Title: The Spacetime of Acceleration

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Collection: Quantum Spacetime in the Cosmos: From Conception to Reality

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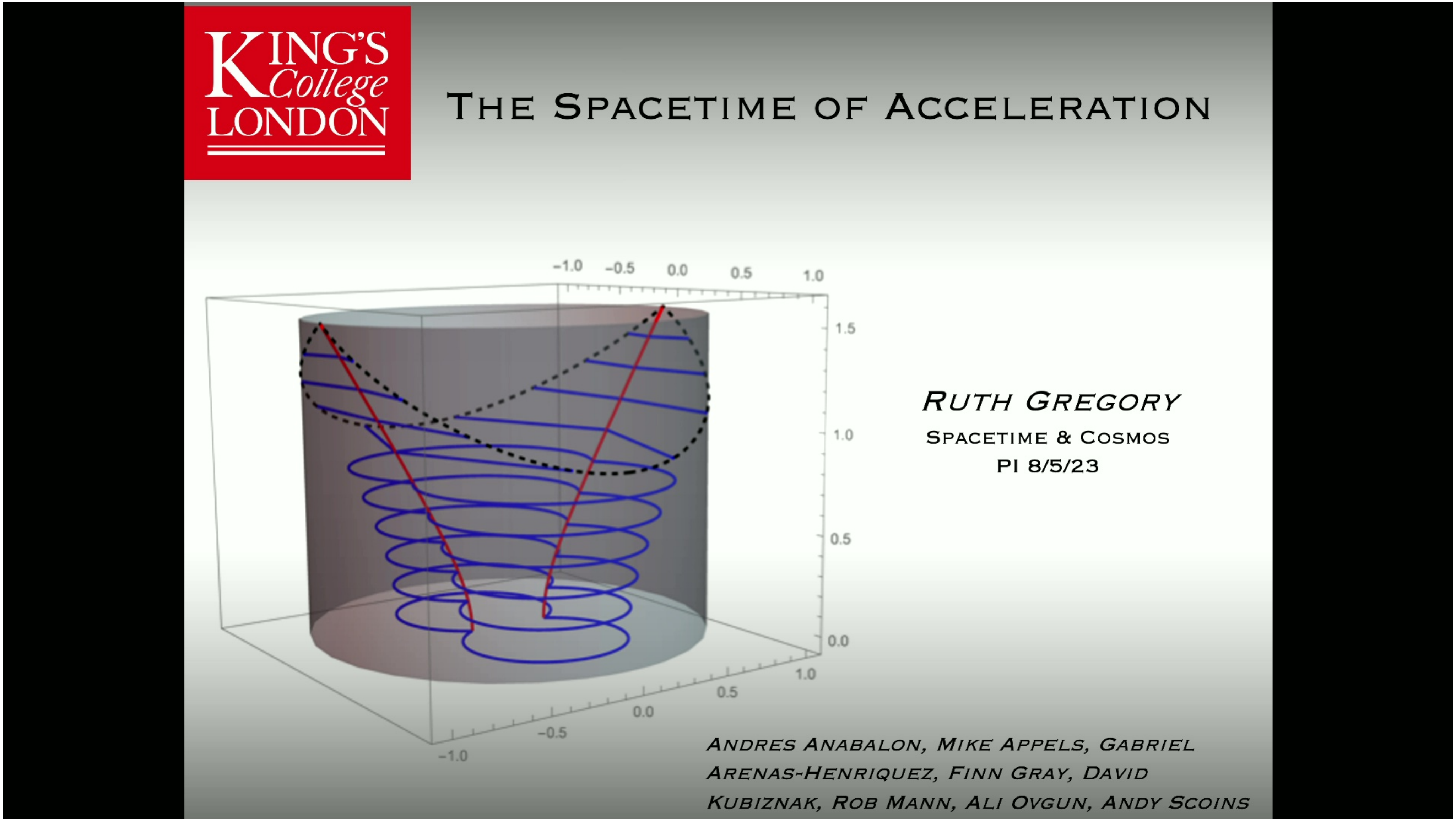
Abstract: I will review recent work on accelerating spacetimes in 3D, explaining how acceleration manifests for point particles and BTZ black holes, as well as a novel BTZ-like solution in a disconnect region of parameter space. I will also discuss some holographic aspects of the solutions.

Zoom Link: https://pitp.zoom.us/j/94575380034?pwd=Y21DMTRqeFFGNnd5dnVBc1dac2tUQT09



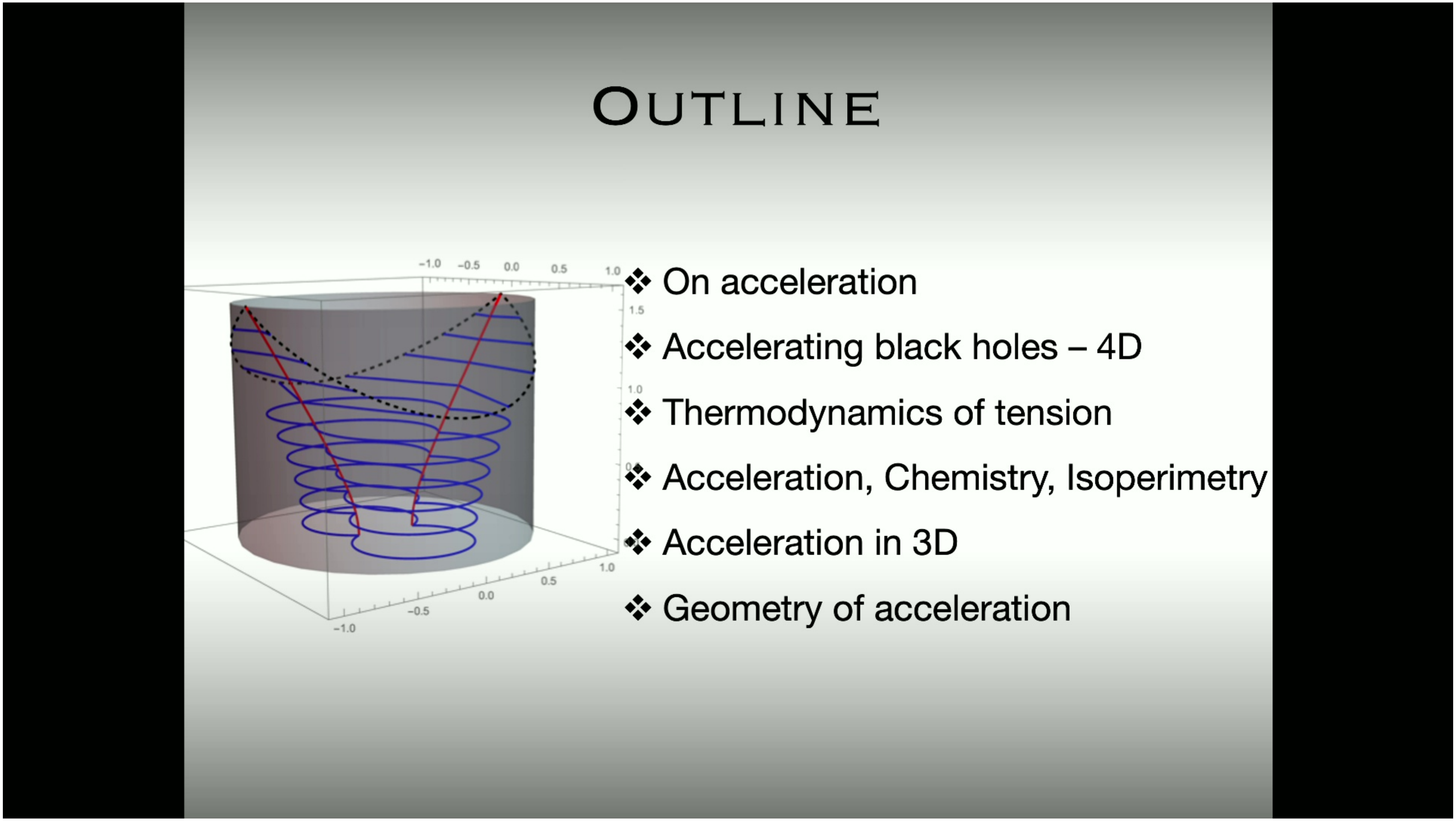
THE SPACETIME OF ACCELERATION

RUTH GREGORY

SPACETIME & COSMOS

PI 8/5/23

ANDRES ANABALON, MIKE APPELS, GABRIEL ARENAS-HENRIQUEZ, FINN GRAY, DAVID KUBIZNAK, ROB MANN, ALI OVGUN, ANDY SCOINS



OUTLINE

- ❖ On acceleration
- ❖ Accelerating black holes – 4D
- ❖ Thermodynamics of tension
- ❖ Acceleration, Chemistry, Isoperimetry
- ❖ Acceleration in 3D
- ❖ Geometry of acceleration

WHAT IS ACCELERATION?

Acceleration is when an object is not travelling on a geodesic.

$$\nabla_T T \not\propto T$$

For example, consider an observer at R=R$_0$ in AdS:

$$ds^2_{AdS} = -\left(1 + \frac{R^2}{\ell^2}\right) dt^2 + \frac{dR^2}{1 + \frac{R^2}{\ell^2}} + R^2 \left(d\Theta^2 + \sin^2 \Theta d\phi^2\right)$$

The tangent vector is purely timelike, but the acceleration is radial:

$$\mathbf{T} = \frac{1}{\sqrt{1 + \frac{R_0^2}{\ell^2}}} \frac{\partial}{\partial t} \qquad \mathbf{A} = \nabla_T T = \frac{R_0}{\ell^2} \frac{\partial}{\partial r}$$

RINDLER WITH NO HORIZON

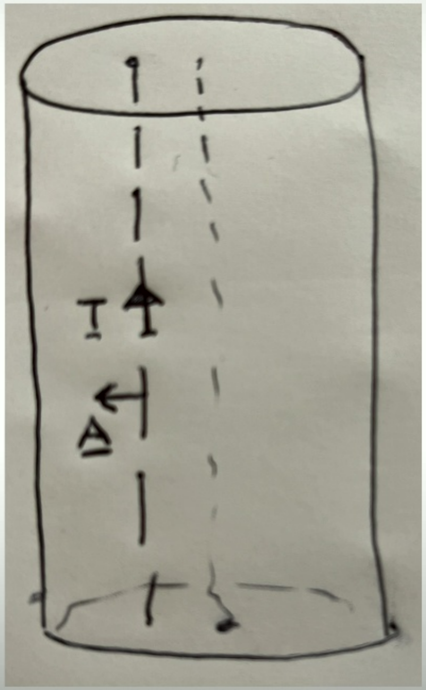
The magnitude of the acceleration is related to R_0

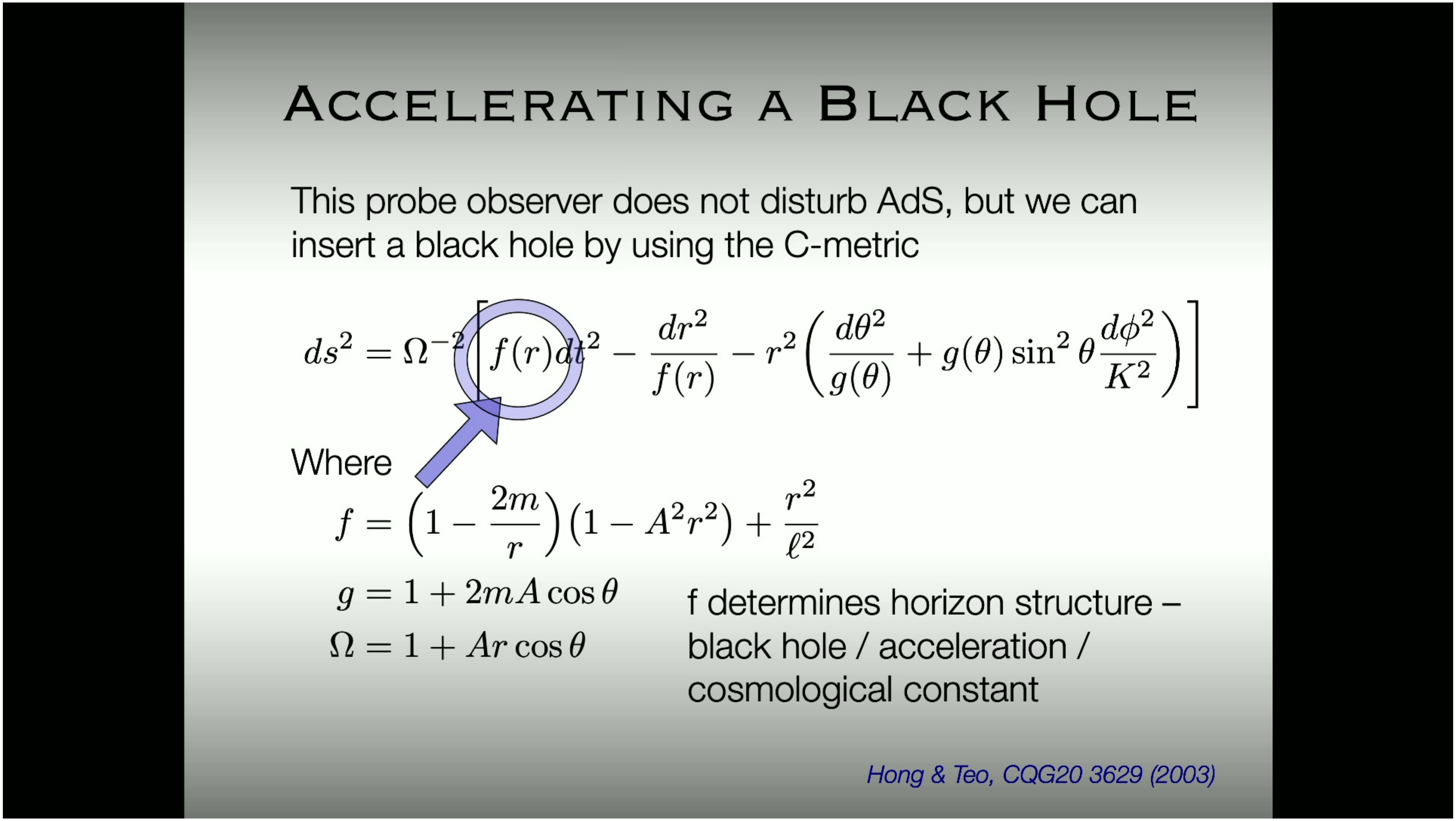
$$\mathbf{T} = \frac{1}{\sqrt{1+\frac{R_0^2}{\ell^2}}}\frac{\partial}{\partial t}$$

$$|\mathbf{A}|^2 = \frac{R_0^2/\ell^4}{1+R_0^2/\ell^2}$$

$$\mathbf{A} = \frac{R_0}{\ell^2}\frac{\partial}{\partial r}$$

$$R_0 = \frac{A\ell^2}{\sqrt{1-A^2\ell^2}}$$





ACCELERATING A BLACK HOLE

This probe observer does not disturb AdS, but we can insert a black hole by using the C-metric

$$ds^2 = \Omega^{-2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

Where

$$f = \left(1 - \frac{2m}{r}\right)(1 - A^2r^2) + \frac{r^2}{\ell^2}$$

$$g = 1 + 2mA\cos\theta$$

$$\Omega = 1 + Ar\cos\theta$$

f determines horizon structure – black hole / acceleration / cosmological constant

Hong & Teo, CQG20 3629 (2003)

SLOWLY ACCELERATING RINDLER

For m=0, there is no horizon, and setting

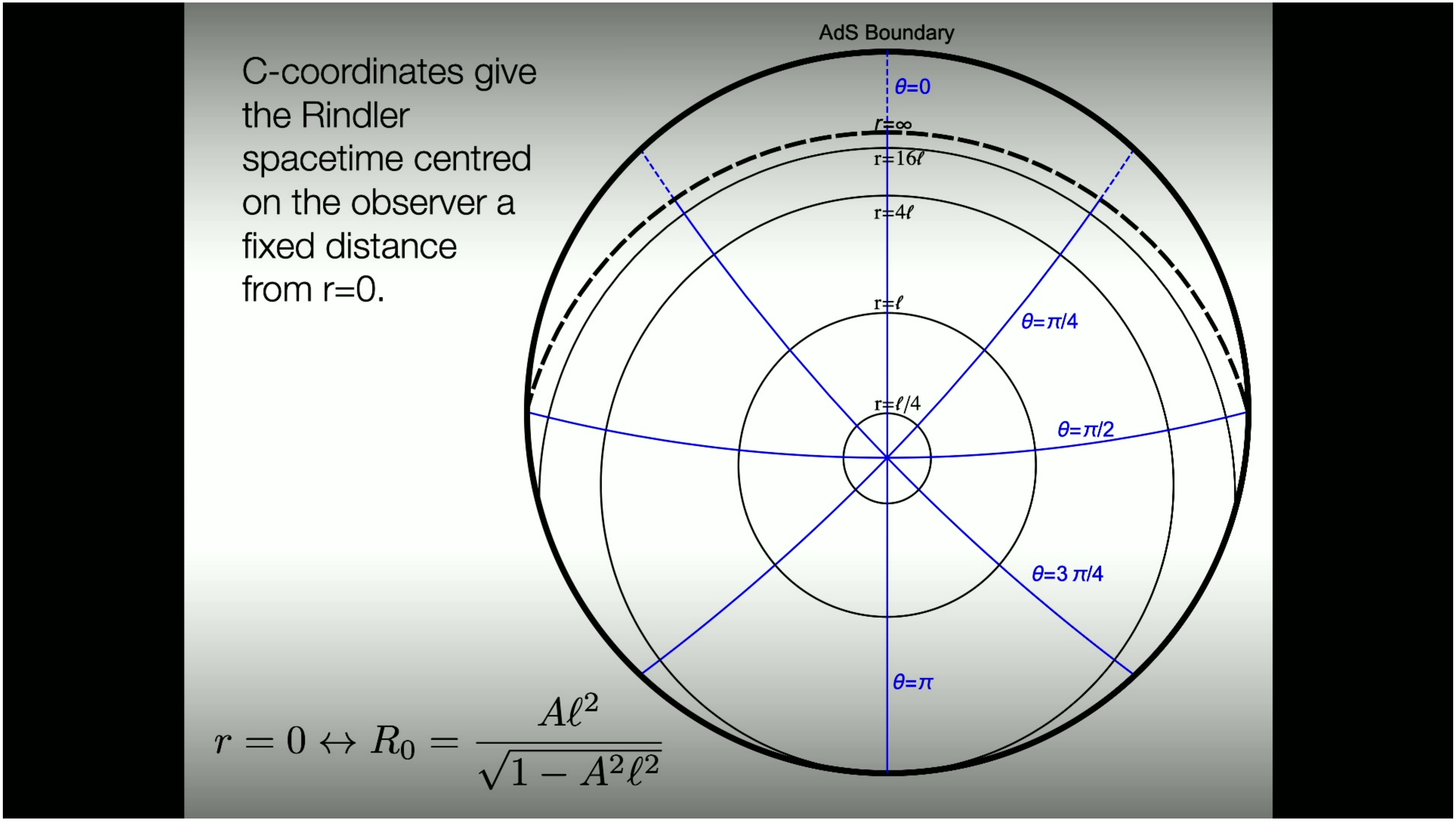
$$1+\frac{R^2}{\ell^2}=\frac{1+(1-A^2\ell^2)r^2/\ell^2}{(1-A^2\ell^2)\Omega^2}\,,\quad R\sin\Theta=\frac{r\sin\theta}{\Omega}$$

We get back to global AdS

$$ds^2_{AdS}=-\left(1+\frac{R^2}{\ell^2}\right)\alpha^2dt^2+\frac{dR^2}{1+\frac{R^2}{\ell^2}}+R^2\left(d\Theta^2+\sin^2\Theta\frac{d\phi^2}{K^2}\right)$$

Modulo the factor of

$$\alpha=\sqrt{1-A^2\ell^2}$$



C-coordinates give the Rindler spacetime centred on the observer a fixed distance from r=0.

$$r = 0 \leftrightarrow R_0 = \frac{A\ell^2}{\sqrt{1-A^2\ell^2}}$$

Acceleration and Black Hole

If m is not zero, A gives rise to an imbalance between North and South axes, which now have (different) conical deficits.

- $\theta \to 0$ $$ds^2_{\theta,\phi} \propto d\theta^2 + \frac{(1+2mA)^2}{K^2}\theta^2 d\phi^2$$
- $\theta \to \pi$ $$ds^2_{\theta,\phi} \propto d\theta^2 + \frac{(1-2mA)^2}{K^2}(\pi-\theta)^2 d\phi^2$$

The axis, if regular, would look locally like the origin in polar coordinates, but here we see different factors in the angular part:

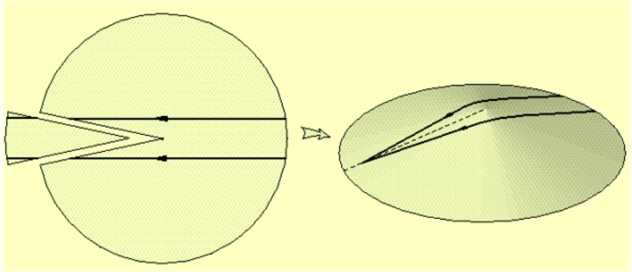
$$\delta_\pm = 2\pi\left(1 - \frac{g(0)}{K}\right) = 2\pi\left(1 - \frac{1 \pm 2mA}{K}\right) = \text{“}8\pi\mu_\pm\text{”}$$

This translates to a conical deficit, interpreted as a cosmic string.

COSMIC STRING

$$T^\mu_\nu \approx \delta^{(2)}(\mathbf{r})\ \mathrm{diag}\,(\mu,\mu,0,0)$$

A string produces a conical deficit, but no long range spacetime curvature (no tidal forces).



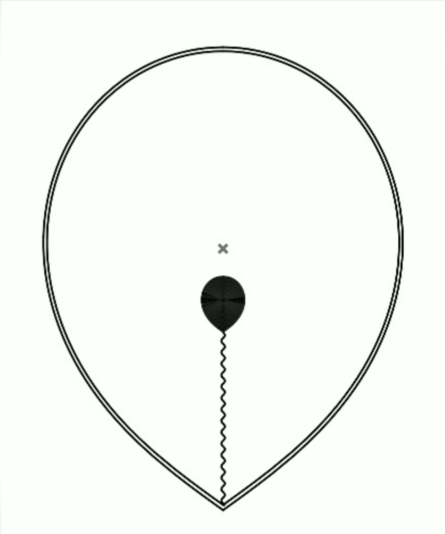
$$\delta = 8\pi G\mu$$

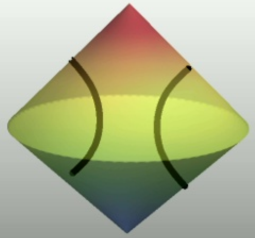
Strings can be threaded onto black holes, where the conical singularity is interpreted as an idealization of some finite width string core.

c.f. Aryal, Ford, Vilenkin: PRD 34, 2263 (1986), Achucarro, Gregory, Kuijken: PRD 52 5729 (1995)

The Slowly Accelerating Black Hole

The slowly accelerating black hole in AdS is displaced from centre. It has a conical deficit running from the horizon to the boundary. The string tension provides the force that hold the black hole off-centre.

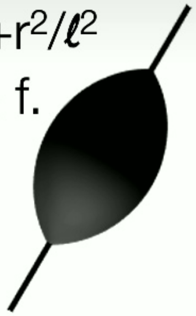




The general C-metric has an acceleration horizon, so we looked at slowly accelerating black holes to have no ambiguity.

THERMODYNAMICS, WITH STRINGS ATTACHED!

Want to do thermodynamics with strings, so start with a semi-familiar case: Schwarzschild-AdS with a deficit: f = 1-2m/r+r^2/ℓ^2



Black hole horizon defined by f=0, look at small changes in f. Horizon still defined by f(r) = 0.

$$f(r_+ + \delta r_+) = f'(r_+)\delta r_+ + \frac{\partial f}{\partial m}\delta m + \frac{\partial f}{\partial \ell}\delta\ell = 0$$

Changes r+,
hence S

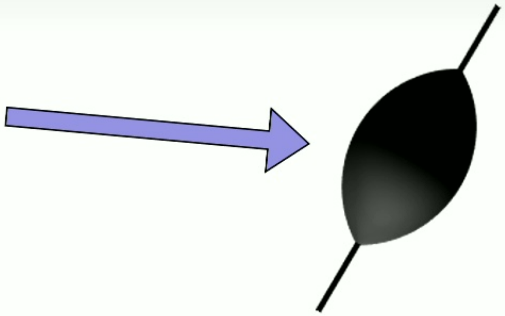
Changes m,
hence M

Changes ℓ,
hence Λ

BLACK HOLE THERMODYNAMICS

Temperature has usual definition, but entropy depends on K:

$$T = \frac{f'(r_+)}{4\pi} \qquad S = \frac{\pi r_+^2}{K}$$



-and we want to do thermodynamics including the string, so we have to take into account varying K.

$$\delta S = \frac{\pi r_+ \delta r_+}{K} - \pi r_+^2 \frac{\delta K}{K^2}$$

(S): Herdeiro, Kleihaus,Kunz,Radu: 0912:3386 [gr-qc]

CHANGING TENSION

Tension is related to K:

$$\mu = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

So easily get

$$\delta\mu = \frac{\delta K}{4K^2}$$

Finally
$$P = -\Lambda = \frac{3}{8\pi\ell^2} \qquad V = \frac{4\pi r_+^3}{3K}$$

First Law with Tension

Putting together:

$$0 = \frac{2K}{r_+}\left(T\delta S + 2(m - r_+)\delta\mu + V\delta P - \delta(\frac{m}{K})\right)$$

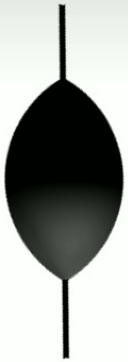
So identify $M = \frac{m}{K}$

Then also get Smarr relation:

$$M = 2TS - 2PV$$

THERMODYNAMIC LENGTH

The term multiplying the variation in tension is a “thermodynamic length”



$$\lambda = r_+ - m$$

Reinforces interpretation of M as **enthalpy,** if black hole grows, it swallows some string, but has also displaced the same amount of energy from environment.

Kastor & Traschen: 1207:5415 [hep-th]

BACK TO THE ACC BLACK HOLE:

Based on experience with the Kerr-AdS metric (and motivated by the coordinate transformation for slowly accelerating Rindler) we introduce a possible rescaling of the time coordinate

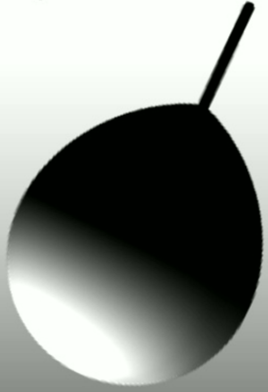
$$ds^2 = \frac{1}{H^2}\left\{\frac{f(r)}{\Sigma}\left[\frac{dt}{\alpha} - a\sin^2\theta\frac{d\varphi}{K}\right]^2 - \frac{\Sigma}{f(r)}dr^2 - \frac{\Sigma r^2}{h(\theta)}d\theta^2 - \frac{h(\theta)\sin^2\theta}{\Sigma r^2}\left[\frac{a\,dt}{\alpha} - (r^2+a^2)\frac{d\varphi}{K}\right]^2\right\}$$

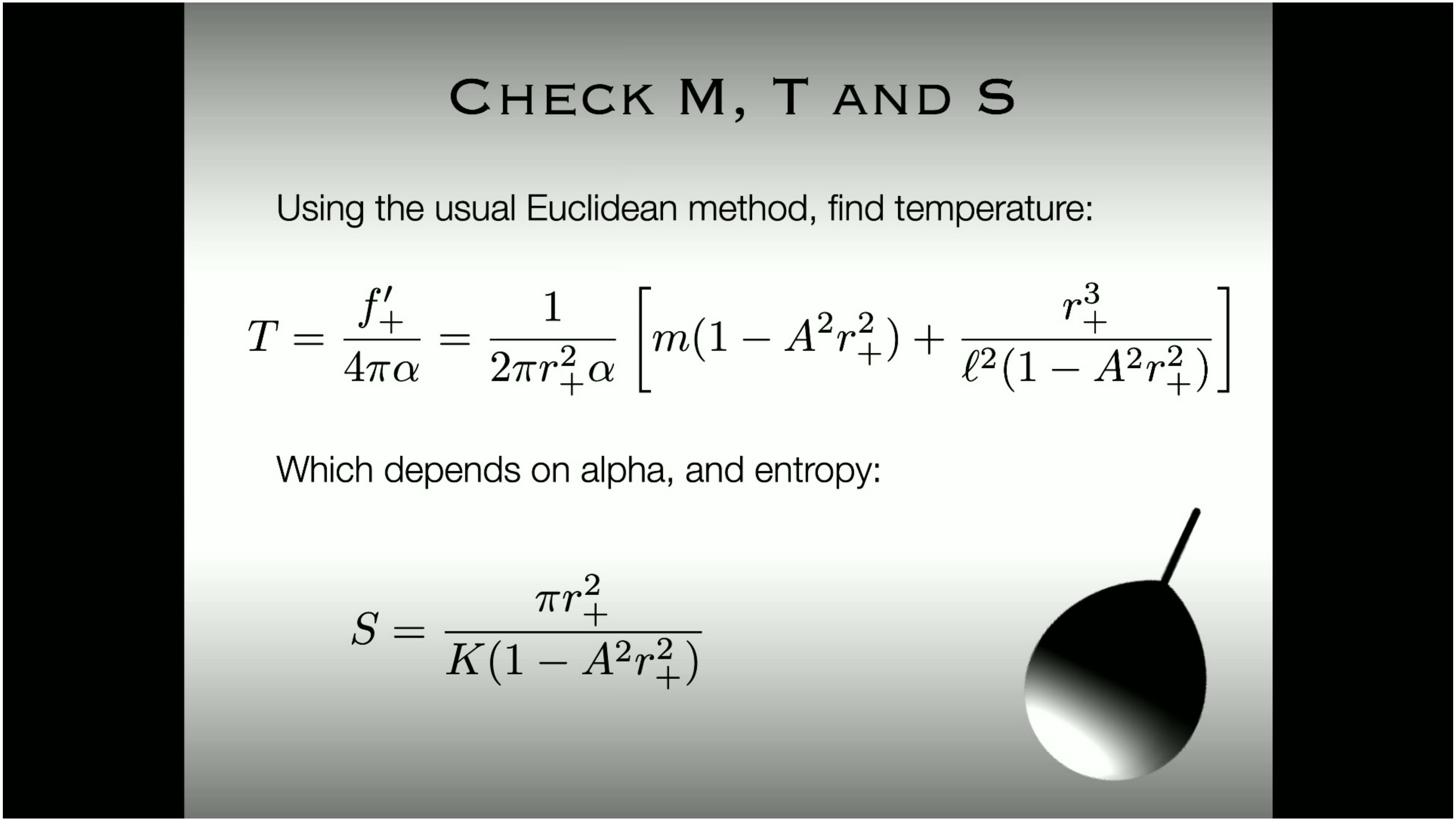
This will rescale temperature, and also changes computations of the mass.

$$f(r) = (1 - A^2r^2)\left[1 - \frac{2m}{r} + \frac{a^2+e^2}{r^2}\right] + \frac{r^2+a^2}{\ell^2}\,,$$

$$h(\theta) = 1 + 2mA\cos\theta + \left[A^2(a^2+e^2) - \frac{a^2}{\ell^2}\right]\cos^2\theta\,,$$

$$\Sigma = 1 + \frac{a^2}{r^2}\cos^2\theta\,, \qquad H = 1 + Ar\cos\theta\,.$$





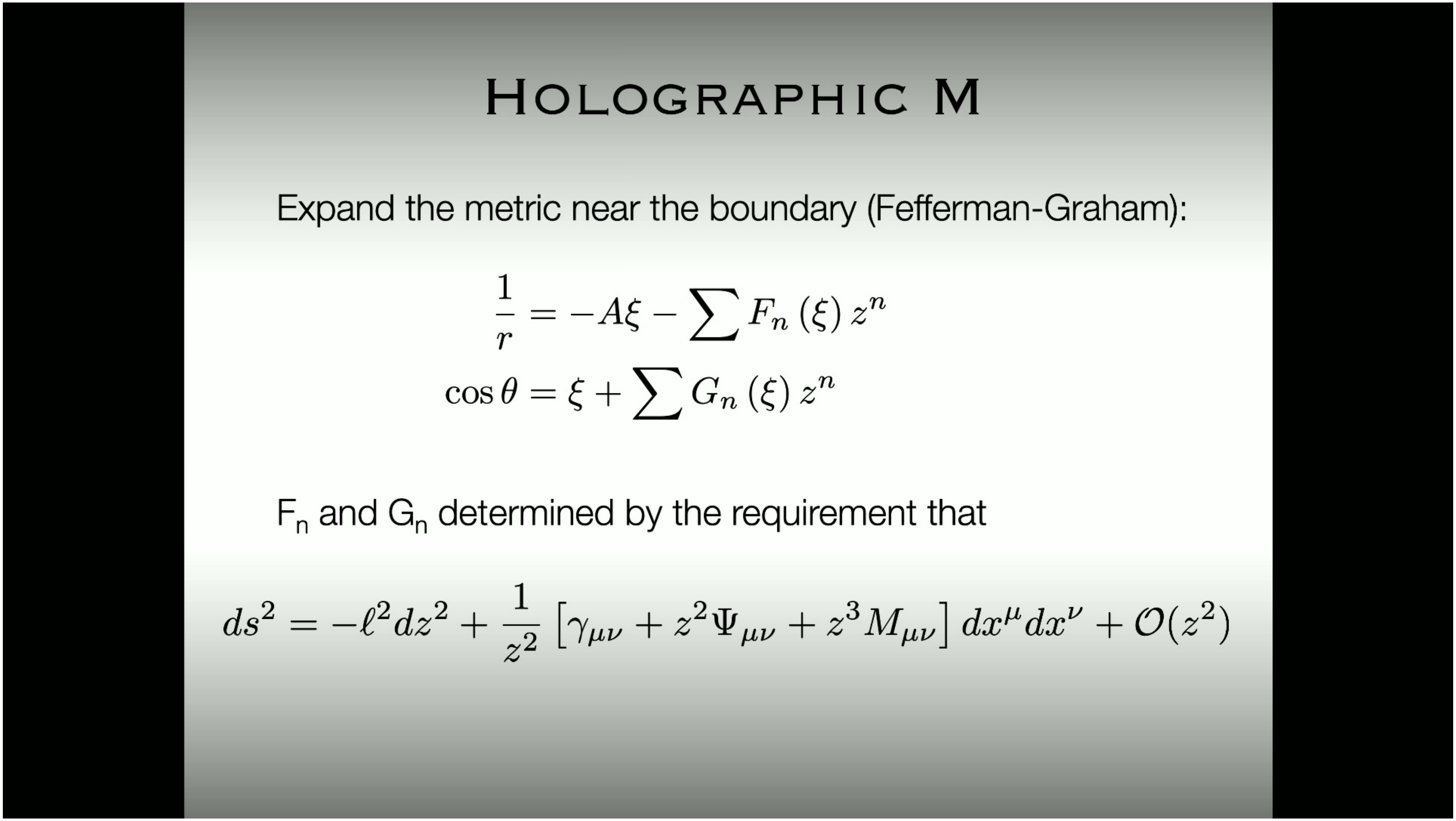
CHECK M, T AND S

Using the usual Euclidean method, find temperature:

$$T = \frac{f'_+}{4\pi\alpha} = \frac{1}{2\pi r_+^2 \alpha}\left[m(1 - A^2 r_+^2) + \frac{r_+^3}{\ell^2(1 - A^2 r_+^2)}\right]$$

Which depends on alpha, and entropy:

$$S = \frac{\pi r_+^2}{K(1 - A^2 r_+^2)}$$



HOLOGRAPHIC M

Expand the metric near the boundary (Fefferman-Graham):

$$\frac{1}{r} = -A\xi - \sum F_n\left(\xi\right) z^n$$

$$\cos\theta = \xi + \sum G_n\left(\xi\right) z^n$$

F_n and G_n determined by the requirement that

$$ds^2 = -\ell^2 dz^2 + \frac{1}{z^2}\left[\gamma_{\mu\nu} + z^2 \Psi_{\mu\nu} + z^3 M_{\mu\nu}\right] dx^\mu dx^\nu + \mathcal{O}(z^2)$$

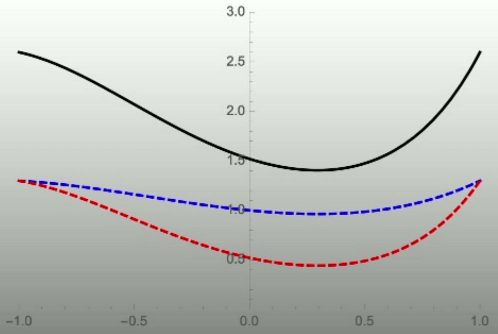
FEFFERMAN-GRAHAM

For the boundary metric, get:

$$\frac{\left(1-A^2\ell^2 g(\xi)\right)^3}{\alpha^2\ell^2 F_1^2(\xi)}d\tau^2-\frac{\left(1-A^2\ell^2 g(\xi)\right)}{F_1^2(\xi)g(\xi)}d\xi^2-\frac{g(\xi)\left(1-A^2\ell^2 g(\xi)\right)^2}{K^2F_1^2(\xi)}d\phi^2$$

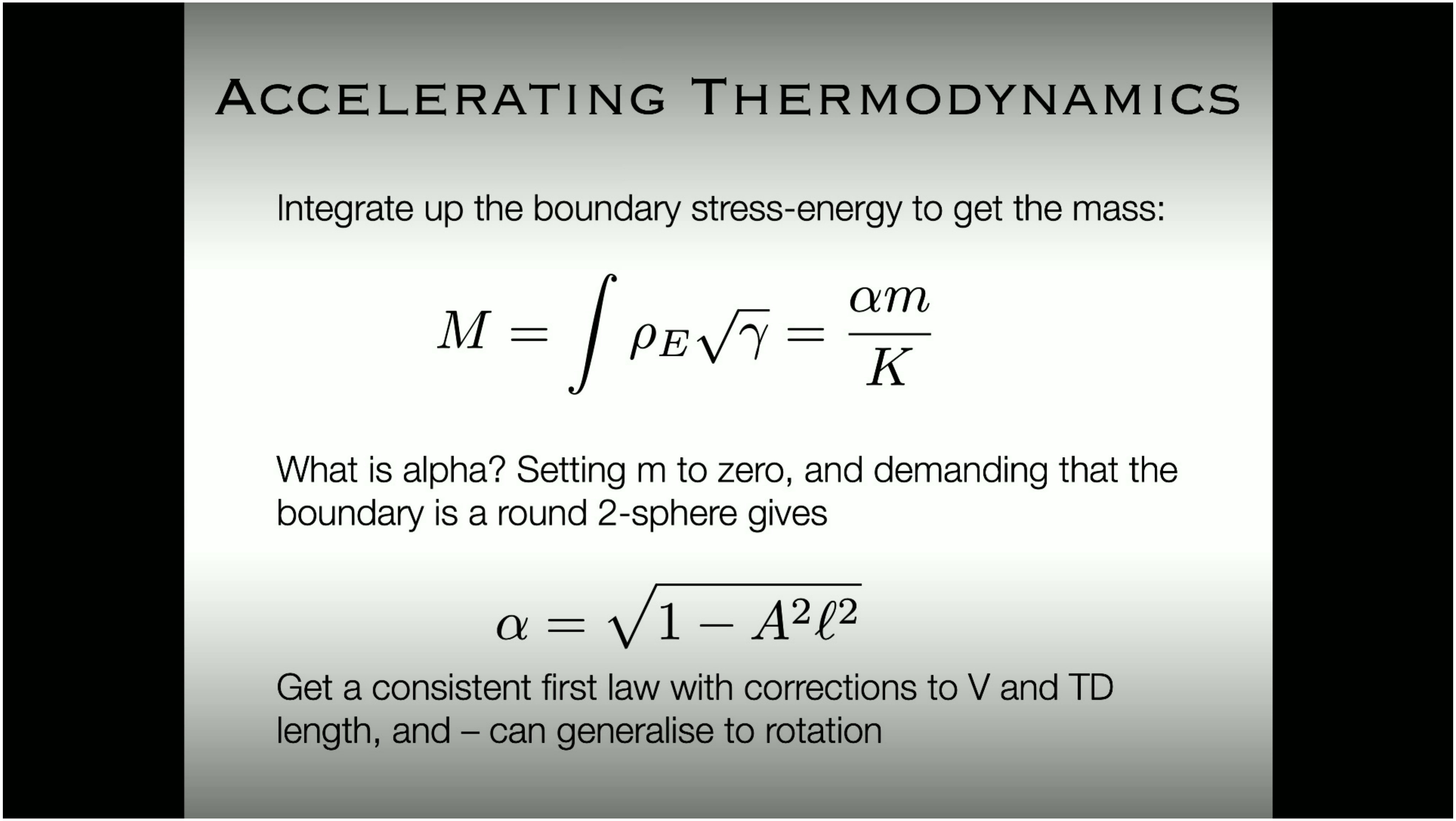
And for the boundary fluid stress tensor:

$$\langle \mathcal{T}^{\mu}_{\nu}\rangle = \mathrm{diag}\left\{\rho_E, -\rho_E/2+\Pi, -\rho_E/2-\Pi\right\}$$



$$\rho_E = \frac{m}{\alpha}(1-A^2\ell^2 g)^{3/2}(2-3A^2\ell^2 g)$$

$$\Pi = \frac{3A^2\ell^2 g m}{2\alpha}(1-A^2\ell^2 g)^{3/2}$$



ACCELERATING THERMODYNAMICS

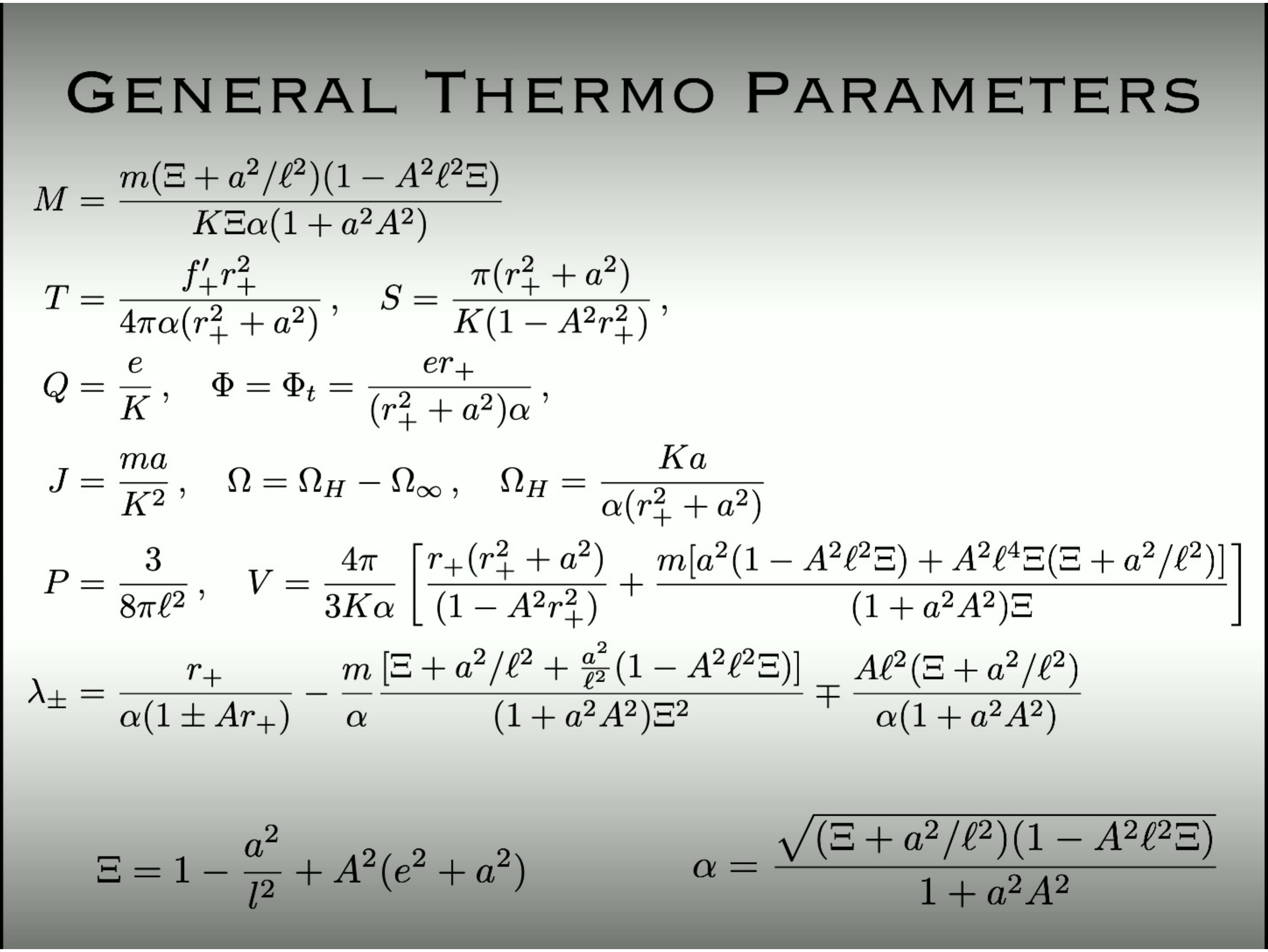
Integrate up the boundary stress-energy to get the mass:

$$M = \int \rho_E \sqrt{\gamma} = \frac{\alpha m}{K}$$

What is alpha? Setting m to zero, and demanding that the boundary is a round 2-sphere gives

$$\alpha = \sqrt{1 - A^2 \ell^2}$$

Get a consistent first law with corrections to V and TD length, and – can generalise to rotation



GENERAL THERMO PARAMETERS

$$M = \frac{m(\Xi + a^2/\ell^2)(1 - A^2\ell^2\Xi)}{K\Xi\alpha(1 + a^2A^2)}$$

$$T = \frac{f'_+ r_+^2}{4\pi\alpha(r_+^2 + a^2)}\,, \quad S = \frac{\pi(r_+^2 + a^2)}{K(1 - A^2r_+^2)}\,,$$

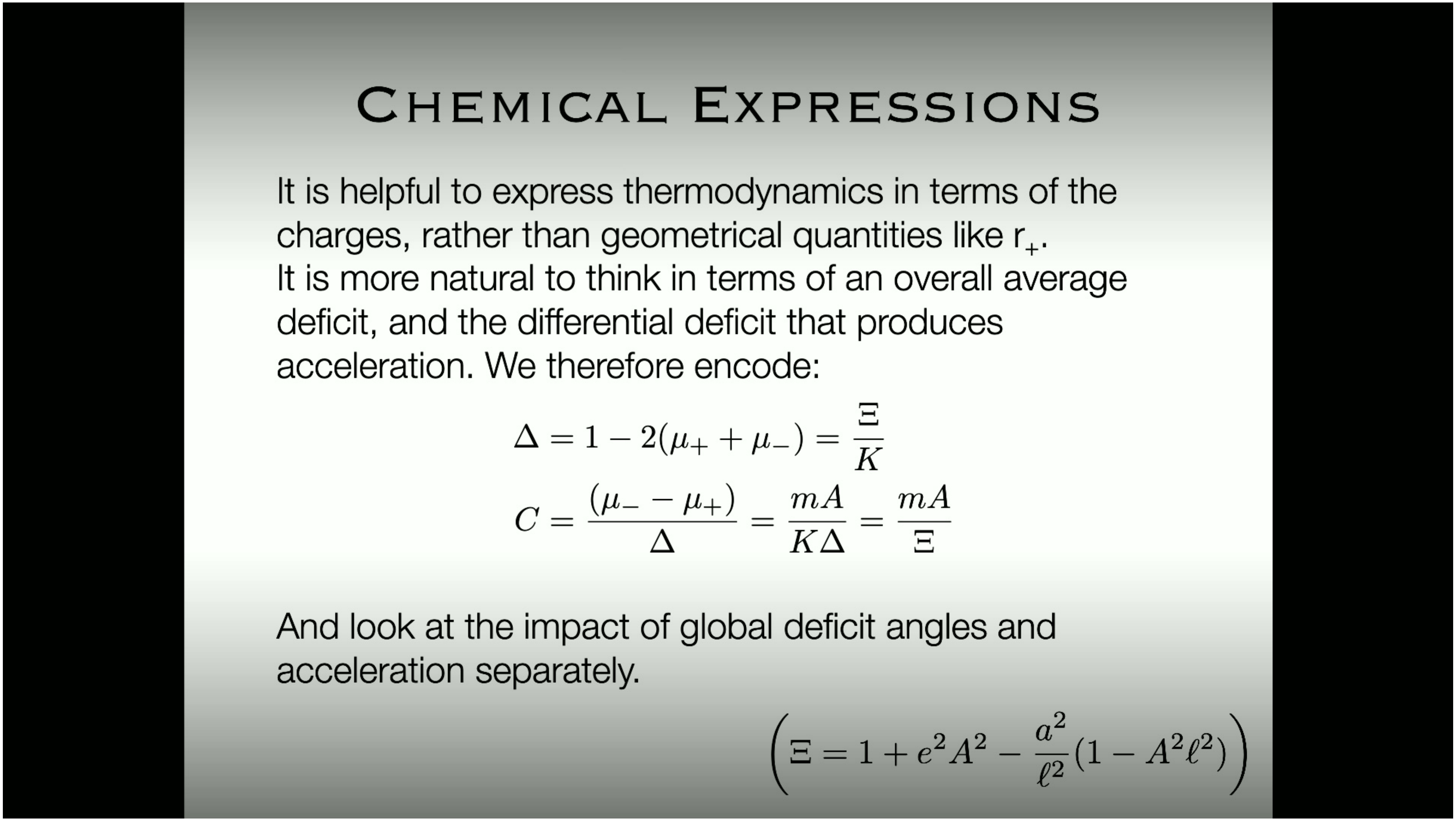
$$Q = \frac{e}{K}\,, \quad \Phi = \Phi_t = \frac{er_+}{(r_+^2 + a^2)\alpha}\,,$$

$$J = \frac{ma}{K^2}\,, \quad \Omega = \Omega_H - \Omega_\infty\,, \quad \Omega_H = \frac{Ka}{\alpha(r_+^2 + a^2)}$$

$$P = \frac{3}{8\pi\ell^2}\,, \quad V = \frac{4\pi}{3K\alpha}\left[\frac{r_+(r_+^2 + a^2)}{(1 - A^2r_+^2)} + \frac{m[a^2(1 - A^2\ell^2\Xi) + A^2\ell^4\Xi(\Xi + a^2/\ell^2)]}{(1 + a^2A^2)\Xi}\right]$$

$$\lambda_\pm = \frac{r_+}{\alpha(1 \pm Ar_+)} - \frac{m}{\alpha}\frac{[\Xi + a^2/\ell^2 + \frac{a^2}{\ell^2}(1 - A^2\ell^2\Xi)]}{(1 + a^2A^2)\Xi^2} \mp \frac{A\ell^2(\Xi + a^2/\ell^2)}{\alpha(1 + a^2A^2)}$$

$$\Xi = 1 - \frac{a^2}{l^2} + A^2(e^2 + a^2) \qquad \alpha = \frac{\sqrt{(\Xi + a^2/\ell^2)(1 - A^2\ell^2\Xi)}}{1 + a^2A^2}$$



CHEMICAL EXPRESSIONS

It is helpful to express thermodynamics in terms of the charges, rather than geometrical quantities like r_+.
It is more natural to think in terms of an overall average deficit, and the differential deficit that produces acceleration. We therefore encode:

$$\Delta = 1 - 2(\mu_+ + \mu_-) = \frac{\Xi}{K}$$

$$C = \frac{(\mu_- - \mu_+)}{\Delta} = \frac{mA}{K\Delta} = \frac{mA}{\Xi}$$

And look at the impact of global deficit angles and acceleration separately.

$$\left(\Xi = 1 + e^2 A^2 - \frac{a^2}{\ell^2}(1 - A^2\ell^2)\right)$$

$$M^2 = \frac{\Delta S}{4\pi}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)^2+\left(1+\frac{8PS}{3\Delta}\right)\left(\frac{4\pi^2 J^2}{(\Delta S)^2}-\frac{3C^2\Delta}{2PS}\right)\right]$$

$$V = \frac{2S^2}{3\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)+\frac{2\pi^2 J^2}{(\Delta S)^2}+\frac{9C^2\Delta^2}{32P^2S^2}\right],$$

$$T = \frac{\Delta}{8\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)\left(1-\frac{\pi Q^2}{\Delta S}+\frac{8PS}{\Delta}\right)-\frac{4\pi^2 J^2}{(\Delta S)^2}-4C^2\right],$$

$$\Omega = \frac{\pi J}{SM\Delta}\left(1+\frac{8PS}{3\Delta}\right),$$

$$\Phi = \frac{Q}{2M}\left(1+\frac{\pi Q^2}{S\Delta}+\frac{8PS}{3\Delta}\right),$$

$$\lambda_\pm = \frac{S}{4\pi M}\left[\left(\frac{8PS}{3\Delta}+\frac{\pi Q^2}{\Delta S}\right)^2+\frac{4\pi^2 J^2}{(\Delta S)^2}\left(1+\frac{16PS}{3\Delta}\right)-(1\mp 2C)^2\pm\frac{3C\Delta}{2PS}\right]$$

RG & Scoins, 1904.09660

NEW REVERSE ISOPERIMETRIC INEQUALITY

Now can manipulate

$$M^2\left(\frac{3V}{4\pi}\right)^2\left(\frac{\pi}{S}\right)^4 \geq \left(\frac{3\pi MV}{4S^2}-\frac{C^2}{x^2}\right)^2 \geq \frac{\pi M^2}{\Delta S}$$

into a new inequality appropriate for conical deficit black holes:

$$\left(\frac{3V}{4\pi}\right)^2 \geq \frac{1}{\Delta}\left(\frac{\mathcal{A}}{4\pi}\right)^3$$

“C” IN 3

We can look for an exact solution in 3D with the same type of structure:

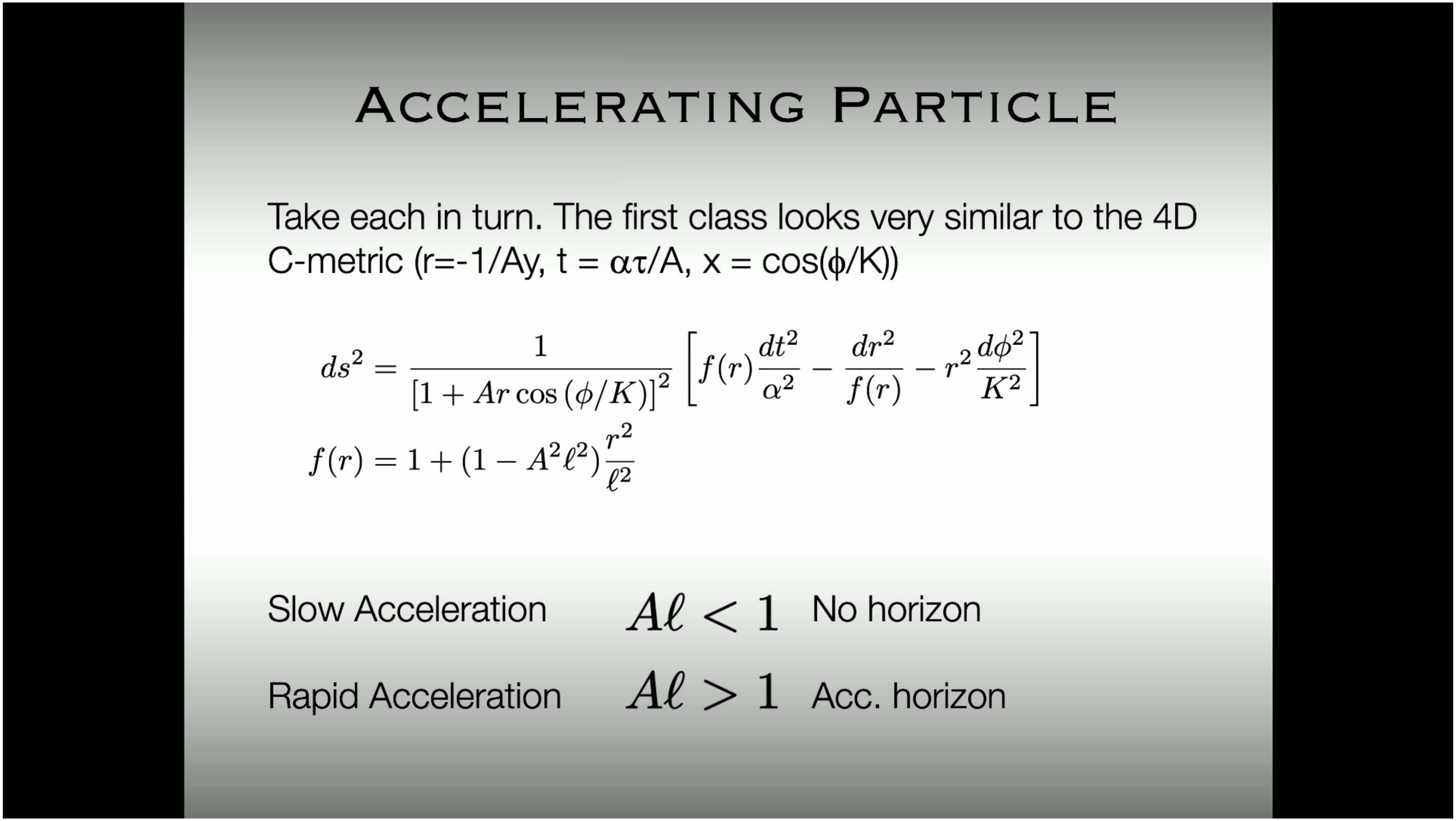
$$ds^2 = \frac{1}{A^2(x-y)^2}\left[P(y)d\tau^2 - \frac{dy^2}{P(y)} - \frac{dx^2}{Q(x)}\right]$$

With general solution: $Q(x) = c + bx + ax^2\,,\qquad P(y) = \frac{1}{A^2\ell^2} - Q(y)$

Which, after coordinate rescaling/shifts reduces to:

Class	$Q(x)$	$P(y)$	Maximal range of x
I	$1-x^2$	$\frac{1}{A^2\ell^2} + (y^2-1)$	$\|x\| < 1$
II	x^2-1	$\frac{1}{A^2\ell^2} + (1-y^2)$	$x > 1$ or $x < -1$
III	$1+x^2$	$\frac{1}{A^2\ell^2} - (1+y^2)$	$\mathbb{R}$

Arenas-Henriquez, RG, Scoins: 2202:08823; Anber 0809:2789; Astorino 1101.2616



ACCELERATING PARTICLE

Take each in turn. The first class looks very similar to the 4D C-metric (r=-1/Ay, t = ατ/A, x = cos(ϕ/K))

$$ds^2 = \frac{1}{\left[1 + Ar\cos\left(\phi/K\right)\right]^2}\left[f(r)\frac{dt^2}{\alpha^2} - \frac{dr^2}{f(r)} - r^2\frac{d\phi^2}{K^2}\right]$$

$$f(r) = 1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}$$

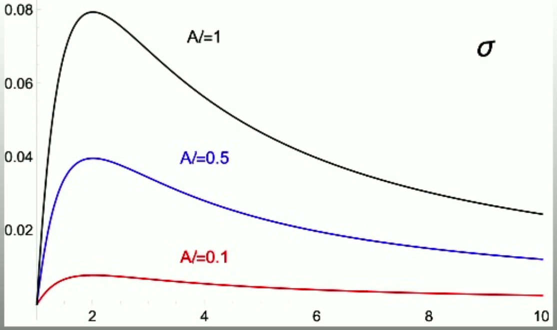
Slow Acceleration	$A\ell < 1$	No horizon
Rapid Acceleration	$A\ell > 1$	Acc. horizon

SLOW ACCELERATION

The presence of K now indicates both a conical deficit (the particle) and a *domain wall* at ϕ=±π, i.e. codimension 1 defect. The conical deficit at r=0 has a natural mass:

$$m_c = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

Because of the nonzero extrinsic curvature along ϕ=± π, (thanks to A) there is a wall of tension



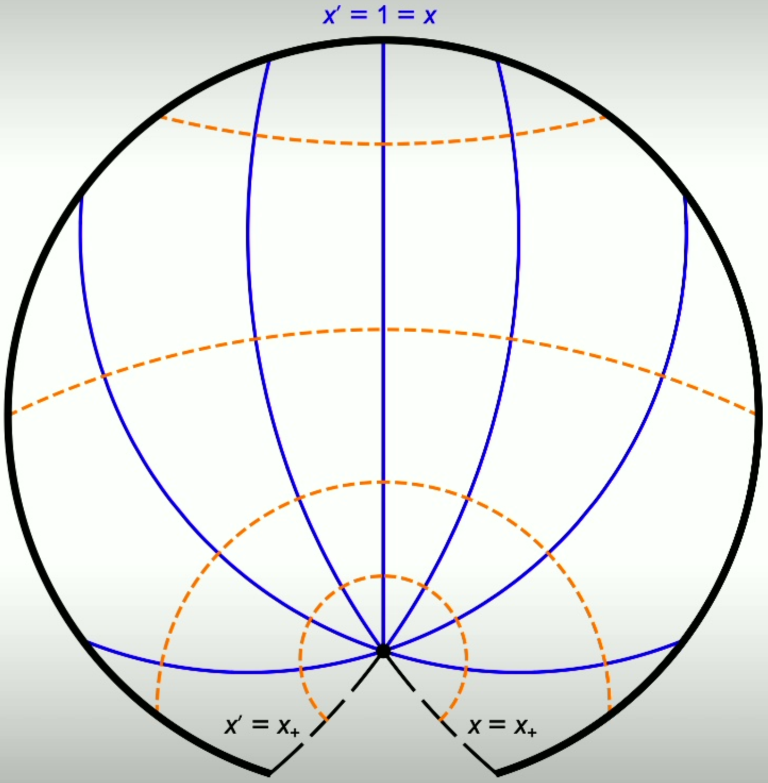
$$\sigma = \frac{A}{4\pi}\sin\left(\frac{\pi}{K}\right)$$

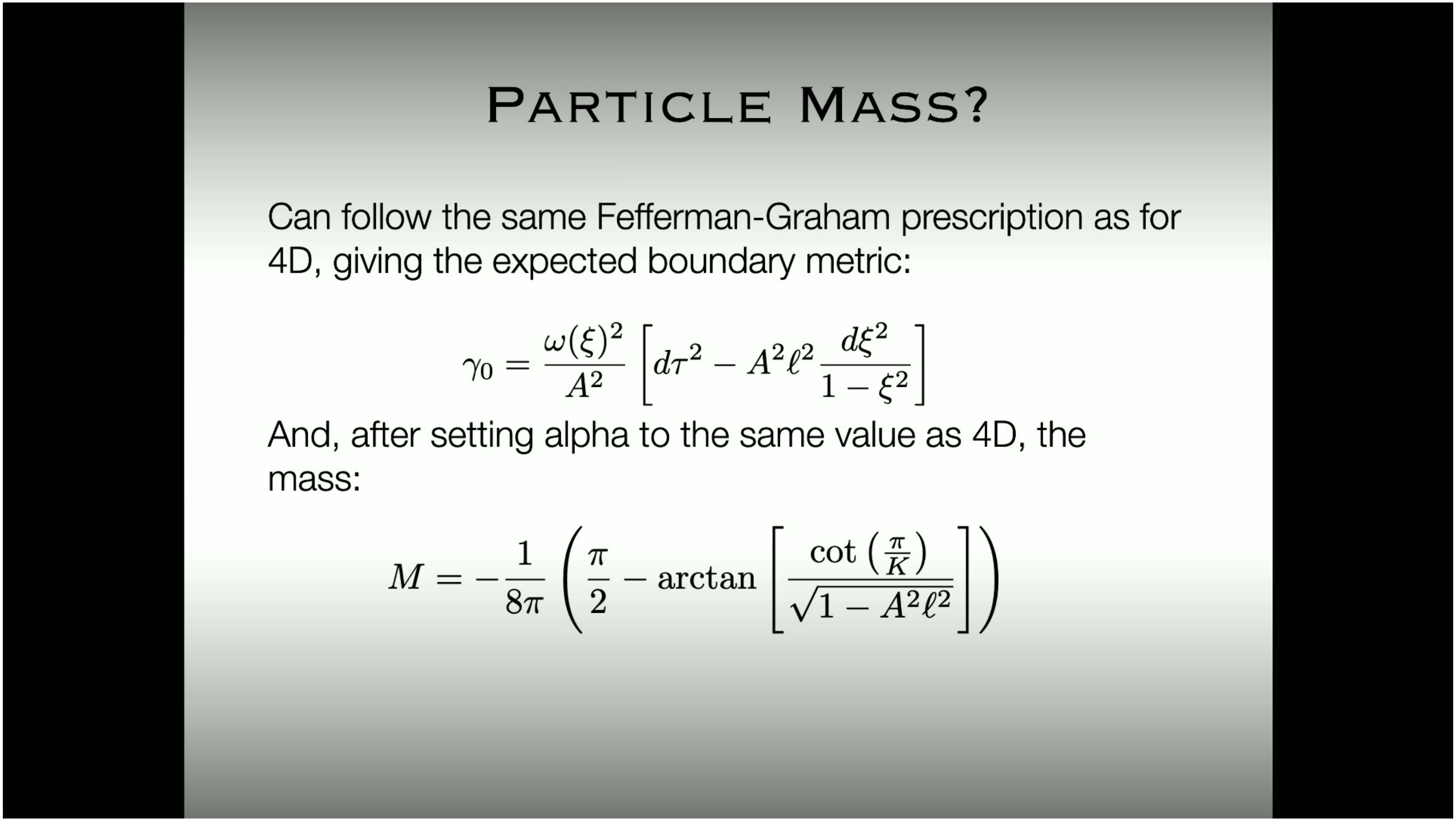
SLOW ACCELERATION

We can do the same coord transformation as in the 4D slow acceleration case, to get the same sort of picture:

$$R_0 = \frac{A\ell^2}{\alpha}$$

A determines the displacement from origin, and x_+ both the particle “mass” and wall tension.





PARTICLE MASS?

Can follow the same Fefferman-Graham prescription as for 4D, giving the expected boundary metric:

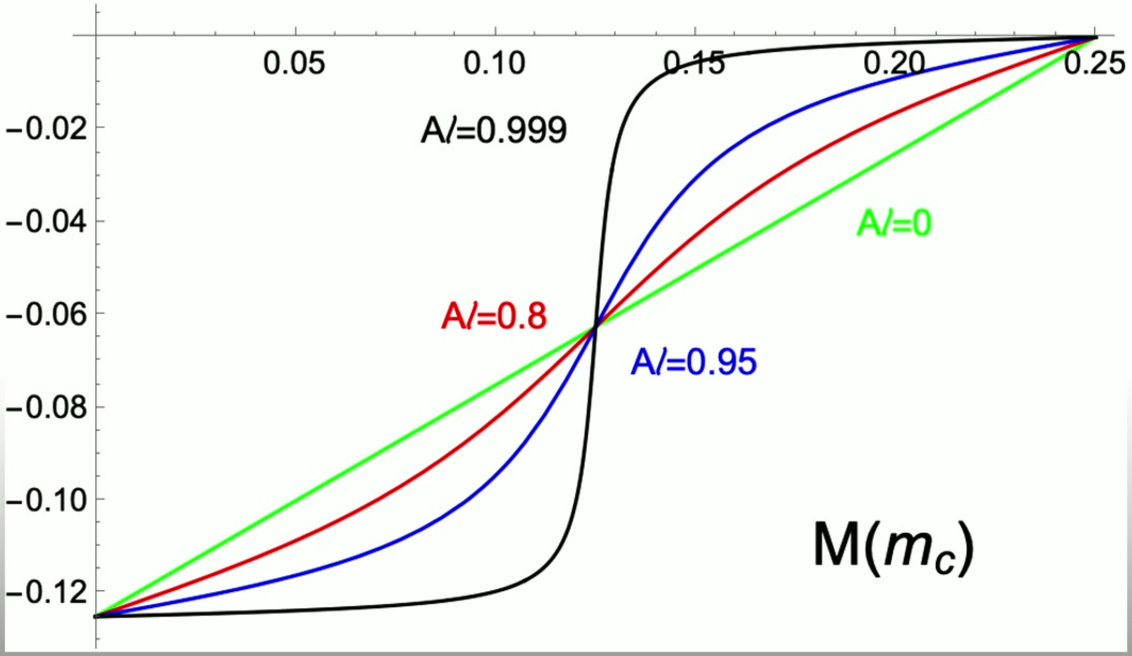
$$\gamma_0 = \frac{\omega(\xi)^2}{A^2}\left[d\tau^2 - A^2\ell^2\frac{d\xi^2}{1-\xi^2}\right]$$

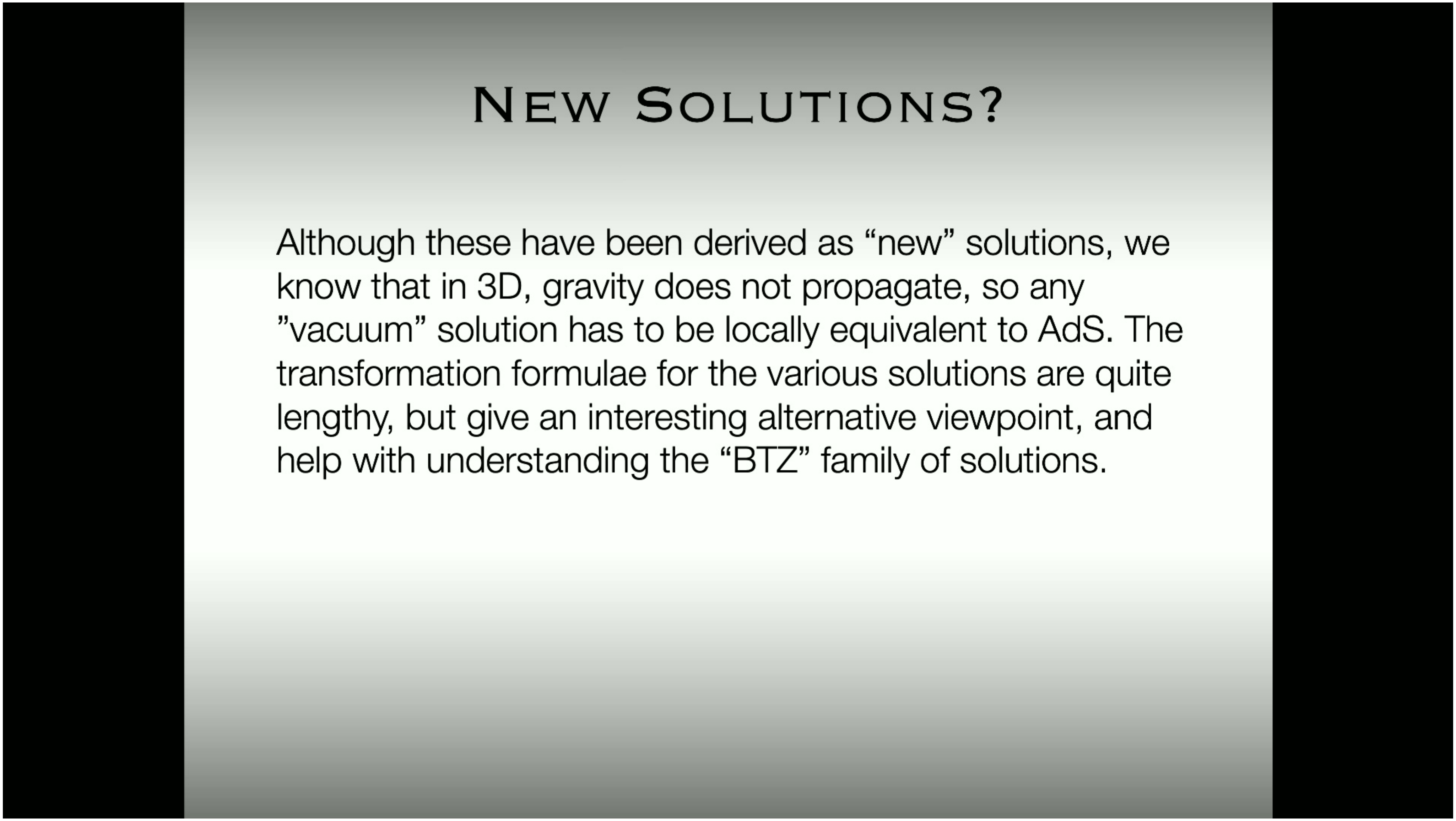
And, after setting alpha to the same value as 4D, the mass:

$$M = -\frac{1}{8\pi}\left(\frac{\pi}{2} - \arctan\left[\frac{\cot\left(\frac{\pi}{K}\right)}{\sqrt{1-A^2\ell^2}}\right]\right)$$

HOLOGRAPHIC MASS

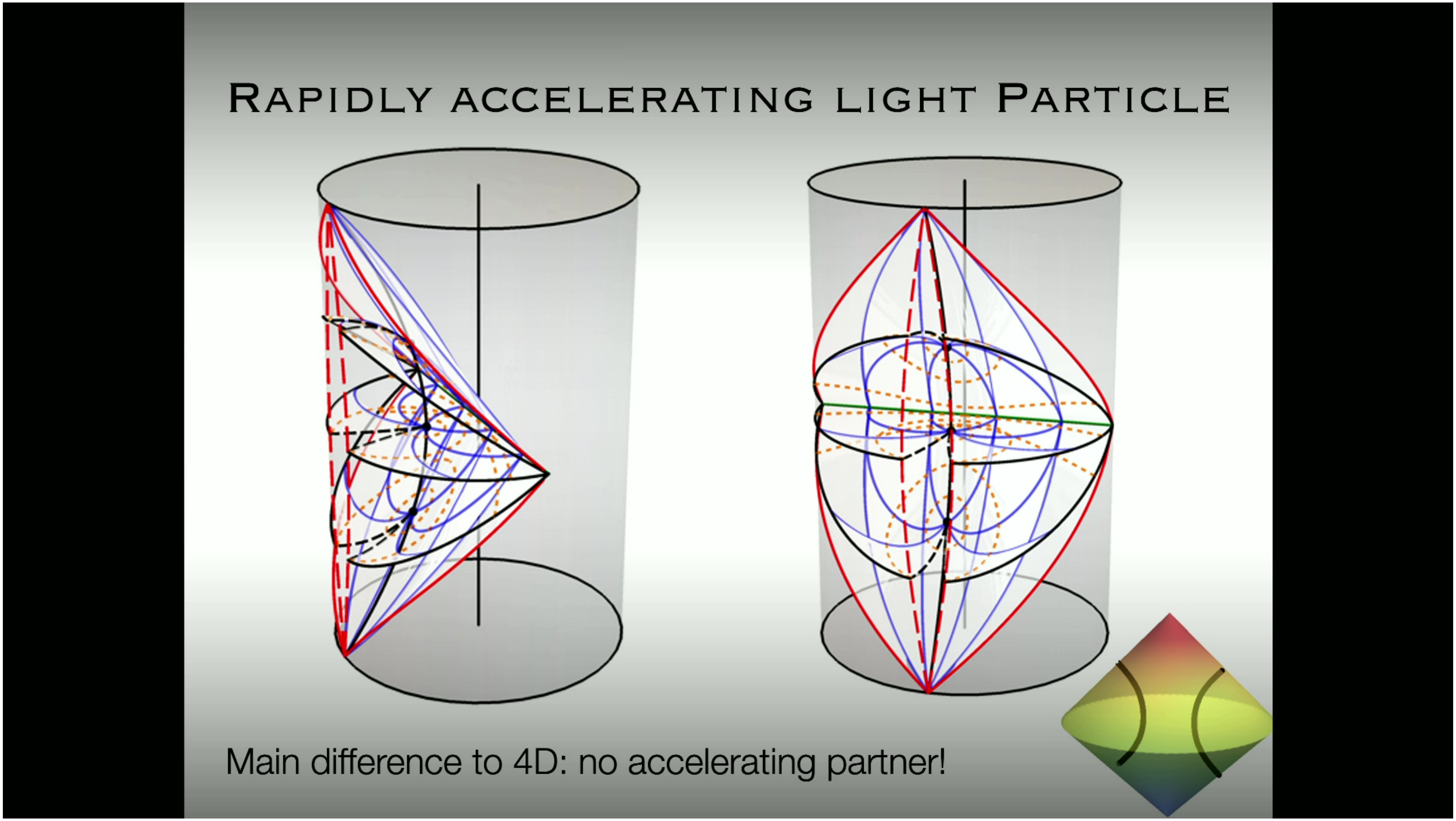
Compare to “particle” mass from conical deficit:





NEW SOLUTIONS?

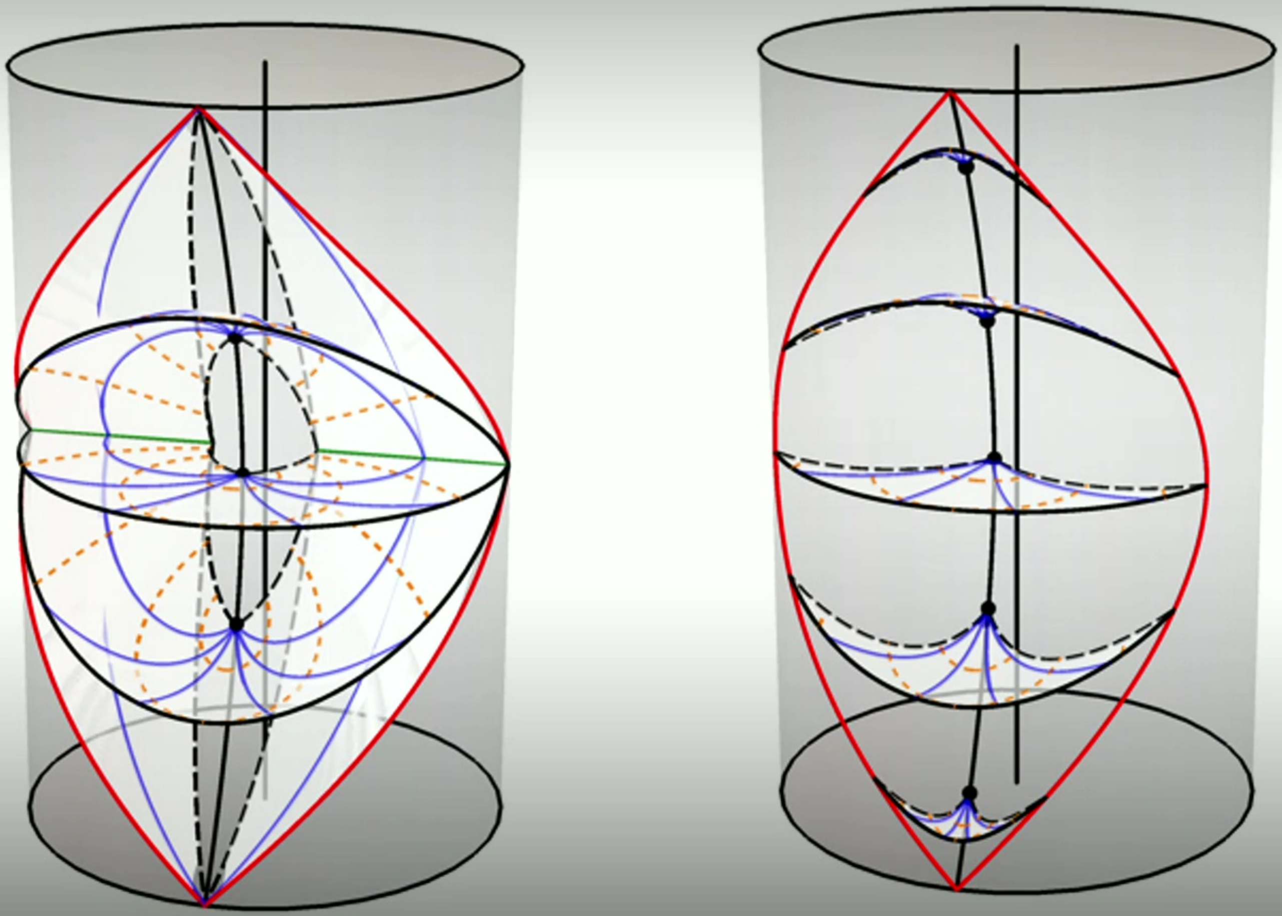
Although these have been derived as “new” solutions, we know that in 3D, gravity does not propagate, so any ”vacuum” solution has to be locally equivalent to AdS. The transformation formulae for the various solutions are quite lengthy, but give an interesting alternative viewpoint, and help with understanding the “BTZ” family of solutions.



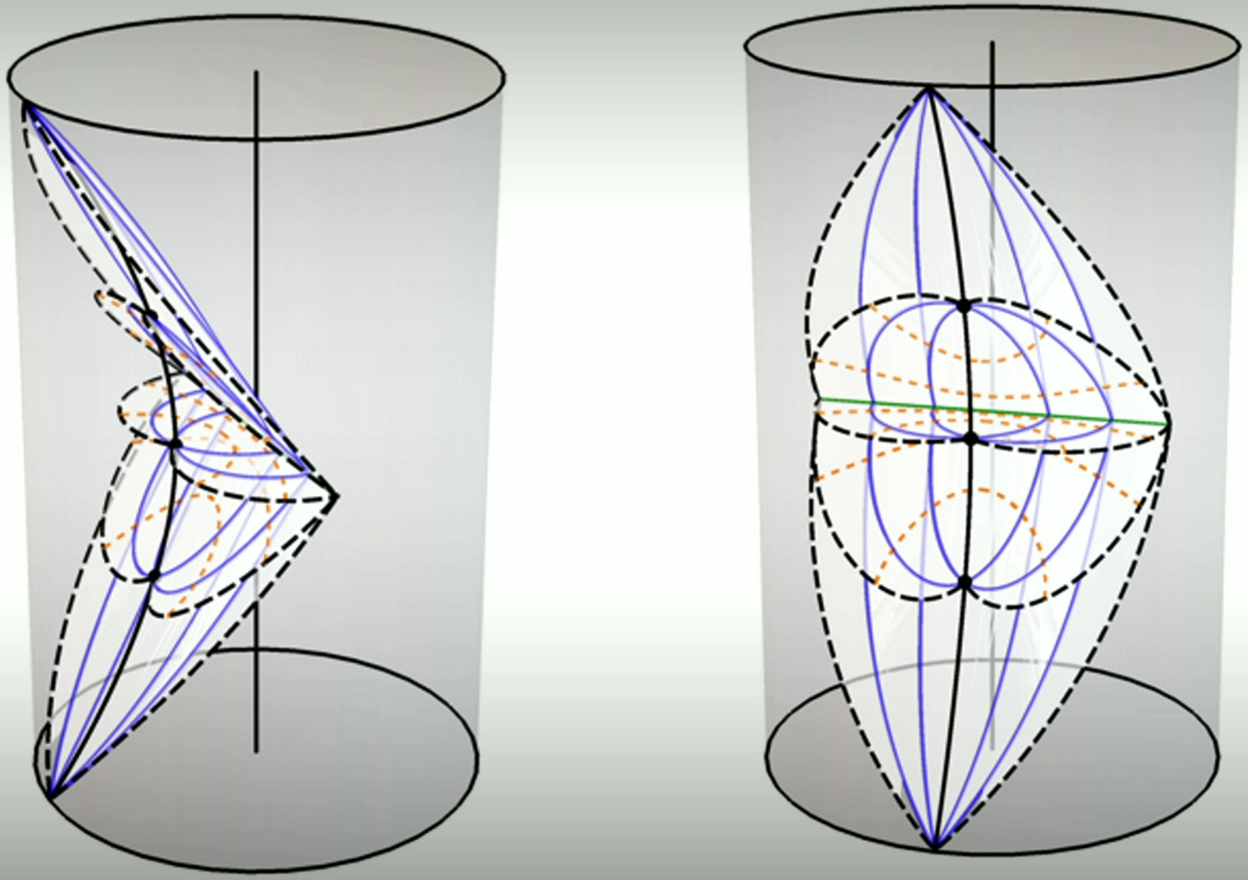
Rapidly accelerating light Particle

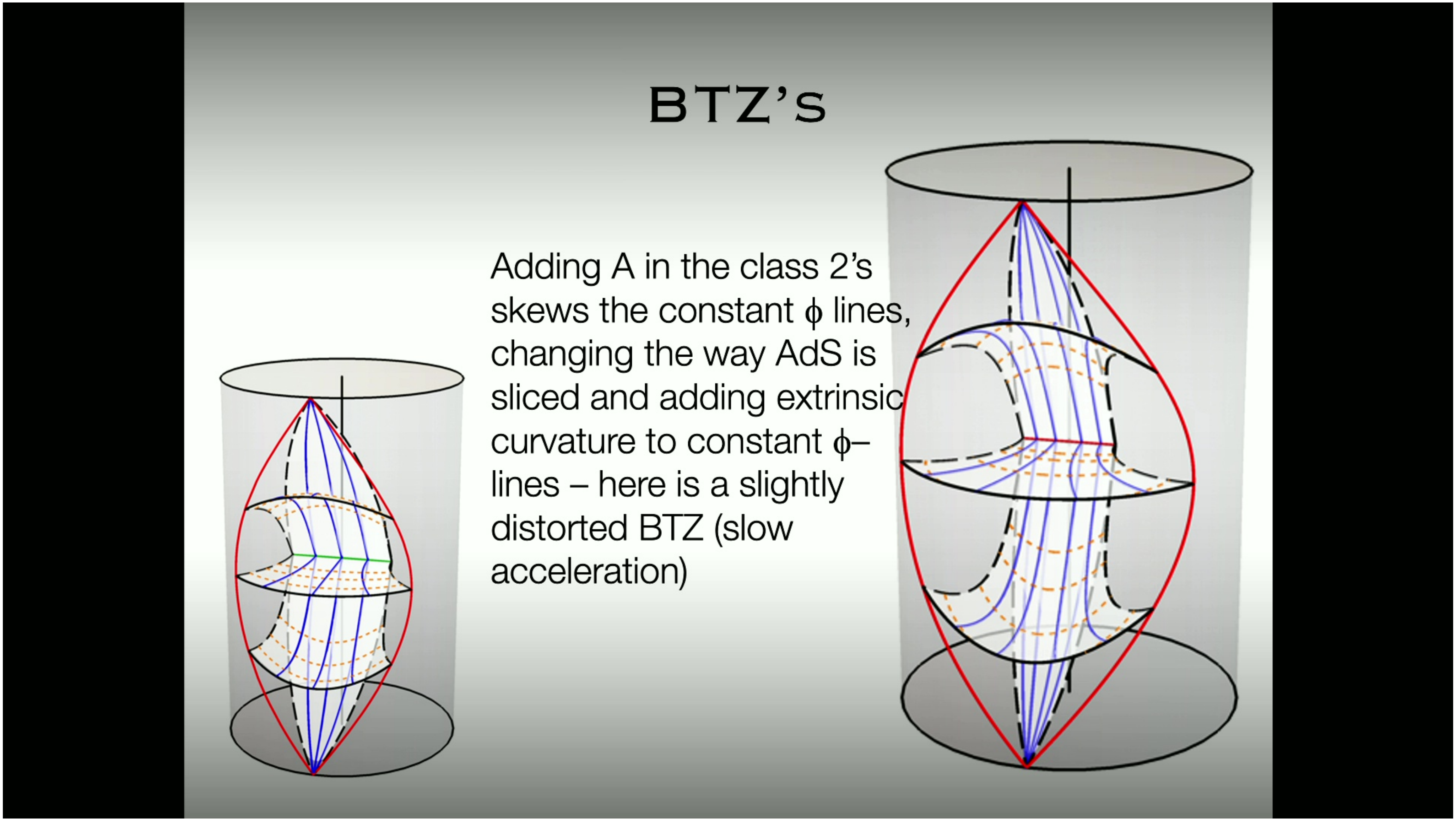
Main difference to 4D: no accelerating partner!

ACCELERATION WITH STRUTS



RAPIDLY ACCELERATING HEAVY PARTICLE



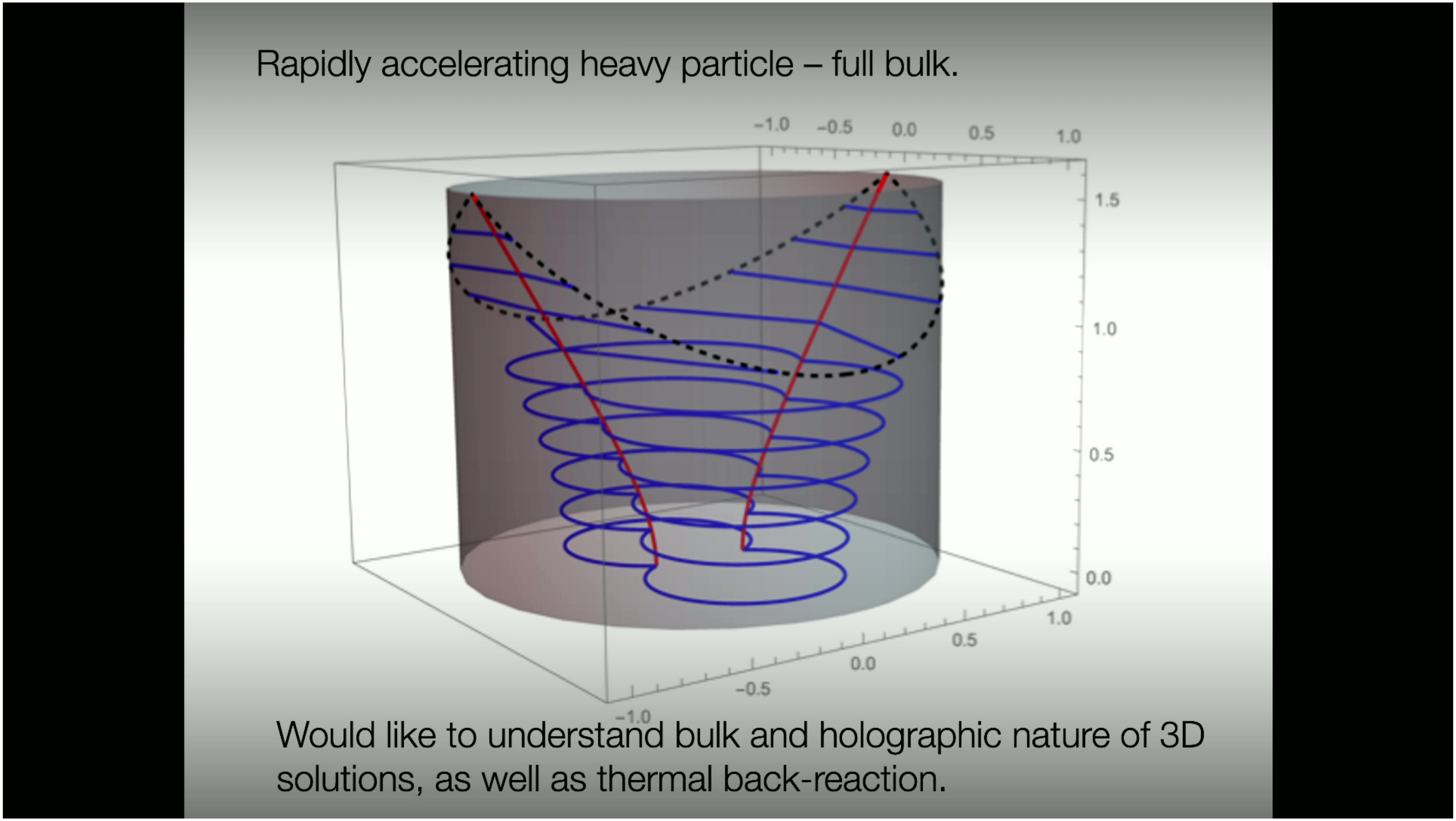


BTZ's

Adding A in the class 2's skews the constant ϕ lines, changing the way AdS is sliced and adding extrinsic curvature to constant ϕ–lines – here is a slightly distorted BTZ (slow acceleration)

RECAP

- Have shown how to allow for varying tension in thermodynamics of black holes.
- Conjugate variable is *Thermodynamic Length*
- Thermodynamics of accelerating black holes is computable – non-static and non-isolated.
- A key technical point is the normalisation of timelike Killing vector
- Have derived extensive expressions for the TD variables and a new Reverse Isoperimetric Inequality.
- Three dimensions is both familiar and new!



Rapidly accelerating heavy particle – full bulk.

Would like to understand bulk and holographic nature of 3D solutions, as well as thermal back-reaction.