Title: Physics with non-perturbative quantum gravity: the end of the Hawking evaporation

Speakers: Carlo Rovelli

Collection: Quantum Spacetime in the Cosmos: From Conception to Reality

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Abstract: Quantum gravitational phenomena dominate the physics of a black hole in the past of a high curvature spacelike surface. Because of the backreaction of the evaporation, this surface crosses the horizon. I describe a recent line of investigation on the possible evolution compatible with nonperturbative quantum gravity, and in particular I illustrate what can be predicted using loop quantum gravity.

loop quantum gravity
and
the end of the life of a black hole

carlo rovelli

Eugenio Bianchi, Marios Chistodoulou, Fabio D'Amborsio, Tommaso De Lorenzo, Pietro Donà, Hal Haggard, Muxin Han, Alejandro Perez, Simone Speziale, Farshid Soltani, Charalampos Theofilis, Ilya Vilensky, Francesca Vidotto.

General relativity

GR $S[g] = \int \sqrt{\det[-g]}R$

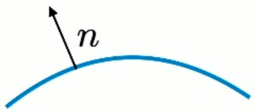
Tetrads $g_{ab} \to e^i_a$ $\quad g_{ab} = e^i_a \; e^i_b$ $\quad e = e_a dx^a \in R^{(1,3)}$

Spin connection $\omega = \omega_a dx^a \in sl(2,C)$ $\quad \omega(e): \quad de + \omega \wedge e = 0$

GR action $S[e,\omega] = \int e \wedge e \wedge F^*[\omega]$

GR Holst action $S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega]$

Canonical variables $\omega, \quad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$ $\quad S[B,\omega] = \int B \wedge F[\omega]$

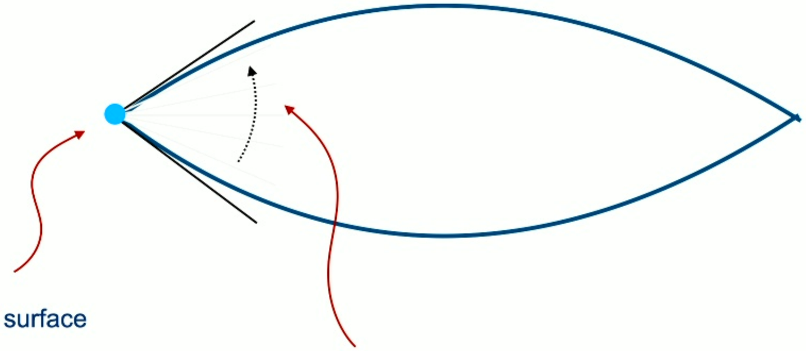


On the boundary $n_i = e^a_i n_a$ $\quad n_i = (1,0,0,0)$ $\quad SL(2,C) \to SU(2)$

$B \to (K = nB, L = nB^*)$

Linear simplicity constraint $\vec{K} + \gamma \vec{L} = 0$

$$\vec{K} = \gamma \vec{L}$$



2d surface

$$L = e \wedge e$$

Is the area element
of the surface

Lorentz boost

$$\vec{K}$$

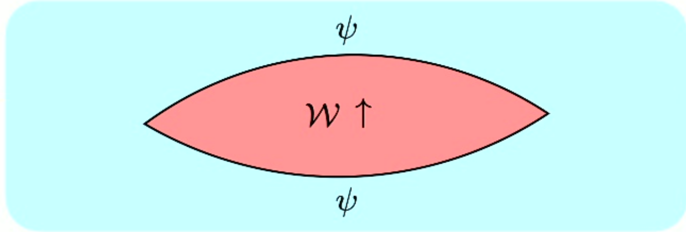
is the generator

The modular

Hamiltonian

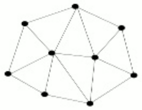
is the area

Structure of the theory



Truncation

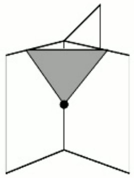
State space $\mathcal{H} \ni \psi$ $\mathcal{H}_\Gamma$



Spin networks

Quantum states of geometry

Operators: $\vec{L}_l$ $\vec{L}_l$



Spin foams

Transition amplitudes: $\mathcal{W}$ $\mathcal{W}_\mathcal{C}$

i. Quantum histories of geometries
ii. Discretized spacetime

Cfr:

i. Lattice gauge theory
ii. Feynman graph expansion

Truncation in diff-invariant theories (Ditt-invariance).

Harmonic oscillator

$$S = \frac{m}{2}\int dt \left(\left(\frac{dq}{dt}\right)^2 - \omega^2 q^2 \right)$$

Discretize $a = t/N$

$$S_N = \frac{m}{2}\sum_n^N a \left(\left(\frac{q_{n+1}-q_n}{a}\right)^2 - \omega^2 q_n^2 \right)$$

$$W(q_f,t_f;q_i,t_i) = \lim_{\substack{\Omega\to 0\\ N\to\infty}} \mathcal{N}\int dQ_n\, e^{iS_{N,\Omega}(Q_n)}$$

Rescale variables

$$Q_n = \sqrt{\tfrac{m}{a\hbar}}\, q_n \qquad \Omega = a\omega$$

$$\frac{S_N}{\hbar} = \frac{1}{2}\sum_n^N \left((Q_{n+1}-Q_n)^2 - \Omega^2 Q_n^2\right) \equiv S_{N,\Omega}(Q_n)$$

Parametrized theory

$$S = \frac{m}{2}\int d\tau \left(\frac{\dot q^2}{\dot t} - \omega^2 \dot t\, q^2 \right)$$

Discretize

$$S_N = \frac{m}{2}\sum_n^N a \left(\frac{\left(\frac{q_{n+1}-q_n}{a}\right)^2}{\frac{t_{n+1}-t_n}{a}} - \omega^2 \frac{t_{n+1}-t_n}{a}\, q_n^2 \right).$$

$$= \frac{m}{2}\sum_n^N \left(\frac{(q_{n+1}-q_n)^2}{t_{n+1}-t_n} - \omega^2 (t_{n+1}-t_n)\, q_n^2 \right)$$

$$W(q_f,t_f;q_i,t_i) = \lim_{N\to\infty} \int dQ_n\, dT_n\, e^{iS_N(Q_n,T_n)}$$

Rescale variables

$$Q_n = \sqrt{\tfrac{m\omega}{\hbar}}\, q_n$$

$$\frac{S_N}{\hbar} = \frac{1}{2}\sum_n^N \left(\frac{(Q_{n+1}-Q_n)^2}{T_{n+1}-T_n} - (T_{n+1}-T_n) Q_n^2 \right)$$

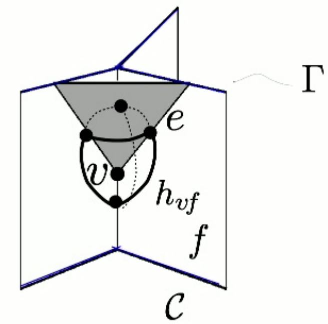
$$\equiv S_N(Q_n, T_n)$$

Quantization of $S[B,\omega] = \int B \wedge F[\omega]$

Partition function $$Z = \int dB\, d\omega\, e^{i\int B\wedge F} \to \int d\omega\, \delta[F[\omega]]$$

Discretization $$\to \int dh_l \prod_f \delta[h_f]$$

Equivalent form $$\to \int dh_{vf} \prod_f \delta[h_f] \prod_v A(h_{vf})$$



2-complex
(vertices, edges, faces)

Vertex amplitude $$A(h_f) = \sum_{j_f} (2j_f+1) \int dg_e\, Tr[h_f g_e g_{e'}^{-1}]$$

Missing $$\vec{K} = \gamma \vec{L}$$

Dynamics

Main tool: SL(2,C) ***unitary*** representations (why so little used in physics?)

SU(2) unitary representations:	$2j \in Z$	$\|j;m\rangle \in \mathcal{H}_j$
SL(2,C) unitary representations:	$2k \in N, \quad \nu \in R$	$\|k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu},$
γ-simple representations:	$\nu = \gamma(k+1)$	
SU(2) → SL(2,C) map:	$Y_\gamma : \quad \mathcal{H}_j \quad \to \mathcal{H}_{j,\gamma j}$ $\|j;m\rangle \mapsto \|(j,\gamma(j+1))\,;\; j,m\rangle$	
Image of Y_γ : minimal weight subspace	$j = k$	
Main property:	$\vec{K} + \gamma \vec{L} = 0$	weakly on the image of Y_γ

Boost generator ($\vec{K}$) Rotation generator ($\vec{L}$)

Dynamics

Transition amplitudes

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

$$h_f = \prod_v h_{vf}$$

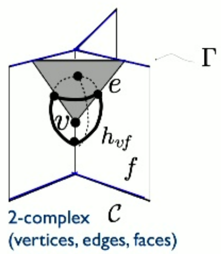
Vertex amplitude

$$A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f + 1)\ Tr_j[h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$$

Simplicity map

$$Y_\gamma : \mathcal{H}_j \rightarrow \mathcal{H}_{j,\gamma j}$$

$$|j;m\rangle \mapsto |j,\gamma(j+1);j,m\rangle$$

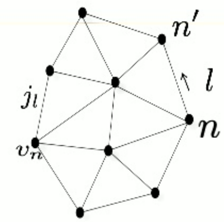


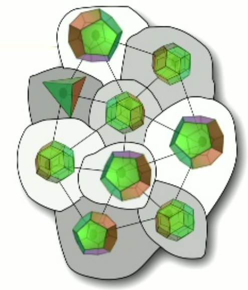
With a cosmological constant $\Lambda > 0$:
Amplitude A^q: SL(2,C) → SL(2,C)_q network evaluation.

- A^q ~ SL(2,C) Chern-Simon expectation value of the boundary graph of the vertex

Engle, Pereira, Livine, Krasnov, Freidel, Speziale, Lewandowski, Kaminski, Han, CR

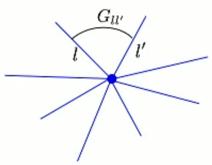
Boundary state space





State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]_\Gamma \ni \psi(h_l) = \psi(\Lambda_n h_l \Lambda_{n'}^{-1})$

Operators: $\vec{L}_l = \{L_l^i\}, i = 1,2,3,$ where $L^i\psi(h) \equiv \left.\dfrac{d}{dt}\psi(he^{t\tau_i})\right|_{t=0}$

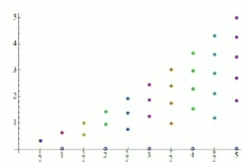


The gauge invariant operator: $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$

area $A_l^2 = G_{ll}$

volume $V_n^2 = \dfrac{2}{9}\,\vec{L}_{l_1}\cdot(\vec{L}_{l_2}\times\vec{L}_{l_3})$

- Area eigenvalues $A = 8\pi\gamma\hbar G\,\sqrt{j_l(j_l+1)}$
- There is an area gap $a_o = 8\pi\gamma\hbar G\,\dfrac{\sqrt{3}}{2}$
- The volume eigenvalues are finite and discrete:



Geometry is quantized:

(i) eigenvalues are discrete
(ii) the operators do not commute
(iii) a generic state is a quantum superposition

→ coherent states theory

Results

1. The classical limit of the vertex amplitude converges (appropriately) to the Regge Hamilton function (with cosmological constant).
   (Conrady-Freidel, Barrett *et al*, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012).

2. The amplitudes with positive cosmological constant are UV and IR finite: $W^q_{\mathcal{C}} < \infty$
   (Han, Fairbairn, Moesburger, 2011).

3. The boundary states represent classical geometries.
   (Canonical LQG 1990', Penrose spin-geometry theorem 1971).

4. Boundary geometry operators have discrete spectra.
   (Canonical LQG main results, 1990').

Connection to observations

1. No super symmetry (naturally suggested at low energy by string theory)..
2. Positive cosmological constant (instead of the negative one naturally suggested by string theory.).
3. No Lorentz invariance violation (predicted by Horava theory.)
4. No difference between speed of gravitational and EM waves (predicted by most alternatives to GR).

the end of the life of a black hole

Boundary state space

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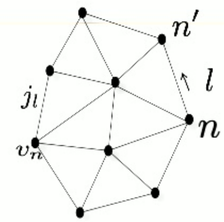

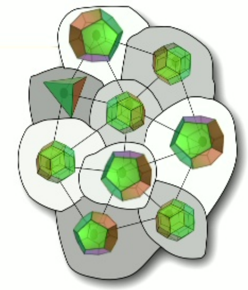

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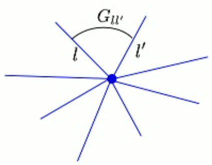

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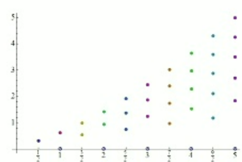

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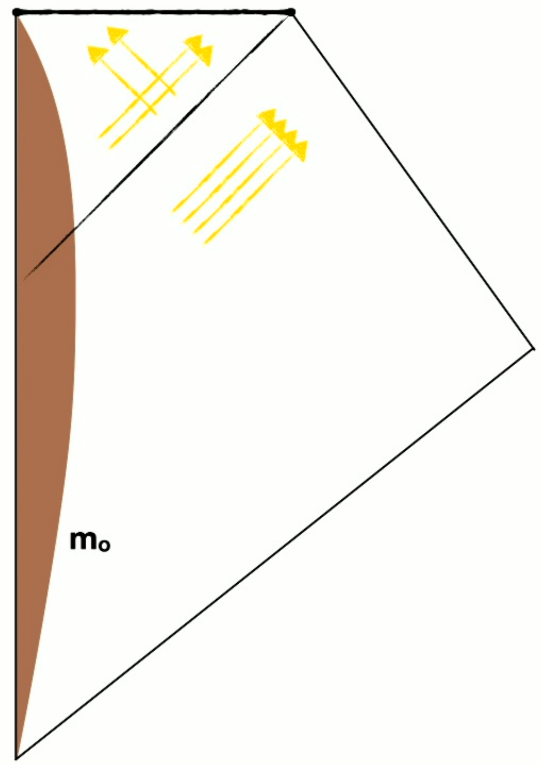
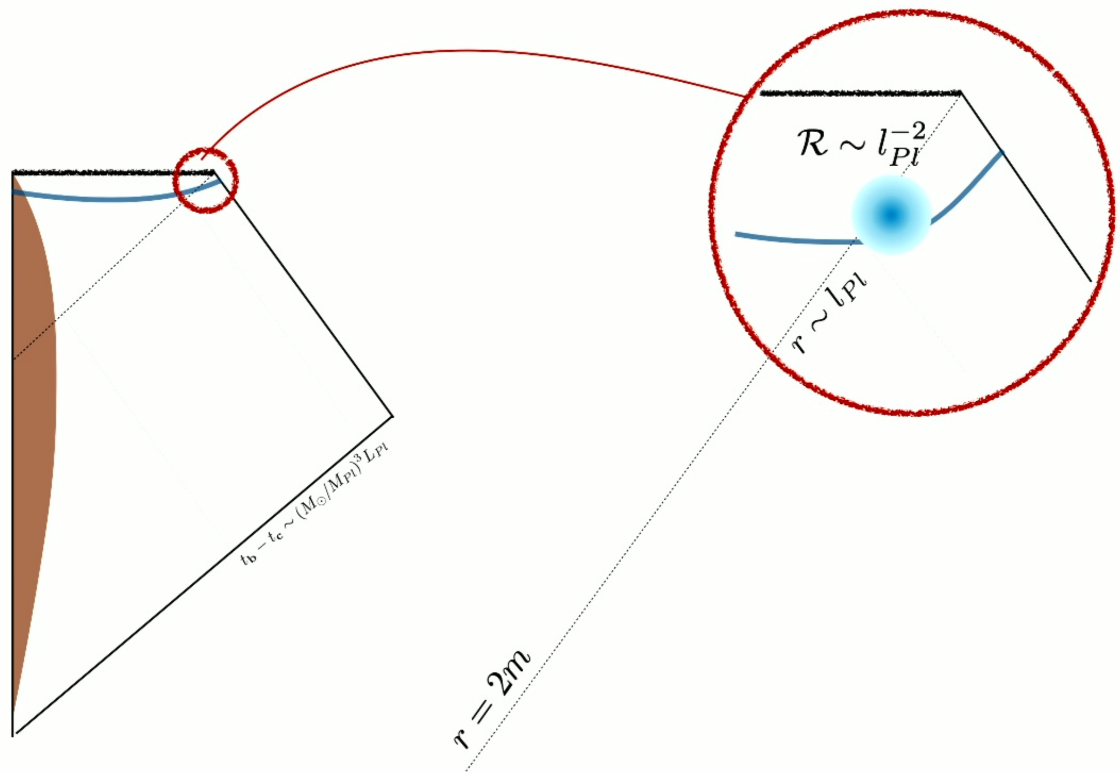
→ coherent states theory

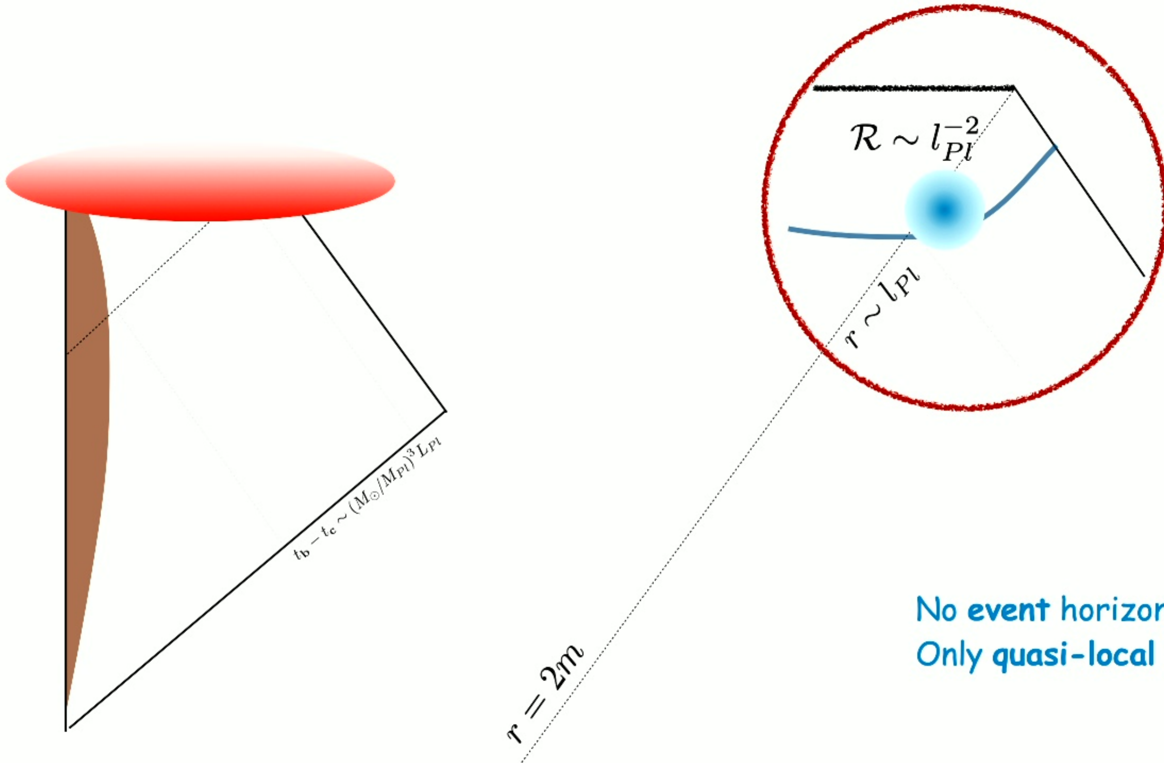
The incomplete picture:

Where is this un reliable?

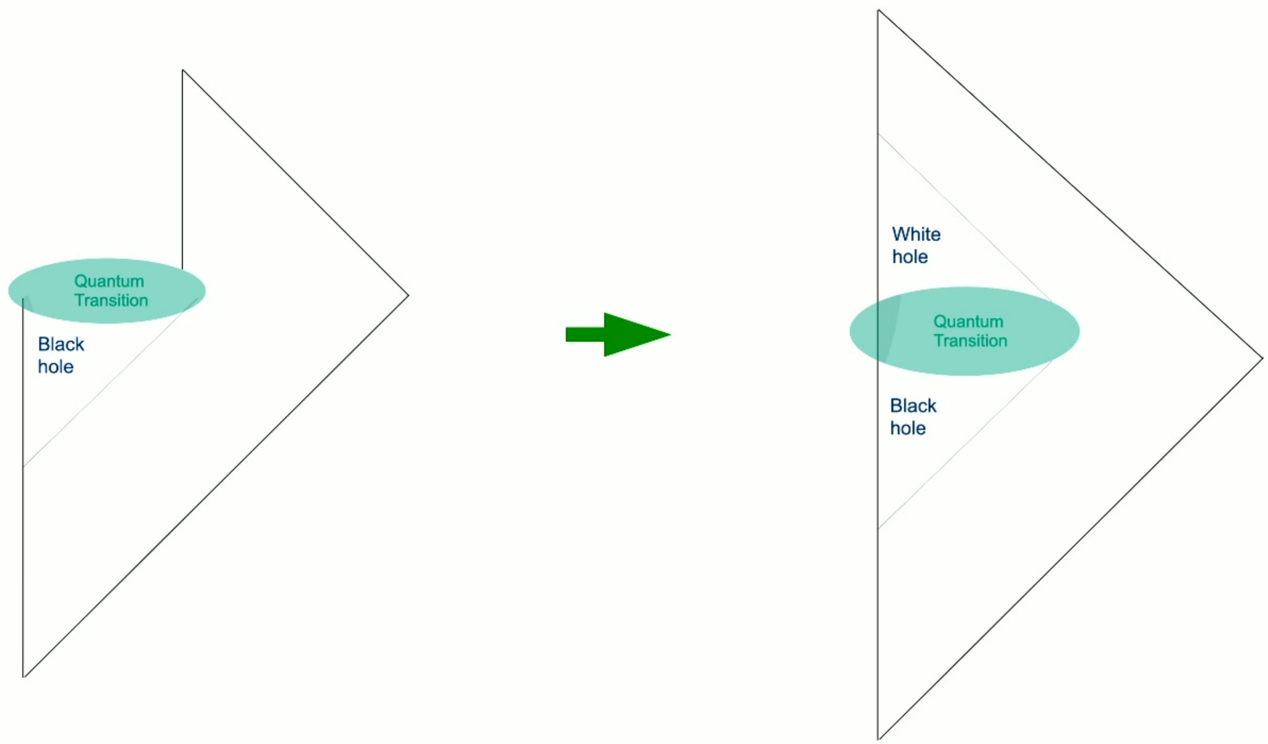
$$\mathcal{R} \sim l_{Pl}^{-2}$$




$\mathcal{R} \sim l_{Pl}^{-2}$
$r \sim l_{Pl}$
$t_{\mathbf{b}} - t_{\mathbf{c}} \sim (M_{\odot}/M_{Pl})^3 L_{Pl}$
$r = 2m$

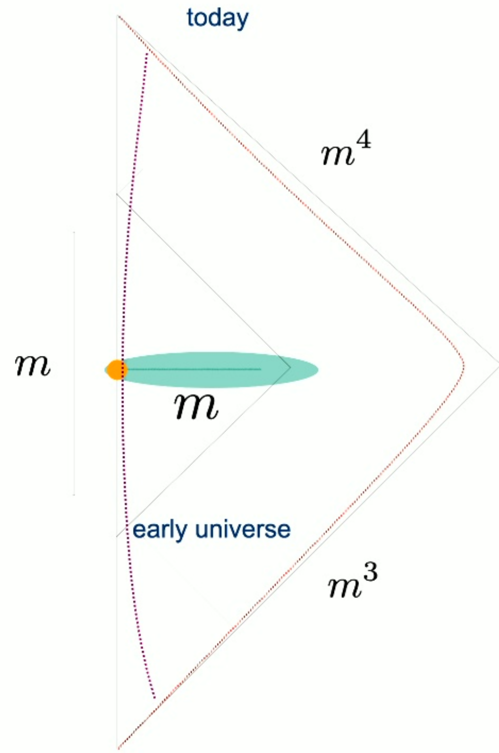


No **event** horizon.
Only **quasi-local** horizon.



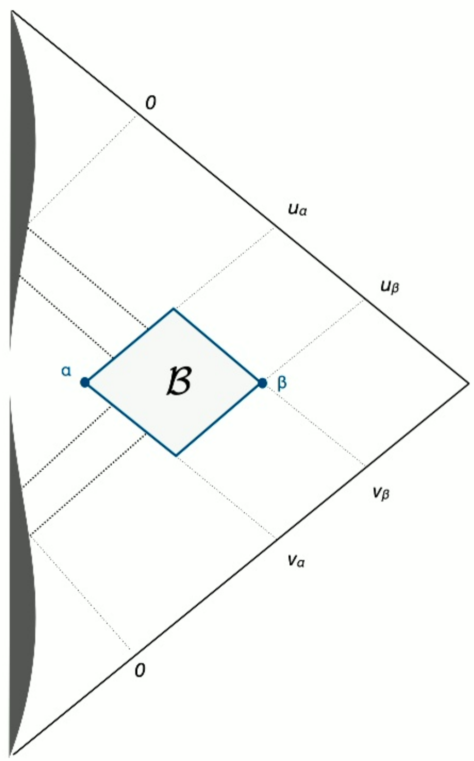
The white hole is a small, long living, remnant, with mass ~Planck slowly emitting a large amount of information.

Black hole can quantum tunnel to small white holes, which are long living remnants



Long living small mass remnant

Dark matter candidate



Star:

$$ds^2 = -dT^2 + a^2(T)(dR^2 + R^2 d\Omega^2)$$

$$a(T) = \left(\frac{9mT^2 + Am}{2}\right)^{\frac{1}{3}}$$

Cfr LQC

Exterior:

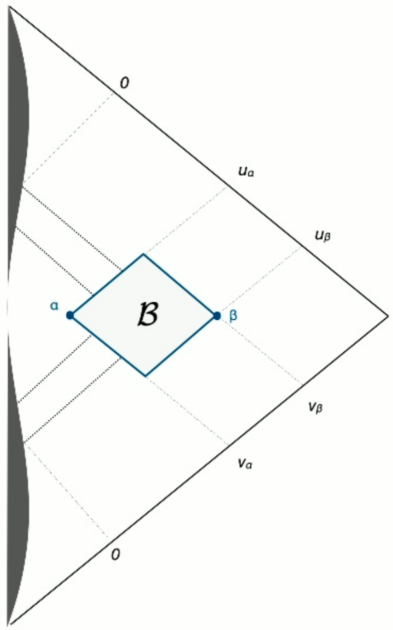
$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega^2$$

$$F(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$

B Region:

of a regular interpolating metric

Han, Soltani, CR, 2023



The full metric outside the star, in a single coordinate chart

$$ds^2 = F(r(u,v)) f(v,u)\, du\, dv + r^2(u,v) d\Omega^2.$$

$$f(v,u) = (1 + S(u)R'(v))\ (1 + S(v)R'(u)),$$

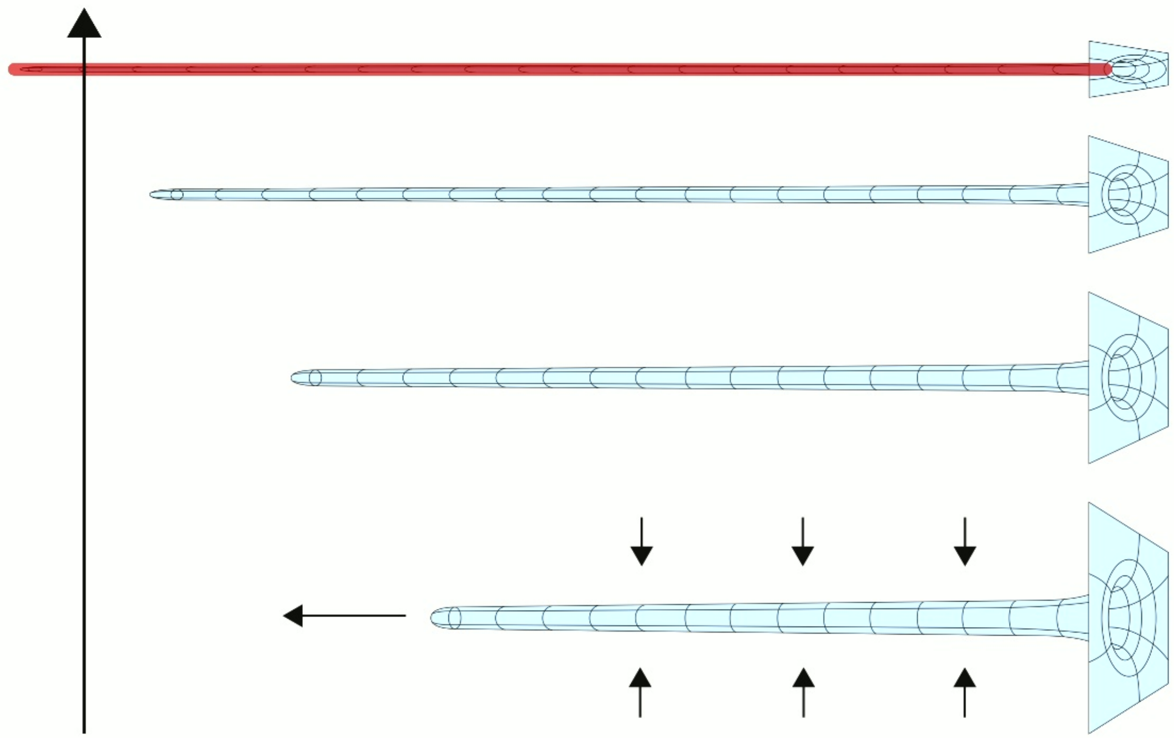
$$2r_*(r) = v + u + R(v + u - v_\alpha).$$

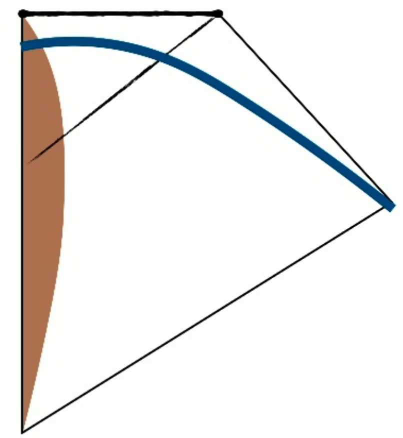
$$S(v) = 1 - S_n((v - v_\alpha)/(v_\beta - v_\alpha)).$$

$$S_2(v) = 6v^5 - 15v^4 + 10v^3.$$

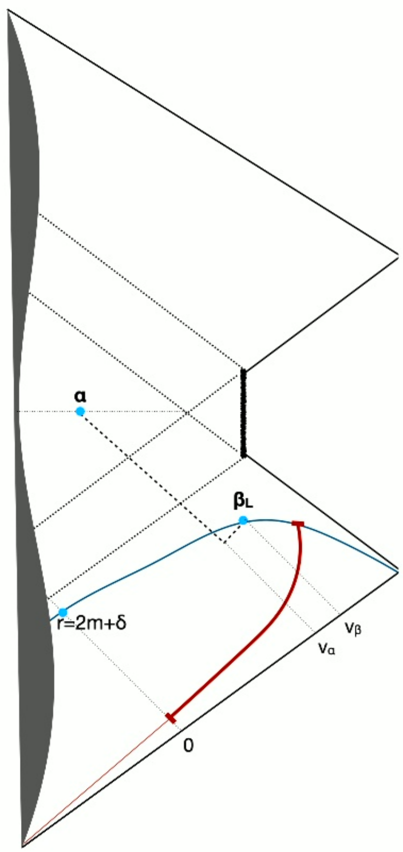
$$r_*(R) = \int_{2m+\delta}^{R} \frac{dr}{F(r)}. \quad F(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$

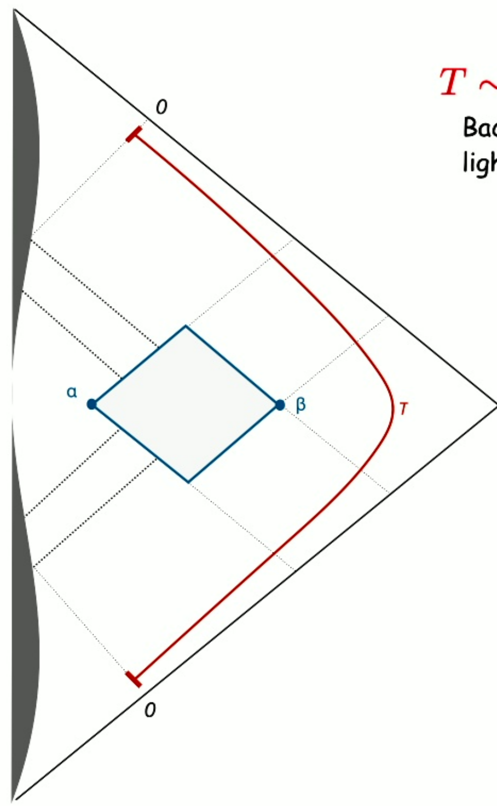
Han, Soltani, CR, 2023





Christodoulou, CR 2015





Relativistic
correction

$$T \sim 2R + 4m\ln(R-2m) - 4m\ln\delta$$

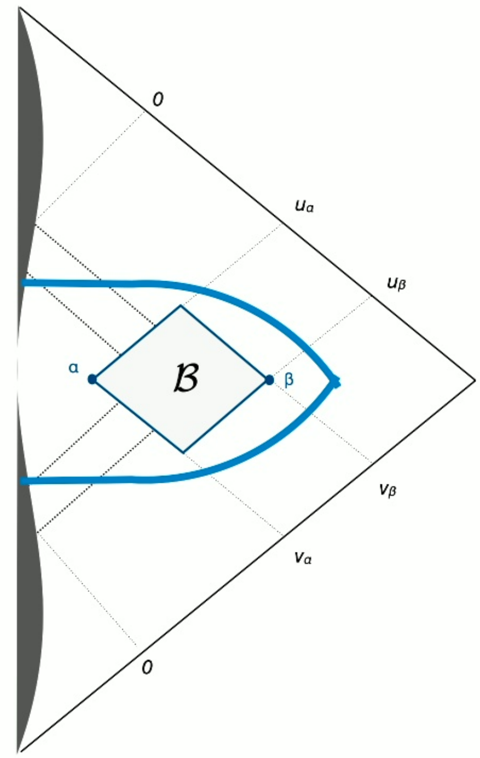
Back and forth
light travel time

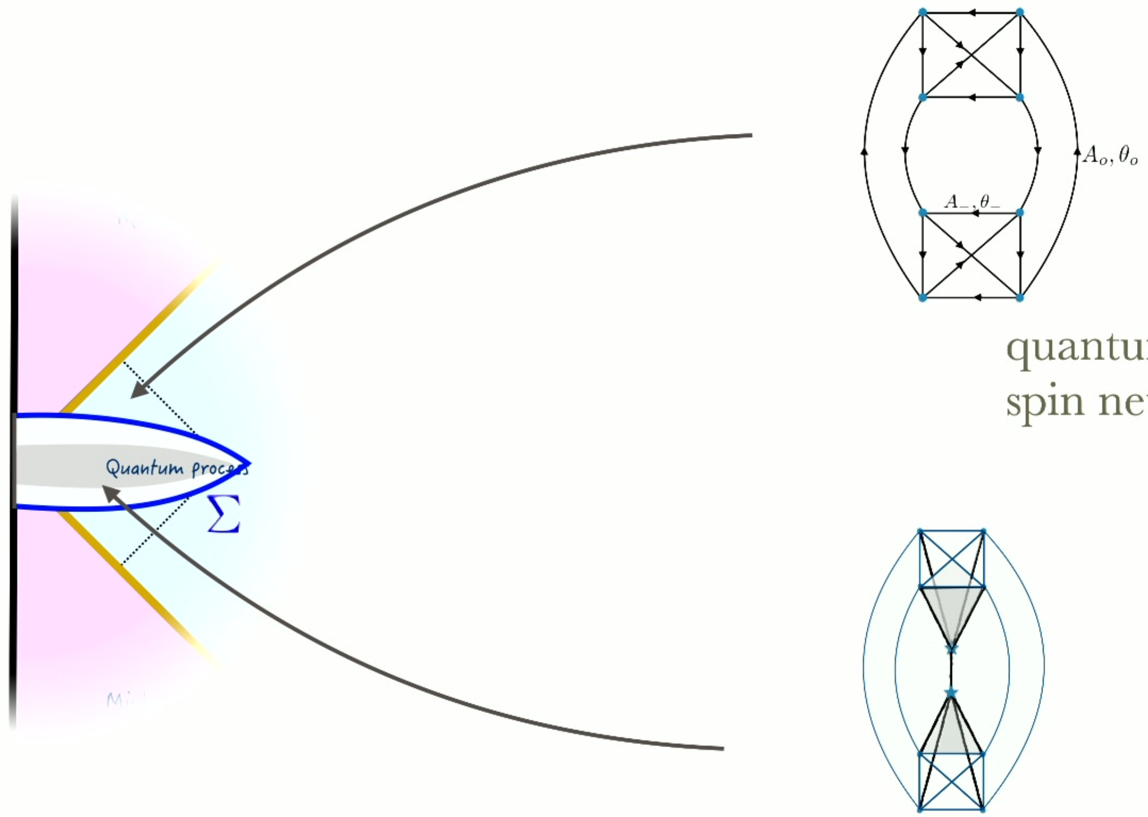
Life time of the
full process

$$\mathcal{T} \equiv -4m\ln\delta$$

The life time of the
full process

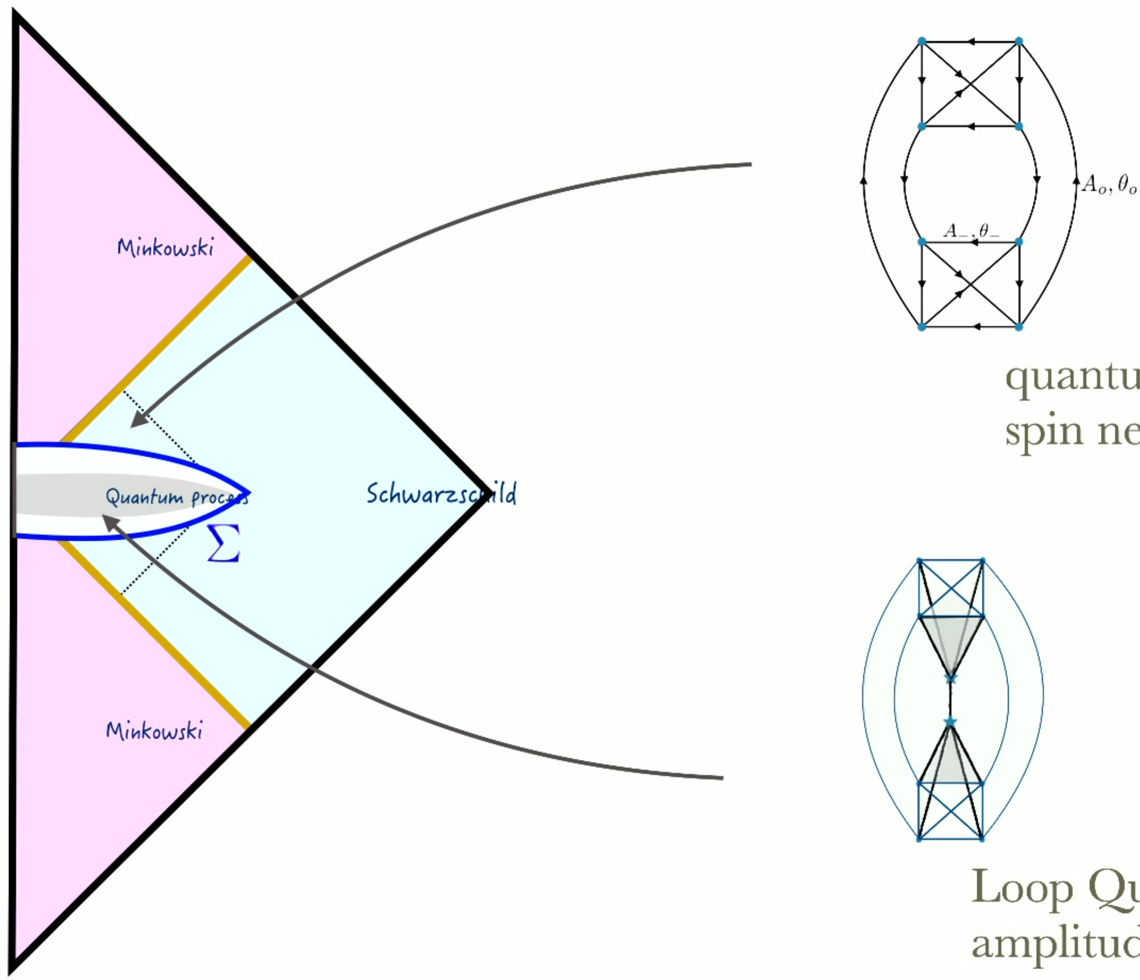
Quantum theory of the transition





quantum state
spin network

Loop Quantum Gravity
amplitude: spinfoam



quantum state
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Loop Quantum Gravity
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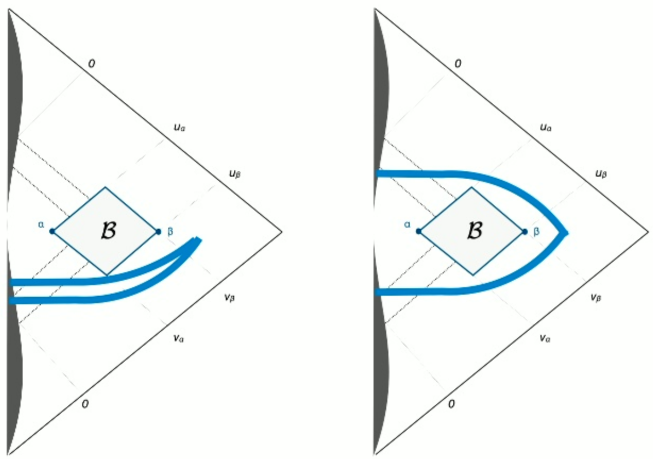
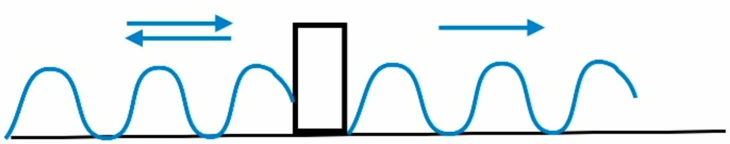
Probability of transition versus probability of non transition





Full expression for T(m):

$$W(m,T) = \sum_{\{j_\ell\}} w(m,T,j_\ell) \sum_{\{J_n\},\{K_n\},\{l_\ell\}} \left(\bigotimes_n N^{J_n}_{\{j_n\}}(\{\nu_n\},\{\alpha_n\})\, f^{J_n,K_n}_{\{j_n\}\{l_n\}} \right) \left(\bigotimes_n i^{K_n,\{l_n\}} \right)_\Gamma$$

$$w(m,T,j_\ell) = c(m) \prod_\ell d_{j_\ell} e^{-\frac{1}{2\eta_\ell}(j_\ell - \frac{(2\eta_\ell^2-1)}{2})^2} e^{i\gamma\zeta_\ell j_\ell} \ , \ \eta_\ell^2 \sim m^2$$

$$N^{J_n}_{\{j_n\}} = \left(\overleftarrow{\bigotimes_{\ell\in n}} D^{j_\ell}_{m_\ell j_\ell}(\{\nu_n\},\{\alpha_n\}) \right) i^{J_n,\{j_n\}}_{\{\vec{m}_n\}}$$

$$f^{K_n,J_n}_{\{j_n\}\{l_n\}} \equiv d_{J_n}\ i^{J_n,\{j_n\}}_{\{\vec{p}_n\}} \left(\int dr_n \frac{\sinh^2 r_n}{4\pi} \overrightarrow{\bigotimes_{\ell\in n}} d_{j_\ell l_\ell p_\ell}(r_n) \right) i^{K_n,\{l_n\}}_{\{\overleftarrow{p}_n\}}\ d_{K_n}$$

$$P \sim e^{-\alpha \frac{m^2}{m_{Pl}^2}} \sim e^{-\alpha \frac{Gm^2}{\hbar}}$$

α is a parameter of order unity (computable) that depends on the boundary coherent state

Chistodoulou, D'Amborsio, CR, Speziale, Vilensky, Soltani, Donà, Theofilis.

Summary

- LQG is a genuine tentative theory of non perturbative quantum gravity that can be used to study quantum gravitational phenomena.

- Black holes can die quantum tunnelling into white holes, stabilized by QG.

- These are long living remnants and are a possible dark matter candidate.