

Title: Simulating one-dimensional quantum chromodynamics on a quantum computer: Real-time evolutions of tetra- and pentaquarks

Speakers: Christine Muschik

Collection: Quantum Simulators of Fundamental Physics

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URL: <https://pirsa.org/23060002>

Abstract: ZOOM: <https://pitp.zoom.us/j/95722860808?pwd=REYwSDdiK3pFamRJcjJwOW5FV1RPZz09>

Teaching quantum computers to simulate particle physics

Christine Muschik



QUANTUM INTERACTIONS THEORY GROUP

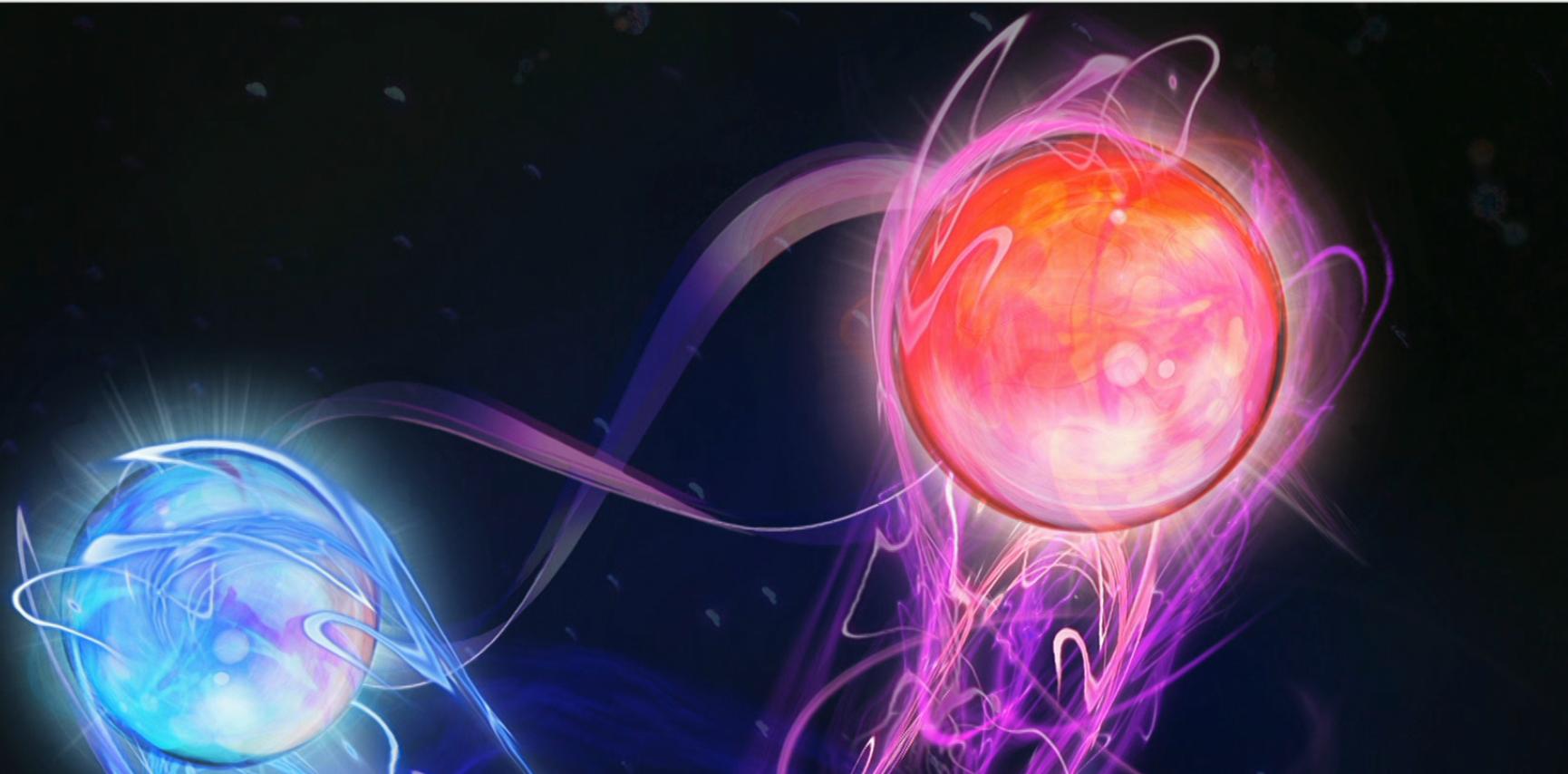


UNIVERSITY OF
WATERLOO

IQC Institute for
Quantum
Computing

PI PERIMETER
INSTITUTE

Quantum Simulation for particle physics



Numerical Simulations of Quantum Systems

Fundamental laws

Complex simulations

Answers to our questions

Condensed matter physics

Material science:
Can we predict and design
solids with certain properties?
npj Computational Materials 6, 85 (2020).

Numerical Simulations of Quantum Systems

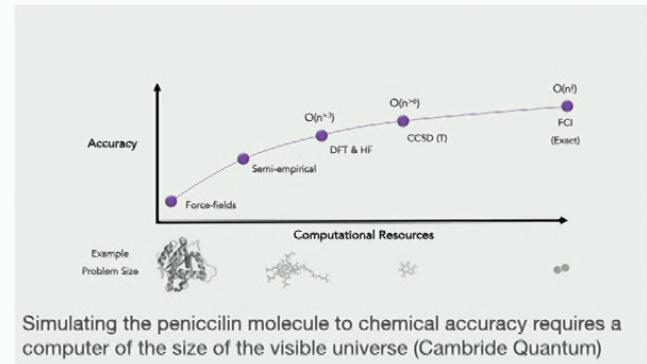
Fundamental laws

Complex simulations →

Answers to our questions

Quantum chemistry

Drug design:
Which types molecules have the
desired properties?



Traditional lattice gauge theory

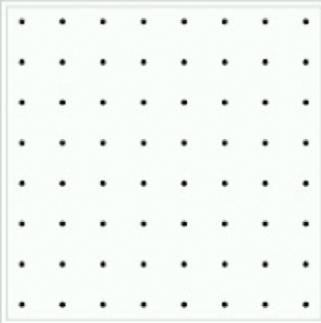
Discretize space and time



Path integral formalism



Monte Carlo Methods



$$\langle \mathcal{O} \rangle = \int \mathcal{D}_{\text{Fields}} \mathcal{O} e^{-S} / \int \mathcal{D}_{\text{Fields}} e^{-S}$$



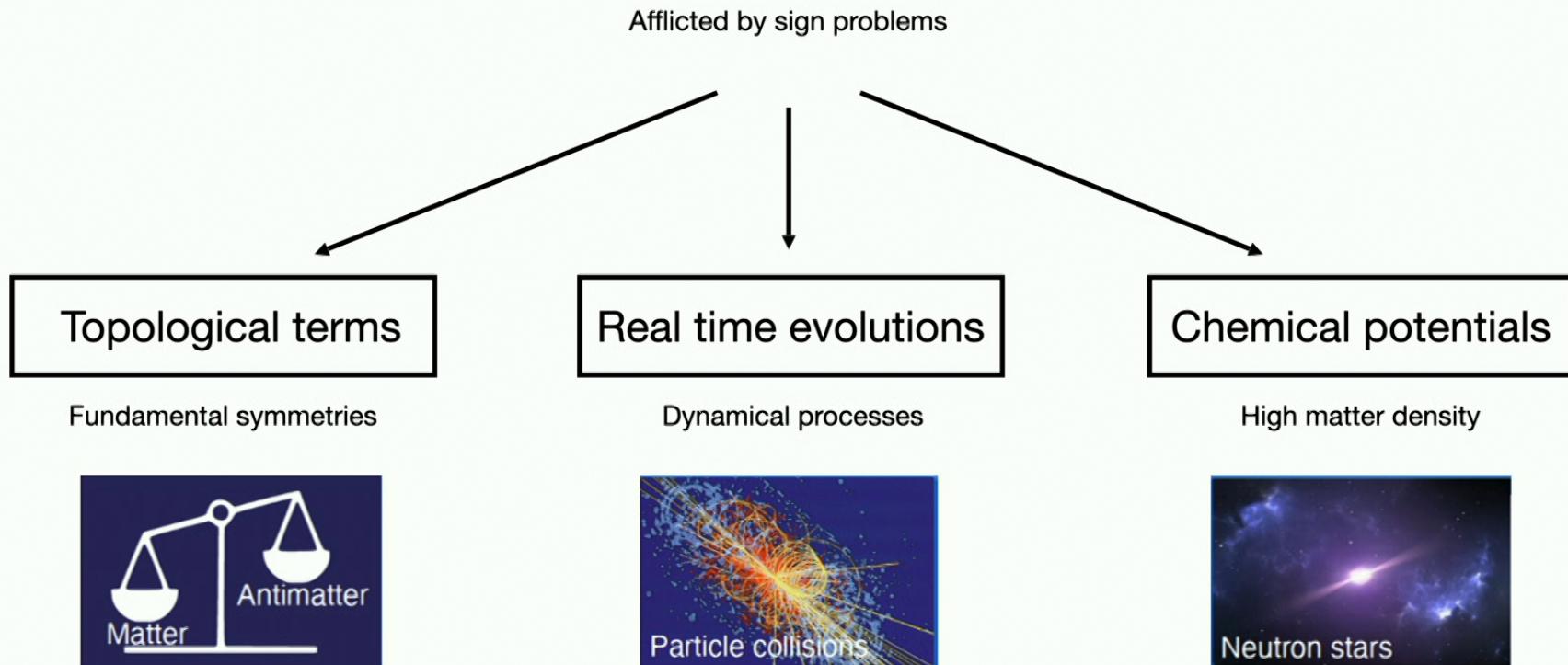
Immensely successful classical approach, despite the quantumness of the problem!



See for example [FLAG review 2021](#)

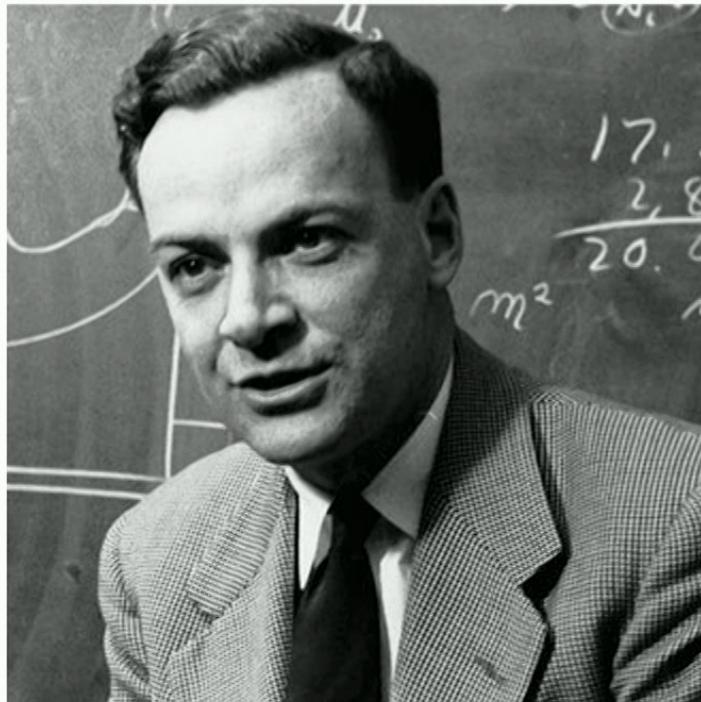
DOE High Energy Physics Exascales Requirement Review

Inaccessible to traditional lattice gauge theory



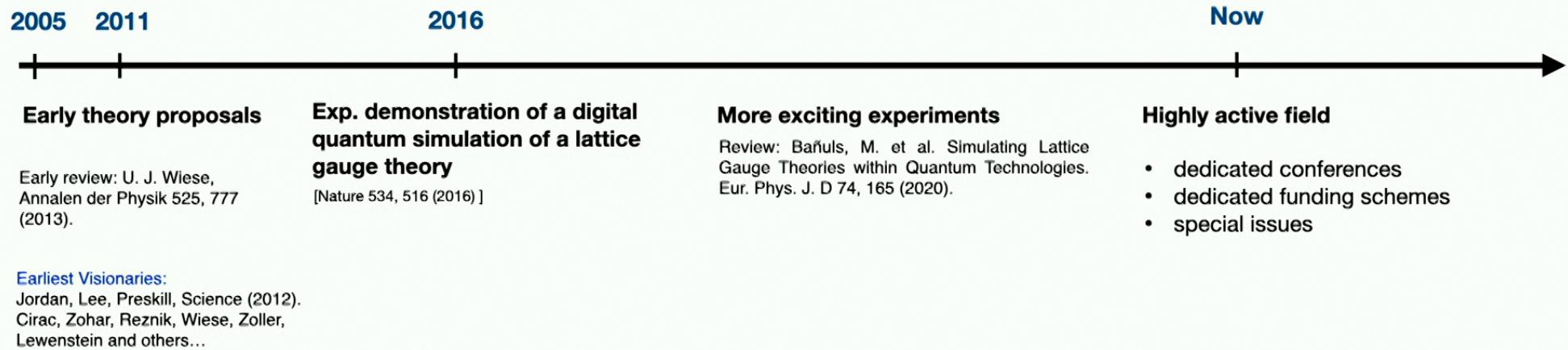
"Nature is quantum, goddamn it!

So if we want to simulate it, we need a quantum computer."



Richard Feynman

Quantum simulations of gauge theories for particle physics



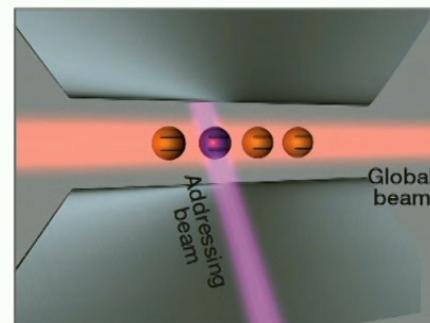
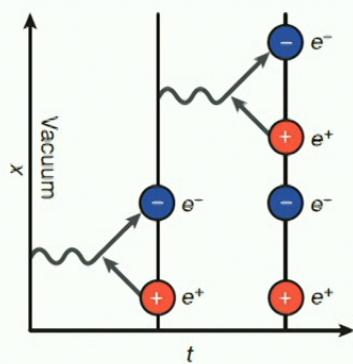
Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

[Esteban A. Martinez](#) , [Christine A. Muschik](#) , [Philipp Schindler](#), [Daniel Nigg](#), [Alexander Erhard](#),

[Markus Heyl](#), [Philipp Hauke](#), [Marcello Dalmonte](#), [Thomas Monz](#), [Peter Zoller](#) & [Rainer Blatt](#)

[Nature](#) 534, 516–519 (2016)

- Trapped ion quantum computer
- Quantum simulation of 1D-QED
- Real-time dynamics of electron-positron pair creation.



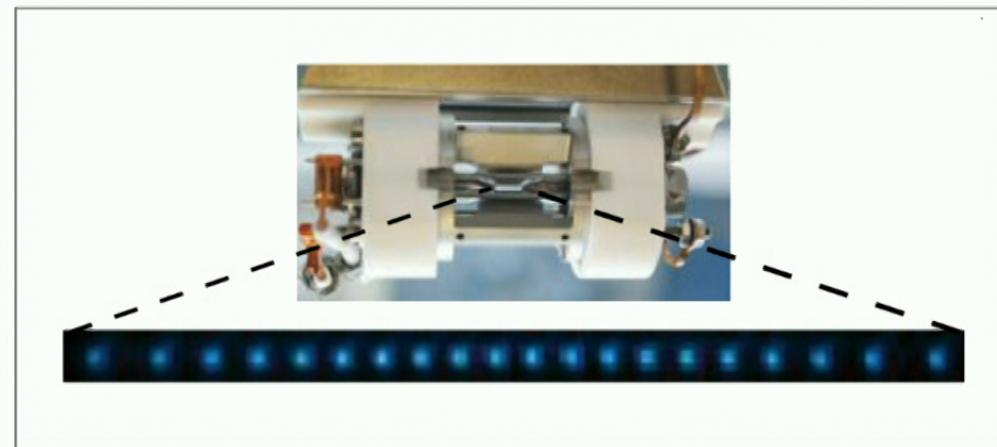
Theory + Experiment (in “Top 10 breakthroughs in physics 2016”)

Self-verifying variational quantum simulation of lattice models

[C. Kokail](#), [C. Maier](#), [R. van Bijnen](#), [T. Brydges](#), [M. K. Joshi](#), [P. Jurcevic](#), [C. A. Muschik](#), [P. Silvi](#), [R. Blatt](#), [C. F. Roos](#) & [P. Zoller](#) [✉](#)

[Nature](#) **569**, 355–360 (2019)

- ➡ VQE on trapped ion quantum simulator
- ➡ Application = 1D-QED



Describing stable matter as we know it requires non-Abelian gauge theories



Gauge Theories

Abelian

Non-Abelian

Gauge singlets involve as much matter as antimatter

Gauge singlets can be built from matter only

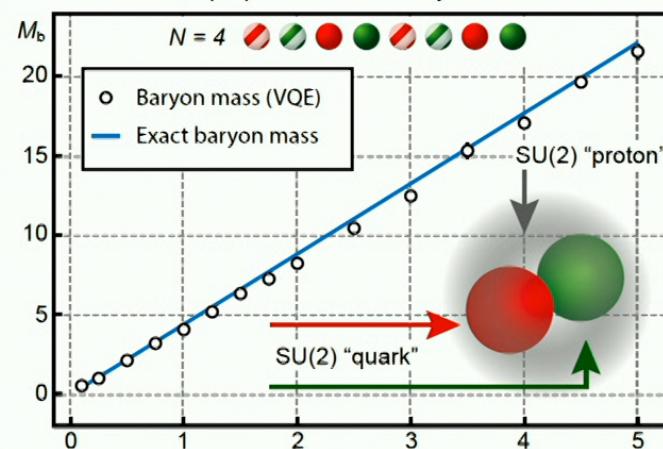


SU(2) hadrons on a quantum computer via a variational approach

[Yasar Y. Atas](#) , [Jinglei Zhang](#) , [Randy Lewis](#), [Amin Jahanpour](#), [Jan F. Haase](#)  & [Christine A. Muschik](#)

Nature Communications 12, Article number: 6499 (2021)

- ➡ Superconducting quantum simulator (IBM)
- ➡ Quantum simulation of 1D-SU(2)

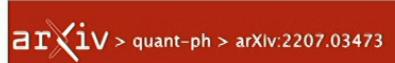


Gauge group of QCD

$SU(3)$

$SU(2)$

$U(1)$



[Submitted on 7 Jul 2022]

Real-time evolution of SU(3) hadrons on a quantum computer

[Yasar Y. Atas](#), [Jan F. Haase](#), [Jinglei Zhang](#), [Victor Wei](#), [Sieglinde M.-L. Pfaendler](#), [Randy Lewis](#), [Christine A. Muschik](#)

The theory of quarks and gluons – quantum chromodynamics – has been known for decades. Yet it is not fully understood. A recent example is the discovery of tetraquarks that led to a new research field. To address the many unsolved questions of the standard model, nonperturbative calculations are crucial. Quantum computers could simulate problems for which traditional QCD methods are inapplicable, such as real-time evolutions. We take a key step in exploring this possibility by performing a real-time evolution of tetraquark physics in one-dimensional SU(3) gauge theory on a superconducting quantum computer. Our experiment represents a first quantum computation involving quarks with three colour degrees of freedom, i.e. with the gauge group of QCD.

Simulating 1D-QCD on a quantum computer

Theory + Experiment:

- Tetraquarks
- Real-time evolution

Simulate quarks with three colours, i.e. the gauge group of QCD



Simulating 1D-QCD on a quantum computer

Related work:

Preparations for Quantum Simulations of Quantum Chromodynamics in 1+1 Dimensions: (I) Axial Gauge

Roland C. Farrell, Ivan A. Chernyshev, Sarah J. M. Powell, Nikita A. Zemlevskiy, Marc Illa, Martin J. Savage

Tools necessary for quantum simulations of 1 + 1 dimensional quantum chromodynamics are developed. When formulated in axial gauge and with two flavors of quarks, this system requires 12 qubits per spatial site with the gauge fields included via non-local interactions. Classical computations and D-Wave's quantum annealer Advantage are used to determine the hadronic spectrum, enabling a decomposition of the masses and a study of quark entanglement. Color edge states confined within a screening length of the end of the lattice are found. IBM's 7-qubit quantum computers, ibmq_jakarta and ibm_perth, are used to compute dynamics from the trivial vacuum in one-flavor QCD with one spatial site. More generally, the Hamiltonian and quantum circuits for time evolution of 1 + 1 dimensional $SU(N_c)$ gauge theory with N_f flavors of quarks are developed, and the resource requirements for large-scale quantum simulations are estimated.



M. Savage,
University of Washington

Experimental demonstration of color neutral objects (gauge singlets/color singlets).

Color neutral states of SU(3): invariant under arbitrary rotations in color space

Involve all the color charges available in the theory i.e. red, green, and blue (and their anticolors).

In contrast to Abelian quantum electrodynamics (QED), where a singlet state involves electron-positron pairs only.

Color singlet states are the relevant physical states,

→ important step towards the understanding, description and prediction of more complex and realistic experiments



Simulating 1D-QCD on a quantum computer



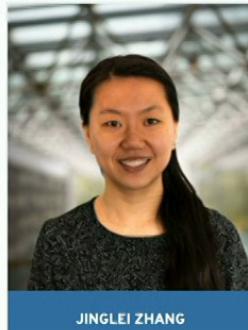
YASAR ATAS

IQC



JAN FRIEDRICH HAASE

IQC



JINGLEI ZHANG

IQC



VICTOR WEI

IQC



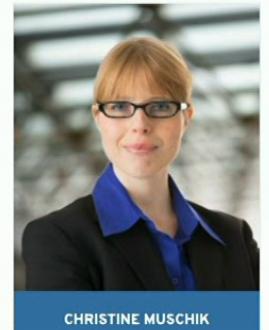
SIEGLINDE PFÄNDLER

IBM



RANDY LEWIS

YORK



CHRISTINE MUSCHIK

IQC

Lattice QCD

in one dimension



Lattice QCD

in one dimension



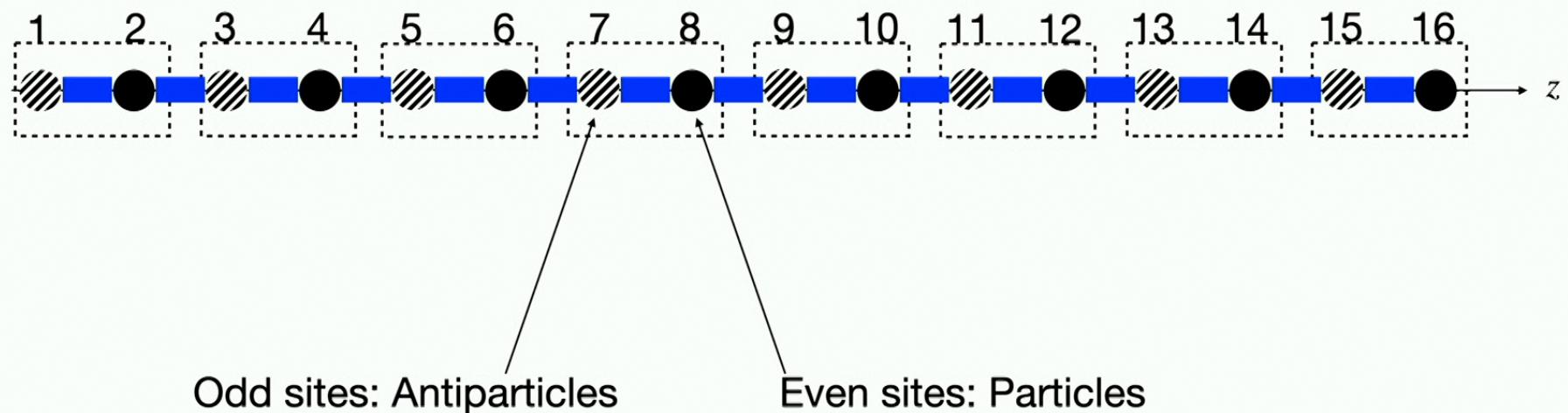
Lattice QCD in one dimension



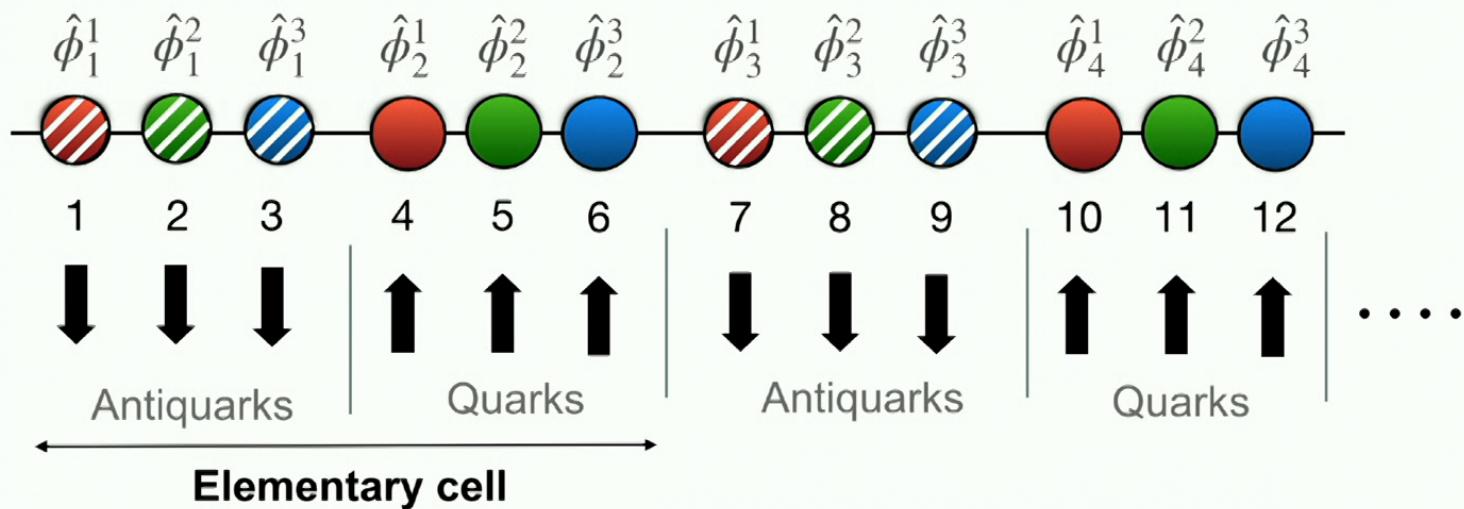
Matter: on vertices

Gauge fields: on links

Lattice QCD in one dimension

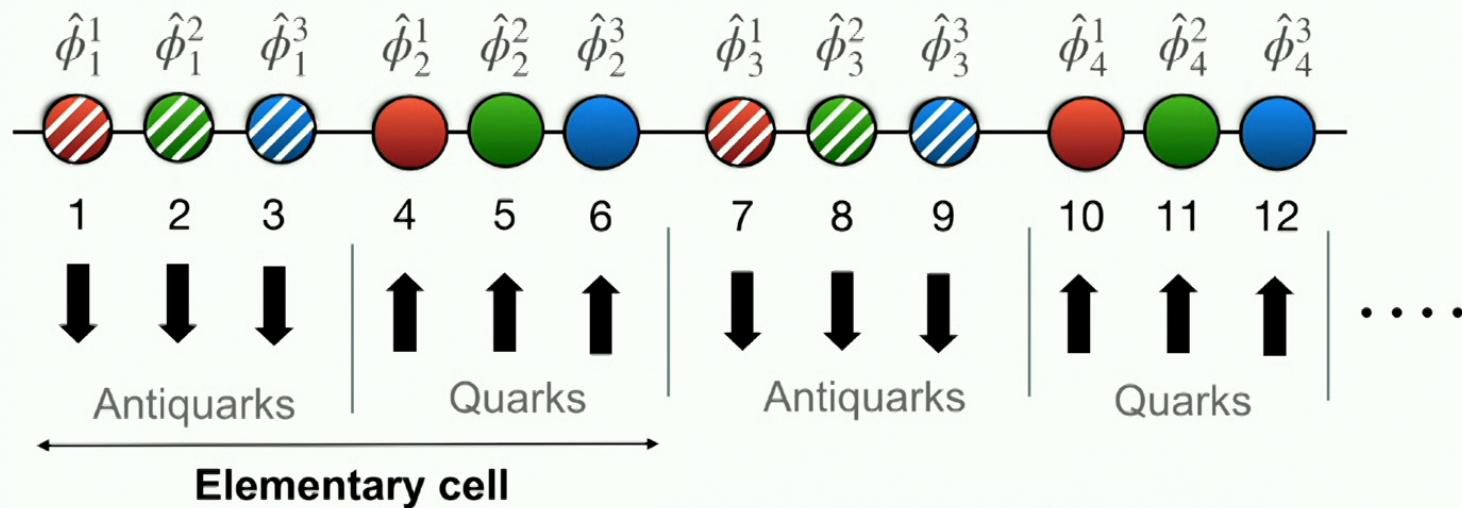


Lattice QCD in one dimension



Lattice QCD in one dimension

$n \bmod 6$	\uparrow	\downarrow
1	Red	Grey
2	Green	Grey
3	Blue	Grey
4	Grey	Red
5	Grey	Green
0	Grey	Blue



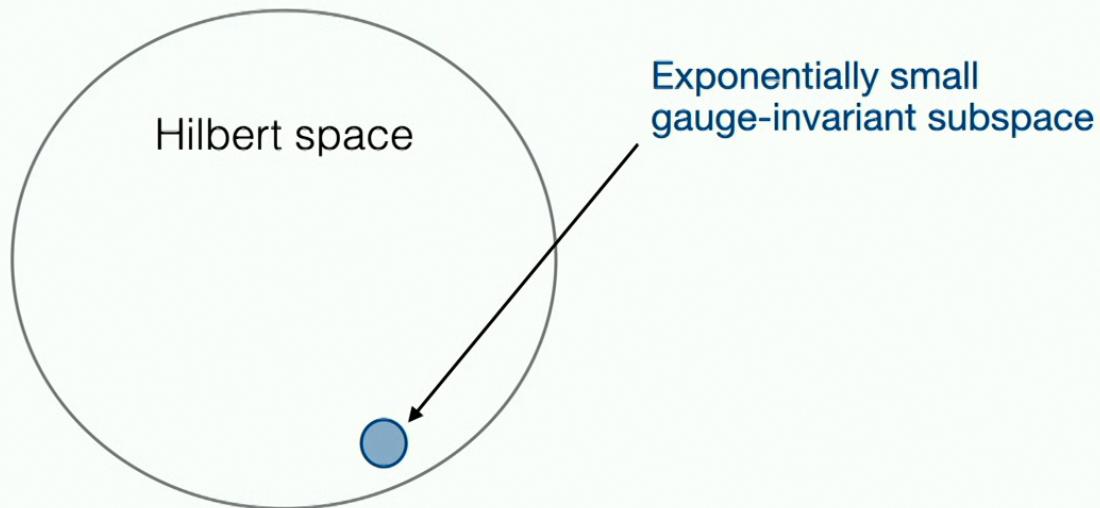
Lattice QCD in one dimension

Open boundary conditions → gauge fields can be eliminated by a gauge transformation*

- Hamiltonian description in terms of matter fields only
- Resource-efficient approach for quantum simulations
- Colour-electric fields contribute
- Colour-electric fields can be accessed in the simulation

*Zohar, E. & Cirac, J. I. Eliminating fermionic matter fields in lattice gauge theories. Phys. Rev. B 98, 075119 (2018).

Resource-efficient encoding of gauge theories



Quantum 5, 393 (2021), PRX Quantum 2, 030334 (2021).

Lattice QCD

in one dimension

$$\hat{H} = \hat{H}_{kin} + \tilde{m}\hat{H}_m + \frac{1}{2x}\hat{H}_e$$

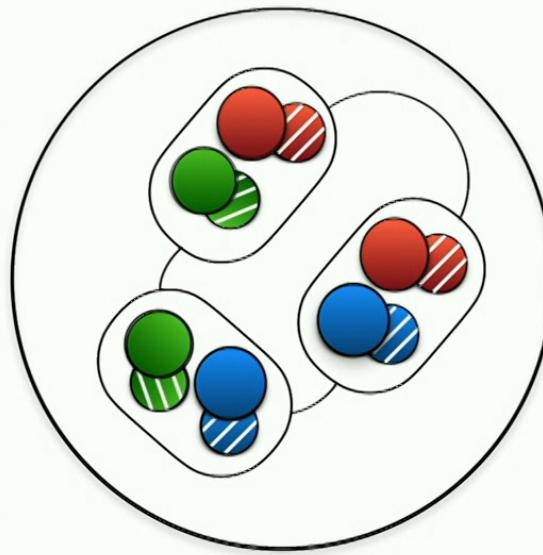
Lattice QCD in one dimension

$$\hat{H} = \hat{H}_{kin} + \tilde{m}\hat{H}_m + \frac{1}{2x}\hat{H}_e$$

$$\begin{aligned}\hat{H}_{kin} = & \frac{1}{2} \sum_{n=1}^{N-1} (-1)^n (\hat{\sigma}_{3n-2}^+ \hat{\sigma}_{3n-1}^z \hat{\sigma}_{3n}^z \hat{\sigma}_{3n+1}^- \\ & - \hat{\sigma}_{3n-1}^+ \hat{\sigma}_{3n}^z \hat{\sigma}_{3n+1}^z \hat{\sigma}_{3n+2}^- \\ & + \hat{\sigma}_{3n}^+ \hat{\sigma}_{3n+1}^z \hat{\sigma}_{3n+2}^z \hat{\sigma}_{3n+3}^- + \text{H. c.})\end{aligned}$$

$$\begin{aligned}\hat{H}_m = & \frac{1}{2} \sum_{n=1}^N [(-1)^n (\hat{\sigma}_{3n-2}^z + \hat{\sigma}_{3n-1}^z + \hat{\sigma}_{3n}^z) + 3] \\ \hat{H}_e = & \sum_{n=1}^{N-1} \left(\sum_{m \leq n} \hat{\mathbf{Q}}_m \right)^2\end{aligned}$$

Tetraquark



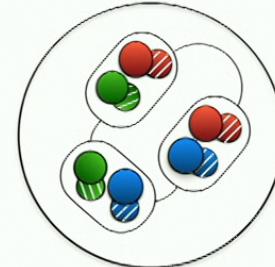
Lattice QCD in one dimension

$$\hat{H} = \hat{H}_{kin} + \tilde{m}\hat{H}_m + \frac{1}{2x}\hat{H}_e$$

$$\tilde{m} = am \text{ and } x = 1/g^2a^2$$

Strong coupling limit: $\tilde{m} \rightarrow \infty$ and $x \rightarrow 0$

Tetraquark in the strong coupling limit

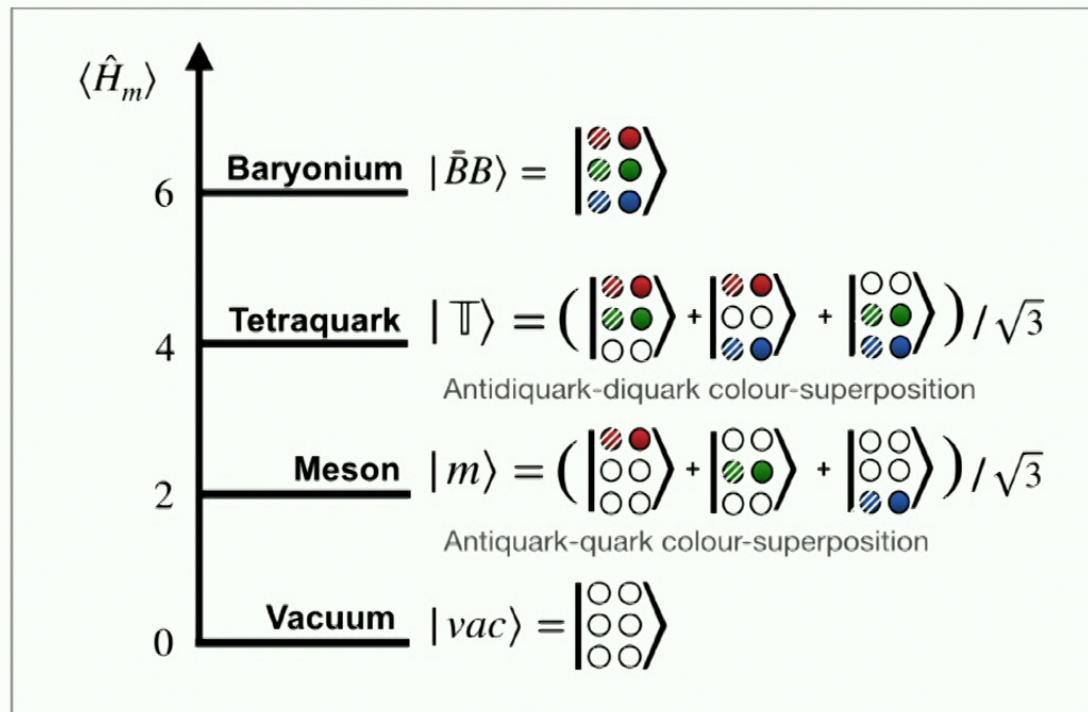


$$|\mathbb{T}\rangle = \left(\left| \begin{smallmatrix} \textcolor{brown}{\bullet} & \textcolor{brown}{\bullet} \\ \textcolor{green}{\bullet} & \textcolor{green}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{grey}{\bullet} & \textcolor{grey}{\bullet} \end{smallmatrix} \right\rangle + \left| \begin{smallmatrix} \textcolor{brown}{\bullet} & \textcolor{brown}{\bullet} \\ \textcolor{green}{\bullet} & \textcolor{green}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{grey}{\bullet} & \textcolor{grey}{\bullet} \end{smallmatrix} \right\rangle + \left| \begin{smallmatrix} \textcolor{brown}{\bullet} & \textcolor{brown}{\bullet} \\ \textcolor{green}{\bullet} & \textcolor{green}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{grey}{\bullet} & \textcolor{grey}{\bullet} \end{smallmatrix} \right\rangle \right) / \sqrt{3}$$

$$|\mathbb{T}\rangle = \frac{1}{\sqrt{3}} \left(|\textcolor{red}{\uparrow} \textcolor{green}{\uparrow} \textcolor{brown}{\downarrow} \textcolor{blue}{\downarrow} \rangle | \textcolor{red}{\downarrow} \textcolor{green}{\downarrow} \textcolor{brown}{\uparrow} \textcolor{blue}{\uparrow} \rangle + |\textcolor{red}{\uparrow} \textcolor{green}{\downarrow} \textcolor{brown}{\uparrow} \textcolor{blue}{\uparrow} \rangle | \textcolor{red}{\downarrow} \textcolor{green}{\uparrow} \textcolor{brown}{\downarrow} \textcolor{blue}{\downarrow} \rangle + |\textcolor{red}{\downarrow} \textcolor{green}{\uparrow} \textcolor{brown}{\uparrow} \textcolor{blue}{\downarrow} \rangle | \textcolor{red}{\uparrow} \textcolor{green}{\downarrow} \textcolor{brown}{\downarrow} \textcolor{blue}{\uparrow} \rangle \right)$$

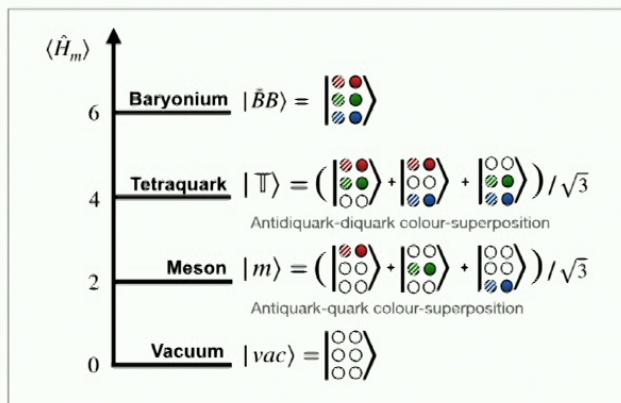
$n \bmod 6$	\uparrow	\downarrow
1		
2		
3		
4		
5		
0		

Basis states in the strong coupling limit in the B=0 subsector

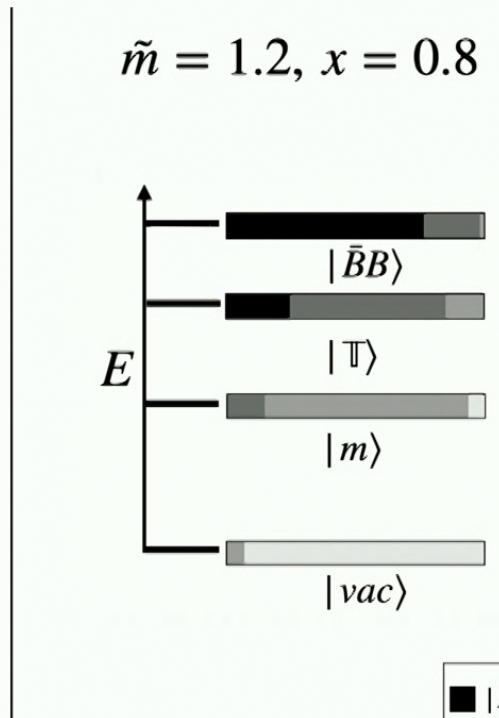


Energy Spectrum

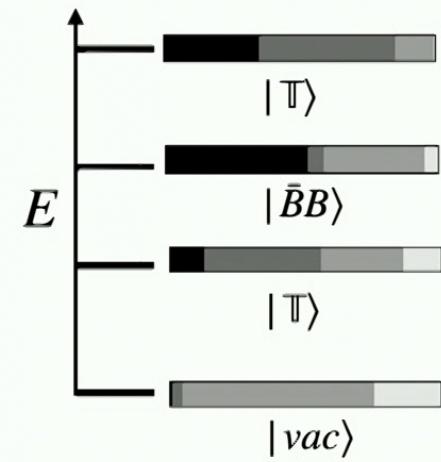
$\tilde{m} \rightarrow \infty$ and $x \rightarrow 0$



$\tilde{m} = 1.2, x = 0.8$



$\tilde{m} = 0.45, x = 0.8$



Quantum time evolution

Trotter protocol

- Up to 8 trotter steps (~80 gates)
- Reduction to 3 qubits
- Observe mean particle number versus time

Experiment

- IBM quantum computer
- Error self-mitigation using randomised compiling

Quantum time evolution

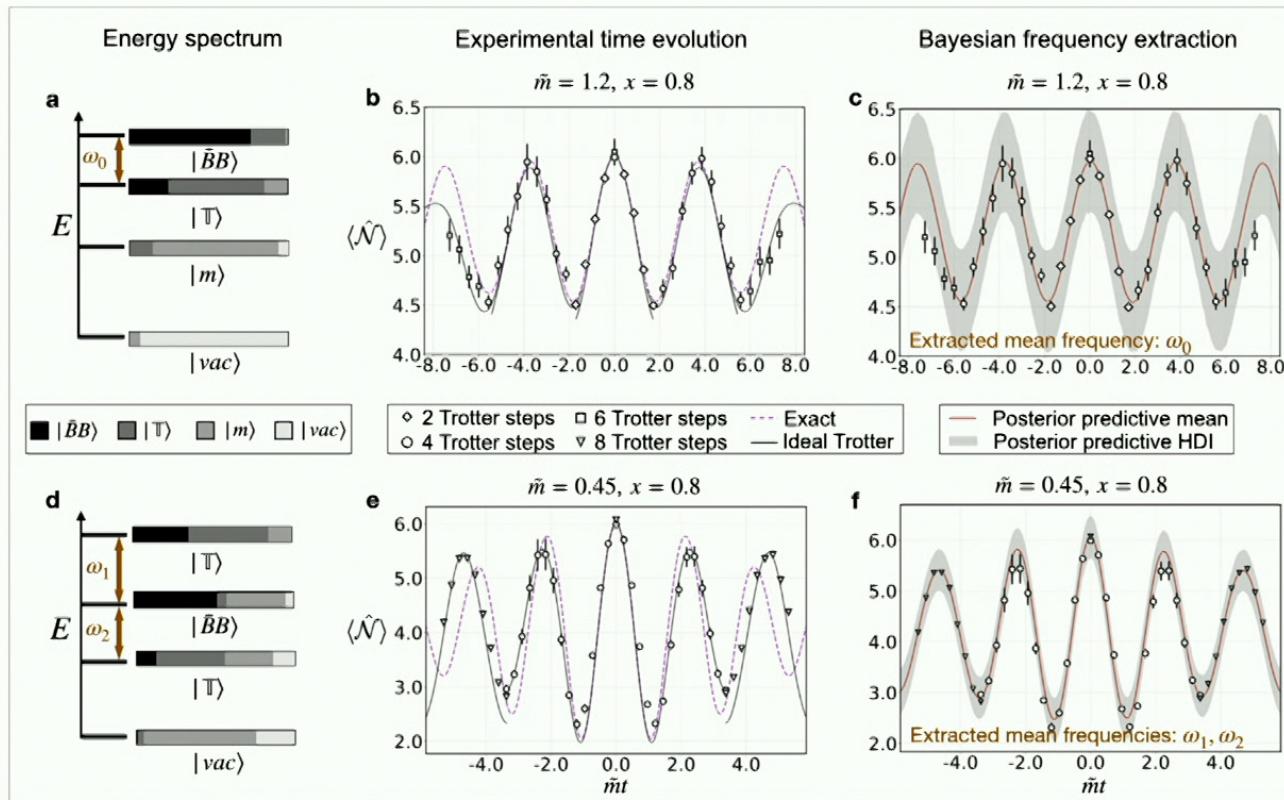
Trotter protocol

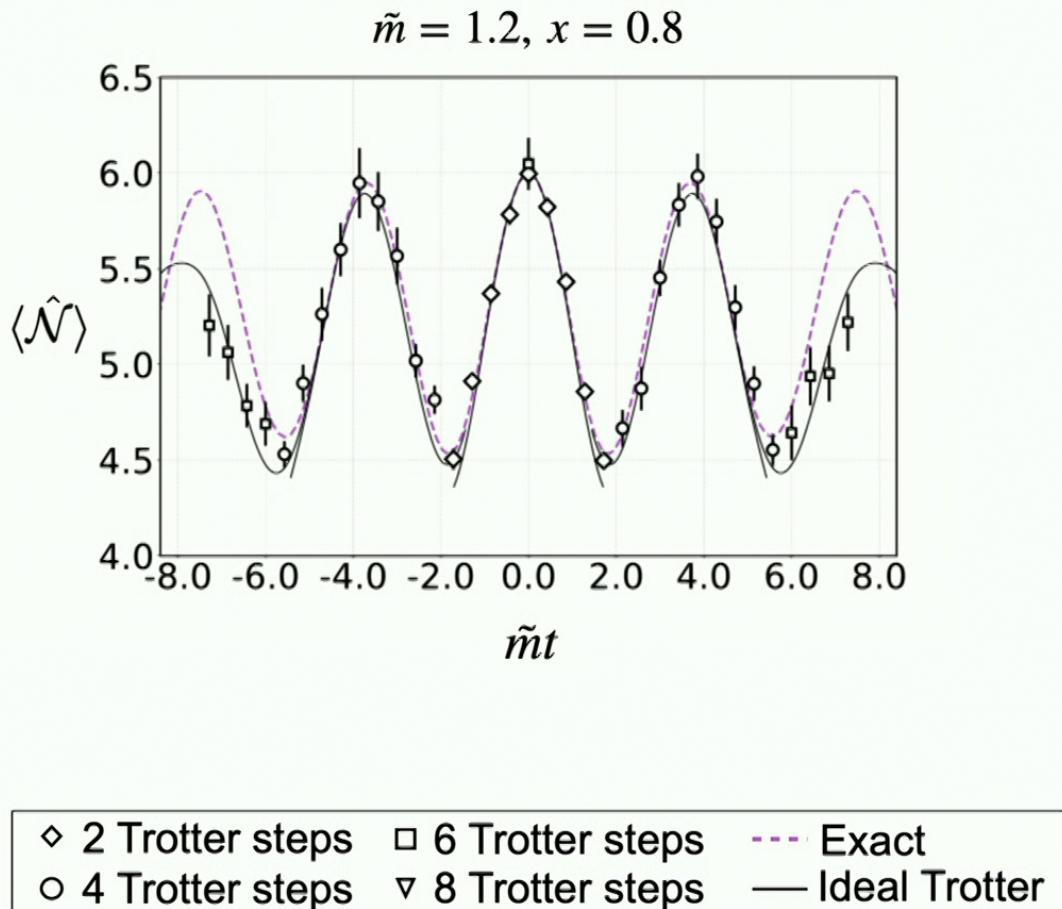
- Up to 8 trotter steps (~80 gates)
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Experiment

- IBM quantum computer
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Results

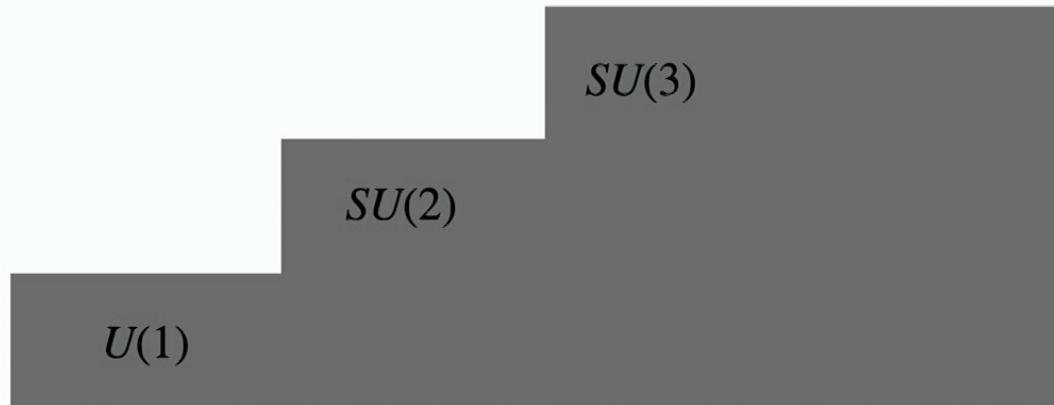




Summary

- Time evolutions cannot be accessed with traditional lattice gauge theory
- Develop quantum protocols to simulate fundamental particle interactions

Gauge group of QCD



Experimental demonstration of color neutral objects (gauge singlets/color singlets).

Color neutral states of SU(3): invariant under arbitrary rotations in color space

Involve all the color charges available in the theory i.e. red, green, and blue (and their anticolors).

In contrast to Abelian quantum electrodynamics (QED), where a singlet state involves electron-positron pairs only.

Color singlet states are the relevant physical states,

→ important step towards the understanding, description and prediction of more complex and realistic experiments



Next steps

- More flavours
- Higher dimensions: 2D* and 3D
- Time evolutions of physical dynamical processes

*Quantum 5, 393 (2021), PRX Quantum 2, 030334 (2021).

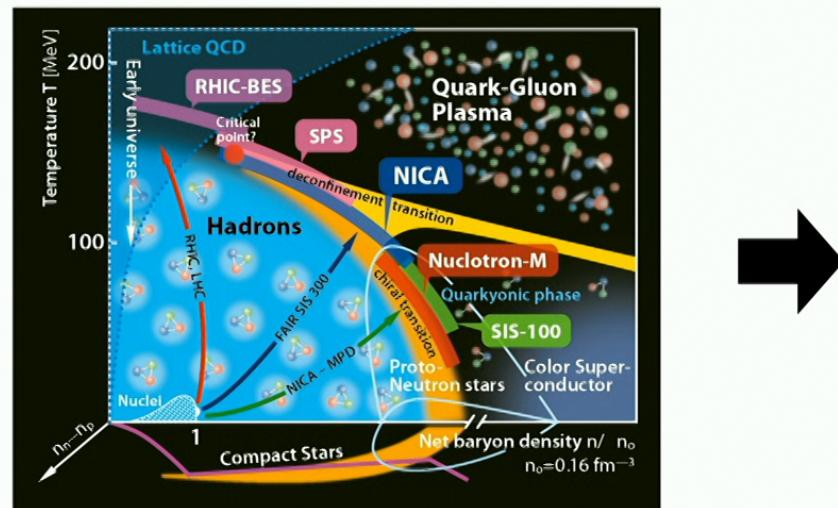
Outlook

Vast problem classes in particle physics are insurmountable for traditional approaches, (but not for quantum computers!)

Examples:

Real-time evolutions

High matter density



- Particle physics
- Nuclear physics
- Cosmology
- Astrophysics
- Condensed matter physics

Quantum enhanced scientific computing

$\langle X, P \rangle = \delta_{X,P}$ $\langle \alpha | \beta \rangle = \overline{\langle \beta | \alpha \rangle} = \delta_{\alpha,\beta}$ $\int_0^t \frac{d}{dt} \left(\frac{E_0 + E_1(t) + \epsilon(t)}{M(t)} \right)^2 dt = \left(\frac{E_0 + E_1(t) + \epsilon(t)}{M(t)} \right)^2$ $\frac{dy}{dt} = \frac{dy}{d\theta} \frac{d\theta}{dt} = \frac{dy}{d\theta} \omega = \frac{dy}{dt} \mu$ $0 < \epsilon < A$ $r = \sqrt{1 - \epsilon^2}$
 $H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2$ $\langle \Psi | \Psi' \rangle = \sqrt{n} \delta_{\Psi,\Psi'}$ $E = \frac{1}{2} \hbar \omega L Q^2$ $\theta = \frac{2\pi}{\hbar \omega}$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 $H|\Psi\rangle = E|\Psi\rangle$ $\langle \Psi | \Psi' \rangle = \sqrt{n} \delta_{\Psi,\Psi'}$ $E = \frac{1}{2} \hbar \omega L Q^2$ $\theta = \frac{2\pi}{\hbar \omega}$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 $\left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m \omega^2 x^2 \right] \Psi(x) = E \Psi(x)$ $\langle \Psi | \Psi' \rangle = \sqrt{n} \delta_{\Psi,\Psi'}$ $E = \frac{1}{2} \hbar \omega L Q^2$ $\theta = \frac{2\pi}{\hbar \omega}$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 $\hat{X} = \sqrt{\frac{m}{2\pi k_B T}} \sum_n e^{-\frac{E_n}{k_B T}}$ $\langle \Psi | \Psi' \rangle = \sqrt{n} \delta_{\Psi,\Psi'}$ $E = \frac{1}{2} \hbar \omega L Q^2$ $\theta = \frac{2\pi}{\hbar \omega}$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 $\hat{H} = \frac{1}{2} (\hat{X}^2 + \hat{P}^2)$ $\langle \Psi | \Psi' \rangle = \sqrt{n} \delta_{\Psi,\Psi'}$ $E = \frac{1}{2} \hbar \omega L Q^2$ $\theta = \frac{2\pi}{\hbar \omega}$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
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 $a = \frac{d}{dt} (k+1)^2$ $\langle \alpha | \beta \rangle = \left(\frac{\alpha}{\beta} \right)^2 e^{-\frac{1}{T}}$ $\frac{dy}{dt} = \left(\frac{\omega}{L} \right)^2$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 $a^2 = \frac{d^2}{dt^2} (k+1)^2$ $\langle \alpha | \beta \rangle = \left(\frac{\alpha}{\beta} \right)^2 e^{-\frac{1}{T}}$ $\frac{dy}{dt} = \left(\frac{\omega}{L} \right)^2$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 $a^3 = \frac{d^3}{dt^3} (k+1)^2$ $\langle \alpha | \beta \rangle = \left(\frac{\alpha}{\beta} \right)^2 e^{-\frac{1}{T}}$ $\frac{dy}{dt} = \left(\frac{\omega}{L} \right)^3$ $\frac{dy}{dt} = \frac{dy}{d\theta} \left(\frac{\omega}{L} \right) + \frac{dy}{d\theta} \frac{\omega}{L} \left(\frac{A}{\mu} \right)$
 \dots \dots \dots \dots

Quantum co-processing units → **New discoveries**
+ regular computing infrastructure



$$\begin{aligned}Q^2) &= \frac{-1}{\pi} e^{-Q^2} \\&\propto \nabla \Psi^* - \Psi \nabla \Psi \\&= \Psi^*(r,t) \Psi(r,t) \\&\quad - \Psi(r,t) \Psi^*(r,t) \\&\stackrel{*}{\nabla} \nabla \Psi - \Psi \nabla \Psi^* \\&\stackrel{*}{\nabla} \nabla \Psi - \Psi \nabla \Psi^* \\&= a_n \left(\ln \frac{\langle f \rangle}{Q^2} + \right) \\&\quad \left(\frac{h_d}{dQ^2} + Q^2 \right)\end{aligned}$$