Title: Primordial fluctuations from quantum gravity

Speakers: Francesca Vidotto

Collection: Quantum Spacetime in the Cosmos: From Conception to Reality

Date: May 12, 2023 - 9:45 AM

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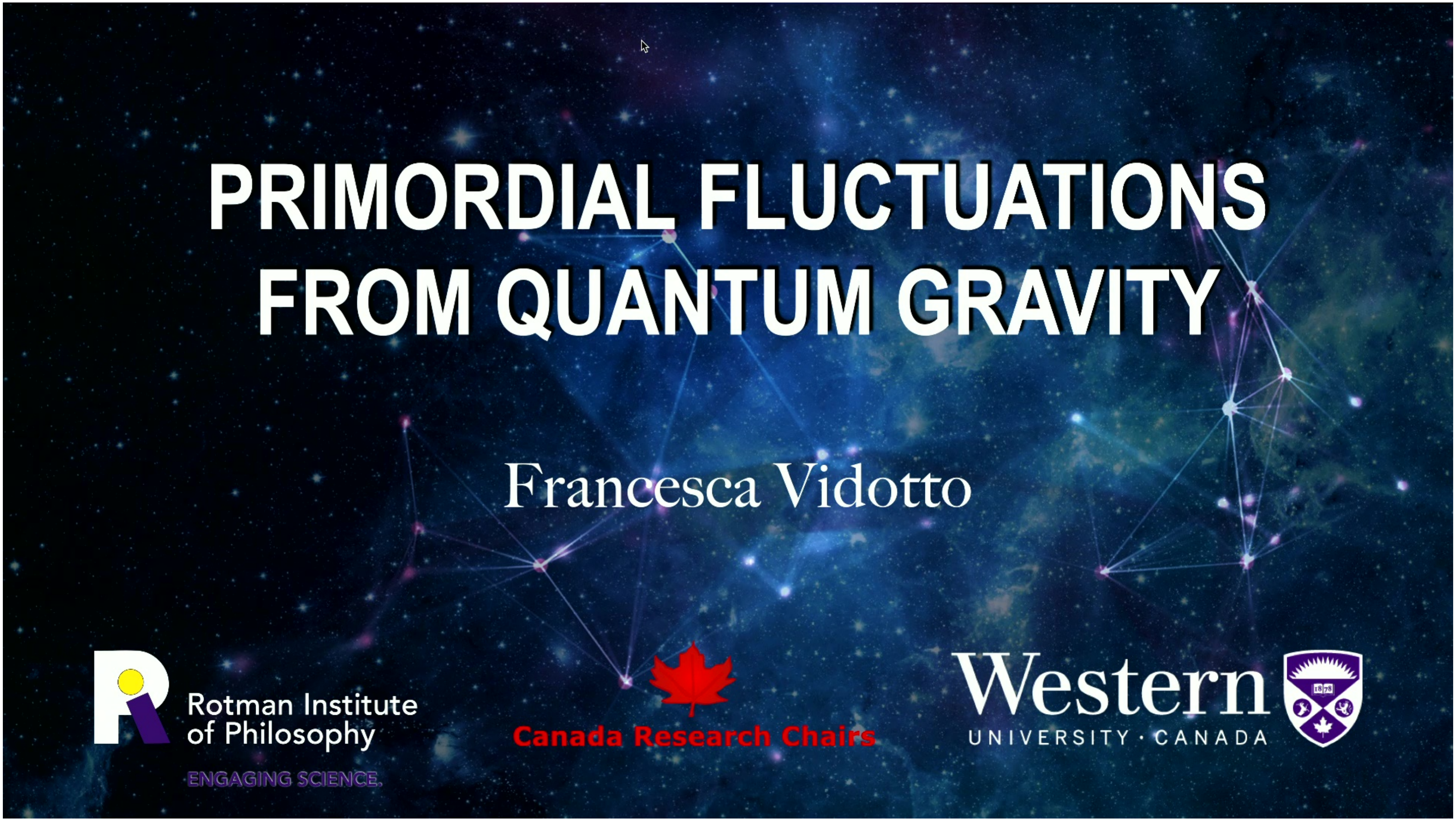
Abstract: In modern cosmology there is an agreement that the seeds of structure formations resides in the quantum fluctuation of the geometry in the early universe, but there is no agreement about how these could be derived from a quantum theory of gravity. In this talk I present a proposal based on the covariant formulation of Loop Quantum Gravity. I describe how to define a wavefunction of the universe in this context, and a how we can study fluctuations and correlations between spacial regions. The results obtained so far has been made possible by recent progress in numerical computations. I discuss the current state of this research program and the possible implications for modeling the early universe.

MAY 12
INTERNATIONAL DAY OF WOMEN IN MATHEMATICS


PRIMORDIAL FLUCTUATIONS
FROM QUANTUM GRAVITY
Francesca Vidotto
Rotman Institute
of Philosophy
ENGAGING SCIENCE.
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UNIVERSITY · CANADA

THE STANDARD SCENARIO

- THE STANDARD MODEL OF COSMOLOGY

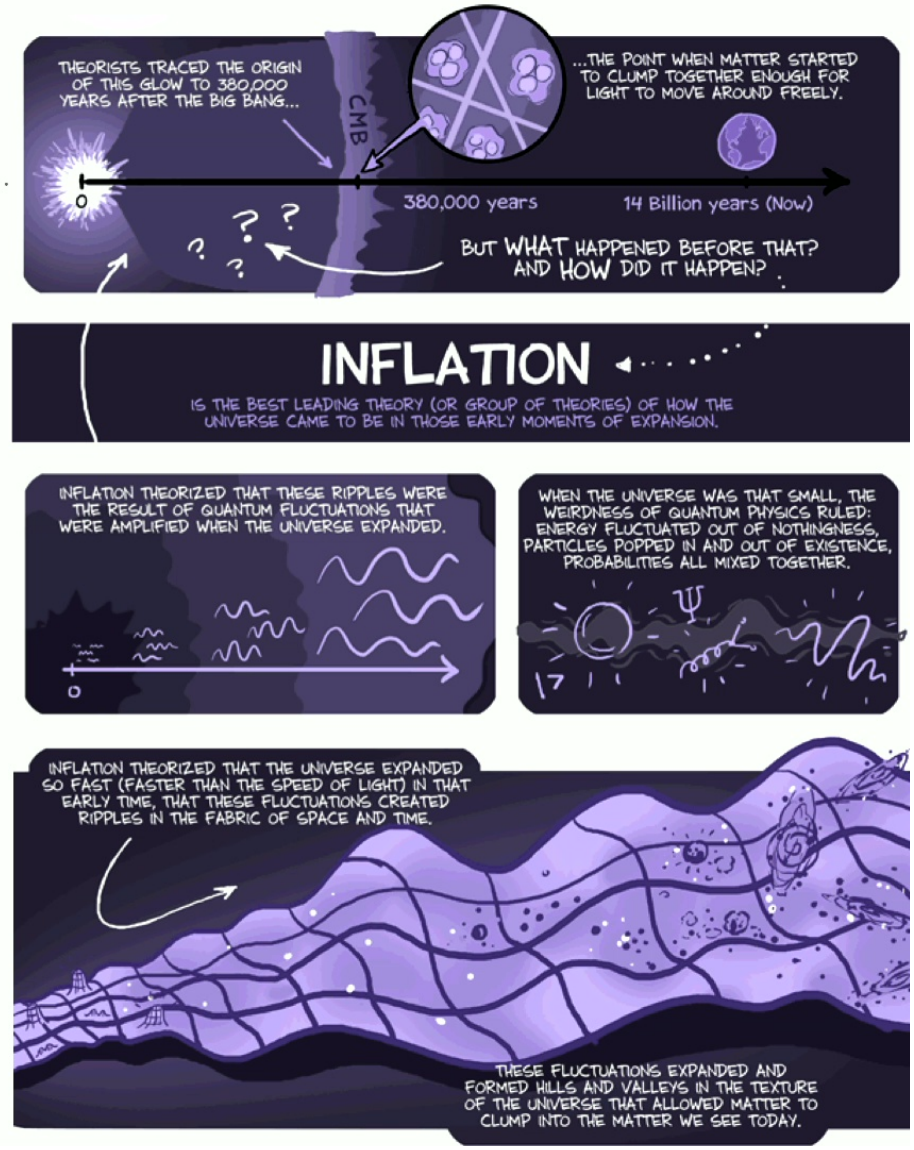
  (a PHDcomics' summary by Jorge Cham: phdcomics.com/comics.php?f=1691)

- BIG BANG: The universe starts hot and dense in a quantum regime ⇒ quantum fluctuations

- INFLATION: a non-identified field governs the dynamics of the universe driving the expansion and putting in place the seeds of structure formation

- INITIAL CONDITION: kinetic energy of the inflaton should dominate over the potential

  ⇒ power spectra depends on the choice of vacuum



THE STANDARD SCENARIO ⟶ LOOP QUANTUM COSMOLOGY

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- NO SINGULARITIES: maximal curvature

  [Agullo, Wang, Wilson-Ewing 2301.10215]

- BIG BOUNCE: maximal energy density deep quantum regime ⇒ tunnelling

- INFLATION IS GENERIC: no fine-tuned initial conditions are required

- INITIAL CONDITION: the contracting phase makes the inflaton to climb up the potential
- VACUUM STATE: it still needs to be chosen!

QUANTUM REGIME

Image credit: ESA

QUANTUM COSMOLOGY

canonical / covariant
quantization

gravity $\longrightarrow$ quantum gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbf{1})$$

QUANTUM COSMOLOGY

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symmetry
reduction

cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

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QUANTIZING GENERAL RELATIVITY

- GR action $$S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma}\int e \wedge e \wedge F[\omega]$$ *(Holst term)*
- BF theory $$S[e,\omega] = \int B[e] \wedge F[\omega]$$
- Canonical variables $$\omega, \quad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$$

$$B^{0k} = \gamma \epsilon^k{}_{ij}\, B^{ij}$$

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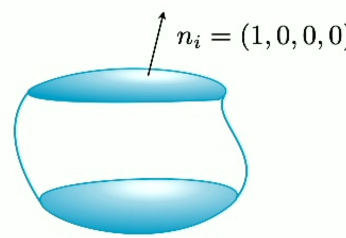
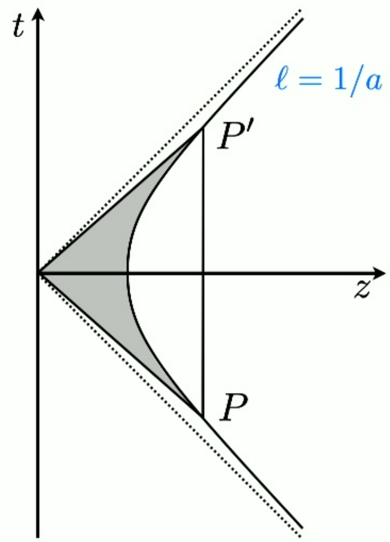
- BF theory $$S[e,\omega] = \int B[e] \wedge F[\omega]$$
- Canonical variables $$\omega, \quad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$$
- Linear simplicity constraint $$B^{0k} = \gamma \epsilon^k{}_{ij}\ B^{ij}$$
- On the boundary $$n_i = e_i^a n_a \qquad n_i e^i = 0 \qquad SL(2,\mathbb{C}) \to SU(2)$$

$$B \to (K = nB, L = nB^*)$$

$$\vec{K} = \gamma \vec{L}$$



MAXIMAL ACCELERATION



$$dA = \frac{\ell^2}{2} d\eta = \frac{1}{2a^2} d\eta$$

η is the boost parameter along the trajectory from P to P′

- Constantly accelerated observer:
  - K generator of boost
  - $E=aK$ generator of proper time evolution
- Lorentzian area: $A = \int_{\mathcal{R}} \frac{1}{\gamma} K^z = \int_{\mathcal{R}} L^z$

$$\ell_{min} = \sqrt{8\pi G\hbar} \qquad A_{min} = 4\pi G\hbar \qquad a_{max} = \sqrt{\frac{1}{8\pi G\hbar}}$$

[Cainiello '81]
[Cainiello, Gasperini, Scarpetta '91]
[Bozza, Feoli, Lambiase, Papini, Scarpetta]

HEURISTICS FOR NO CURVATURE SINGULARITIES IN LQG

QUANTUM COSMOLOGY

canonical / covariant quantization

gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

quantum gravity

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symmetry reduction

cosmology

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+ perturbations

quantum cosmology

IN THIS TALK

- THEORY: *Covariant* Loop Quantum Gravity (*Spinfoam*)
- STATE: Cosmological Lorentzian Spinfoam State — with Rovelli & Bianchi
- QUANTUM FLUCTUATIONS: Numerical Evaluation — with Gozzini & Frisoni
- FUTURE ROADMAP

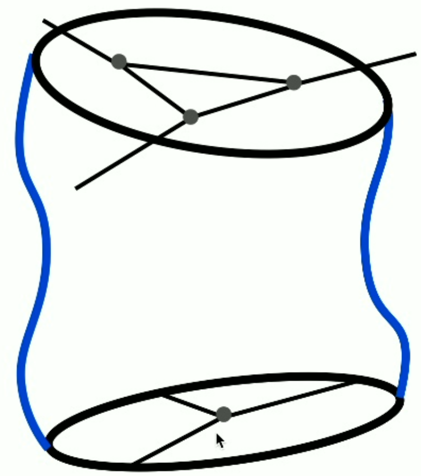
SPINFOAM AMPLITUDES

[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov '08]

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

for a state Ψ associated to the boundary of a 4d region

$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq\, e^{iS}$$



- Superposition $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Local vertex expansion $W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbf{1})$
- UV and IR finite (with Λ)
- Classical limit: discretized GR (with Λ) Barrett et al. '09

www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf

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Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

for a state Ψ associated to the boundary of a 4d region

$$W(q) \approx \int_{\partial g = q} Dq \; e^{iS[q]}$$

- Superposition $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Local vertex expansion $W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbf{1})$
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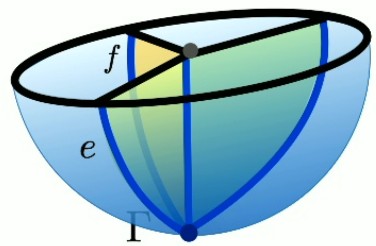
Spinfoam Hartle-Hawking state



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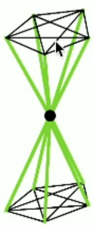
1^{ST}-ORDER FACTORIZATION

[Vidotto 1107.2633]

- classical dynamics $$S_H = const \int \mathrm{d}t\, (a\dot{a}^2 + \frac{\Lambda}{3}a^3)\Big|_{\dot{a}=\pm\sqrt{\frac{\Lambda}{3}}a} = const\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$$
- quantum dynamics $$W(a_f, a_i) = e^{\frac{i}{\hbar}\, S_H(a_f, a_i)} = W(a_f)\overline{W(a_i)}$$
- loop dynamics $$\langle W|\psi_{H_{(z,z')}}\rangle = W(z,z') = W(z)\,\overline{W(z')}$$

order (1)



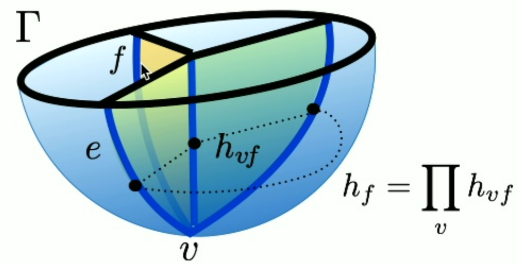
$$W_{\mathcal{C}_\infty}(z',z) = \int h_\ell \int h'_\ell\ \overline{\psi_{z'}(h'_\ell)}\ W_1(h'_\ell, h_\ell)\ \psi_z(h'_\ell)$$

SPINFOAM HARTLE-HAWKING STATES

[Bianchi, Rovelli, Vidotto'10]

- Hartle-Hawking states:

$$\psi_H(q) = \int_{\partial g = q} Dg\, e^{iS[g]}$$



- Spinfoam HH states:

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

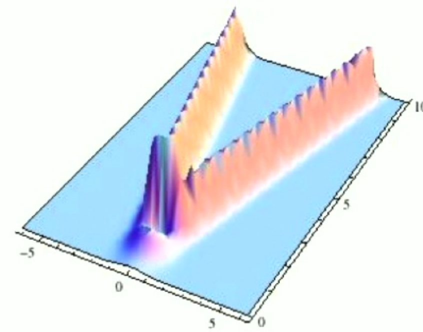
SEMICLASSICAL REGIME

[Bianchi, Krajewski, Rovelli, Vidotto'11]

- LQG coherent states
  peaked on a homogenous and isotropic geometry

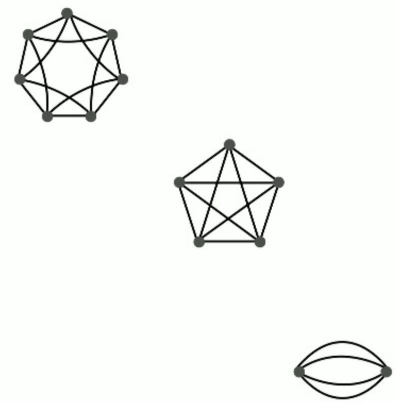
- Spinfoam amplitude with an effective Λ:

$$Z_{\mathcal{C}} = \sum_{j_f, \mathtt{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda \mathtt{v}_e} \prod_v A_v(j_f, \mathtt{v}_e)$$



GRAPH STATES

- Restrict the states to a fixed graph with a finite number N of nodes.
  This defines an approximated kinematics of the universe,
  inhomogeneous but truncated at a finite number of cells.



- The graph captures the large scale d.o.f. obtained averaging the metric over the faces of a cellular decomposition formed by N cells.
- The full theory can be regarded as an expansion for growing N.
  For instance FRW cosmology corresponds to the lower order where there is only a regular cellular decomposition: the only d.o.f. is given by the volume.
- Different graphs can be useful to model different physical situations.

FEW-NODE THEORY: REGGE CALCULUS

[Collins & Williams '72]

- IDEA — Evolve one or few tetrahedra, triangulating a 3-sphere.
- PROBLEM — Compare the evolution for 5, 16 and 600 tetrahedra.
- RESULT — The qualitative behavior is the same!

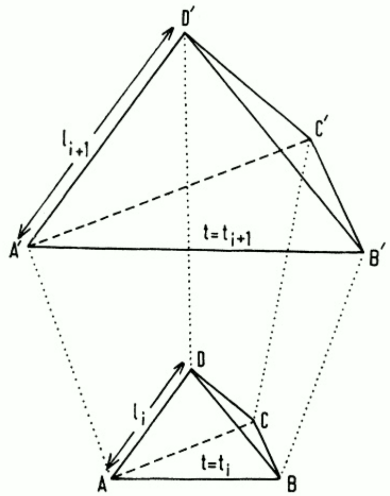


FIG. 1. Diagram illustrating a 4-dimensional block.

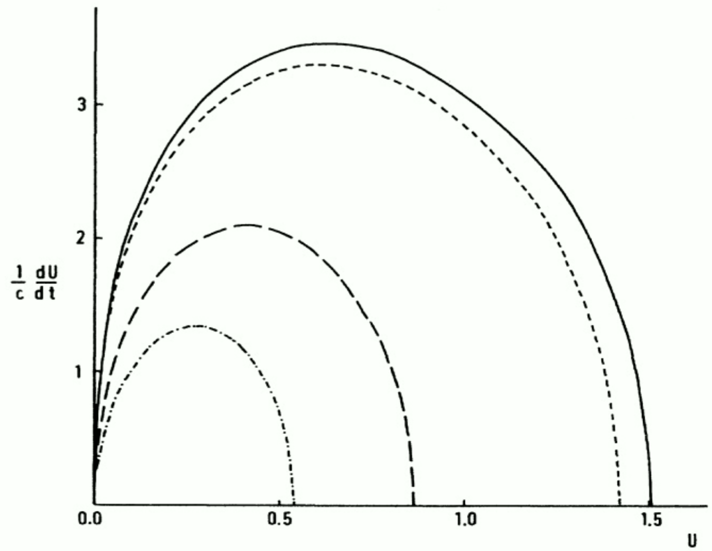


FIG. 2. Rate of change of the volume of the universe plotted against the volume for $MG/c^2=1$; analytic solution ———— , 600-tetrahedron model - - - - - - - - , 16-tetrahedron model — — — — , 5-tetrahedron model —·—·—· .

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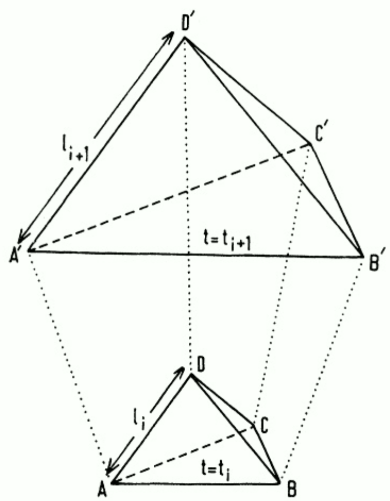

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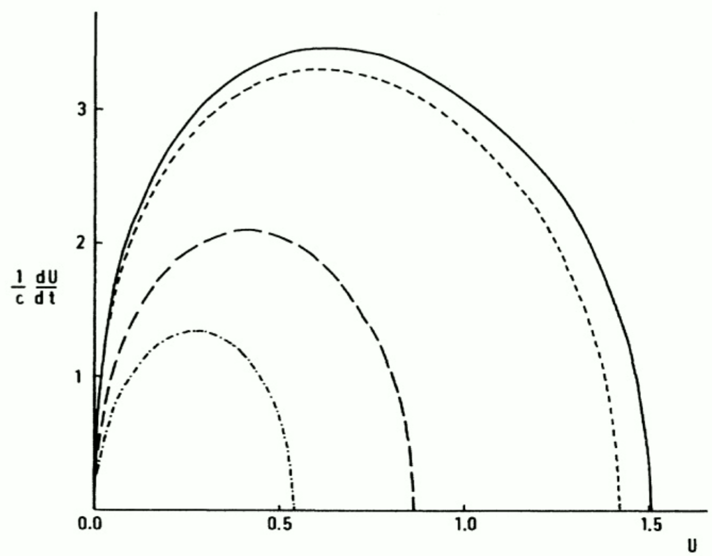

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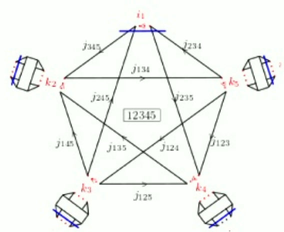
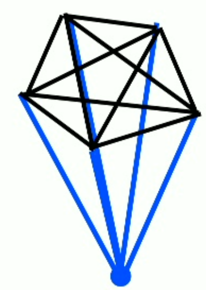
5-CELL PENTACHORDS

Frisoni, Gozzini, Vidotto 2207.02881

- Simplest regular 4-polytope



- Regular triangulation of S_3



OBSERVABLES

- Area
- Volume $\langle O \rangle = \langle \psi_o | O | \psi_o \rangle$
- Dihedral Angles $\Rightarrow$ Curvature *spread*
- **Correlations** $C(O_1, O_2) = \dfrac{\langle \psi_o | O_1 O_2 | \psi_o \rangle - \langle O_1 \rangle \langle O_2 \rangle}{(\Delta O_1)\ (\Delta O_2)}$ $\Delta O = \sqrt{\langle \psi_o | O^2 | \psi_o \rangle - \langle O \rangle^2}$
- **Entanglement**

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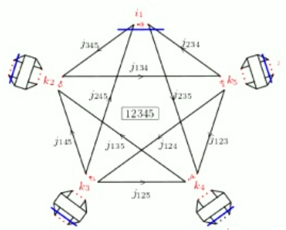

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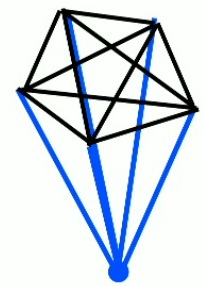

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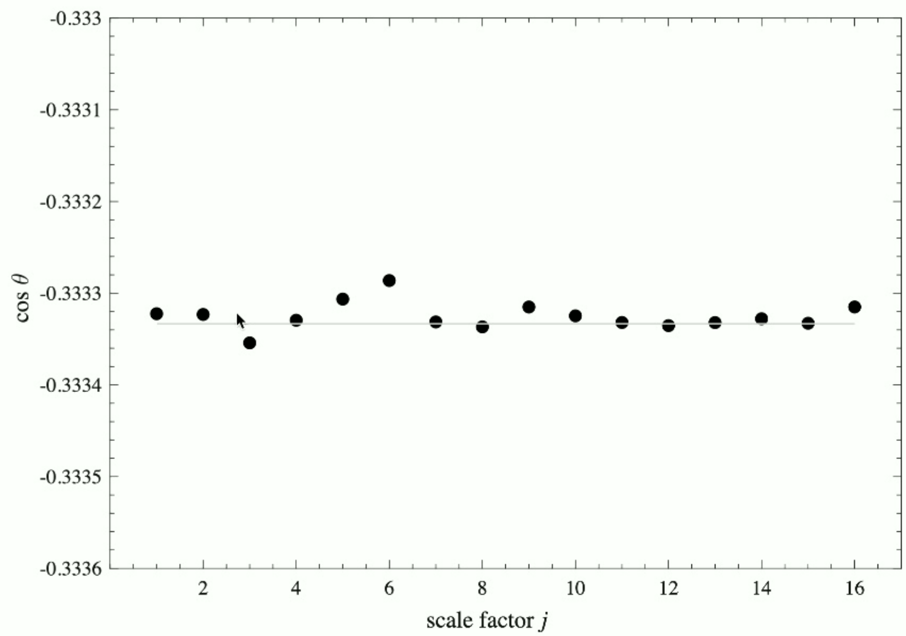
RESULTS

Gozzini, Vidotto 1906.02211

1. 3-sphere as emerging geometry

2. large fluctuations

3. large correlations



Primordial fluctuations from quantum gravity, with F. Gozzini

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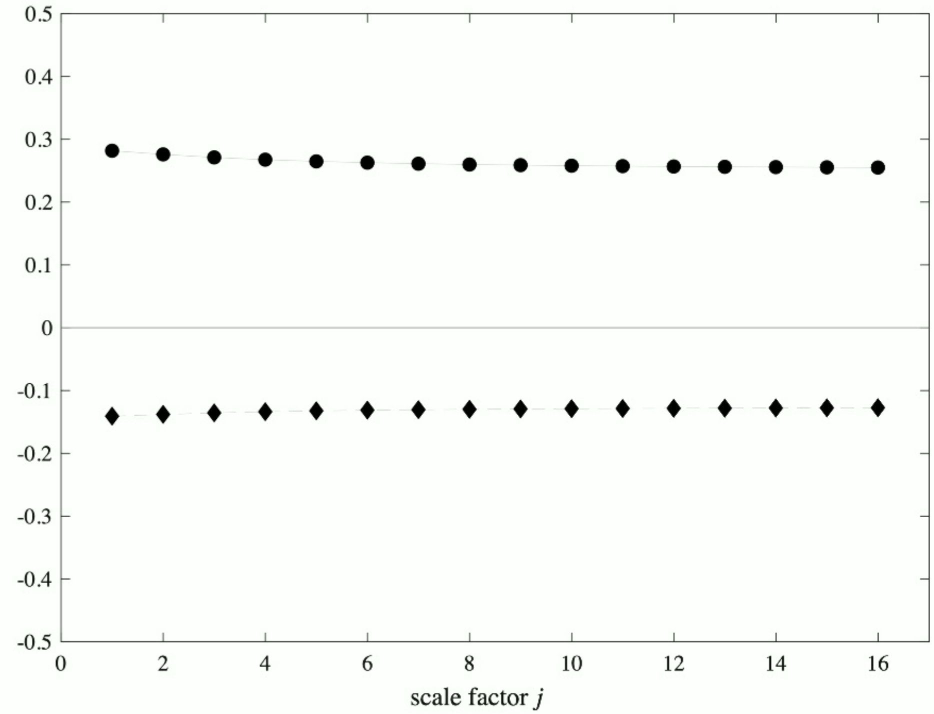
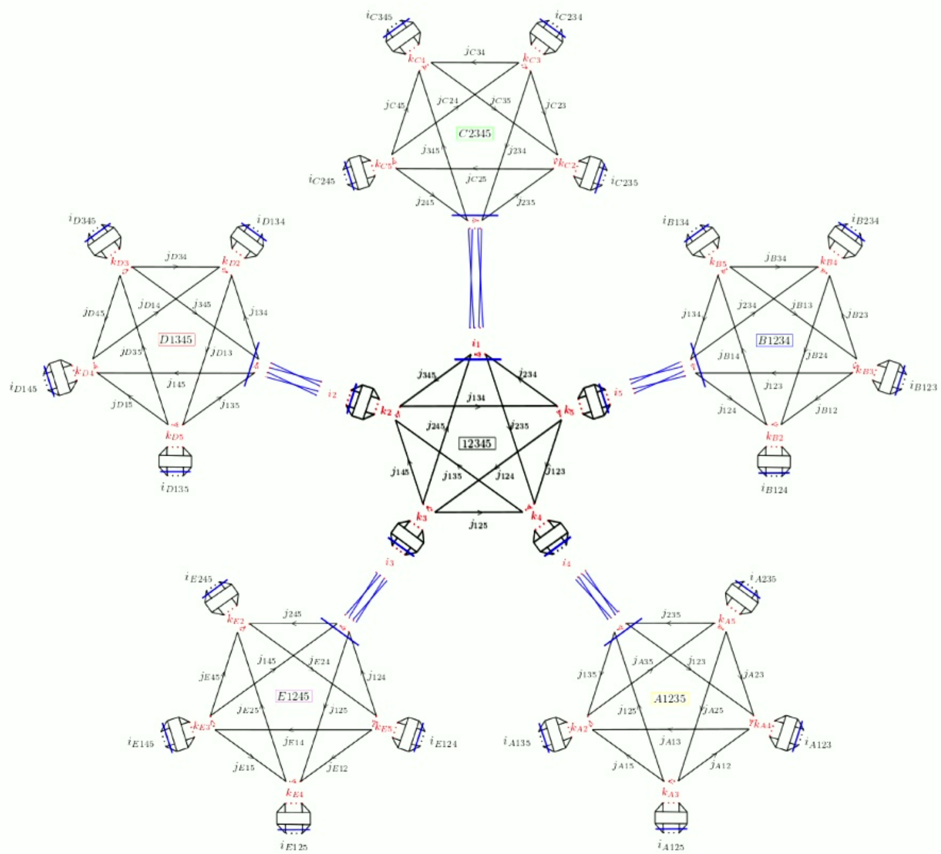
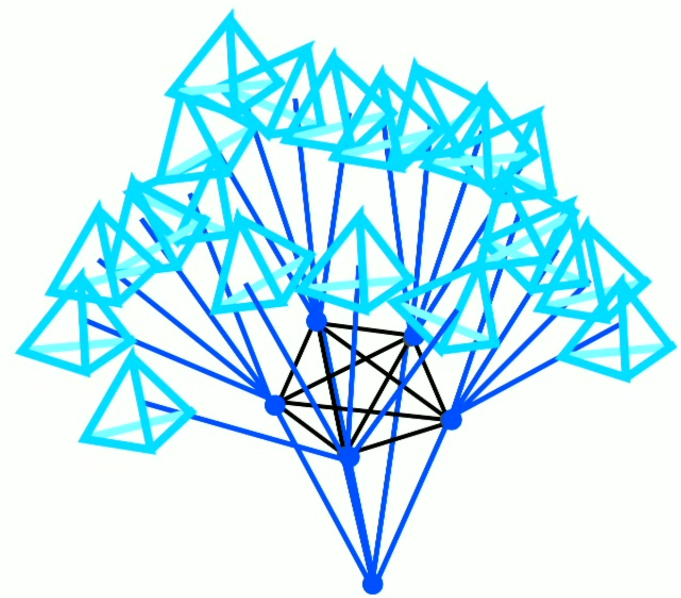


Primordial fluctuations from quantum gravity, with F. Gozzini

GRAPH REFINEMENT

Frisoni, Gozzini, Vidotto 2207.02881

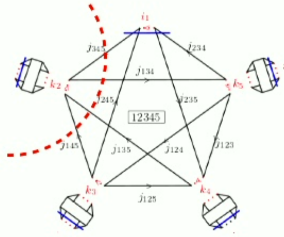




ENTANGLEMENT ENTROPY

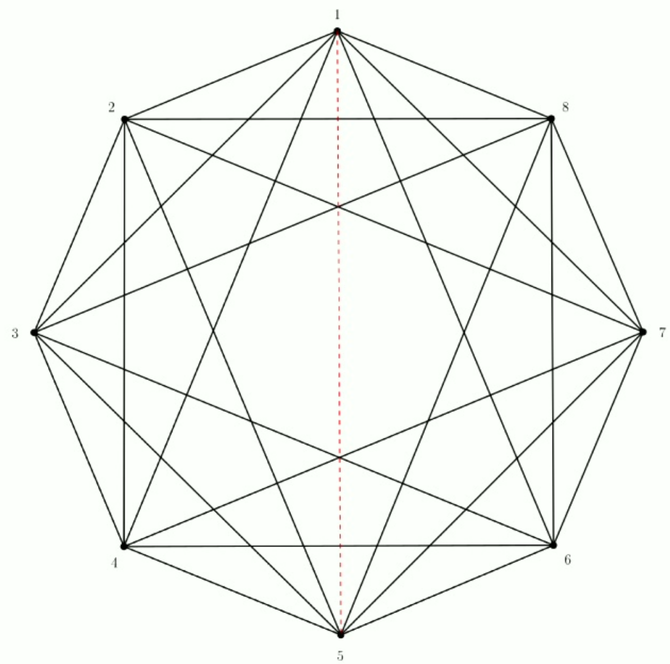
Frisoni, Gozzini, Vidotto 2207.02881

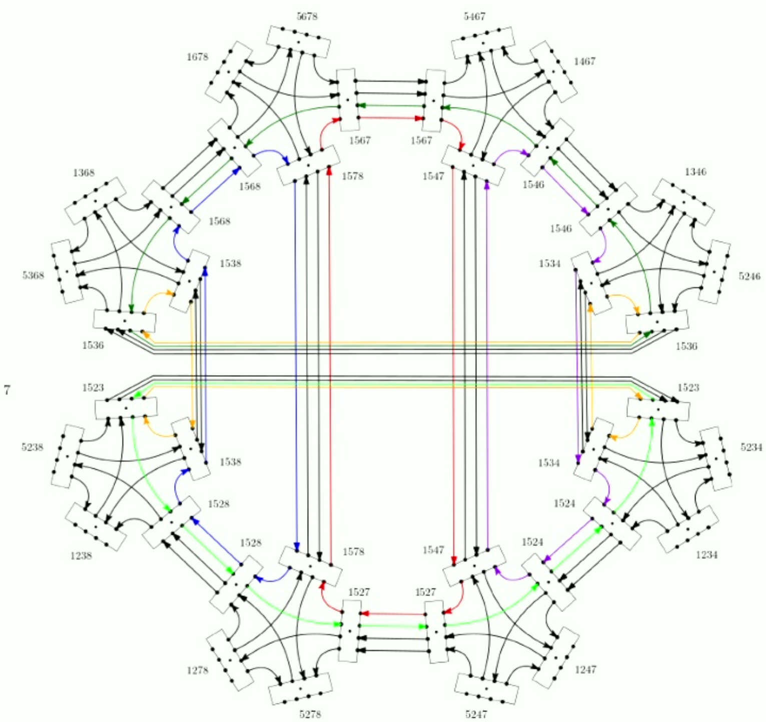
- Partition: $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_{\bar{A}}$
- Reduced density matrix: $\rho_A = \dfrac{1}{Z} Tr_{\bar{A}} \, |\psi_0\rangle\langle\psi_0|$
- Entanglement entropy: $S_A = - Tr\left(\rho_A \log \rho_A\right)$



BF 16-CELL MODEL

Frisoni, Gozzini, Vidotto TBA





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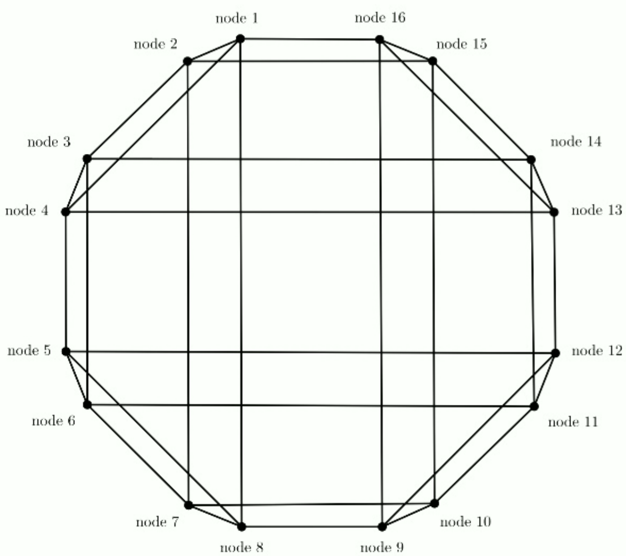

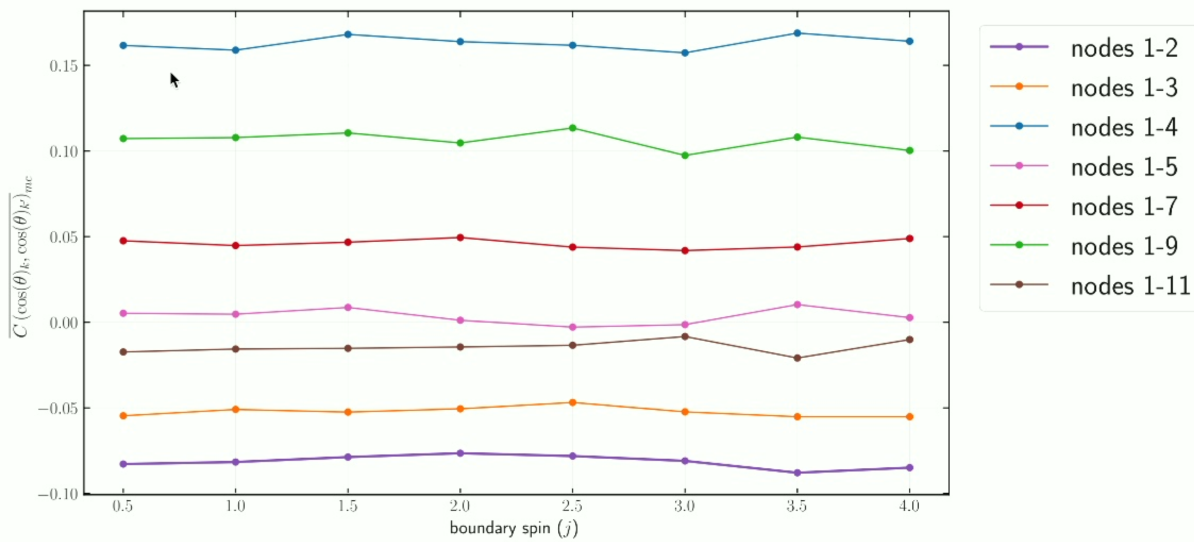

SUMMARY

- Computing primordial quantum fluctuations from the full theory is one of the main goals of a quantum theory of gravity!

- Proposal: use Spinfoam Hartle-Hawking States

- Graph truncation: 5-cell (full) ✓, 20-cell (refinement) ✓, 16-cell (topological) ✓

- Computational challenge: compute expectation values for observables

- Results:
  1. emerging S_3 geometry
  2. large fluctuations
  3. large correlations (for adjacent nodes) $\longrightarrow$ 16-cell needed for richer structure

COLLABORATIONS AND FUTURE DIRECTIONS

FIRST SIMPLE MODEL

- 1 vertex
- 5-cells boundary graph
- computation of observables
- high correlations

with Francesco Gozzini

RELATION TO COSMOLOGICAL VACUUM

- properties of standard cosmological vacua
- QFT on a triangulated 3-sphere
- entanglement entropy

with Sofie Ried

MORE COMPLEX RELIABLE MODELS

- 1 vertex, 6 vertices
- 16-cells and 20-cells boundary graphs
- MCMC to compute observables
- rich behaviour of correlations

with Pietropaolo Frisoni

NON-INFLATIONARY MODELS

- cosmological perturbations from an effective highly-correlated vacuum states
- matter bounce as an alternative to the inflationary models

with Mateo Pascual