Title: Two loops and Higgs field space geometries; pushing the interpretation of LHC data forward in the SMEFT

Speakers: Michael Trott

Series: Particle Physics

Date: May 12, 2023 - 11:00 AM

URL: https://pirsa.org/23050084

Abstract: Getting the strongest physics conclusions from collider particle physics experiments regarding the (in)consistency of the Standard Model with actual measurements requires Effective Field Theory techniques. This approach (known as the SMEFT) has been rapidly advanced in recent years, leading to new analyses of the data being executed by Atlas and CMS. The current state of the theoretical art is not precise enough as such EXP studies are continued into the future, as the measurements continue to become more precise. We need to be able to calculate more precisely in the SMEFT to keep up. After an intro to this area of research, I will discuss some recent calculations that have pushed things to the two loop level in precision for higgs production/decay in the SMEFT, and how thinking geometrically (in terms of field space connections and the resulting Higgs geometries) in EFT is the key to keep advancing the theoretical state of the art.

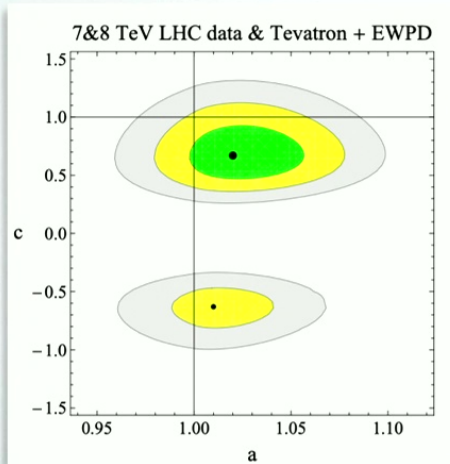
Zoom link: https://pitp.zoom.us/j/99931154202?pwd=SUMzK2JIS0prNk5KaGZWakphckZhdz09

Two loops and Higgs Field Space geometries.

Mike Trott (Perimeter/Caltech)

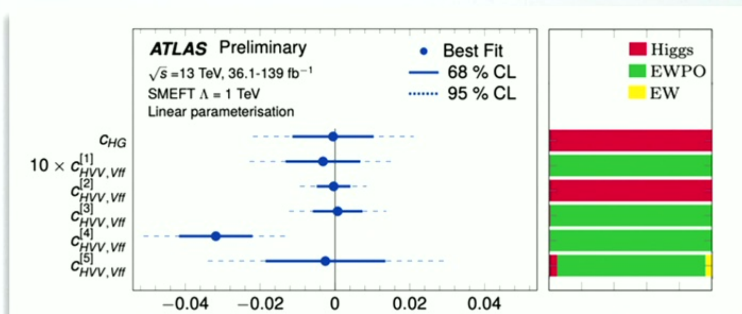
Related paper this week: Martin and Trott https://arxiv.org/abs/2305.05879

VILLUM FONDEN



Pre-Higgs discovery Kappa/HEFT :**2012**!

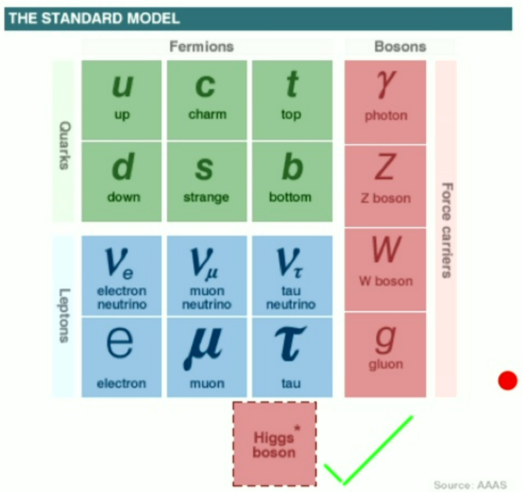
Higgs@10

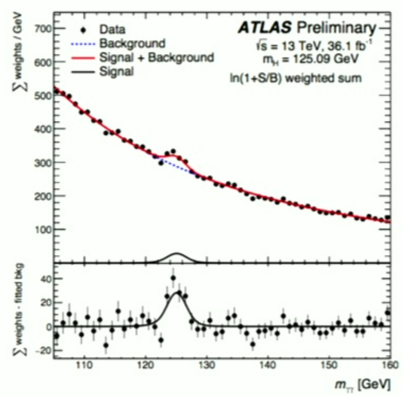


Modern SMEFT Analysis: ATLAS-PHYS-PUB-**2022**-037

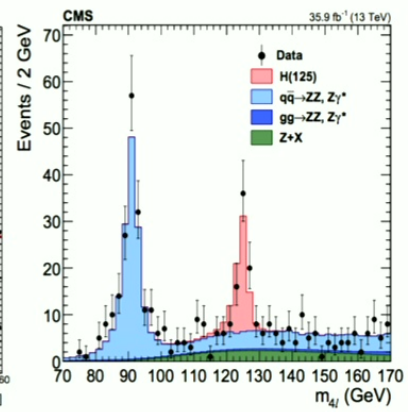
What was discovered at LHC, a particle

- Discovery of (Higgs like) Meaning $J^P \sim 0^+$ particle in 2012





ATLAS-CONF-2017-045 (2017)

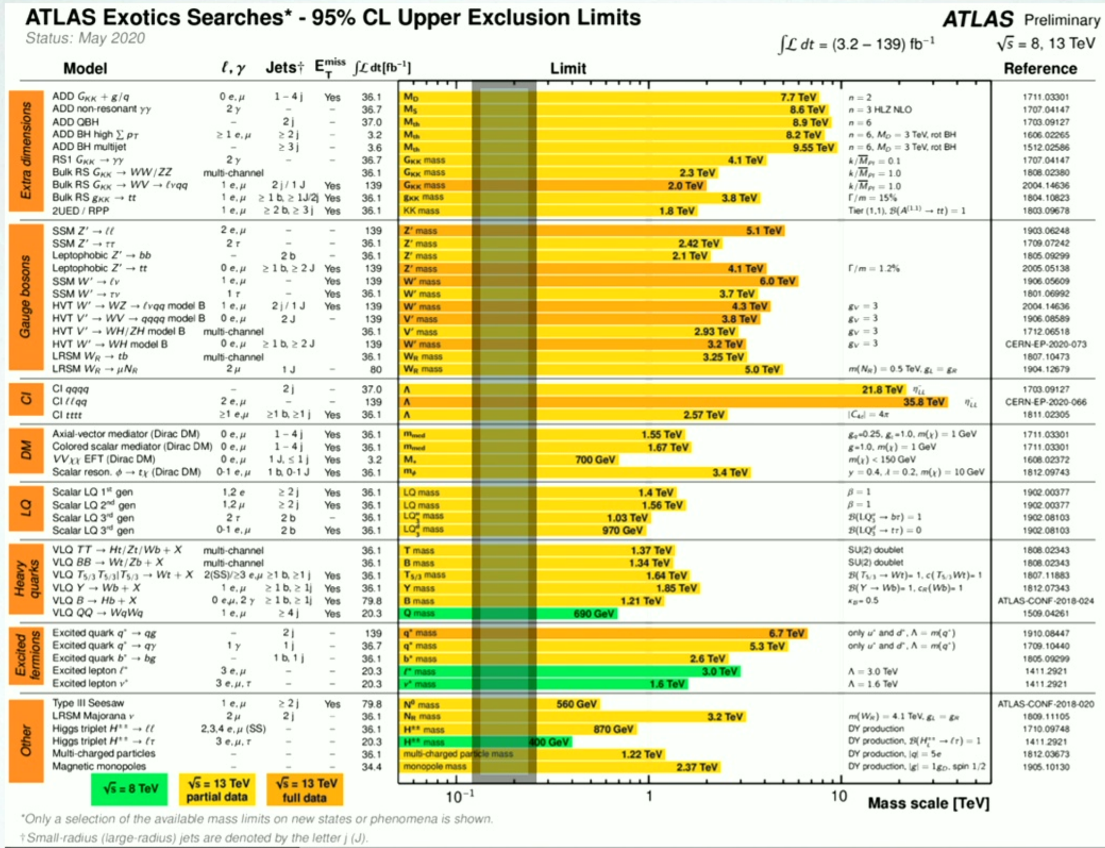


CMS-PAS-HIG-16-041 (2017)

- The SM, an SU(3) xSU(2)xU(1) ***linearly realized gauge theory*** :

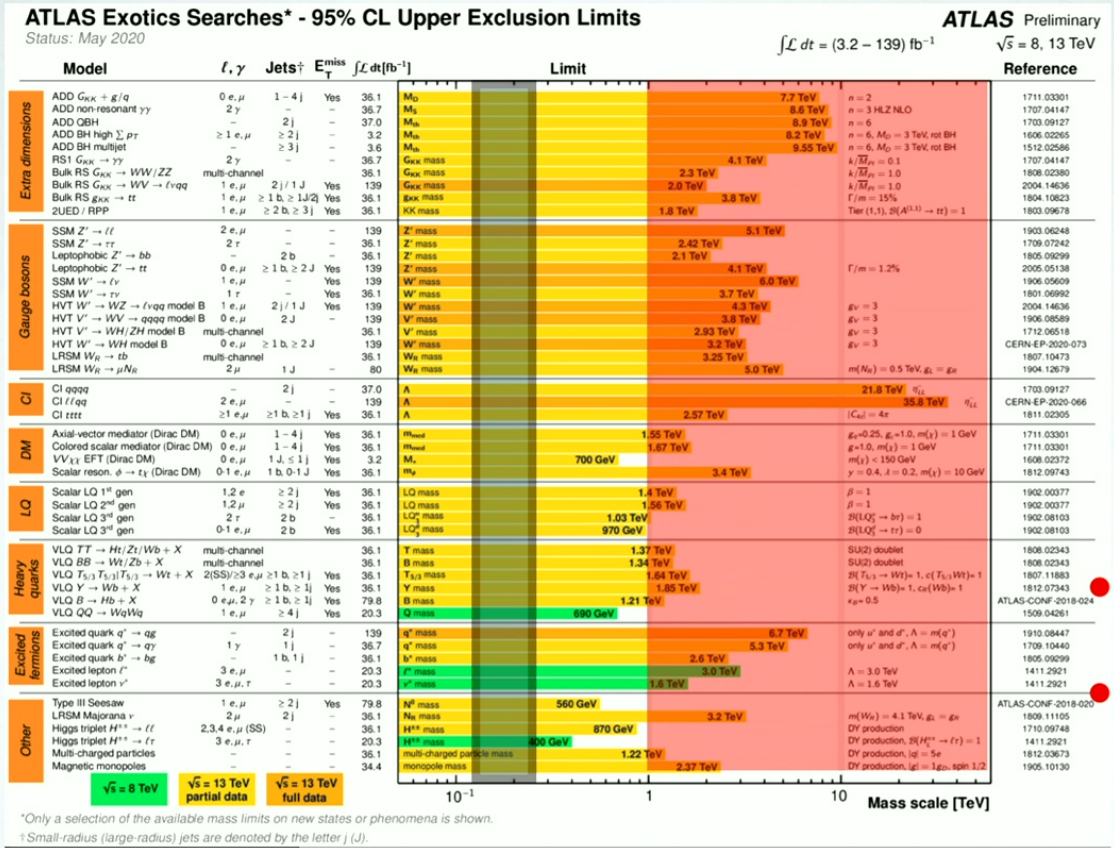
$$\mathcal{L}_{\rm SM} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,l,e} \overline{\psi}\, i \not{D}\, \psi$$
$$-\lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - \left[H^{\dagger j}\overline{d}\, Y_d\, q_j + \widetilde{H}^{\dagger j}\overline{u}\, Y_u\, q_j + H^{\dagger j}\overline{e}\, Y_e\, l_j + \text{h.c.}\right],$$

What wasn't discovered at LHC



Masses of EW scale ($\sim g\, v$) states m_W, m_Z, m_t, m_h

What wasn't discovered at LHC



Bounds have been pushed away from

$v \sim m_h$

USE that

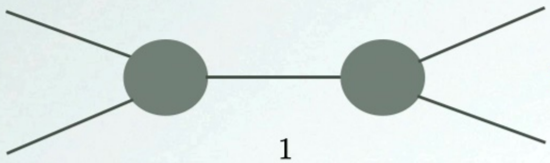
$v/M < 1$

to simplify/for stronger conclusions:

- bound many models at once
- bound multiple resonances at same time

Deviations then look like local contact operator effects in EFT

When you do measurements below a particle threshold..



$$\sim \frac{1}{s - m^2 + i\Gamma(s)\, m}$$

IF the collision probe does not reach $\sim m^2_{heavy}$
THEN observable's dependence on that scale simplified

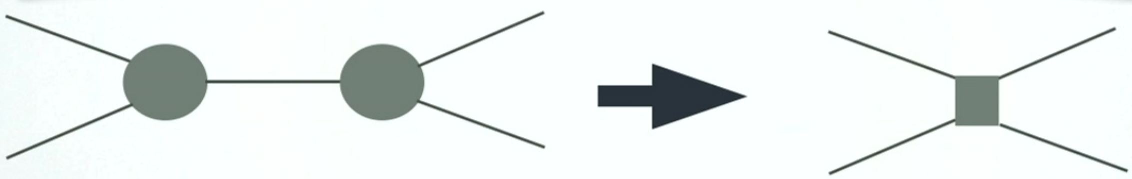
Infrared singularities and small distance behavior analysis
K. Symanzik (DESY) (Apr, 1973)
Published in: *Commun.Math.Phys.* 34 (1973) 7-36

Infrared singularities and massive fields
Thomas Appelquist and J. Carazzone
Phys. Rev. D **11**, 2856 – Published 15 May 1975

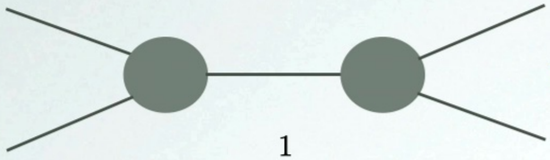
- The effects of heavy physics are localised, essentially, by the uncertainty principle

$$\Delta t\, \Delta E \sim \Delta t\, M > 1 \rightarrow \Delta t \sim \frac{1}{M}, \qquad \Delta|x|\, \Delta|p| \sim \Delta|x|\, M > 1 \rightarrow \Delta|x| \sim \frac{1}{M}.$$

$\hbar = 1 = c.$



When you do measurements below a particle threshold



$$\sim \frac{1}{s - m^2 + i\Gamma(s)\, m}$$

IF the collision probe does not reach $\sim m_{heavy}^2$
THEN observable's dependence on that scale simplified

Infrared singularities and small distance behavior analysis
K. Symanzik (DESY) (Apr, 1973)
Published in: Commun.Math.Phys. 34 (1973) 7-36

Infrared singularities and massive fields
Thomas Appelquist and J. Carazzone
Phys. Rev. D **11**, 2856 – Published 15 May 1975

- You can Taylor expand in LOCAL functions (operators)

$$\langle\rangle \sim O^0_{SM} + \frac{f_1(s,t,u)}{M^2_{heavy}} + \frac{f_2(s,t,u)}{M^4_{heavy}} + \cdots$$

IR operator form

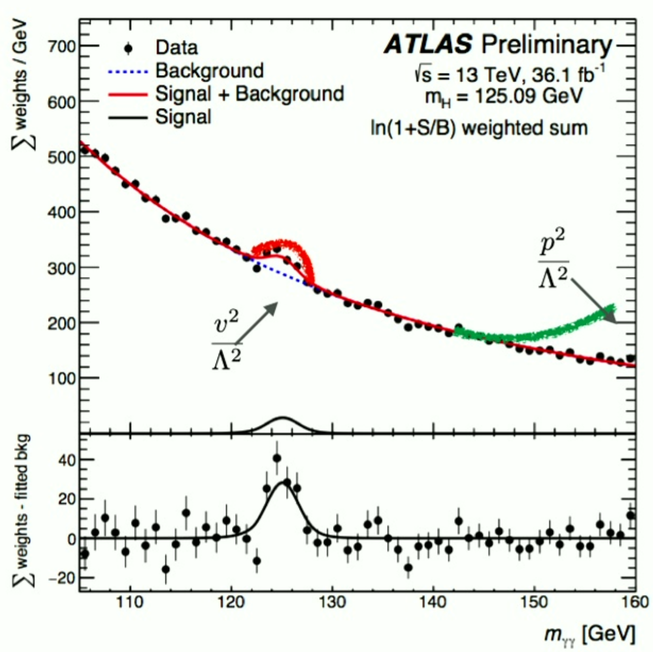
$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots, \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

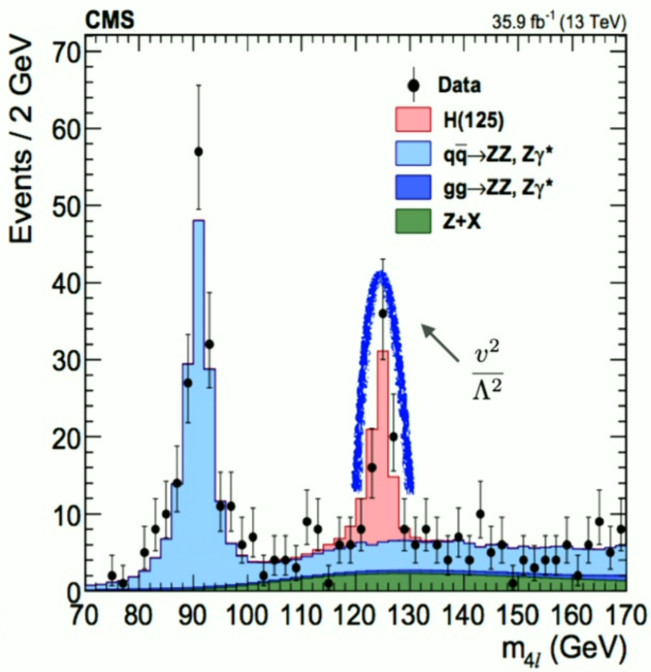
Λ v.s. M_{heavy}

UV dependent Wilson coefficient
and suppression scale

How does it help to have this simplification?

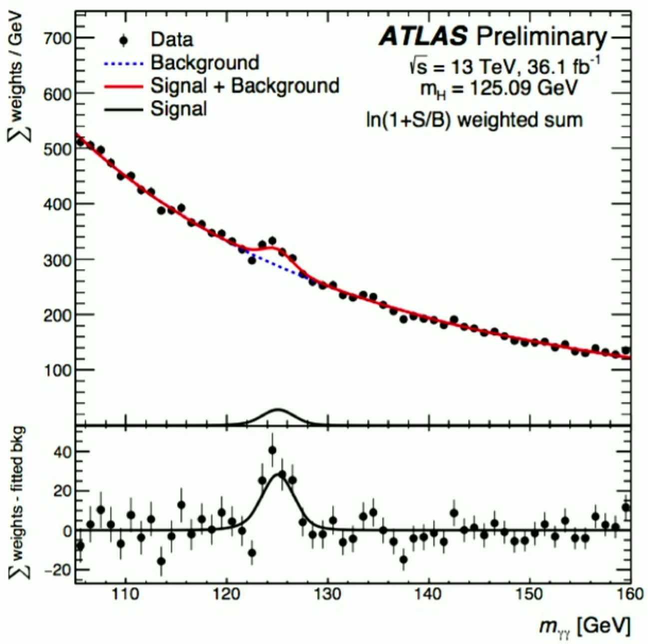
- What sort of deviations are then allowed experimentally?

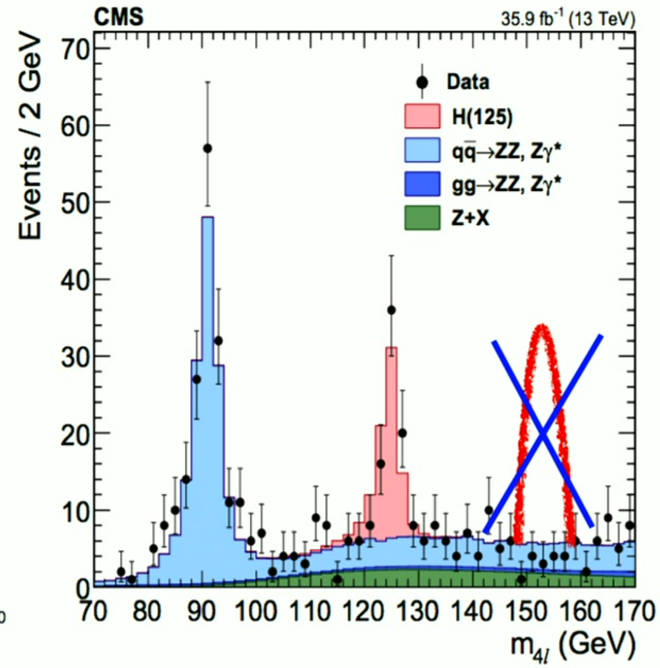




How does it help to have this simplification?

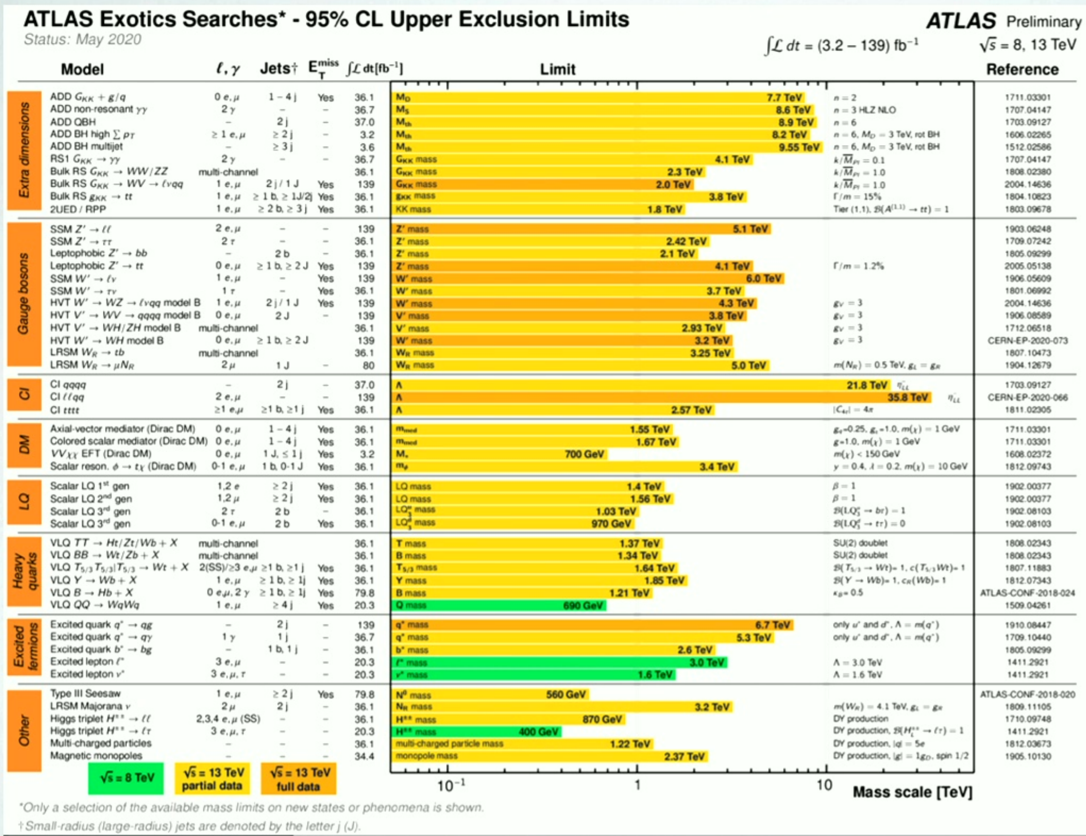
- What sort of deviations are then allowed experimentally?





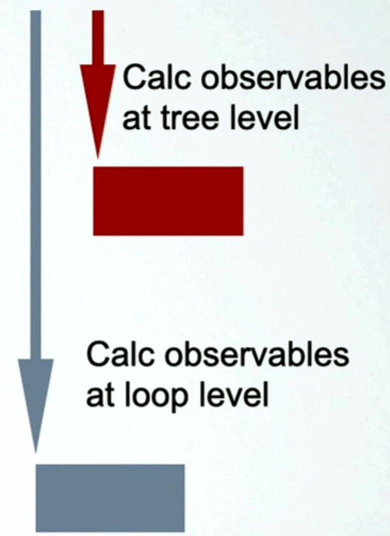
- BY FAR the majority of experimental analysis effort has been about bumps

This simplification is extremely helpful!

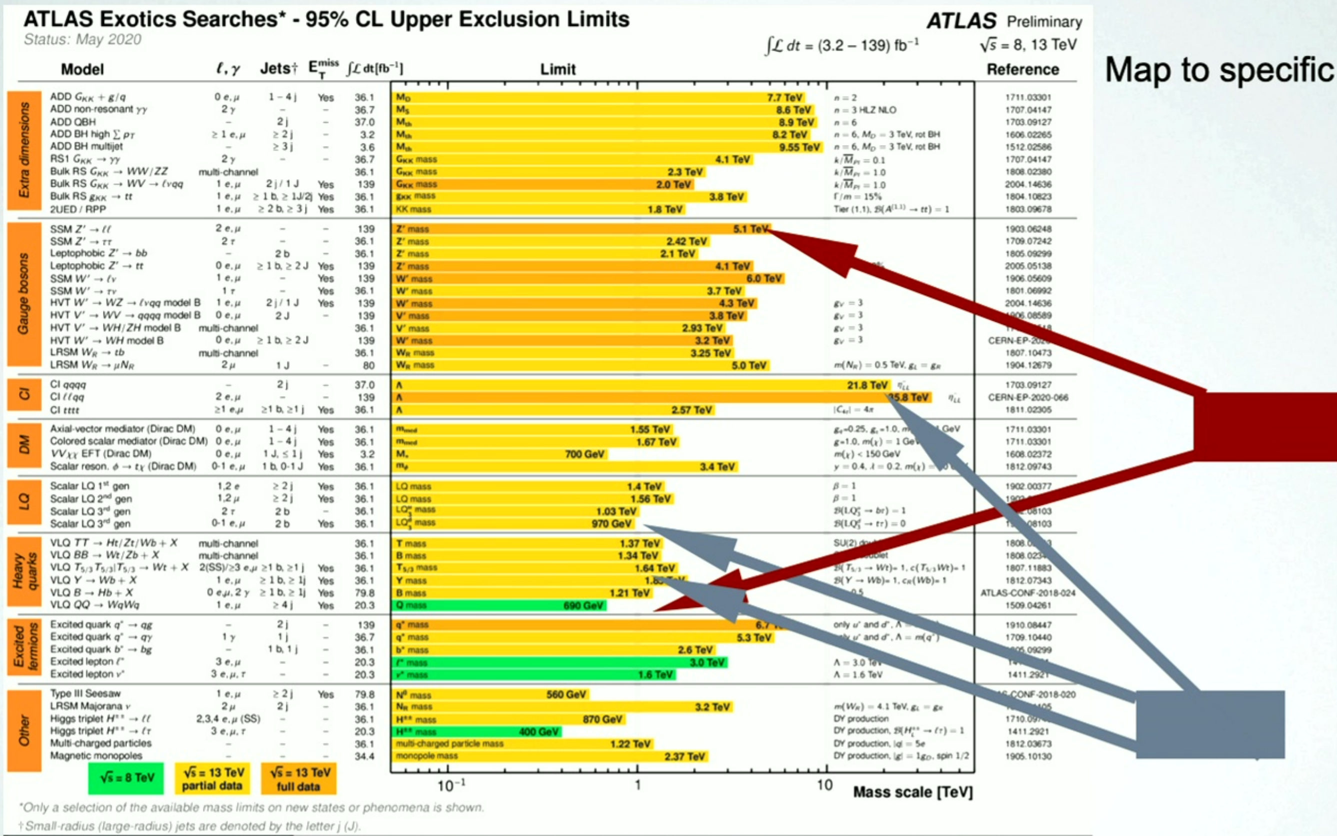


One can compare against data in a model dependent way:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{model}$$

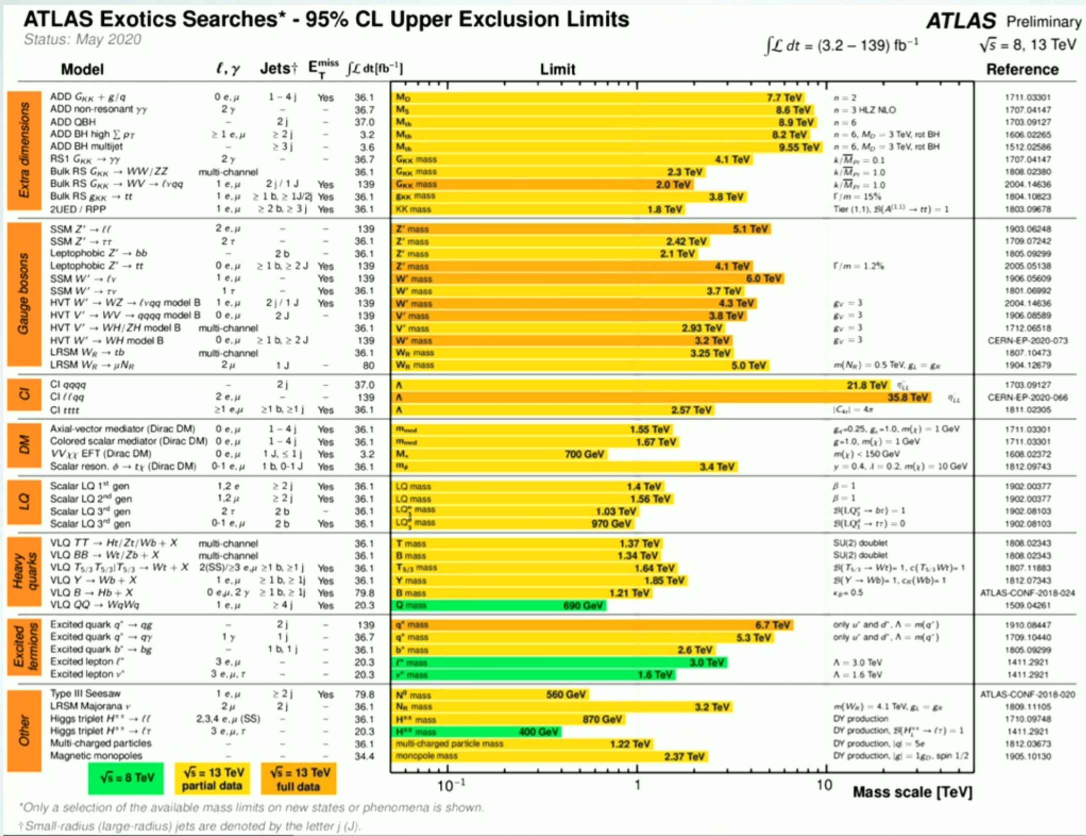


This simplification is extremely helpful!



Map to specific studies:

This simplification is extremely helpful!



Alternate approach

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{model}$$

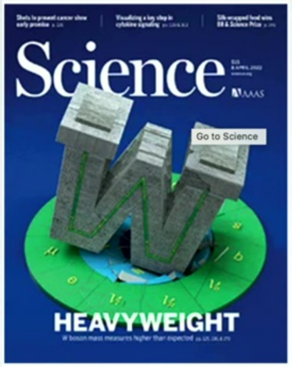
$$\downarrow$$

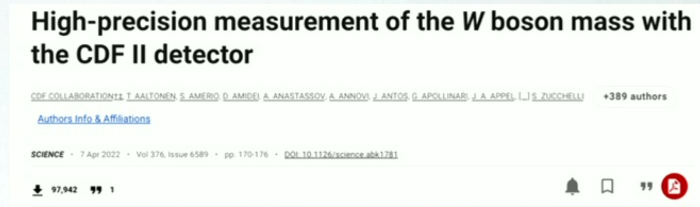
$$\mathcal{L}_{SMEFT}$$

And perform a global SMEFT fit once and for all.

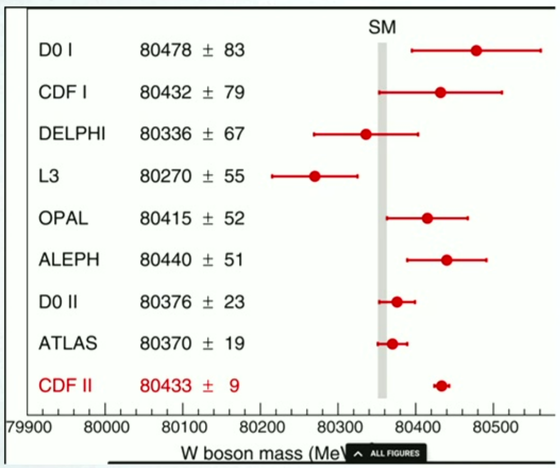
Benefit: many IR physics Parts of calc are the SAME And this is captured in EFT

The SMEFT is a key tool for interpreting ?deviations? like:





- Any one measurement will just dictate a parameter in a theory. But a PATTERN of measurements can falsify a theory.
  We need to study the Global data set in SMEFT.



- SMEFT allows the experimental pattern to deviation From the SM expectation - while still doing well Defined field theory.

Find deviations → Map to pattern in SMEFT
↓
Follow pattern to underlying model

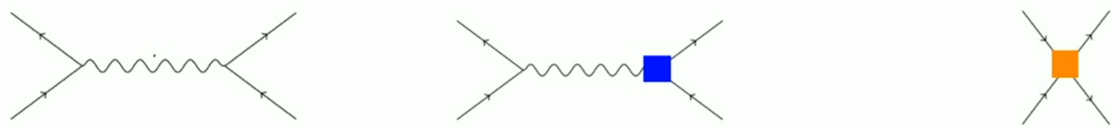
Inputs also needed -SMEFT Muon decay

- Decay of $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ still measured far below the W pole.

1312.2014 Alonso, Jenkins, Manohar, Trott

- Still probes the effective lagrangian

$$\mathcal{L}_{G_F} = -\frac{4\mathcal{G}_F}{\sqrt{2}} \left(\bar{\nu}_\mu \gamma^\mu P_L \mu\right) \left(\bar{e}\, \gamma_\mu P_L \nu_e\right)$$



So now
$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{v_T^2} + \left(\frac{C_{\mu e e \mu}^{ll}}{\Lambda^2} + \frac{C_{e \mu\mu e}^{ll}}{\Lambda^2}\right) - 2\left(\frac{C_{Hl,ee}^{(3)}}{\Lambda^2} + \frac{C_{Hl,\mu\mu}^{(3)}}{\Lambda^2}\right)$$

δG_F

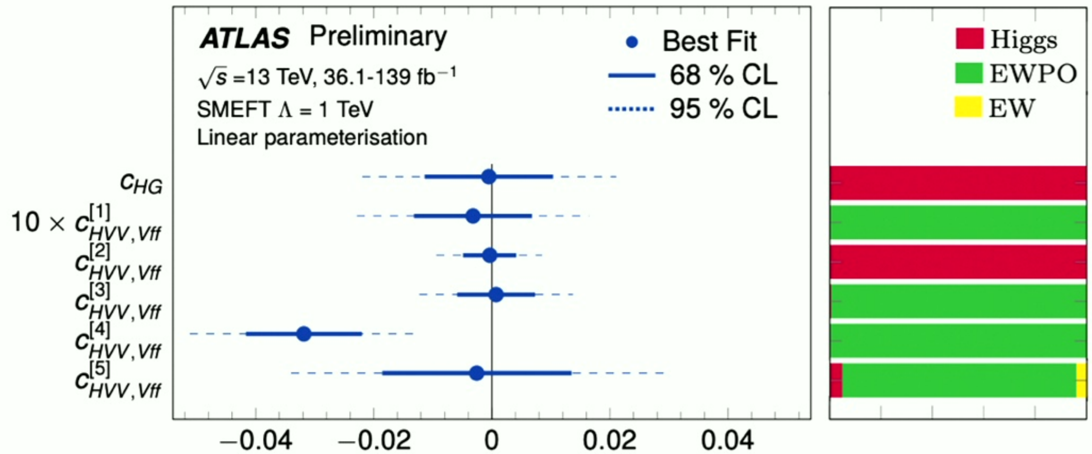
- Tons of work to redefine things at dim 6, can we go to dim 8?

Due to SMEFTsim the experimentalists have stepped up

ATLAS-PHYS-PUB-2022-037.pdf

$\sigma(\mathcal{G}\mathcal{G} \to h)$,

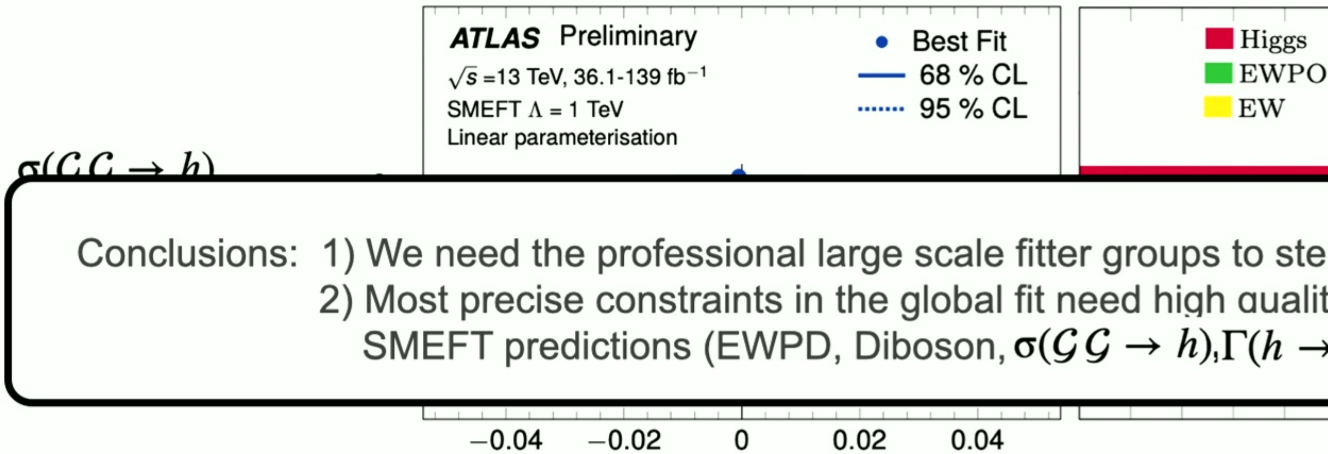
$\Gamma(h \to \gamma\gamma)$



- As this evolves forward, we need precise and consistent SMEFT results for these LHC two processes, and EWPD in particular.

Due to SMEFTsim the experimentalists have stepped up

ATLAS-PHYS-PUB-2022-037.pdf



Conclusions: 1) We need the professional large scale fitter groups to step up
2) Most precise constraints in the global fit need high quality SMEFT predictions (EWPD, Diboson, $\sigma(\mathcal{G}\mathcal{G} \to h)$,$\Gamma(h \to \gamma\gamma)$)

- As this evolves forward, we need precise and consistent SMEFT results for these LHC two processes, and EWPD in particular.

An instant pay off of "geoSMEFT"

- Growth in operator forms in connections
  Always saturate to fixed number, this is just the simplest organization exploiting this

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

2001.01453 Helset, Martin, Trott

- Once we have things to dim eight it is sufficient in many observables

Mases
Couplings and mixing angles
TGC, Higgs to ZZ,WW
QGC,TGC + Higgs

Yukawas

Dipoles

W,Z couplings to fermions +higgs

Field coord. invariance leads to field space geometry

$$\mathcal{L}_{SMEFT} = \frac{1}{2} h_{IJ}(\phi)(D_\mu \phi)^I (D_\mu \phi)^J - \frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu\nu} \mathcal{W}^B_{\mu\nu} + \cdots$$

- Dimensionless expansion into operator bases $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

$$\sqrt{h}^{IJ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{4}\tilde{C}_{HD} & 0 \\ 0 & 0 & 0 & 1 + \tilde{C}_{H\Box} - \frac{1}{4}\tilde{C}_{HD} \end{bmatrix}$$

$$\sqrt{g}^{AB} = \begin{bmatrix} 1 + \tilde{C}_{HW} & 0 & 0 & 0 \\ 0 & 1 + \tilde{C}_{HW} & 0 & 0 \\ 0 & 0 & 1 + \tilde{C}_{HW} & -\frac{\tilde{C}_{HWB}}{2} \\ 0 & 0 & -\frac{\tilde{C}_{HWB}}{2} & 1 + \tilde{C}_{HB} \end{bmatrix}$$

(Small perturbations so positive semi-definite matrix and unique square root)

- Geometric field space quantities are useful (True independent of mass dimension of ops) Amp. perturb. are:

Fun. of 4 vectors (kinematics)

$$\mathcal{A} \simeq \mathcal{A}_{SM} + \langle \mathcal{O} \rangle_1 N_1 + \langle \mathcal{O} \rangle_2 N_2 + \cdots$$

Defined by field space geometries

Simple all orders results for the vev expansion

- Glue Glue higgs

$$\langle h|\mathcal{G}\mathcal{G}\rangle = -\frac{\sqrt{h}^{44}}{4}\,\langle h\,\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\rangle\,\frac{\delta\kappa_{AA}}{\delta\phi_4}$$

- Higgs to gamma gamma

$$\langle h|\mathcal{A}(p_1)\mathcal{A}(p_2)\rangle = -\langle hA^{\mu\nu}A_{\mu\nu}\rangle\,\frac{\sqrt{h}^{44}}{4}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1^2}\right],$$

- Where the geometric electric charge is $\bar{e} = g_2\left(s_{\bar{\theta}}\sqrt{g}^{33} + c_{\bar{\theta}}\sqrt{g}^{34}\right)$

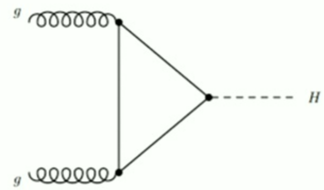
$$s_{\bar{\theta}}^2 = \frac{(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2g_1g_2\sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

LO SMEFT perturbation to the SM predictions

- **Modifications to the properties of the Higgs boson**

  Aneesh V. Manohar (UC, San Diego), Mark B. Wise (Caltech) (Jan, 2006)

  Published in: *Phys.Lett.B* 636 (2006) 107-113 • e-Print: hep-ph/0601212 [hep-ph]



$$\frac{\sigma(gg\to h)}{\sigma^{\rm SM}(gg\to h)} \simeq \frac{\Gamma(h\to gg)}{\Gamma^{\rm SM}(h\to gg)} \simeq \left|1-\frac{8\pi^2 v^2 c_G}{\Lambda^2 I^g}\right|^2 .$$

$$\delta\mathcal{L} = -\frac{c_G g_3^2}{2\Lambda^2} H^\dagger H G^A_{\mu\nu} G^{A\mu\nu}$$

$$I^g = I_f(m_h^2/(4m_t^2),0)\left(1+\frac{11}{4}\frac{\alpha_s}{\pi}\right),$$ ⟵ Partial 2 loop result, the 2 loop matching

$$I_f(a,b) = \int_0^1 {\rm d}x \int_0^{1-x} {\rm d}y \, \frac{1-4xy}{1-4(a-b)xy-4by(1-y)-i0^+}.$$

SM results at LO:

H. M. Georgi, S. L. Glashow, M. E. Machacek and D. V. Nanopoulos, *Higgs Bosons from Two Gluon Annihilation in Proton Proton Collisions*, *Phys. Rev. Lett.* **40** (1978) 692.

M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, *Low-Energy Theorems for Higgs Boson Couplings to Photons*, *Sov. J. Nucl. Phys.* **30** (1979) 711.

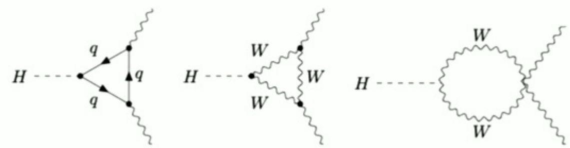
LO SMEFT perturbation to the SM predictions

- **Modifications to the properties of the Higgs boson**

  Aneesh V. Manohar (UC, San Diego), Mark B. Wise (Caltech) (Jan, 2006)

  Published in: *Phys.Lett.B* 636 (2006) 107-113 • e-Print: hep-ph/0601212 [hep-ph]

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} \simeq \left|1 - \frac{4\pi^2 v^2 c_{\gamma\gamma}}{\Lambda^2 I^\gamma}\right|^2 .$$



$$c_{\gamma\gamma} = c_W + c_B - c_{WB}$$

$$\delta\mathcal{L} = -\frac{c_B g_1^2}{2\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} - \frac{c_{WB} g_1 g_2}{2\Lambda^2} H^\dagger \tau^a H B_{\mu\nu} W^{a\mu\nu} - \frac{c_W g_2^2}{2\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a\mu\nu} - \frac{c_G g_3^2}{2\Lambda^2} H^\dagger H G^A_{\mu\nu} G^{A\mu\nu}$$

- We have improved both of these processes to consistent dimension 8 and one more loop order. (A short 17 years later!)

LO SMEFT perturbation to the SM predictions

- **Modifications to the properties of the Higgs boson**

  Aneesh V. Manohar (UC, San Diego), Mark B. Wise (Caltech) (Jan, 2006)

  Published in: *Phys.Lett.B* 636 (2006) 107-113 • e-Print: hep-ph/0601212 [hep-ph]

$$\frac{\Gamma(h\to\gamma\gamma)}{\Gamma^{\rm SM}(h\to\gamma\gamma)} \simeq \left|1-\frac{4\pi^2 v^2 c_{\gamma\gamma}}{\Lambda^2 I^\gamma}\right|^2 .$$

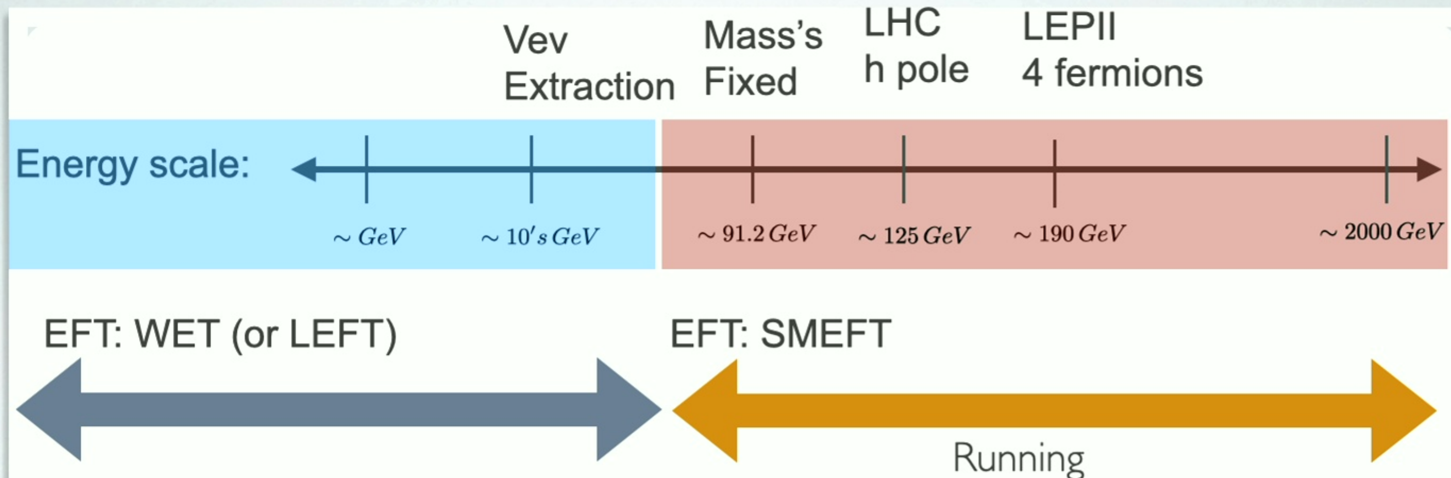
$$c_{\gamma\gamma} = c_W + c_B - c_{WB}$$

$$\delta\mathcal{L} = -\frac{c_B g_1^2}{2\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} - \frac{c_{WB} g_1 g_2}{2\Lambda^2} H^\dagger \tau^a H B_{\mu\nu} W^{a\mu\nu} - \frac{c_W g_2^2}{2\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a\mu\nu} - \frac{c_G g_3^2}{2\Lambda^2} H^\dagger H G^A_{\mu\nu} G^{A\mu\nu}$$

- Doing this in that timeframe is actually not so bad!

  Need to include SMEFT loop corrections and operator corrections to actual processes AND the input parameter processes used to fix the Lagrangian terms.

The Relevant Tower of EFT's



Complete LEFT RGE:

1711.05270 Jenkins, Manohar, Stoffer

HEFT/SMEFT matching LEFT:

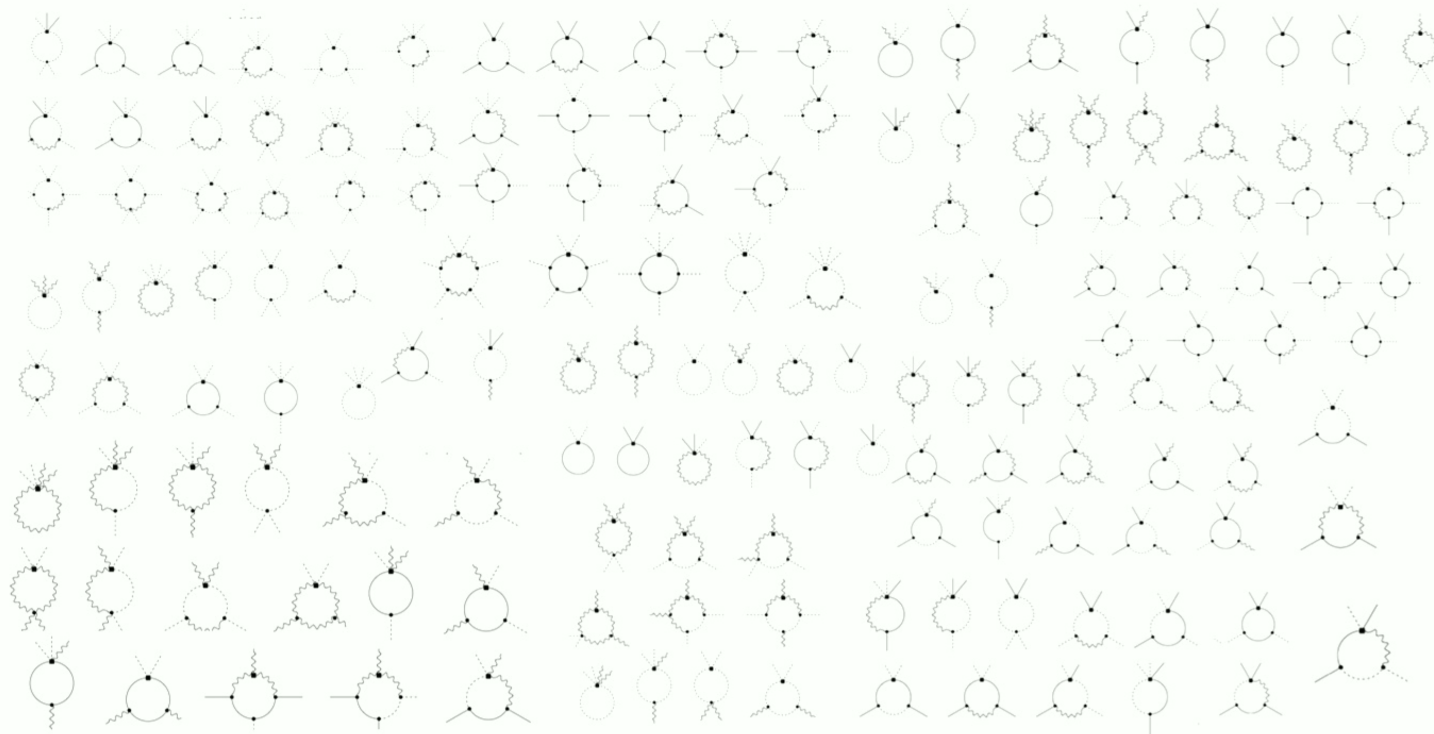
1709.04486 Jenkins, Manohar, Stoffer

Complete SMEFT RGE:

arXiv:1301.2588 Grojean, Jenkins, Manohar, Trott
arXiv:1308.2627, 1309.0819, 1310.4838 Jenkins, Manohar, Trott
arXiv: 1312.2014 Alonso, Jenkins, Manohar, Trott

SMEFT renormalised.. as was LEFT

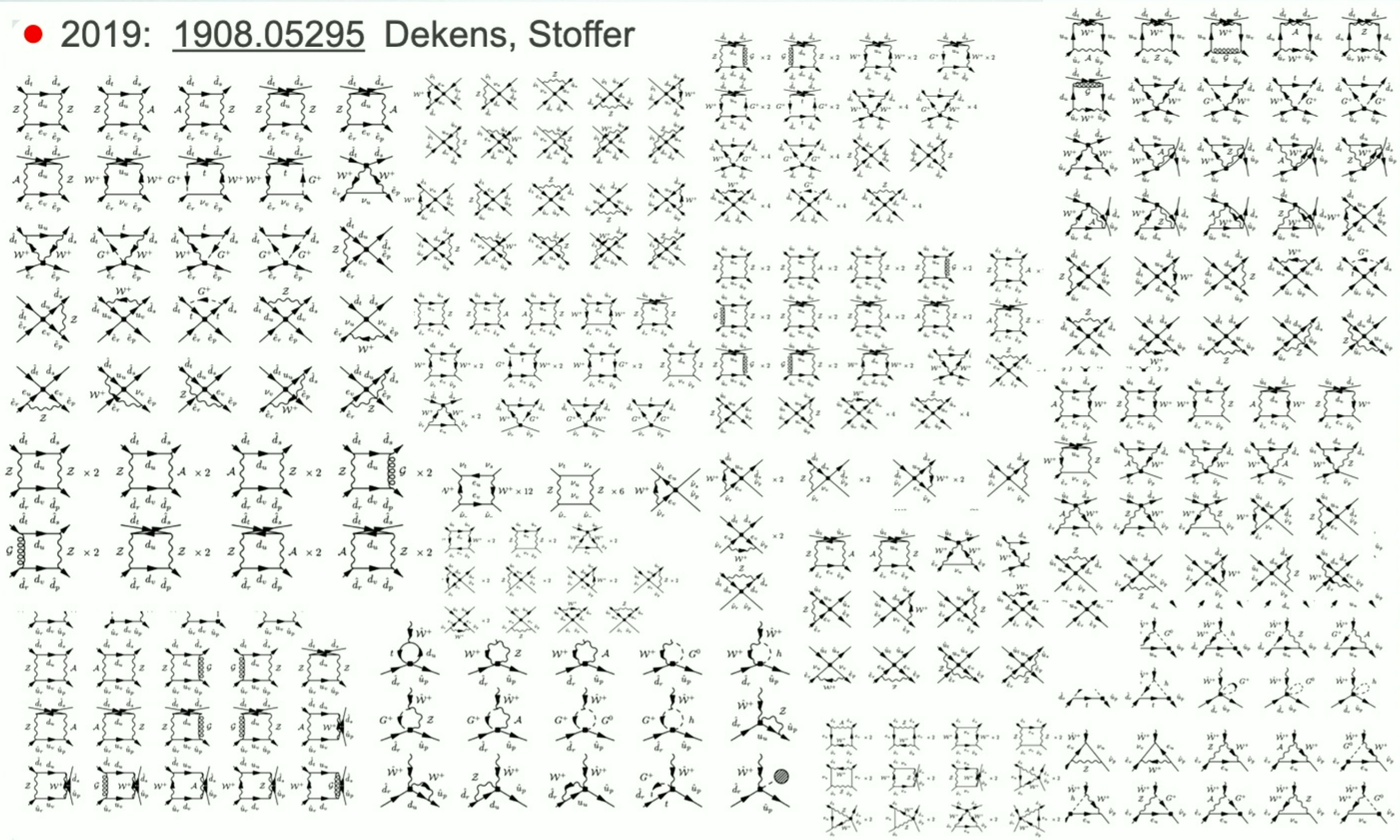
- SMEFT RGE's



- Each dot can be 59 types operator

Subset of the one loop matching..

- 2019: 1908.05295 Dekens, Stoffer

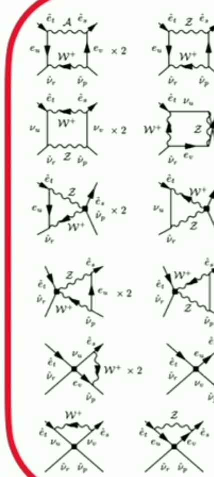


One loop vev extraction

One loop matching onto the LEFT operator: $\mathcal{L}_{LEFT} \supset L^{V,LL}(\bar{\nu}_{L,\mu}\gamma^{\mu}\nu_{L,e})(\bar{e}_L\gamma_{\mu}\mu_L).$

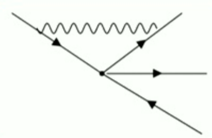
Δ : Loop expansion

δ : Higher dimensional op (vev) expansion



$$\bar{v}_T^2\,\Delta L^{V,LL} = \frac{\left(7\bar{m}_h^4 + \bar{m}_h^2\left(2m_t^2 N_c - 5\left(2\bar{m}_W^2 + \bar{m}_Z^2\right)\right) + 4\left(-4\,m_t^4 N_c + 2\,\bar{m}_W^4 + \bar{m}_Z^4\right)\right)}{16\pi^2\,\bar{m}_h^2\,\bar{v}_T^2},$$
$$+ \frac{3(\bar{m}_h^4 - 2\bar{m}_h^2\,\bar{m}_W^2)}{8\pi^2\,\bar{v}_T^2(\bar{m}_h^2 - \bar{m}_W^2)}\log\left(\frac{\mu^2}{\bar{m}_h^2}\right) + \frac{m_t^2 N_c(\bar{m}_h^2 - 4m_t^2)}{4\pi^2\bar{m}_h^2\,\bar{v}_T^2}\log\left(\frac{\mu^2}{m_t^2}\right),$$
$$+ \frac{3((\bar{m}_h^2(\bar{m}_Z^4 - 2\bar{m}_W^2\bar{m}_Z^2) + 2\bar{m}_Z^4(\bar{m}_W^2 - \bar{m}_Z^2))}{8\pi^2\bar{m}_h^2\bar{v}_T^2(\bar{m}_W^2 - \bar{m}_Z^2)}\log\left(\frac{\mu^2}{\bar{m}_Z^2}\right),$$
$$- \frac{3\,\bar{m}_W^2\left(\bar{m}_h^4\left(\bar{m}_W^2 - 2\bar{m}_Z^2\right) + \bar{m}_h^2\left(7\,\bar{m}_W^2\bar{m}_Z^2 - 6\bar{m}_W^4\right) + 4\bar{m}_W^4(\bar{m}_W^2 - \bar{m}_Z^2)\right)}{8\pi^2\bar{m}_h^2\bar{v}_T^2(\bar{m}_h^2 - \bar{m}_W^2)(\bar{m}_W^2 - \bar{m}_Z^2)}\log\left(\frac{\mu^2}{\bar{m}_W^2}\right).$$

Need to add back photon loops canceling in matching:



$$\Delta L_{ew}^{V,LL} = -\frac{\alpha}{4\pi}\left(\pi^2 - \frac{25}{4}\right).$$

G. Kallen, *Radiative corrections in elementary particle physics*, Springer Tracts Mod. Phys. **46** (1968) 67.

End result: $$-\frac{4\,\hat{G}_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2}\left(1 + \Delta L_{ew}^{V,LL}\right) + \Delta L^{V,LL} - 2\sqrt{2}\,\frac{\delta G_F}{\bar{v}_T^2}$$

GeoSMEFT based loop corrections.

- Many groups calculate in the background field gauge fixing with a geoSMEFT gauge fixing term

$$\mathcal{L}_{\rm GF} = -\frac{\hat{g}_{AB}}{2\,\xi}\mathcal{G}^A\,\mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}^X_{CD}\hat{\mathcal{W}}^C_\mu \mathcal{W}^{D,\mu} + \frac{\xi}{2}\hat{g}^{XC}\phi^I\,\hat{h}_{IK}\,\tilde{\gamma}^K_{C,J}\hat{\phi}^J.$$

1803.08001 Helset, Paraskevas, Trott.

- Immediate BFM Ward Identities were derived:

$$0 = \left(\partial^\mu \delta^A_B - \tilde{\epsilon}^A_{\ BC}\ \hat{\mathcal{W}}^{C,\mu}\right)\frac{\delta \Gamma}{\delta \hat{\mathcal{W}}^\mu_A} - \frac{\tilde{\gamma}^I_{B,J}}{2}\hat{\phi}^J\frac{\delta\Gamma}{\delta\hat{\phi}^I} + \sum_j\left(\bar{f}_j\bar{\Lambda}^j_{B,i}\,\frac{\delta\Gamma}{\delta\bar{f}_i} - \frac{\delta\Gamma}{\delta f_i}\Lambda^i_{B,j}f_j\right).$$

1909.08470 Corbett, Helset, Trott

And checked - at one loop in the results. It works.

2010.08451 Corbett, Trott 2010.15852 Corbett

Consistency checks at one loop/dim8

Benefits of the Background Field method one loop approach in SMEFT.

- Cross checks/understanding afforded (Ward identities and more).
- One loop redefinition of input parameters INDIVIDUALLY gauge independent.
- Cross checks of

$$\Delta Z_e = -\tfrac{1}{2}\Delta Z_{\hat{\mathcal{A}}},$$
$$\Delta R_e = -\tfrac{1}{2}\Delta R_{\hat{\mathcal{A}}}.$$

Our calc in 2107.07470
Stoffer/Denkens in 1908.05295

$$\Delta R_{\hat{\mathcal{A}}} = \frac{\bar{g}_1^2 \bar{g}_2^2}{(\bar{g}_1^2+\bar{g}_2^2)}\left[-\frac{7}{16\pi^2}\log\left(\frac{\mu^2}{\bar{m}_W^2}\right)+\sum_\psi \frac{N_c^\psi Q_\psi^2}{12\pi^2}\log\left(\frac{\mu^2}{\bar{m}_\psi^2}\right)-\frac{1}{24\pi^2}\right].$$

Cross checks worked out

Cancelation of large mt dependent logs in relations between observables:
Expected and anticipated in 1505.02646 Hartmann,Trott

- Expected cancelation confirmed in 2107.07470 and 1908.05295

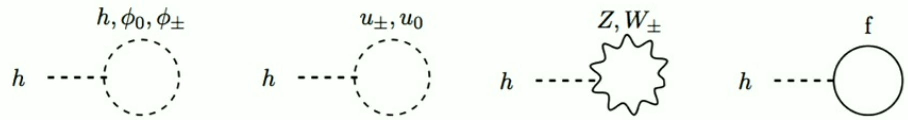
$$\bar{v}_T = \hat{v}_T \left[1 + \frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2}\left[1 + \log\left(\frac{\mu^2}{m_f^2}\right)\right] + \cdots\right].$$

$$\frac{\Delta v}{\bar{v}_T} \propto -\frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2}\left[1 + \log\left(\frac{\mu^2}{m_f^2}\right)\right].$$

- Cancelation in single Higgs, single dev observables with tadpole term and GF extraction. We both use the FJ tadpole scheme.

NLO EFT - fix finite terms

- Define vev of the theory as the one point function vanishing - fixes Δv



$$
\begin{aligned}
T = m_h^2\, h\, v\, \frac{1}{16\pi^2} \Bigg[& -16\pi^2\, \frac{\Delta v}{v} + 3\,\lambda \left(1 + \log\left[\frac{\mu^2}{m_h^2}\right]\right) + \frac{m_W^2}{v^2}\xi \left(1 + \log\left[\frac{\mu^2}{\xi\, m_W^2}\right]\right), \\
& + \frac{1}{2}\frac{m_Z^2}{v^2}\xi \left(1 + \log\left[\frac{\mu^2}{\xi\, m_Z^2}\right]\right) - \frac{1}{2}\sum_i y_i^4\, N_c \frac{1}{\lambda} \left(1 + \log\left[\frac{\mu^2}{m_i^2}\right]\right), \\
& + \frac{g_2^2}{2}\frac{m_W^2}{m_h^2} \left(1 + 3\log\left[\frac{\mu^2}{m_W^2}\right]\right) + \frac{1}{4}(g_1^2 + g_2^2)\frac{m_Z^2}{m_h^2} \left(1 + 3\log\left[\frac{\mu^2}{m_Z^2}\right]\right)\Bigg].
\end{aligned}
$$

- How do we deal with tadpoles? FJ tadpole scheme

J. Fleischer and F. Jegerlehner, *Radiative Corrections to Higgs Decays in the Extended Weinberg-Salam Model*, *Phys. Rev.* **D23** (1981) 2001.

One point function vanishes, so drop tadpoles. Include Δv when expanding around min.

Consistency checks at one loop/dim8

- Gauge independence of a common partial matrix element in single Higgs processes

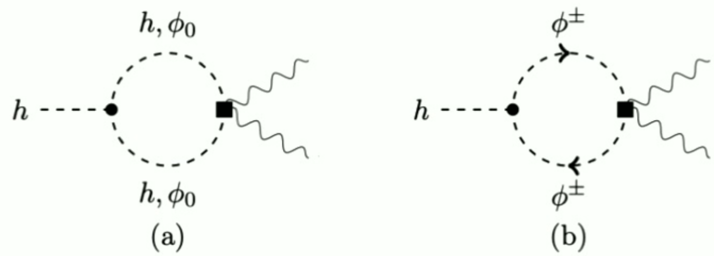


Figure 2. One loop contributions to $\langle \phi_4 | \, F \, F \rangle \langle \frac{\delta M_{AB}}{\delta \phi_4} \rangle$.

$$\frac{\langle \phi_4 F(p_1) F(p_2) \rangle^1}{\langle \phi_4 F^{\mu\nu} F_{\mu\nu} \rangle^0 \langle \frac{\delta M_{AB}(\phi)}{\delta \phi_4} \rangle^0} \propto M_1$$

- This common sub diagram contribution to $\sigma(\mathcal{G}\mathcal{G} \to h), \Gamma(h \to \gamma\gamma)$ is gauge independent:

$$M_1 \equiv \left(\frac{\Delta R_h}{2} + \frac{\Delta v}{v} + \frac{(\sqrt{3}\pi - 6)\lambda}{16\pi^2} + \frac{1}{16\pi^2} \left(\frac{\bar{g}_1^2}{4} + \frac{3\bar{g}_2^2}{4} + 6\lambda \right) \log \left[\frac{\bar{m}_h^2}{\mu^2} \right] \right),$$
$$+ \frac{1}{16\pi^2} \left(\frac{\bar{g}_1^2}{4} \mathcal{I}[\bar{m}_Z] + (\frac{\bar{g}_2^2}{4} + \lambda)(\mathcal{I}[\bar{m}_Z] + 2\mathcal{I}[\bar{m}_W]) \right).$$

Best practice example in SMEFT (3 schemes)

$$\frac{\Gamma_{SMEFT}}{\hat{\Gamma}_{\rm SM}} \simeq 1 + S_1 \left[f_1 + \left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4} \right) f_1 + f_2 \right] + S_2\, f_1^2 + S_3\, (\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB})^2 + S_4\, \delta G^{(6)}_F\, \tilde{C}^{(6)}_{HB},$$
$$+ S_5\, \delta G^{(6)}_F\, \tilde{C}^{(6)}_{HW} + S_6\, \delta G^{(6)}_F\, \tilde{C}^{(6)}_{HWB} + S_7\, \tilde{C}^{(6)}_{HD}\, \tilde{C}^{(6)}_{HB} + S_8\, \tilde{C}^{(6)}_{HD}\, \tilde{C}^{(6)}_{HW} + S_9\, \tilde{C}^{(6)}_{HD}\, \tilde{C}^{(6)}_{HWB},$$
$$+ S_{10}\, \tilde{C}^{(6)}_{HWB}\, \tilde{C}^{(6)}_{HB} + S_{11}\, \tilde{C}^{(6)}_{HWB}\, \tilde{C}^{(6)}_{HW} + S_{12}\, (\tilde{C}^{(6)}_{HWB})^2 + S_{13}\, \tilde{C}^{(6)}_{HB} + S_{14}\, \tilde{C}^{(6)}_{HW},$$
$$+ \left[S_{15} + S_{16} \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{HWB} + \left[S_{17} + S_{18} \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{W},$$
$$+ \left[S_{19} + S_{20} \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] {\rm Re}\, \tilde{\rm C}^{(6)}_{\substack{uB\\33}} + \left[S_{21} + S_{22} \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] {\rm Re}\, \tilde{\rm C}^{(6)}_{\substack{uW\\33}} + S_{23}\, {\rm Re}\, \tilde{\rm C}^{(6)}_{\substack{uH\\33}},$$
$$+ S_{24}\, {\rm Re}\, \tilde{C}^{(6)}_{\substack{dH\\33}} + S_{25}\, (\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4}) + S_{26}\, \tilde{C}^{(6)}_{HD} + S_{27}\, \tilde{C}^{(6)}_{HWB} + S_{28}\, \sqrt{2}\, \delta G^{(6)}_F.$$

- Here

$$\delta G^{(6)}_F = \frac{1}{\sqrt{2}} \left(\tilde{C}^{(3)}_{\substack{Hl\\ee}} + \tilde{C}^{(3)}_{\substack{Hl\\\mu\mu}} - \frac{1}{2} (\tilde{C}'_{\substack{ll\\\mu ee\mu}} + \tilde{C}'_{\substack{ll\\e\mu\mu e}}) \right),$$
$$f_1^{\hat{m}_W} = \left[\tilde{C}^{(6)}_{HB} + 0.29\, \tilde{C}^{(6)}_{HW} - 0.54\, \tilde{C}^{(6)}_{HWB} \right],$$
$$f_2^{\hat{m}_W} = \left[\tilde{C}^{(8)}_{HB} + 0.29\, (\tilde{C}^{(8)}_{HW} + \tilde{C}^{(8)}_{HW,2}) - 0.54\, \tilde{C}^{(8)}_{HWB} \right]$$
$$f_3^{\hat{m}_W} = \left[\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB} - 0.66\, \tilde{C}^{(6)}_{HWB} \right].$$

- Significant input parameter Dependence in what you get. This is expected. 2305.05879

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
$\hat{M}_W$	-753	1.41×10^5	321	2041	586	-1093	897	721	-914	1880
$\hat{\alpha}_{ew}^{(\hat{M}_Z)}$	-724	1.30×10^5	-320	1402	-126	-269	149	-149	95.0	297
$\hat{\alpha}_{ew}^{(0)}$	-794	1.56×10^5	-317	1447	-105	-274	138	-138	97.0	227

	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}
$\hat{M}_W$	1587	-1843	-100	-21.2	46.2	1.87	-0.51	3.28	24.4	-25.6	13.1
$\hat{\alpha}_{ew}^{(\hat{M}_Z)}$	-297	320	-199	32.0	-16.0	1.80	-0.49	3.25	23.9	-25.0	43.6
$\hat{\alpha}_{ew}^{(0)}$	-227	317	-222	30.3	-20.7	1.95	-0.45	3.32	25.1	-26.3	48.4

	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}
$\hat{M}_W$	-13.7	0.51	-0.28	2	-3.49	-7.5	$-3\sqrt{2}$
$\hat{\alpha}_{ew}^{(\hat{M}_Z)}$	-45.7	0.51	-0.28	2	0	0	$-\sqrt{2}$
$\hat{\alpha}_{ew}^{(0)}$	-50.7	5.04	-1.22	2	0	0	$-\sqrt{2}$

Table 3. Numerical coefficients for SMEFT perturbations to $\Gamma(h \to \mathcal{A}\mathcal{A})$ in three input parameter schemes, including two loop QCD interference effects.

Best practice example in SMEFT (3 schemes)

$$\frac{\Gamma_{SMEFT}}{\hat{\Gamma}_{\rm SM}} \simeq 1 + S_1 \left[f_1 + \left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4} \right) f_1 + f_2 \right] + S_2 f_1^2 + S_3 (\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB})^2 + S_4 \, \delta G_F^{(6)} \, \tilde{C}^{(6)}_{HB},$$

$$+ S_5 \, \delta G_F^{(6)} \, \tilde{C}^{(6)}_{HW} + S_6 \, \delta G_F^{(6)} \, \tilde{C}^{(6)}_{HWB} + S_7 \, \tilde{C}^{(6)}_{HD} \, \tilde{C}^{(6)}_{HB} + S_8 \, \tilde{C}^{(6)}_{HD} \, \tilde{C}^{(6)}_{HW} + S_9 \, \tilde{C}^{(6)}_{HD} \, \tilde{C}^{(6)}_{HWB},$$

$$+ S_{10} \, \tilde{C}^{(6)}_{HWB} \, \tilde{C}^{(6)}_{HB} + S_{11} \, \tilde{C}^{(6)}_{HWB} \, \tilde{C}^{(6)}_{HW} + S_{12} \, (\tilde{C}^{(6)}_{HWB})^2 + S_{13} \, \tilde{C}^{(6)}_{HB} + S_{14} \, \tilde{C}^{(6)}_{HW},$$

$$+ \left[S_{15} + S_{16} \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{HWB} + \left[S_{17} + S_{18} \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{W},$$

$$+ \left[S_{19} + S_{20} \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \mathrm{Re} \, \tilde{C}^{(6)}_{\substack{uB \\ 33}} + \left[S_{21} + S_{22} \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \mathrm{Re} \, \tilde{C}^{(6)}_{\substack{uW \\ 33}} + S_{23} \, \mathrm{Re} \, \tilde{C}^{(6)}_{\substack{uH \\ 33}},$$

$$+ S_{24} \, \mathrm{Re} \, \tilde{C}^{(6)}_{\substack{dH \\ 33}} + S_{25} \, (\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4}) + S_{26} \, \tilde{C}^{(6)}_{HD} + S_{27} \, \tilde{C}^{(6)}_{HWB} + S_{28} \, \sqrt{2} \, \delta G_F^{(6)}.$$

- The various contributions

$$|\mathcal{A}^{a,ij}_{SM} + \mathcal{A}^{a,ij}_{SMEFT}|^2 = |\mathcal{A}^{a,ij}_{SM} + \frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + \frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \cdots|^2$$

$$= |\mathcal{A}^{a,ij}_{SM}|^2 + \mathcal{A}^{a,ij}_{SM} \frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + |\frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2}|^2 + \mathcal{A}^{a,ij}_{SM} \frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \mathrm{h.c} + \cdots$$

All was not perfect as yet….

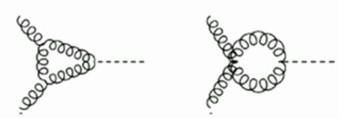
$$\sigma(\mathcal{G}\,\mathcal{G} \to h)$$

- The following challenges in 2107.07470:

1) SM results and literature are NOT in the BFM- but that seemed essential!?
2) 2 loop SM amplitudes were not presented in any transparent fashion
3) Two contributions:

$$\mathcal{O}\left(\alpha_s^2/(4\pi)^2\right) \longrightarrow \langle \mathcal{G}\mathcal{G}|h\rangle^2_{SM} \times \langle \mathcal{G}\mathcal{G}|h\rangle^0_{\tilde{C}_{HG}} \longleftarrow \mathcal{O}\left(v^2/\Lambda^2\right) C_{HG}$$

$$\mathcal{O}\left(\alpha_s/(4\pi)\right) \longrightarrow \langle \mathcal{G}\mathcal{G}|h\rangle^1_{SM} \times \langle \mathcal{G}\mathcal{G}|h\rangle^1_{\tilde{C}_{HG}} \longleftarrow \mathcal{O}\left(\alpha_s/(4\pi)\, v^2/\Lambda^2\right) C_{HG}$$



Improving $\sigma(\mathcal{G}\,\mathcal{G} \to h)$

- These contributions are the same in the $m_t \to \infty$ limit: 2305.05879 Martin, Trott

$$\langle \mathcal{G}\mathcal{G}|h\rangle^2_{SM} \times \langle \mathcal{G}\mathcal{G}|h\rangle^0_{\tilde{C}_{HG}}$$

$$\lim_{m_t \to \infty} \langle \mathcal{G}\mathcal{G}|h\rangle^2_{SM} \frac{\tilde{C}_{HG}}{\bar{v}_T^0\, \Delta C^{SM}_{h\mathcal{G}\mathcal{G}}} \to \langle \mathcal{G}\mathcal{G}|h\rangle^1_{\tilde{C}_{HG}}$$

$$\lim_{m_t \to \infty} \langle \mathcal{G}\mathcal{G}|h\rangle^1_{SM} \equiv \langle \mathcal{G}\mathcal{G}|h\rangle^0_{\tilde{C}_{HG}} \times \frac{\bar{v}_T^0\, \Delta C^{SM}_{h\mathcal{G}\mathcal{G}}}{\tilde{C}_{HG}}.$$

$$\langle \mathcal{G}\mathcal{G}|h\rangle^1_{SM} \times \langle \mathcal{G}\mathcal{G}|h\rangle^1_{\tilde{C}_{HG}}$$

- After established by brute force!

Example of the utility of EFT, same composite operator form.

Renormalisation issue $\sigma(\mathcal{G}\,\mathcal{G} \to h)$

- The two loop amplitude in the $m_t \to \infty$ analytically with expansion in ϵ explicit: C. Anastasiou, N. Deutschmann and A. Schweitzer, *Quark mass effects in two-loop Higgs amplitudes*, *JHEP* **07** (2020) 113 [2001.06295].

  Gives analytically:

  $S_\varepsilon = (4\pi)^\varepsilon \exp(-\varepsilon\gamma_E)$
  (Mbar factor)

$$\mathcal{A}^0_{gg\to H} = \frac{2i}{v^0}\frac{\alpha_s^0 S_\varepsilon \mu^{-2\varepsilon}}{4\pi}\left(-\frac{s}{\mu^2}\right)^{-\varepsilon}\delta_{ab}\left(s\left(\varepsilon_1\cdot\varepsilon_2\right)-2\left(\varepsilon_1\cdot p_2\right)\left(\varepsilon_2\cdot p_1\right)\right)\times\left(M^0_{\rm LO}+\frac{\alpha_s^0 S_\varepsilon\mu^{-2\varepsilon}}{4\pi}\left(-\frac{s}{\mu^2}\right)^{-\varepsilon}M^0_{\rm NLO}+\mathcal{O}\left((\alpha_s^0)^3\right)\right)$$

- LO result:

$$\Delta C^{SM,m_t\to\infty}_{h\mathcal{G}\mathcal{G}} = -\frac{\alpha_s^{(r)}}{\bar{v}_T^0\,16\pi}\left(\frac{\hat{m}_t^2}{\hat{\mu}^2}\right)^{-\epsilon}M^{(0),m_t\to\infty}_{t,SM},$$
$$= -\frac{\alpha_s^{(r)}}{\bar{v}_T^0\,16\pi}\left[-\frac{4}{3}\left(1+\frac{\pi^2}{12}\epsilon^2-\epsilon\,L_{\hat{m}_t}+\frac{1}{2}L^2_{\hat{m}_t}\epsilon^2+\mathcal{O}(\epsilon^3)\right)\right],$$

  (Log defn)
  $L_m = \log\left(m^2/\hat{\mu}^2\right)$

- NLO result supplied as: $M^{(1)}_{t,SM} = M_{UV} + M_{UV,m} + M_{IR} + M_{fin} + M_{fin,s}\log\left(-\frac{s}{\hat{\mu}^2}\right)$

 NOT in the BFM, so QFT surgery required 

Where is the 2 loop matching ?

- 2 loop matching part of answer: (after typo corrections)

$$\langle \mathcal{G}\mathcal{G}|h\rangle^{2,F}_{SM} \equiv i\,\delta_{ab}\,\frac{K_{ab}}{\bar{v}_T^0}\left[\left(-\frac{s}{\hat{\mu}^2}\right)^{-\epsilon}\frac{\alpha_s^0\,S^\epsilon\,\hat{\mu}^{-2\epsilon}}{4\pi}\right]^2\left(M^{(1)}_{t,SM} - M_{UV} - M_{UV,m} - M_{IR}\right)$$

Explicitly:

$$\langle \mathcal{G}\mathcal{G}|h\rangle^{2,F}_{SM} = \frac{\alpha_s^{(r)}}{4\pi}\left[11 + c_1\,\epsilon + (-\beta_0 + c_2\,\epsilon)\log\left(-\frac{\hat{m}_h^2}{\hat{\mu}^2}\right)\right]\langle \mathcal{G}\mathcal{G}|h\rangle^1_{SM,\epsilon\to 0},$$

Where:

$$c_1 = \left[-\frac{\pi^2\,\beta_0}{12} + 28\,\log(z) + 12\,\zeta_3 - \frac{40}{3}\right],$$

$$c_2 = \left[-\frac{1}{2}\beta_0\,\log\left(\frac{-s}{\mu^2}\right) - 2\beta_0\,\log(z) + 8\right].$$

(Log defn)

$$\log(z) = \log(-s/m_t^2)/2.$$

The 2 loop matching result is not a good approximation .

Renormalisation soln $\sigma(\mathcal{G}\,\mathcal{G} \to h)$

- SM lit, and past SMEFT results (including 2107.07470) followed a mixed scheme

$$Z_g^2\, Z_{\hat{\mathcal{G}}} \left(-\frac{s}{\hat{\mu}^2}\right)^{-\epsilon} i\,\delta_{ab}\,K_{ab}\,\frac{1}{\bar{v}_T^{(r)}}\,\frac{\alpha_s^{(r)}}{4\pi}\,M_{t,SM}^{(0)} = -\left[\frac{\alpha_s^{(r)}}{4\pi}\right]^2 \frac{\beta_0}{\epsilon}\left(-\frac{s}{\hat{\mu}^2}\right)^{-\epsilon} i\,\delta_{ab}\,K_{ab}\,\frac{1}{\bar{v}_T^{(r)}}\,M_{t,SM}^{(0)}.$$

Renormalise as:

$$Z_{\alpha_s} = 1 - \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\left(\beta_0 - \frac{2}{3}\left(\frac{\mu^2}{m_t^2}\right)^{\varepsilon}\right)$$

$$Z_g = 1 + \frac{\alpha_s}{4\pi}\frac{2}{3\varepsilon}\left(\frac{\mu^2}{m_t^2}\right)^{\varepsilon}.$$

But in the BFM: $\mu^{2\epsilon}\, Z_g^2 Z_{\hat{\mathcal{G}}} \equiv 1,$ how do we modify to the BFM?

Treat the EFT, as an EFT: Just renormalise the composite operator.

$$\langle \mathcal{G}\mathcal{G}|h\rangle^0_{\mathcal{O}(v^2/\Lambda^2)} \to Z_{HG}\,\frac{\tilde{C}_{HG}^{(6)}}{\bar{v}_T}\langle \mathcal{G}^{\mu\nu}\mathcal{G}_{\mu\nu}h\rangle_0. \qquad Z_{HG} = 1 - \frac{\beta_0\,\alpha_s}{4\pi\,\epsilon} + \cdots$$

IR problems $\sigma(\mathcal{G}\,\mathcal{G} \to h)$

- Such renormalisation (re) introduces a β_0 IR pole:

(Log defn)
$L_+ = L_{\hat{m}_h} + L_{\hat{m}_t}$

$$\frac{\Delta^2 \delta\sigma(\mathcal{G}\,\mathcal{G} \to h)}{\Delta^2\, \hat{\sigma}^{SM}_{LO,\epsilon\to 0}(\mathcal{G}\,\mathcal{G} \to h; z)} = 6\left[-\frac{6}{\epsilon^2} - \frac{\beta_0}{\epsilon} + 6\frac{L_+}{\epsilon} - \frac{6}{\epsilon} + \beta_0\, L_{\hat{m}_t} + 3\,\pi^2 + 5 - \beta_0 - 3L_+^2 + 6L_+\right] \tilde{C}^{(6)}_{HG},$$

This is why it seemed a mixed scheme was/is required to many in the lit.

This IR behavior is consistent with the Catani-Seymour subtraction:

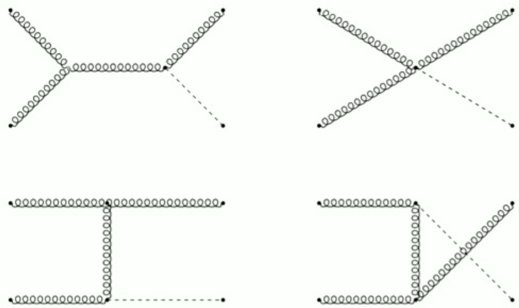
$$\mathcal{M}^{(1)}_{t,IR} = \frac{-e^{\epsilon\,\gamma_E}}{\Gamma(1-\epsilon)}\left[\frac{2N_c}{\epsilon^2} + \frac{\beta_0}{\epsilon}\right] M^{(0)}_{t,SM}.$$

Z. Kunszt, A. Signer and Z. Trocsanyi, *Singular terms of helicity amplitudes at one loop in QCD and the soft limit of the cross-sections of multiparton processes*, *Nucl. Phys. B* **420** (1994) 550 [hep-ph/9401294].

S. Catani and M. H. Seymour, *A General algorithm for calculating jet cross-sections in NLO QCD*, *Nucl. Phys. B* **485** (1997) 291 [hep-ph/9605323].

Canceled by considering IR limit of $\sigma(\mathcal{G}\mathcal{G} \to \mathcal{G}h)$

IR problems $\sigma(\mathcal{G}\,\mathcal{G} \to h)$



- Appropriate re-interpretation of:

S. Dawson, *Radiative corrections to Higgs boson production*, *Nucl. Phys.* **B359** (1991) 283.

Higgs boson production at the LHC

M. Spira (Hamburg U.), A. Djouadi (Montreal U. and DESY), D. Graudenz (CERN), P.M. Zerwas (DESY) (Feb, 1995)

Published in: *Nucl.Phys.B* 453 (1995) 17-82 • e-Print: hep-ph/9504378 [hep-ph]

$$\Delta\delta|\mathcal{A}(\mathcal{G}\,\mathcal{G} \to h\mathcal{G})|^2 = \frac{768\pi\alpha_s^{(0)}}{\bar{v}_T^0} 2\mathrm{Re}\left(\frac{\Delta C^{SM}_{h\mathcal{G}\mathcal{G}}}{\mu^{2\epsilon}}\tilde{C}_{HG}\right)\frac{(\hat{m}_h^8 + s^4 + t^4 + u^4)(1-2\epsilon) + \frac{1}{2}\epsilon\,(\hat{m}_h^4 + s^2 + t^2 + u^2)^2}{s\,t\,u},$$

- Results in ratio: (IR cancels poles in both inf. terms)

$$6\left[\frac{6}{\epsilon^2} - 6\frac{L_+}{\epsilon} + \frac{6}{\epsilon} + 3\,L_+^2 - 6\,L_+ - \pi^2 + 6\right]\delta(1-z)\tilde{C}_{HG},$$

$$+6\,\left[(12\,f_1(z)\,(L_{\hat{m}_h} - \log(z)) - 11 f_1(z) + 11 z)\,f_1(z) + 11\,(1-z)^2\,z\right]\left(\frac{1}{1-z}\right)_+\tilde{C}_{HG}$$

$$+\;144\,f_1^2(z)\left(\frac{\log(1-z)}{1-z}\right)_+\tilde{C}_{HG} - 72\,f_1^2(z)\left[\frac{1}{\epsilon} + 1 - L_{\hat{m}_t}\right]\left(\frac{1}{1-z}\right)_+\tilde{C}_{HG}.$$

SMEFT splitting functions

- To cancel all poles two steps, regulate end-point singularities: (on-shellness equiv?) add $\sigma(\mathcal{G}\mathcal{G} \to \mathcal{G}h), \sigma(\mathcal{G}\mathcal{G} \to h)$ AND

$$(1-z)^{-1-2\epsilon} = \left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{1}{2\epsilon}\delta(1-z)$$

introduce counter-term for AP splitting fund: (this means SMEFT pdf's)

$$\Delta^2 \delta\sigma^{AP}_{DR\,c.t} \equiv 36\,\Delta^2 \hat{\sigma}^{SMEFT}_{LO,\epsilon\to 0}(\mathcal{G}\,\mathcal{G} \to h)\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\right](4\pi)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)}\left[\frac{1}{\epsilon}\right] z\,p_{\mathcal{G}\mathcal{G}}(z)\tilde{C}_{HG}$$

Where: $$p_{\mathcal{G}\mathcal{G}}(z) = 2\,z\left(\left(\frac{1}{1-z}\right)_+ - z + \frac{f_1(z)}{z^2}\right) + \frac{\beta_0}{6}\delta(1-z).$$

- Need to upgrade the α_s input, certainly if extracted from PDF's

NLO Compact Final Answer

$$\frac{\Delta^2 \delta\sigma^{SMEFT}}{\Delta^2\,\hat{\sigma}^{SM}_{LO,\epsilon\to 0}}\frac{1}{2\,\tilde{C}^{(6)}_{HG}} = 12\left[\pi^2 + \frac{11}{2}\right]\delta(1-z) - 66(1-z)^3 + 144\, f_1^2(z)\left(\frac{\log(1-z)}{1-z}\right)_+,$$
$$+ 72\, f_1^2(z)\,[L_+ - \log(z) - 1]\left(\frac{1}{1-z}\right)_+ + 36\, z\, p_{\mathcal{GG}}(z)\,\log\left(\frac{\hat{\mu}^2}{\mu_F^2}\right).$$

Notation:

$$L_m = \log\left(m^2/\hat{\mu}^2\right)$$
$$L_+ = L_{\hat{m}_h} + L_{\hat{m}_t}$$
$$z = \hat{m}_h^2/s$$
$$f_1(z) = z^2 - z + 1$$

$$p_{\mathcal{GG}}(z) = 2\,z\left(\left(\frac{1}{1-z}\right)_+ - z + \frac{f_1(z)}{z^2}\right) + \frac{\beta_0}{6}\delta(1-z).$$

$$\int_0^1 d\,x\frac{f(x)}{(x)_+} = \int_0^1 dx\frac{f(x)-f(0)}{x},$$
$$\int_0^1 d\,f(x)\left(\frac{log(x)}{x}\right)_+ = \int_0^1 dx\frac{(f(x)-f(0))log(x)}{x}.$$

SMEFT $\sigma(\mathcal{G}\,\mathcal{G} \to h)$ perturbation

- Numerically the result is:

$$\frac{\sigma^{\hat{\alpha}}_{\rm SMEFT}(\mathcal{G}\mathcal{G} \to h)}{\hat{\sigma}_{{\rm SM},m_t \to \infty}(\mathcal{G}\mathcal{G} \to h)} \simeq 1 + 289\,\tilde{C}^{(6)}_{HG}$$
$$+ 289\,\tilde{C}^{(6)}_{HG}\left(\tilde{C}^{(6)}_{H\Box} - \frac{1}{4}\tilde{C}^{(6)}_{HD}\right) + 4.68 \times 10^4\,(\tilde{C}^{(6)}_{HG})^2 + 289\,\tilde{C}^{(8)}_{HG}$$
$$+ 0.85\left(\tilde{C}^{(6)}_{H\Box} - \frac{1}{4}\tilde{C}^{(6)}_{HD}\right) + 369\,\tilde{C}^{(6)}_{HG} - 0.91\,\tilde{C}^{(6)}_{uH} - 7.26\,{\rm Re}\,\tilde{C}^{(6)}_{uG}$$
$$- 0.60\,\delta G^{(6)}_F - 4.42\,{\rm Re}\,\tilde{C}^{(6)}_{uG}\,\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right) - 0.126\,{\rm Re}\,\tilde{C}^{(6)}_{dG}\,\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)$$
$$- 0.057\,{\rm Re}\,\tilde{C}^{(6)}_{dG} + 2.06\,\tilde{C}^{(6)}_{dH}.$$

2305.05879 Martin, Trott

- Operator Definitions:

$$\mathcal{L}_{SMEFT} \supset \frac{C^{(6)}_{HG}}{\Lambda^2}\,H^\dagger H G^{\mu\nu}_A G^A_{\mu\nu} + \frac{C^{(8)}_{HG}}{\Lambda^4}\,(H^\dagger H)^2 G^{\mu\nu}_A G^A_{\mu\nu} + \frac{C^{(6)}_{H\Box}}{\Lambda^2}\,(H^\dagger H)\Box(H^\dagger H) + \frac{C^{(6)}_{HD}}{\Lambda^2}\,\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D_\mu H\right)$$
$$+ \frac{C^{(6)}_{uH}}{\Lambda^2}\,(H^\dagger H)(\bar{q}_p u_r \widetilde{H}) + \frac{C^{(6)}_{dH}}{\Lambda^2}\,(H^\dagger H)(\bar{q}_p d_r H) + \frac{C^{(6)}_{uG}}{\Lambda^2}\,(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\,G^A_{\mu\nu} + \frac{C^{(6)}_{dG}}{\Lambda^2}\,(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\,G^A_{\mu\nu}$$

SMEFT $\sigma(\mathcal{G}\mathcal{G} \to h)$ perturbation

- Numerically the result is:

$$\frac{\sigma^{\hat{\alpha}}_{\rm SMEFT}(\mathcal{G}\mathcal{G} \to h)}{\hat{\sigma}_{{\rm SM}, m_t \to \infty}(\mathcal{G}\mathcal{G} \to h)} \simeq 1 + 289\, \tilde{C}^{(6)}_{HG}$$
$$+ 289\, \tilde{C}^{(6)}_{HG}\left(\tilde{C}^{(6)}_{H\Box} - \frac{1}{4}\tilde{C}^{(6)}_{HD}\right) + 4.68 \times 10^4\, (\tilde{C}^{(6)}_{HG})^2 + 289\, \tilde{C}^{(8)}_{HG}$$
$$+ 0.85\left(\tilde{C}^{(6)}_{H\Box} - \frac{1}{4}\tilde{C}^{(6)}_{HD}\right) + 369\, \tilde{C}^{(6)}_{HG} - 0.91\, \tilde{C}^{(6)}_{uH} - 7.26\, {\rm Re}\, \tilde{C}^{(6)}_{uG}$$
$$- 0.60\, \delta G^{(6)}_F - 4.42\, {\rm Re}\, \tilde{C}^{(6)}_{uG} \log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right) - 0.126\, {\rm Re}\, \tilde{C}^{(6)}_{dG} \log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)$$
$$- 0.057\, {\rm Re}\, \tilde{C}^{(6)}_{dG} + 2.06\, \tilde{C}^{(6)}_{dH}.$$

2305.05879 Martin, Trott

- The various contributions

$$|\mathcal{A}^{a,ij}_{SM} + \mathcal{A}^{a,ij}_{SMEFT}|^2 = |\mathcal{A}^{a,ij}_{SM} + \frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + \frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \cdots|^2$$
$$= |\mathcal{A}^{a,ij}_{SM}|^2 + \mathcal{A}^{a,ij}_{SM}\, \frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + |\frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2}|^2 + \mathcal{A}^{a,ij}_{SM}\, \frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + {\rm h.c} + \cdots$$