Title: Quantum Superpositions of Black Holes

Speakers: Robert Mann

Collection: Quantum Spacetime in the Cosmos: From Conception to Reality

Date: May 10, 2023 - 11:45 AM

URL: https://pirsa.org/23050123

Abstract: If relativistic gravitation has a quantum description, it must be meaningful to consider a spacetime metric in a genuine quantum superposition. Here I present a new operational framework for studying "superpositions of spacetimes" via model particle detectors. After presenting the general approach, I show how it can be applied to describe a spacetime generated by a BTZ black hole in a superposition of masses and how such detectors would respond. The detector exhibits signatures of quantum-gravitational effects reminiscent of Bekenstein's seminal conjecture concerning the quantized mass spectrum of black holes in quantum gravity. I provide further remarks in distinguishing spacetime superpositions that are genuinely quantum-gravitational, notably with reference to recent proposals to test gravitationally-induced entanglement, and those in which a putative superposition can be re-expressed in terms of dynamics on a single, fixed spacetime background.

Zoom Link: https://pitp.zoom.us/j/98277900018?pwd=SW92OWYrRFpkWC9QOS9NeTlQWkY5dz09

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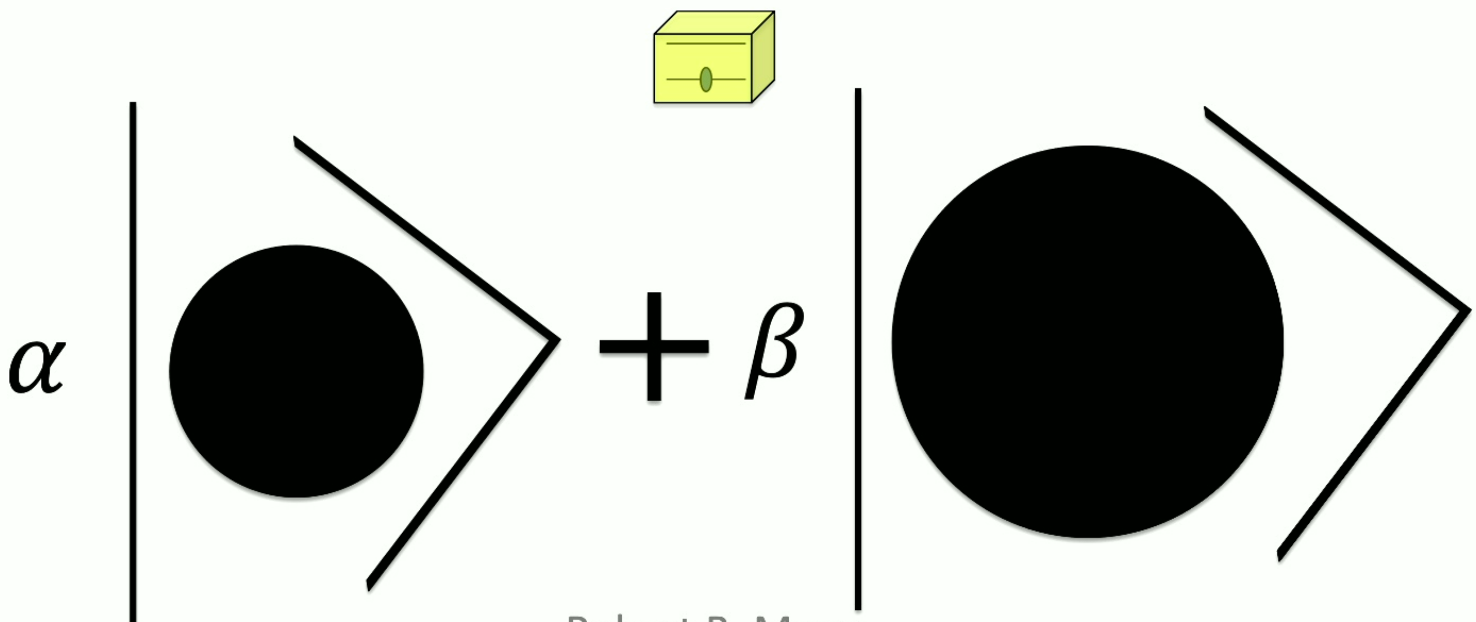
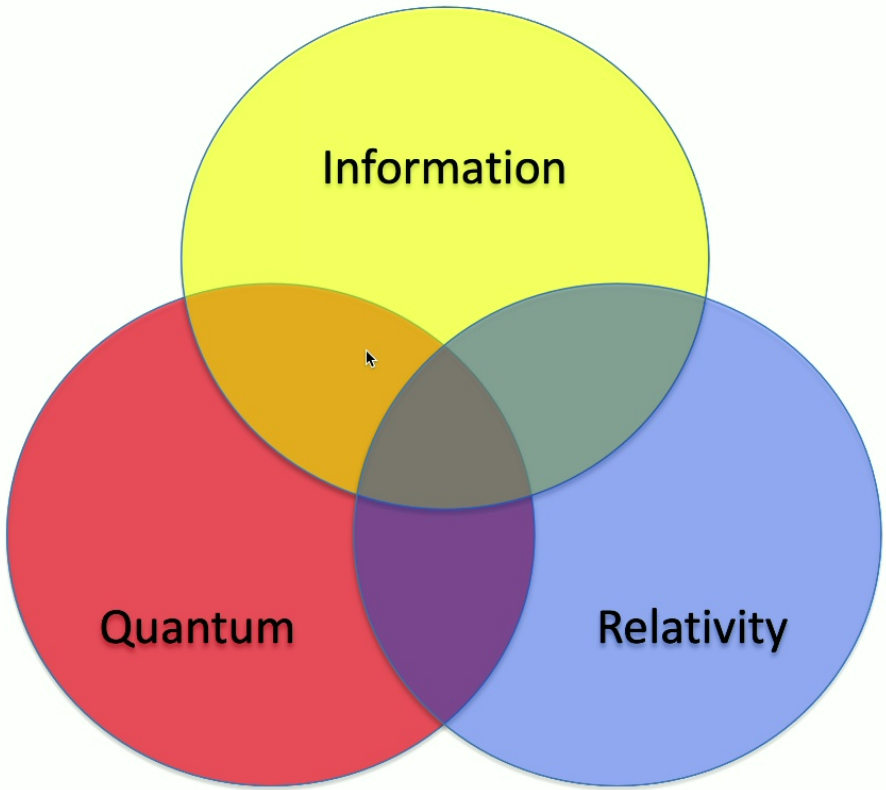
Quantum Superpositions of Black Holes



Robert B. Mann
J. Foo C. Arabaci M. Robbins C. Suryaatmadja M. Zych

Physical Review Letters **129** (2022) 181301 2111.13315

Relativistic Quantum Information



Some Big Picture Questions

- What is the relationship between Information and Spacetime?
- How do relativistic effects influence quantum information tasks?
  - And how can we exploit this?

Some Big Picture Questions

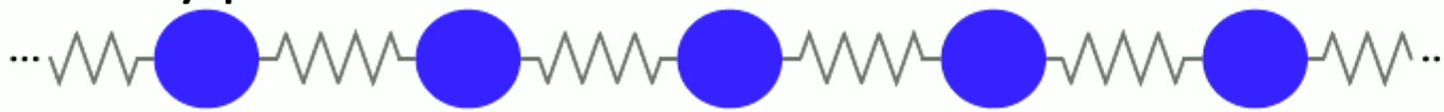
- What is the relationship between Information and Spacetime?
- How do relativistic effects influence quantum information tasks?
  - And how can we exploit this?
- What can we learn about spacetime through studying quantum information?

Probing Quantum Fields in Curved Spacetime(s)

- QFT in curved spacetime treats both quantum matter fields and gravitation as having equal physical significance
- Provides clues as to what can be expected from a `true' quantum gravity theory
- Many issues
  - Causal structure
  - Black hole entropy
  - Cosmic evolution
  - Vacuum entanglement
  - Black Hole Superposition

The Quantum Vacuum

- Empty space isn't empty!
  - It is filled with fields (electric, magnetic, scalar,...)
- These fields are like a set of coupled springs, one at every point



- Vacuum (ground) state is a global field state
  - springs vibrate with zero-point energy
  - entanglement between local modes
  - Bell-like inequality can be violated by local fluctuations
- And we can extract the entanglement!

Summers/Werner PL A110 (1985) 5; J. Math Phys 28 (1987) 2440

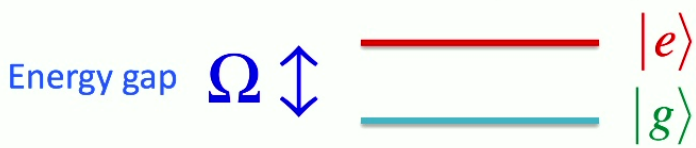
Reznik Fnd Phys 33 (2003) 167

Valentini PLA153(1991) 321 Salton/RBM/Menicucci NJP17 (2015) 035001

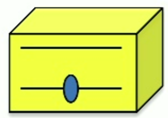
Quantum Detectors

- Model systems that couple to quantum fields
- Operational approach to probing quantum fields
- Example: Atoms – respond to EM field (photons!)
  - Absorb a photon → electron jumps up a level
  - Emit a photon → electron drops down a level
- Simplest atom? A qubit!

Unruh de Witt (UdW) detector (qubit)



B. deWitt in *General Relativity: An Einstein Centenary Survey* (CUP 1980)



UdW detector
→ like a quantum dot

$P \rightarrow$ Probability that the detector gets excited

Quantum Detectors

S-Y Lin, B.L.Hu PRD73 (2006) 124018
PRD76 (2007) 064008

$$S_I = \lambda_0 \int d\tau \int d^4x Q(\tau)\Phi(x)\delta^4\left(x^\mu - z^\mu(\tau)\right)$$

Vacuum

$$S = \frac{m_0}{2}\int d\tau\left[\left(\partial_\tau Q\right)^2 - \Omega_0^2 Q^2\right] - \int d^4x\sqrt{-g}\,\frac{1}{2}\left(\nabla\Phi(x)\right)^2 + S_I$$

detector field interaction

Cavity

$$\hat{H} = \Omega_d \hat{a}_d^\dagger \hat{a}_d + \frac{dt}{d\tau}\sum_n \omega_n \hat{a}_n^\dagger \hat{a}_n + H_I$$

$$H_I = \lambda(\tau)(\hat{a}_d e^{-i\Omega\tau} + \hat{a}_d^\dagger e^{i\Omega\tau})\sum_n\left(\hat{a}_n u_n[x(\tau),t(\tau)] + \hat{a}_n^\dagger u_n^*[x(\tau),t(\tau)]\right)$$

E.G. Brown, E. Martin-Martinez, N. Menicucci, RBM PRD87 (2013) 084062
D. Bruschi, A. Lee, I Fuentes J. Phys A46 (2013) 165303

2-Detector Formalism

$$S=-\int d^4x\sqrt{-g}\left[R+\frac{1}{2}\partial_\mu\Phi(x)\partial^\mu\Phi(x)-\xi R\Phi^2(x)\right]$$

$$+\int d\tau\left\{\frac{m_0}{2}\left[\left(\partial_\tau Q\right)^2-\Omega_0^2Q^2\right]+\sum_D\lambda_D\int d^4x Q_D(\tau)\Phi(x)\delta^4\left(x^\mu-z_D^\mu(\tau)\right)\right\}$$

$$U=\mathcal{T}e^{-i\int dt\left[\sum_D\frac{d\tau_D}{dt}H_{ID}(\tau_D)\right]}$$

$$H_{ID}(\tau)=\chi_D(\tau)\left[e^{i\Omega_D\tau}\sigma_D^+ + e^{-i\Omega_D\tau}\sigma_D^-\right]\Phi[z_D(\tau)]$$

switcher monopole operator

$$\chi_D=\exp\left(-\frac{(\tau-\tau_D)^2}{2\sigma_D^2}\right)$$

$$\sigma_D^+ := |1_D\rangle\langle 0_D| = |e_D\rangle\langle g_D|$$

$$\sigma_D^+ := |0_D\rangle\langle 1_D| = |g_D\rangle\langle e_D|$$

$$\rho_{ij} := \mathrm{Tr}_\Phi(U|\Psi\rangle_i\langle\Psi|_i U^\dagger)=\begin{pmatrix}1-P_A-P_B & 0 & 0 & X\\ 0 & P_B & C & 0\\ 0 & C^* & P_A & 0\\ X^* & 0 & 0 & 0\end{pmatrix}+\mathcal{O}(\lambda^4)$$

$$\rho = \begin{pmatrix} 1-P_A-P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$W(x,x') := \langle 0|\phi(x)\phi(x')|0\rangle$$

$$\tau_D = \gamma_D t$$

$$P_D = \lambda^2 \int_{-\infty}^{\infty} d\tau_D \int_{-\infty}^{\infty} d\tau_{D'}\, \chi_D(\tau_D)\chi_{D'}(\tau_{D'}) e^{-i\Omega_D(\tau_D-\tau_{D'})} W\big(x_D(\tau_D), x_D(\tau_{D'})\big) \quad D = A,B$$

Local excitations

$$C = \lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A-\Omega_B\tau_B)} W\big(x_A(t), x_B(t')\big)$$

Local correlations

$$X = -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \Big[\frac{d\tau_A}{dt'}\frac{d\tau_B}{dt} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_B\tau_B+\Omega_A\tau_A)} W\big(x_A(t'), x_B(t)\big) + \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A+\Omega_B\tau_B)} W\big(x_B(t'), x_A(t)\big)\Big]$$

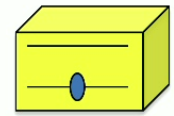
Non-Local correlations

Entanglement measure:

Concurrence $\rightarrow$ $\mathcal{C} = 2\mathcal{N} = \max\{0, |X| - \sqrt{P_A P_B}\} + \mathcal{O}(\lambda^4)$

Accelerating Detectors Get Hot!

$$\xrightarrow{\quad a \quad}$$



$$P \rightarrow \text{ thermal} \quad T = \frac{a}{2\pi}\left(\frac{\hbar}{k_B c}\right)$$

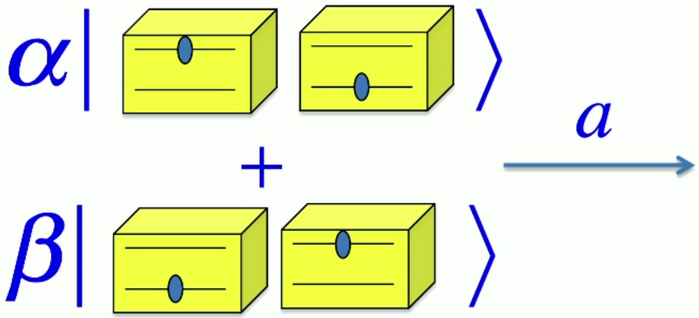
S.A. Fulling PRD7 (1973) 2850 P.C.W. Davies J Phys A8 (1975) 609
W. G. Unruh PRD14 (1976) 3251

- A single detector, accelerating uniformly forever, will respond as though it is in a heat bath
- Idealized conditions can be relaxed
  - Finite time acceleration
  - Motion within cavities
  - Non-uniform motion
- A cold vacuum to one observer is a hot vacuum to another
- Effect is robust

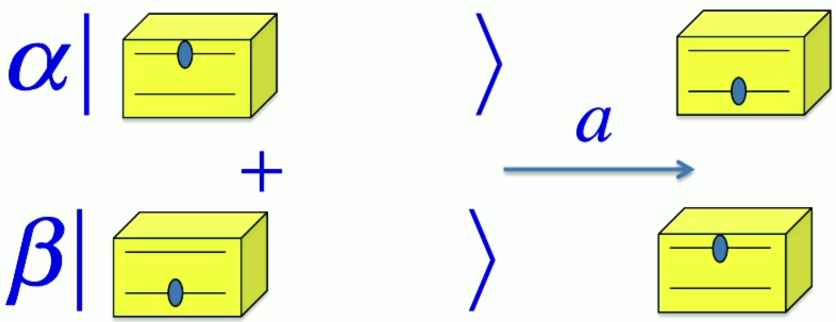
Can even turn this into a heat engine!

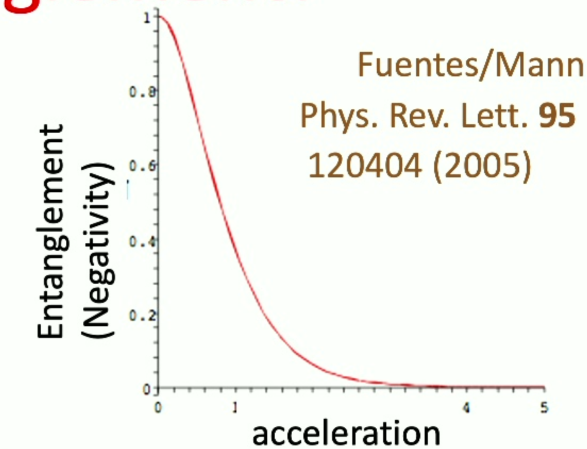
Arias/ Oliveira/ Sarandy JHEP **18** (2018) 168
F. Gray/RBM JHEP **11** (2018) 174

Entangled Accelerating Detectors
→ Degraded Entanglement!



Entangled Accelerating Detectors → Degraded Entanglement!



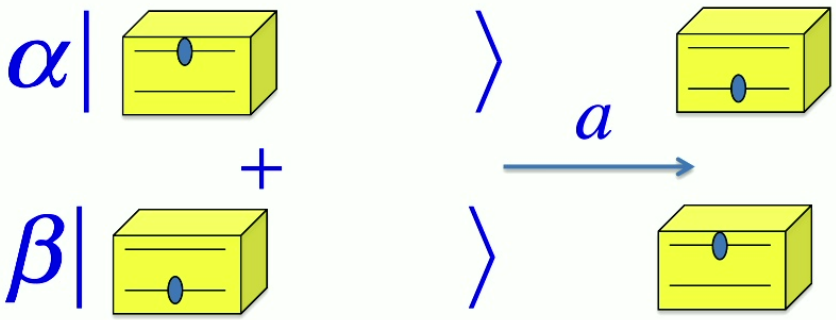


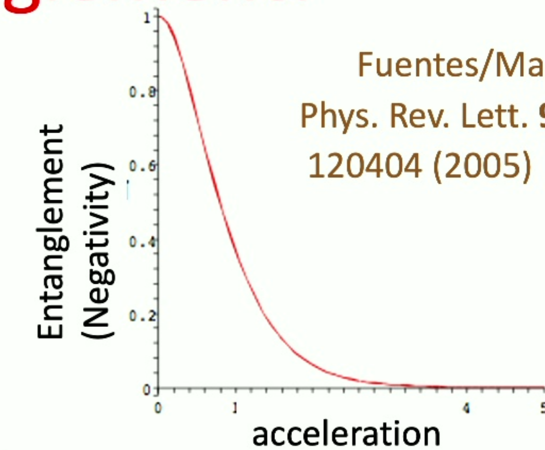
Fuentes/Mann Phys. Rev. Lett. **95** 120404 (2005)

Alsing/Fuentes/RBM/Tessier Phys. Rev. A**74** 032326 (2006)

- 2 maximally entangled detectors lose entanglement if one of them accelerates!
- More acceleration → less entanglement!
- But gravity and acceleration are locally the same!

Entangled Accelerating Detectors → Degraded Entanglement!



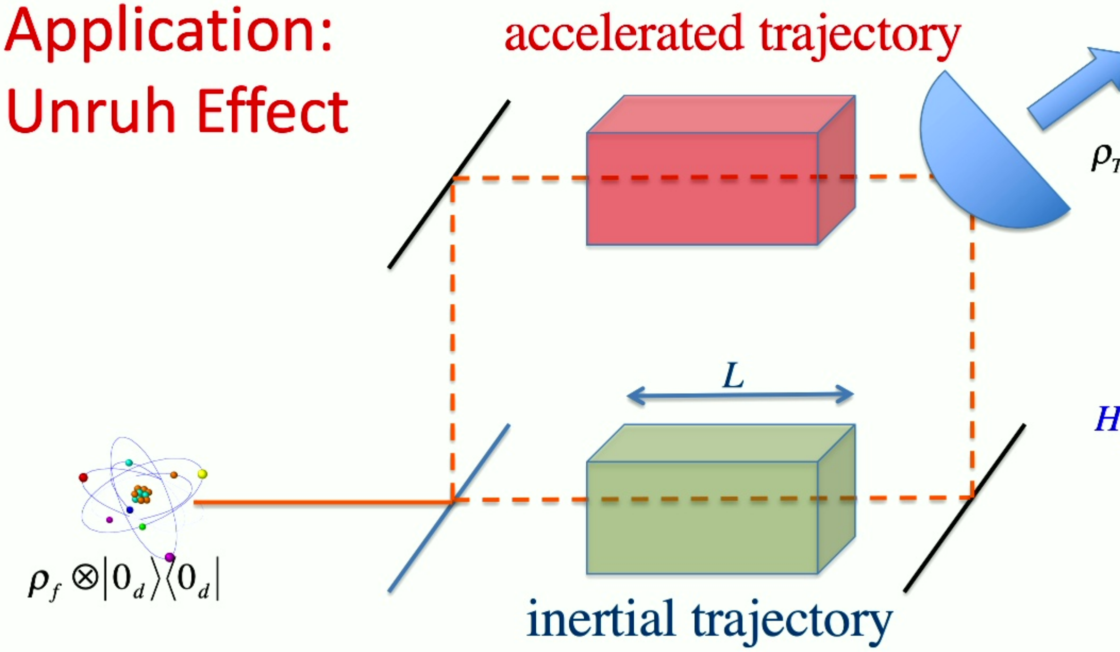


Fuentes/Mann
Phys. Rev. Lett. **95**
120404 (2005)

Alsing/Fuentes/RBM/Tessier
Phys. Rev. A**74**
032326 (2006)

- 2 maximally entangled detectors lose entanglement if one of them accelerates!
- More acceleration → less entanglement!
- But gravity and acceleration are locally the same!
- → Gravity (curved spacetime) should affect quantum entanglement!

Application: Unruh Effect



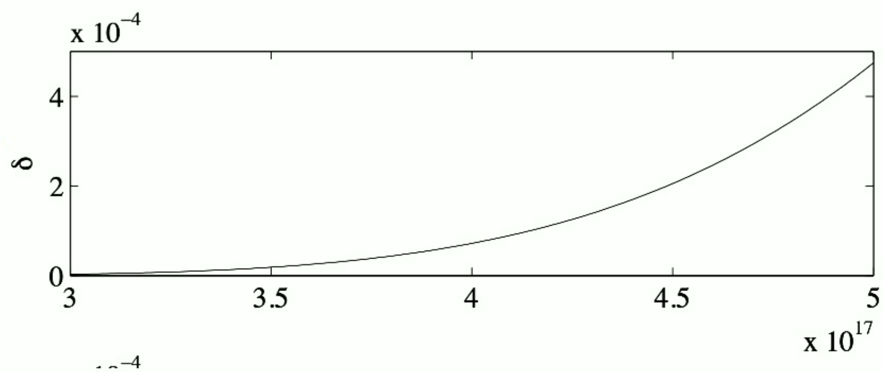
Phase difference

$$\rho_T = U_R^\dagger\left(\rho_f \otimes |0_d\rangle\langle 0_d|\right)U_R$$

$$H_T = \Omega_a a^\dagger a + \Omega_b b^\dagger b + \lambda(b + b^\dagger)[a^\dagger e^{i(kx-\Omega_a t)} + a e^{-i(kx-\Omega_a t)}]$$

Difference in Berry Phase

$$\delta = \gamma_a - \gamma_I$$



$\Omega_a \simeq 2.0\text{GHz}$

$\Omega_b \simeq 2.0\text{GHz}$

$\lambda \simeq 34\text{Hz}$

Quantum Controlled Detectors

Foo/Onoe/Zych
PRD **102** (2020) 085013
Foo/Onoe/RBM/Zych
PRR **3** (2021) 043056

Recall: Single Detector

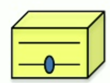
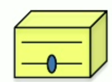
$$H_{ID}(\tau) = \chi_D(\tau)\left[e^{i\Omega_D\tau}\sigma_D^+ + e^{-i\Omega_D\tau}\sigma_D^-\right]\Phi[z_D(\tau)]$$

$$W(x,x') := \langle 0|\phi(x)\phi(x')|0\rangle$$

$$P_D = \lambda^2\int_{-\infty}^{\infty} d\tau_D \int_{-\infty}^{\infty} d\tau_{D'}\,\chi_D(\tau_D)\chi_{D'}(\tau_{D'})e^{-i\Omega_D(\tau_D-\tau_{D'})}W\left(x_D(\tau_D), x_D(\tau_{D'})\right) \quad D = A,B$$

$$\tau_D = \gamma_D t$$

Superposed Detector → use Quantum Control

$\alpha|$  $\psi_1\rangle + \beta|$  $\psi_2\rangle$

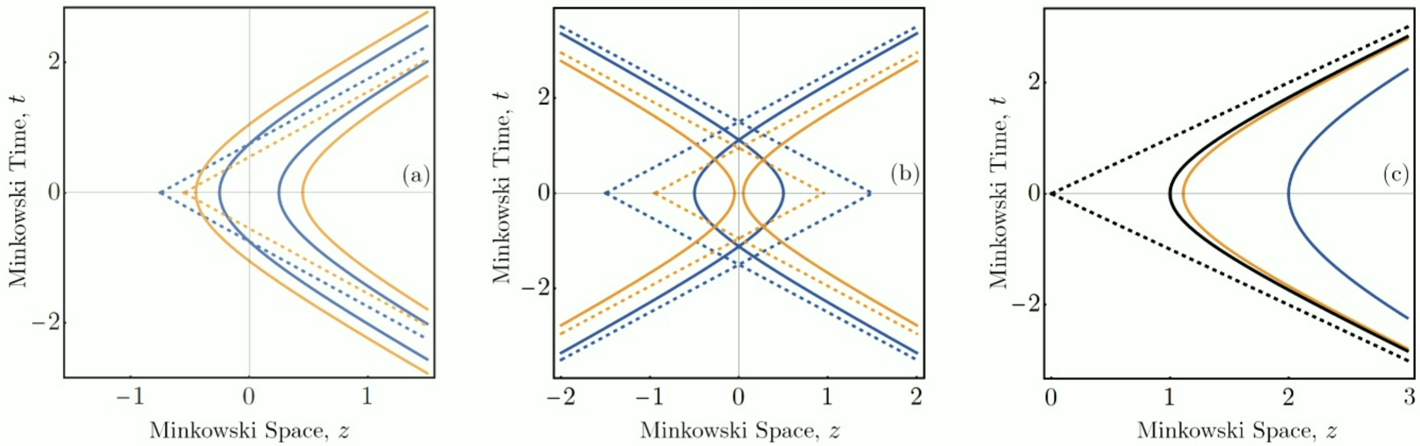
$$\hat{H}_I(\tau) = \lambda\left[e^{i\Omega\tau}\sigma^+ + e^{-i\Omega\tau}\sigma^-\right]\sum_{i=1}^{N}\chi_i(\tau)\Phi(\mathsf{z}_i(\tau)) \otimes |c_i\rangle\langle c_i|$$

Measure in control state $|c\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|c_i\rangle$

$$P_D = \frac{\lambda^2}{N^2}\sum_{i,j=1}^{N}\int_{-\infty}^{\infty} d\tau'\chi_i(\tau')e^{-i\Omega\tau'}\int_{-\infty}^{\infty} d\tau''\overline{\chi_j}(\tau'')e^{-i\Omega\tau''}\mathcal{W}_+^{ji}(\tau',\tau'')$$

Superposed trajectories exhibit interference

$$P_D = \frac{\lambda^2}{N^2} \sum_{i,j=1}^{N} \int_{-\infty}^{\infty} d\tau' \chi_i(\tau') e^{-i\Omega\tau'} \int_{-\infty}^{\infty} d\tau'' \overline{\chi_j}(\tau'') e^{-i\Omega\tau''} \mathcal{W}_+^{ji}(\tau', \tau'')$$



Superposed
Parallel Accelerations

Superposed
Anti-parallel Accelerations

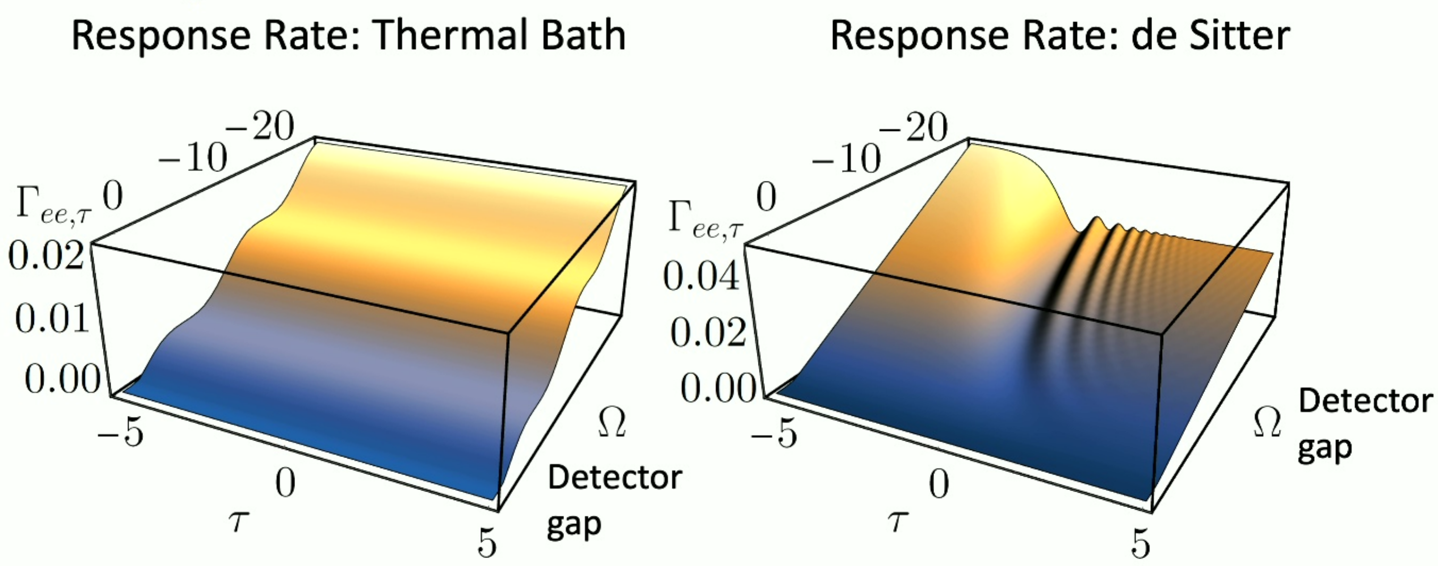
Superposed
Accelerations of
Different Magnitude

$$T_U \neq \frac{a}{2\pi}$$

Superposed Detectors with same uniform acceleration do NOT have a thermal response!

A Superposed Detector gains information about the field!

$$\Gamma \propto \sum_{i,j=1}^{N} \int_{-\infty}^{\tau} d\tau' \chi_j(\tau') e^{-i\Omega\tau'} \int_{-\infty}^{\tau} d\tau'' \overline{\chi}_i(\tau'') e^{-i\Omega\tau''} \mathcal{W}_{+}^{ji}(\tau', \tau'')$$



- Superposed detector separated by a distance *L*
- Single detector cannot distinguish between a thermal bath and conformal vacuum of expanding universe at same temperature
- Superposed detector CAN distinguish these two settings

Schroedinger's Cat in de Sitter Space

Superpositions of stationary detector trajectories in a single spacetime ⟷ A single detector in a superpositions of diffeomorphic spacetimes

$$|\Psi\rangle_{\text{iFD}} = \frac{1}{\sqrt{2}}\big(|\xi\rangle + |\xi + \mathcal{L}\rangle\big)|g\rangle|0_{\text{dS}}\rangle$$
$$= \underbrace{\frac{1}{\sqrt{2}}\big(\mathbb{I} + \hat{\mathcal{T}}(\mathcal{L})\big)|\xi\rangle}_{\text{Detector in superposed trajectory}}|g\rangle|0_{\text{dS}}\rangle$$

Time evolution

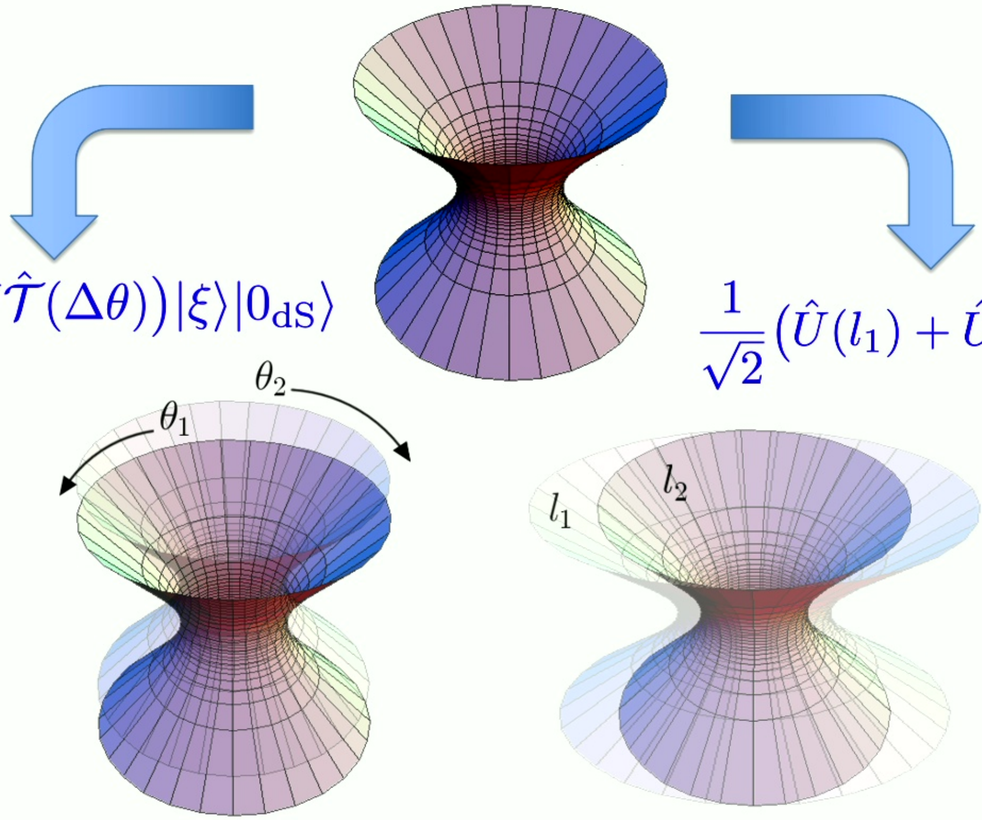
$$\hat{U}|\Psi\rangle_{\text{iFD}} = \frac{1}{\sqrt{2}}\big(\hat{U} + \hat{U}\hat{\mathcal{T}}(\mathcal{L})\big)|\xi\rangle|0_{\text{dS}}\rangle|g\rangle$$

$$|\Psi\rangle_{FD} = \frac{1}{2}\underbrace{\big(\hat{U}(\xi) + \hat{U}(\xi + \mathcal{L})\big)|0_{\text{dS}}\rangle}_{\text{superposed spacetime/field}}|g\rangle$$

Measure in control basis

Foo/RBM/Zych CQG **38** (2021) 115010

de Sitter Spacetime

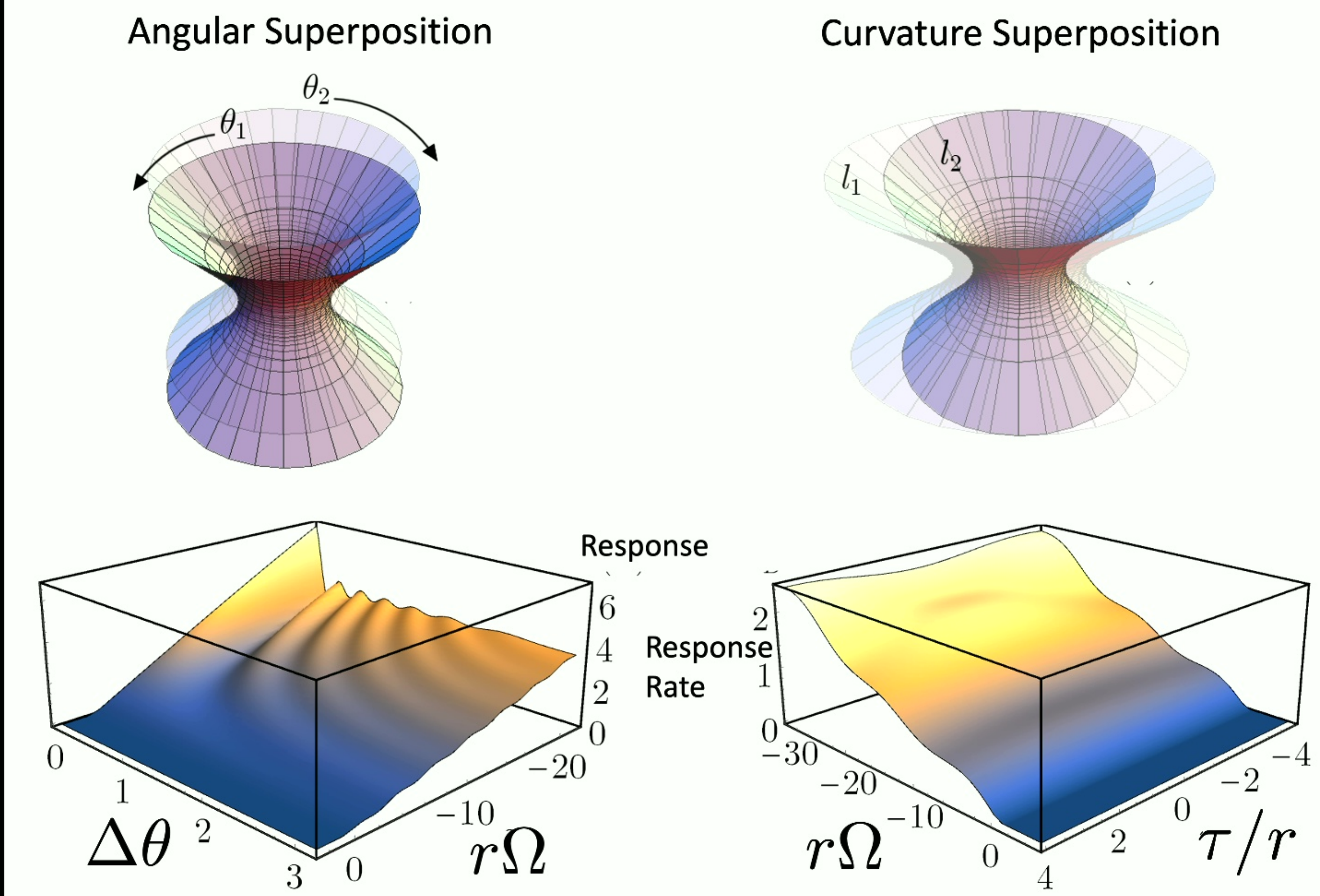


$$\frac{1}{\sqrt{2}}\big(\hat{U} + \hat{U}\hat{\mathcal{T}}(\Delta\theta)\big)|\xi\rangle|0_{\rm dS}\rangle$$

$$\frac{1}{\sqrt{2}}\big(\hat{U}(l_1) + \hat{U}(l_2)\big)|\xi\rangle|0_{\rm dS}\rangle$$

Superposed
de Sitter spacetimes
with different angular separations

Superposed
de Sitter spacetimes
with different curvatures



$l = 1.0 \quad R_D = 0.5$

$l_A/l_B = \sqrt{19}/4 \quad R_D = 1.0$

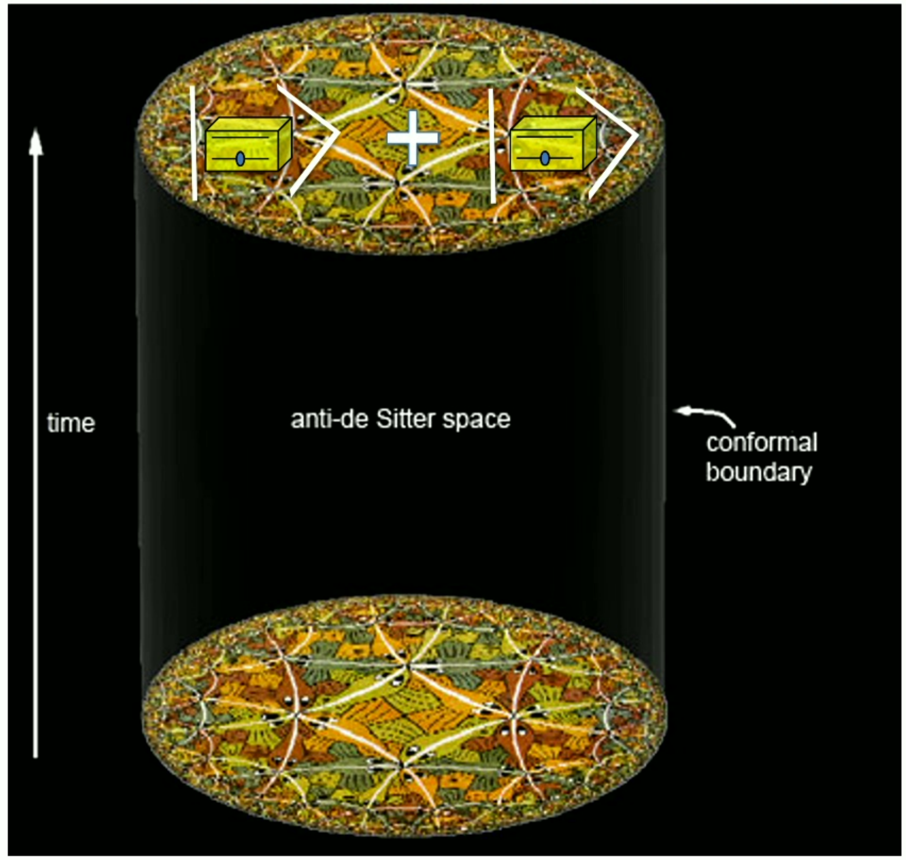
Black Hole Superposition

- Still lack a quantum theory of gravity
- General Expectation: Spacetime superposition
- Specifically: black hole superposition
  - Complicated in general: curvature not constant
  - Wightman functions are mode superpositions
- Test lab: BTZ black hole
  - Constant curvature black hole
  - Superpose using methods from de Sitter space

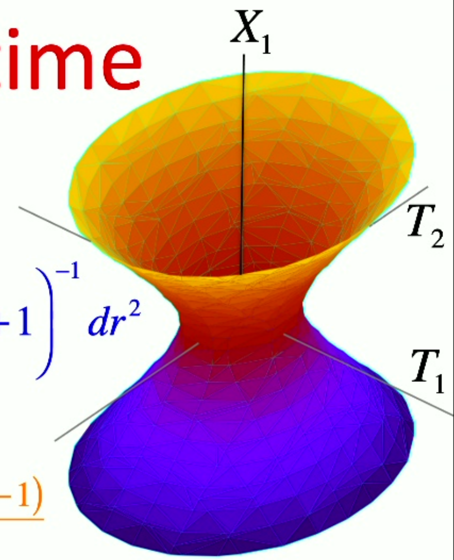
Two (superposed) Detectors in Anti de Sitter Space

2+1: Henderson/Hennigar/Smith/Zhang/RBM JHEP **05** (2019) 178

3+1: Ng/Martin-Martinez/RBM PRD98 (2018) 125005



Anti de Sitter Spacetime



$$\sum_{J=1}^{D-1} X_J^2 - T_1^2 - T_2^2 = -\ell^2$$

$$ds^2 = -dT_1^2 - dT_2^2 + \sum_{J=1}^{D-1} dX_J^2$$

Hyperboloid in flat spacetime

$$ds^2 = -\left(\frac{r^2}{\ell^2}+1\right)dt^2 + \left(\frac{r^2}{\ell^2}+1\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

$$\Lambda = -\frac{(D-2)(D-1)}{2\ell^2}$$

Conformal scalar coupling

$$W_{AdS}^{(\zeta)}(x,x') = \frac{1}{4\pi\ell\sqrt{2}}\left(\frac{1}{\sqrt{\sigma_\epsilon(x,x')}} - \frac{\zeta}{\sqrt{\sigma_\epsilon(x,x')+2}}\right) \quad 2+1$$

Geodesic length

$\zeta = 1$ (Dirichlet)

$\zeta = 0$ (Transparent)

$\zeta = -1$ (Neumann)

$$W_{AdS}^{(\zeta=1)}(x,x') = \sum_{\omega=0}^{\infty}\sum_{lm}\frac{1}{2\omega}e^{-i\omega(t-t')}\varphi_{\omega lm}(x)\bar{\varphi}_{\omega lm}(x') \quad 3+1$$

Sum over modes

$$\Phi_{\omega lm}(t,x) = \frac{1}{\sqrt{2\omega}}e^{-i\omega t}\varphi_{\omega lm}(x)$$

The BTZ Black Hole

$$ds^2 = -dT_1^2 - dT_2^2 + dX_1^2 + dX_2^2$$ covering space metric

$$-l^2 = -T_1^2 - T_2^2 + X_1^2 + X_2^2$$ hyperbolic constraint surface

$$T_1 = l\sqrt{\frac{r^2}{l^2}}\cosh\phi \qquad X_1 = l\sqrt{\frac{r^2}{l^2}}\sinh\phi$$

$$T_2 = l\sqrt{\frac{r^2}{l^2} - 1}\sinh\frac{t}{l} \qquad X_2 = l\sqrt{\frac{r^2}{l^2} - 1}\cosh\frac{t}{l}$$

$$ds^2 = -\left(\frac{r^2}{l^2} - 1\right)dt^2 + \left(\frac{r^2}{l^2} - 1\right)dr^2 + r^2 d\phi^2$$

The BTZ Black Hole

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- ϕ unidentified $\Rightarrow$ AdS Rindler space
- $\phi \to \phi + 2\pi\sqrt{M} \Rightarrow$ BTZ black hole

The BTZ Black Hole

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- ϕ unidentified $\Rightarrow$ AdS Rindler space
- $\phi \to \phi + 2\pi\sqrt{M} \Rightarrow$ BTZ black hole

$$r = \tilde{r}/\sqrt{M} \quad t = \tilde{t}\sqrt{M} \quad \phi = \tilde{\phi}\sqrt{M}$$

$$ds^2 = -\left(\frac{\tilde{r}^2}{l^2} - M\right)d\tilde{t}^2 + \left(\frac{\tilde{r}^2}{l^2} - M\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2$$

$\tilde{\phi} \to \tilde{\phi} + 2\pi$

BTZ black hole

The Superposed BTZ Black Hole

Arabaci/Foo/RBM/Zych
PRL (to appear)

$$ds^2 = -\left(\frac{r^2}{l^2} - 1\right)dt^2 + \left(\frac{r^2}{l^2} - 1\right)dr^2 + r^2 d\phi^2 \quad \Gamma : \phi \to \phi + 2\pi\sqrt{M}$$

Superpose the identifications:

$$\alpha \left| M_A \right\rangle + \beta \left| M_B \right\rangle$$

$$\Gamma_A : \phi \to \phi + 2\pi\sqrt{M_A} \qquad \Gamma_B : \phi \to \phi + 2\pi\sqrt{M_B}$$

AdS Wightman fn

$$W^{(\zeta)}_{AdS}(x,x') = \frac{1}{4\pi\ell\sqrt{2}}\left(\frac{1}{\sqrt{\sigma_\epsilon(x,x')}} - \frac{\zeta}{\sqrt{\sigma_\epsilon(x,x')+2}}\right)$$

$\eta = \pm 1$ (untwisted / twisted)

$$W_{\mathrm{BTZ}}(\mathsf{x},\mathsf{x}') = \frac{1}{\sum_k \eta^{2k}} \sum_n \sum_m \eta^n \eta^m W_{\mathrm{AdS}}(\Gamma^n \mathsf{x}, \Gamma^m \mathsf{x}')$$

BTZ Wightman fn

$$W^{(AB)}_{\mathrm{BTZ}}(\mathsf{x},\mathsf{x}') = \frac{1}{\sum_k \eta^{2k}} \sum_{n,m} \eta^n \eta^m W_{\mathrm{AdS}}(\Gamma^n_A \mathsf{x}, \Gamma^m_B \mathsf{x}')$$

Superposed BTZ Wightman fn

$$W_{\mathrm{BTZ}}^{(AB)}(\mathsf{x},\mathsf{x}') = \frac{1}{\sum_k \eta^{2k}} \sum_{n,m} \eta^n \eta^m W_{\mathrm{AdS}}(\Gamma_A^n \mathsf{x}, \Gamma_B^m \mathsf{x}')$$

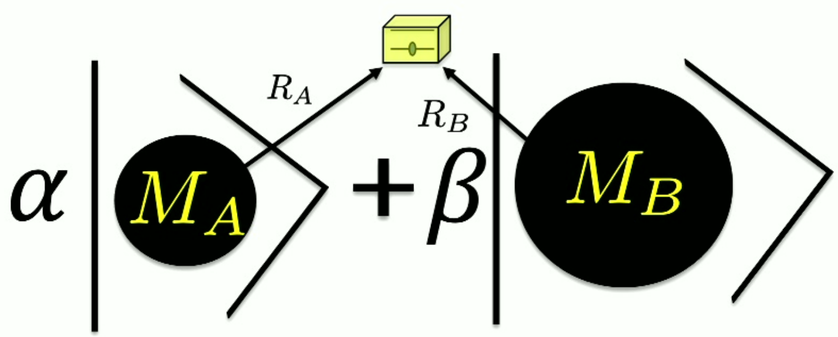
$$W_{AdS}^{(\zeta)}(x,x') = \frac{1}{4\pi\ell\sqrt{2}}\left(\frac{1}{\sqrt{\sigma_\epsilon(x,x')}} - \frac{\zeta}{\sqrt{\sigma_\epsilon(x,x')+2}}\right)$$

$\zeta = 1$ (Dirichlet)

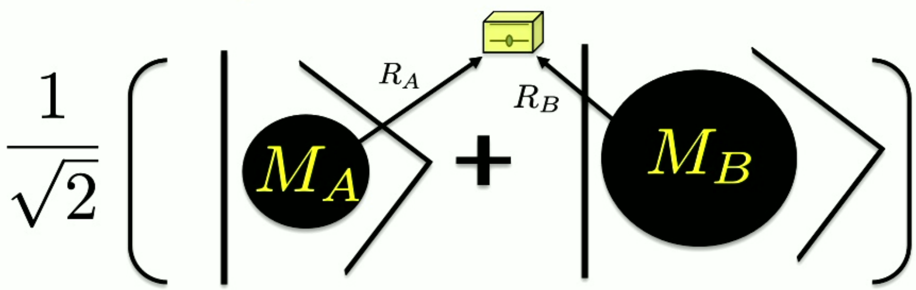
$\zeta = 0$ (Transparent)

$\zeta = -1$ (Neumann)

$$\sigma(\Gamma_A^n \mathsf{x}, \Gamma_B^m \mathsf{x}') = \sqrt{\frac{R_A^2}{l^2}}\sqrt{\frac{R_B^2}{l^2}} \cosh\left[2\pi(m\sqrt{M_A} - n\sqrt{M_B})\right] - 1$$
$$- \sqrt{\frac{R_A^2}{l^2} - 1}\sqrt{\frac{R_B^2}{l^2} - 1} \cosh\left(\frac{t-t'}{l}\right)$$



Dynamical Evolution



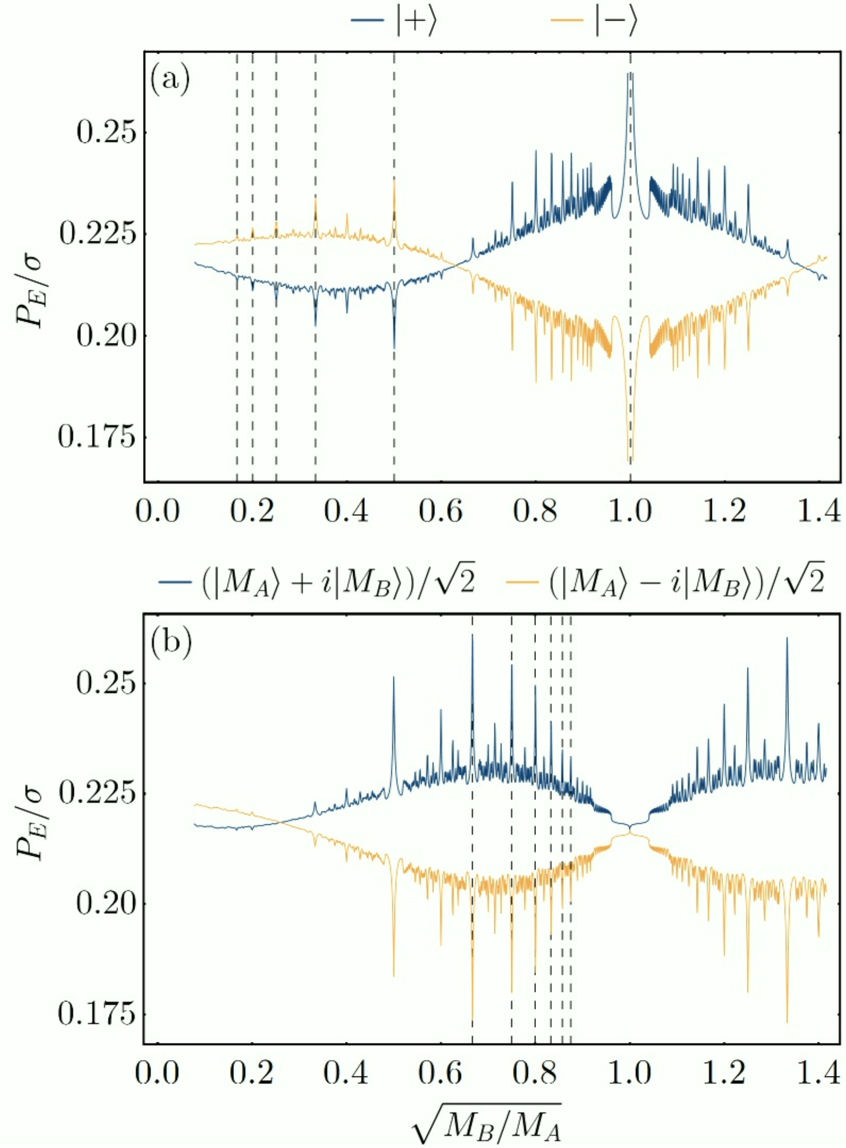
$$|\psi(t_i)\rangle = \frac{1}{\sqrt{2}}(|M_A\rangle + |M_B\rangle)|0\rangle|g\rangle \Longrightarrow |\psi(t_f)\rangle = e^{-iH_{0,S}t_f}\hat{U}e^{iH_{0,S}t_i}|\psi(t_i)\rangle$$

Condition on $|\pm\rangle = (|M_A\rangle \pm |M_B\rangle)/\sqrt{2}$ and trace out the field

$$\mathrm{Tr}_\phi\Big[\langle\pm|\psi(t_f)\rangle\langle\psi(t_f)|\pm\rangle\Big] = \frac{|g\rangle\langle g|}{2}P_G^{(\pm)} + \lambda^2\frac{|e\rangle\langle e|}{2}P_E^{(\pm)}$$

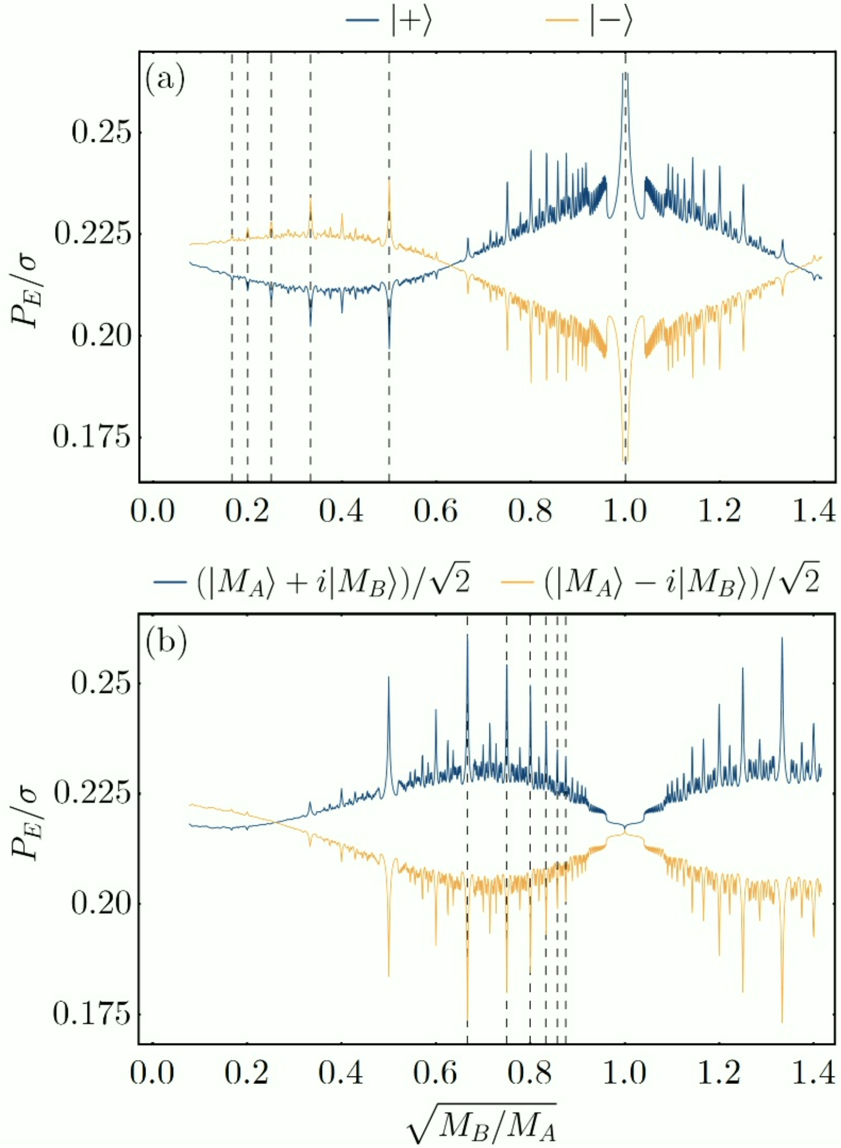
$$= \frac{|g\rangle\langle g|}{2}\Big(1 \pm \cos(\Delta E \Delta t)\Big)\left[1 - \frac{\lambda^2}{2}\int_{-t_f}^{t_f} d\tau \int_{-t_f}^{t_f} d\tau' \eta(\tau)\eta(\tau')e^{-i\Omega(\tau-\tau')}\Big(W(x_A, x'_A) + W(x_B, x'_B)\Big)\right]$$

$$+\frac{\lambda^2|e\rangle\langle e|}{4}\int_{-t_f}^{t_f} d\tau \int_{-t_f}^{t_f} d\tau' \eta(\tau)\eta(\tau')e^{-i\Omega(\tau-\tau')}\Big(W(x_A, x'_A) + W(x_B, x'_B) \pm 2\cos(\Delta E \Delta t)W(x_A, x'_B)\Big)$$



Arabaci/Foo/RBM/Zych
PRL **129** (2022) 181301
2111.13315

Dashed lines:
$\sqrt{M_B/M_A} = (n-1)/n$
where $n = \{3, \ldots 8\}$



Arabaci/Foo/RBM/Zych
PRL **129** (2022) 181301
2111.13315

Dashed lines:
$\sqrt{M_B/M_A} = (n-1)/n$
where $n = \{3, \dots 8\}$

Resonant peaks at integer values of (mass ratios)$^{1/2}$!

Consistent with Bekenstein's black hole mass quantization conjecture

Rotating Black Holes

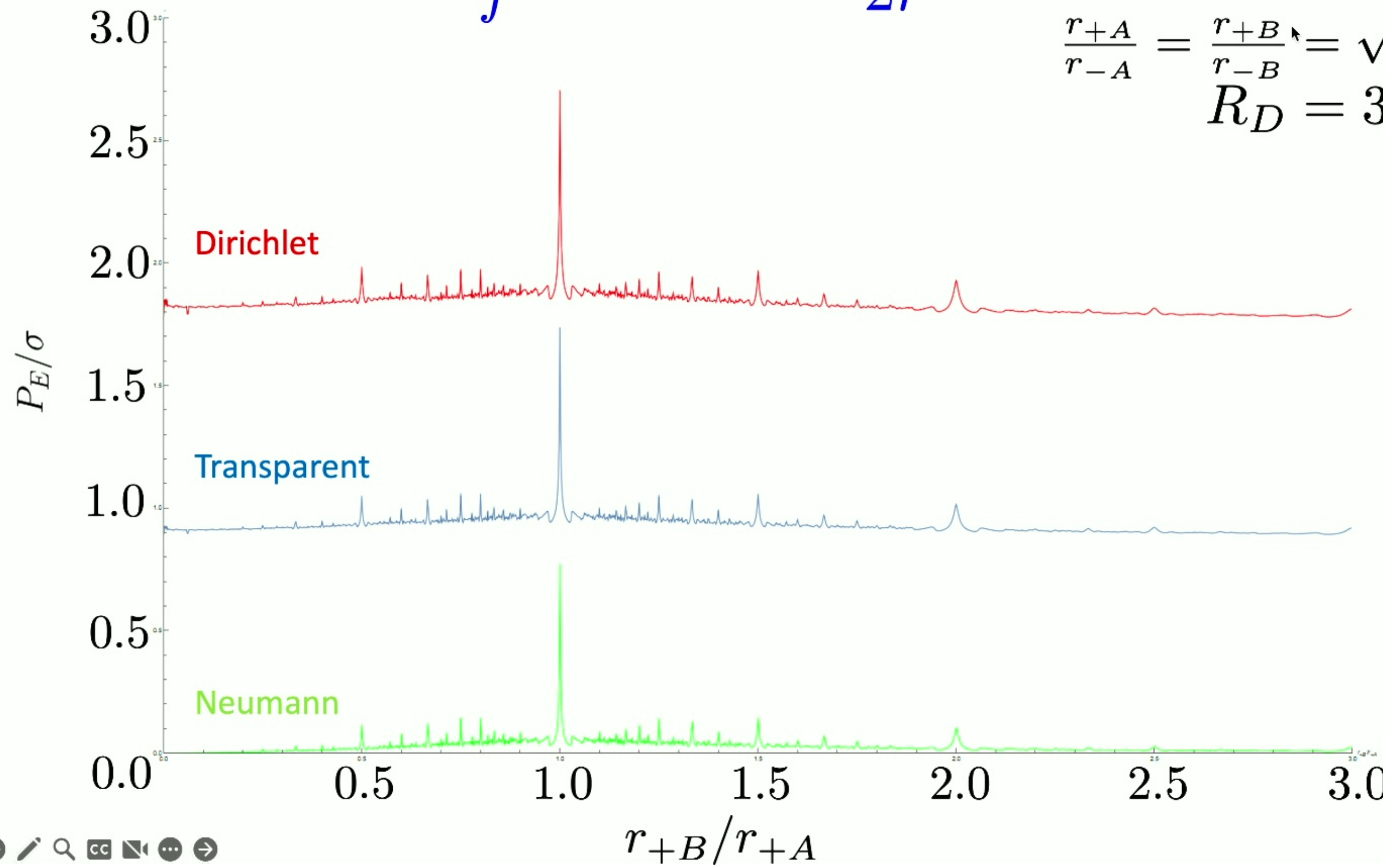
$$f = -M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}$$

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2(d\phi - \frac{J}{2r^2}dt)^2$$

$$r_{+A} = \ell/4$$

$$\frac{r_{+A}}{r_{-A}} = \frac{r_{+B}}{r_{-B}} = \sqrt{7}$$

$$R_D = 3.0$$



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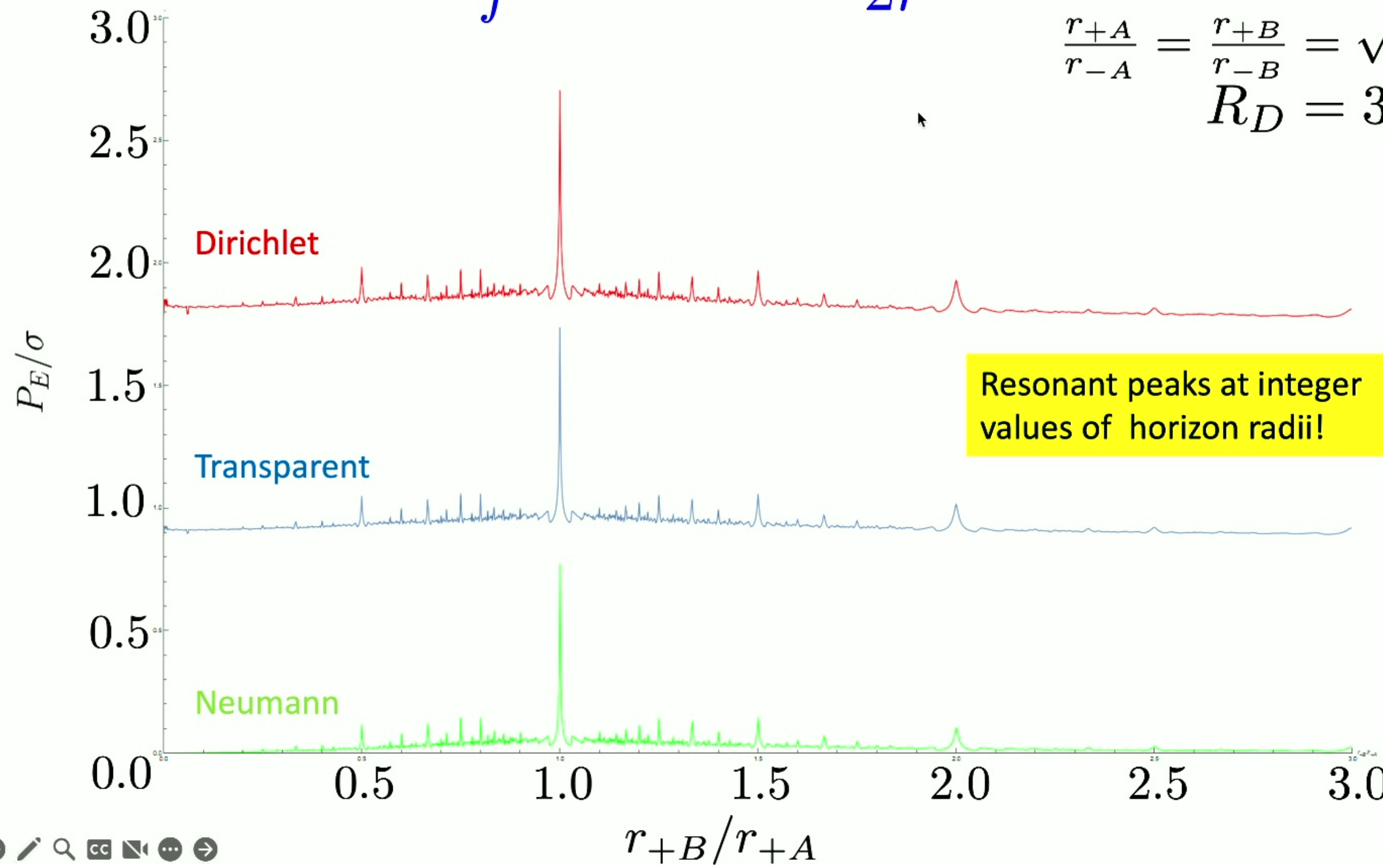

Resonant peaks at integer values of horizon radii!

Superposed Metric?

Classically:

$$g_{\mu\nu}(x) = -\lim_{x \to x'} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} \sigma(\mathsf{x}, \mathsf{x}')$$

Knowledge of Synge geodesic distance
→ metric

Quantum Mechanically:

$$g_{\mu\nu} = -\frac{1}{2} \left(\frac{\Gamma(d/2-1)}{(4\pi^{d/2})} \right)^{\frac{2}{d-2}} \lim_{x \to x'} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} W(\mathsf{x}, \mathsf{x}')^{\frac{2}{d-2}}$$

Saravani/Aslanbeigi/Kempf
PRD 93 (2016) 045026
Kempf
Front Phys **9** (2021) 247

Superposed Black Hole Metric?

$$g_{\mu\nu} = \Lambda(d) \lim_{x \to x'} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} \sum_{D,D'} f_D f^\star_{D'} W(x_D, x'_{D'})^{\frac{2}{d-2}}$$

Summary

- Construction of superposed spacetimes
  - curvature-superposed de Sitter
  - mass-superposed black hole
  - Operational description via Wightman function
  - Generalizable to other spacetimes

Arabaci/Foo/RBM/Zych
PRD **107** (2023) 045014
2208.12083

- Detector response
  - Peaks at rational values of horizon ratio
  - Consistent with Bekenstein' Conjecture
- Provides a pathway for understanding effects of quantum gravitational phenomena even without a quantum theory of gravity!