

Title: From wave function collapse to non-abelian anyons on a quantum processor

Speakers: Ruben Verresen

Series: Colloquium

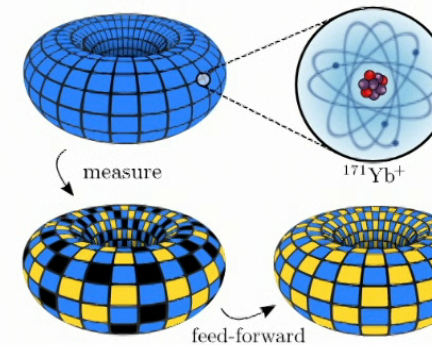
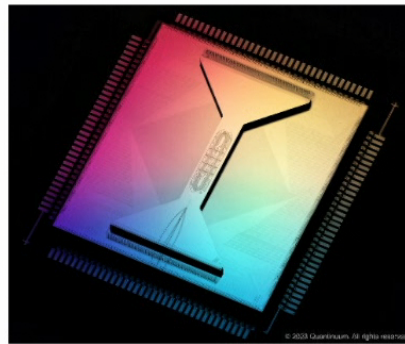
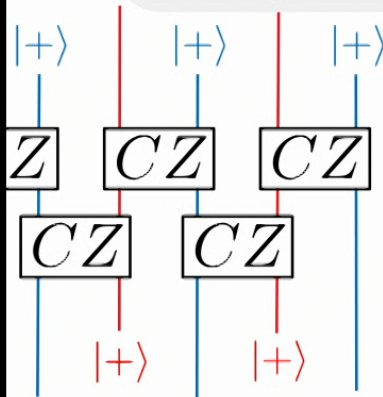
Date: May 17, 2023 - 2:00 PM

URL: <https://pirsa.org/23050145>

Abstract: Schrodinger's thought experiment famously illustrates the dramatic effect of measuring a quantum state. The resulting wave function collapse is often thought to make states more classical and familiar. However, in this colloquium, we explore how measurements can be used as a chisel to efficiently build exotic forms of quantum entanglement. We focus on topological states of matter, whose quasiparticles exhibit generalized 'anyonic' exchange statistics with potential relevance to quantum computation. We use these ideas to experimentally realize the first controlled realization of non-Abelian anyons, which can remember the sequence in which they are exchanged. The smoking gun signature of this experiment is inspired by the coat of arms of the House of Borromeo.

Zoom Link: <https://pitp.zoom.us/j/98167813390?pwd=aG5vcklVZzBWT1BRSjI4RVRtbDhBUT09>

From **wave function collapse** to **non-abelian anyons** on a **quantum processor**



Ruben Verresen
Harvard University

(Funding: Harvard Quantum Initiative and UQM Simons collaboration)

Colloquium @ Perimeter Institute (May 17, 2023)

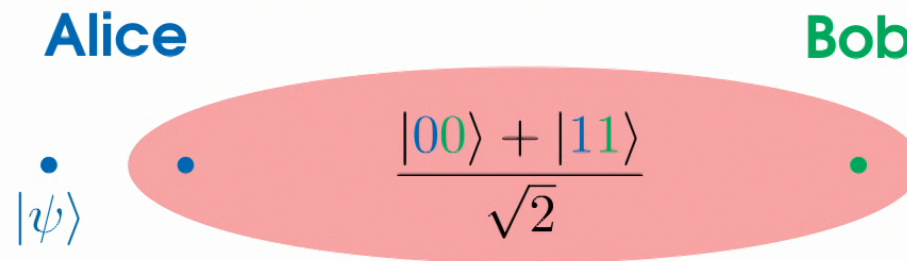
Measuring in quantum mechanics is a violent act



$$\left| \begin{array}{c} \text{Awake} \\ \text{Cat} \end{array} \right\rangle + \left| \begin{array}{c} \text{Dead} \\ \text{Cat} \end{array} \right\rangle \xrightarrow[\text{Collapse}]{\text{Measure}} \left| \begin{array}{c} \text{Dead} \\ \text{Cat} \end{array} \right\rangle$$

Schrödinger, 1935

Measurement as a tool: Quantum Teleportation



Bennett et al, 1993

Measurement as a tool: Quantum Teleportation

N.B.: We write X,Y,Z for the spin-1/2 Pauli matrices

Alice



$$|\Psi_{a,b}\rangle = Z_1^a X_2^b \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Measure in Bell pair basis
→ outcome $a, b \in \{0, 1\}$

Bob



Bennett et al, 1993

Measurement as a tool: Quantum Teleportation

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Measure in Bell pair basis
→ outcome $a, b \in \{0, 1\}$

Bob



$$X^b Z^a |\psi\rangle$$

Measurement allows us to efficiently perform
quantum channels **via a classical** channel!

Bennett et al, 1993

Measurement as a tool: preparing long-range entanglement

Measurement can collapse Schrödinger's cat ...
but it can also **create a 'cat state'**!

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|000 \dots 0\rangle + |111 \dots 1\rangle)$$

The GHZ state is hard/slow to prepare without measurement due to massive entanglement

(Greenberger, Horne, Zeilinger, 1989)

Measurement as a tool: preparing long-range entanglement

Measurement can collapse Schrödinger's cat ...
but it can also **create a 'cat state'**!

Start with a **product state**:

$$|+\rangle^{\otimes N} \propto |000 \cdots 0\rangle + |111 \cdots 1\rangle + \cdots$$

Briegel, Raussendorf, '00

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Measure whether neighboring spin are (anti)aligned

Outcome: $Z_n Z_{n+1} = (-1)^{a_n}$ with $a_n \in \{0, 1\}$

$$|\psi\rangle = \prod_{n=2}^N X_n^{\sum_{m=1}^{n-1} a_m} |\mathbf{GHZ}\rangle$$

Briegel, Raussendorf, '00

Measurement as a tool: preparing long-range entanglement

Measurement and non-local feedback seem to massively *speed up* state preparation!
Can we use this to create *exotic phases of matter*???

Start with a *product state*:

$$|+\rangle^{\otimes N} \propto |000 \cdots 0\rangle + |111 \cdots 1\rangle + \cdots$$



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Briegel, Raussendorf, '00

Quantum devices enter the many-body era!

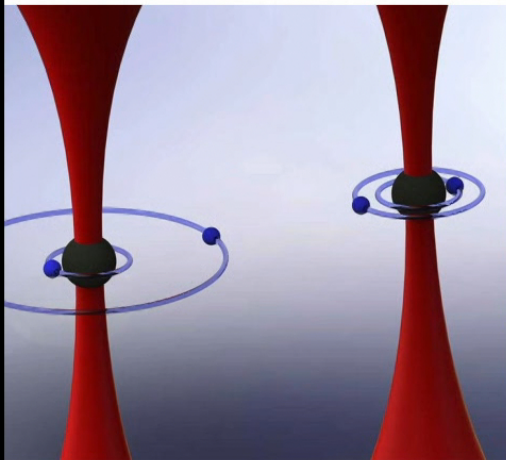
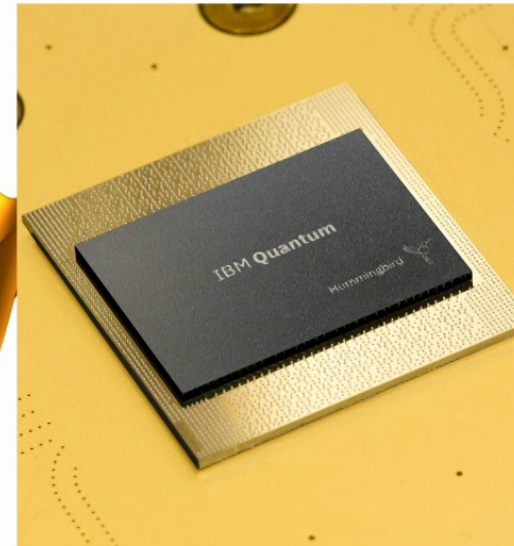
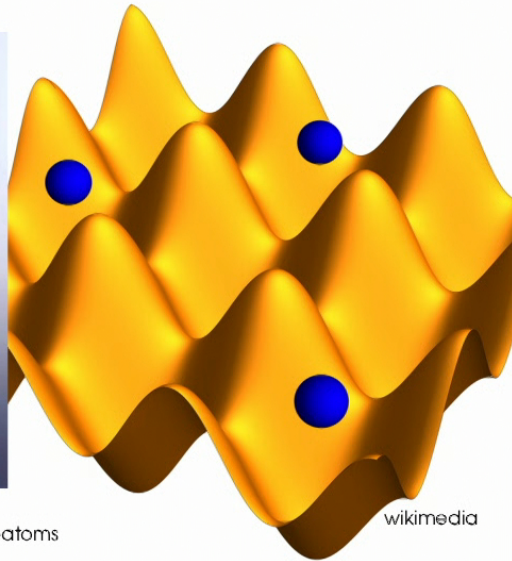


Fig: <https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms>

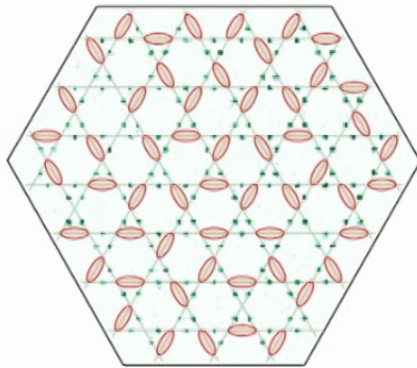


- significant **knowledge** of interactions
- extreme **control** (tune lattice, parameters, ...)
- resolved and multi-body **measurements**

Signatures of Z_2 topological order in quantum devices

Rydberg atom tweezers

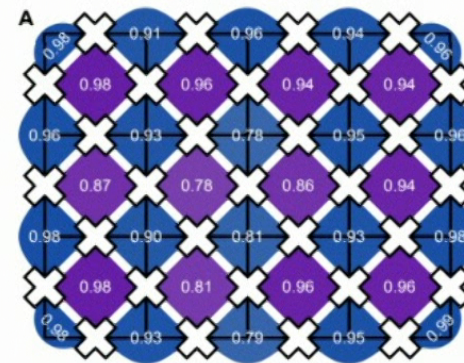
(Lukin lab)



Theory: RV, Lukin, Vishwanath, PRX (2021)
Experiment: Semeghini et al, Science (2021)

Superconducting qubits

(Google)



Theory: Liu et al, PRX Quantum (2022)
Experiment: Satzinger et al, Science (2021)

Emergent low-energy physics

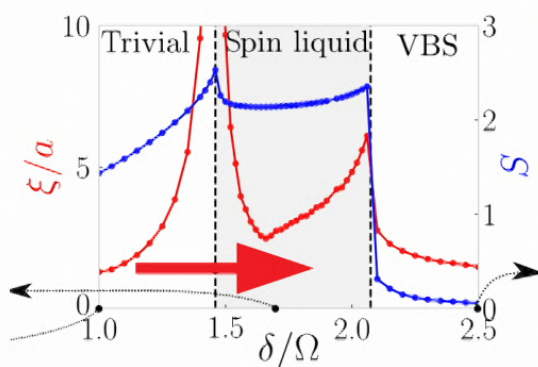
(with important non-equilibrium aspects: Sahay et al, arxiv:2211.01381)

Constructed
using unitary circuits

Experiments highlight new challenges

Rydberg atom tweezers

(Lukin lab)



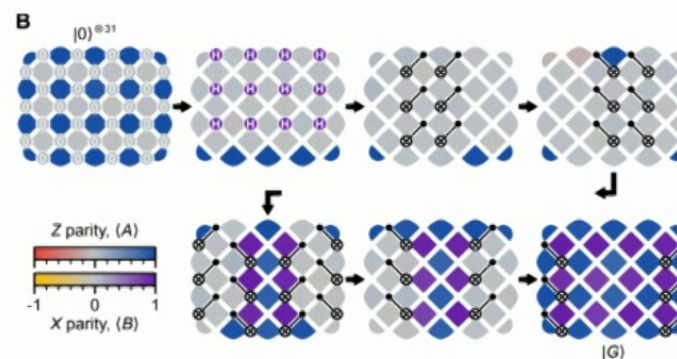
Theory: RV, Lukin, Vishwanath, PRX (2021)

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'Adiabatic' parameter sweep
through critical point
→ **time ~ system size!**

Superconducting qubits

(Google)



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Experiment: Satzinger et al, Science (2021)

Depth of unitary circuit
scales with system size!

(Bravyi, Hastings, Verstraete '06)

Experiments highlight new challenges

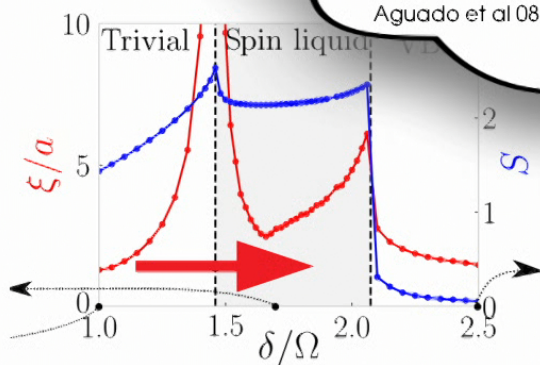
Rydberg atoms

(Lukin)

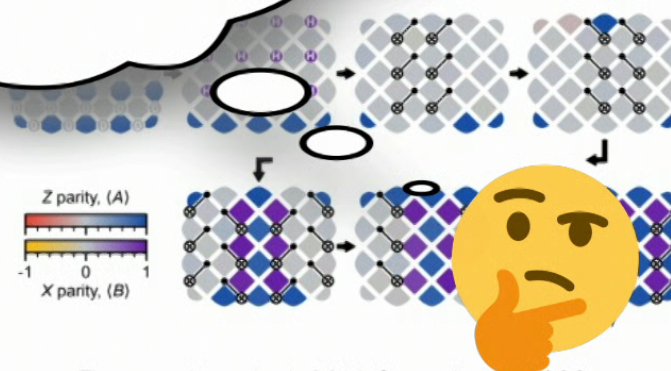
Can we use wavefunction collapse and feedback to scalably prepare topological states of matter???

(Briegleb, Raussendorf '01; Raussendorf, Bravyi, Harrington '05; Aguado et al '08; Bolt et al '16; Piroli, Styliaris, Cirac '21; etc)

storing qubits



Theory: RV, Lukin, Vishwanath, PRX (2021)
Experiment: Semeghini et al, Science (2021)



Theory: Liu et al, PRX Quantum (2022)
Experiment: Satzinger et al, Science (2021)

'Adiabatic' parameter sweep
through critical point
→ time ~ system size!

Depth of unitary circuit
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(Bravyi, Hastings, Verstraete '06)

Last week: new experimental implementation!

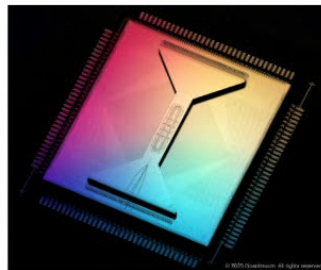
[Submitted on 5 May 2023]

Creation of Non-Abelian Topological Order and Anyons on a Trapped-Ion Processor

Mohsin Iqbal, Nathanan Tantivasadakarn, Ruben Verresen, Sara L. Campbell, Joan M. Drelling, Caroline Figgatt, John P. Gaebler, Jacob Johansen, Michael Mills, Steven A. Moses, Juan M. Pino, Anthony Ransford, Mary Rowe, Peter Siegfried, Russell P. Stutz, Michael Foss-Felg, Ashvin Vishwanath, Henrik Dreyer

Non-Abelian topological order (TO) is a coveted state of matter with remarkable properties, including quasiparticles that can remember the sequence in which they are exchanged. These anyonic excitations are promising building blocks of fault-tolerant quantum computers. However, despite extensive efforts, non-Abelian TO and its excitations have remained elusive, unlike the simpler quasiparticles or defects in Abelian TO. In this work, we present the first unambiguous realization of non-Abelian TO and demonstrate control of its anyons. Using an adaptive circuit on Quantinuum's H2 trapped-ion quantum processor, we create and move anyons on a 32-qubit processor, with fidelity per site exceeding 98.4%. By creating and moving anyons along Borromean rings in space, we demonstrate a braiding process. Furthermore, tunneling non-Abellions around a torus creates all 22 ground states, as well as Abelian TO. This work illustrates the counterintuitive nature of non-Abellions and enables their study in quantum computing.

On new 32-qubit processor
of Quantinuum/Honeywell



arxiv:2305.03828

nature

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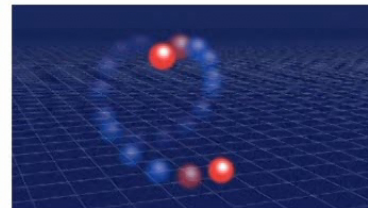
NEWS | 09 May 2023

Physicists create long-sought topological quantum states

Exotic particles called nonabellions could fix quantum computers' error problem.

Davide Castelvecchi

Quanta magazine



QUANTUM COMPUTING

Physicists Create Elusive Particles That Remember Their Pasts

By CHARLIE WOOD | MAY 9, 2023 | 3 | 1

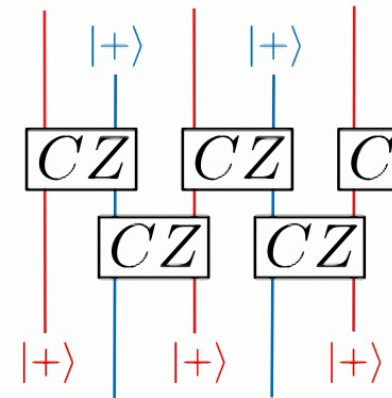
In two landmark experiments, researchers used quantum processors to engineer exotic particles that have captivated physicists for decades. The work is a step toward crash-proof quantum computers.

Outline

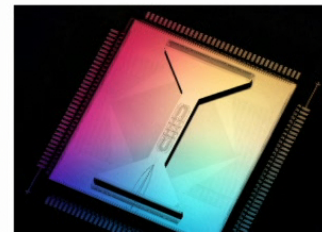
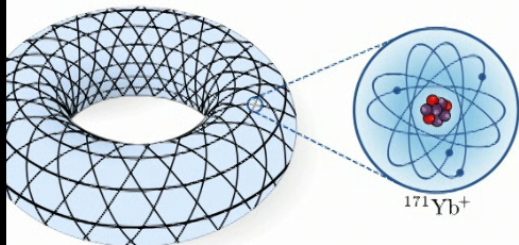
$$|\psi\rangle = \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \end{array} \right\rangle + \dots$$

1) Topological Order

2) Measurement-based Protocols



3) Non-Abelions in a Trapped-Ion Processor



Entanglement = the 'right amount' of superposition

$$|\uparrow\uparrow\rangle$$

$$\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$$

$$\frac{|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle}{2} = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \otimes \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

Entanglement = the 'right amount' of superposition

$$\cancel{|\uparrow\uparrow\rangle}$$

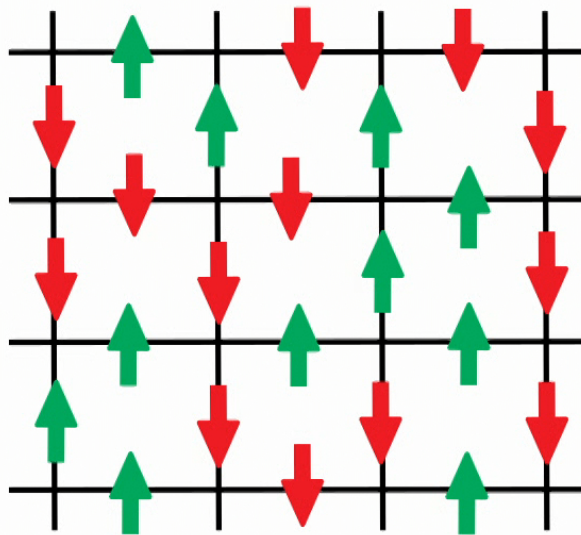
$$\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} = \text{maximal superposition}$$

with the constraint $\sigma_1^z \sigma_2^z = 1$

$$\cancel{\frac{|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle}{2}} = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \otimes \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

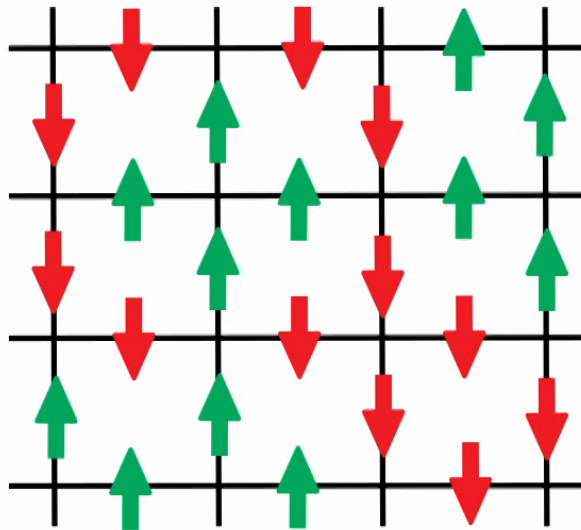
Towards (interesting) many-body entanglement

Consider qubits (i.e., spin-1/2) on bonds of square lattice







Towards (interesting) many-body entanglement

Constraint: even number of down spins around every vertex

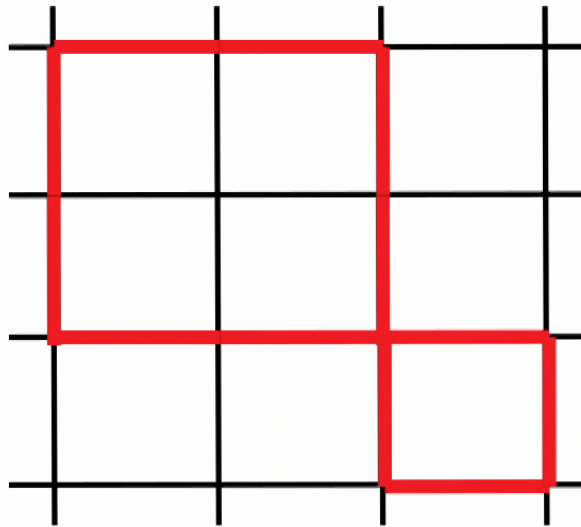


$$\sigma^z \sigma^z = 1$$





Towards (interesting) many-body entanglement

Graphical notation:  =   = 

Constraint: only closed loops!



Towards (interesting) many-body entanglement

Graphical notation:  =   = 





Consider the **superposition** of all **constrained** states:

$$|\psi\rangle = \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{red} & & \\ \hline \text{red} & \text{red} & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{red} & & \\ \hline & \text{red} & \text{red} & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & \text{red} & \text{red} & \\ \hline & \text{red} & \text{red} & \\ \hline \end{array} \right\rangle + \dots$$

This is the ground state of the **toric code** Hamiltonian:

$$H = - \underbrace{\sum \sigma^z \sigma^z \sigma^z \sigma^z}_{\text{constraint}} - \underbrace{\sum \sigma^x \sigma^x \sigma^x \sigma^x}_{\text{superposition}} \quad (\text{Kitaev 1997})$$

Towards (interesting) many-body entanglement





Graphical notation:  =   = 

Consider the **superposition** of all **constrained** states:

$$|\psi_{\text{exc}}\rangle = \left| \begin{array}{ccc} \text{red string} & & e \\ \text{red string} & & \\ \text{red string} & & \\ \text{red string} & & e \end{array} \right\rangle + \left| \begin{array}{ccc} \text{red string} & & e \\ \text{red string} & & \\ \text{red string} & & \\ \text{red string} & & e \end{array} \right\rangle + \left| \begin{array}{ccc} \text{red string} & & e \\ \text{red string} & & \\ \text{red string} & & \\ \text{red string} & & e \end{array} \right\rangle + \dots$$

e-particles are created at end of X-string

Towards (interesting) many-body entanglement

Graphical notation:  =   = 

Consider the **superposition** of all **constrained** states:

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e-particles are created at end of **X-string**

Similarly can create '**m**' and '**f**' particles at ends of strings

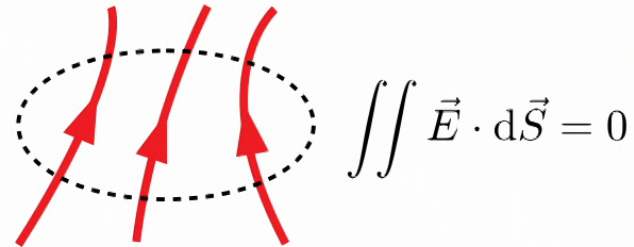
These are NOT bosons! In fact, '**f**' is a **fermion**!!

This is a **stable** property of a (topological) **phase** of matter!

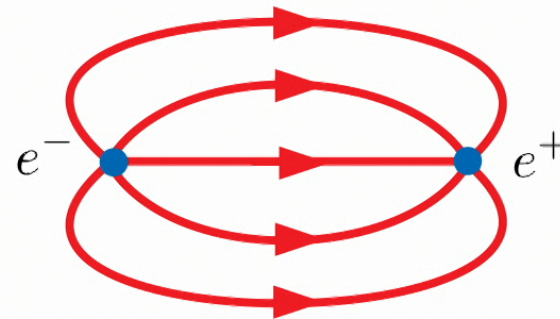
(Fradkin-Shenker '79, Arovas et al '84, Einarsson '84, Read-Sachdev '91, Wen '91, ...)

Deeper perspective: loop states \approx gauge theory!

Electric field lines are **closed**
in electromagnetic vacuum
= Gauss's law



Electric field lines can only
end and start at **charges**
→ charges created in **pairs**!



Only difference for toric code: gauge group is \mathbb{Z}_2 not $U(1)$

Discrete gauge groups give **topological** order! (Witten '89, Wen '89)

Anyons: generalized exchange statistics

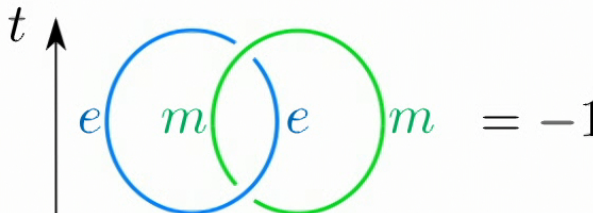
Particles in 2+1D need not be bosons or fermions!

(Leinaas, Myrheim (1977), Wilczek (1982), Halperin; Arovas, et al (1984),)

Anyons = point-like objects with generalized exchange statistics

Abelian anyons pick up a **phase factor** upon braiding

E.g., for Z_2 topological order:


$$e \quad m \quad e \quad m = -1$$

Realized in fractional quantum Hall states + Google's processor

Nakamura et al., Bartolomei et al., (2020))

Satzinger et al. (2021)

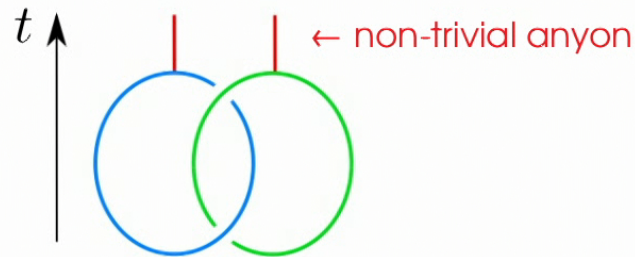
Non-Abelian anyons

More exotic '**non-Abelian**' anyons have an internal space which transforms non-trivially under braiding

(Witten; Moore and Seiberg; Wen; Moore and Read (1989))

Similar to the internal 'color' label of quarks (SU(3) symmetry)

Consequence:
need not fuse
back to vacuum!



= working principle of **topological quantum computation**

(Kitaev '97)

Many models known (Levin-Wen '04),

no clear-cut realization: $\nu = 5/2$ FQH comes closest (Willett et al., (2023))

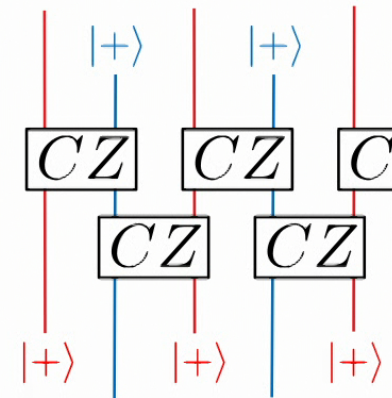
Although see recent Google paper for non-Abelian *defects* (2023)

Outline

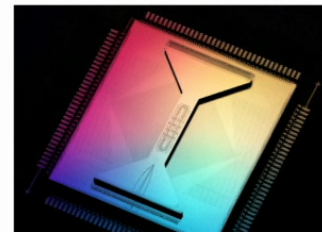
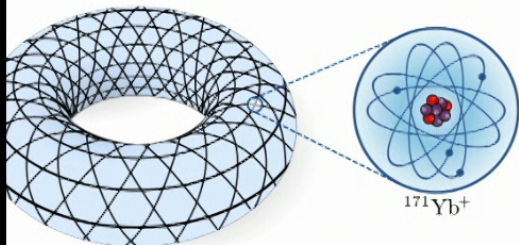
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✓ 1) Topological Order

2) Measurement-based Protocols



3) Non-Abelions in a Trapped-Ion Processor

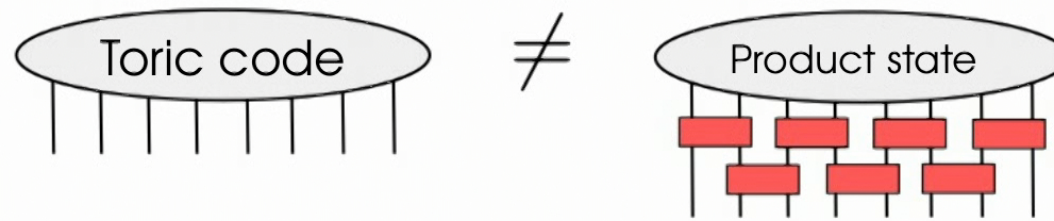


Constraints imposed by unitarity and locality

Unitary protocols cannot efficiently create topological order

E.g., a finite-depth circuit cannot prepare a toric code!

(Bravyi, Hastings, Verstrate, PRL (2006))

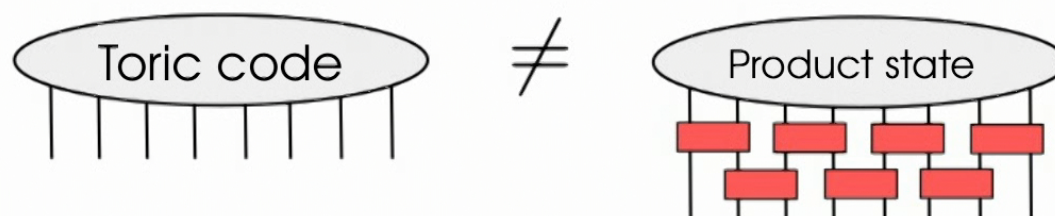


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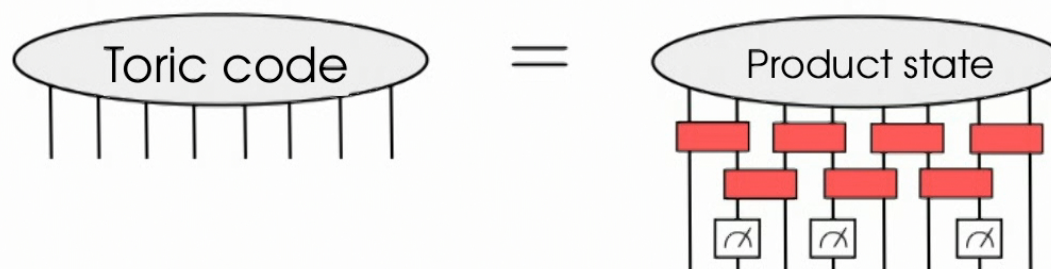
E.g., a finite-depth circuit cannot prepare a toric code!

(Bravyi, Hastings, Verstrate, PRL (2006))



But with measurements we can avoid unitarity constraints!

(Raussendorf, Bravyi, Harrington, '05)



Towards (interesting) many-body entanglement

Graphical notation: $\text{---}\uparrow\text{---} = \text{---}$ $\text{---}\downarrow\text{---} = \text{---}$

Consider the **superposition** of all **constrained** states:

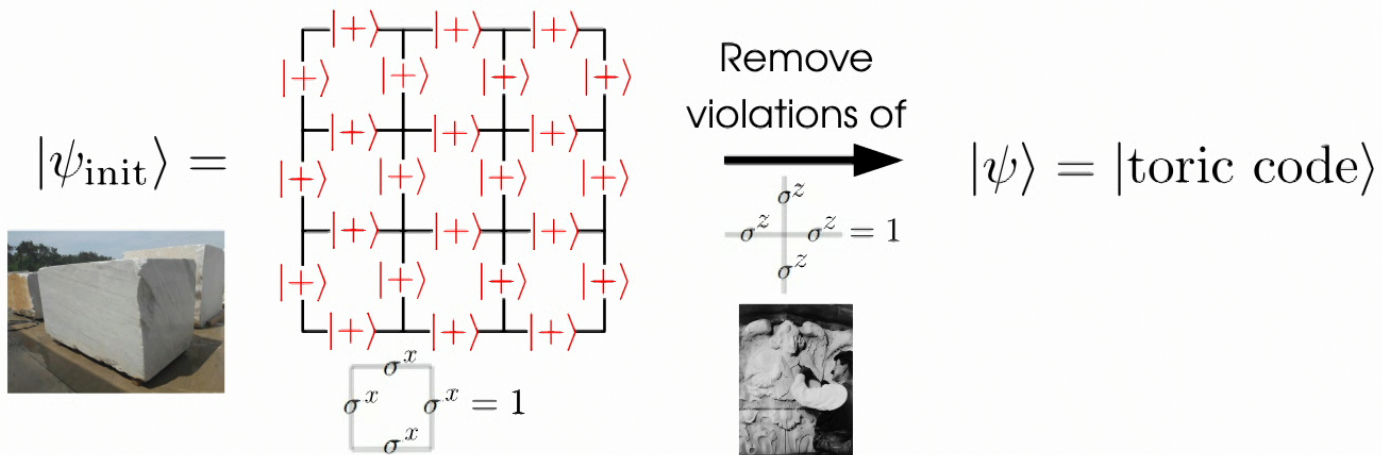
$$|\psi\rangle = \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \text{red} & \text{red} & \text{red} & \text{red} \\ \hline \end{array} \right\rangle + \dots$$

This is the ground state of the **toric code** Hamiltonian:

$$H = - \underbrace{\sum \sigma^z \sigma^z \sigma^z \sigma^z}_{\text{constraint}} - \underbrace{\sum \sigma^x \sigma^x \sigma^x \sigma^x}_{\text{superposition}} \quad (\text{Kitaev 1997})$$

A little reminder...

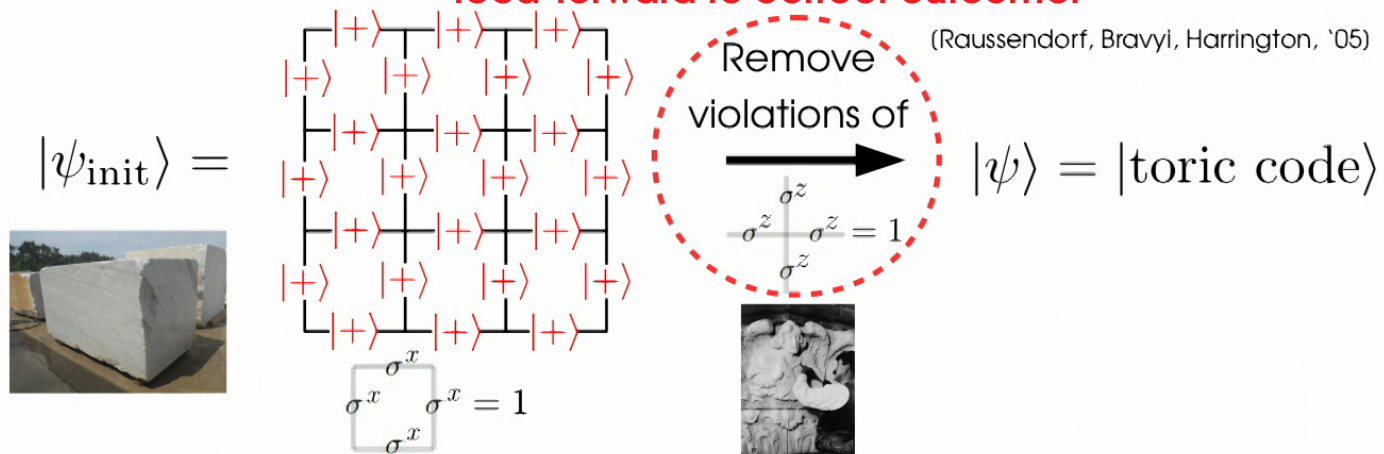
The 'stone carving route' to topological order



The 'stone carving route' to topological order



Can be achieved via measurement and
feed-forward to correct outcome!



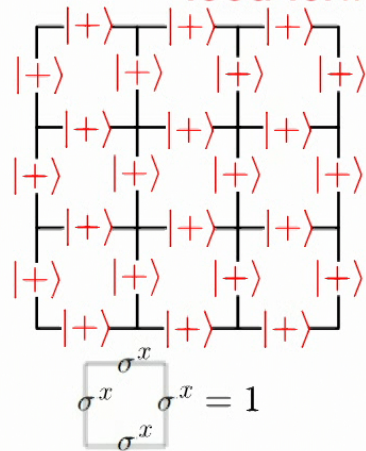
The 'stone carving route' to topological order



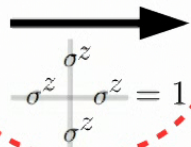
Can this really be done
in an *experiment*?
What types of states *can* one
(not) prepare this way?

Can be achieved via measurement and
feed-forward to correct outcome!

$|\psi_{\text{init}}\rangle =$



Remove
violations of



(Raussendorf, Bravyi, Harrow)

$|\psi\rangle = |\text{toric code}\rangle$



The 'stone carving route' to topological order

Can this really be done
in an *experiment*?
What types of states *can* one
(not) prepare this way?



arXiv > quant-ph > arXiv:2302.01917

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Quantum Physics

[Submitted on 3 Feb 2023]

Topological Order from Measurements and Feed-Forward on a Trapped Ion Quantum Computer

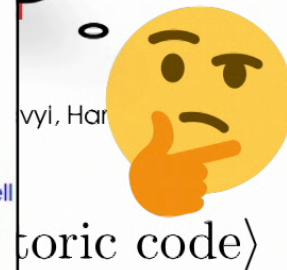
Mohsin Iqbal, Nathanan Tantivasadakarn, Thomas M. Gatterman, Justin A. Gerber, Kevin Gilmore, Dan Gresh, Aaron Hankin, Nathan Hewitt, Chandler V. Horst, Mitchell Matheny, Tanner Mengle, Brian Neyenhuis, Ashvin Vishwanath, Michael Foss-Felg, Ruben Verresen, Henrik Dreyer

Quantum systems evolve in time in one of two ways: through the Schrödinger equation or wavefunction collapse. So far, deterministic control of quantum many-body systems in the lab has focused on the former, due to the probabilistic nature of measurements. This imposes serious limitations: preparing long-range entangled states, for example, requires extensive circuit depth if restricted to unitary dynamics. In this work, we use mid-circuit

$$|\psi_{\text{init}}\rangle =$$



$$\sigma^x \sigma^x = 1$$



Non-Abelian states are more difficult to create

The above protocol amounts to:

1. **measuring** e-anyons of Z_2 toric code
2. **pairing** up e-anyons to produce clean state

The latter step crucially relies on there being a simple (Pauli) string operator which creates/annihilates e-anyons

This set-up **fails for non-Abelian** states!
Non-Abelian anyons cannot be paired up
by **finite-depth** unitary string operator (Shi, '19)

What states can be efficiently created by measurement?

We found a broad **generalization**
beyond toric-code-type states!
Includes certain **non-Abelian** states!

Tantivasadakarn, Thorngren, Vishwanath, RV, arxiv:2112.01519



Nat
Tantivasadakarn

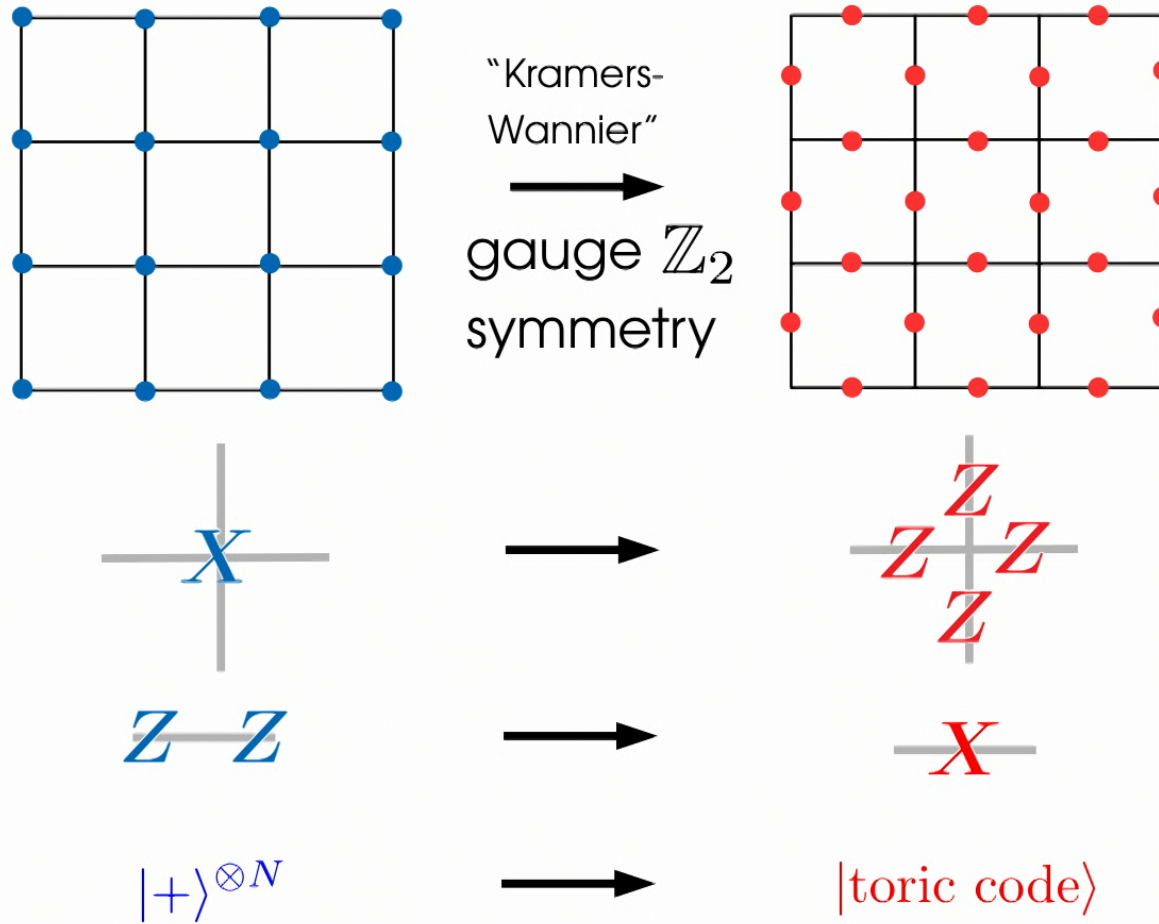
Ryan
Thorngren

Ashvin
Vishwanath

Measurements+feedforward can efficiently
implement the **gauging map** (Kogut 1979)
(Sometimes also called 'Kramers-Wannier transformation')

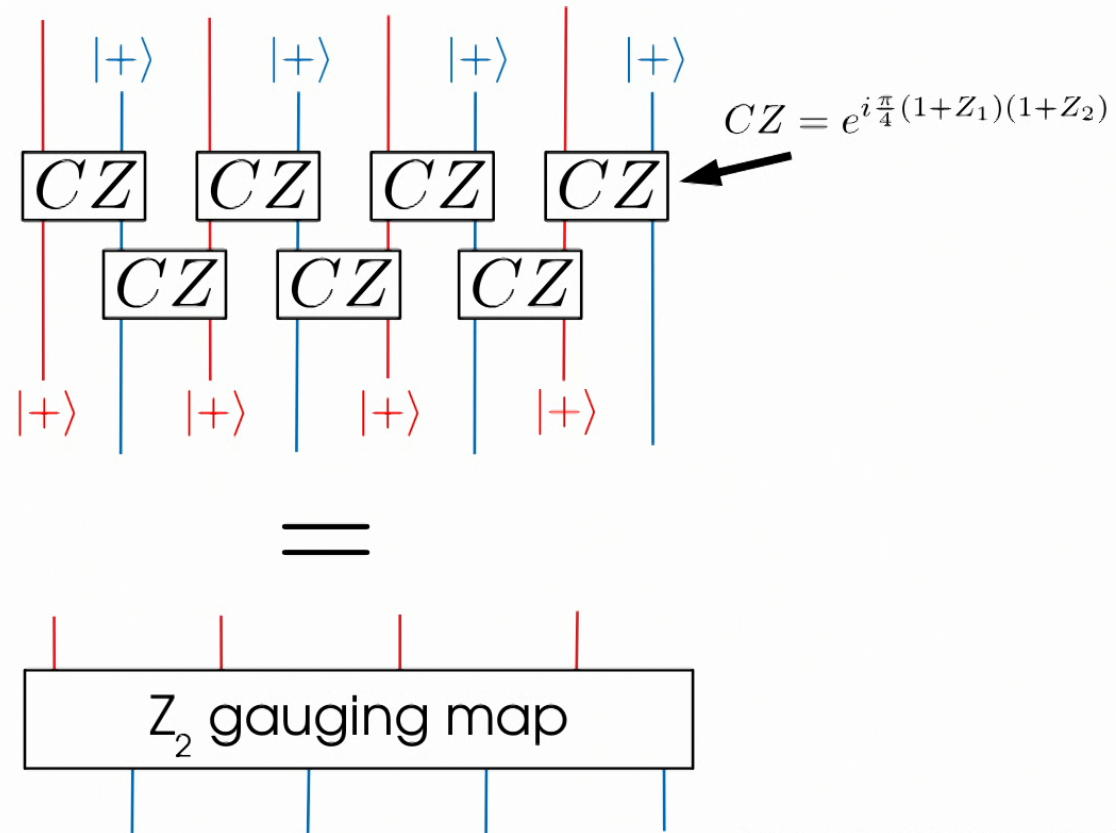
Example of '**many-body quantum teleportation**'

An example of a 'gauging map'



Implementing the gauging map efficiently!

Nat Tantivasadakarn, Ryan Thorngren, Ashvin Vishwanath, **RV**, arxiv:2112.01519

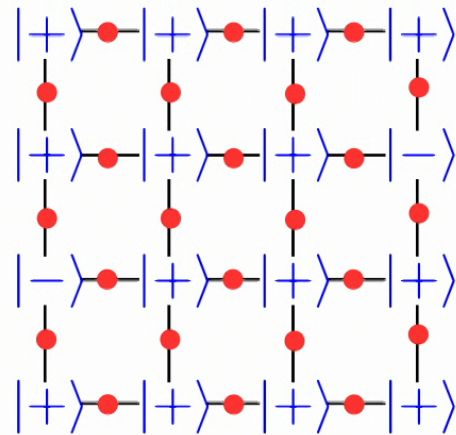


(see also: S. Ashkenazi, E. Zohar,
arxiv:2111.04765)

(for any Abelian group we can efficiently fix measurement outcomes!)

Example: gauging map on the square lattice

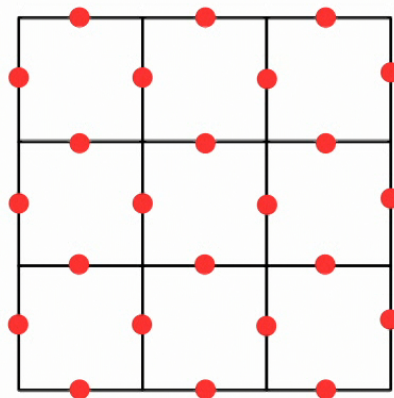
Nat Tantivasadakarn, Ryan Thorngren, Ashvin Vishwanath, RV, arxiv:2112.01519



Step 3: measure vertices

Example: gauging map on the square lattice

Nat Tantivasadakarn, Ryan Thorngren, Ashvin Vishwanath, RV, arxiv:2112.01519

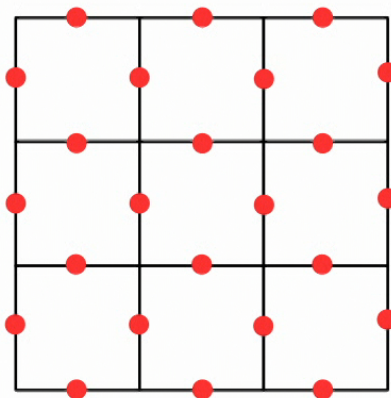


Result: deterministic **gauging** of input state!

Special case: if input state = product state ,
we recover the known toric code protocol (Raussendorf et al. '05)

Example: gauging map on the square lattice

Nat Tantivasadakarn, Ryan Thorngren, Ashvin Vishwanath, RV, arxiv:2112.01519



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State preparation of non-Abelian topological states

Non-Abelian anyons allow for **universal** quantum computation!

(Kitaev '97, Mochon '03)

Were believed to be **inaccessible** via measurement!



State preparation of non-Abelian topological states

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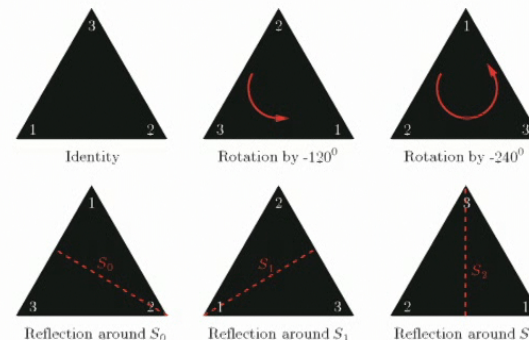
(Kitaev '97, Mochon '03)

Were believed to be **inaccessible** via measurement!

Multiple applications of our protocol can create them!

Intuition: $S_3 = \mathbb{Z}_3 \rtimes \mathbb{Z}_2$

(Follow-up work by Bravyi et al, arXiv:2205.01933 extends this logic to efficiently moving anyons!)



State preparation of non-Abelian topological states

Non-Abelian anyons allow for **universal** quantum computation!

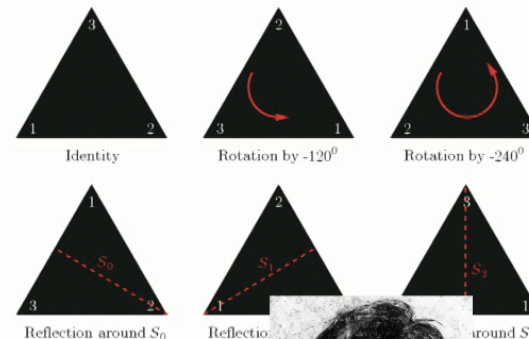
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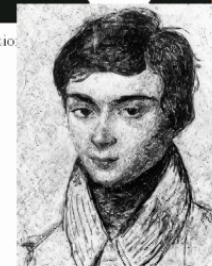
We argue it is **impossible** to prepare non-Abelian states associated to **non-solvable** symmetry groups!



For possible no-go results, see:

Nat Tantivasadakarn, **RV**, Ashvin Vishwanath, arXiv:2209.06202

However, see Lu, Lessa, Kim, Hsieh, PRX Quantum (2022)
for log-depth protocols for non-solvable cases

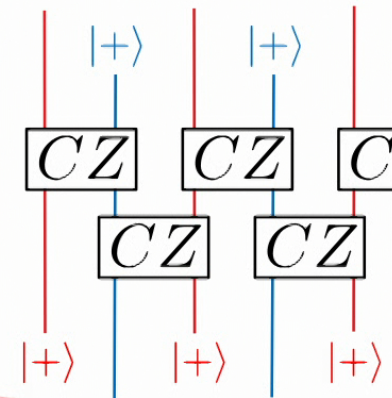


Outline

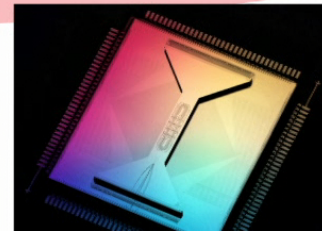
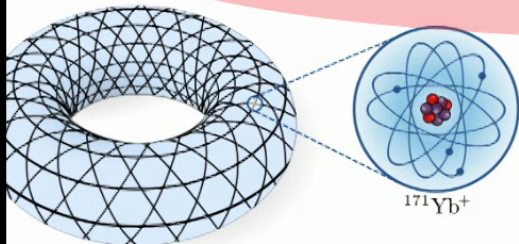
$$|\psi\rangle = \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|c|c|} \hline \text{red} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \text{black} & \text{black} & \text{black} & \text{black} \\ \hline \end{array} \right\rangle + \dots$$

✓ 1) Topological Order

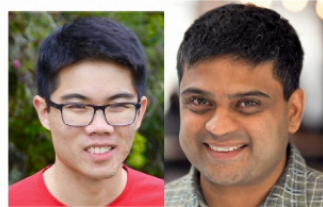
✓ 2) Measurement-based Protocols



3) Non-Abelions in a Trapped-Ion Processor



Implementation on Quantinuum's trapped ion processor



Nat Tantisadakarn
Ashvin Vishwanath

+

Mohsin Iqbal



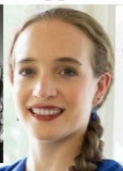
Sara Campbell



Joan Dreiling



Caroline Figgatt



John Gaebler



Jacob Johansen



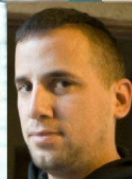
Michael Mills



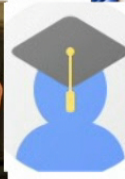
Steven Moses



Juan Pino



Anthony Ransford



Mary Rowe



Peter Siegfried



Russell Stutz

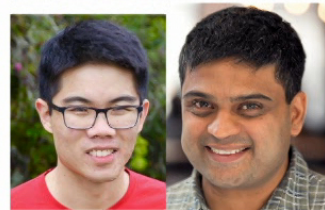


Michael Foss-Feig



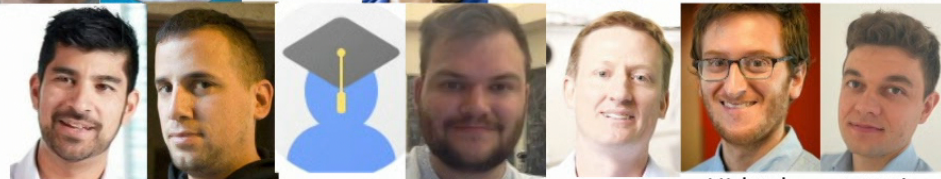
Henrik Dreyer

Implementation on Quantinuum's trapped ion processor



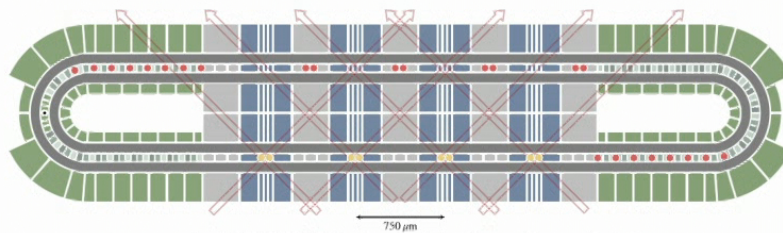
Nat Tantisadakarn
Ashvin Vishwanath

+

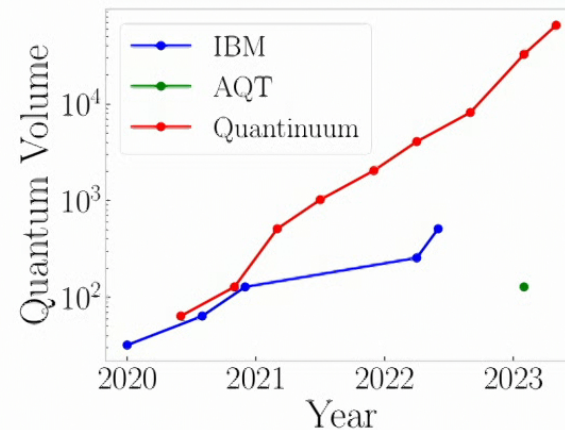


Juan Pino
Anthony Ransford
Mary Rowe
Peter Siegfried
Russell Stutz
Michael Foss-Feig
Henrik Dreyer

- 32 qubits
- Gate fidelity 99.8%
- Measurement and feed-forward
- All-to-all connectivity



arxiv:2305.03828 (graphic from Foss-Feig)

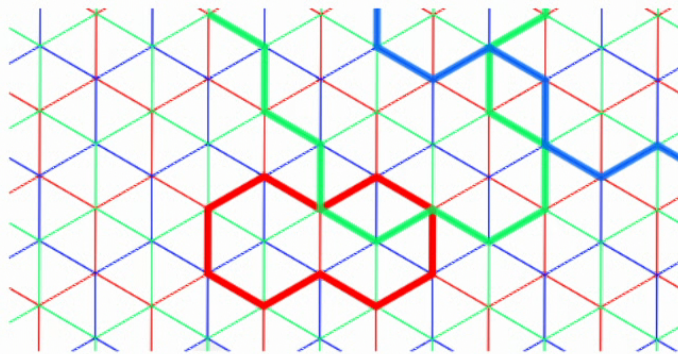


Simplest non-Abelian topological order

We can create non-Abelian topological order with D_4 gauge group by applying our gauging map to a “ \mathbb{Z}_2^3 SPT phase”

See also: Yoshida (2016)

There is simple loop model picture: consider three loop models on red, green, blue honeycomb lattices



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See also: Yoshida (2016)

There is simple loop model picture: consider three loop models on red, green, blue honeycomb lattices

$$|\psi\rangle = \left| \begin{array}{c} \text{red, green, blue loops on honeycomb lattice} \end{array} \right\rangle - \left| \begin{array}{c} \text{red, green, blue loops on honeycomb lattice} \end{array} \right\rangle + \dots$$

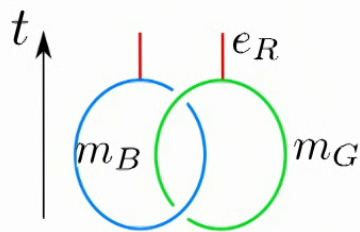
Coupled: blue resonance is negative if red and green intersect!

Simplest non-Abelian topological order

$$|\psi\rangle = \left| \begin{array}{c} \text{Diagram 1: A triangular lattice with red and green strings forming a path through a central shaded hexagon.} \end{array} \right\rangle - \left| \begin{array}{c} \text{Diagram 2: A triangular lattice with red, green, and blue strings forming a path through a central shaded hexagon.} \end{array} \right\rangle + \dots$$

Breaking strings open creates non-Abelian anyons: m_R, m_G, m_B

Plaquette resonance defines Abelian anyons: e_R, e_G, e_B

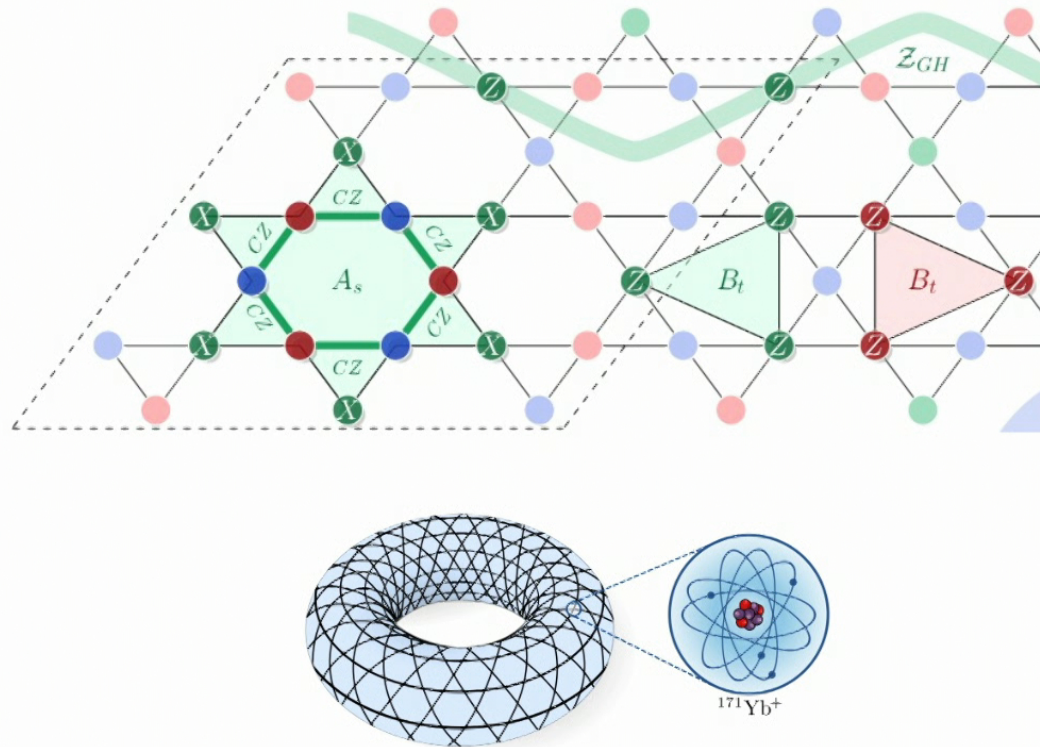


Braiding two of the fluxes
creates a charge for the third color!

Fusion rules: $m_R \times m_R = 1 + e_G + e_B + e_G e_B$

Kagome torus geometry on ion trap

These coupled honeycomb loop models correspond to qubits on a kagome lattice:



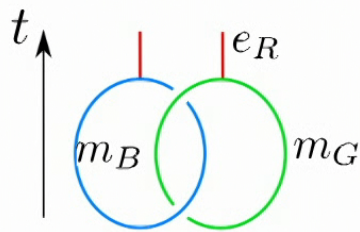
lqbal et al, arxiv:2305.03766

Simplest non-Abelian topological order

$$|\psi\rangle = \left| \begin{array}{c} \text{Diagram 1: A 3D-like lattice with red, green, and blue strings. A central cube is highlighted in grey, with red strings forming its edges.} \end{array} \right\rangle - \left| \begin{array}{c} \text{Diagram 2: Similar lattice, but the central cube is formed by blue strings.} \end{array} \right\rangle + \dots$$

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Plaquette resonance defines Abelian anyons: e_R, e_G, e_B

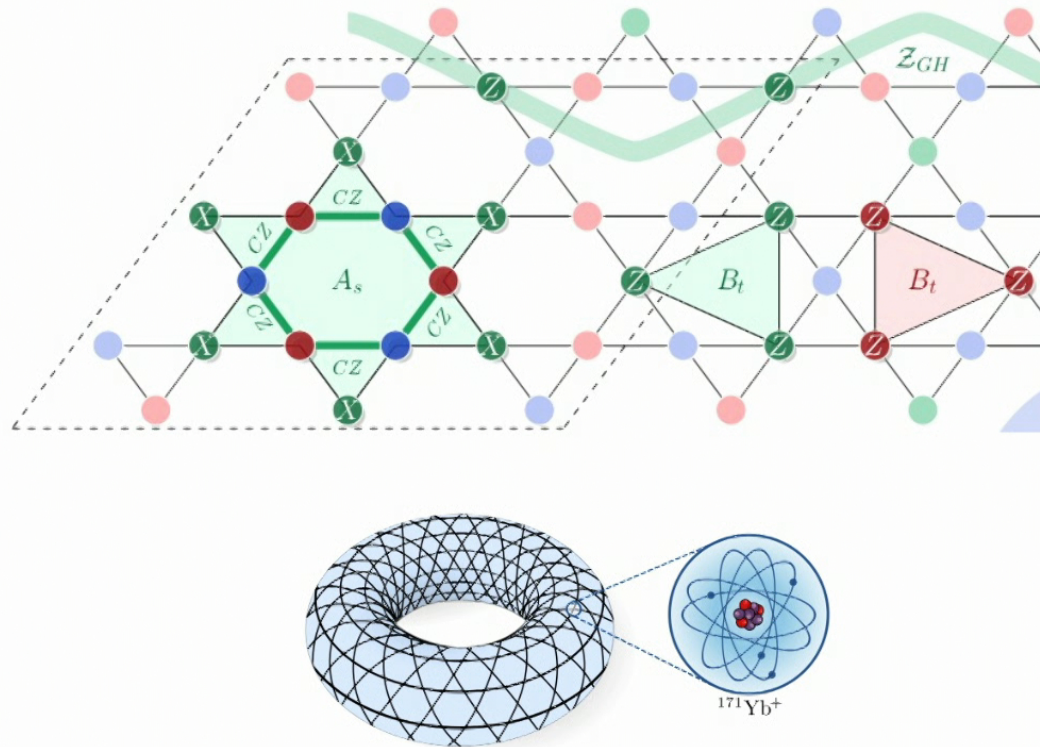


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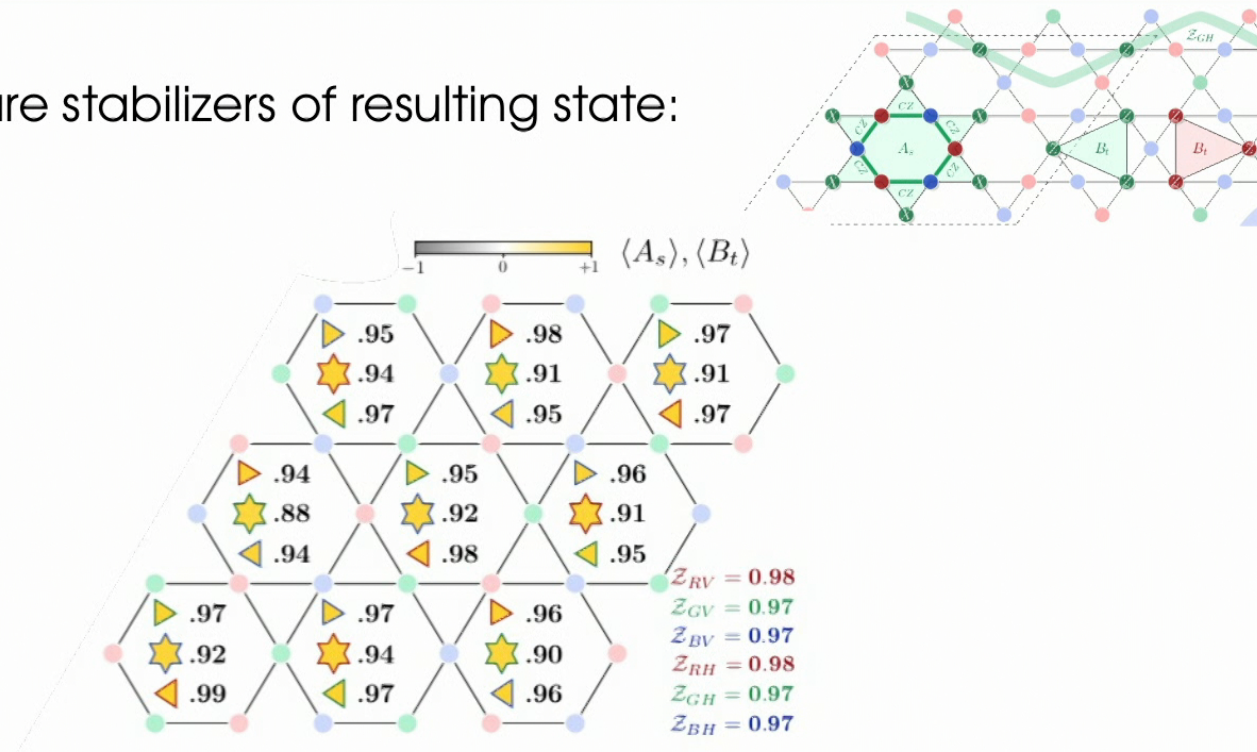
These coupled honeycomb loop models correspond to qubits on a kagome lattice:



lqbal et al, arxiv:2305.03766

Preparing non-Abelian state on cold-ion trap

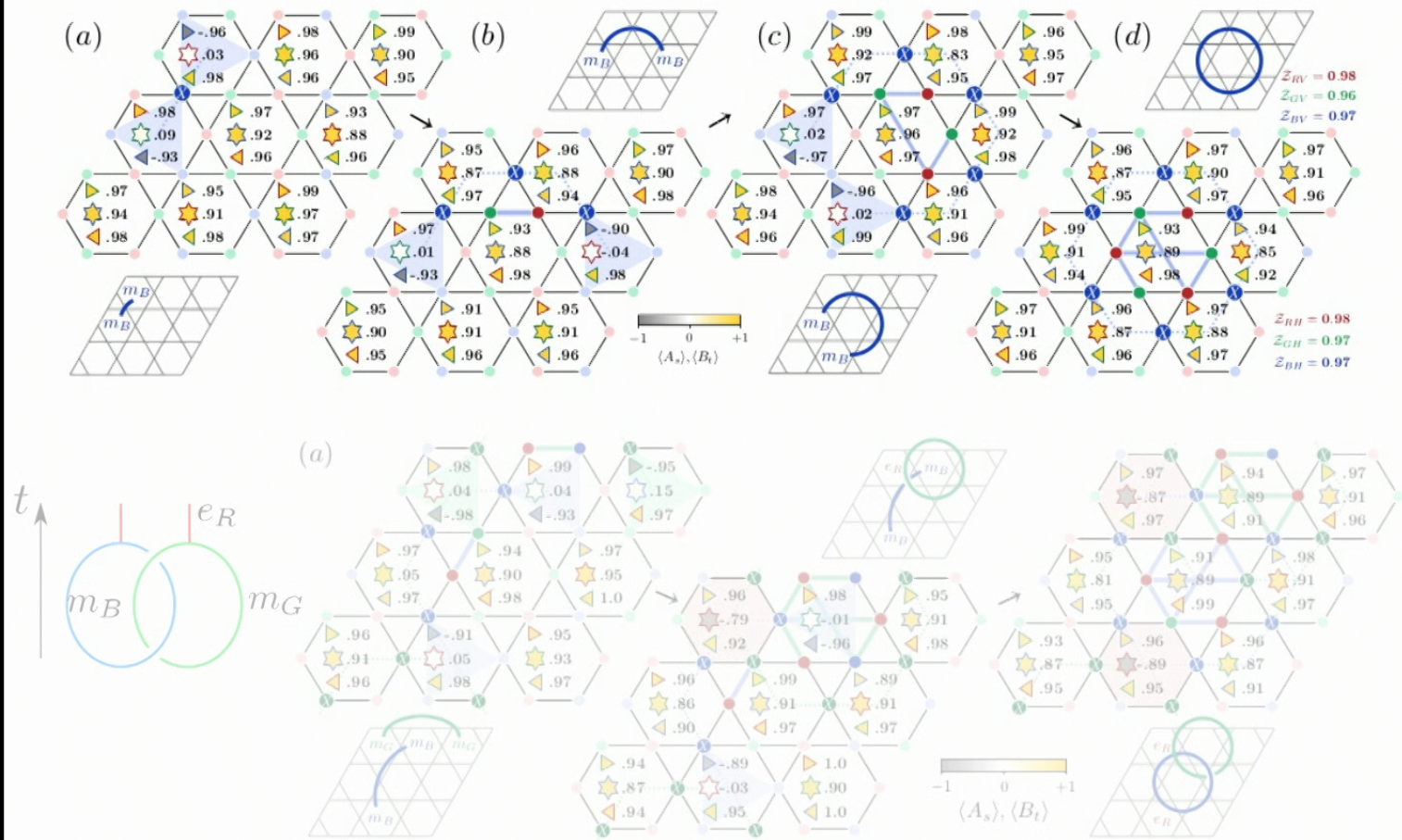
Measure stabilizers of resulting state:



Global fidelity with ideal state lower bounded by 75% !

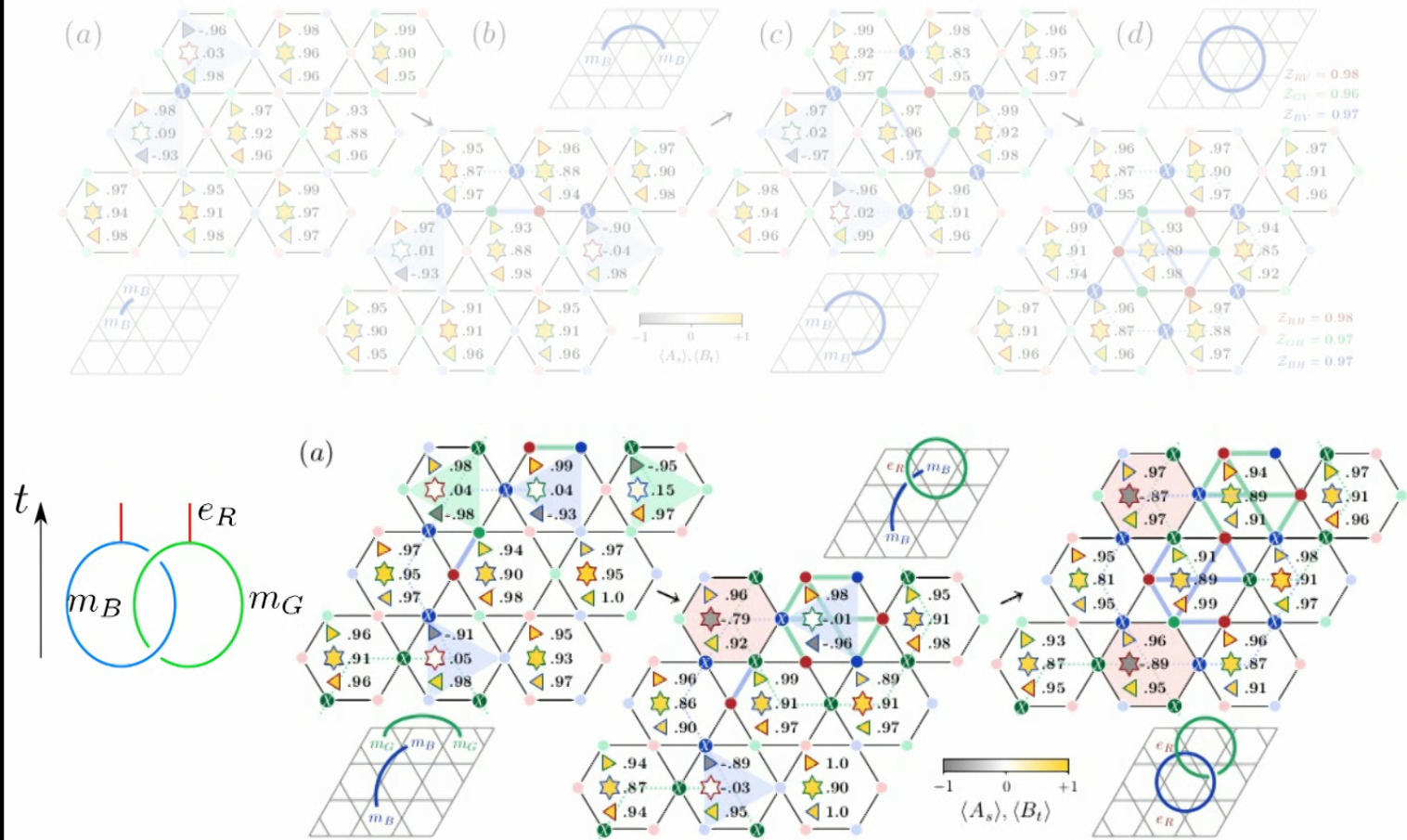
Iqbal et al, arxiv:2305.03766

Creating, braiding and annihilating non-Abelions



lqbal et al, arxiv:2305.03766

Creating, braiding and annihilating non-Abelions



lqbal et al, arxiv:2305.03766

Non-Abelions have a PhD in knot theory

Abelian anyons are only sensitive to pairwise **linking**:


$$m \text{ and } e \text{ linked} = -1$$

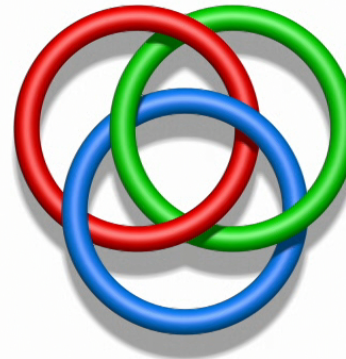
lqbal et al, arxiv:2305.03766

Non-Abelions have a PhD in knot theory

Abelian anyons are only sensitive to pairwise **linking**:

$$\text{link}(m, e) = -1$$


But there exist non-trivial knots without pairwise linking!



Iqbal et al, arxiv:2305.03766

Conclusions and Collaborators

Using **measurement** opens door to preparing exotic states!

→ achieved in **experiment** (so far: toric code and D_4 order)

→ we also have proposals for more **exotic** states (e.g., S_3 order)

Open questions and next steps:

→ link between computational **power** and difficulty in preparing?

→ **no-go** proofs for states we cannot create?

→ active error correction and topological **qubits**!

→ non-Abelian **defects** (Anderson et al, Nature (2023)): difference for computation?

