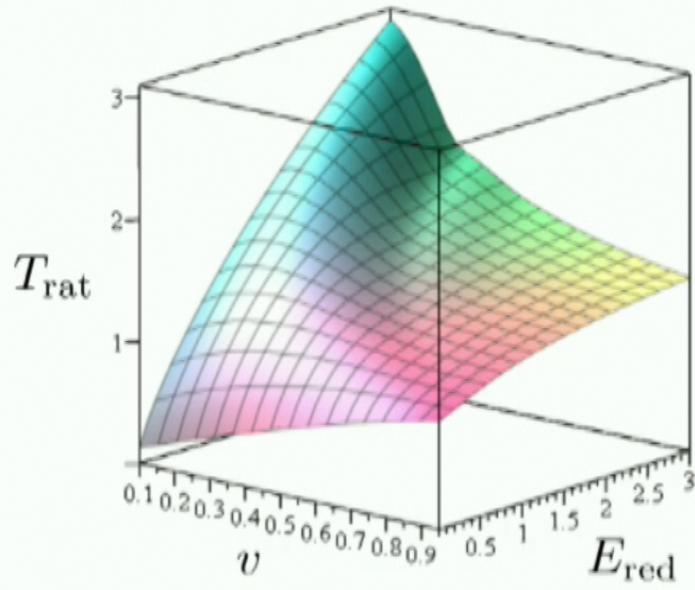


3 + 1 dimensions

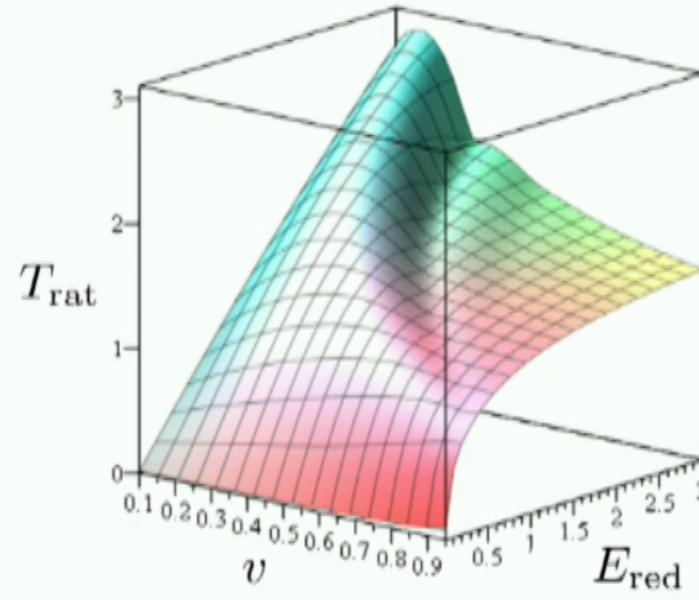


2 + 1 dimensions

$$T_{\text{rat}} := \frac{T_{\text{circ}}}{T_{\text{lin}}} \begin{cases} \rightarrow \frac{E}{\ln\left(\frac{\mathcal{P}(-E)}{\mathcal{P}(E)}\right)} \\ \rightarrow \frac{a}{2\pi} \end{cases}$$



3 + 1 dimensions



2 + 1 dimensions

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Let's build an experiment

Defining the theory (From a theorist's point of view)

1) W r i t e d o w n

Examples

Gravity waves

$$\omega^2 = \left(gk + \frac{\sigma}{\rho} k^3 \right) \tanh(hk)$$

BEC (Боголюбов) dispersion

$$\omega^2 = \frac{k^2}{2m} \left(\frac{k^2}{2m} + \mu \right)$$

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$$\omega^2 \approx ghk^2 =: c_{\text{eff}}^2 k^2 \text{ for } hk \ll 1$$

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$$\omega^2 \approx \frac{\mu}{2m} k^2 =: c_{\text{eff}}^2 k^2 \text{ for } k \ll 1$$



$$\omega^2 \approx c_{\text{eff}}^2 k^2 \quad \text{a linear regime, matching with QFT}$$

Examples

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Defining the theory

2) Initial state

$$\mathcal{P}(E) = \lambda^2 |\langle E | \hat{\mu}(0) | 0 \rangle|^2 \int_{\mathbb{R}} d\tau' \int_{\mathbb{R}} d\tau'' e^{-iE(\tau' - \tau'')} \langle \hat{\phi}(\tau') \hat{\phi}(\tau'') \rangle$$

$$\mathcal{P}(E; \beta) = \lambda^2 |\langle E | \hat{\mu}(0) | 0 \rangle|^2 \int_{\mathbb{R}} d\tau' \int_{\mathbb{R}} d\tau'' e^{-iE(\tau' - \tau'')} \langle \hat{\phi}(\tau') \hat{\phi}(\tau'') \rangle_{\beta}$$

Vacuum?

Thermal?



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Labs have $T > 0$

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$$\langle \hat{O} \rangle_{\beta} = \frac{\text{Tr}(e^{-\hat{H}\beta} \hat{O})}{\text{Tr}(e^{-\hat{H}\beta})}$$

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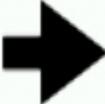
$$\langle \hat{O} \rangle_{\beta} = \frac{\text{Tr}(e^{-\beta \hat{H}} \hat{O})}{\text{Tr}(e^{-\beta \hat{H}})}$$

~~Lorentz invariance~~

Defining the theory

3) Boundary conditions?

No lab is infinite*:

 Constrain the field to a finite size

Experimentally-informed choice

*except for Poincaré disc-mapped ones

Dirichlet?

Neumann?

Mixed/Robin?

Defining the theory

4) Finite time

Have to turn the experiment on and off

Infinite interaction time easier analytically

Can we salvage something?

For a *stationary* system, infinite time can give us finite time

$$W(\tau', \tau'') \stackrel{!}{=} \langle \hat{\phi}(\tau') \hat{\phi}(\tau'') \rangle = W(\tau' - \tau'', 0)$$

Define a **probability rate**

$$\mathcal{F}(E) = \int_{\mathbb{R}} d\tau e^{-iE\tau} W(\tau, 0)$$

$$\text{Finite time } \mathcal{F}(E) = \frac{1}{2\pi} \int_{\mathbb{R}} d\omega |\hat{\chi}(\omega)|^2 \int_{\mathbb{R}} d\tau e^{-i(E+\omega)\tau} W(\tau, 0)$$

“Waiting for Unruh”, Fewster et al. *Class.Quant.Grav.* 33 (2016)

Analogue Setting

- ➔ Lab frame time t and energy gap \bar{E} instead of proper time τ and energy E
- ➔ Analogue (condensed matter) spacetime such as BEC or superfluid helium
- ➔ Reduced spacetime dimension ($2 + 1$)
- ➔ Analogue scalar fields: BEC density perturbations/helium surface waves

Gooding et al. *Phys.Rev.Lett.* 125 (2020)

Bunney et al. *arXiv[2302.12023]*

- Circular motion trajectory
 $x^\mu(t) = (t, R \cos(\Omega t), R \sin(\Omega t))$
- Spectrum no longer exactly thermal - but still has acceleration-dependent response
- Only stationary situations; deal with response function $\mathcal{F}(\bar{E})$

Putting it all together

b

Unruh-DeWitt detector in circular motion in a $(2 + 1)$ analogue spacetime of finite size (Dirichlet boundary conditions at $r = a$), interacting with scalar field with modified* dispersion relation prepared in a thermal state for a finite time

Unruh-DeWitt detector in **circular motion** in a $(2 + 1)$ analogue spacetime of **finite size** (Dirichlet boundary conditions at $r = a$), interacting with scalar field with **modified*** **dispersion relation** prepared in a **thermal state** for a **finite time**

$$\mathcal{F}[\omega_{mn}, \chi](\bar{E}; a, \beta) = \frac{1}{2\pi} \int_{\mathbb{R}} d\varpi |\hat{\chi}(\varpi)|^2 \frac{(\bar{E} + \varpi)^2}{a^2} \sum_{n \in \mathbb{N}} \sum_{m \in \mathbb{Z}} \left[\left(1 + \frac{1}{e^{\omega_{mn}\beta} - 1} \right) \frac{J_{|m|}^2(\omega_{mn}R)}{\omega_{mn} J_{|m|+1}^2(\omega_{mn}a)} \delta(\bar{E} + \varpi + \omega_{mn} - m\Omega) \right. \\ \left. + \frac{1}{e^{\omega_{mn}\beta} - 1} \frac{J_{|m|}^2(\omega_{mn}R)}{\omega_{mn} J_{|m|+1}^2(\omega_{mn}a)} \delta(\bar{E} + \varpi - \omega_{mn} + m\Omega) \right]$$

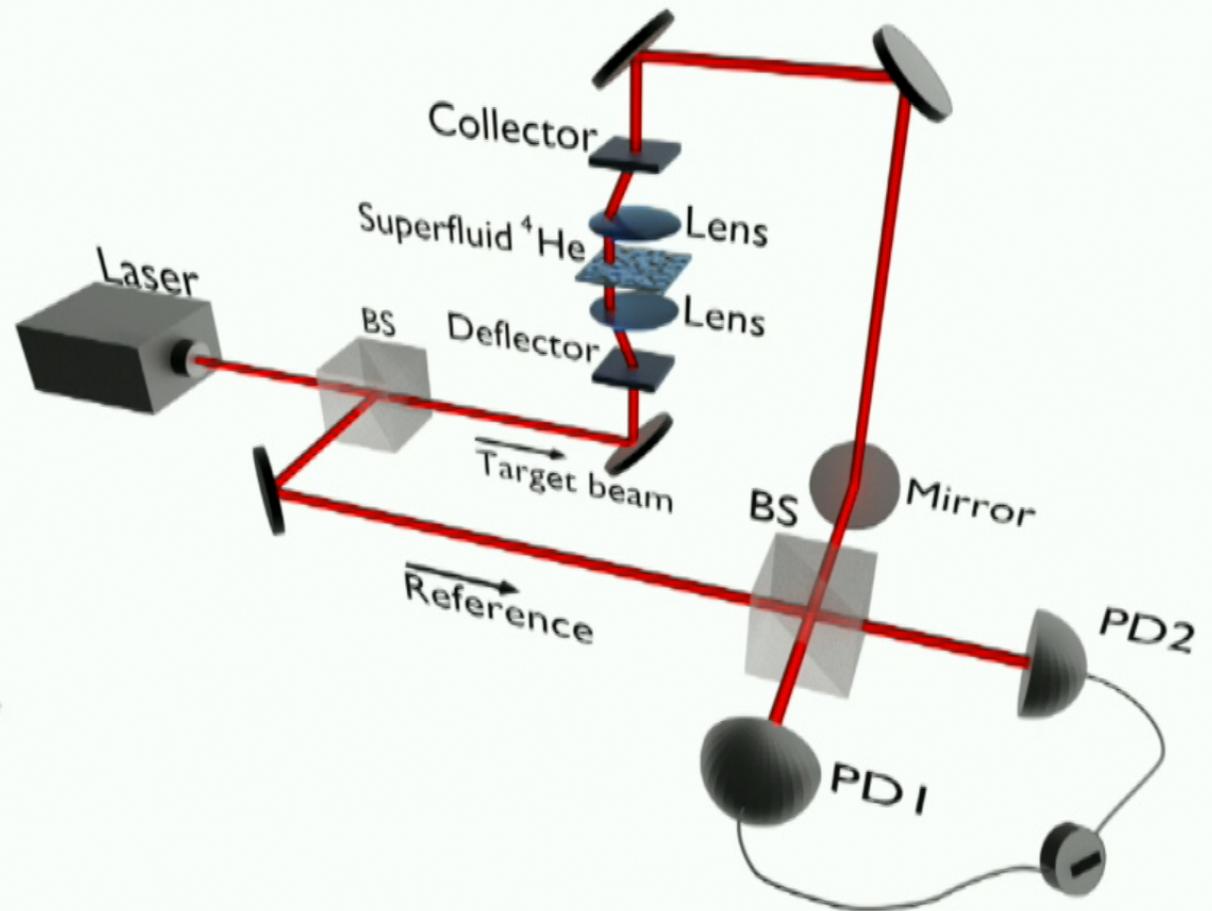
*Bunney et al. *arXiv[2303.12690]*

Unruh-DeWitt detector in **circular motion** in a $(2 + 1)$ analogue spacetime of **finite size** (Dirichlet boundary conditions at $r = a$), interacting with scalar field with **modified*** **dispersion relation** prepared in a **thermal state** for a **finite time**

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*Bunney et al. *arXiv*[2303.12690]

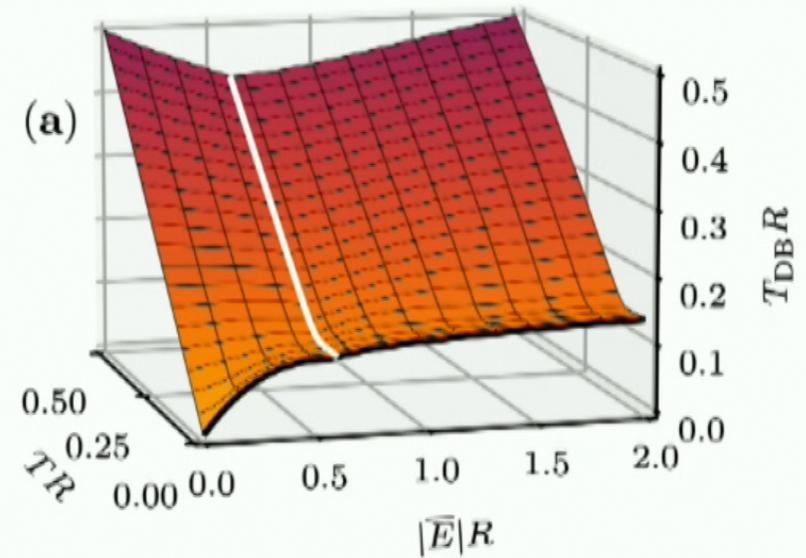
Initial Thermal State

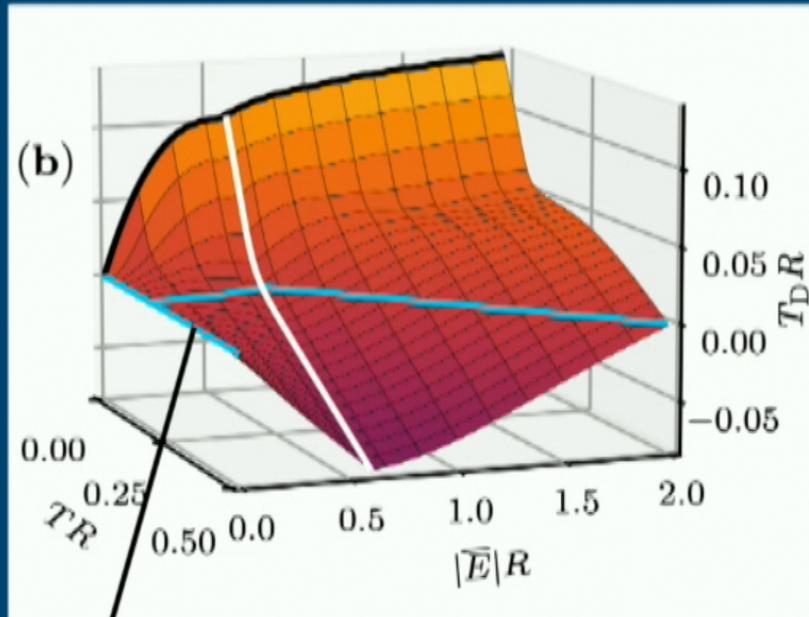


Bunney et al. *arXiv[2302.12023]*, image produced by Radivoje Prizia

Acceleration-dependent signal?

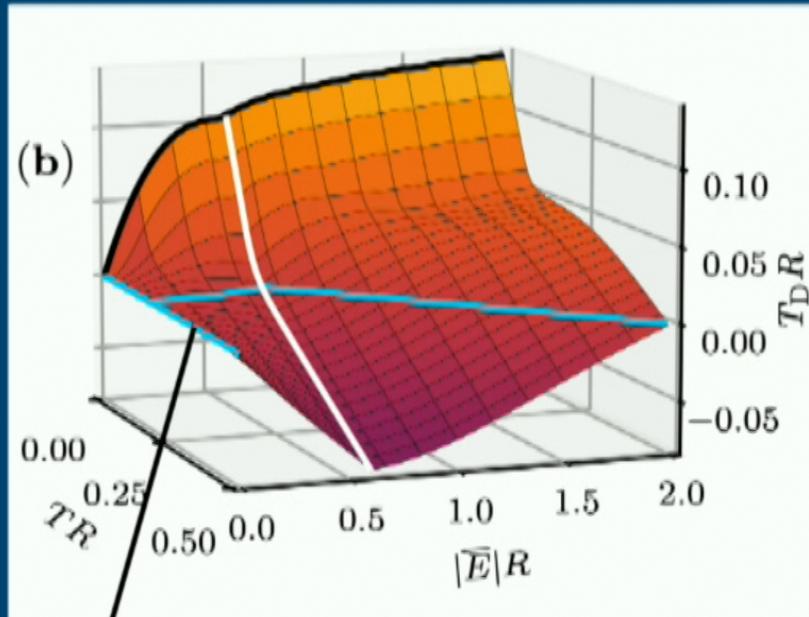
Quantifier of detector response $T_{DB} = \frac{\bar{E}}{\ln\left(\frac{\mathcal{F}(-\bar{E}; \beta)}{\mathcal{F}(\bar{E}; \beta)}\right)}$





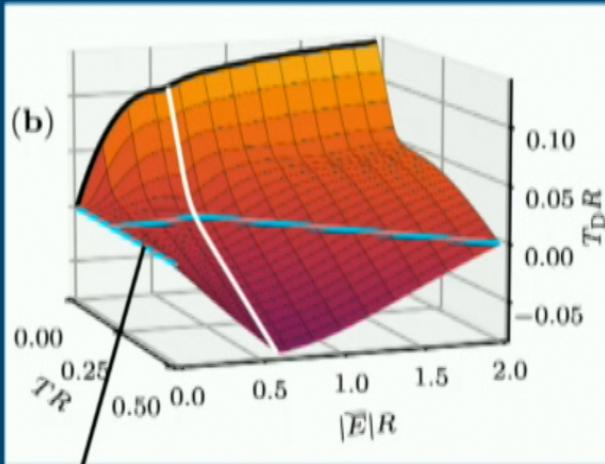
$$T_D = T_{DB} - T_{\text{thermal state}}$$

Line: $T_{DB} = T_{\text{initial state}}$



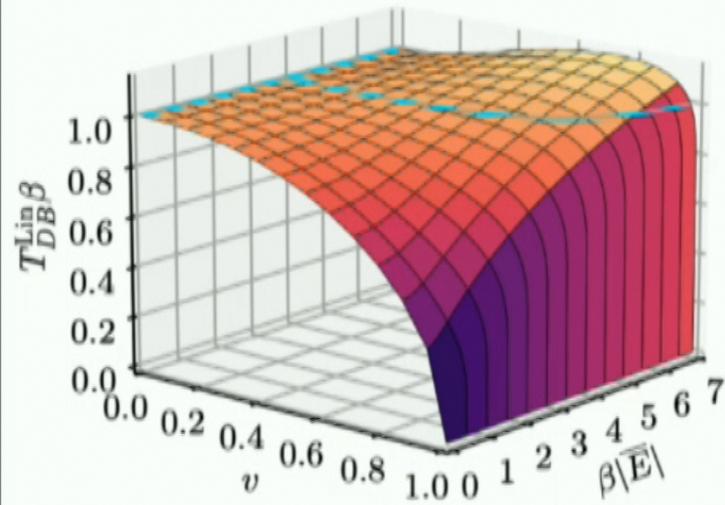
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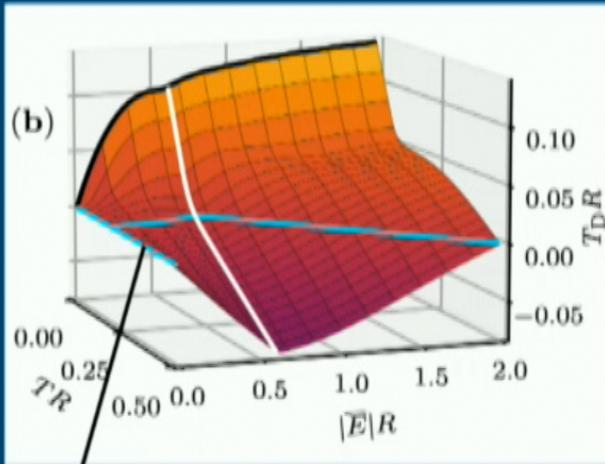
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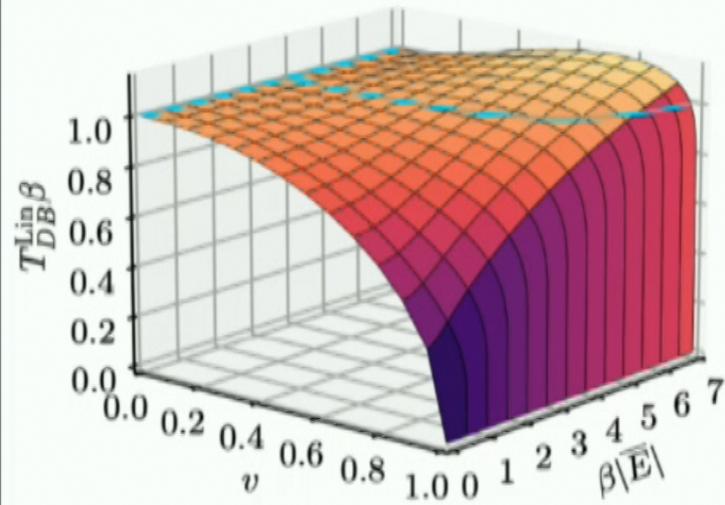
Linear moti





$$T_D = T_{DB} - T_{\text{thermal state}}$$

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Linear moti



Velocity vs. Acceleration

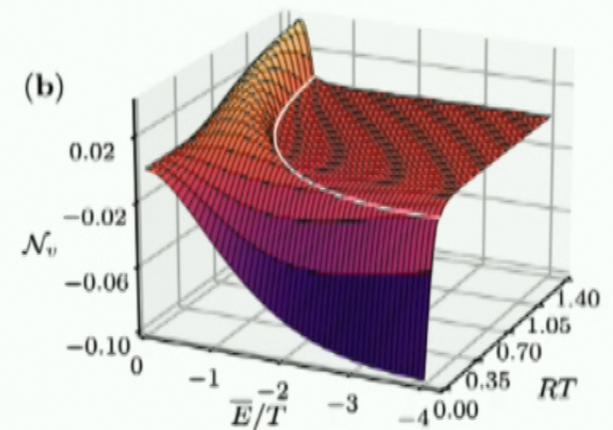
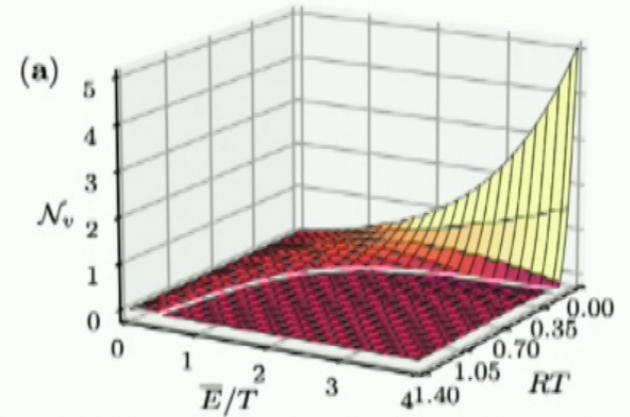
- Circular motion: 2 free parameters
 - Choose acceleration and velocity
- Special case $a = 0$, $v = \text{const}$
 - Straight line motion
- $\mathcal{F}(\bar{E}; \beta) - \mathcal{F}_{\text{Lin}}(\bar{E}; \beta)$ removes velocity contribution

Velocity vs. Acceleration

- Circular motion: 2 free parameters
 - Choose acceleration and velocity
- Special case $a = 0$, $v = \text{const}$
 - Straight line motion

- $\mathcal{F}(\bar{E}; \beta) - \mathcal{F}_{\text{Lin}}(\bar{E}; \beta)$ removes velocity contribution

$$\mathcal{N}_v(\bar{E}\beta, R|\beta) = \frac{\mathcal{F}(\bar{E}; \beta) - \mathcal{F}_{\text{Lin}}(\bar{E}; \beta)}{\mathcal{F}_{\text{Lin}}(\bar{E}; \beta)}$$

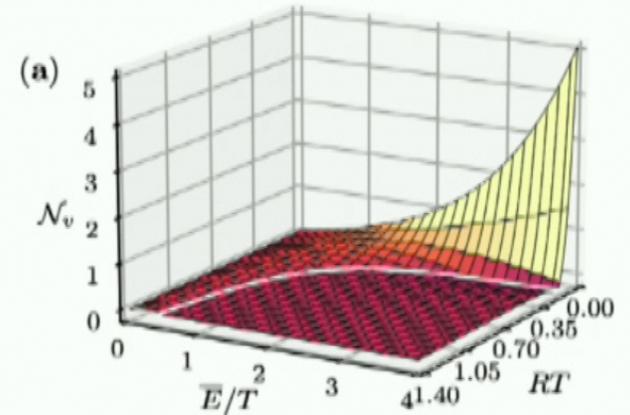


Velocity vs. Acceleration

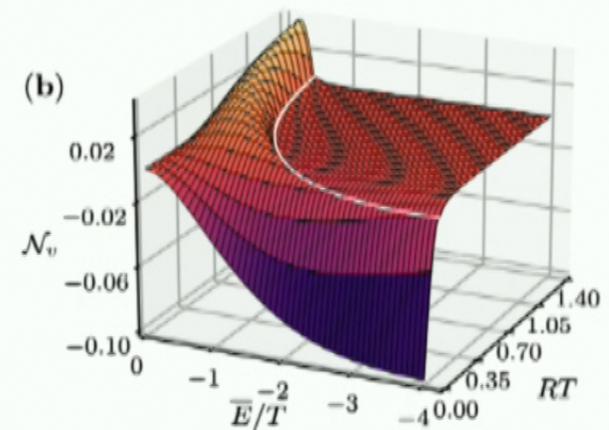
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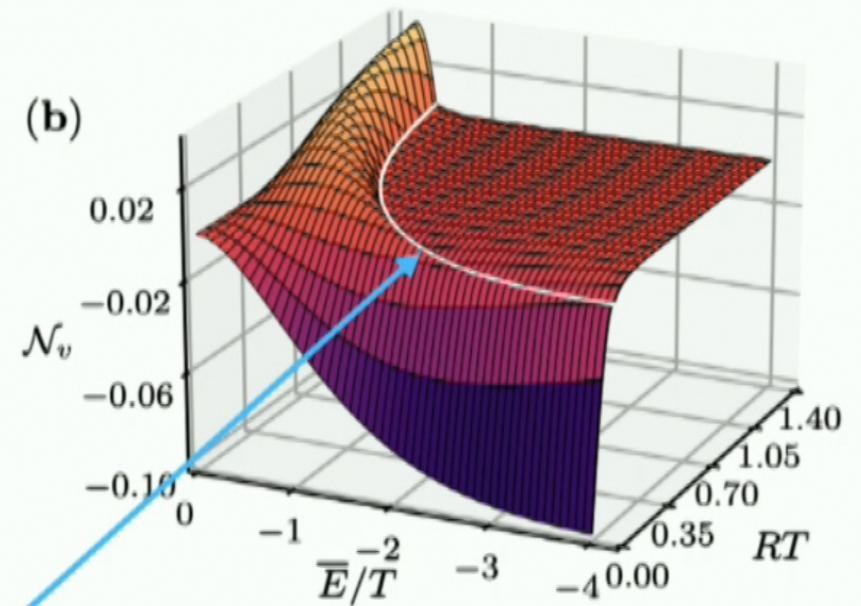
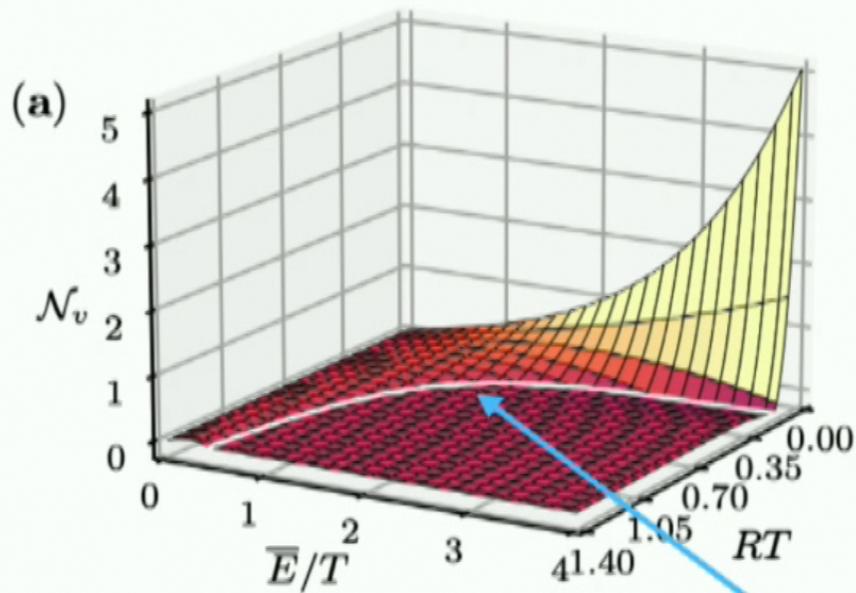
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Structure independent of temperature!





$$|\bar{E}| = \Omega$$

Resonant frequency

Take-home:

Probing below resonance, non-negligible acceleration contribution

Helium setup

Focus on an initial thermal state

- Thin film superfluid helium-4
 - Equilibrium height $h_0 = 100\text{nm}$

Helium setup

Focus on an initial thermal state

- Thin film superfluid helium-4
 - Equilibrium height $h_0 = 100\text{nm}$
 - **Analogue scalar field:** long wavelength surface perturbations in superfluid component; *decoupled from normal component*
- Probe the field with a continuous laser
- Laser samples height fluctuations
↳ phase fluctuations ψ



signal extraction (in progress)

Post signal extraction

Dimensionfully, power spectral density proportional to response function

$$S_{\psi}(\omega) = \frac{\hbar^2(n^2 - 1)k_L^2}{g\rho} \mathcal{F}(\omega)$$

Helium setup

Focus on an initial thermal state

- Thin film superfluid helium-4
 - Equilibrium height $h_0 = 100\text{nm}$
 - **Analogue scalar field:** long wavelength surface perturbations in superfluid component; *decoupled from normal component*
- Probe the field with a continuous laser
- Laser samples height fluctuations
↳ phase fluctuations ψ



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Diagram illustrating the components of the power spectral density $S_{\psi}(\omega)$ equation:

- \hbar^2 is associated with the **Index of refraction**.
- $(n^2 - 1)$ is associated with the **Index of refraction**.
- k_L^2 is associated with the **Laser wave number**.
- $g\rho$ is associated with **Van der Waals-dominated effective gravity** and **Mass density**.

Read-off and acceleration isolation

$$S_{\psi}(\omega) = \kappa \mathcal{F}(\omega; T) + \sigma_{\text{sn}}^2$$

$$S_{\delta}(\omega) = S_{\psi}(\omega) - \sigma_{\text{sn}}^2 - \kappa \mathcal{F}_{\text{Lin}}(\omega; T)$$



Read-off and acceleration isolation

$$S_{\psi}(\omega) = \kappa \mathcal{F}(\omega; T) + \sigma_{\text{sn}}^2$$

$$S_{\delta}(\omega) = S_{\psi}(\omega) - \sigma_{\text{sn}}^2 - \kappa \mathcal{F}_{\text{Lin}}(\omega; T)$$

$$\text{Recall } \mathcal{N}_v(\bar{E}\beta, R/\beta) = \frac{\mathcal{F}(\bar{E}; \beta) - \mathcal{F}_{\text{Lin}}(\bar{E}; \beta)}{\mathcal{F}_{\text{Lin}}(\bar{E}; \beta)}$$

Read-off and acceleration isolation

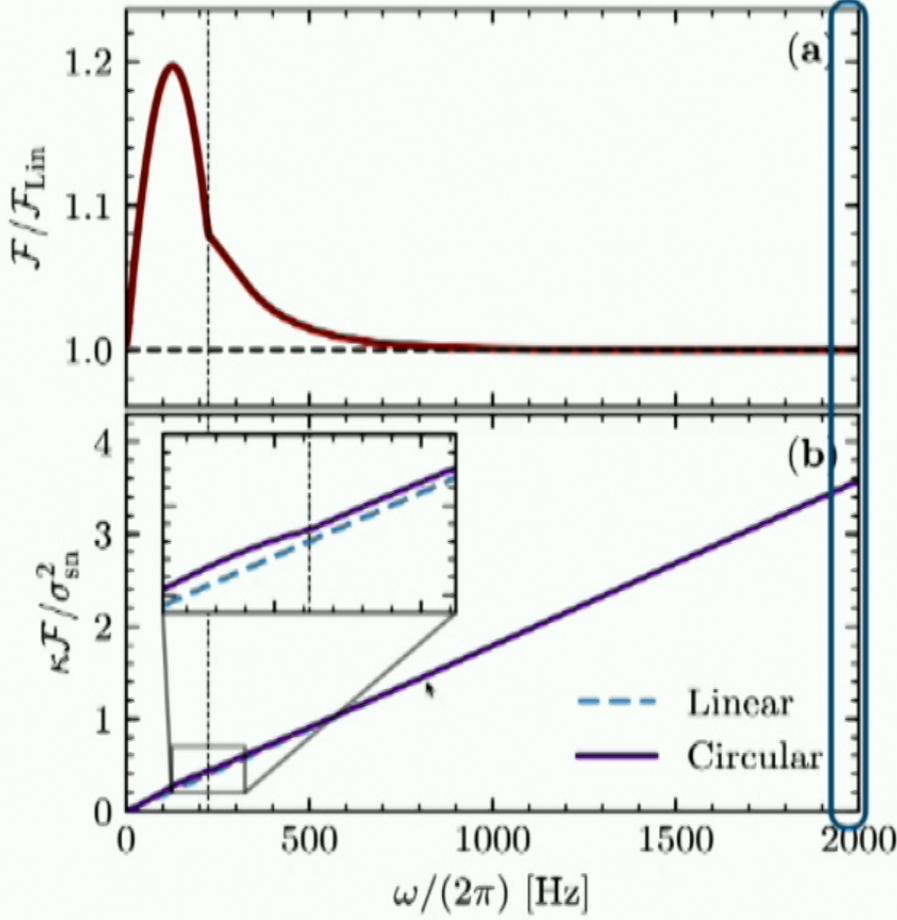
$$S_{\psi}(\omega) = \kappa \mathcal{F}(\omega; T) + \sigma_{\text{sn}}^2$$

$$S_{\delta}(\omega) = S_{\psi}(\omega) - \sigma_{\text{sn}}^2 - \kappa \mathcal{F}_{\text{Lin}}(\omega; T)$$

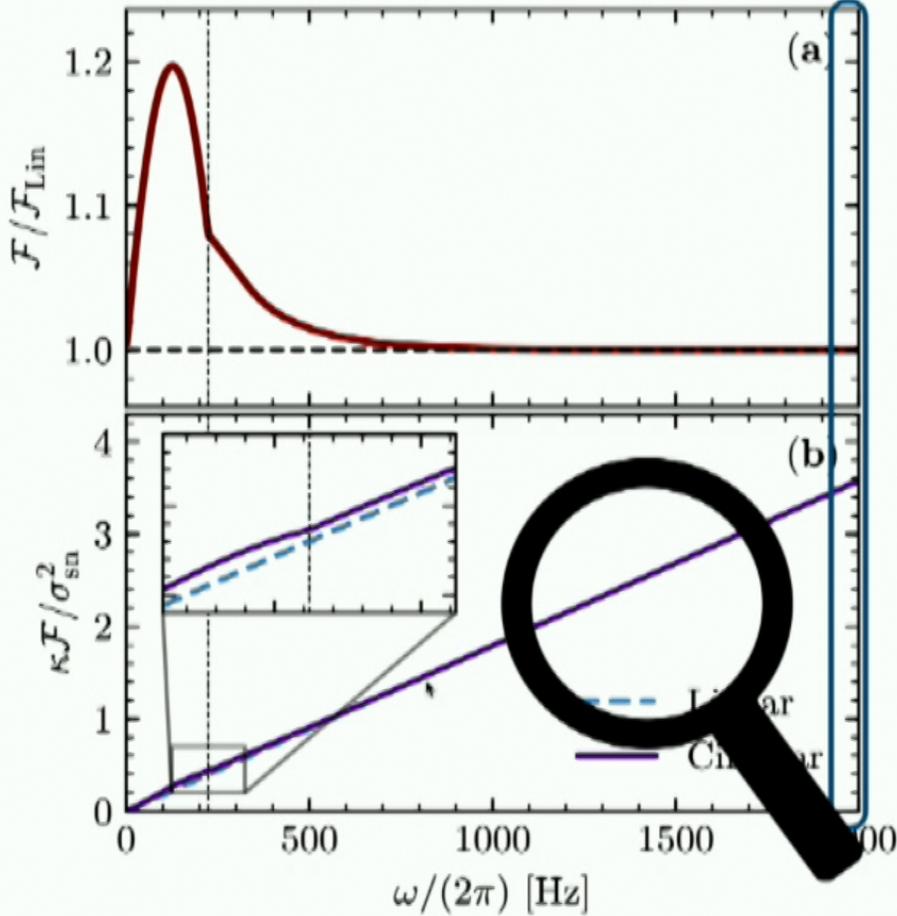
$$\text{SNR} \propto \frac{S_{\delta}(\omega)}{\sigma_{\text{sn}}^2 \sqrt{1 + 2 \frac{S_{\delta}(\omega)}{\sigma_{\text{sn}}^2} + 2 \frac{S_{\delta}^2(\omega)}{\sigma_{\text{sn}}^2}}}$$

Gooding et al. *Phys.Rev.Lett.* 125 (2020)

Linear dispersion cutoff



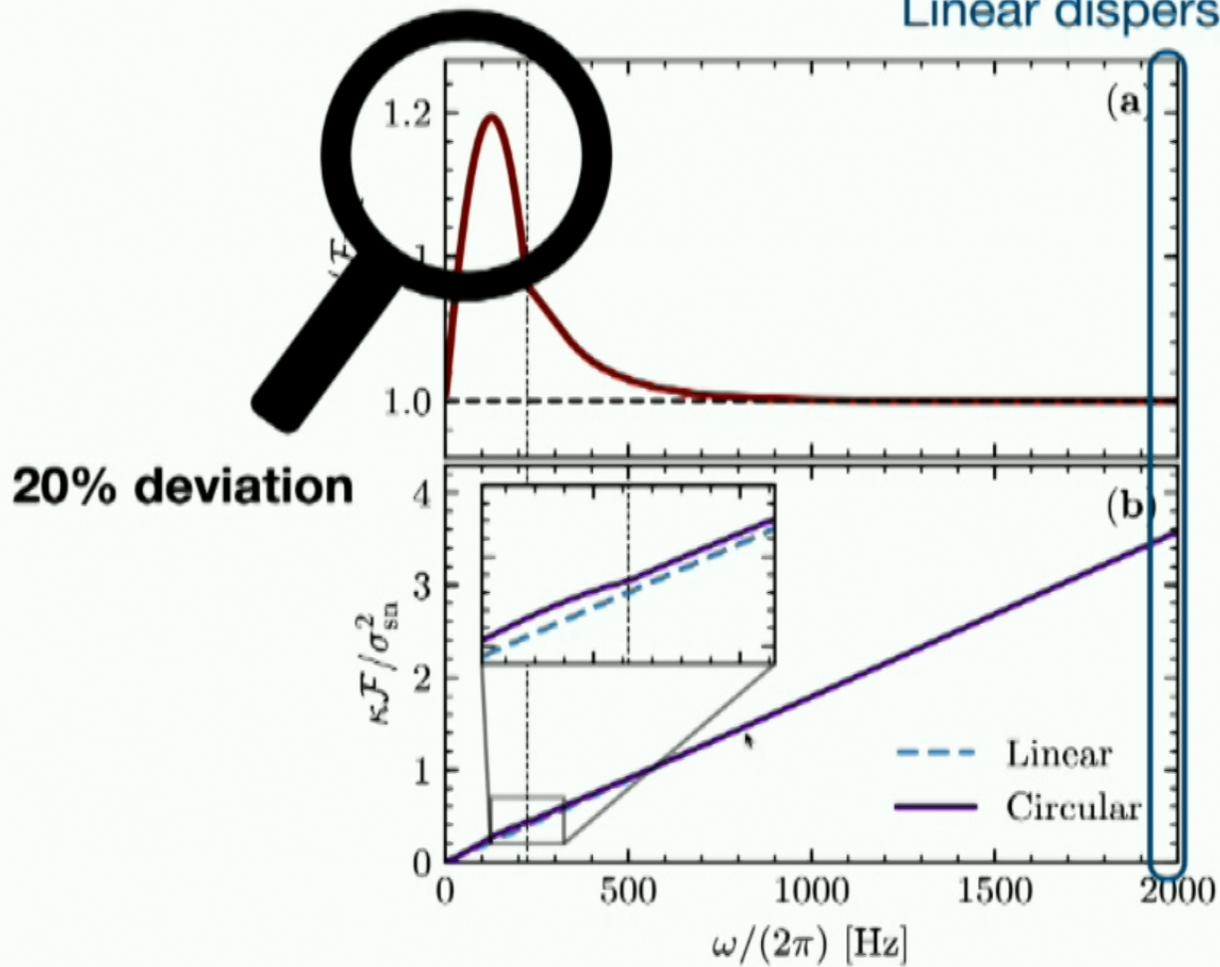
Linear dispersion cutoff



“Linear” has linear regime
 $\mathcal{F}_{Lin}(\omega) \propto \omega, (\omega > 0)$

Can approximate
 linear response

Linear dispersion cutoff



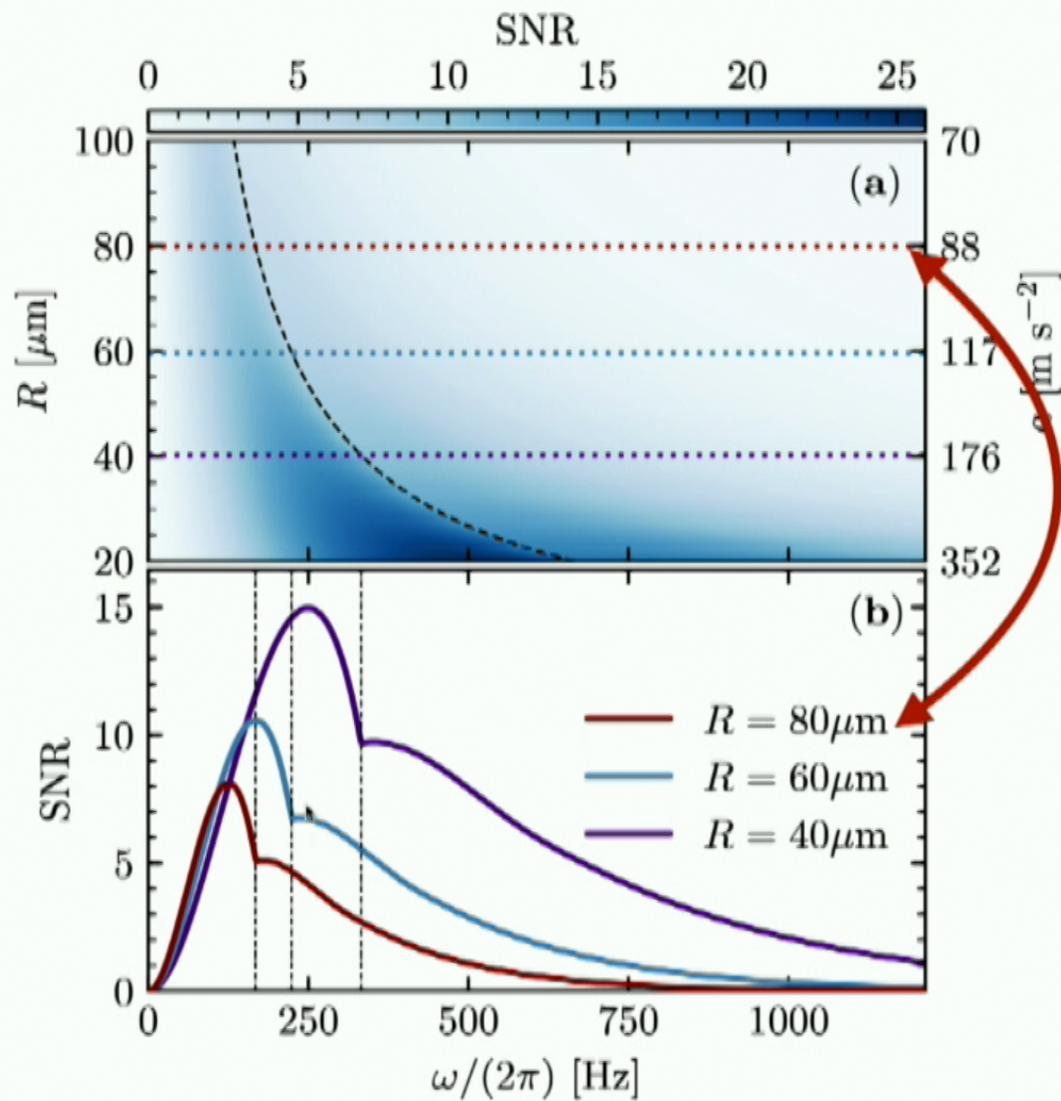
“Linear” has linear regime
 $\mathcal{F}_{\text{Lin}}(\omega) \propto \omega, (\omega > 0)$

Can approximate
linear response

(Parameter) Space Exploration

b

$T = 10 \text{ mK}$
 $\nu = 0.95c_3$ fixed

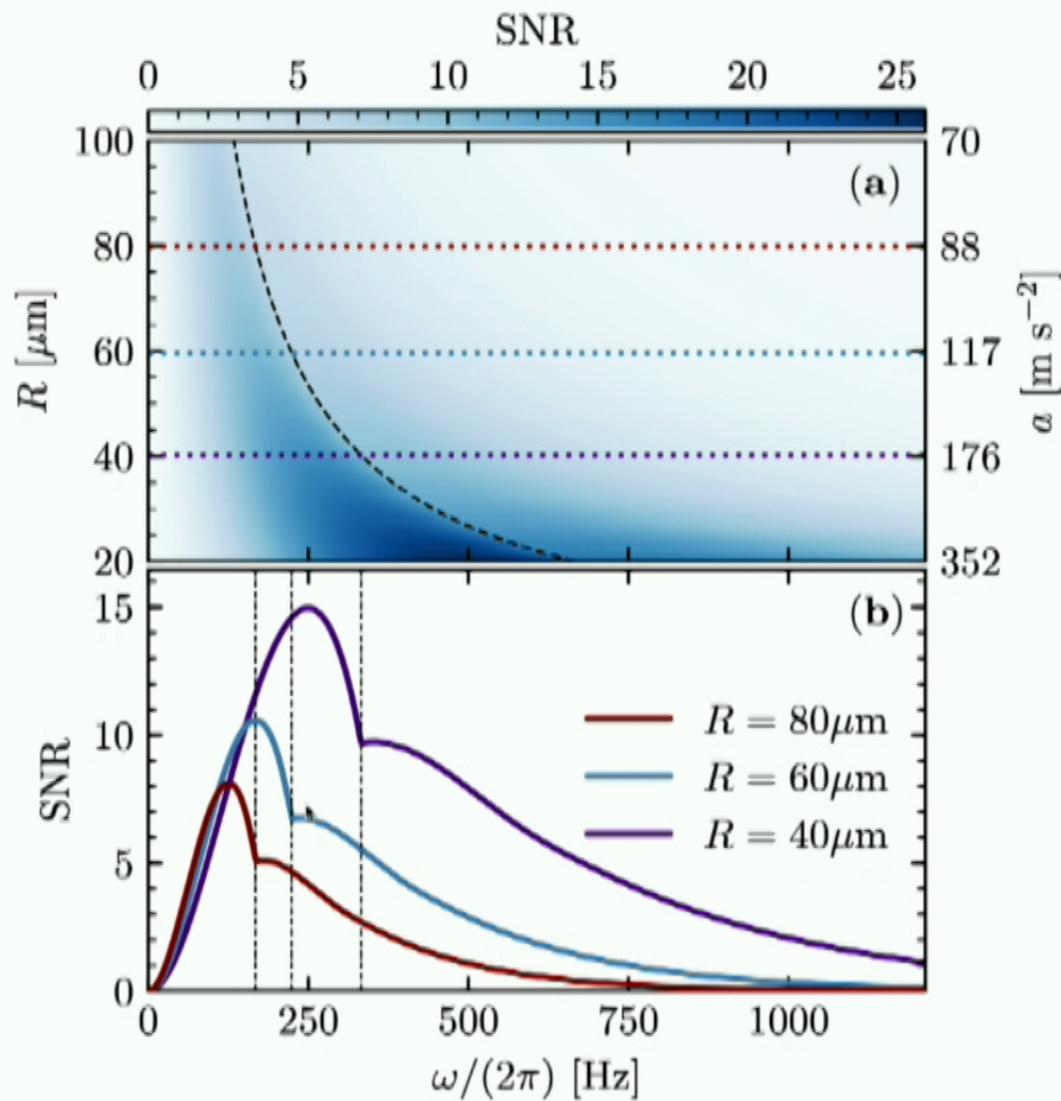


Bunney et al. arXiv[2302.12023]

$T = 10 \text{ mK}$
 $\nu = 0.95c_3$ fixed



Resonant probing
 $\omega = \Omega$

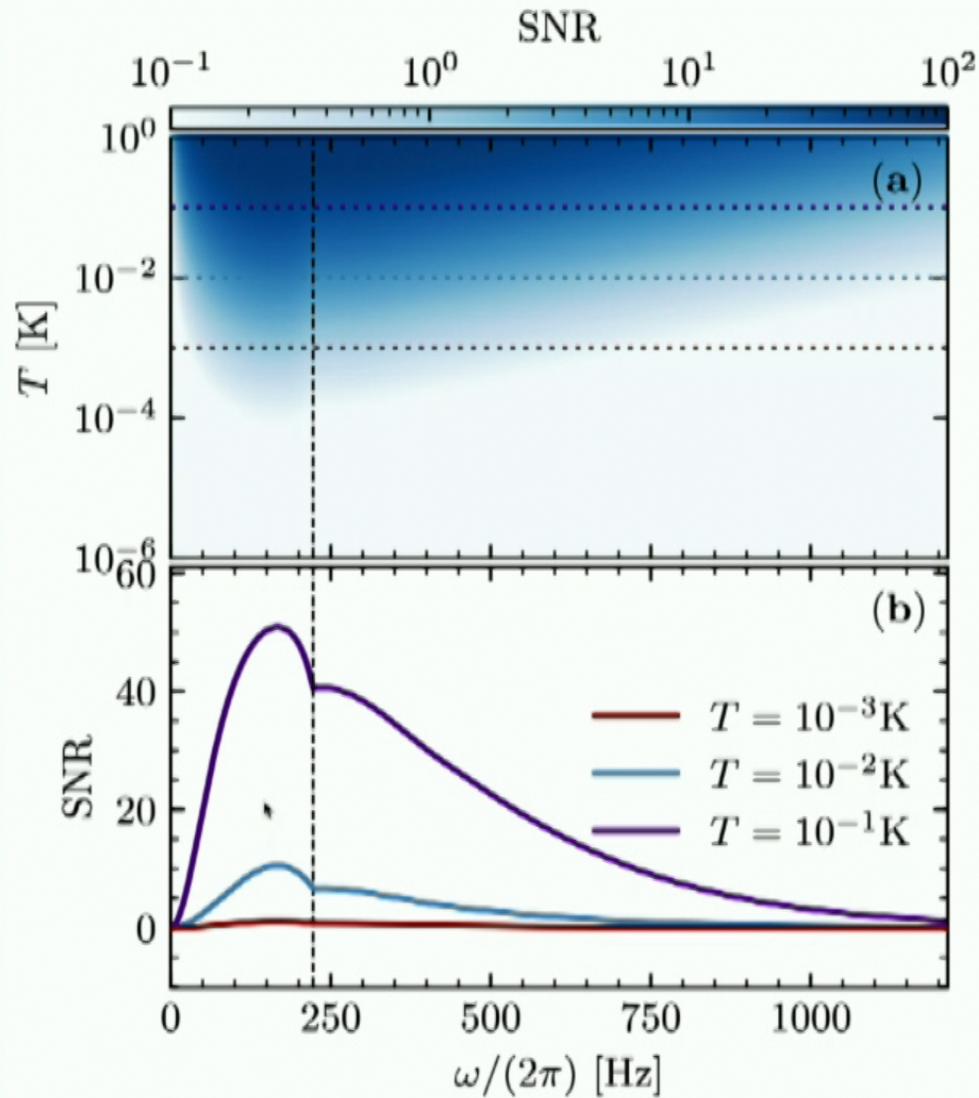


Bunney et al. *arXiv*[2302.12023]

$R = 60 \mu\text{m}$
 $v = 0.95c_3$ fixed



Resonant probing
 $\omega = \Omega$



Operational measuring
of $\mathcal{F}_{\text{Lin}}(\omega)$ over wide
parameter space

- **Even when $\hbar\omega < k_B T$ (or $\hbar\omega \ll k_B T$), can still find non-negligible acceleration-dependent signal**

- **Higher initial thermal state temperatures emphasise the acceleration-dependent structure**

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Thank you for listening!

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