

Title: LECTURE: Tensor networks and quantum algorithms

Speakers: Martin Ganahl

Collection: Quantum and AI Career Trajectories Mini-Course: Computational Methods and their Applications

Date: May 10, 2023 - 9:30 AM

URL: <https://pirsa.org/23050099>

Abstract: TBC

## Today

- MPS gauge & exp. values
- area law for MPS
- truncation of MPS
- real time evolution

## MPS review

- efficient parametrization of many body wf
- systematic way to explore Hilbert space
- area law



# MPS review

ues

- efficient parametrization of many body wf

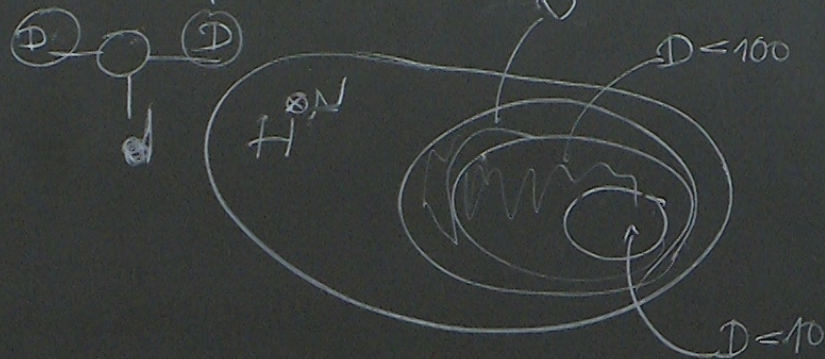
- systematic way to explore Hilbert space

- area low

$147 \text{ on } N \text{ spins}$

$\rightarrow \# \text{ params } \sim e^N$

MPS:  $\# \text{ params } \sim N$





$$|4\rangle^{\wedge} = \text{---} \circ \text{---} \circ \text{---} \overset{-1}{\circ} \text{---} \circ \text{---}$$

$$= \text{---} \circ \text{---} \circ \text{---} \circ \text{---}$$

3 types of gauges

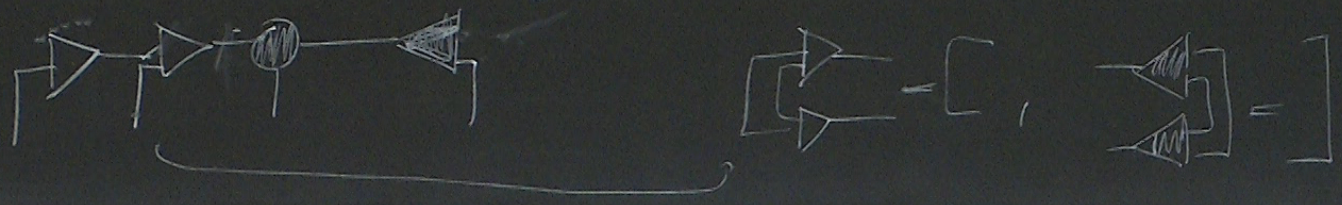
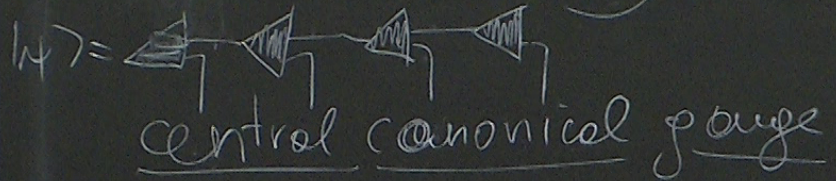
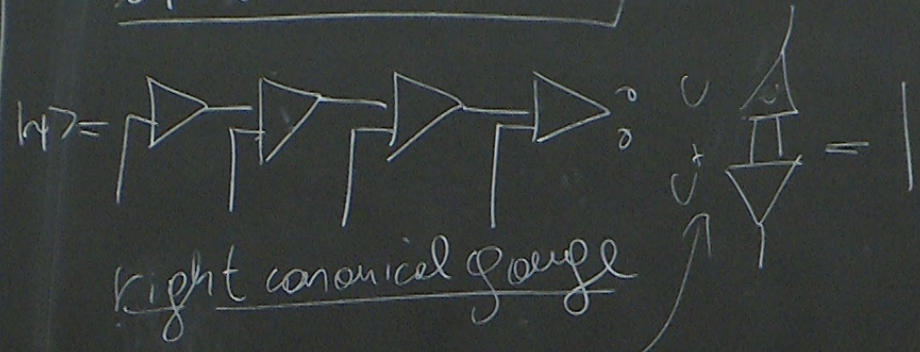
- left canonical

- right canonical

- central canonical gauge



left canonical MPS

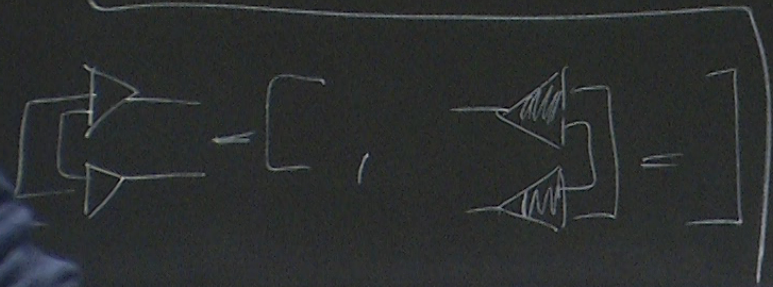




Exp. value  $\Pi$

$$\langle \sigma_1 \sigma_2 \rangle = 0$$

- 1): simplification of contraction
- 2): stability of optimization

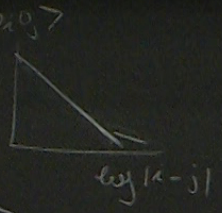




### Correlation functions

$$\langle \psi | \sigma_i \sigma_j | \psi \rangle = \frac{1}{|i-j|}$$

Some critical state  $|\psi\rangle$

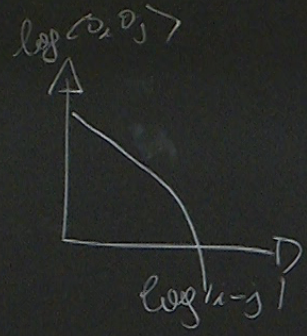


$|\psi\rangle = \text{MPS}$

can be cast

$$\langle \psi | \sigma_i \sigma_j | \psi \rangle \sim e^{-\frac{|i-j|}{\xi}}$$

MPS-correlations

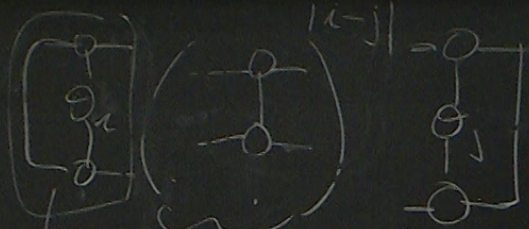


$$\langle (\sigma_i \sigma_j) - \langle \sigma_i \rangle \langle \sigma_j \rangle \rangle$$





$$D=10$$



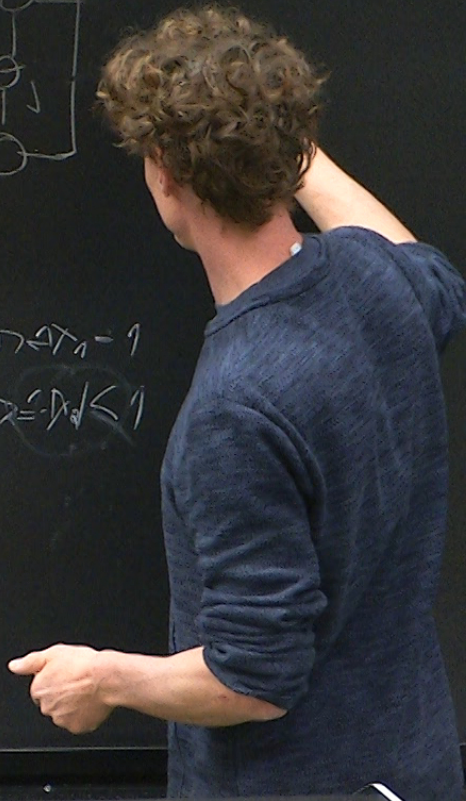
Transfer operator

$$|K| = |A|$$

$$|K| = \sum_{n=0}^{\infty} A_n |K|$$

$$|\lambda - 1| < 1 \Rightarrow |\lambda| < 2$$

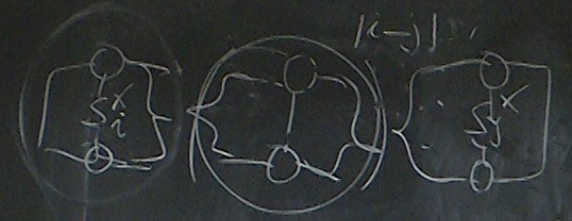
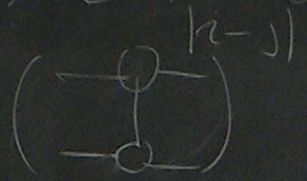
$$|\lambda - 2| < 1 \Rightarrow |\lambda| < 3$$





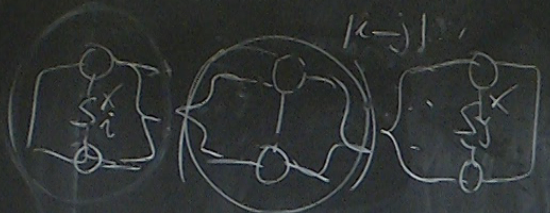
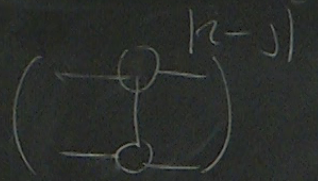
$D=10$

MPS have area law



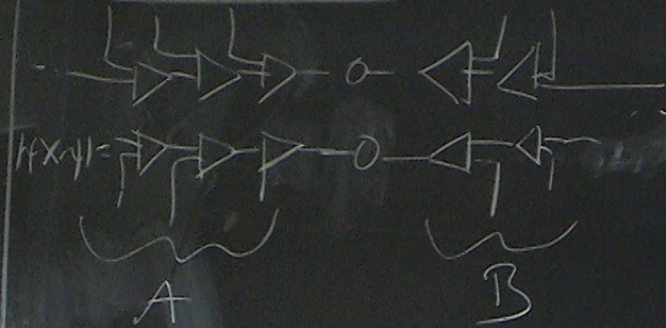


$D=10$



0 0 0 0 0 0 0 0

MPS area

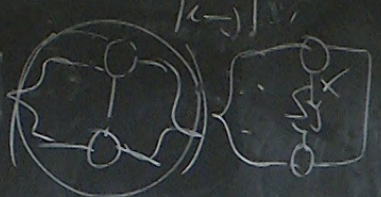
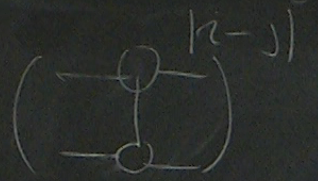


$$\hat{S}_A = \text{tr}_B \hat{S}$$



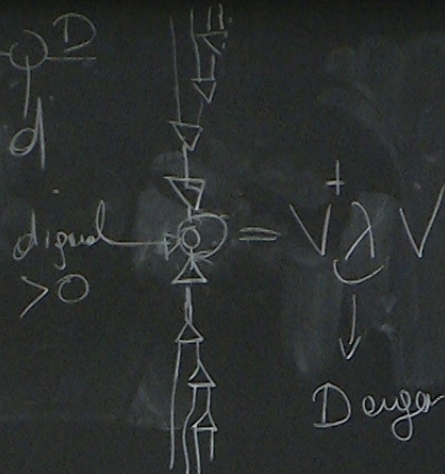
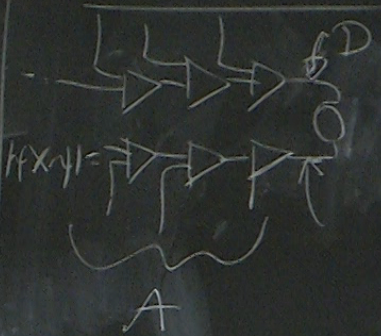


$D=10$



0 0 0 0 0 0 0 0

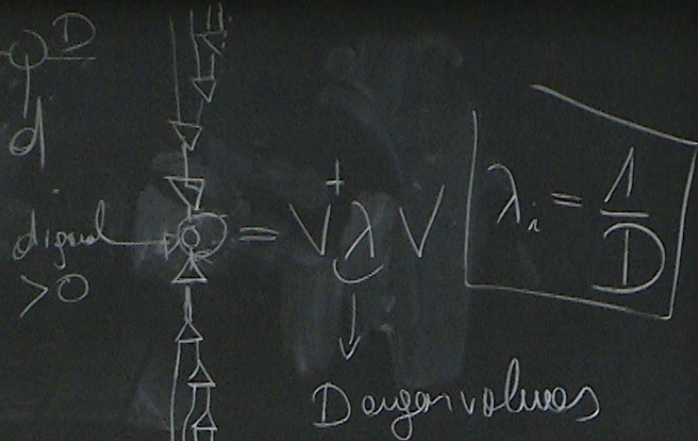
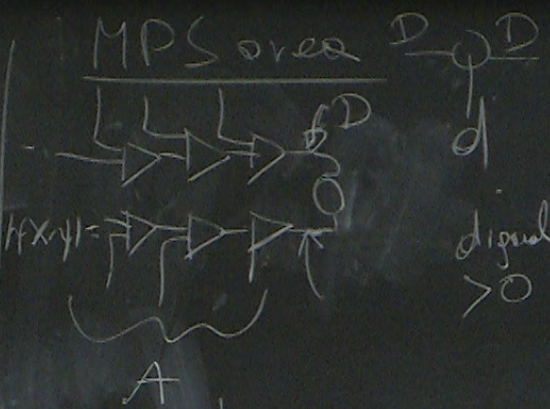
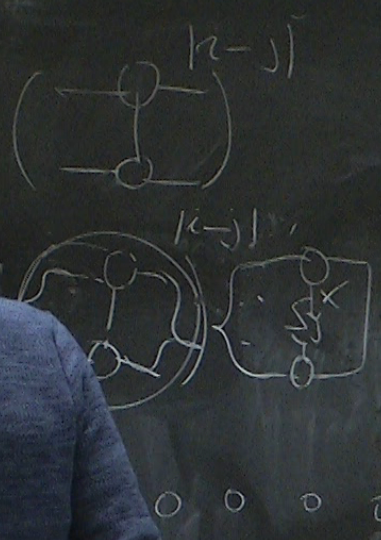
MPS area  $D \times D$



$$S(\hat{S}_A) = -\text{tr} S \log_2 \hat{S}_A = \sum_{\lambda=1}^D \lambda \log \frac{1}{\lambda}$$



$D=10$



$$\hat{S}_A = \text{tr}_B \hat{S}$$

$$S(\hat{S}_A) = -\text{tr} S \log_2 \hat{S}_A = \sum_{\lambda=1}^D \lambda \log \frac{1}{\lambda}$$

$\leq \log D$



$D=10$



$$\hat{S}_A = \text{tr}_B S$$

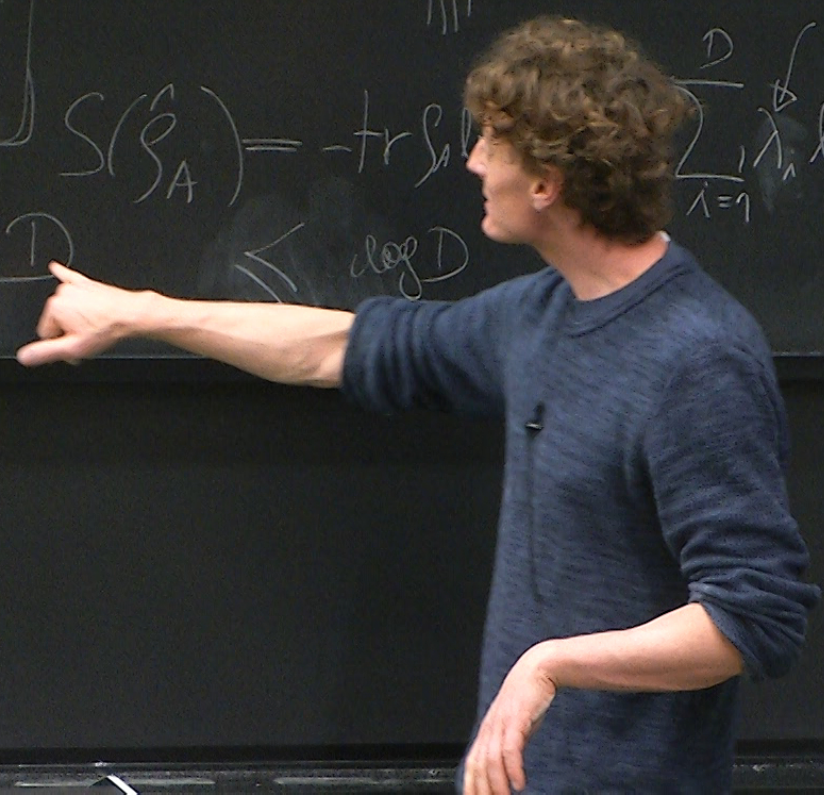
Dagonal values

$$\sum_{\lambda=1}^D \lambda_i \log \frac{1}{\lambda_i}$$

$$S(\hat{S}_A) = -\text{tr} S_A \log S_A$$

$$S(S) \leq \log_2 D \iff D \leq 2^S \leq D$$

$$\leq \log D$$



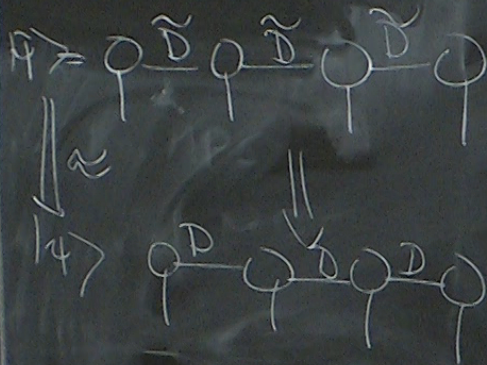


- central canonical gauge



orthogonality center

### truncation of MPS

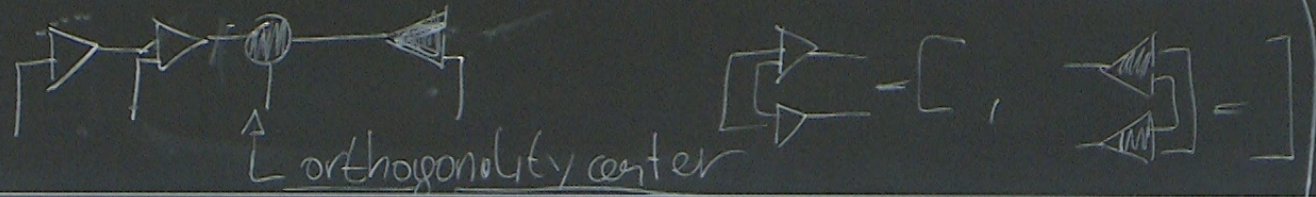


$$\min_{\{D\}} \|\tilde{|\psi\rangle} - |\psi\rangle\|$$

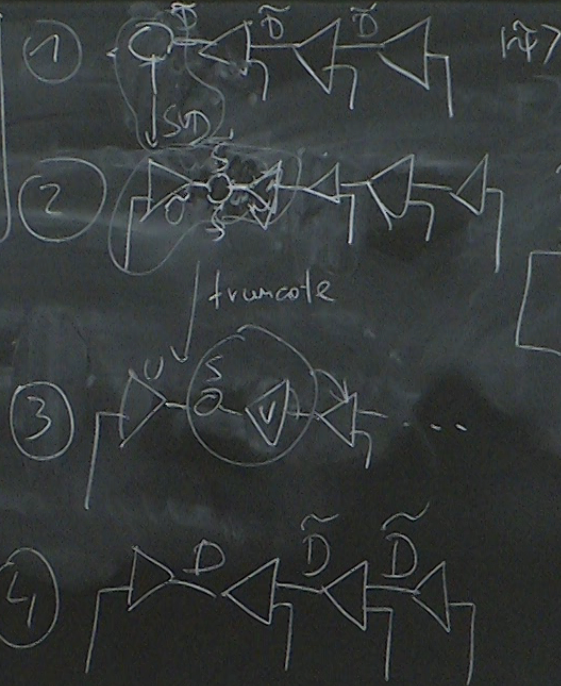


usual  
canonical gauge

central canonical gauge



MPS  
 $\tilde{D} \gg D$



$\sum_{s_i} \dots \text{has } \tilde{D} \text{ SV}$

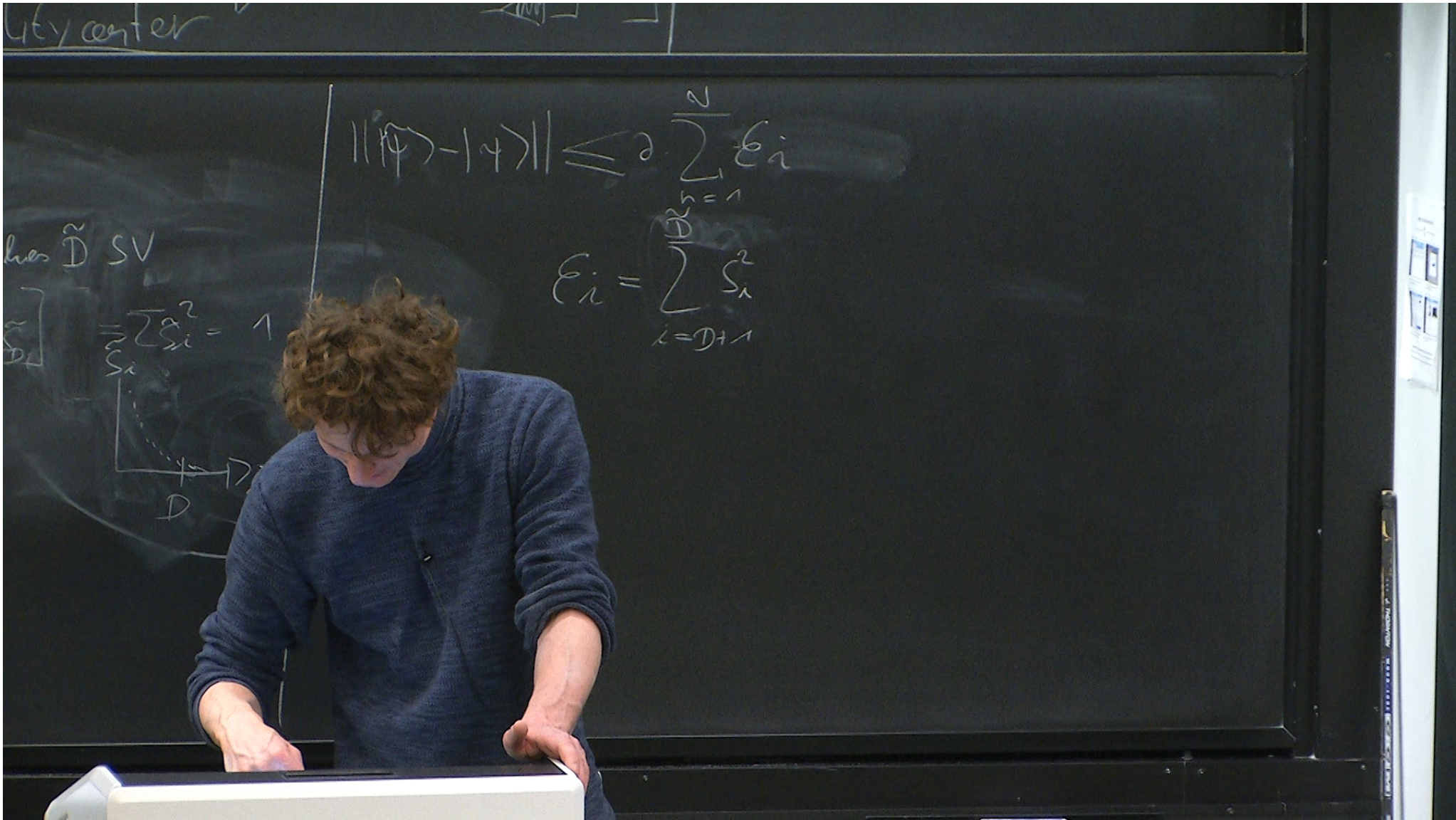
$\sum_{s_i} \tilde{D}^2 = 1$

$\sum_{s_i} \tilde{D}^2 = 1$

$\tilde{D}$









city center

$$\| | \psi \rangle - | \phi \rangle \| \leq a \sum_{i=1}^D \epsilon_i$$

has  $\tilde{D}$  SV

$$\sum_{i=1}^{\tilde{D}} s_i^2 = 1$$

$$\epsilon_i = \sum_{l=D+1}^{\tilde{D}} s_{li}^2$$

