Title: Universality of black hole thermodynamics

Speakers:

Collection: Quantum Spacetime in the Cosmos: From Conception to Reality

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Universality of black hole thermodynamics

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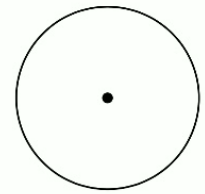
The Ohio State University

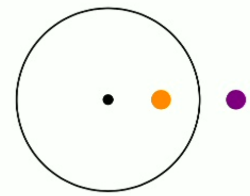
When black hole thermodynamics was discovered in the 1970s, black holes looked very puzzling

(a) The entropy appears to be $S_{bek} = \frac{A}{4G}$



But what is this entropy counting ?

(b) Black holes radiate by the creation of entangled pairs



This leads to the black hole information paradox

It is often said these days that black holes continue to be mysterious ....

I will argue that this is not so; we do understand black holes well
(if we use string theory as our theory of gravity)

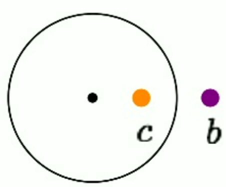
The information puzzle

Suppose we burn a piece of coal

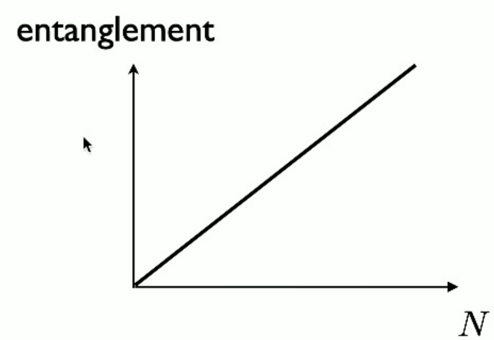
$|\psi_i\rangle$  → $|\psi_f\rangle$  

The radiation is in a definite state, so a device that checks if the radiation is in this state, the device will give the answer "yes" with 100% probability

Black holes radiate by the production of entangled pairs from the vacuum



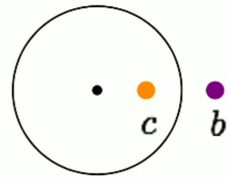
$$\frac{1}{\sqrt{2}}\left(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c\right)$$



Collide two high energy particles

$|\psi_i\rangle$

Make a black hole, evaporate through Hawking pairs



Radiation is in an entangled state, but there is nothing left that it is entangled with

$$|\Psi\rangle = \sum_{i=1}^{M} C_i \; |\chi_i\rangle_{bh} \otimes |\psi_i\rangle_{rad}$$

Measuring device will not give a unique answer for ANY choice of state for this radiation

This is the black hole information paradox

Hawking's original argument has now been made into a rigorous statement:

The small corrections theorem (SDM arxiv:0909.1038)

The effective small corrections theorem

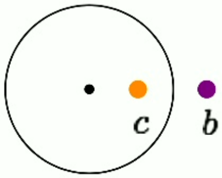
(Guo, Hughes, Mehta, SDM arxiv:2111.05295)

(i) If the EXACT quantum gravity theory is unitary

(ii) 'Normal' laboratory physics is required to hold to a better and better approximation as we recede far from the black hole

(iii) We require that the radiation is in a pure state (no remnants)

Then we cannot obtain low energy semiclassical physics around the horizon in any approximation



No !!

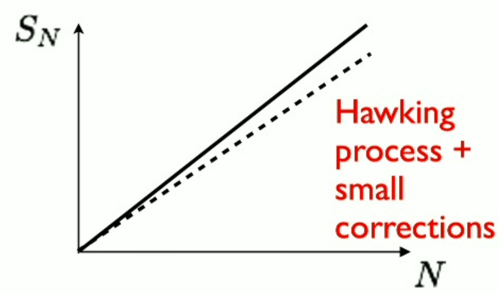
The essential technical step:

In the semiclassical effective theory the pairs have a state

$$\frac{1}{\sqrt{2}}(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c) \;\to\; \frac{1}{\sqrt{2}}(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c) + |\delta\psi_k\rangle \;, \qquad |\delta\psi_k\rangle| < \epsilon$$

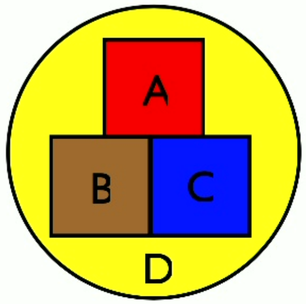
This forces the entanglement at step N in the exact theory to keep growing as

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$



Thus any small correction to low energy dynamics around the horizon cannot resolve the information paradox, we need an order unity correction

Proving this fact uses the strong subadditivity of quantum entanglement entropy, and so may not be very intuitive ...



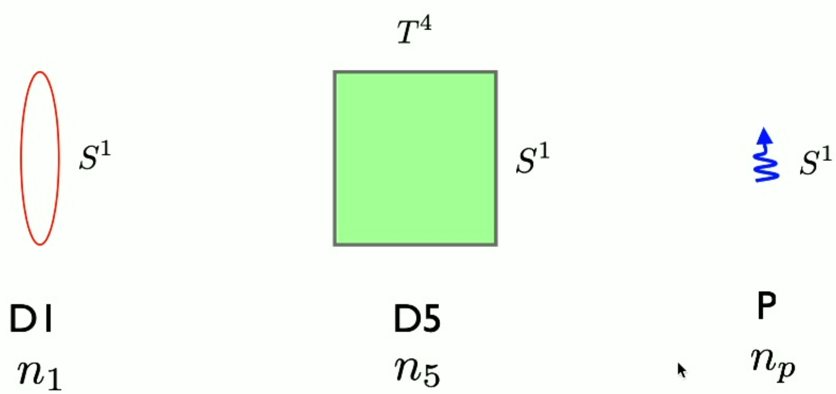
In string theory we make a black hole by taking a bound state of the fundamental objects in the theory: gravitons, strings, branes, ...

We compactify some dimensions

$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$

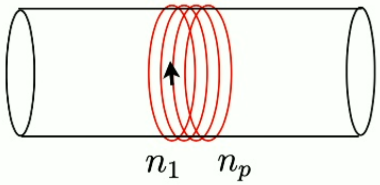
$$M_{9,1} \to M_{4,1} \times K3 \times S^1$$

We wrap branes etc. on the compact dimensions, and take a bound state of these objects



A bound state of these branes gives a black hole

The simplest hole is the 2-charge extremal hole ('small black hole')



String winding charge and momentum charge

At weak coupling, this is just a vibrating string, so we can find the entropy

$$S_{micro} = 4\pi\sqrt{n_1 n_2}$$
$$(K3 \times S^1)$$

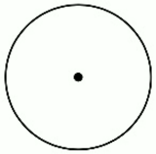
$$S_{micro} = 2\sqrt{2}\pi\sqrt{n_1 n_2}$$
$$(T^4 \times S^1)$$

(Sen 94, Vafa 95)

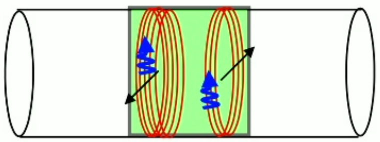
At strong coupling, we expect a black hole, so we find the Bekenstein entropy for a hole with the same charges

$$S_{micro} \sim S_{bek}$$ (Sen 95)

$$S_{micro} = S_{bek-wald} = \frac{A}{2G}$$ (Dabholkar Kallosh Maloney 04)



3 charge extremal $S_{micro} = 2\pi\sqrt{n_1 n_2 n_3}$



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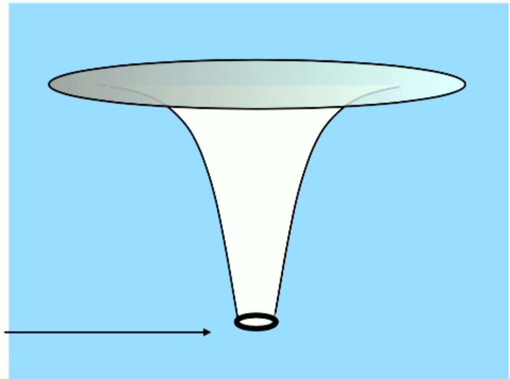


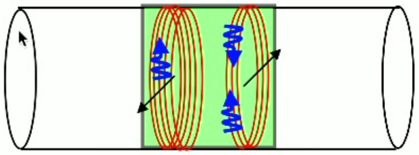
(Strominger + Vafa 96)

Similar expressions for near extremal, also extrapolate to far from extremal ...

3 charge near-extremal

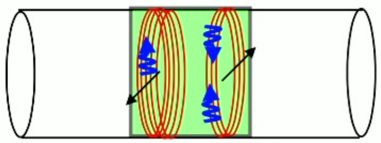
$$S_{micro} = 2\pi\sqrt{n_1 n_2}(\sqrt{n_3} + \sqrt{\bar{n}_3})$$

(Callan + Maldacena 96)

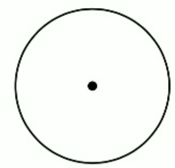


Thus we have support for the idea that the Bekenstein entropy counts the microstates of the black hole

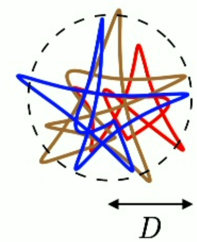
Weak coupling picture



Is this the strong coupling picture ?



Estimates of the bound state size suggested that due to 'fractionation' the size of the bound state grows with the number of branes and the coupling



$$D \sim \left[\frac{\sqrt{n_1 n_5 n_p}\, g^2\, \alpha'^4}{V\, L} \right]^{\frac{1}{3}} \sim R_H$$

(SDM hep-th/9706151)

Can we make this idea more precise ?

3 charge extremal

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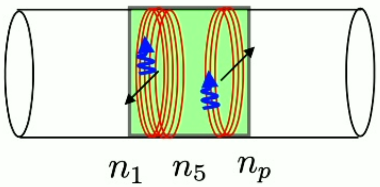

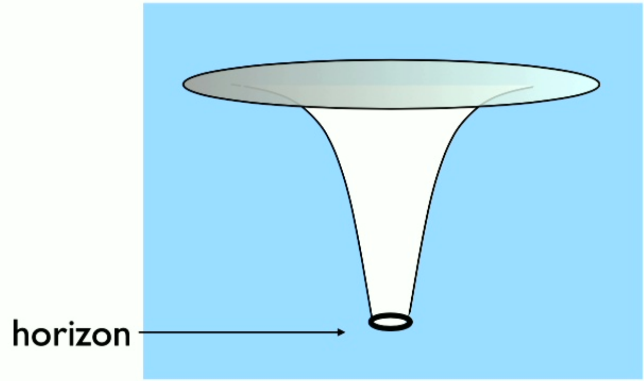

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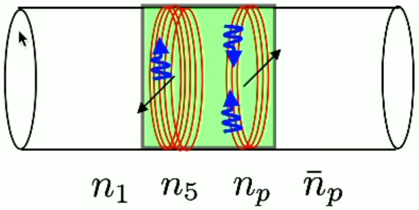

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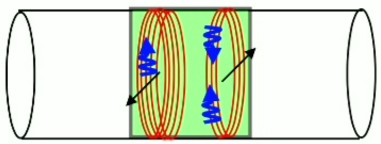

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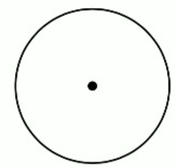

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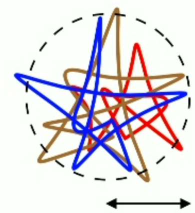

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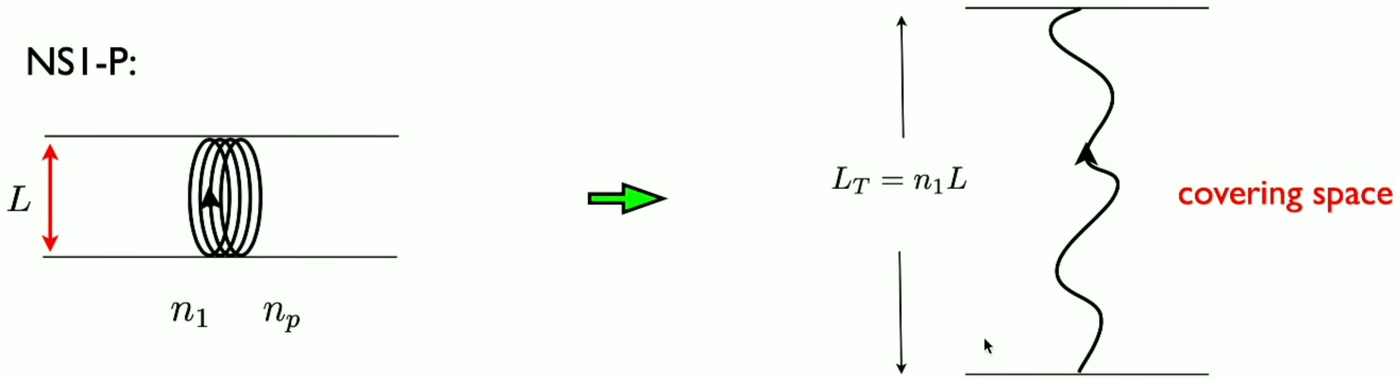
Can we make this idea more precise ?

Start with the simplest system: 2-charge extremal

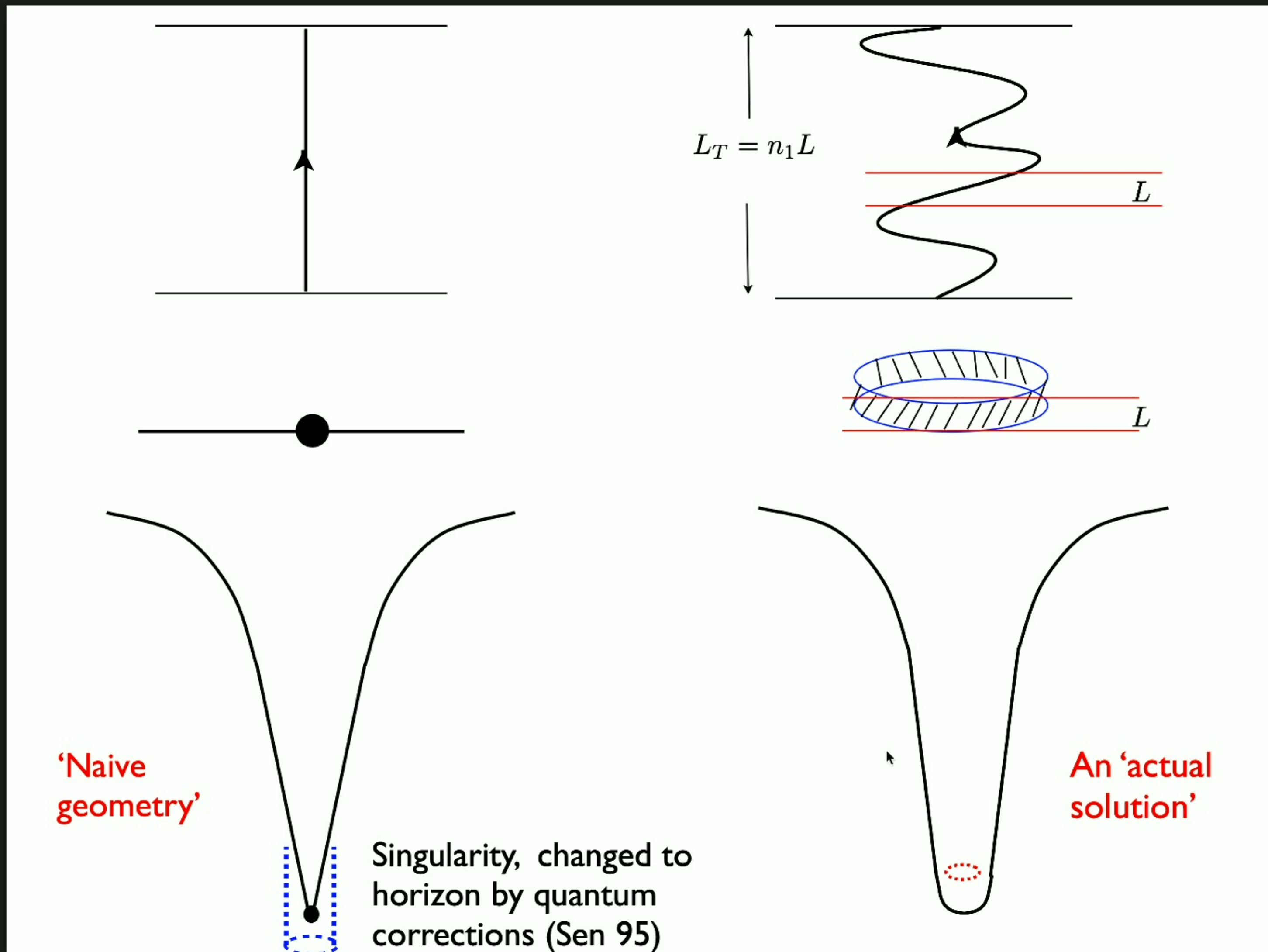
Bound state of strings: A multiwound string

Bound state of strings with momentum: Traveling waves on the multiwound string

NS1-P:



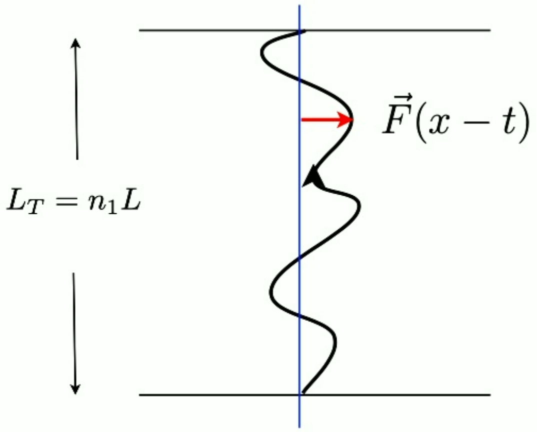
Different microstates will correspond to different vibration profiles ...


$L_T = n_1 L$
L
L
'Naive geometry'
Singularity, changed to horizon by quantum corrections (Sen 95)
An 'actual solution'

$$ds^2_{string} = H[-dudv + Kdv^2 + 2A_i dx_i dv] + \sum_{i=1}^{4} dx_i dx_i + \sum_{a=1}^{4} dz_a dz_a$$

$$B_{uv} = \frac{1}{2}[H-1], \qquad B_{vi} = HA_i$$

$$e^{2\phi} = H$$



$$H^{-1} = 1 + \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2}$$

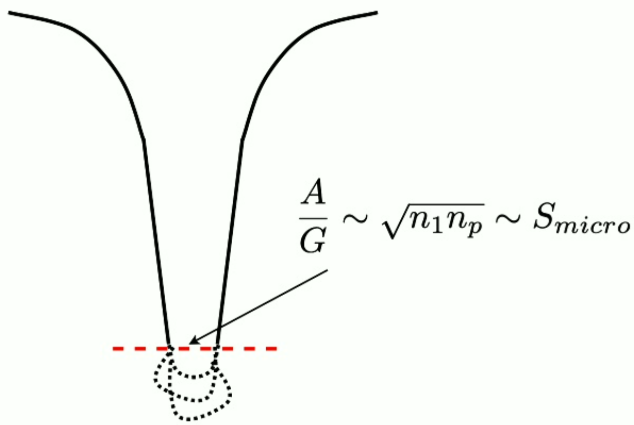
$$K = \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2}$$

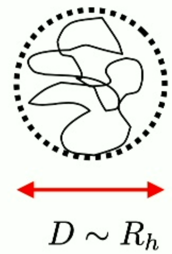
$$A_i = -\frac{Q_1}{L_T}\int_0^{L_T} \frac{dv\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

(Lunin+SDM 01, Lunin+Maldacena+Maoz 02, Skenderis+Taylor 07)

(a) No microstate has a horizon

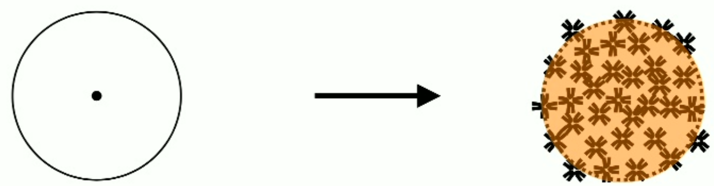
(b) The surface area of the generic state satisfies a Bekenstein-type relation

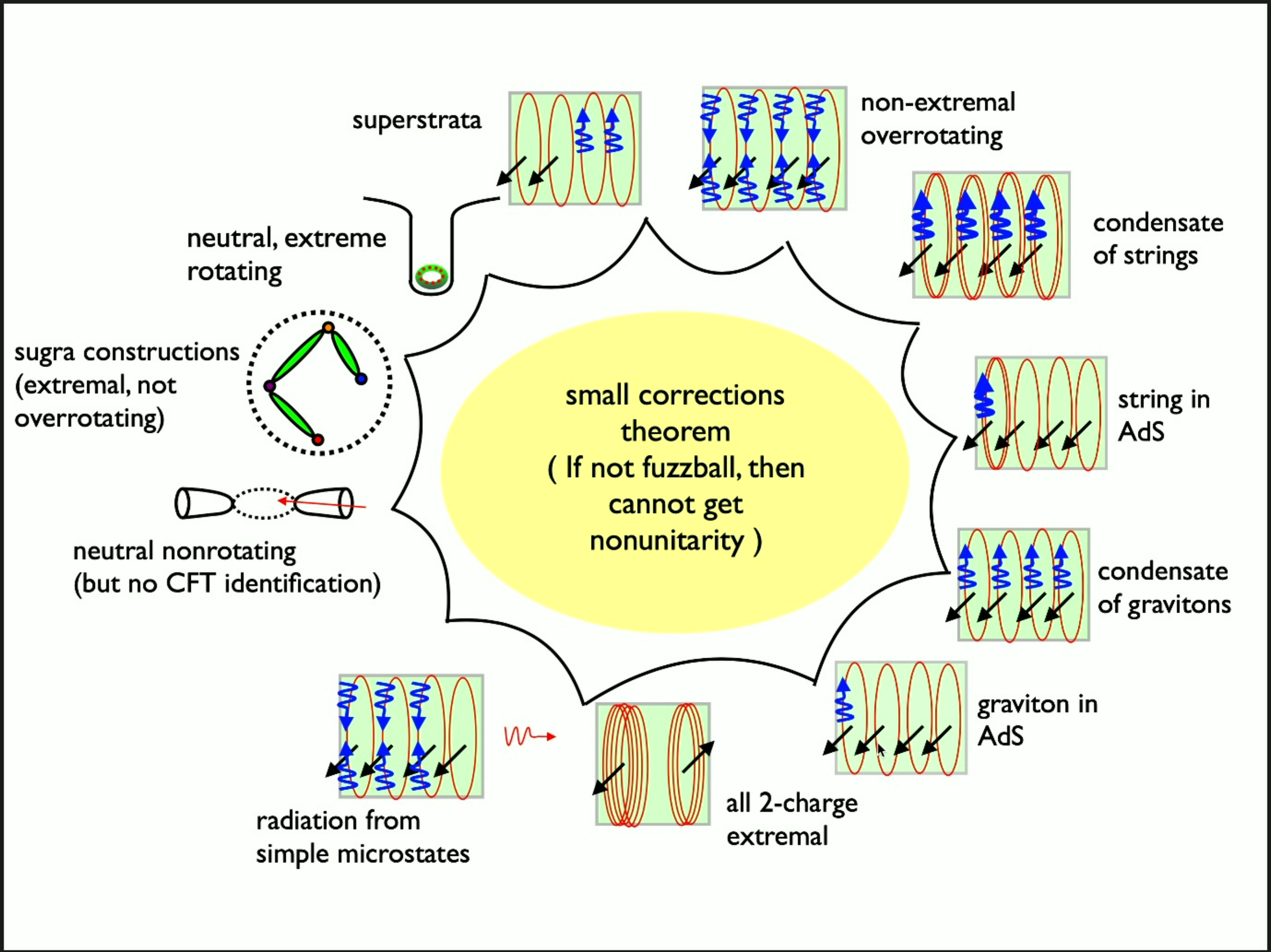




(Lunin+SDM 02)

(c) The microstates are not spherically symmetric ... imposing a spherically symmetric ansatz gives a solution with horizon and singularity, which is not actually realized




superstrata
non-extremal
overrotating
condensate
of strings
neutral, extreme
rotating
sugra constructions
(extremal, not
overrotating)
small corrections
theorem
(If not fuzzball, then
cannot get
nonunitarity)
string in
AdS
neutral nonrotating
(but no CFT identification)
condensate
of gravitons
graviton in
AdS
radiation from
simple microstates
all 2-charge
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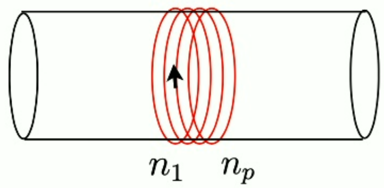
The simplest hole is the 2-charge extremal hole ('small black hole')



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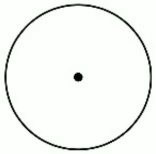
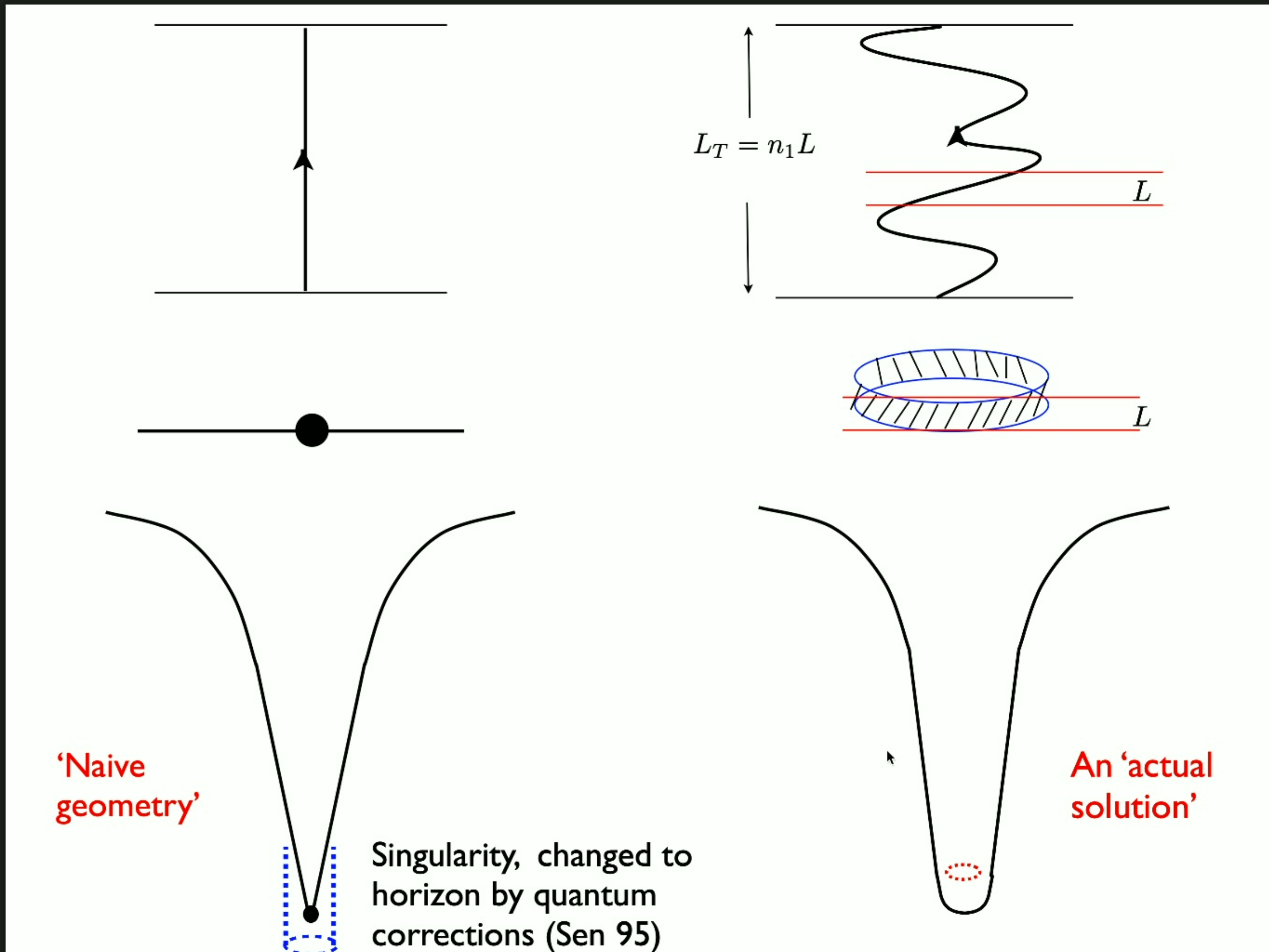
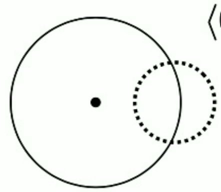


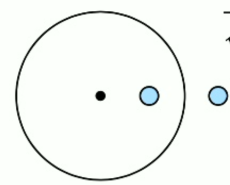

$L_T = n_1 L$
L
L
'Naive geometry'
An 'actual solution'
Singularity, changed to horizon by quantum corrections (Sen 95)

Thus we have support for the idea that in string theory we never form a horizon ⟶ The fuzzball paradigm

Traditional picture:

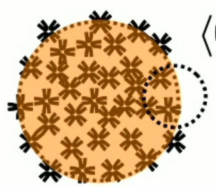


$\langle 0|\psi\rangle = 1-\epsilon$

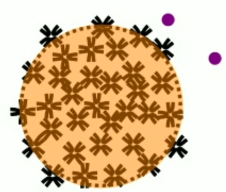


$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) + O(\epsilon)$

Fuzzball:



$\langle 0|\psi\rangle = \epsilon$



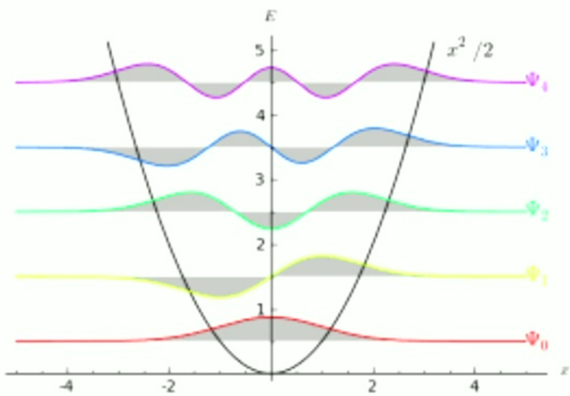
~~$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) + O(\epsilon)$~~

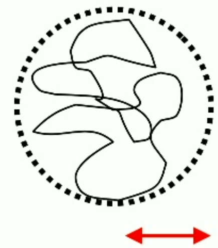
(Evaporates like burning paper, radiated quanta not modified at $r \gg M$)

(Avery, Balasubramanian, Bena, Bobev, Bossard, Carson, Ceplak, Chowdhury, de Boer, Gimon, Giusto, Guo, Hampton, Heidmann, Jejjala, Katmadas, Kanitscheider, Keski-Vakkuri, Kraus, Levi, Lunin, Madden, Maldacena, Maoz, Martinec, Mayerson, Niehoff, Park, Peet, Potvin, Puhm, Ross, Ruef, Rychkov, Saxena, Shigemori, Simon, Skenderis, Srivastava, Taylor, Titchner, Turton, Vasilakis, Wang, Warner ... and many others)

(A) Why are string theory microstates as big as the horizon scale ?

Wavefunctionals for different states need to be orthogonal to each other





$D \sim R_h$

The string states are packed to the maximal density allowed by phase space... this many states will not fit in a smaller region

$D = \mu R_h \ , \ \mu < 1$

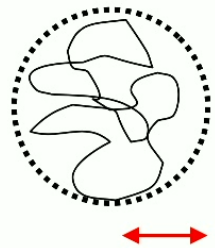
Count only those states that fit in a smaller region, Find a Bekenstein type relation again

$$S[\mu] \sim \frac{A[\mu]}{G}$$

(SDM 0706.3884)

Some qualitative observations

(A) Why are string theory microstates as big as the horizon scale ?



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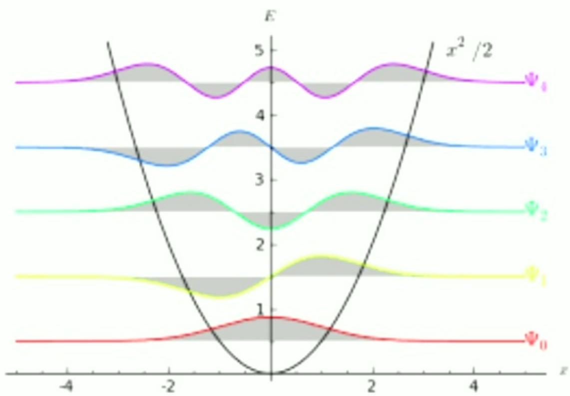

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(SDM 0706.3884)

(B) How do we evade all the no-hair arguments ?

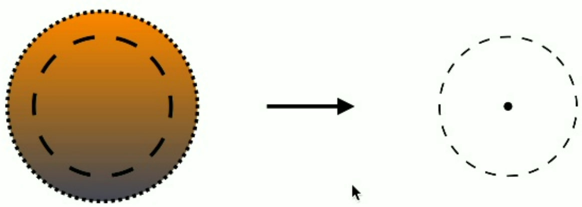
A detailed analysis was done By Gibbons and Warner

There are special features of a theory like string theory which has extra dimensions/extended objects/Chern-Simmons terms etc.

(Gibbons+Warner 1307.0957)

What about Buchdahl theorem? Fluid sphere with pressure decreasing outwards must collapse if

$$R < \frac{9}{4} M$$

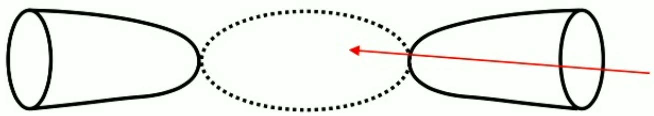


Consider a toy model of the fuzzball

Toy model: Euclidean Schwarzschild plus time ('neutral fuzzball')

$$ds^2 = -dt^2 + (1 - \frac{r_0}{r})d\tau^2 + \frac{dr^2}{1 - \frac{r_0}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$0 \le \tau < 4\pi r_0$$



We can reduce on the direction τ to get a scalar field Φ in 3+1 gravity.

The stress tensor is the standard one for a scalar field

$$T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g^E_{\mu\nu}\Phi_{,\lambda}\Phi^{,\lambda}$$

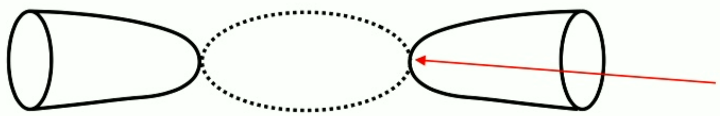
Why does this shell of scalar field not collapse inwards ?

We can compute the stress tensor of this scalar field in the 3+1spacetime

$$T^\mu{}_\nu = \mathrm{diag}\{-\rho, p_r, p_\theta, p_\phi\} = \mathrm{diag}\{-f, f, -f, -f\}\,,$$

$$f = \frac{3r_0^2}{8r^4(1-\frac{r_0}{r})^{\frac{3}{2}}}$$

Pressure diverges at tip of cigar so a Buchdahl type analysis would call this a singularity



But the 4+1 dimensional solution is completely regular

(g_{tt} never changes sign, so there is no horizon)

Roughly speaking, the compact dimensions are not trivially tensored with the noncompact directions ... different solutions of this kind give the black hole entropy

(SDM 1609.05222)

Consequences of fuzzball structure

Interesting observation by Chua and Afshordi that low energy electromagnetic waves will reflect off the plasma produced by electron-positron plasma in the near horizon radiation

Also works for low energy gravitational waves

(Chua and Afshordi: 2103.05790)

Does it also work for nonlinear waves? The echoes may be trapped and not escape back to us ....

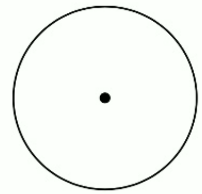
(Guo, Hampton,SDM 1711.01617, Guo, SDM 2205.10921)

Black hole thermodynamics

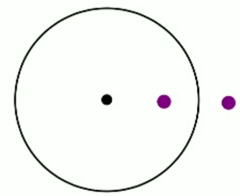
(SDM+Madhur Mehta, to appear)

Black holes have a very well defined thermodynamical behavior

(1) The entropy is $S_{bek} = \frac{A}{4G}$



(2) The temperature is $T_H = \frac{1}{8\pi G M}$



(3) The emission rate satisfies the law of black body radiation

$$\Gamma_{BH}(l,m,\omega)d\omega = \frac{\mathcal{P}(l,m,\omega)}{e^{\frac{\omega}{T_H}} - 1}\frac{d\omega}{2\pi}$$

$\Gamma_{BH}(l,m,\omega)d\omega$ Number of quanta in spherical harmonic emitted per unit time in energy range

$\mathcal{P}(l,m,\omega)$ Absorption probability of spherical wave

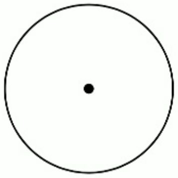
Since the information paradox was an important puzzle, any resolution must give us some new deep lessons about quantum gravity

(a) On ground of units, we think that the length scale relevant to quantum gravity is

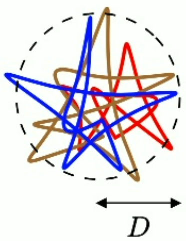
$$l_p \sim \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-33}\, cm$$

But a black hole is made of a large number of quanta N

So we need to ask if the relevant length scale is l_p or $N^\alpha l_p$

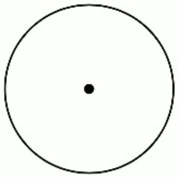


In string theory it seems that the latter is true ...



$$D \sim \left[\frac{\sqrt{n_1 n_5 n_p}\, g^2\, \alpha'^4}{V\, L}\right]^{\frac{1}{3}} \sim R_H$$

(b) Why is the semiclassical approximation violated ?



A path integral has a classical action and a measure term

$$Z = \int D[g]\; e^{-\frac{1}{\hbar} S_{cl}[g]}$$

For normal classical processes we extremize the classical action while ignoring the measure term

For a black hole $\quad S_{cl} \sim E \times T, \qquad E = M \sim \hbar \, \frac{R^{d-2}}{l_p^{d-1}} \quad , \qquad T \sim R$

$$\longrightarrow \quad \frac{1}{\hbar} S_{cl} \sim \left(\frac{R}{l_p}\right)^{d-1}$$

$$D[g] \sim e^{S_{bek}} \quad , \qquad S_{bek} \sim \frac{A}{G} \sim \left(\frac{R}{l_p}\right)^{d-1}$$

Thus for a black hole, the measure term seems to be as important as the classical action

(c) The vacuum of quantum gravity has virtual fluctuations of fuzzball type configurations of all sizes

The probability of any large size fluctuation is small

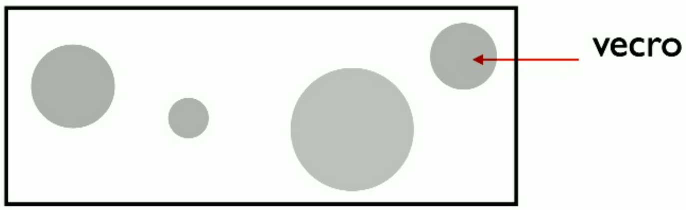
$$P \sim e^{-S} \sim e^{-ET}$$

$$E \sim M \sim \frac{R^{d-2}}{l_p^{d-1}}, \quad T \sim R \quad \longrightarrow \quad S \sim \left(\frac{R}{l_p}\right)^{d-1}$$

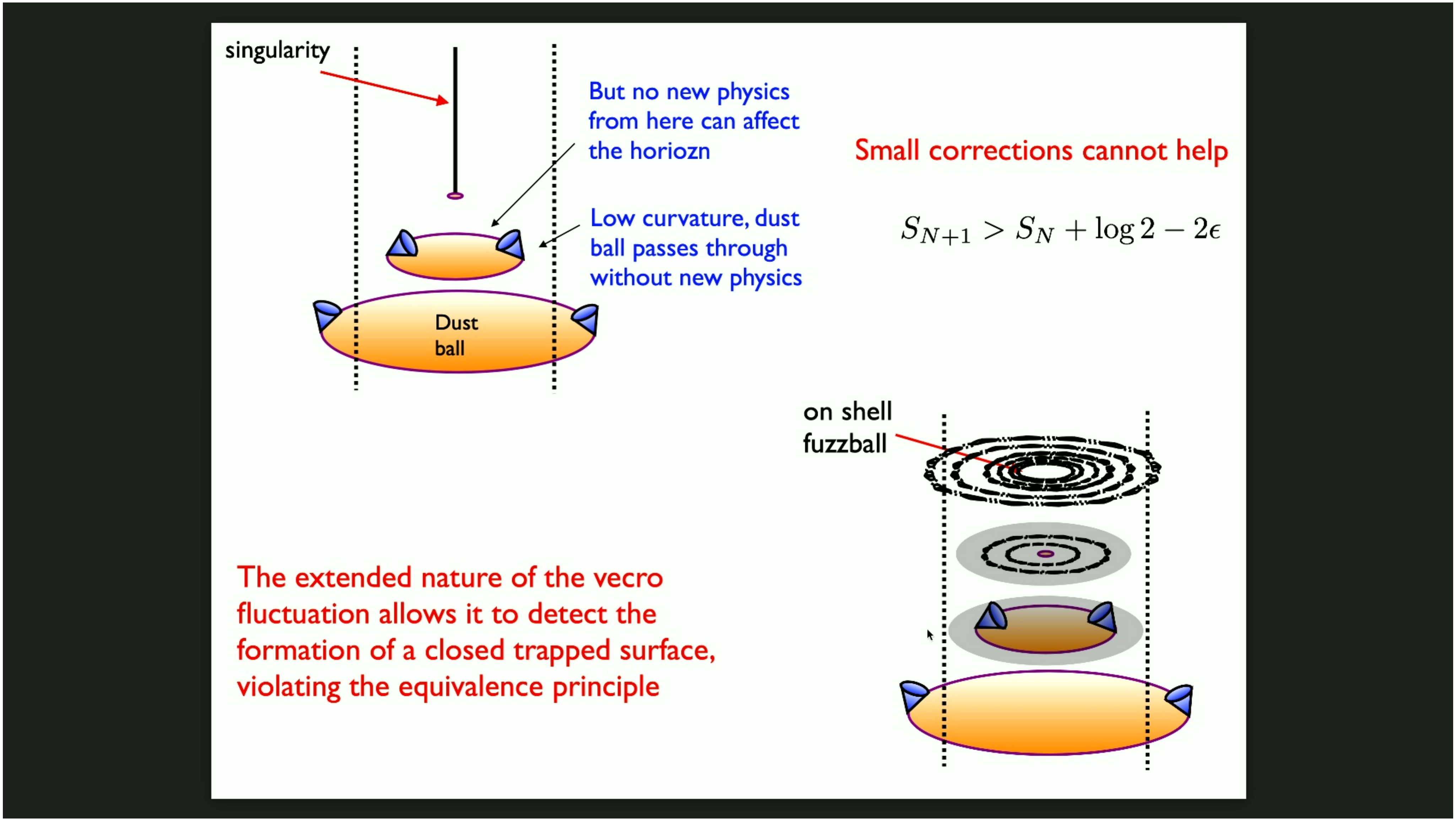
But there are a very large number of such fuzzball type configurations:

$$\mathcal{N} \sim e^{S_{bek}} \quad , \qquad S_{bek} \sim \frac{A}{G} \sim \left(\frac{R}{l_p}\right)^{d-1}$$

Thus we can have $P\mathcal{N} \sim 1$ (the suppression is offset by the large degeneracy)



(Virtual Extended Compression-Resistant Objects)


singularity
But no new physics
from here can affect
the horiozn
Low curvature, dust
ball passes through
without new physics
Dust
ball
Small corrections cannot help
$S_{N+1} > S_N + \log 2 - 2\epsilon$
on shell
fuzzball
The extended nature of the vecro
fluctuation allows it to detect the
formation of a closed trapped surface,
violating the equivalence principle

SUMMARY

In string theory, we seem to have a coherent picture of black holes through the fuzzball paradigm ...

This resolution of the information puzzle provides deep lessons about how quantum gravity seems to work (at least in string theory)

We now need to apply these lessons to learn about cosmology

Black hole horizon → Cosmological horizon

Black hole singularity → Big Bang singularity

(SDM 2105.06963, Brandenberger + Mitchell 2302.12924)