Title: Analogical reasoning in quantum gravity

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Abstract: Analogical reasoning has a long and distinguished history as a method for making discoveries in physics. I will discuss novel uses of formal analogies in twentieth century particle physics and condensed matter physics. I will then offer some reflections on how methodological lessons from these cases could inform the use of analogies in discoveries related to quantum gravity.

Analogical reasoning in quantum gravity

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My Perspective

My research:

- History and Philosophy of Physics
- Foundations of QFT (particle physics)
- Applicability of mathematics
- Analogical reasoning

* *Does not include discovering QG*

Big Picture History and Philosophy of Science Questions

1. Has it taken too long to find QG? Do we need to be patient? (what does that look like?)
2. We believe that a paradigm-shift is needed. Does this involve abandoning our current best theories entirely?
3. Are there *methods* that can be used to make discoveries in physics? *methods* = strategies that produce models/theories that are pursuit-worthy (e.g., worth testing experimentally, developing theoretically)
   - correspondence principles (Smeenk)
     e.g., Lorentz transformations → Galilean transformations
   - unification (Dawid)
   - analogical reasoning
     Examples from this week (lots!):
     a black hole horizon is similar to the cosmological horizon
     analogue experiments (Johnson)
     analytic continution (different examples, some of which supply analogical mappings)

Big Picture History and Philosophy of Science Questions

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2. We believe that a paradigm-shift is needed. Does this involve abandoning our current best theories entirely?
3. Are there *methods* that can be used to make discoveries in physics?
   - analogical reasoning
4. Science is a community-based activity. How to structure scientific communities to encourage diverse membership and diverse ideas, which improves the quality of knowledge produced by scientific communities? (Holbrook)

Has it taken too long to find QG? Do we need to be patient?

We need to be patient. What does that look like?

Experimental side: looking for discrepancies from theoretical predictions using precision tests (Koberinski, Smeenk)

Historical example: Newtonian gravity (Smith, Stein, Smeenk)

Theoretical side: try to use current theories (GR, QFT) to model scenarios at the presumed limits of its applicability and see how it breaks down (Crowther)

Historical example of theoretical development taking a long time: The long history of classical mechanics

1687 Newton, *Principia Mathematica*

1700s attempted applications to other types of interactions and more complex systems; some new principles

c.1800 Lagrange's abstract mathematical development of classical mechanics

Late 1800s Analytical mechanics applied to practical problems

The development of Hamiltonian and Lagrangian mechanics mattered for the theoretical development of QT.

Is further theoretical development of GR, QFT needed as input for theoretical development of QG?

A method for discovery: Analogical reasoning

Idealized Example: Application of the wave equation in different domains

Source domain: Water	**Target domain: Electromagnetism**
✓ finite speed of propagation	✓ finite speed of propagation
✓ wave eq: $\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$	**?** wave eq: $\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$
✓ $f(x,t)$ = displacement of medium	**?** $f(x,t)$ = displacement of medium

A method for discovery: Analogical reasoning

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Retrospective analysis:

- One hypothesis panned out; one did not.
- What kinds of similarities are relevant? The successful hypothesis is supported by an analogy that is both *formal* and *physical*:
  formal: wave equation (same or similar mathematical equation)
  physical: similar causal mechanisms of propagation—something (water or field strengths) varies in a cycle (in direction(s) perpendicular to the direction in which the wave propagates)

Historical example of a purely formal analogy: Wilson-Kogut (1974) application of RG methods to particle physics

Source: a classical statistical mechanical Ising model for a ferromagnet

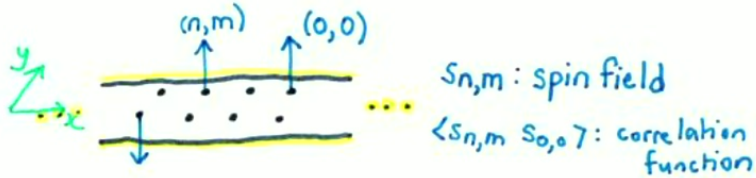
Target: QFT model for scalar ϕ^4 interactions

Immediate Goal: method for constructing renormalized, continuum model for ϕ^4 interactions

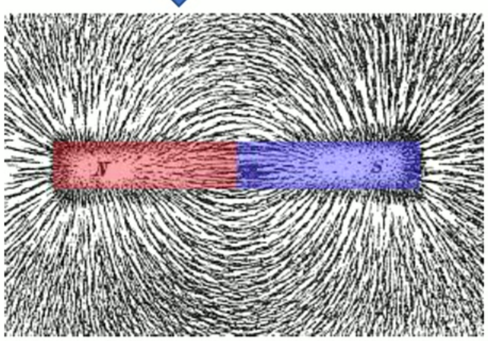
DF (2020), "The development of renormalization group methods for particle physics: Formal analogies between classical statistical mechanics and quantum field theory," *Synthese*

DF (2022), "Justifying the use of purely formal analogies in physics," http://philsci-archive.pitt.edu/20494/

Ising model

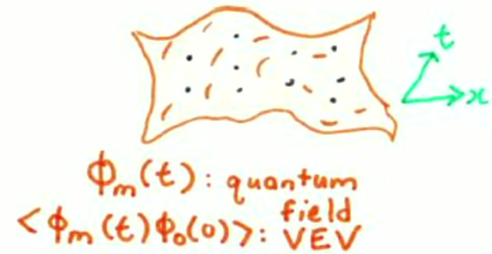


RG transformation, limit of infinite correlation length



Ferromagnet

ϕ^4 QFT model



RG transformation, continuum limit

CMS detector, LHC, CERN

Wick rotation

CSM system: a classical Ising model for a ferromagnet
QFT system: scalar ϕ^4 self-interaction

Wick rotation: $t \to -it$

After Wick rotating the QFT,

$$\Gamma_{n,m} = \zeta^2 D_m(-in\tau) \tag{1}$$

$$\frac{\langle s_{n,m} s_{0,0} \exp(-\beta\mathcal{H})\rangle}{\langle \exp(-\beta\mathcal{H})\rangle} = \zeta^2 \langle\Omega| T\phi_m(t)\phi_0(0)|\Omega\rangle \tag{2}$$

where ζ, τ are constants

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Analogical mappings

CSM	**QFT**
spin field s_m	quantum field ϕ_m
space x_d	space*time* $(x_{d-1}, -it)$
corr function $\Gamma_{n,m}$	VEV $D_m(t)$

Wilson's formal analogy

Problems in statistical mechanical models for critical phenomena and the problem of renormalizing QFT models both involve *statistical continuum limits*

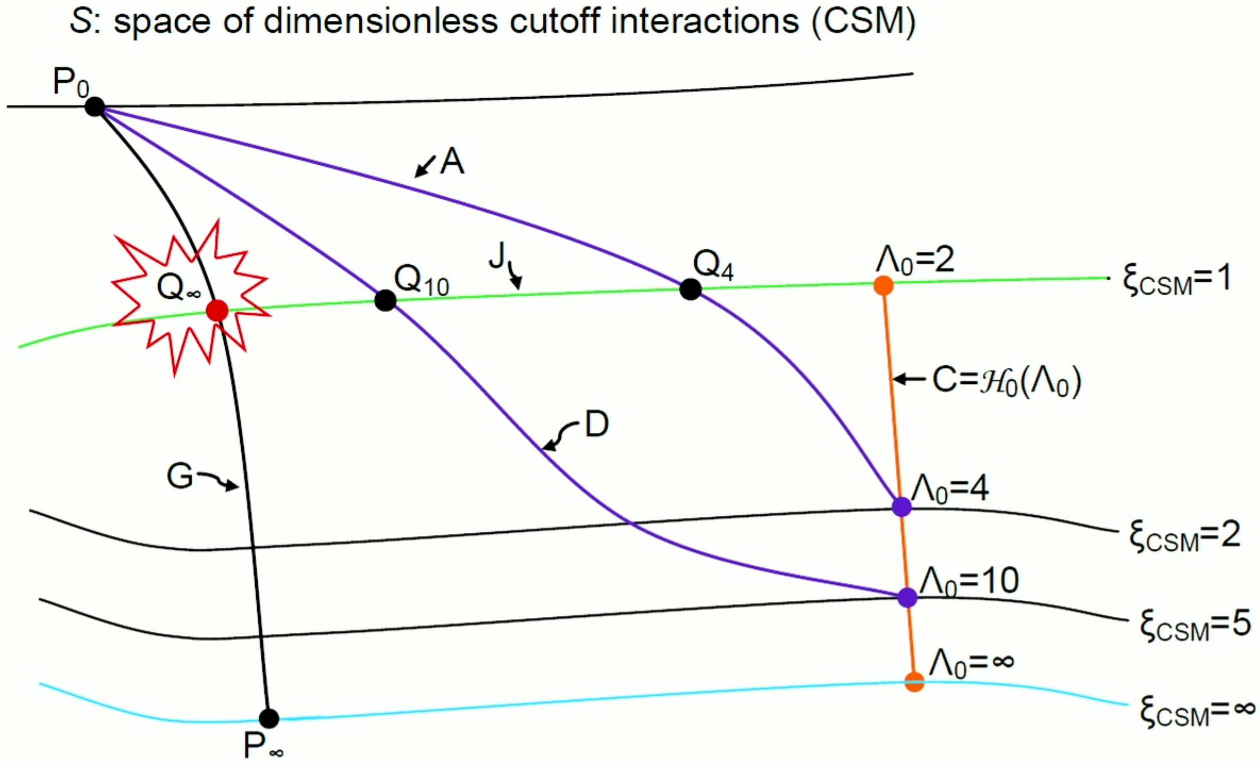
CSM: correlation length $\xi \to \infty$

QFT: lattice spacing $a \to 0$ (or momentum cutoff $\Lambda \to \infty$)
(inverse renormalized mass $\mu_R^{-1} = const$)

Analogical mappings

CSM	**QFT**
corr length $\xi \to \infty$	momentum cutoff $\Lambda \to \infty$

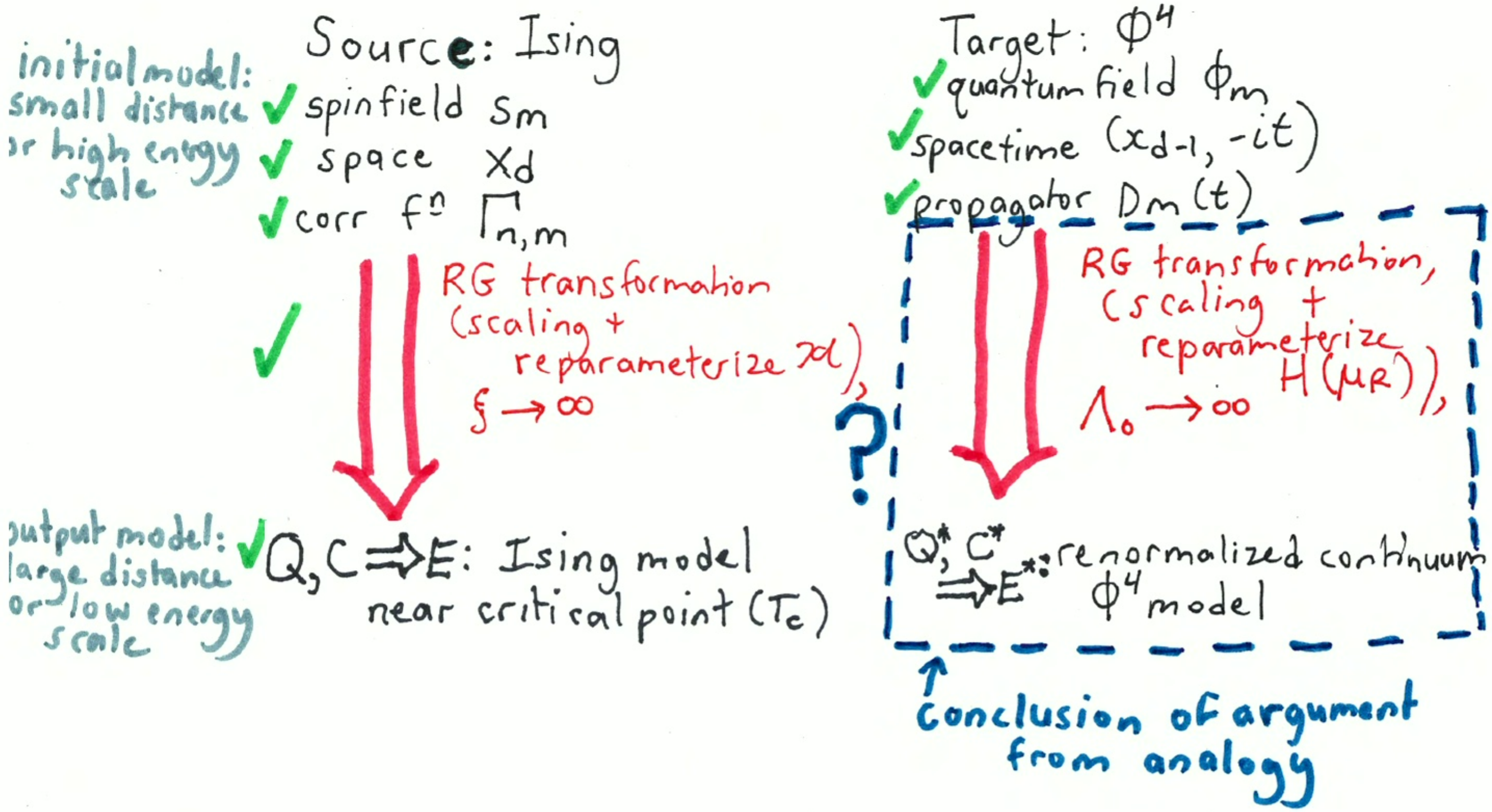
Construction of effective, renormalized continuum QFT



Constraint fixing curve C: $\mu_R^{-1} = \frac{\xi_{CSM}}{\Lambda_0}$

RG transformation: $\frac{\partial \mathcal{H}_t}{\partial t} = U[\mathcal{H}_t]$; $\Lambda_t = e^{-t}\Lambda_0$

(UV fixed point P_∞ assumed to exist)



Source: Ising
initial model: small distance or high energy scale
spinfield Sm
space Xd
corr f^n Γn,m
RG transformation (scaling + reparameterize ℋ), ξ → ∞
output model: large distance or low energy scale
Q, C ⇒ E: Ising model near critical point (Tc)
?
Target: Φ^4
quantum field φm
spacetime (xd-1, -it)
propagator Dm(t)
RG transformation, (scaling + reparameterize H(μR)), Λ0 → ∞
Q*, C* ⇒ E*: renormalized continuum φ^4 model
conclusion of argument from analogy

The Wilson-Kogut argument from formal analogy

- conclusion: model construction procedure + renormalized, continuum ϕ^4 model
- the plausibility of the mathematical model construction strategy is supported by the *mathematical* similarities to the initial Ising model and the model construction strategy for the macro-level model for critical phenomena
- *physical* similarities between the systems represented are **not** needed to support the conclusion
- the conclusion is a hypothesis that is worth pursuing (e.g., by trying to construct models for other QFTs using similar techniques, experimentally testing application of ϕ^4 model to concrete systems)

Moral for methodology of discovery: Purely formal analogies can be useful heuristics for the purpose of obtaining new model construction strategies.

- The physical interpretation cannot be transferred between source and target models.

Conclusions

1. The long history of classical mechanics suggests that development of theories can take a long time. Refined formulations of theories can be important inputs to successor theories (e.g., Hamiltonian, Lagrangian mechanics)

   $\Rightarrow$ Is further theoretical development of GR, QFT needed as input for theoretical development of QG?

2. Analogical reasoning is a method for discovery. What kind of analogy is relevant depends on the circumstances:
   - *physical* analogies are relevant when the obstacle to progress is gaps in understanding of the physics
   - purely *formal* analogies are relevant when the obstacle to progress is inability to infer predictions from hypothesized theoretical principles (e.g., calculational intractability, no techniques for constructing concrete models)
   - these obstacles to progress are not mutually exclusive! are both types of obstacles standing in the way of QG?