Title: Asymptotic-safety inspired results and ideas in causal set quantum gravity

Speakers: Astrid Eichhorn

Series: Quantum Gravity

Date: May 25, 2023 - 2:30 PM

URL: https://pirsa.org/23050152

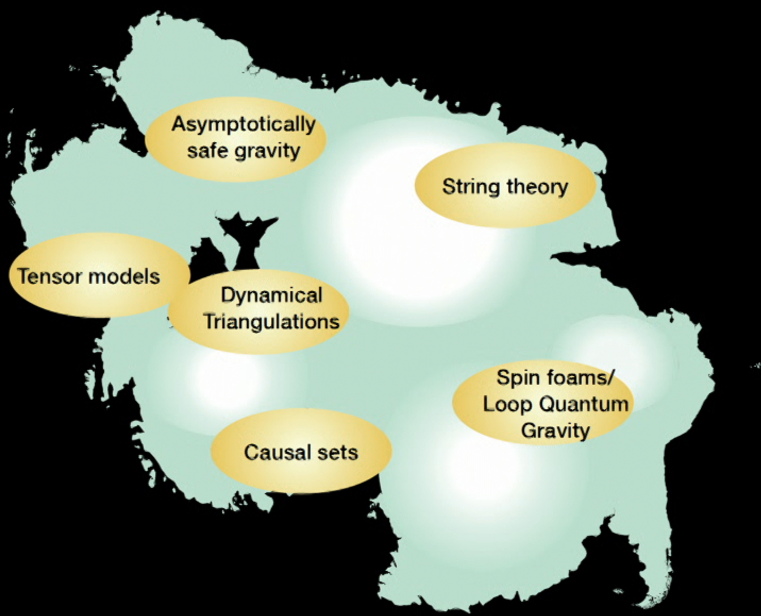
Abstract:

I will argue that a fruitful strategy to make progress in quantum gravity is to connect distinct approaches and transfer methods and ideas from one approach to another. As a concrete example, I will explain recent results in causal-set quantum gravity. The first, namely the construction of a higher-order curvature operator in causal sets, is motivated by the idea to use causal sets as a Lorentzian regularization of the gravitational path integral, in which one can search for asymptotic safety. The second, namely an upper bound on the mass of scalars is inspired by the "matter matters" program in asymptotically safe quantum gravity, in which observational tests of quantum gravity are based on gravity's interplay with matter. I will argue that a similar program for causal sets can provide new, observationally motivated constraints on causal set quantum gravity.To provide background and motivation for these results, I'll provide brief and pedagogical introductions into the key features and key open challenges of both asymptotically safe quantum gravity as well as causal set quantum gravity.

Zoom Link: https://pitp.zoom.us/j/99855484685?pwd=M0IvOFk1OWNCdVlZVDdkUFQ4MzZKUT09


Asymptotic-safety inspired results and ideas in causal-set quantum gravity
Quantum gravity seminar,
Perimeter Institute, May 25, 2023
Astrid Eichhorn, CP3-Origins, University of Southern Denmark
VILLUM FONDEN
CP3
SDU

The current status of quantum gravity



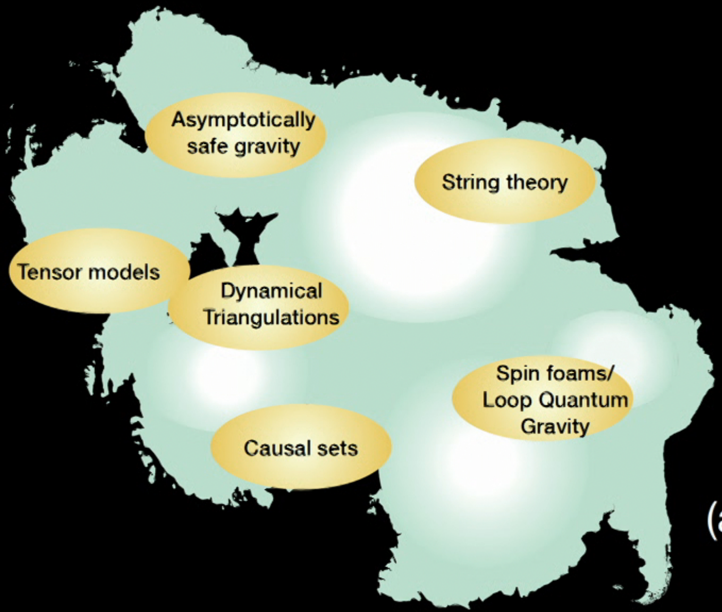
Scientific perspective:

Many approaches exist; each faces challenges.

Sociological perspective:

Many sub-communities exists, methods/ideas are only sometimes "exported/imported"

Connecting quantum gravity approaches



How to address challenges in the various approaches?

- Use tools/techniques from another approach
- Borrow inspiration for which questions to ask

(also requires a change in the “sociology of quantum gravity”)

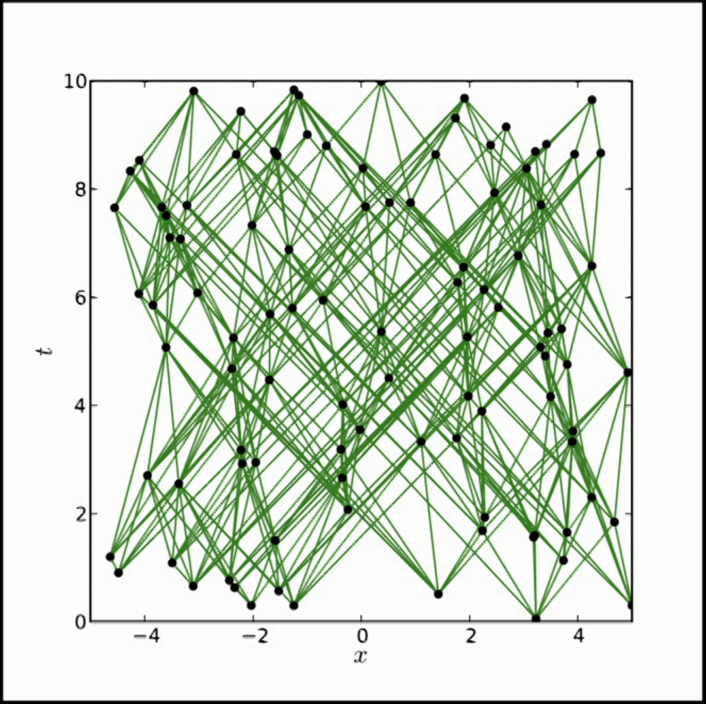
Here: causal set quantum gravity and asymptotically safe quantum gravity

Lightning introduction to causal set quantum gravity

Key assumptions:

- Lorentzian signature (and Lorentz invariance)
- Spatiotemporal discreteness

$\Rightarrow$ spacetime is a discrete, causal network



A causal set corresponding to a "chunk" of 1+1 d Minkowski spacetime

Quantum theory of causal sets:

$$Z = \int \mathcal{D}C \, e^{i S[C]}$$

Configuration space: all causal sets C, i.e., sets of spacetime points with relation $\prec$ ("precedes"), such that:

- $\forall x, y, z \in C : x \prec y \text{ and } y \prec z \Rightarrow x \prec z$ (transitivity)
- $\forall x, y \in C :$ if $x \prec y$ and $y \prec x \Rightarrow x = y$ (no closed timelike curves)
- $\forall x, y \in C : |\{z \in C : x \prec z \prec y\}| < \infty$ (local finiteness)

Lightning introduction to asymptotically safe quantum gravity

Key assumptions:

- Metric carries the fundamental degrees of freedom
- Path integral has a finite number of free parameters

$\Rightarrow$ quantum scale symmetry

$$\int \mathscr{D} g_{\mu\nu}\, e^{iS}$$

$$S = S_{\text{Einstein-Hilbert}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, (R - 2\Lambda)$$

Problem:

perturbative non-renormalizability $\rightarrow$ breakdown of predictivity

1-loop: $\left(a\, R^2 + b\, R_{\mu\nu} R^{\mu\nu}\right)$

2-loop: $c\, R_{\mu\nu\kappa\lambda} R^{\kappa\lambda\rho\sigma} R_{\rho\sigma}{}^{\mu\nu}$....

...

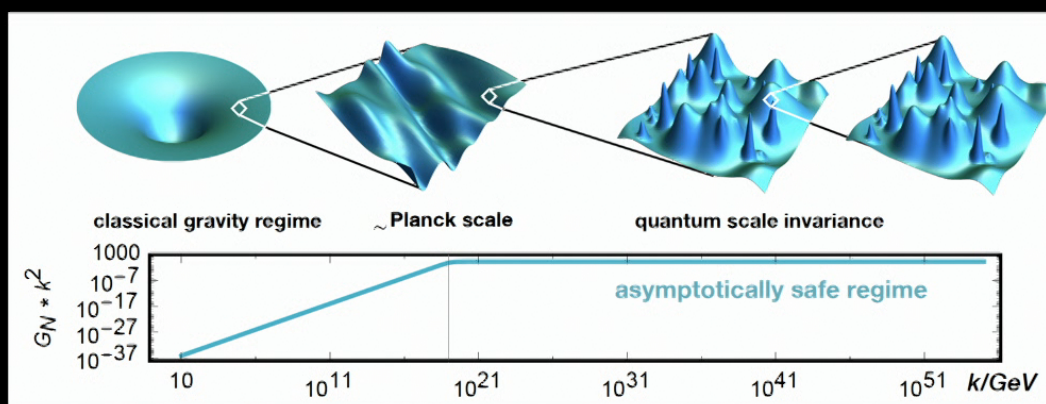
Solution:

Quantum scale symmetry $\rightarrow$ relations between the couplings restore predictivity

Lightning introduction to asymptotically safe quantum gravity

Presence of quantum fluctuations: Theory is scale dependent (Renormalization Group flow)

Scale symmetry:
Renormalization Group fixed point



A challenge in causal set quantum gravity and how to use results in asymptotically safe gravity to address it

- Observational tests: particle "swerves"; Everpresent Λ
- New proposal: "Matter matters" program
  - (Observed) properties of elementary particles (e.g., masses and interaction strengths) constrain the underlying quantum gravity theory

    Examples in asymptotically safe quantum gravity:

    Higgs mass [Shaposhnikov, Wetterich '09]

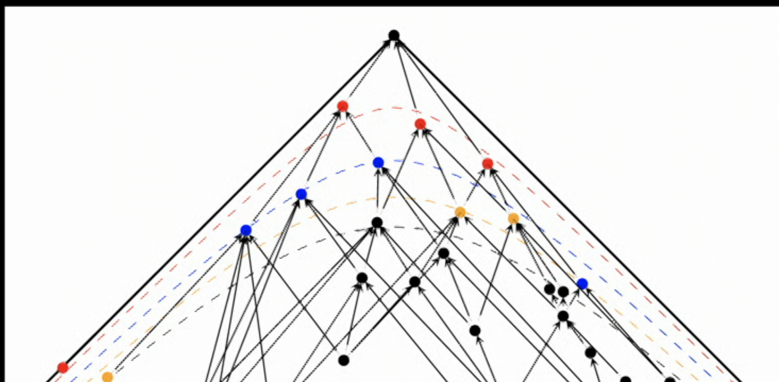
    Abelian gauge coupling [Harst, Reuter '11; AE, Versteegen '17]

    Top quark mass [AE, Held '17]

    Top-bottom mass difference [AE, Held '18]

    …

Towards an upper bound on the Higgs mass from causal set quantum gravity



Propagator for a scalar field: [Sorkin '07, Benincasa, Dowker '10]

summing over (causal) nearest neighbors:

$$B\phi(x) = \frac{-4}{\sqrt{6}\ell_d^2}\left(\phi(x) - \sum_{y\in L_1(x)}\phi(y) + 9\sum_{y\in L_2(x)}\phi(y) - 16\sum_{y\in L_3(x)}\phi(y) + 8\sum_{y\in L_4(x)}\phi(y)\right)$$

$L_n(x)$: points with $(n-1)$ points in the causal interval with x

$$\lim_{\ell_d\to 0}\langle B\phi(x)\rangle = \left(\Box - \frac{1}{2}R\right)\phi(x)$$

expectation value taken over sprinklings into a manifold

Fluctuations about the average increase with decreasing ℓ_d : replace ℓ_d by non-locality scale ℓ [Dowker, Glaser '13]

→in Minkowski spacetime:

propagator $P = (\Box)^{-1} \sim 1/p^2$ in the IR

$\sim \text{const} - \frac{1}{p^4}$ in the UV

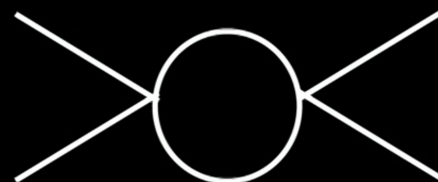
[Aslanbeigi, Saravani, Sorkin '14]

Towards an upper bound on the Higgs mass from causal set quantum gravity

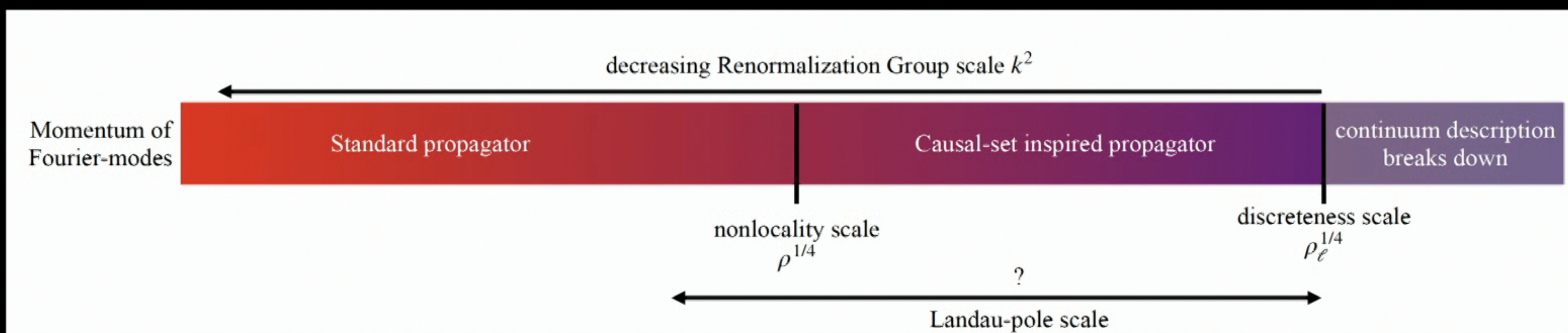
$\rightarrow$propagator $P = (\Box)^{-1} \sim 1/p^2$ in the IR

$\sim \text{const} - \dfrac{1}{p^4}$ in the UV

affects quantum corrections to quartic scalar coupling λ

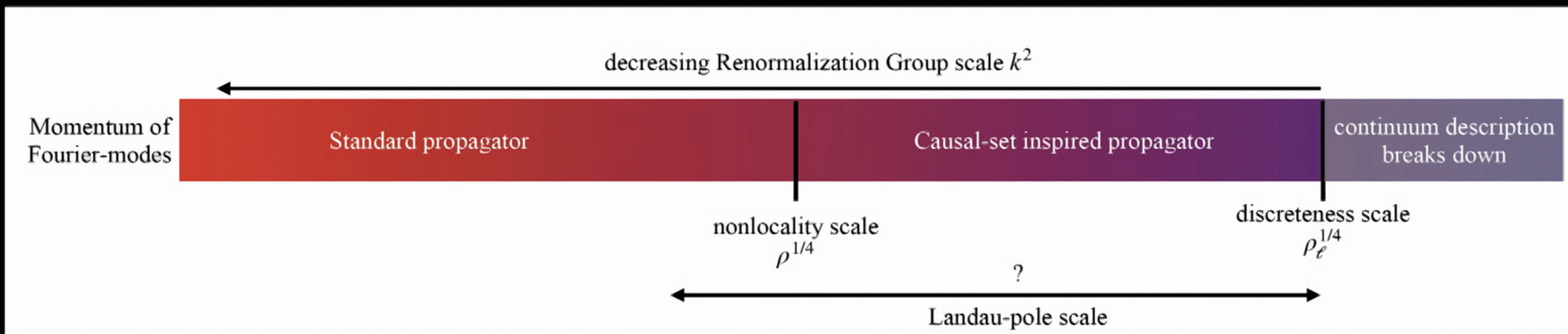


$\rightarrow$ affects scale-dependence of λ



Towards an upper bound on the Higgs mass from causal set quantum gravity

use functional RG techniques to calculate running of the quartic scalar coupling [de Brito, AE, Fausten '23]

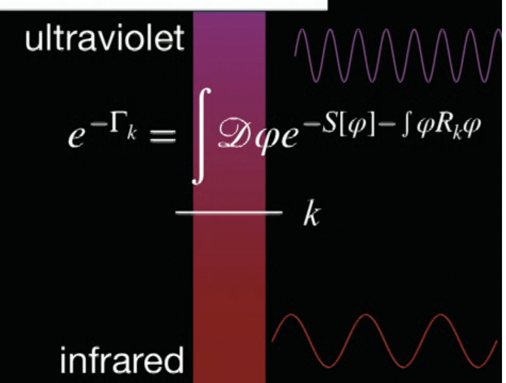


functional Renormalization Group ≈ mathematical microscope

Γ_k: analog of classical action, but with quantum fluctuations above k included

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\,\partial_k R_k\right] \quad \rightarrow \beta_\lambda = k\partial_k\,\lambda(k)$$

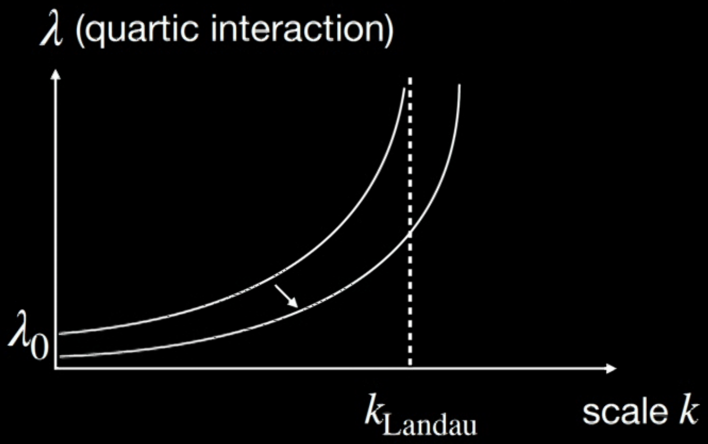
[Wetterich '93; Reuter '96]



Towards an upper bound on the Higgs mass from causal set quantum gravity

standard local QFT: $\beta_\lambda = \frac{9}{8\pi^2}\lambda^2 + \ldots$

Landau pole in quartic coupling



To shift k_{Landau} further into the UV, must lower λ_0.

Mass of the scalar $M = 3\lambda_0\, v$

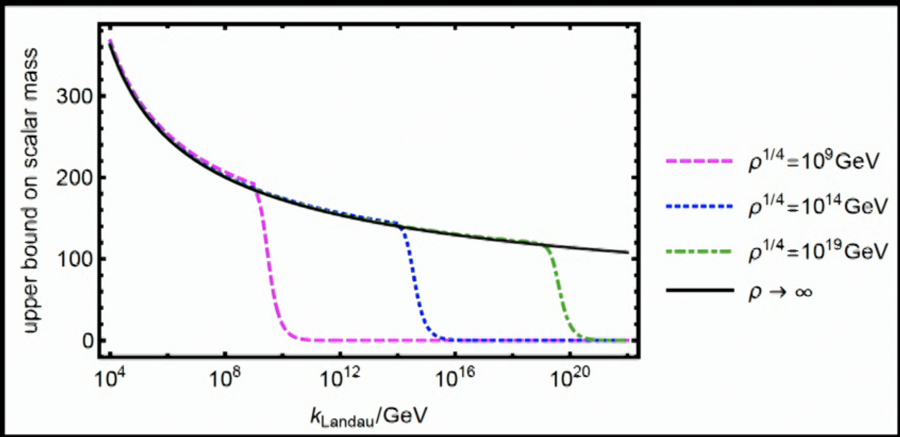
$\Rightarrow$ the further the theory extends into the UV, the lower the scalar mass

causal-set inspired case:

Landau pole in quartic coupling persists

Landau pole occurs at nonlocality scale

$\Rightarrow$ QFT description breaks down at nonlocality scale, so nonlocality scale is not a viable mechanism to suppress fluctuations



$$k_{\text{Landau}} \gtrsim \rho^{1/4}$$

Outlook: including the other fields of the Standard Model will result in upper bound on Higgs mass as a function of ρ

Key challenge of asymptotically safe quantum gravity

- How to test asymptotic safety in Lorentzian signature?

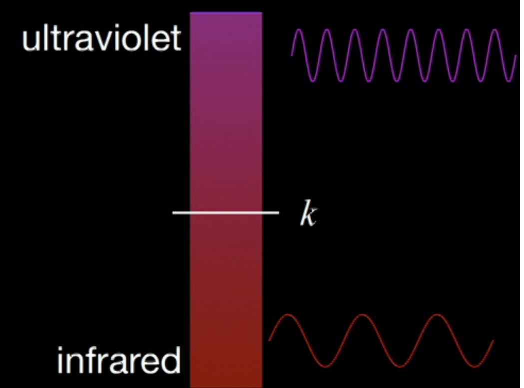
functional Renormalization Group ≈ mathematical microscope
for Euclidean quantum gravity

$$e^{-\Gamma_k} = \int \mathcal{D}\varphi \, e^{-S[\varphi] - \int \varphi R_k \varphi}$$

Γ_k: analog of classical action, but with quantum fluctuations above k included

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\,\partial_k R_k\right] \quad \to \beta_g = k\partial_k\, g(k)$$

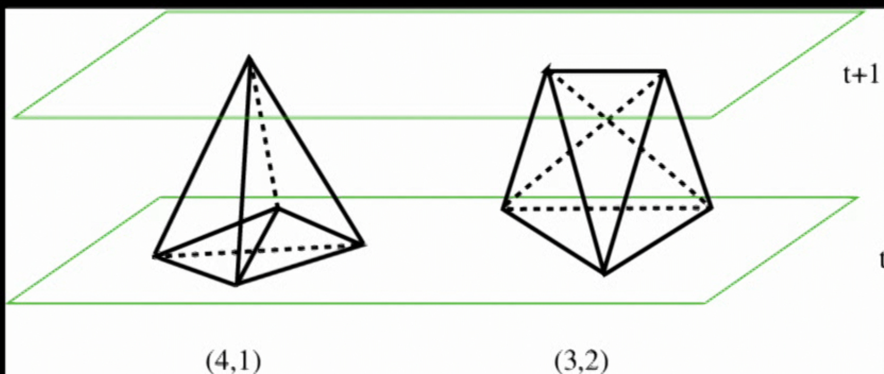
[Wetterich '93; Reuter '96]



Idea: causal sets not as fundamentally discrete, but as regularization of the Lorentzian path integral

Asymptotic safety in discretized settings

Example: Causal Dynamical Triangulations



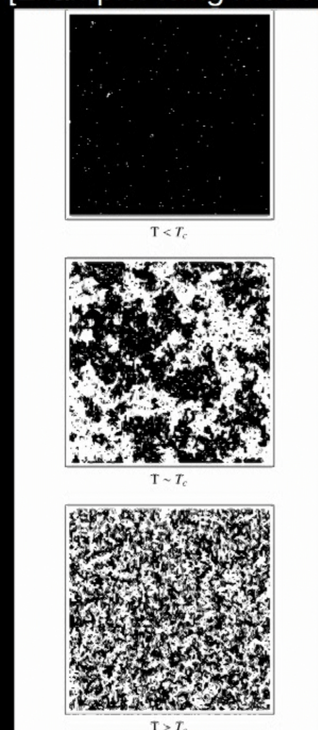
Discrete building blocks

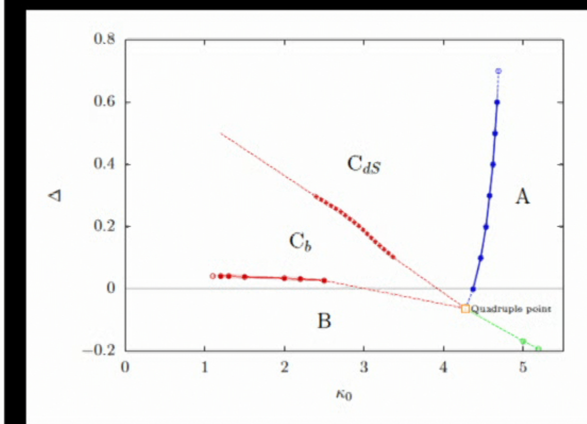
Asymptotic safety: can take continuum limit, because at fixed-point values of the couplings, there is (quantum) scale symmetry

Scale symmetry in lattice theories: second order phase transition

→ interacting RG fixed points: ubiquitous in statistical physics

[Example: Ising model]





Asymptotic safety in discretized settings

Idea: causal sets provide a discretization/regularization of Lorentzian geometries

[AE '17]

→ search for Lorentzian asymptotic safety by:

- searching for second-order phase transition in causal sets
- if found, compare critical exponents to FRG results

Existing results:

in d=1+1 and d=2+1, MCMC simulations with curvature term in the action yield a phase diagram with a phase boundary [Surya '11; Cunningham, Surya '19]

in d=1+1, the phase transition is of first order [Glaser, O' Connor, Surya '17]

→ What is missing?

→ Euclidean FRG studies predict three relevant directions, i.e., three parameters that need tuning to reach the phase transition

Quantum scale symmetry: Parameters that require tuning to see a second-order phase transition

Truncations of the full dynamics

selected examples:

- Einstein-Hilbert $+a\,R^2 + b\,R_{\mu\nu}R^{\mu\nu}$

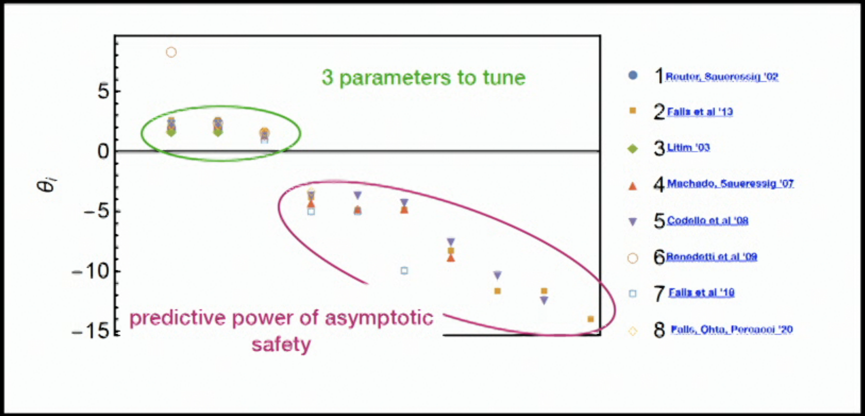
  Falls, Ohta, Percacci '20

- Einstein-Hilbert $+a\,R^2 + \ldots + a_{70}R^{70}$

  Falls, Litim, Schroeder '18

- Einstein-Hilbert $+c\,R_{\mu\nu\kappa\lambda}R^{\kappa\lambda\rho\sigma}R_{\rho\sigma}{}^{\mu\nu}$

  Gies, Knorr, Lippoldt, Saueressig '16



Parameters to tune:

Newton coupling, cosmological constant, combination of curvature-squared couplings

access higher-order curvature terms in causal sets!

Constructing higher-order curvature terms in causal set quantum gravity

Causal sets do not have tangent space ⇒ not known how to construct Riemann tensor

But: action contains curvature invariants, e.g., (Riemann)^2 etc

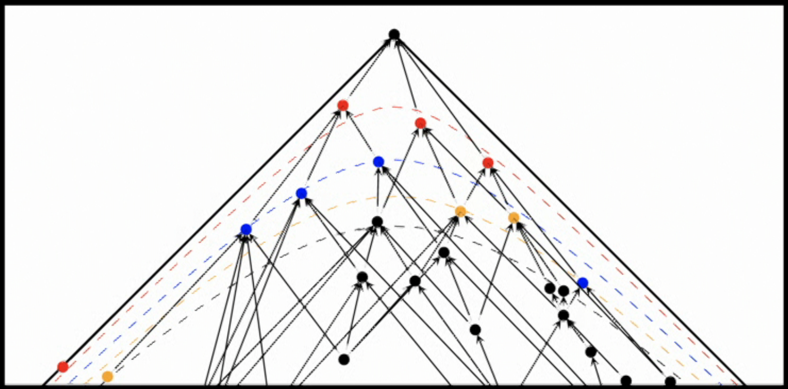
Hawking-Malament theorem: Under mild conditions on the spacetime, the causal structure encodes the metric up to a conformal factor.
In causal sets, conformal factor is recovered by counting number of elements.

⇒(discrete counterparts of) curvature invariants must be encoded in causal sets

→Concrete example: $R^2 - 2\Box R$ [de Brito, AE, Pfeiffer '23]

→Speculation about Riemann invariants

$R^2 - 2\Box R$ in causal sets



Propagator for a scalar field: [Sorkin '07]
summing over (causal) nearest neighbors:

$$B\phi(x) = \frac{-4}{\sqrt{6}\ell_d^2}\left(\phi(x) - \sum_{y\in L_1(x)}\phi(y) + 9\sum_{y\in L_2(x)}\phi(y) - 16\sum_{y\in L_3(x)}\phi(y) + 8\sum_{y\in L_4(x)}\phi(y)\right)$$

$L_n(x)$: points with $(n-1)$ points in the causal interval with x

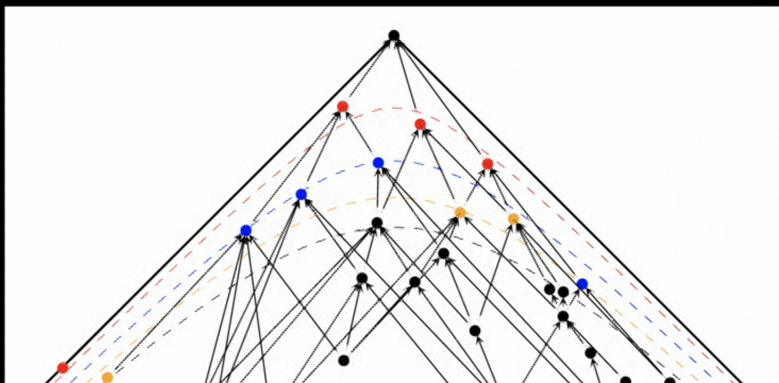
$$\lim_{\ell\to 0}\langle B\phi(x)\rangle = \left(\Box - \frac{1}{2}R\right)\phi(x) \xrightarrow{\phi=\text{const}} -\frac{1}{2}R\,\text{const}$$

$\psi(x) = B\phi(x)$ is a scalar field

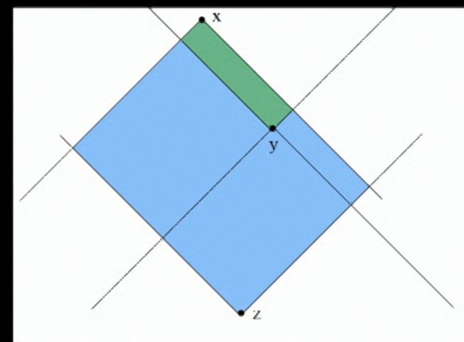
$$\Rightarrow \lim_{\ell\to 0} B\left(\psi(x)\right) = \left(\Box - \frac{1}{2}R\right)\psi(x) = \left(\Box - \frac{1}{2}R\right)\left(\Box - \frac{1}{2}R\right)\phi(x) \xrightarrow{\phi\to\text{const}} \left(\frac{1}{4}R^2 - \frac{1}{2}\Box R\right)\text{const}$$

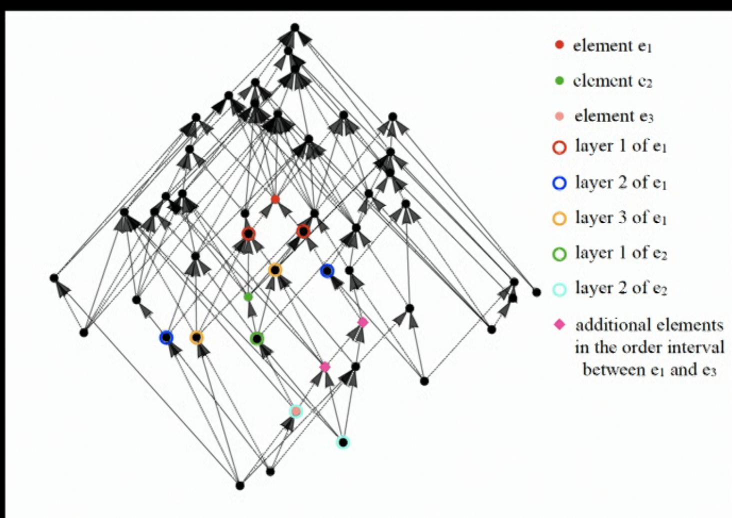
[de Brito, AE, Pfeiffer '23]

$R^2 - 2\Box R$ in causal sets

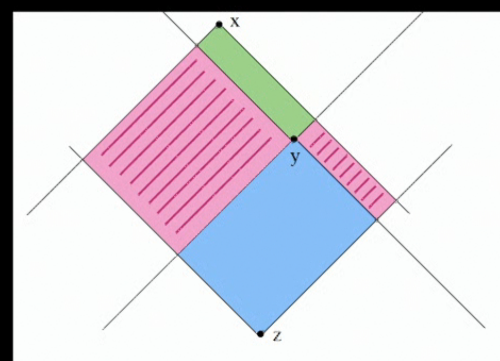


R constructed from causal orders
(nearest neighbors, next-to-nearest neighbors… in causal sense)





$R^2 - 2\Box R$ constructed from "stacked" causal orders



Speculation:
By summing over causal orders,
stacked causal orders…
from a point & varying coefficients
can obtain curvature invariants
at that point

Summary:

- There is physics to be discovered in the space ***between*** quantum gravity approaches
- (Some) challenges in individual approaches can be met by "importing" tools/ideas from other approaches

- Concrete example: Asymptotically safe quantum gravity and causal set quantum gravity
  - "Matter matters" approach with FRG techniques from asymptotic safety:
    - Nonlocality scale in causal sets induces a Landau pole in scalar quartic coupling

    $\Rightarrow$ may not be possible to choose nonlocality scale far below discreteness scale

  - Causal sets as Lorentzian regularization of the gravitational path integral
    - Opportunity to test whether there is asymptotic safety in Lorentzian signature
    - First step: construct necessary higher-order terms in the causal-set action
    - Outcome for causal sets: further quantities to characterize geometry & distinguish manifoldlike causal sets