Title: Grad Student Seminar with Bruno Torres

Speakers: Bruno de Souza Leao Torres
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Abstract: Bruno Torres, Perimeter Institute & University of Waterloo

Optimal coupling for local entanglement extraction from a quantum field

The entanglement structure of quantum fields is of central importance in various aspects of the connection between spacetime geometry and quantum field theory. However, it is challenging to quantify entanglement between complementary regions of a quantum field theory due to the formally infinite amount of entanglement present at short distances. We present an operationally-motivated way of analyzing entanglement in a QFT by considering the entanglement which can be transferred to a set of local probes coupled to the field. In particular, using a lattice approximation to the field theory, we show how to optimize the coupling of the local probes with the field in a given region to most accurately capture the original entanglement present between that region and its complement. This coupling prescription establishes a bound on the entanglement between complementary regions that can be extracted to probes with finitely many degrees of freedom.

Optimal coupling for entanglement extraction from a quantum field

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Motivation

- Entanglement is a type of correlation between subsystems which cannot be explained classically

- It plays a variety of roles across different areas in theoretical physics

  - In quantum information → entanglement is an important resource
  - In condensed matter → can be used to characterized quantum phases of matter
  - In quantum gravity → is connected to the emergence of a classical geometry from quantum degrees of freedom

- Overall, entanglement has become central in our understanding of foundational aspects of quantum field theory.

Bare minimum of entanglement in QFT

- However, it is hard to characterize entanglement quantitatively in QFT.
- If we try using the entanglement entropy between two complementary regions,

$$\mathcal{H} \simeq \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}},$$

$$|\psi\rangle \in \mathcal{H},$$

$$\rho_A = \mathrm{tr}_{\bar{A}} \left(|\psi\rangle \langle\psi| \right),$$

$$S(\rho_A) = -\,\mathrm{tr}\left(\rho_A \log \rho_A\right) = S(\rho_{\bar{A}}) = \text{“}\infty\text{”}$$

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we always obtain UV divergences for any “reasonable” physical state.

Bare minimum of entanglement in QFT

- In order to make progress, we can proceed in two different ways:

  1. Embrace the fact that subregions of a QFT are type III von Neumann algebras

  2. Impose some regularization to the theory in order to render the divergences finite

- Operationally, there are good reasons to go with the latter

  - Motivated by the task of extracting entanglement from the field to a set of probes which provide a cutoff at some finite lengthscale

Coupling a probe to a quantum field

Coupling a probe to a quantum field

- A particle detector is a localized system that can couple to a quantum field in a finite region of spacetime

Paradigmatic form of interaction set by

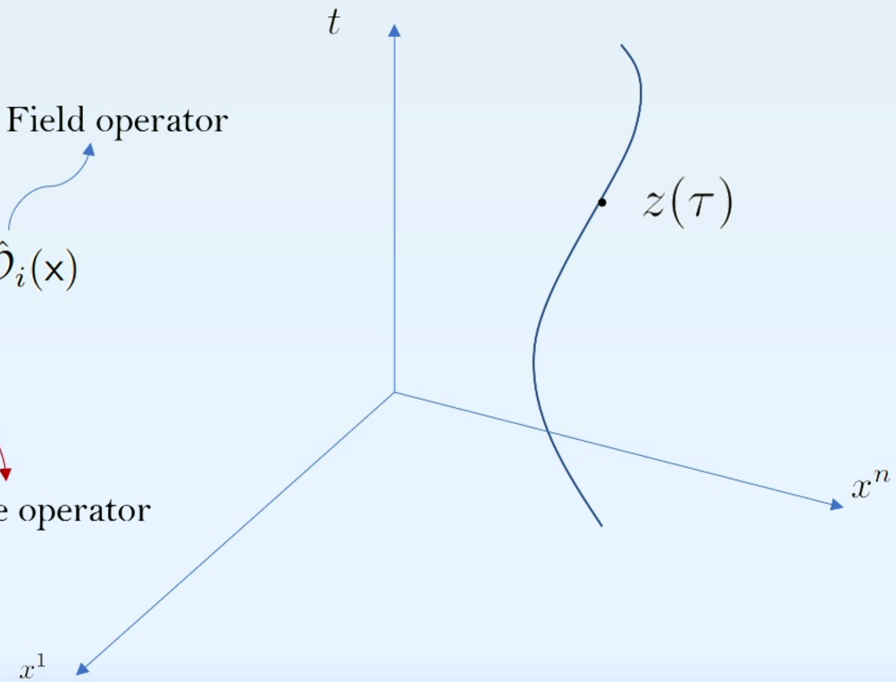
$$\hat{H}_I(t) = \lambda \sum_i \int_{\Sigma_t} \mathrm{d}^n \boldsymbol{x}\, \sqrt{-g(\mathsf{x})}\, \Lambda_i(\mathsf{x}) \hat{\mu}_i[\tau(\mathsf{x})] \hat{\mathcal{O}}_i(\mathsf{x})$$

Field operator

Coupling constant

Spacetime smearing

Probe operator



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Setup for the field

- For concreteness, we will take

Field → real free scalar field

$$S_\phi = -\frac{1}{2}\int \mathrm{d}^D x\sqrt{-g}\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right)$$

Field state → vacuum $|\Omega\rangle$

➢ Pure Gaussian state fully determined by the spacetime two-point function

$$\langle\phi(\mathsf{x})\phi(\mathsf{y})\rangle = \langle\Omega|\,\phi(\mathsf{x})\phi(\mathsf{y})\,|\Omega\rangle$$

Setup for the field

- In practice, the probes always come with a finite resolution; we thus approximate the field at any given time by a lattice of harmonic oscillators

$$\hat{\phi}(\mathsf{x}) \mapsto \hat{\boldsymbol{\Xi}} = (\hat{Q}^1, \hat{P}_1, \ldots, \hat{Q}^N, \hat{P}_N)^\intercal$$

Field on space*time*

Phase space variables (quadratures) over *space*

$$\left[\hat{\phi}(t, \boldsymbol{x}), \hat{\pi}(t, \boldsymbol{x}')\right] = \mathrm{i}\delta^{(n)}(\boldsymbol{x} - \boldsymbol{x}')\mathbb{1} \mapsto \left[\hat{\Xi}^\alpha, \hat{\Xi}^\beta\right] = \mathrm{i}\tilde{\Omega}^{\alpha\beta}\mathbb{1}$$

$$(\tilde{\Omega}^{\alpha\beta}) = \bigoplus_{i=1}^{N} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Symplectic matrix

$$\left\langle \hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y}) \right\rangle \mapsto \sigma^{\mu\nu} = \left\langle \hat{\Xi}^\mu \hat{\Xi}^\nu + \hat{\Xi}^\nu \hat{\Xi}^\mu \right\rangle$$

Covariance matrix

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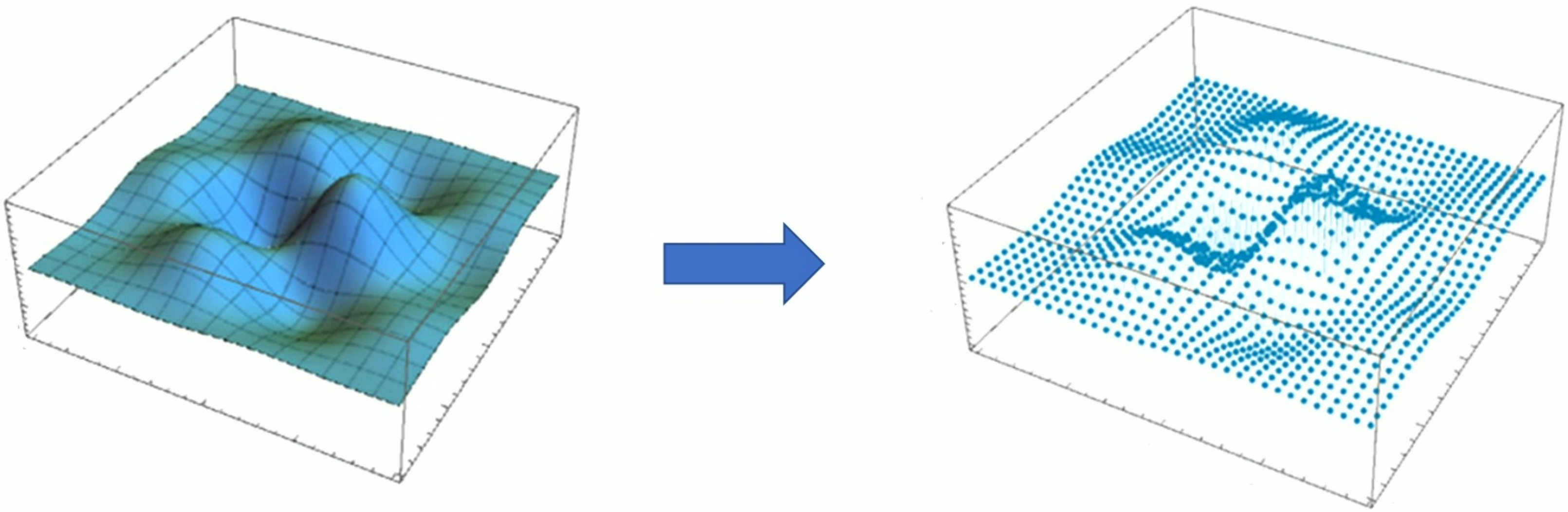
Setup for the field



Adapted from Rivat, S., Effective theories and infinite idealizations: a challenge for scientific realism. *Synthese* **198**, 12107–12136 (2021).

Subsystems in Gaussian states

Each pair of canonically conjugated observables $(\hat{Q}, \hat{P})$ such that $\left[\hat{Q}, \hat{P}\right] = i\mathbb{1}$ defines a *mode.*

By splitting a N-mode Gaussian state into $(n + m)$ modes corresponding to subsystems A and B,

Subsystem A

$$\sigma_{AB} = \begin{pmatrix} \sigma_A & \gamma \\ \gamma^{\intercal} & \sigma_B \end{pmatrix}$$

Subsystem B

we immediately identify the covariance matrix for each subsystem.

Effectively, going from the Hilbert space to phase space turns tensor products into direct sums.

Mixedness and entanglement in Gaussian states

Measures of mixedness and entanglement for Gaussian states are fully characterized by the covariance matrix.

For any covariance matrix, there always exists a change of basis on phase space

$$\hat{\boldsymbol{\Xi}} \mapsto \boldsymbol{S}\hat{\boldsymbol{\Xi}} \qquad S\tilde{\Omega}S^{\mathsf{T}} = \tilde{\Omega}$$

Such that the covariance matrix becomes diagonal,

$$\sigma \mapsto \sigma_D = \bigoplus_{i=1}^{N} \begin{pmatrix} \nu_i & 0 \\ 0 & \nu_i \end{pmatrix}$$

We call this the basis of normal (Williamson) modes.

The von Neumann entropy of a Gaussian state is given in terms of the symplectic eigenvalues by

$$S(\hat{\rho}) \equiv -\operatorname{Tr}(\hat{\rho}\log\hat{\rho}) = \sum_i \left[\frac{\nu_i+1}{2}\log\left(\frac{\nu_i+1}{2}\right) - \frac{\nu_i-1}{2}\log\left(\frac{\nu_i-1}{2}\right)\right]$$

Mixedness and entanglement in Gaussian states

If the system AB is in an overall pure state, the von Neumann entropy of A is a genuine measure of entanglement.

For entanglement in mixed states, it is customary to use the logarithmic negativity,

$$E_{\mathcal{N}}(\hat{\rho}) = \log\left(\mathrm{Tr}\sqrt{(\hat{\rho}^{T_B})^{\dagger}\hat{\rho}^{T_B}}\right)$$

which, for Gaussian states, takes the form

$$E_{\mathcal{N}}(\hat{\rho}) = \sum_i F(\tilde{\nu}_i), \qquad F(x) = -\log(x) \text{ for } x \in (0,1] \text{ and } F(x) = 0 \text{ for } x > 1.$$

$$\tilde{\sigma}_{AB} = (\mathbb{1}_{2N_A} \oplus T_B)\,\sigma_{AB}\,(\mathbb{1}_{2N_A} \oplus T_B) \qquad T_B = \bigoplus_{i=1}^{N_B}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

Optimal mode for entanglement extraction

Now we are ready to state our main result:

Consider a subregion A of a globally pure Gaussian state. For any integer N, **the set of N modes in A that is most entangled (in terms of logarithmic negativity) with the complement of A is given by the first n normal modes of the covariance matrix of A.**

Any normal mode can be expressed in terms of the local modes of the field. This corresponds precisely to the spatial part of the spacetime smearing that a probe can couple to!

Optimal mode for entanglement extraction

Now imagine that the probes are made of a set of harmonic oscillators,

$$\hat{H}_{\mathrm{d}} = \frac{\Omega}{2}\left(\hat{p}_{\mathrm{d}}^2 + \hat{q}_{\mathrm{d}}^2\right)$$

$$[\hat{q}_{\mathrm{d}}, \hat{p}_{\mathrm{d}}] = \mathrm{i}\mathbb{1}$$

For any field mode of interest, we can couple the probe according to

$$\hat{H}_{\mathrm{I}} = -\frac{\pi}{2}\delta(t)\left(\hat{q}\hat{P} - \hat{p}_{\mathrm{d}}\hat{Q}\right)$$

$$\begin{aligned} \hat{q}_{\mathrm{d}} &\mapsto \hat{Q} \\ \hat{Q} &\mapsto -\hat{q}_{\mathrm{d}} \\ \hat{p}_{\mathrm{d}} &\mapsto \hat{P} \\ \hat{P} &\mapsto -\hat{p}_{\mathrm{d}} \end{aligned}$$

Which gives us precisely a swap operator between the mode and the probe.

This way, we guarantee that the probe has extracted as much entanglement with the complement of its coupling region as possible, given the finite set of degrees of freedom of the probes!

Simple results in flat space

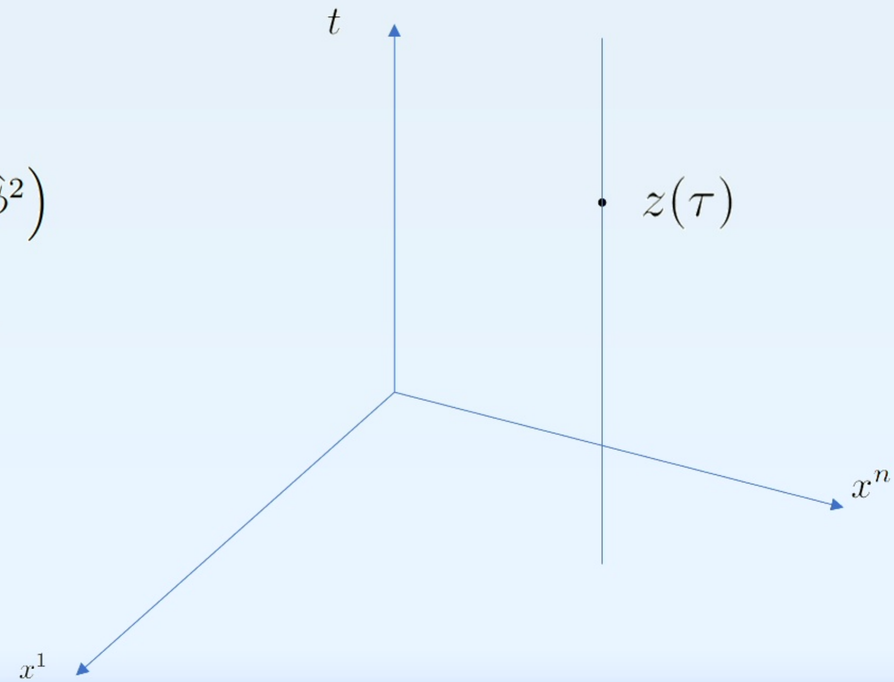
Example in flat space

For concreteness, take inertial probes in Minkowski:

Field → real free scalar field

$$\hat{H}_\phi = \frac{1}{2}\int \mathrm{d}^n \boldsymbol{x} \left(\hat{\pi}^2 + (\nabla\hat{\phi})^2 + m^2\hat{\phi}^2\right)$$

$$\left[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{x}')\right] = \mathrm{i}\delta^{(n)}(\boldsymbol{x}-\boldsymbol{x}')\mathbb{1}$$



Lattice approximation to the field

Discrete approximation to the field is done by

$$\int \mathrm{d}^n x \mapsto \varepsilon^n \sum_i,$$

$$\delta^{(n)}(\boldsymbol{x}-\boldsymbol{y}) \mapsto \frac{1}{\varepsilon^n}\delta_{ij},$$

$$(\nabla\hat{\phi}(\boldsymbol{x}))^2 \mapsto \frac{1}{2\varepsilon^2}\sum_j \left(\hat{\phi}_i - \hat{\phi}_j\right)^2$$

$$\hat{H}_\phi = \frac{1}{2}\sum_i \omega\left(\hat{q}_i^2 + \hat{p}_i^2\right) - \frac{\alpha}{2}\sum_{\langle i,j\rangle}\hat{q}_i\hat{q}_j$$

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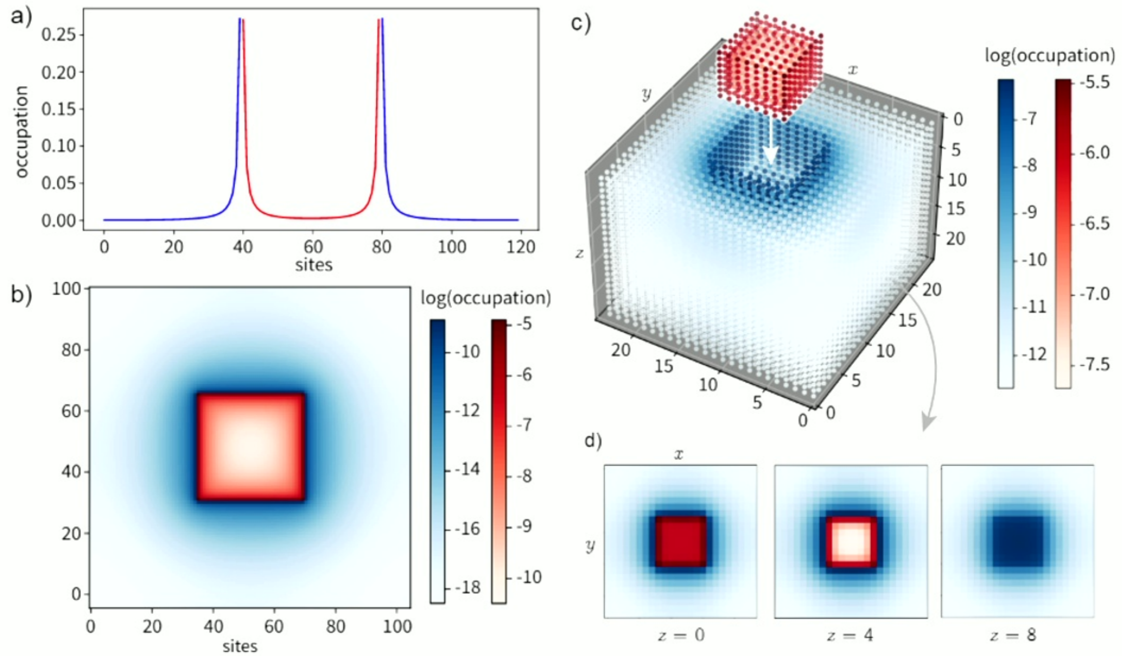
$$\hat{\phi}_i = \frac{1}{\sqrt{\omega\varepsilon^n}}\hat{q}_i,$$

$$\hat{\pi}_i = \sqrt{\frac{\omega}{\varepsilon^n}}\hat{p}_i,$$

$$\omega^2 = m^2 + \frac{2n}{\varepsilon^2}$$

$$\alpha = \frac{1}{\omega\varepsilon^2}$$

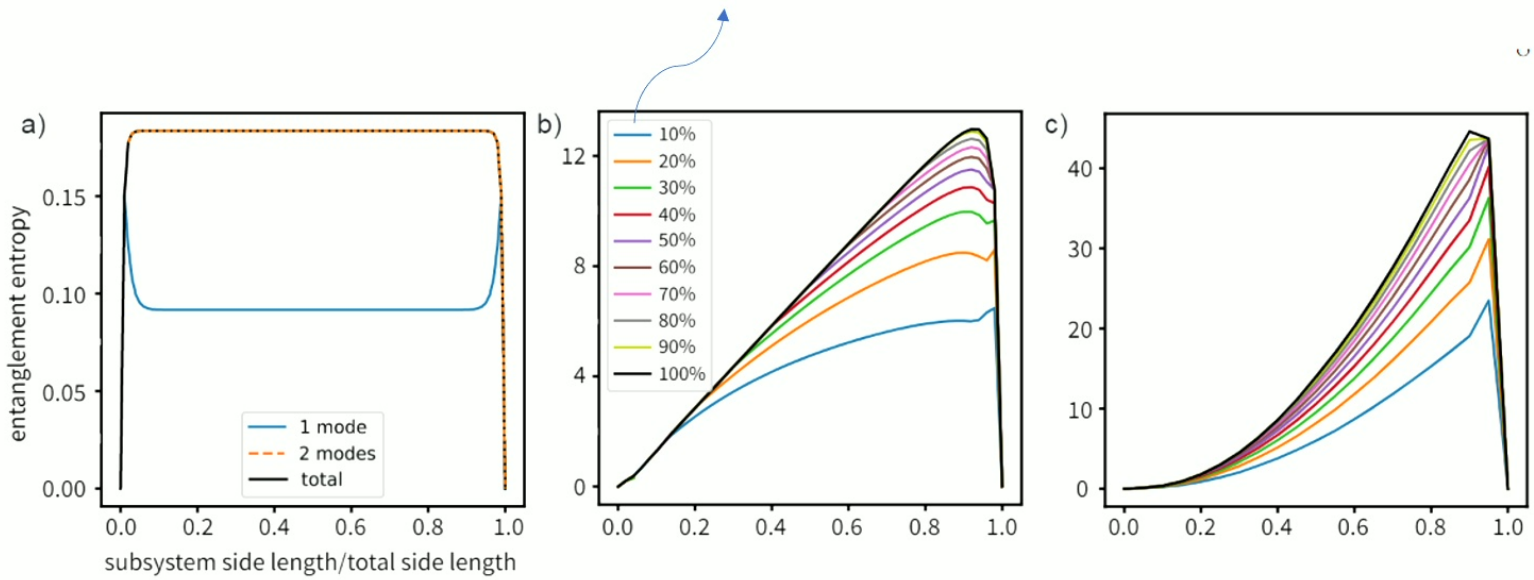
Spatial profile of most mixed normal modes



➢ Most mixed normal modes strongly supported near the boundary of regions.

Entanglement entropy area law with subset of degrees of freedom

(Number of normal modes)/(number of modes supported solely in the boundary of the subregion)



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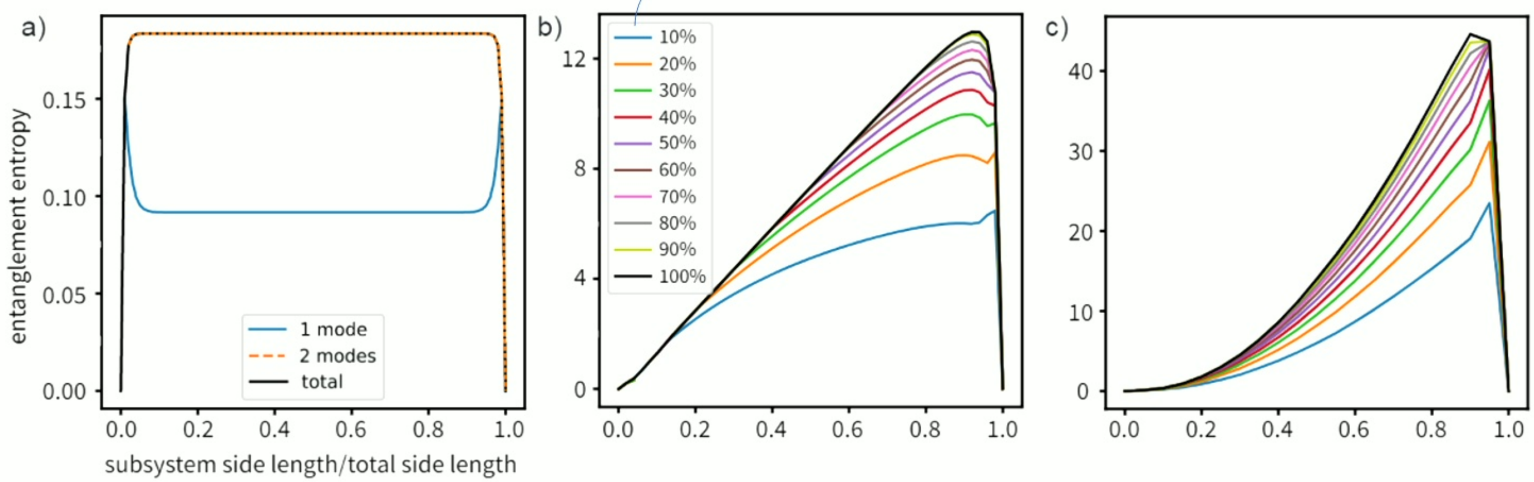

➢ Full entanglement with the complement is achieved when the number of normal modes included equals the number of boundary sites!

Conclusions

- The entanglement that can be extracted by probes from a quantum field can be characterized by normal modes of subregions

- This prescribes the optimal form of coupling with the probes in a given region in order to capture most of the entanglement with the region's complement

- Things to think about:

➢ Generalize this to noncomplementary regions (most useful for entanglement harvesting in Relativistic Quantum Information)
➢ Adapt this to smoother switchings – maybe relate to timelike tube theorem?

Thank you!