Title: Analogue quasinormal mode oscillations of optical solitons

Speakers:

Collection: Quantum Simulators of Fundamental Physics

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Analogue Quasinormal Mode Oscillations of Optical Solitons

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P.I. Workshop Waterloo June 5-9

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SUPA QSimFP EPSRC Engineering and Physical Sciences Research Council

Contents

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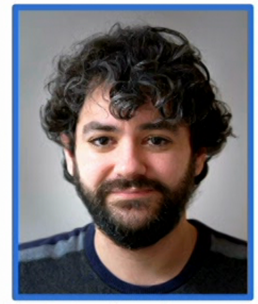

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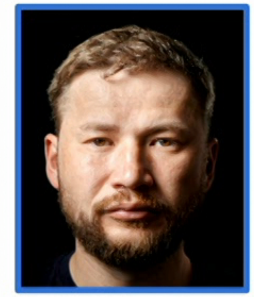

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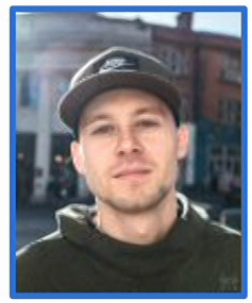

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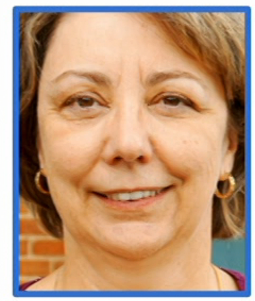

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Systems with Ringdown

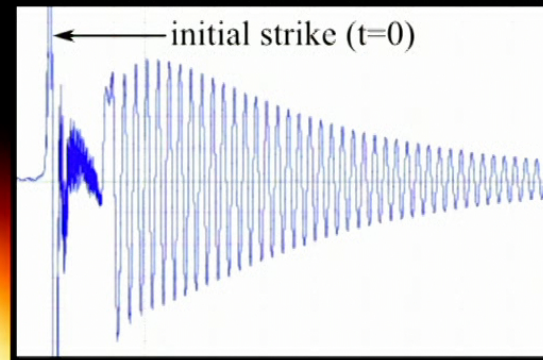
Many perturbative systems relax in oscillatory way

church bells	*stringed instruments*
leaky cavities	*electrical circuits with resistance*
black holes	*water vortex*
etc.	

- ❖ **Use in string theory, brane-world models, quantum gravity**

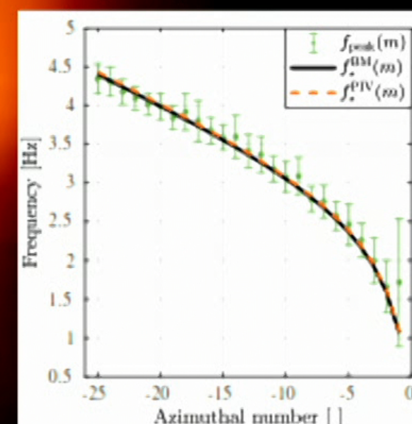
R. A. Konoplya and A. Zhidenko, Rev. Mod. Phys. **83**, 793 (2011).
E. Berti et al., Class Quantum Grav. **26,** 163001 (2009).
C. Sauvan et al., Opt. Ex., **30**, 6846 (2022).

Resonator Ringdown Following an Initial Perturbation
Lalanne, P., Yan, W., Vynck, K., Sauvan, C., & Hugonin, J. P. (2018)



Helmholtz Resonator

Spectrum of light ring modes in a water vortex



S. Patrick et al., PRL **121**, 061101 (2018).
Th. Torres et al. PRL **125**, 011301 (2020).

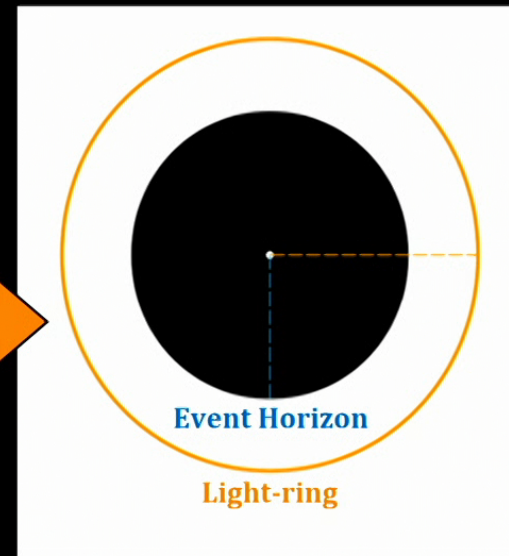
Sagittarius A*, the black hole at the center of the Milky Way galaxy. EHT Collaboration

Black Hole Oscillations

Perturbed black hole "rings down" like a church bell

The bell rings from its material parts
The black hole rings from the spacetime near its light-ring

Both have a *characteristic sound*



$c = G = 1$

Black Hole Spacetime

Schwarzschild $ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega$

(Kerr, Reissner–Nordström, Kerr–Newman)
for any asymptotic curvature

Perturbation Field

e.g. scalar, electromagnetic, gravitational → ***emission "ringdown" waves*** [5]

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu}$$

[5] LIGO, 2016

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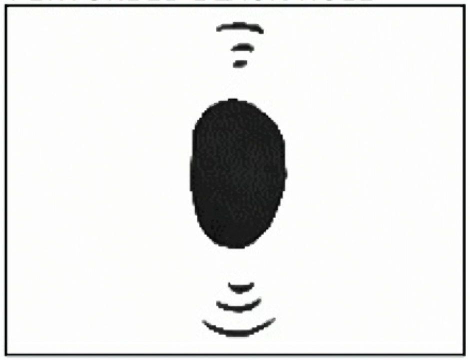
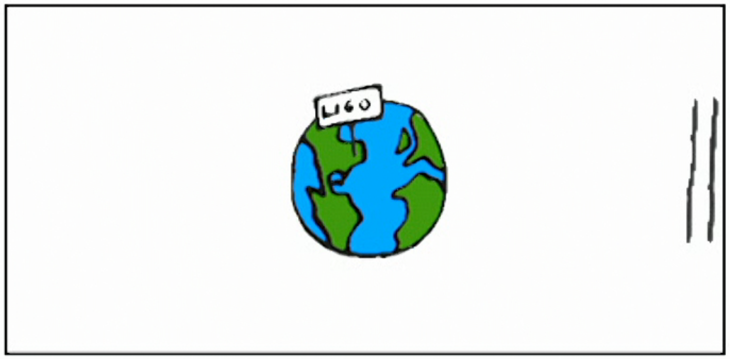
Supermassive Black Hole M87*
The Event Horizon Telescope Collaboration et al. ApJL 875 L4 (2019)

Black Hole Oscillations

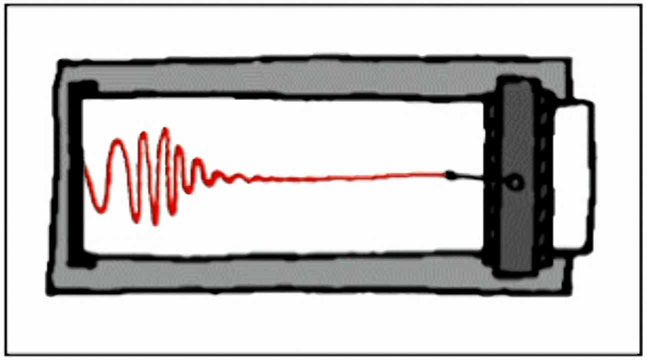
PERTURBED BLACK HOLE



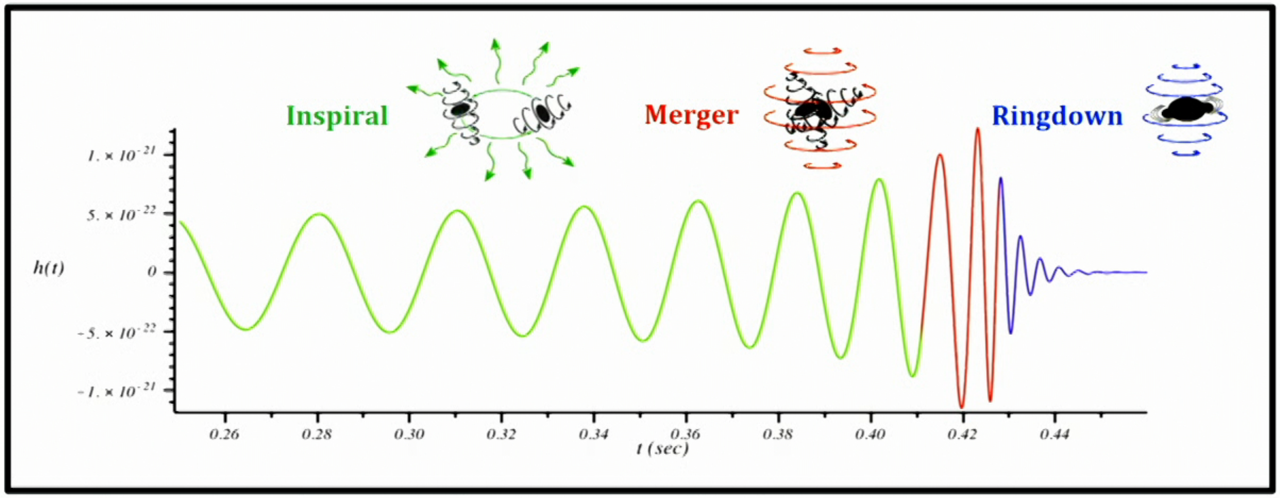
LIGO DETECTOR



LIGO SIGNAL



LIGO Detection of a Black Hole Merger



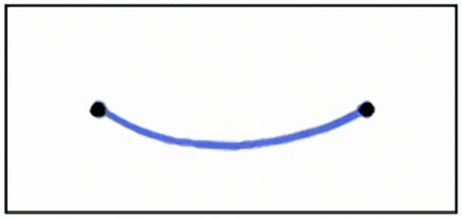
Best-Fit Theoretical Waveform to LIGO data *PRL 116, 061102 (2016)*

"Hearing the shape of a drum"

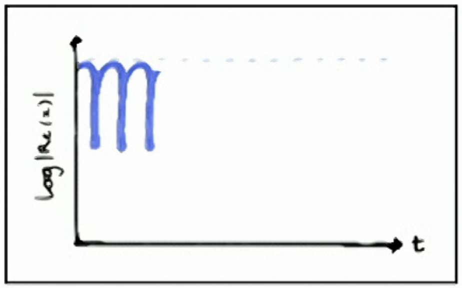
Can we hear the shape of a black hole?

Normal Modes v Quasinormal Modes

NORMAL MODE



$x(t) \sim \underbrace{e^{-i\omega t}}_{\text{oscillation}}$



→ ***closed***
→ ***conservative***
→ ***Hermitian***

Examples

harmonic oscillators
quantum fields

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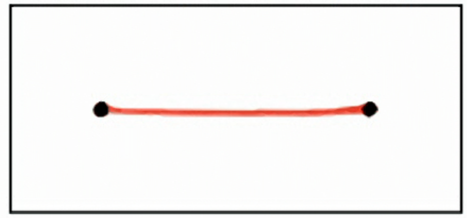
QUASINORMAL MODE (QNM)



$x(t) \sim \underbrace{e^{-\Gamma t}}_{\text{decay}} \underbrace{e^{-i\omega t}}_{\text{oscillation}}$

$\Omega = \omega - i\Gamma$

→ ***open***
→ ***non-conservative***
→ ***non-Hermitian***

Examples

plasmonic resonators [1]
fluid vortices [2]
black holes [3]
optical solitons

[1] Lalanne, 2018
[2] Torres, 2018
[3] Berti, 2009

www.st-Andrews.ac.uk/~qoi

Quasinormal Modes of Fields

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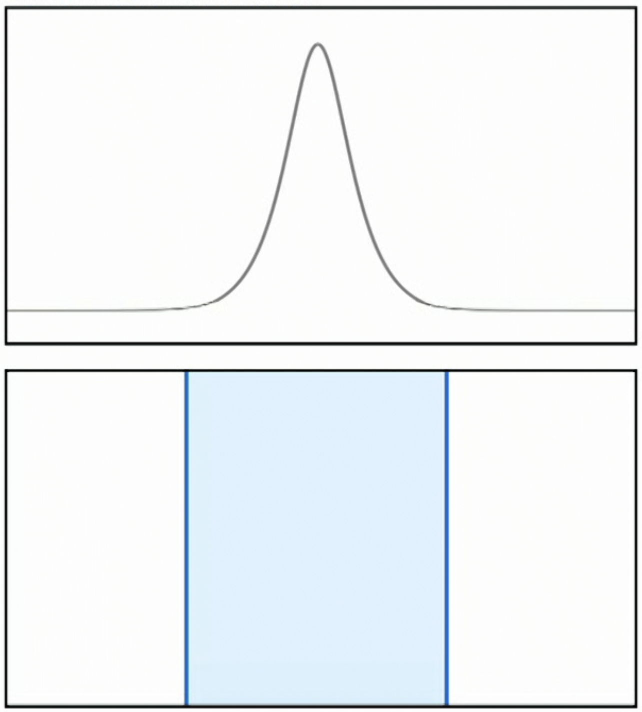
LOCALISED SYSTEMS



System

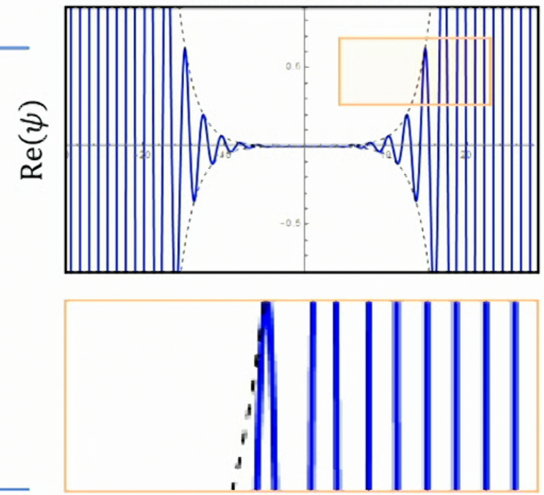
→ ***localised system***
e.g. time-independent potential
cavity / resonator

→ ***imperfect store of energy***
QNMs are "leaky modes"

Quasinormal Modes

→ ***outgoing energy flux***
→ ***decaying in time***
→ ***growing in space***
→ ***unphysical***

QUASINORMAL MODE SHAPE



$$\psi \sim e^{-i\omega(t \pm x/v)}$$

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[4] Kokkotas, Schmidt, L. Rev. Rel. 1999

Ringdown and Quasinormal Modes

SLIDE 8

Ringdown as relaxation of a perturbed system

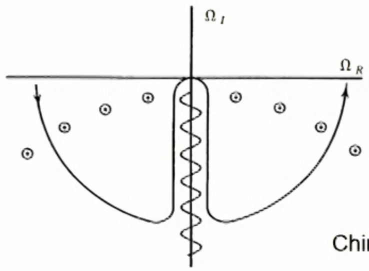
$$\partial_t^2\psi(t,x) + (-\partial_x^2 + V(x))\psi(t,x) = 0$$

Initial conditions $\psi(0,x),\ \partial_t\psi(0,x)$

Solution with Greens function:

$$\psi(t,x) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{(a+i\Omega)t}\int_{-\infty}^{\infty} G(a+i\Omega,x,x')\, j(a+i\Omega,x')\quad dx'd\Omega$$

Greens function   Initial data



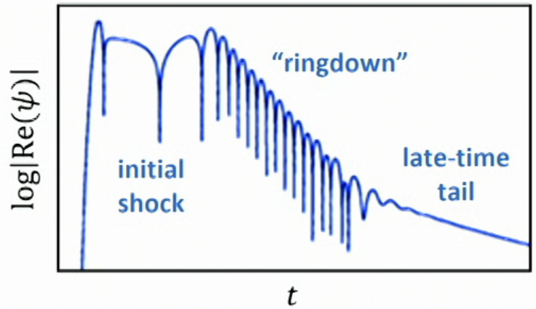
Ching et al., 1995

→ ***QNMs manifest in linear response***

$$\psi(t,x) \sim \sum_{n=0} a_n e^{i\Omega_n t} u(\Omega_n,x) + \psi_{rest}(t,x)$$

QNMs

QNM expansion develops over finite spaces[4]



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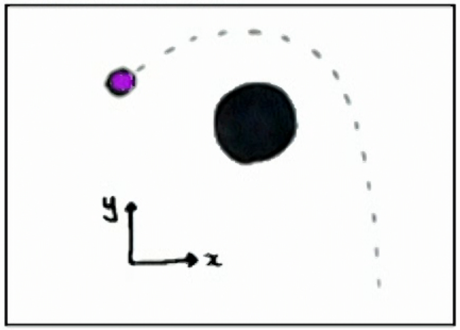
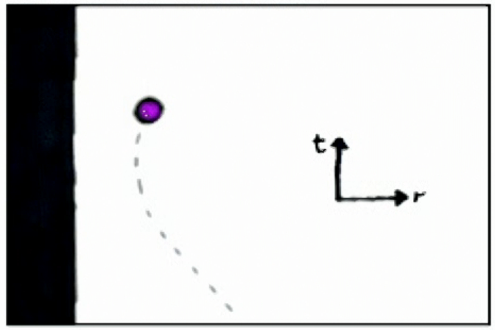
[4] Kokkotas, Schmidt, L. Rev. Rel. 1999

Effective Gravitational Potential

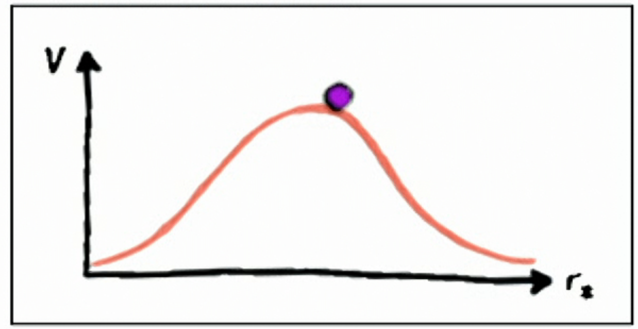
ORBITAL PLANE



RADIAL CO-ORDINATE



CENTRIFUGAL BARRIER



Perturbation Field

e.g. *Massless Scalar Field* $\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu})\partial_\nu\psi = 0$

Radial Wavefunction

$$\psi(t,r,\theta,\phi) = e^{-i\omega t}\,\frac{\Psi(r)}{r}\,Y_l(\theta,\phi)$$

1D Schrödinger Equation

$$\partial_{r_*}^2\Psi + (\omega^2 - V)\Psi = 0$$

Effective Gravitational Potential $V(r) = \left(1 - \frac{r_s}{r}\right)\left[\frac{l(l+1)}{r^2} + \frac{r_s}{r^3}\right]$

(peaked around light-ring)

Tortoise co-ordinate $\frac{dr}{dr_*} = \left(1 - \frac{r_s}{r}\right)$

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Mashoon: Pöschl-Teller approximation

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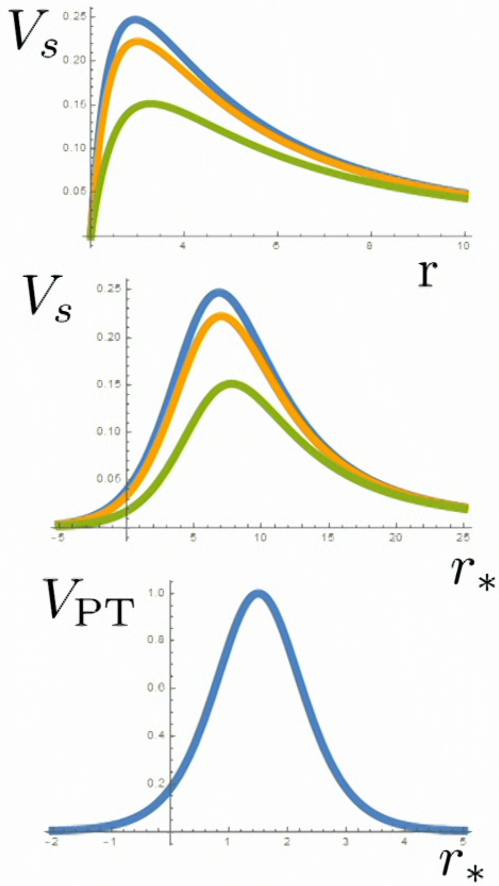
$$\frac{d^2\Psi_s}{dr_*^2} + (\omega^2 - V_s)\Psi_s = 0$$

$$V_s = \left(1 - \frac{2M}{r}\right)\left[\frac{l(l+1)}{r^2} + (1-s^2)\frac{2M}{r^3}\right]$$



`Low lying' QNMs are dominated by the peak of the potential.

Exact solutions available for Pöschl-Teller potential [1]:

$$\omega_n = \pm\sqrt{V_0 - \alpha^2/4} - i\alpha(2n+1)/2$$

$$n = 0, 1, 2, \ldots$$

$$V_{\rm PT} = \frac{V_0}{\cosh^2\alpha(r_* - r_{*0})}$$

[1] H.-J. Blome and B. Mashhoon, Phys. Lett. **110A**, 231 (1984).

Black Hole QNM Spectrum

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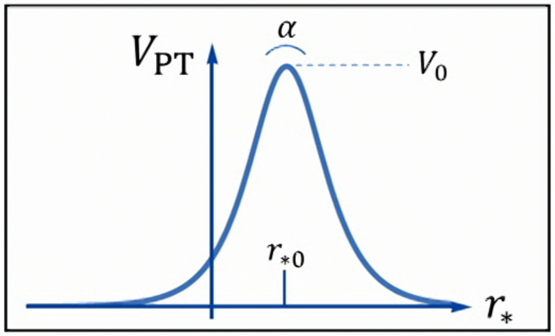
Pöschl-Teller Approximation[6]

$$V \approx V_{\mathrm{PT}} = V_0 \,\mathrm{sech}^2\, \alpha(r_* - r_{*0})$$

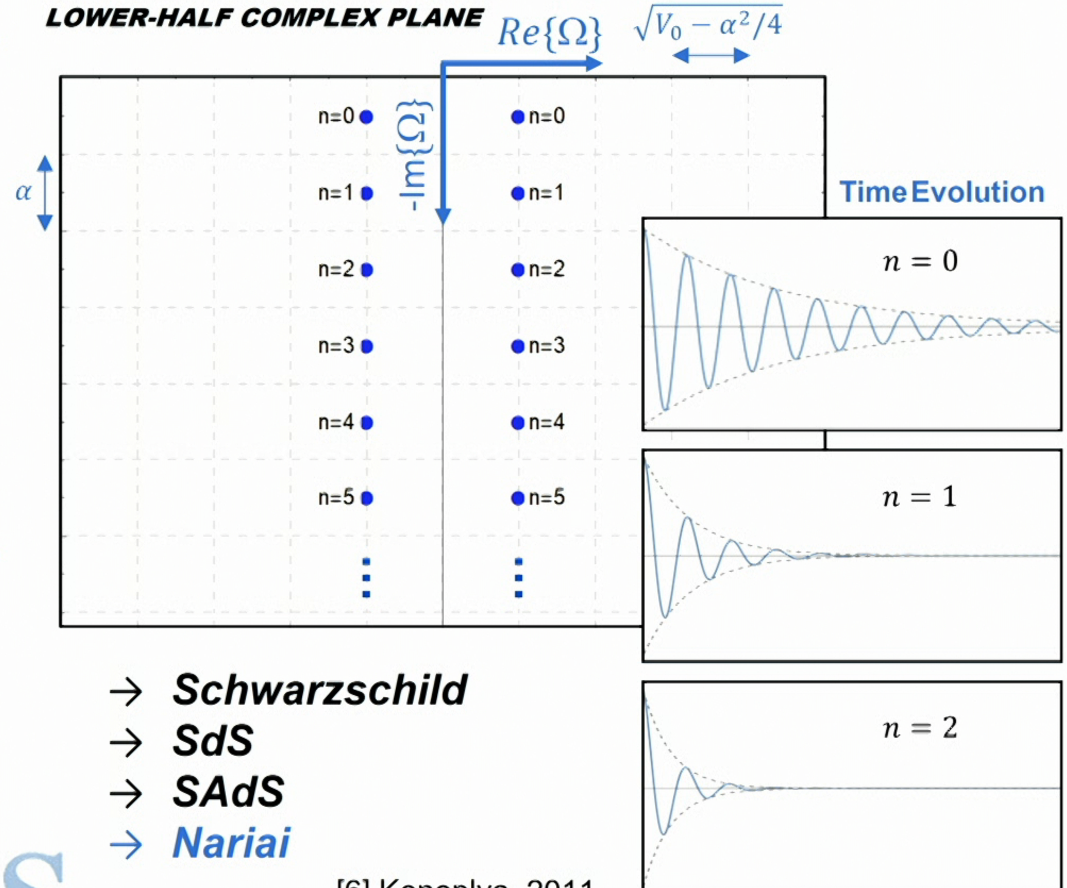


QNM Spectrum

$$\omega_n = \pm\sqrt{V_0 - \alpha^2/4} - i\alpha\left(n + \frac{1}{2}\right)$$

$$n = 0, 1, 2, \ldots$$

LOWER-HALF COMPLEX PLANE



→ ***Schwarzschild***
→ ***SdS***
→ ***SAdS***
→ ***Nariai***

[6] Konoplya, 2011

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Optical Solitons

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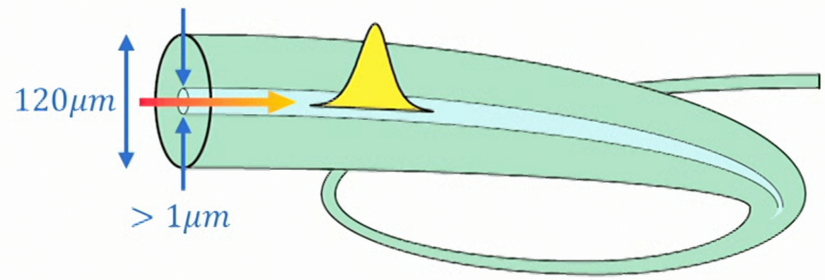
Pulse Propagation in Dispersive Kerr Media[7]

Optical Kerr effect

$$n(\omega, t) = n_0(\omega) + n_2 I(t)$$



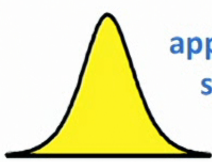
Nonlinear Schrödinger Equation (higher-order dispersion)

$$\partial_z A - i[\beta(i\partial_t + \omega_s) - \beta(\omega_s)]A - i\gamma |A|^2 A = 0$$

$\beta(\omega) = \frac{n_{\mathrm{eff}}(\omega)\,\omega}{c}$	propagation constant
ω_s	carrier frequency
γ	fibre nonlinear coefficient

Soliton Solution

$$A_s(z, t) = \sqrt{P_0}\ \mathrm{sech}\left(\frac{t - z/v}{T_0}\right) e^{i\gamma P_0 z}$$



approximate solution

P_0	soliton pulse peak power
v	group velocity at soliton frequency
T_0	soliton temporal pulse width

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[7] Agrawal, 2003

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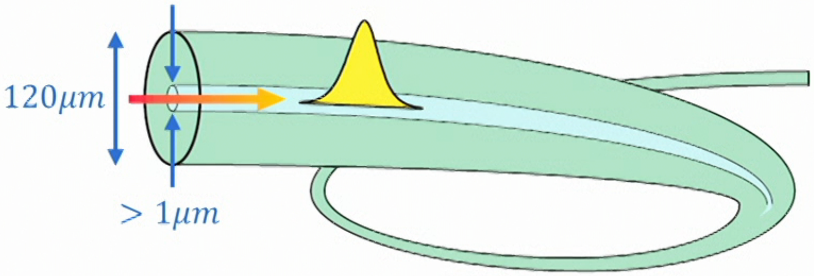
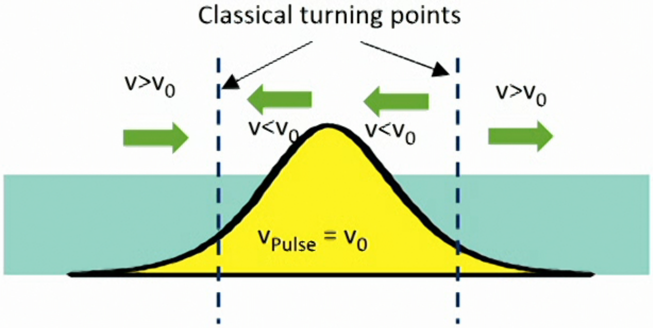
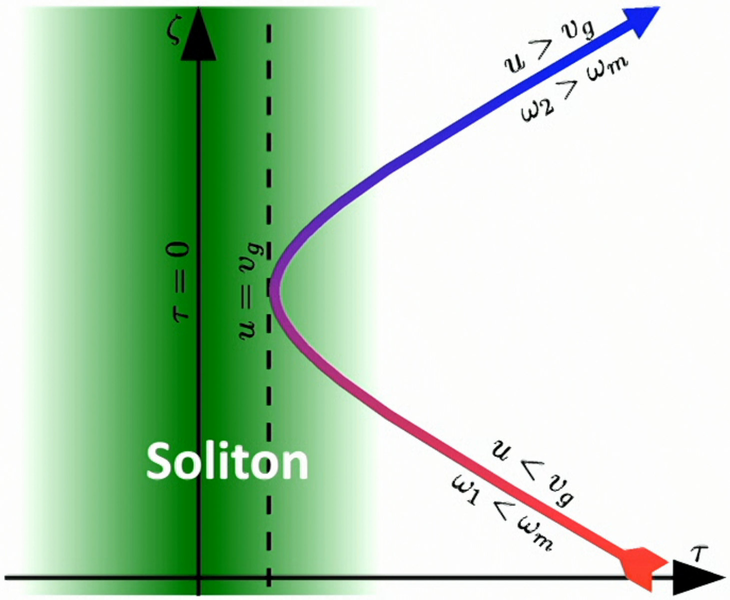
Optical analogue to QNMs?

Wave velocity in moving frame: optical pulse in Kerr medium

A pulse moves at velocity v_0 and locally modifies the refractive index (i.e. wave velocity) by the cross-Kerr effect.







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Soliton as Optical Potential

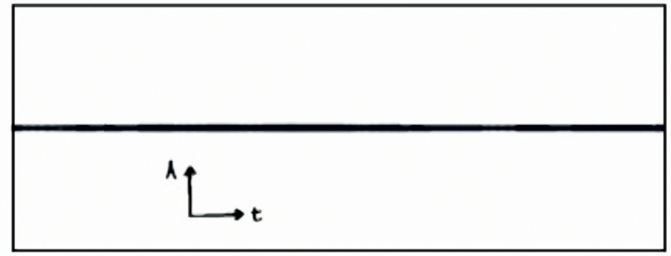
Linear Perturbation Equation

$$\partial_z A_p - i[\beta(i\partial_t + \omega_s) - \beta(\omega_s)]A_p - 2i\gamma|A_s|^2 A_p - \underline{i\gamma A_s^2 A_p^*} = 0$$

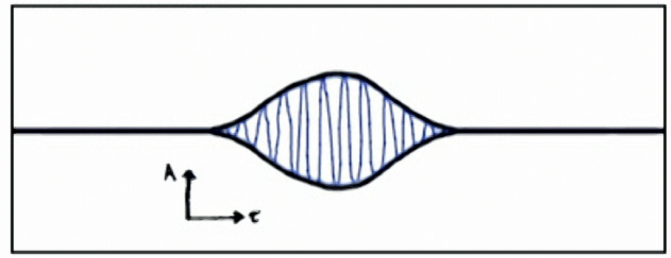
fast-oscillating

$$A = A_s + A_p \qquad \left(|A_p|^2 \ll P_0\right)$$

LABORATORY FRAME



MOVING FRAME



Group-Velocity Matched Perturbation Equation

- → ***spectrally distinct*** → *neglect* $i\gamma A_s^2 A_p^*$
- → ***narrowband*** → $\beta(\omega) = \sum_{k=0}^{2} \frac{\beta_{ak}}{k!}(\omega - \omega_a)^k$
- → ***group velocity matched*** → $\beta_{a1} = \beta_{s1} = 1/v$

$$\beta_{s1}\partial_\zeta a_p + \frac{i}{2}\frac{\beta_{a2}}{T_0^2}\partial_\tau^2 a_p - \underline{2i\gamma|A_s|^2} a_p = 0$$

→ soliton intensity as potential

ω_a perturbation freq.

moving frame

$$\tau = \frac{t - z/v}{T_0} \qquad \zeta = z/v$$

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Soliton Quasinormal Modes

Soliton GVM Modes

$$a_p(\zeta,\tau) = u(\tau)e^{-i\Omega\zeta}$$

$$\partial_\tau^2 u + \left[-\frac{2|\beta_{s2}|\Omega}{\beta_{a2}} - \frac{4|\beta_{s2}|}{\beta_{a2}}\operatorname{sech}^2(\tau)\right]u = 0$$

QNM Solution

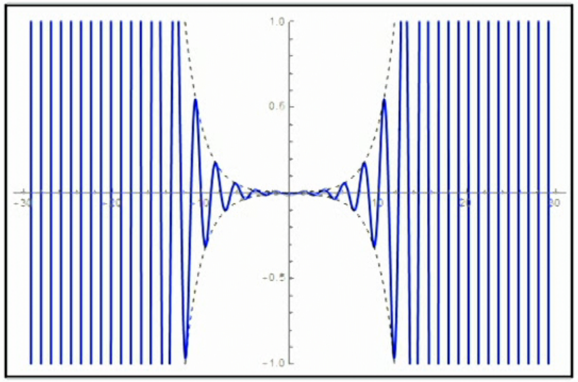
$$a_{p,n}(\zeta,\tau) = \cosh(\tau)^{n+1/2} \times \exp(\mathrm{Im}(\Omega_n)\zeta) \times \exp(-i\,\mathrm{Im}(\lambda)\log\cosh(\tau) - i\mathrm{Re}(\Omega_n)\zeta) \times {}_2F_1[-n, 1+2\lambda-n; 1+\lambda-n; [1-\tanh(\tau)]/2]$$

discrete spectrum
$n = 0, 1, 2, \ldots$

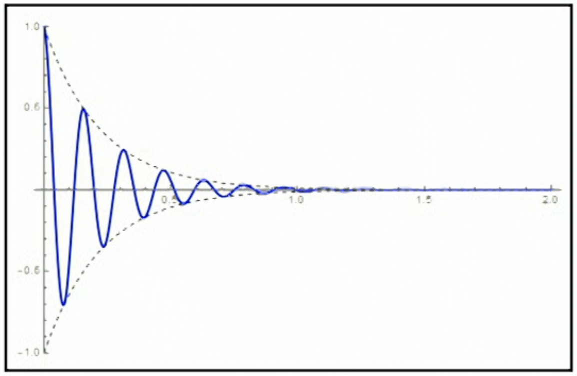
$$\lambda = -\frac{1}{2} + i\sqrt{\frac{4|\beta_{s2}|}{\beta_{a2}} - \frac{1}{4}}$$

1. Divergence in τ ('space')
2. Decay in the ζ ('time')
3. Directs phase velocity
4. Order-n polynomial in $\tanh(\tau)$ controls parity of solution

'SPATIAL' PROFILE



'TEMPORAL' PROFILE



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Soliton QNM Spectrum

QNM Frequencies

$$\Omega_n = \frac{\beta_{a2}}{2|\beta_{s2}|}\left[\left(n+\frac{1}{2}\right)^2 - \left(\frac{4|\beta_{s2}|}{\beta_{a2}} - \frac{1}{4}\right)\right] - i\frac{\beta_{a2}}{|\beta_{s2}|}\left(n+\frac{1}{2}\right)\sqrt{\frac{4|\beta_{s2}|}{\beta_{a2}} - \frac{1}{4}}$$

discrete spectrum
$n = 0, 1, 2, \ldots$

→ ***QNM overtones fall along a parabola***

$$\mathrm{Im}(\Omega)^2 - \left(8 - \frac{\beta_{a2}}{2|\beta_{s2}|}\right)\mathrm{Re}(\Omega) + \frac{1}{4}\left(8 - \frac{\beta_{a2}}{2|\beta_{s2}|}\right)^2 = 0$$

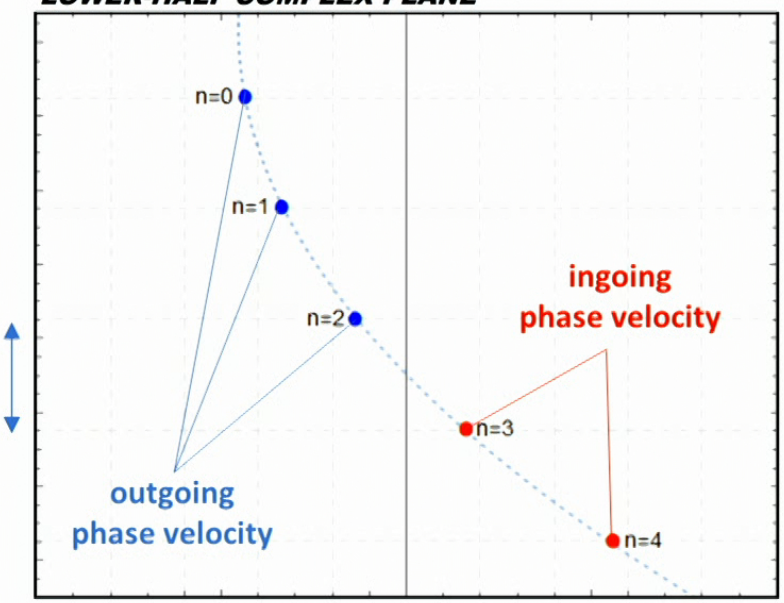
→ ***Finitely many outgoing QNMs***

$$N_{\mathrm{out}} = \left\lfloor \frac{1}{2} + \sqrt{\frac{4|\beta_{s2}|}{\beta_{a2}} - \frac{1}{4}} \right\rfloor$$

→ ***Infinitely many ingoing QNMs***

$$\frac{\beta_{a2}}{|\beta_{s2}|}\sqrt{\frac{4|\beta_{s2}|}{\beta_{a2}} - \frac{1}{4}}$$

LOWER-HALF COMPLEX PLANE



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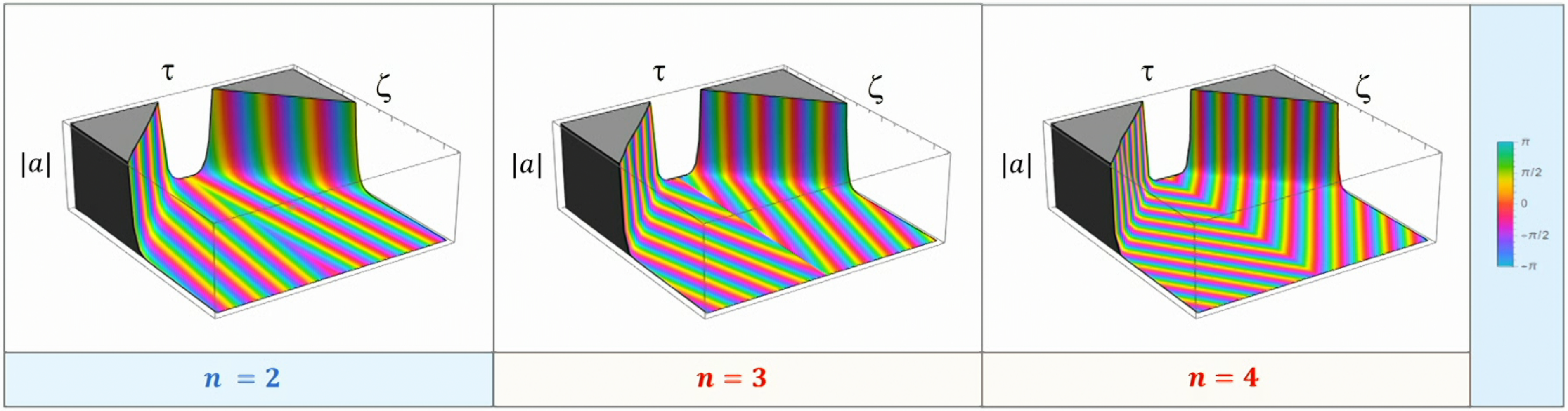
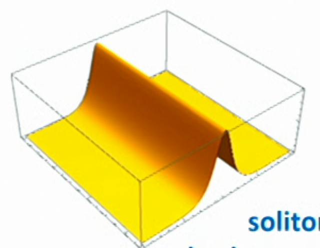
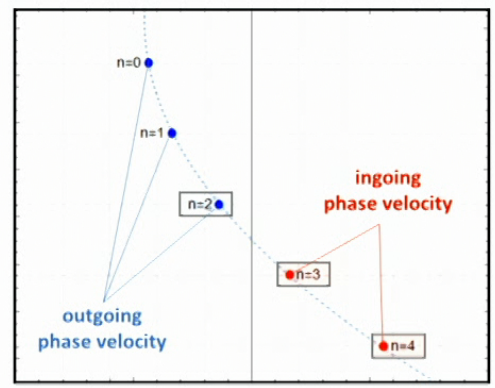
Soliton QNM Representations II



1. Higher modes diverge in τ more quickly.
2. Higher modes decay in ζ more quickly.
3. **Phase velocities turn around and then increase.**





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Optical Soliton / Black Hole QNM Analogy

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Optical Soliton QNMs

→ ***new black hole analogue system***

Formally Equivalent Mode Equations

$$\partial_\tau^2 u + \left[-\frac{2|\beta_{s2}|\Omega}{\beta_{a2}} - \frac{4|\beta_{s2}|}{\beta_{a2}} \operatorname{sech}^2(\tau) \right] u = 0$$

GVM Soliton QNMs

⇕

$$\partial_\rho^2 \Psi + \left[\frac{\omega^2}{\alpha^2} - \frac{V_0}{\alpha^2} \operatorname{sech}^2(\rho) \right] \Psi = 0$$

Black Hole QNMs

→ ***identical spatial profiles***
- *identical field expansions*

$$\Omega \sim -\omega^2$$

→ ***different mode frequencies***
- *different temporal profiles*
- *different field evolution*

(dispersionless limit may solve this)

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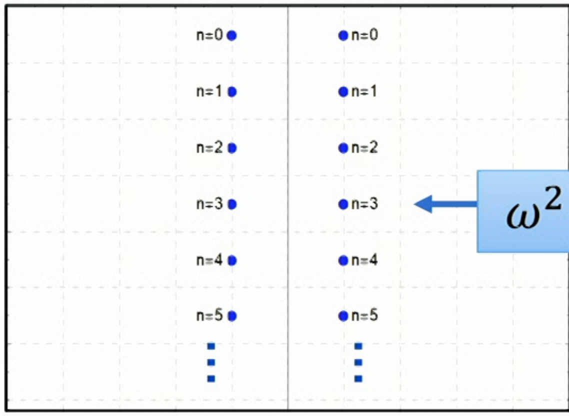
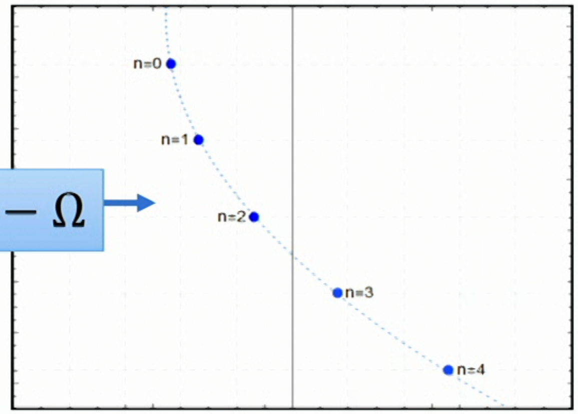
Soliton QNMs: Dispersionless Limit

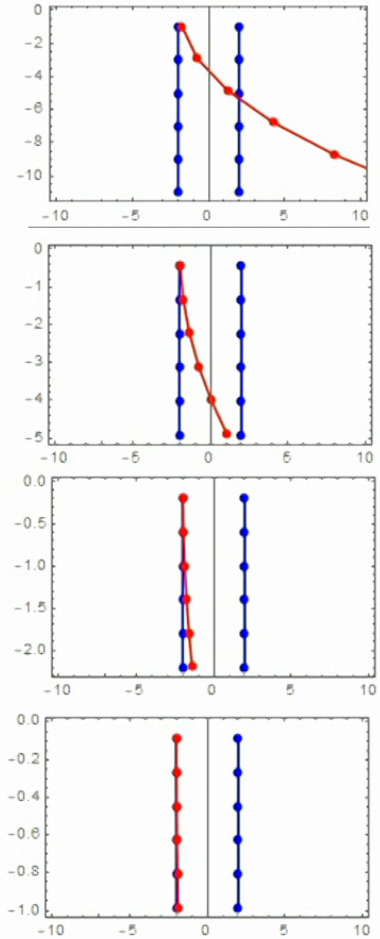
BLACK HOLE QNM SPECTRUM



$$\omega^2 \sim -\Omega$$

OPTICAL SOLITON QNM SPECTRUM





MORE DISPERSION

LESS DISPERSION

QNM frequencies become equivalent for small dispersion at the perturbation:

$$|\beta_{s2}| \gg \beta_{a2}$$

$$\omega_n = \pm\sqrt{V_0 - \frac{\alpha^2}{4}} - i\alpha\left(n + \frac{1}{2}\right) \qquad \Omega_n \approx -2 - i\sqrt{\frac{4\,\beta_{a2}}{|\beta_{s2}|}}\left(n + \frac{1}{2}\right)$$

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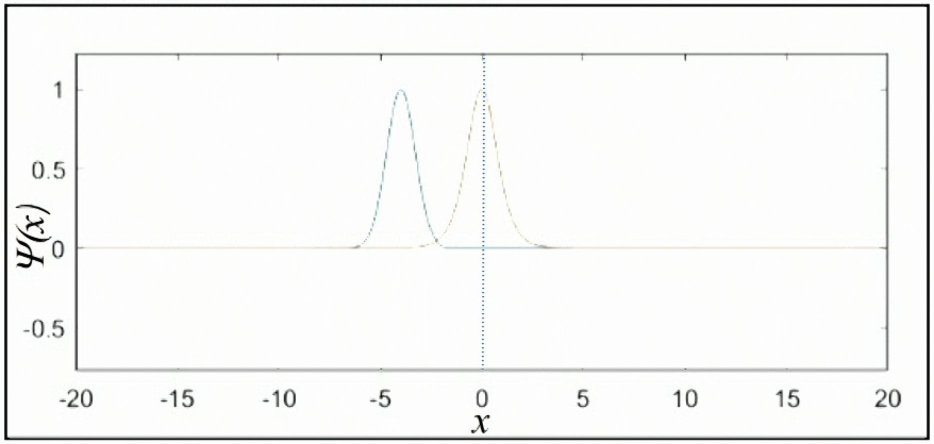
Simulation: Ringdown in Optical Solitons

SLIDE 21

SCATTERING OF DISPERSIVE WAVE



→ ***near-GVM pulse on soliton***

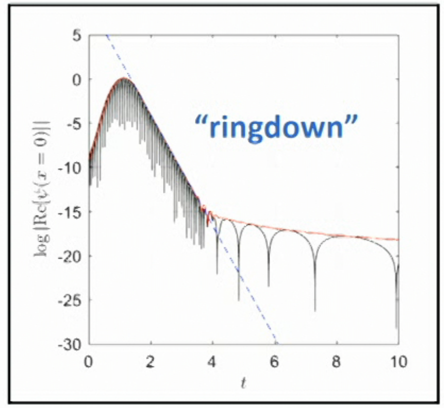
all pulse shapes excite QNMs

best: *super-gaussian and chirped*

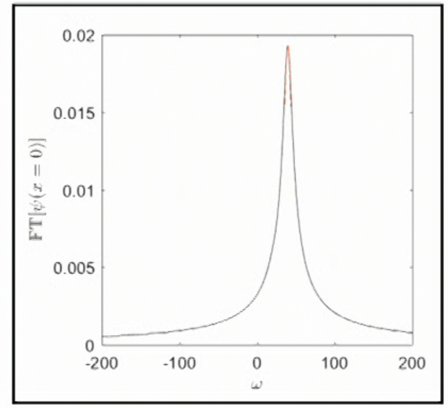
$$V \equiv \frac{4|\beta_{s2}|}{\beta_{a2}}$$

→ ***ringdown in wake of transmission/reflection***

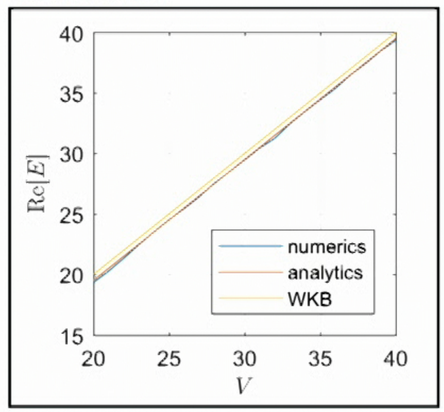
RINGDOWN SIGNAL



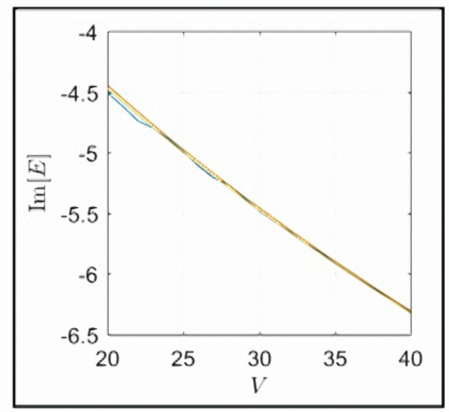
RINGDOWN SPECTRUM



REAL PART



IMAGINARY PART



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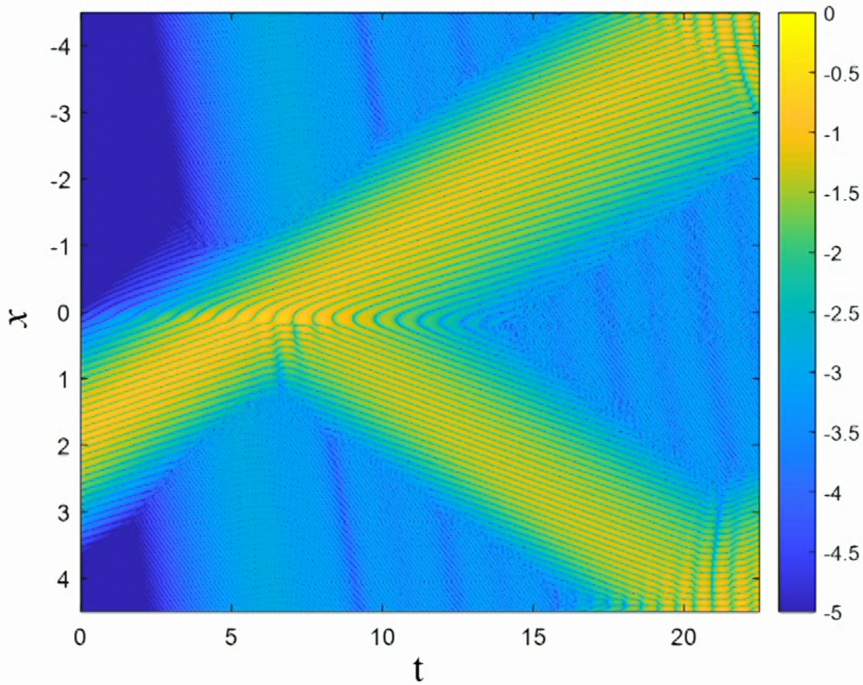
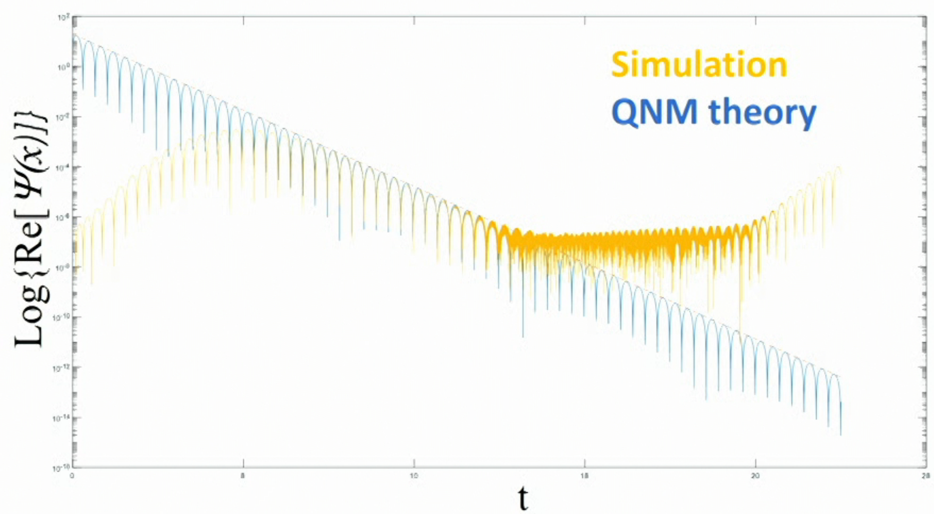
QNM with evolving soliton

Rerturbation field (real part, soliton background removed)



RINGDOWN SIGNAL



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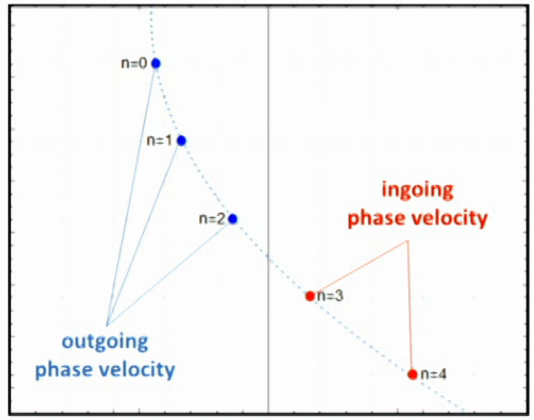
Summary and Outlook

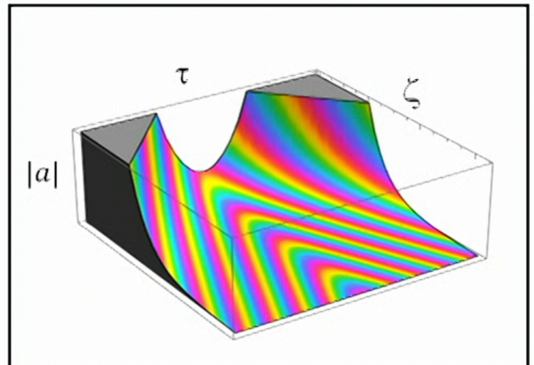
Summary

- Soliton acts as an optical potential
- NLS solitons exhibit "ringdown" described by QNMs
- New experimentally accessible black hole analogue
- First soliton QNM system & first with "ingoing" QNMs

Outlook

- Soliton ringdown to be observed experimentally
- Generalizations beyond GVM QNMs
- QNMs of other solitons (e.g. KdV, sine-Gordon)
- Quantum effects with QNMs





$n = 0$ (Fundamental)

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