

Title: Fireslides

Speakers:

Collection: Quantum Simulators of Fundamental Physics

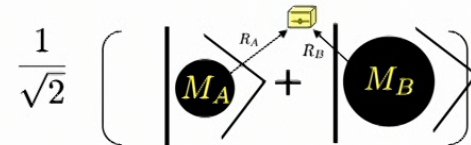
Date: June 06, 2023 - 2:00 PM

URL: <https://pirsa.org/23060010>

Quantum Superposition of Black Holes

- Quantum theory of gravity → Spacetime superposition → Black Hole Superpositions
- How might one observe this with a model detector?

Arabaci/Foo/RBM/Zych
PRL **129** (2022) 181301
2111.13315



$$|\psi(t_i)\rangle = \frac{1}{\sqrt{2}}(|M_A\rangle + |M_B\rangle)|0\rangle|g\rangle \longrightarrow |\psi(t_f)\rangle = e^{-iH_0, st_f} \hat{U} e^{iH_0, st_i} |\psi(t_i)\rangle$$

Condition on $|\pm\rangle = (|M_A\rangle \pm |M_B\rangle)/\sqrt{2}$ and trace out the field

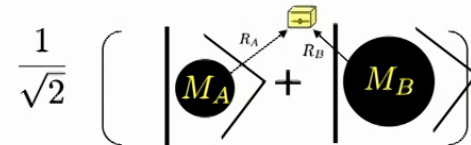
$$\begin{aligned} \text{Tr}_\phi \left[\langle \pm | \psi(t_f) \rangle \langle \psi(t_f) | \pm \rangle \right] &= \frac{|g\rangle\langle g|}{2} P_G^{(\pm)} + \lambda^2 \frac{|e\rangle\langle e|}{2} P_E^{(\pm)} \\ &= \frac{|g\rangle\langle g|}{2} (1 \pm \cos(\Delta E \Delta t)) \left[1 - \frac{\lambda^2}{2} \int_{-t_f}^{t_f} d\tau \int_{-t_f}^{t_f} d\tau' \eta(\tau) \eta(\tau') e^{-i\Omega(\tau-\tau')} (W(x_A, x'_A) + W(x_B, x'_B)) \right] \\ &\quad + \frac{\lambda^2 |e\rangle\langle e|}{4} \int_{-t_f}^{t_f} d\tau \int_{-t_f}^{t_f} d\tau' \eta(\tau) \eta(\tau') e^{-i\Omega(\tau-\tau')} (W(x_A, x'_A) + W(x_B, x'_B) \pm 2 \cos(\Delta E \Delta t) W(x_A, x'_B)) \end{aligned}$$

Reduced Density
Matrix of the
Detector

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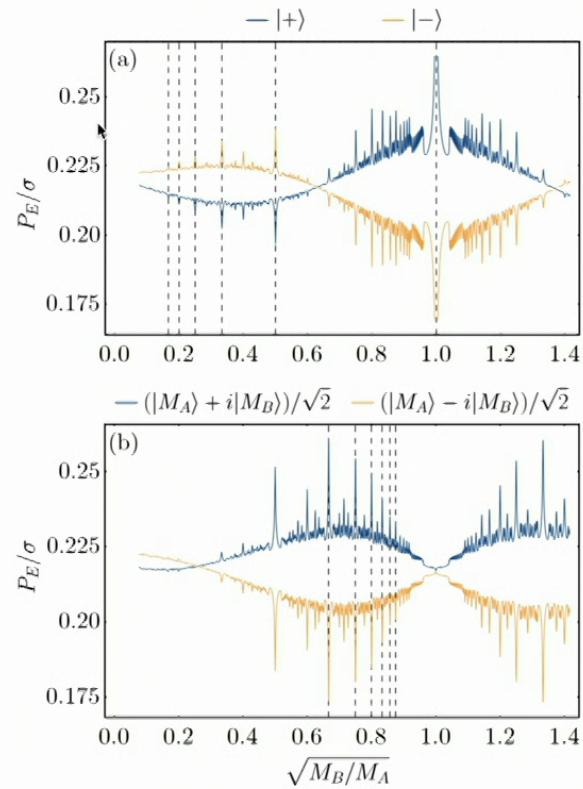
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Result



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Dashed lines:
 $\sqrt{M_B/M_A} = (n-1)/n$
where $n = \{3, \dots, 8\}$

Resonant peaks at integer values of (mass ratios)^{1/2} !

Consistent with Bekenstein's black hole mass quantization conjecture

Could this be simulated in analogue gravity?

Event horizons as quantum catastrophes (Duncan O'Dell, McMaster)

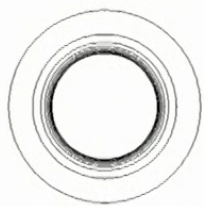
Liam Farrell
(McMaster)



Chris Howls
(Southampton)



Event horizon is a **phase singularity**



$$k \sim \frac{\omega}{z - z_h}$$

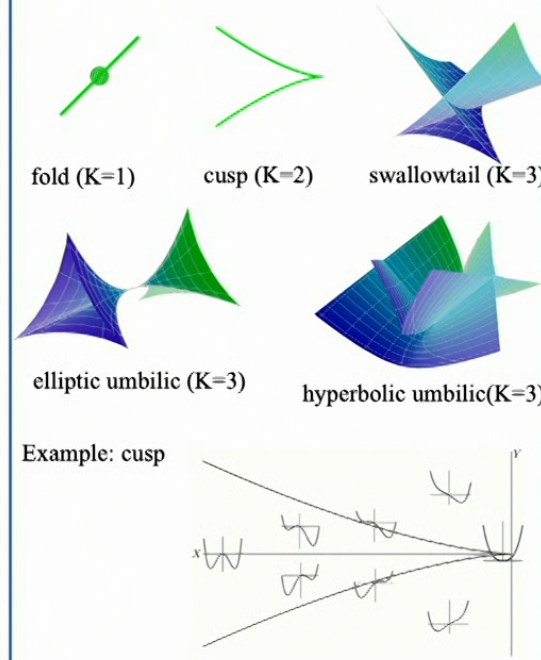
Expect: $\psi \sim e^{i \int [k(z) dz - \omega t]}$
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Flowing BEC, or black hole with dispersion:

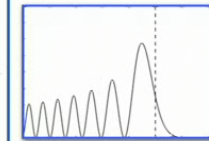
$$\psi(z, \omega) \sim \int_{-\infty}^{+\infty} \frac{1}{k} e^{i[k^3/3 + kz - \omega \log(k)]} dk$$

Logged Airy function

Catastrophe theory

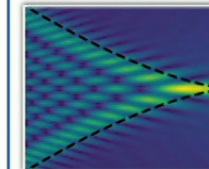


Wave catastrophes



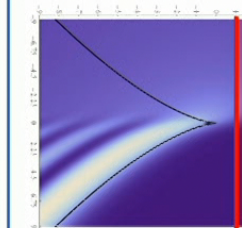
Airy function

$$\Psi_{\text{fold}}(C) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(s^3/3 + Cs)} ds$$



Pearcey function

$$\Psi_{\text{cusp}}(C_1, C_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(s^4/4 + C_2 s^2/2 + C_1 s)} ds$$



Logged Airy
Particle production??

Event horizon

Logarithmic catastrophes and Stokes's phenomenon in waves at horizons, Farrell, Howls, O'Dell, J. Phys. A : Math. Theor. 56, 044001 (2023)

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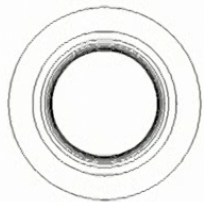
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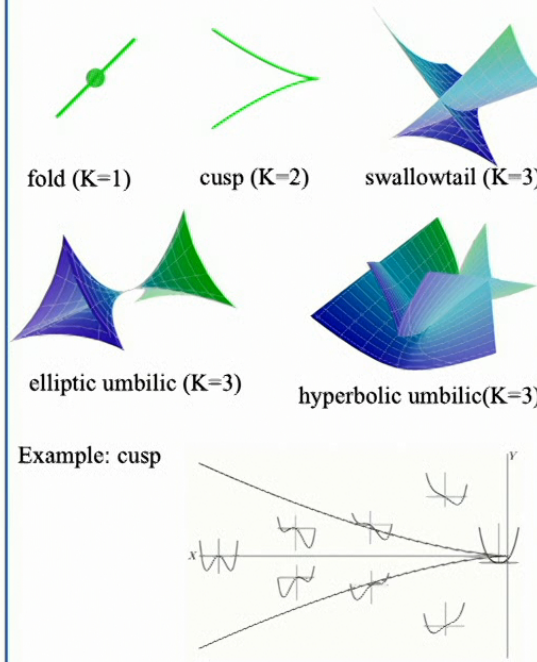
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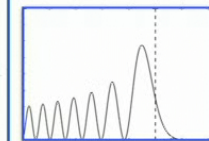
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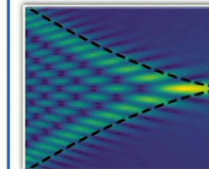


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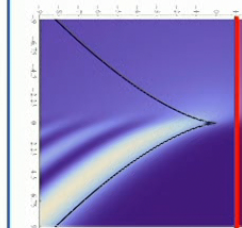
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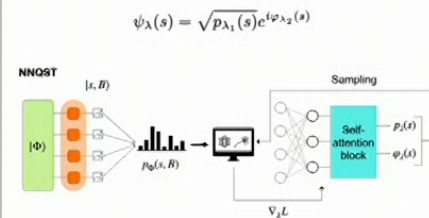
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Victor Wei^{1,2†}, W. A. Coish¹, Pooya Ronagh^{2,3,4,5}, Christine A. Muschik^{2,3,4}

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1. Neural network quantum state tomography (NNQST)

➤ NNQST aims to reconstruct the target quantum state in a neural network ansatz.



➤ The loss function to minimize is the cross-entropy between the two distributions (target state and neural network quantum state) summed over a set of bases.

$$L_\lambda = -\frac{1}{|B|} \sum_{B \in \mathcal{B}} \sum_{s \in \{0,1\}^n} p_B(s, B) \ln p_{\psi_\lambda}(s, B).$$

2. Classical shadows

$$\hat{\rho}_i(U_i, b_i) := \mathcal{M}^{-1}(|\phi_i\rangle\langle\phi_i|) = (2^n + 1)|\phi_i\rangle\langle\phi_i| - \mathbb{I}.$$

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➤ Classical shadows (Clifford version shown above) can be used to predict observables with very few number of measurements.

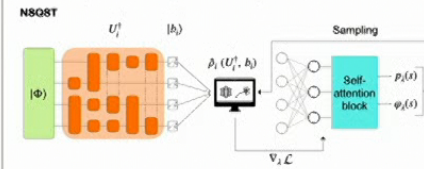
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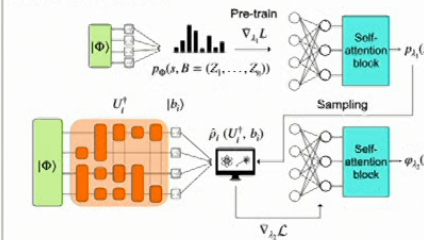
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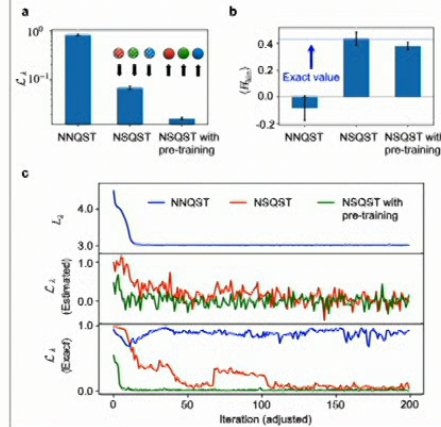
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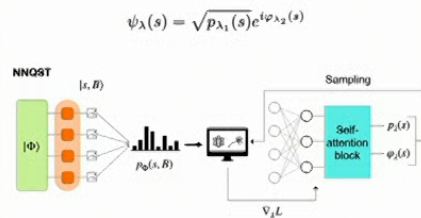
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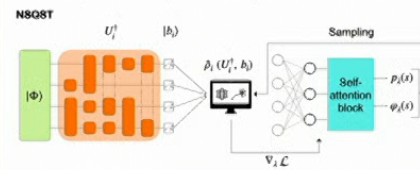
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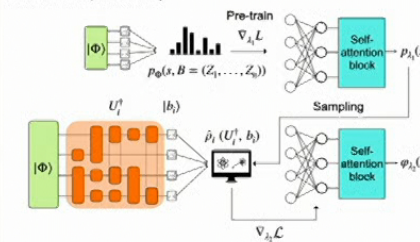
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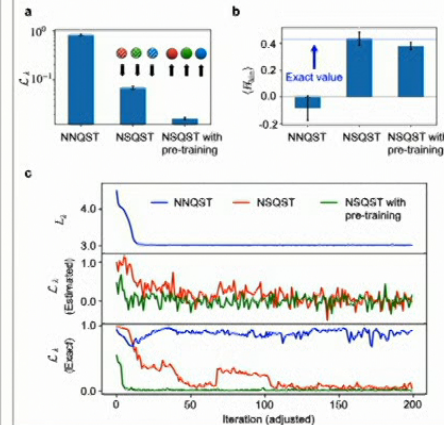
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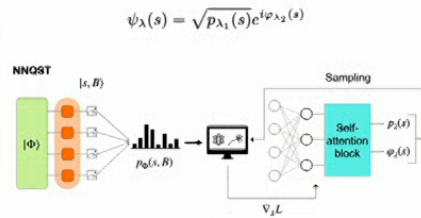
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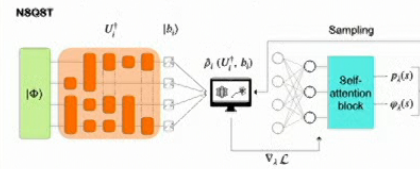
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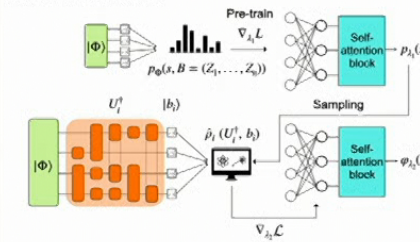
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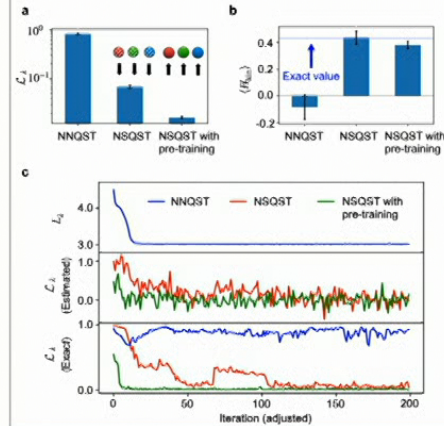
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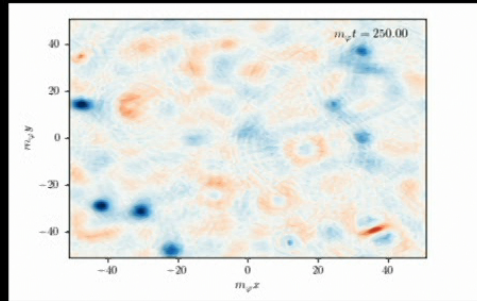


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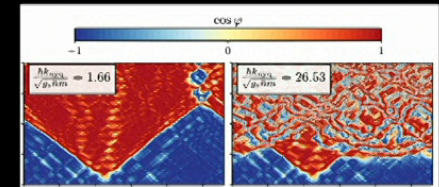
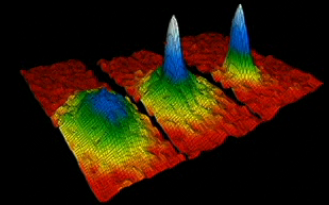
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The NL and NG Quantum Early Universe

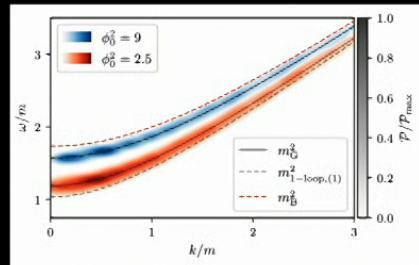
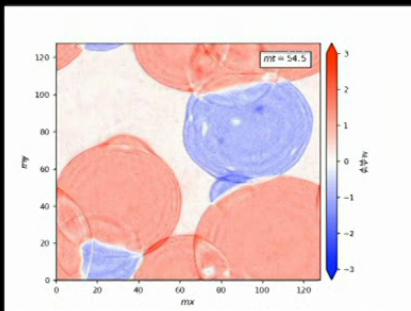
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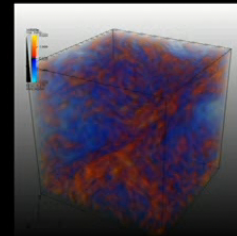
Analog Early Universe



Relativistic Phase Transitions



End-of-Inflation / Nonlinearity During Inflation



$$\zeta = \zeta_G + F_{NL}(\chi)$$

$$\zeta = \zeta_G + \sum \zeta_{\text{peak}}(x - x_i)$$

Novel Forms of NonGaussianity, motivated by detailed numerical calculations, characterization w/ ML techniques



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