Title: Measurement Quantum Cellular Automata and Anomalies in Floquet Codes

Speakers: Zhi Li

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Abstract: We investigate the evolution of quantum information under Pauli measurement circuits. We focus on the case of one- and two-dimensional systems, which are relevant to the recently introduced Floquet topological codes. We define local reversibility in context of measurement circuits, which allows us to treat finite depth measurement circuits on a similar footing to finite depth unitary circuits. In contrast to the unitary case, a finite depth locally reversible measurement sequence can implement a translation in one dimension. A locally reversible measurement sequence in two dimensions may also induce a flow of logical information along the boundary. We introduce "measurement quantum cellular automata" which unifies these ideas and define an index in one dimension to characterize the flow of logical operators. We find a Z_2 bulk invariant for Floquet topological codes which indicates an obstruction to having a trivial boundary. We prove that the Hastings-Haah honeycomb code belong to a class with such obstruction, which means that any boundary must have either non-local dynamics, period doubled, or admits boundary flow of quantum information.

Zoom Link: https://pitp.zoom.us/j/96083249406?pwd=MnhYbTEyU05ybVdyUlE3UGZrdEhPdz09

# Measurement Quantum Cellular Automata and Anomalies in Floquet Codes

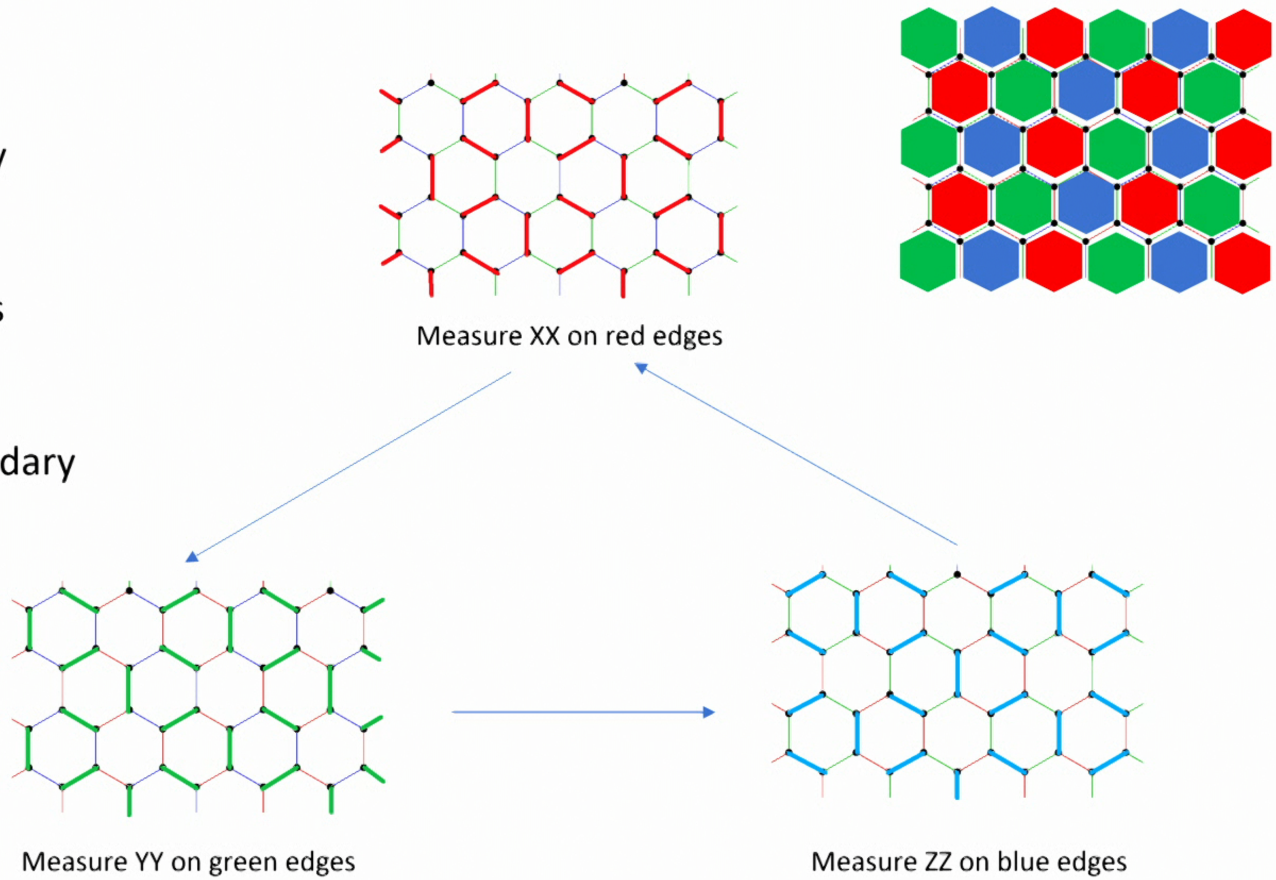
Zhi Li

May 17th, Perimeter Institute

Ref: 2304.01277 with David Aasen, Jeongwan Haah, Roger Mong

# A puzzle in Floquet Code

- Fault tolerant quantum memory by periodic measurement
- toric code, 2 logical qubits on torus
- on plane
    - static toric code: gapped boundary
    - Floquet code: no way?



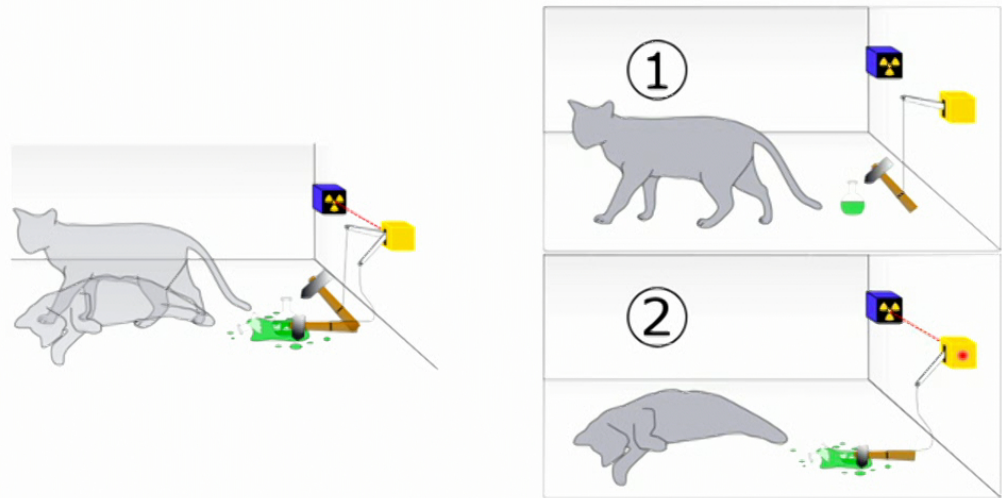
# Outline

- Periodic (Locally) Reversible Measurement Circuits
  - Floquet Codes
- Measurement Quantum Cellular Automata
- Anomalous Boundary of Floquet Codes

# Measurements

$$|\psi\rangle \to \frac{P|\psi\rangle}{\langle\psi|P|\psi\rangle}$$

"collapse"



# Reversible measurements

| | State | Stabilizer Operators | Logical Operators (up to stabilizers) |
|---|---|---|---|
| $\|\psi\rangle \ \|+\rangle$ | $(a\|\uparrow\rangle + b\|\downarrow\rangle)(\|\uparrow\rangle + \|\downarrow\rangle)$<br>$\propto (a\|\uparrow\rangle\|\uparrow\rangle + b\|\downarrow\rangle\|\downarrow\rangle) + (a\|\uparrow\rangle\|\downarrow\rangle + b\|\downarrow\rangle\|\uparrow\rangle)$ | $X_2$ | $Z_1, \quad X_1$ |
| Measure $Z_1 Z_2$ | $P(ZZ = 1) = P(ZZ = -1) = \frac{1}{2}$<br>(**no information** about a, b) | | $X_1 \equiv X_1 X_2$ |
| | $a\|\uparrow\rangle\|\uparrow\rangle + b\|\downarrow\rangle\|\downarrow\rangle$<br>$\propto (a\|\uparrow\rangle + b\|\downarrow\rangle)\|+\rangle + (a\|\uparrow\rangle - b\|\downarrow\rangle)\|-\rangle$ | $Z_1 Z_2$ | $Z_1, \quad X_1 X_2$ |
| Measure $X_2$ | $P(X_2 = 1) = P(X_2 = -1) = \frac{1}{2}$<br>(**no information** about a, b) | | |
| $\|\psi\rangle \ \|+\rangle$ | $(a\|\uparrow\rangle + b\|\downarrow\rangle)\|+\rangle$ | $X_2$ | $Z_1, \quad X_1 X_2 \equiv X_1$ |

# Reversible measurements

- **Theorem**:

no-information via measurement

$\Updownarrow$

well-defined, reversible dynamics of equivalent classes of logical operators

- stabilizer groups: $S_1, S_2$

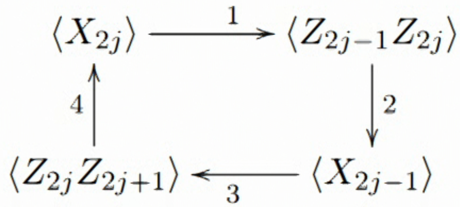
  logical groups (up to equiv): $C_P(S_1)/S_1$, $C_P(S_2)/S_2$

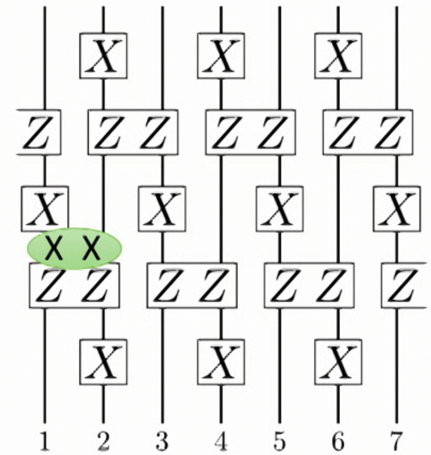
  dynamics: $C_P(S_1)/S_1 \rightarrow C_P(S_2)/S_2$

  $\forall L \in C_P(S_1)$, find $A \in S_1$ s.t. $LA \in C_P(S_2)$, then $[L] \mapsto [LA]$

# Periodic Reversible Measurement Circuits

- $S_1 \rightarrow S_2 \rightarrow \cdots \rightarrow S_n$
  - each step is reversible
  - periodic: $S_n = S_1$
- Example: shift by measurements

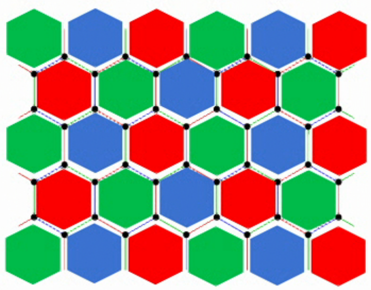


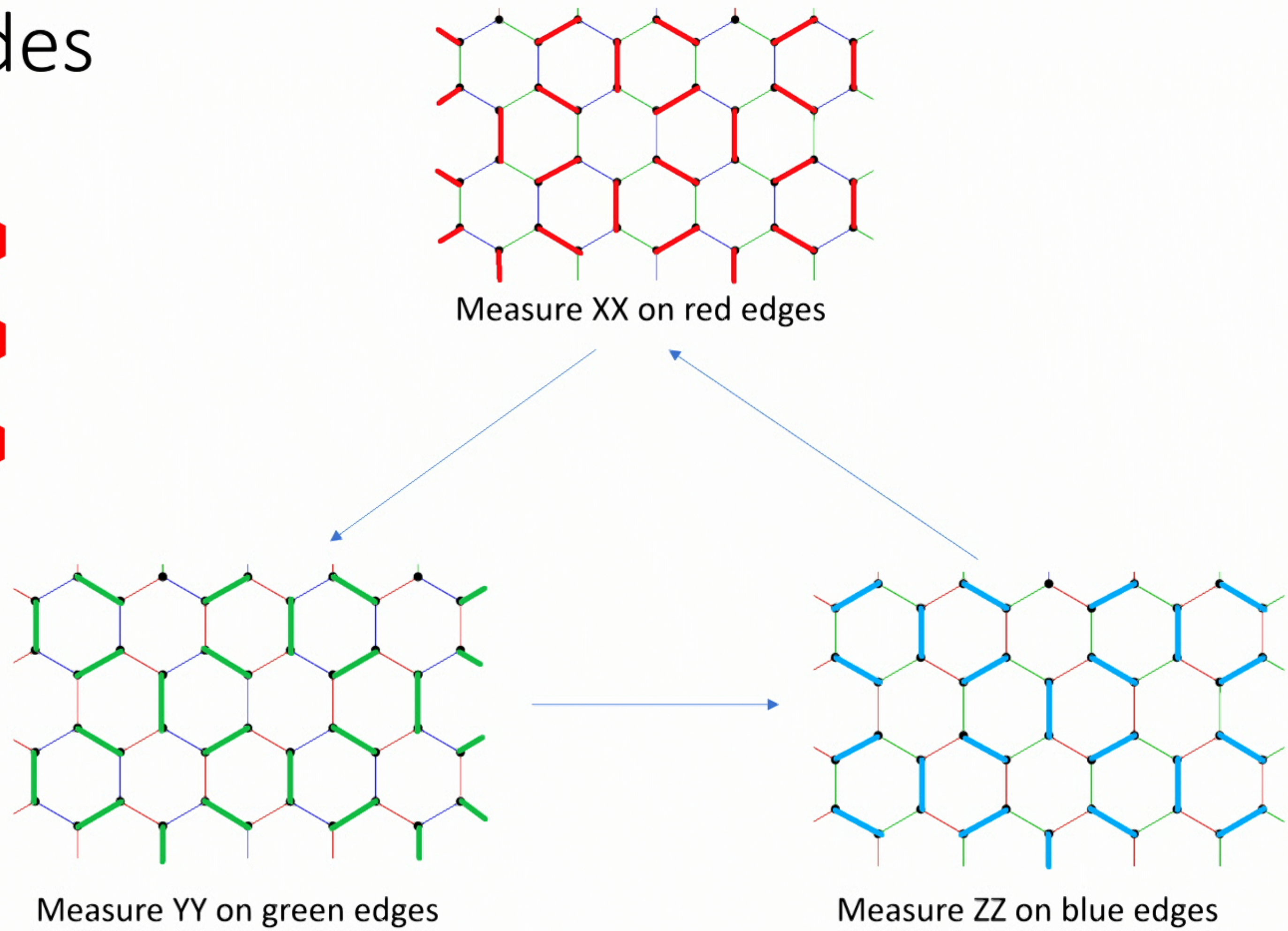




$\xrightarrow{\text{Measure } X_2} X_1 \equiv X_1 X_2 \xrightarrow{\text{Measure } Z_1 Z_2} X_1 X_2 \xrightarrow{\text{Measure } X_1, X_3} X_1 X_2 \equiv X_2 X_3 \xrightarrow{\text{Measure } Z_2 Z_3} X_2 X_3 \xrightarrow{\text{Measure } X_2} X_2 X_3 \equiv X_3$

$\xrightarrow{\text{Measure } X_2} Z_1 \xrightarrow{\text{Measure } Z_1 Z_2} Z_1 \equiv Z_2 \xrightarrow{\text{Measure } X_1, X_3} Z_2 \xrightarrow{\text{Measure } Z_2 Z_3} Z_2 \equiv Z_3 \xrightarrow{\text{Measure } X_2} Z_3$

# Floquet Codes





# Floquet Codes

Instantaneous stabilizers

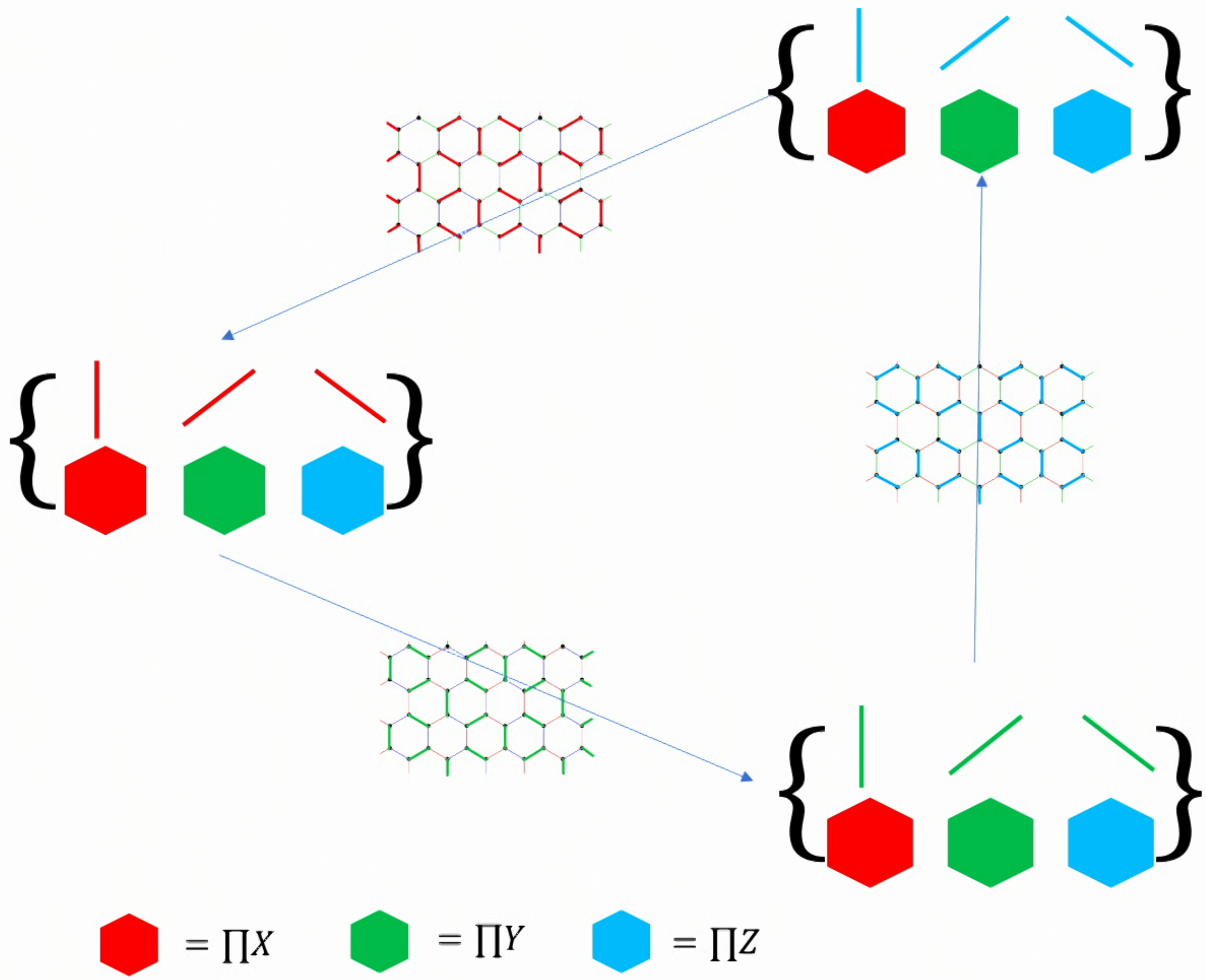
- each step ≡ toric code

Instantaneous logical operators

- "inner": $\prod$edge along loops
- "outer": $\prod$(YY or ZZ) along red loops

Dynamics of logical operators:

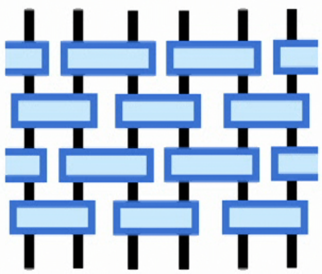
- period 6, not 3



# Quantum Cellular Automata (QCA)

- Def: QCA = unitary & locality preserving

  (locality preserving: $U^\dagger O U$ always close to $O$)



finite depth unitary circuits

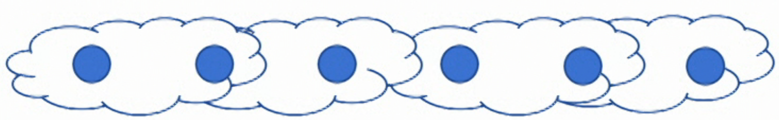
translation/shift

- 1D index theory
  - essentially, only these two possibilities
  - GNVW index $\in \mathbb{Q}^+$, $\log(\mathrm{GNVW}) \sim$ number of shifts

# Measurement Quantum Cellular Automata

| | QCA | MQCA |
|---|---|---|
| Operator algebra | $\otimes\,(Pauli\ algebra)$ | $L/S$<br>$L$: a group of Pauli operators<br>$S$: normal subgroup of $L$ |
| Dynamics | $\alpha(O) = U^{\dagger}OU$<br>$U$ unitary<br>locality preserving | $\alpha: L/S \to L/S$<br>$\alpha$ automorphism<br>locality preserving |
| Index theory in 1D | $\log(\mathrm{GNVM}) \in \mathbb{Z}$ | $Ind^{M} \in \frac{1}{2}\mathbb{Z}$ |

Motivation: the algebra of operators does not always factorize



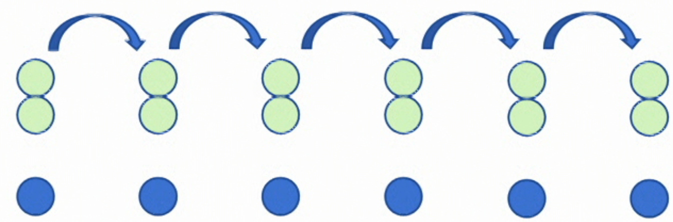
# Measurement Quantum Cellular Automata

- Example 1: usual QCA

  $L$ = ⊗Pauli algebra, $S$ = {1}

  MQCA: $X_i \to X_{i+1}, Z_i \to Z_{i+1}$

  $Ind^M = \log(\text{GNVW}) = 1$

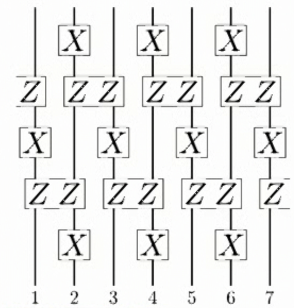


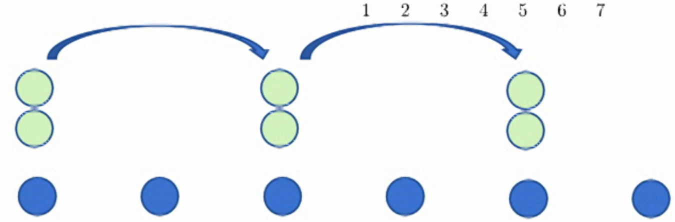
- Example 2: periodic reversible measurement circuit

  $L = C_P(S), S = \{X_{2i}\}$

  MQCA: $X_{2i-1} \to X_{2i+1}, Z_{2i-1} \to Z_{2i+1}$

  $Ind^M = 1$



In 1D:

finite depth unitary circuit

$\subsetneq$

quantum cellular automaton

$\subseteq$ (actually, =)

periodic locally reversible measurement circuit

$\subsetneq$

measurement quantum cellular automaton

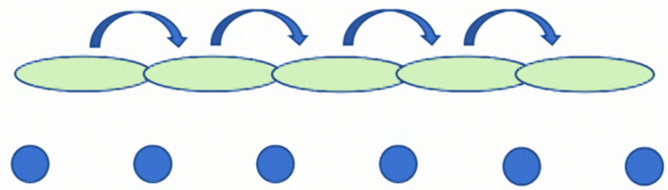
# Measurement Quantum Cellular Automata

- Example 3: half shift

  $L = \{X_i Z_{i+1}\}, S = \{1\}$

  MQCA: $X_i Z_{i+1} \to X_{i+1} Z_{i+2}$

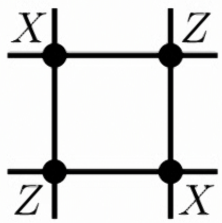
  $Ind^M = \frac{1}{2}$



- **Theorem:** 1D periodic reversible measurement circuit, viewed as an MQCA, has $Ind^M \in \mathbb{Z}$

# MQCA as boundary

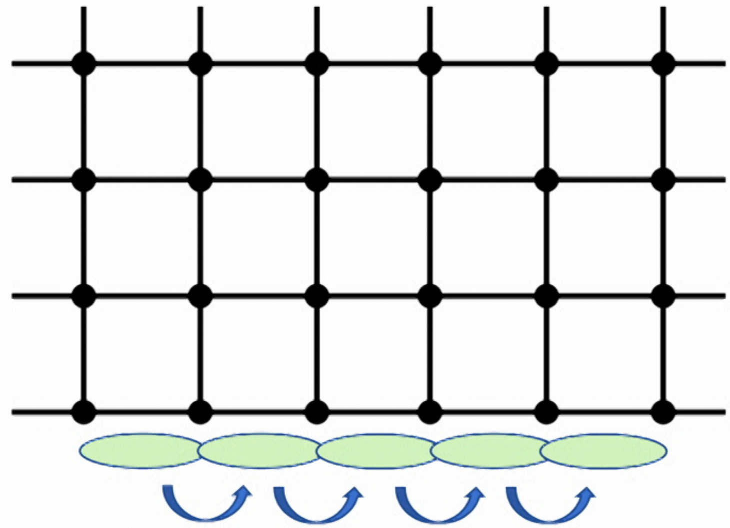
- Wen's plaquette model




- Boundary algebra

$$L = \{X_i Z_{i+1}\}, S = \{1\}$$

- ∃ 2D periodic reversible measurement circuit that induces the half shift



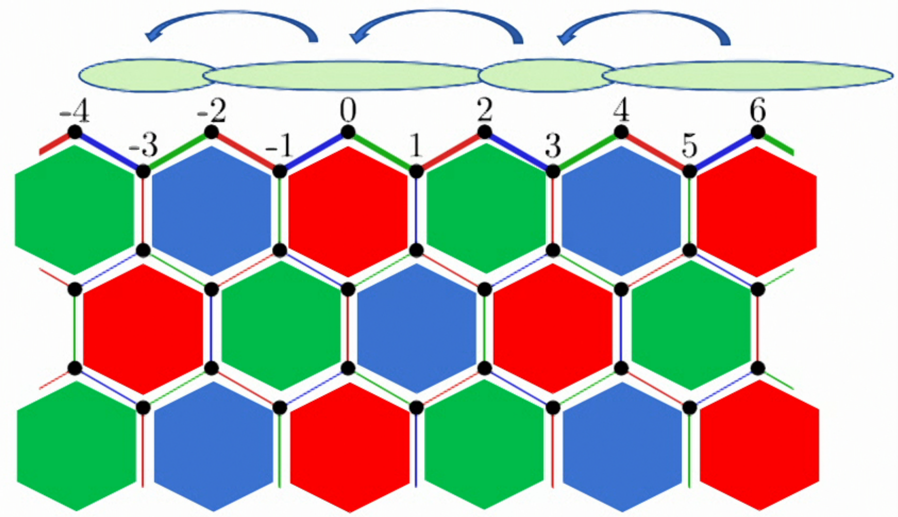
# MQCA as boundary

- Floquet Code boundary:

$$L = \{Z_{-2}Z_{-1}Z_1Z_2, X_2X_3X_4, \dots\}$$

- Dynamics after 3 rounds
  $Z_{-2}Z_{-1}Z_1Z_2 \to X_{-4}X_{-3}X_{-2}$
  $X_2X_3X_4 \to Z_{-2}Z_{-1}Z_1Z_2$

- $Ind^M = -\frac{1}{2}$



# MQCA as boundary

- **Theorem:** $Ind^M$ (mod $\mathbb{Z}$) of the boundary 1D MQCA induced by 2D periodic reversible measurement circuits is independent of the choices of boundaries
- **Corollary:**
  - Bulk-edge correspondence
  - $\mathbb{Z}_2$ anomaly
    - (edge) obstruction for realizing on 1D
    - (bulk) obstruction for trivial boundary

# Conclusion

- Periodic (Locally) Reversible Measurement Circuits
  - Floquet Codes
- Measurement Quantum Cellular Automata
  - Index theory
- Bulk-edge correspondence:
  - MQCA as boundary of measurement circuits
  - $Ind^M$ (mod $\mathbb{Z}$) is determined by bulk
- Floquet Code has $\mathbb{Z}_2$ anomaly