Title: Gravity from chiral algebras

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Series: Mathematical Physics

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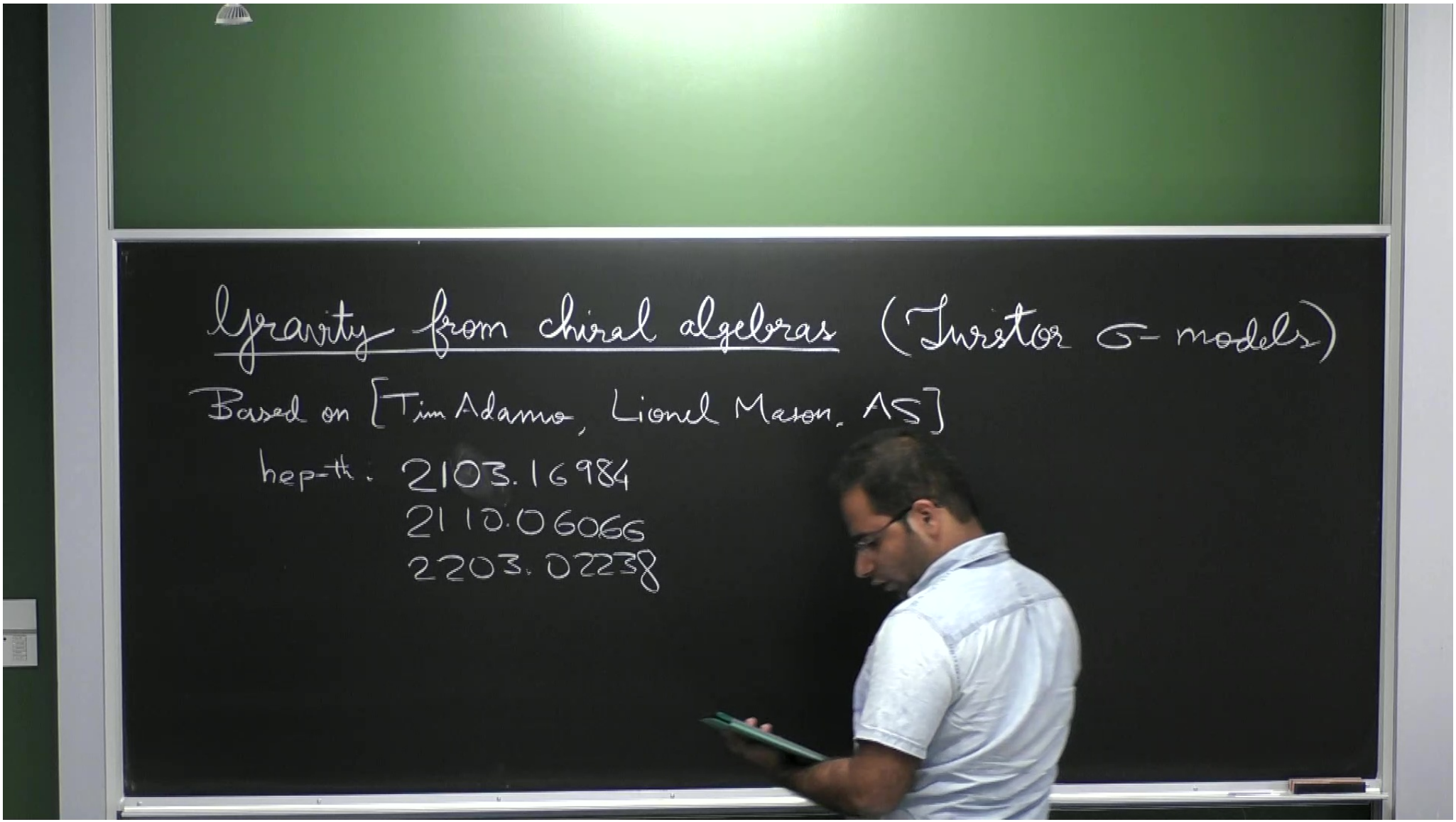
Abstract: I will describe a correspondence between hyperkähler/quaternion-Kähler geometry and two dimensional chiral algebras that arises from twistor theory. As an application, I will explain how to use correlators of these chiral algebras to compute gravitational scattering amplitudes in four dimensional general relativity.

Zoom Link: https://pitp.zoom.us/j/98706048161?pwd=a3k1U3l2UXJlUjlqUElkOEZhZGwwUT09


Gravity from chiral algebras (Twistor σ-models)
Based on [Tim Adamo, Lionel Mason, AS]
hep-th: 2103.16984
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Summary: Hyperkähler (HK) or Quaternion-Kähler (QK) $\longleftrightarrow$ 2d chiral algs living in twistor space

Punchline 1: 2d On-shell actions = Kähler pot. (HK)
Swann scalars (QK)

Punchline 2: 2d corr. = tree level graviton MHV amps.

Review of twistor thy.

$4k$ dim. Riemannian (M, g), $k \geq 1$

Manifold	Holonomy	Curvature	4d
HK	$Sp(k)$	$Ric = 0$	Self-dual vacuum
QK	$\frac{Sp(k) \times Sp(1)}{\mathbb{Z}_2}$	$Ric = \Lambda g$	Self-dual Einstein

These are encoded in twistor thy

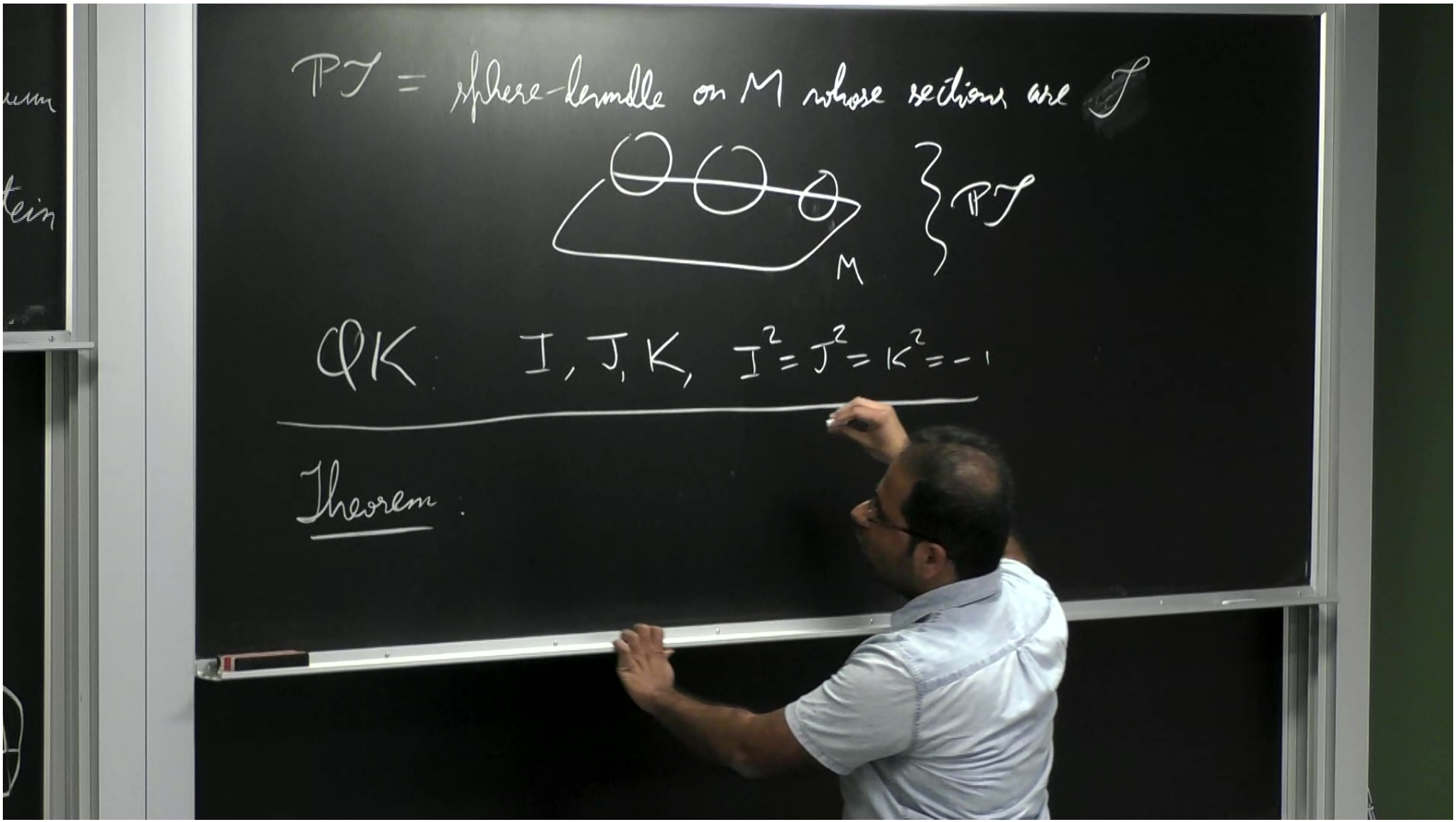
$$\boxed{\begin{array}{ccc} 4k \text{ real dim'l} & \longleftrightarrow & 2k+1 \text{ dim'l complex mani.} \\ HK/QK\ M & & \text{twistor space } \mathbb{P}\mathcal{Z} \end{array}}$$

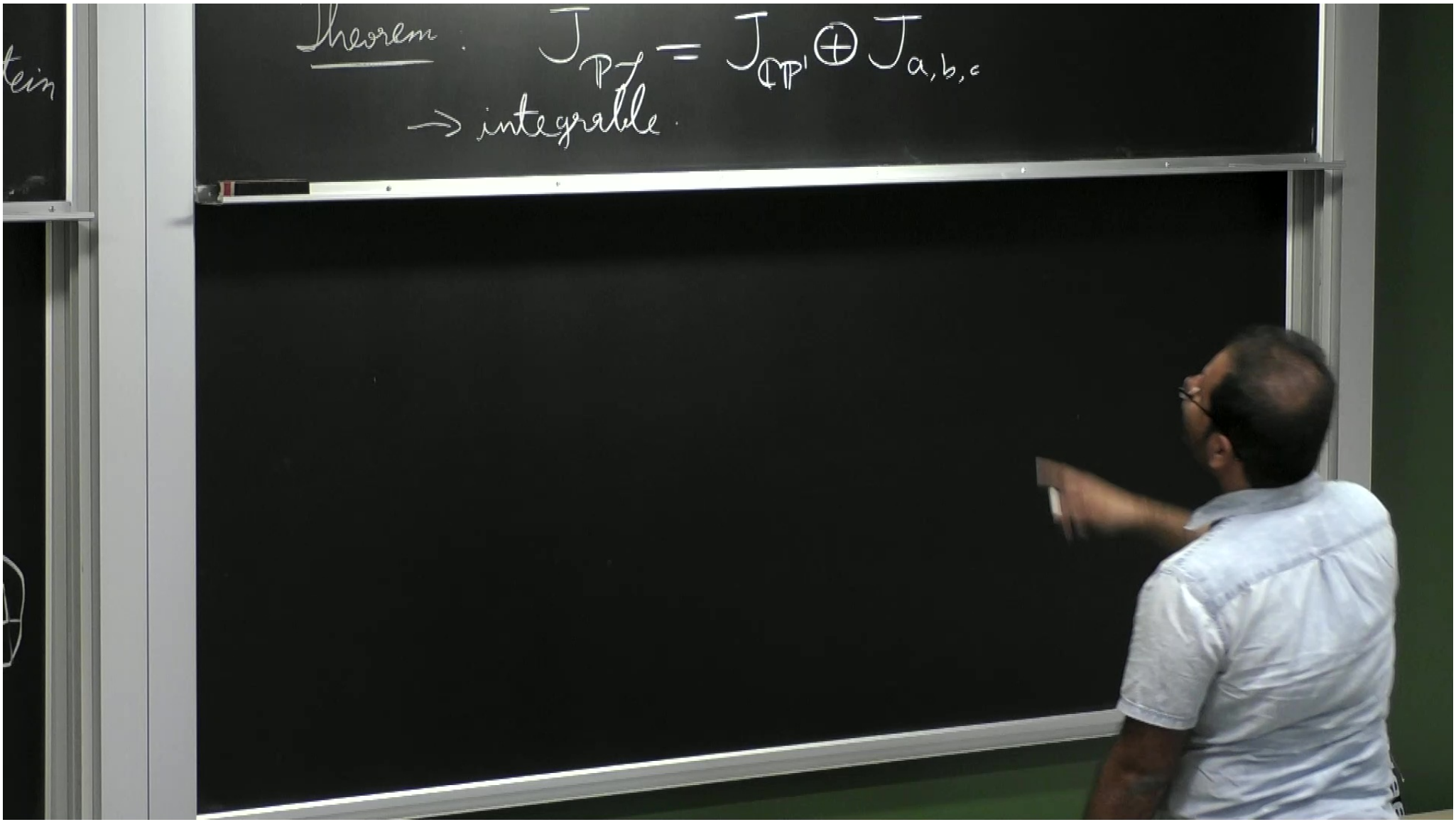
<u>HK</u>. $I, J, K \longrightarrow$ quaternions

$$I^2 = J^2 = K^2 = IJK = -1$$

$(a,b,c) \in \mathbb{R}^3$, $a^2+b^2+c^2=1$

$$J = aI + bJ + cK$$


$\mathbb{P}\mathcal{J}$ = sphere-bundle on M whose sections are $\mathcal{J}$
M
$\mathbb{P}\mathcal{J}$
$\mathbb{Q}K$: I, J, K, $I^2 = J^2 = K^2 = -1$
Theorem :


Theorem: $J_{\mathbb{P}^1} = J_{\mathbb{CP}^1} \oplus J_{a,b,c}$
$\rightarrow$ integrable.

Data on M	Data on $\mathbb{P}\mathcal{Y}$
Points $x \in M$	Rational curves $L_x \subset \mathbb{P}\mathcal{Y}$ with normal bundle $\mathcal{O}(1)^{\oplus 2k}$
HK metric	$\mathbb{P}\mathcal{Y} \longrightarrow \mathbb{CP}^1$ with a fiberwise $K_{\mathbb{P}\mathcal{Y}}^{-1/2}$-valued symplectic form Σ
CK metric	Contact form $\tau \in K_{\mathbb{P}\mathcal{Y}}^{-1/2}$

$\Rightarrow$ maps of conformal weight $(\frac{1}{2}, 0)$ or $\mathcal{O}(-1)$-valued

$v^i(\sigma) : \mathbb{CP}^1 \longrightarrow \mathbb{PT}$ $\leftarrow v^i$ coords

σ

Ward, Penrose

<u>QK case</u>: $$S = \frac{1}{\Lambda} \int_{\mathbb{CP}^1} d\sigma \wedge \tau$$ $\leftarrow$ pullback to $\mathbb{CP}^1$

<u>HK case</u> $$\Sigma = d_2 \Theta$$

$$S = \int d\sigma \wedge \Theta$$

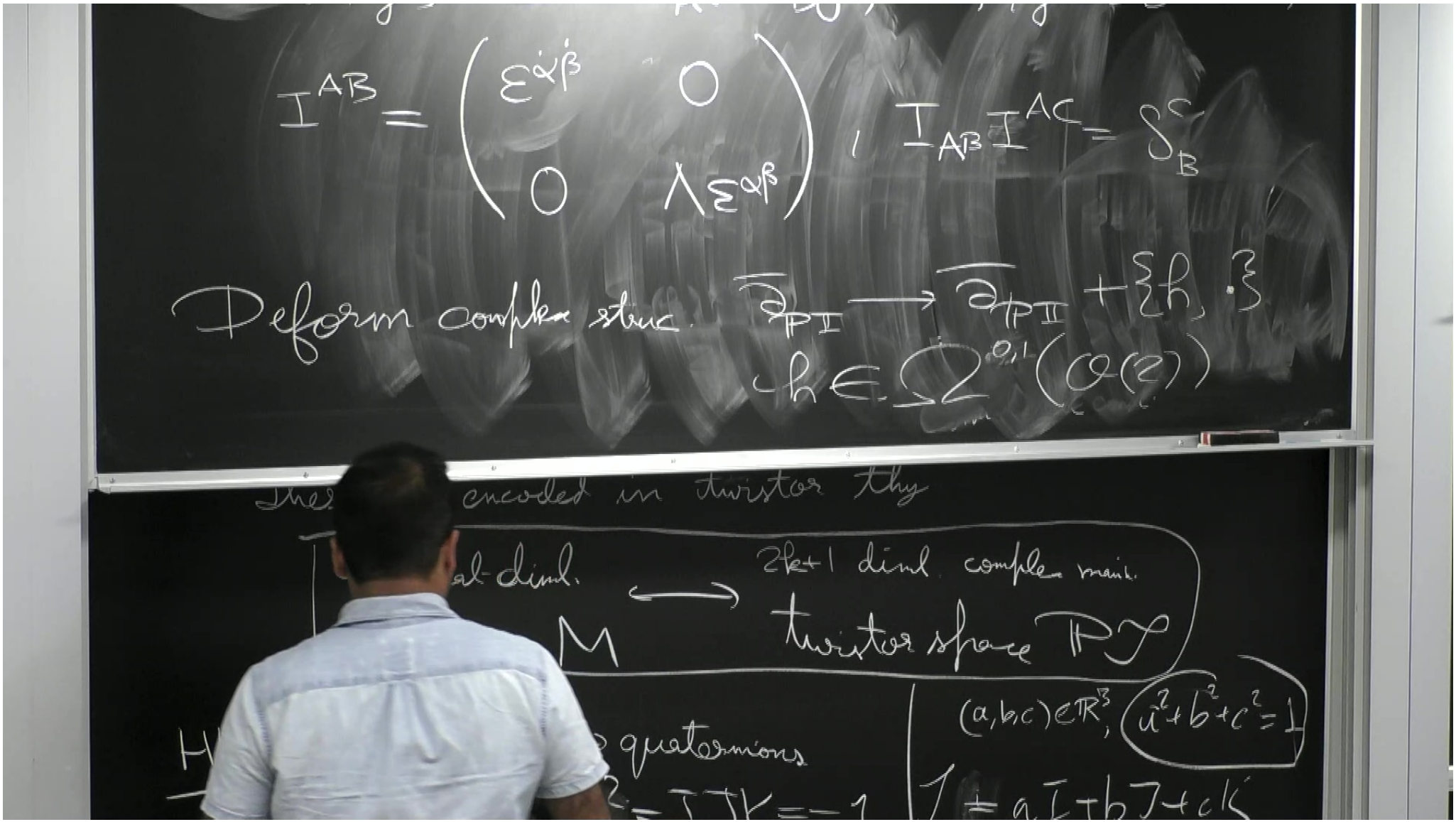
$$S = \int d\sigma \wedge \Omega^{k}$$

Flat space $(\mathbb{R}^{4k}, \delta)$

$$\mathbb{PT} = \mathbb{CP}^{2k+1} - \mathbb{CP}^{2k-1} \simeq \mathcal{O}(1)^{\oplus 2k} \longrightarrow \mathbb{CP}^{1}$$

$$Z^{A} = (\mu^{\dot\alpha}, \lambda_{\alpha})$$

$A = 1, \ldots, 2k+2 \quad \dot\alpha = 1, \ldots, 2k$


$I^{AB} = \begin{pmatrix} \varepsilon^{\dot\alpha\dot\beta} & 0 \\ 0 & \Lambda\varepsilon^{\alpha\beta} \end{pmatrix}$, $I_{AB} I^{AC} = \delta^C_B$
Deform complex struc. $\bar\partial_{PII} \longrightarrow \bar\partial_{PII} + \{h, \cdot\}$
$h \in \Omega^{0,1}(\mathcal{O}(2))$
encoded in twistor thy
2k+1 diml complex mani.
M
twistor space $\mathbb{P}^\infty$
quaternions
$(a,b,c) \in \mathbb{R}^3$, $a^2+b^2+c^2=1$
$J = aI + bJ + cK$

Deform complex struc. $\bar\partial_{\mathbb{P}^{II}} \longrightarrow \bar\partial_{\mathbb{P}^{II}} + (h, \cdot)$

$h \in \Omega^{0,1}(\mathcal{O}(2))$

$\mathcal{O}(2)$-valued

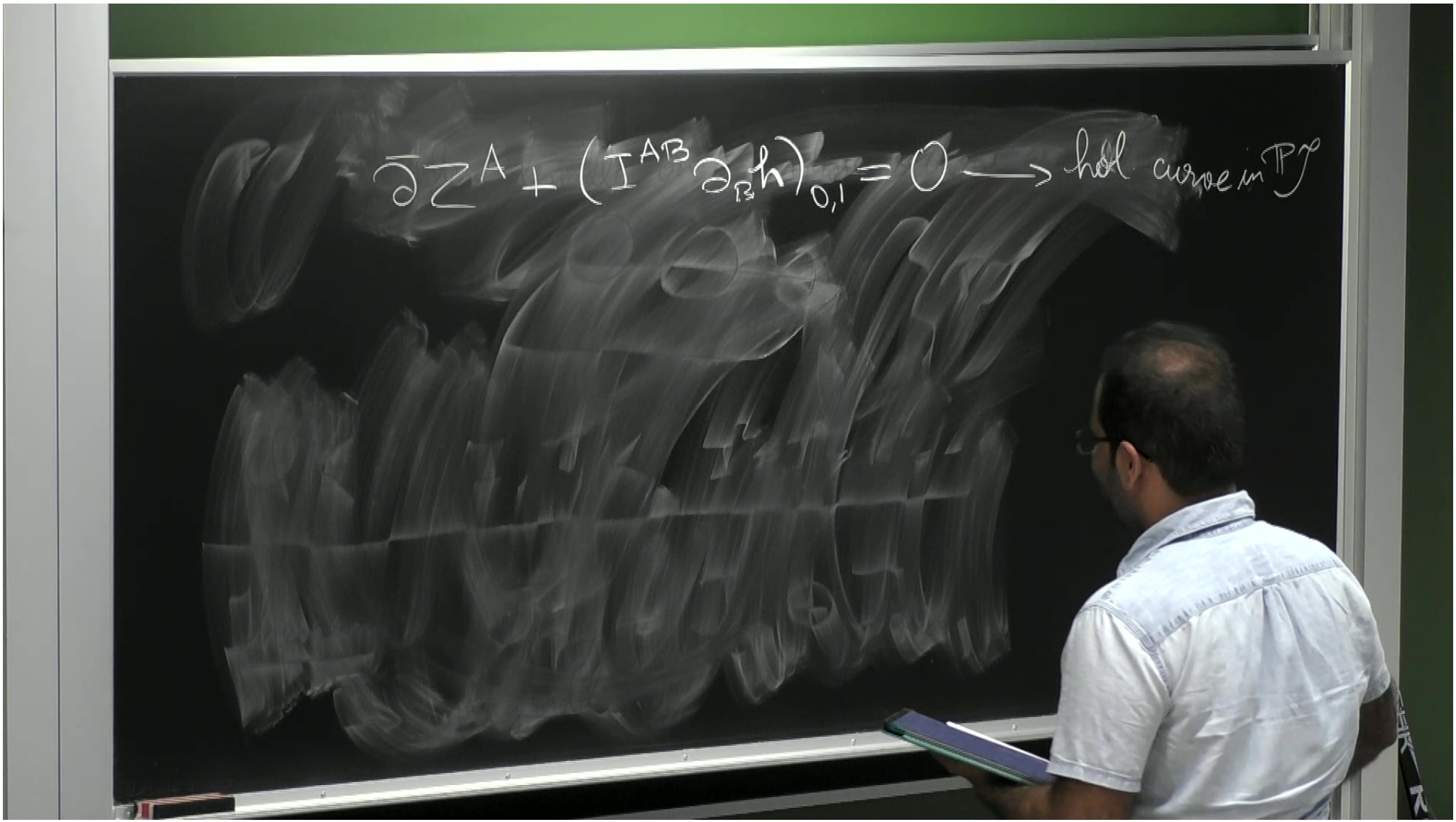
$$(\bar\partial_{\mathbb{P}^{II}} + h)^2 = 0 \iff \tau = \frac{1}{2} I_{AB} \tau^A d\tau^B - h$$

is holomorphic

$\tau^A(\sigma)$

$$\boxed{S = \frac{1}{2\lambda} \int d\sigma \wedge \left(I_{AB} \tau^A \bar\partial \tau^B - h \right)}$$

QK


$\bar{\partial} Z^A + (I^{AB} \partial_B h)_{0,1} = 0 \longrightarrow$ hol. curve in $\mathbb{P}^{\infty}$

HK case: $\Lambda \longrightarrow 0$

$$S = \int d\sigma \left(\frac{1}{2\Lambda} \varepsilon^{\alpha\beta} \lambda_\alpha \bar{\partial} \lambda_\beta + \frac{1}{2} \varepsilon_{\dot\alpha\dot\beta} \mu^{\dot\alpha} \bar{\partial} \mu^{\dot\beta} - h \right)$$

$$\Lambda \to 0 \Rightarrow \bar{\partial} \lambda_\beta = 0 \Rightarrow \lambda_\beta = \left(\frac{1}{\sigma}, 1 \right)$$

$$S = \int d\sigma \left(\frac{1}{2} \varepsilon_{\dot\alpha\dot\beta} \mu^{\dot\alpha} \bar{\partial} \mu^{\dot\beta} - h \right)$$

$K_{\mathbb{P}^1}$-valued symplectic form Σ

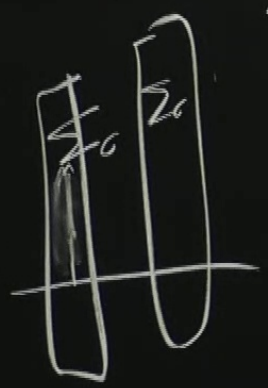
Contact form $\tau \in K_{\mathbb{P}^1}^{-1/2}$

CK metric

$$\Lambda \to 0 \Rightarrow \bar\partial \lambda_\beta = 0 \Rightarrow \lambda_\beta = \left(\frac{1}{\sigma}, 1\right)$$

$$S = \int d\sigma \left(\frac{1}{2} \Sigma_{\alpha\beta} \mu^\alpha \bar\partial \mu^\beta - h\right) \qquad D\sigma \wedge Z^A(\sigma) \bar\partial Z^B(\sigma)$$

Hitchin. $K_{\mathbb{P}\mathcal{Y}}^{1/2}$ exists



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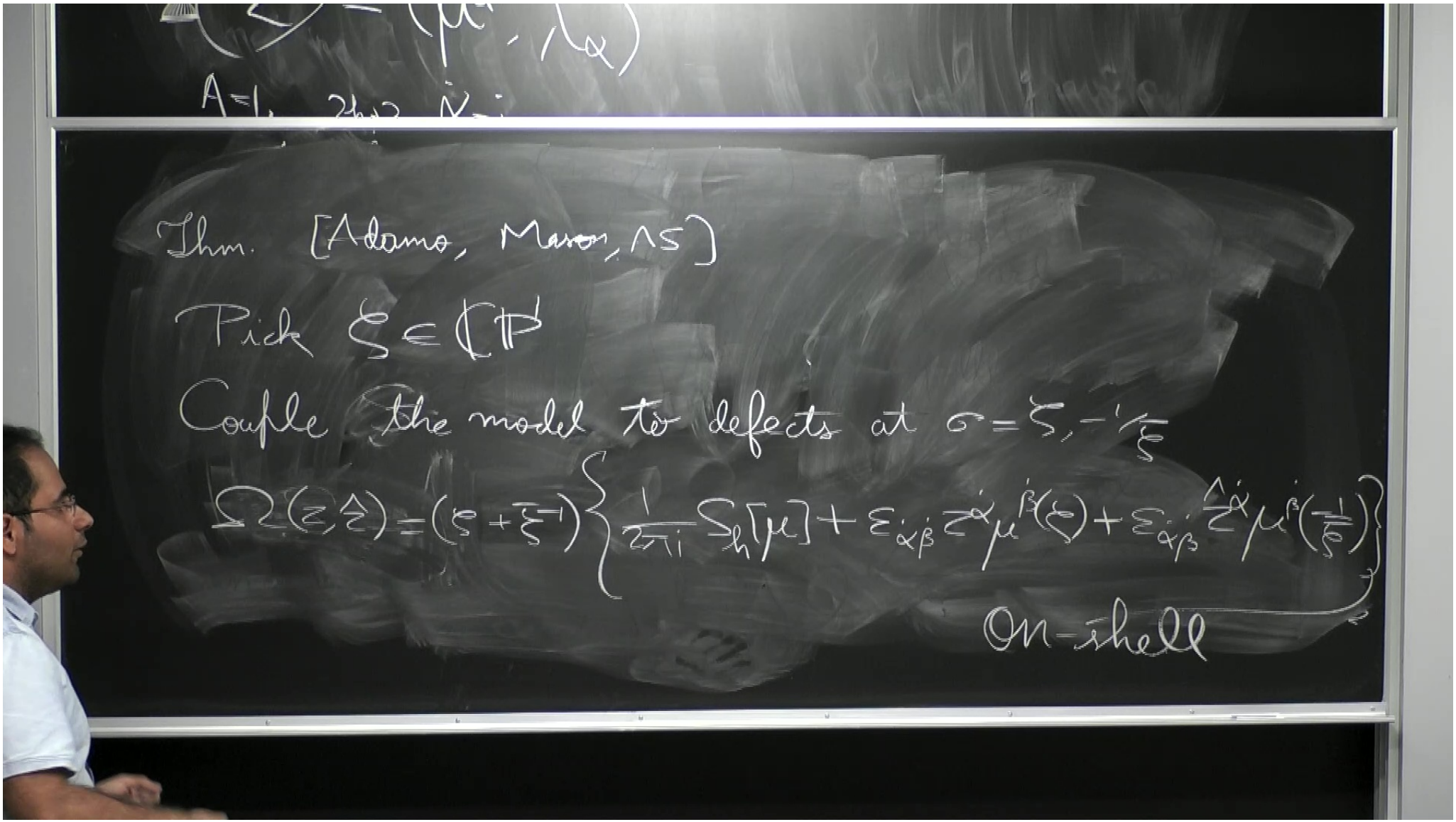

Thm. [Adamo, Mason, AS]
Pick $\xi \in \mathbb{CP}^1$
Couple the model to defects at $\sigma = \xi, -\frac{1}{\xi}$
$\Omega(z,\hat{z}) = (\xi + \xi^{-1})\left\{ \frac{1}{2\pi i} S_h[\mu] + \varepsilon_{\dot\alpha\dot\beta} z^{\dot\alpha} \mu^{\dot\beta}(\xi) + \varepsilon_{\dot\alpha\dot\beta} \hat{z}^{\dot\alpha} \mu^{\dot\beta}\left(-\frac{1}{\xi}\right) \right\}$
On-shell

$$\Omega(z,\hat{z}) = (\zeta + \bar{\zeta}^{-1})\left\{ \frac{1}{2\pi i} S_h[\mu] + \varepsilon_{\dot\alpha\dot\beta} z^{\dot\alpha} \mu^{\dot\beta}(\zeta) + \varepsilon_{\dot\alpha\dot\beta} \hat{z}^{\dot\alpha} \mu^{\dot\beta}\left(-\frac{1}{\bar\zeta}\right) \right\}$$

Kahler potential for Ric = 0 metric ON-shell

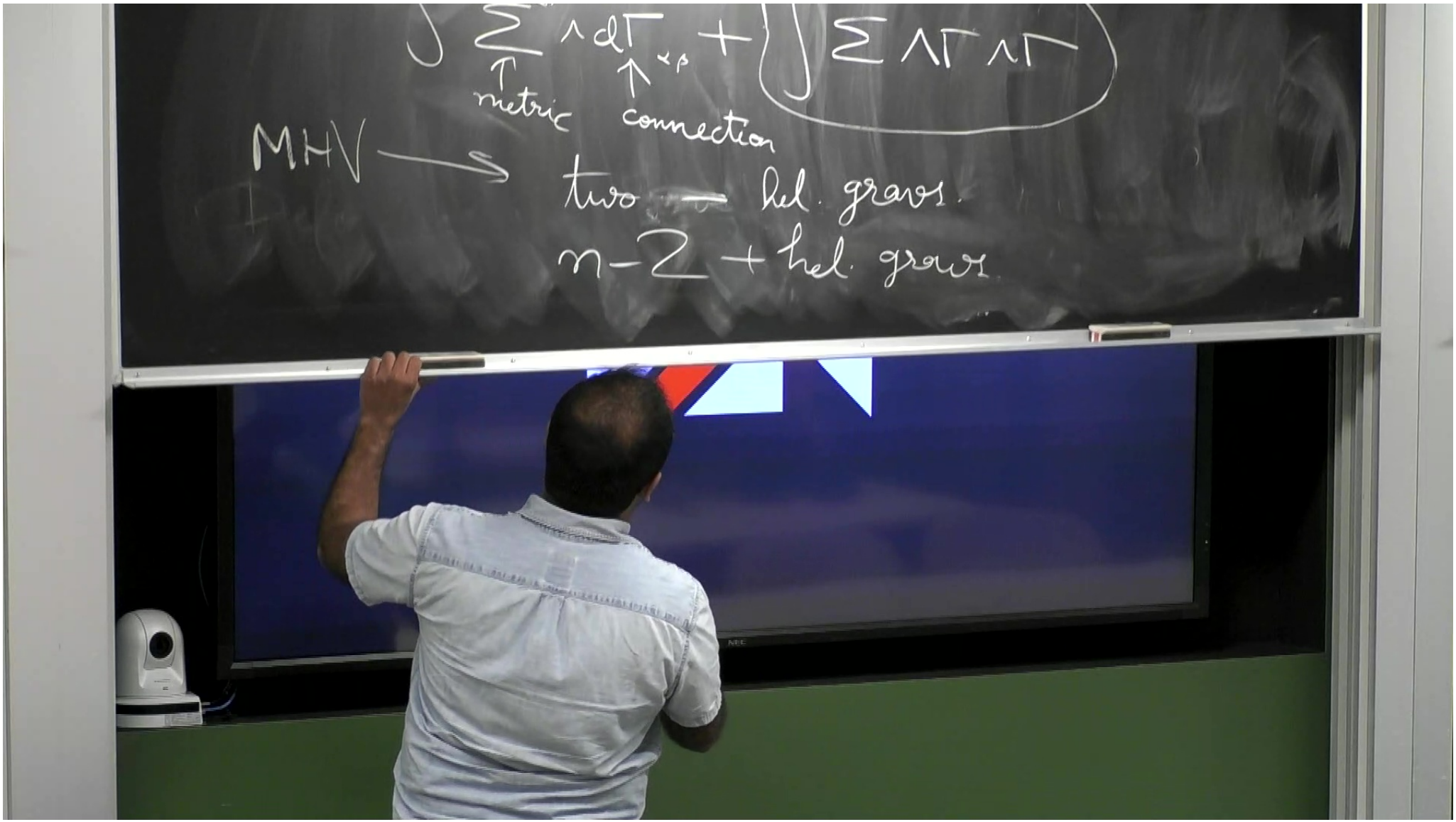
$$g = \partial\bar\partial\Omega, \quad \partial = dz^{\dot\alpha}\frac{\partial}{\partial z^{\dot\alpha}}, \quad \hat{z}^{\dot\alpha} = (\bar{z}\cdot\varepsilon)^{\dot\alpha}$$

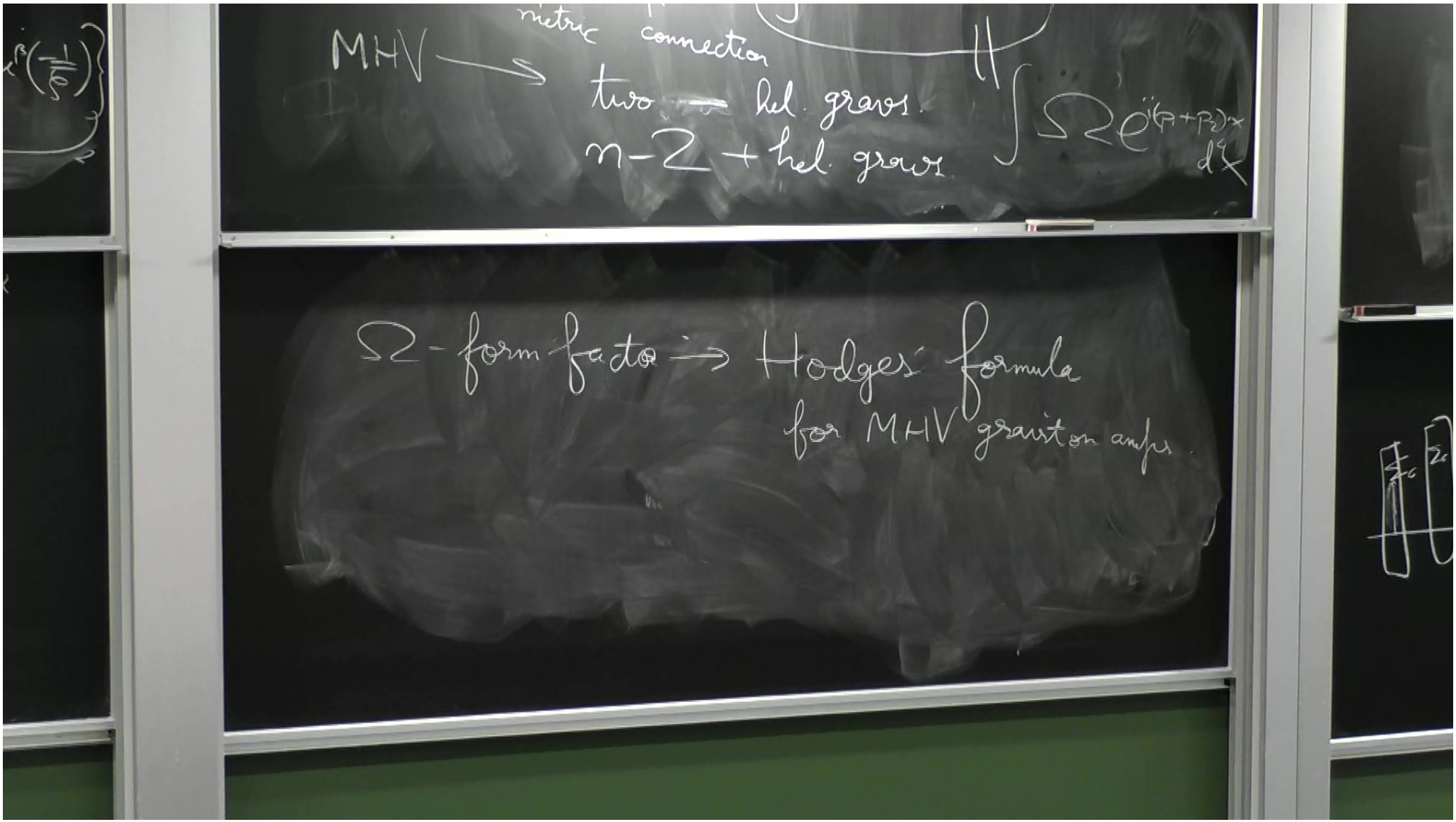
Example: $\mathbb{R}^{4+k}$, $\Omega = \|z\|^2$

$$S_{h=0} = \int d\sigma \, \frac{1}{2} \mu^{\dot\alpha} \bar\partial \mu^{\dot\beta} \varepsilon_{\dot\alpha\dot\beta}$$

eom. $\bar\partial \mu^{\dot\alpha} = z^{\dot\alpha}\delta(\sigma - \zeta) + \hat{z}^{\dot\alpha}\delta(\sigma + \bar\zeta^{-1})$

$$\Rightarrow \mu^{\dot\alpha}(\sigma) = \frac{z^{\dot\alpha}}{\sigma - \zeta} + \frac{\hat{z}^{\dot\alpha}}{\sigma + \bar\zeta^{-1}}$$


metric
connection
MHV
two hel. gravs.
n-2 + hel. gravs


metric connection
MHV
two hel. gravs.
n-2 + hel. gravs
Ω - form factor → Hodges' formula
for MHV graviton amps.