

Title: Symmetries and anomalies at the IR fixed point of gravity

Speakers: Kurt Hinterbichler

Series: Cosmology & Gravitation

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Abstract: I will discuss symmetries that arise at the infrared fixed point of the RG flow of Einstein gravity, including conformal vs. scale invariance in various dimensions, as well as 1-form generalized global symmetries and new anomalies that arise among them.

Zoom link: <https://pitp.zoom.us/j/93217255461?pwd=MXdzWDdVbWZrQzQ5UmVYVkx3US8zZz09>

Symmetries and anomalies at the IR fixed point of gravity

Kurt Hinterbichler (Case Western)

PI, May 30, 2023

arxiv:2110.10160 w/ Kara Farnsworth, Ondra Hulik

arxiv:2205.12272 w/ Diego Hofman, Austin Joyce, Greg Mathys

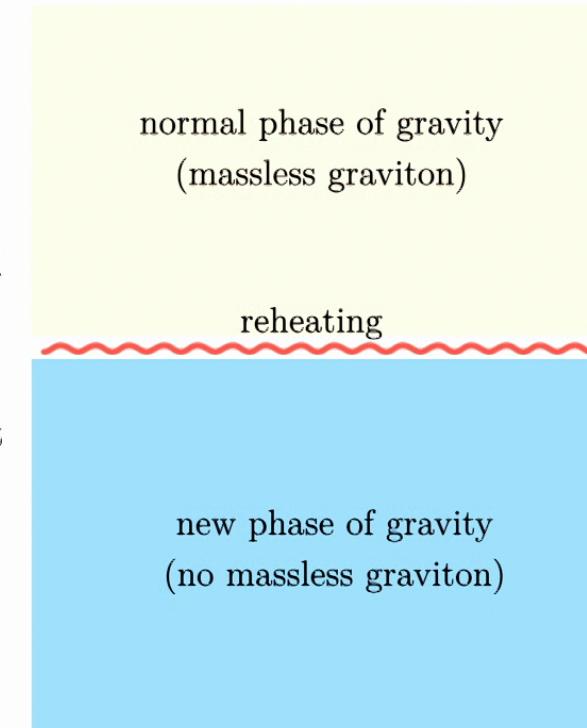
arxiv:23xx.xxxx w/ Austin Joyce, Greg Mathys

Motivation: gravity in the early universe

Could the early universe be in some kind of different phase of gravity?
(topological phase? Higgs phase? dual phase?)

e.g. Agrawal, Gukov, Obied, Vafa (2020)

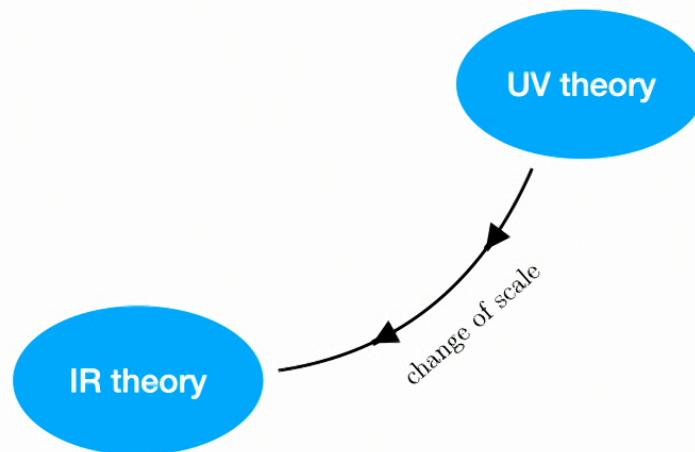
Is there anything we can do to get a model independent handle on such a phase?



QFT RG flows

A quantum field theory is supposed to be a renormalization group flow:

UV fixed point → IR fixed point



$$x \rightarrow 0 \quad \langle \phi(x) \phi(0) \rangle \sim \frac{1}{x^{2\Delta_{\text{UV}}}}$$

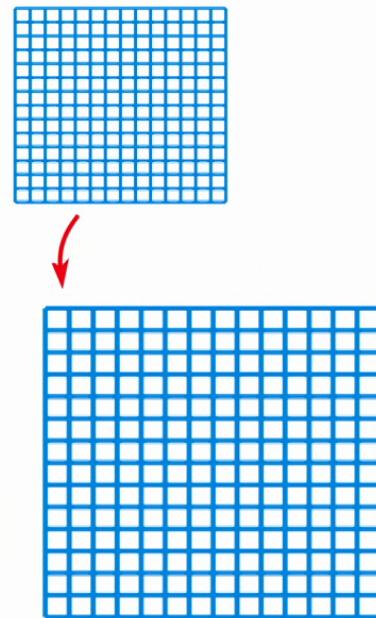
$$x \rightarrow \infty \quad \langle \phi(x) \phi(0) \rangle \sim \frac{1}{x^{2\Delta_{\text{IR}}}}$$

Scale symmetry

Endpoints are fixed points of RG \rightarrow scale invariant theories

Scale transformation: $\mathcal{D} = -(x^\mu \partial_\mu + \Delta)$

scaling weight



Scale vs. conformal

Does scale invariance imply conformal invariance?

- Proven in $D = 2$ for unitary theories with a stress tensor
- Not yet fully proven in $D > 2$

Polchinski (1987)

Review: Nakayama (2013)

Scale vs. conformal: E&M as a counter-example

El-Showk, Nakayama, Rychkov (2011)

$$S = \int d^D x - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Action is invariant under scale transformation in any D :

$$\delta A_\mu = \mathcal{D} A_\mu , \quad \Delta = \frac{D}{2} - 1 \quad \rightarrow \quad \delta S = 0$$

Under special conformal transformation:

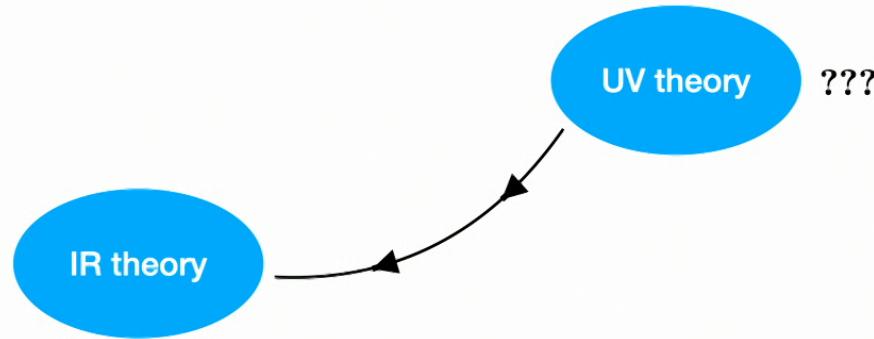
$$\delta A_\mu = (b \cdot K) A_\mu \quad \rightarrow \quad \delta S = \frac{1}{2} \int d^D x (D-4) [b \cdot A \partial \cdot A - b^\nu V^\mu \partial_\mu V_\nu]$$

↑
constant parameters

Correlators of $F_{\mu\nu}$ conformally invariant in $D = 4$, not in $D > 4$

RG flow for quantum gravity

Gravity is not supposed to be a QFT (no local operators) but there are other observables (i.e. S-matrix on flat space) we can ask about RG flows of:



Low energy EFT for gravity coupled to any massive matter:

$$S = \frac{M_P^{D-2}}{2} \int d^D x \sqrt{-g} R + \mathcal{O}(\partial^4).$$

Higher derivative terms from
integrating out UV stuff

Linearized gravity

IR fixed point is Fierz-Pauli theory:

$$S = \int d^D x \left[-\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\lambda h \partial^\lambda h \right]$$

↑
linearized metric $h_{\mu\nu} = \frac{1}{2} M_P^{1/(D-2)} (g_{\mu\nu} - \eta_{\mu\nu})$

Gauge symmetry (linearized diffs): $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Manifestly scale invariant: $\delta h_{\mu\nu} = - \left(x^\lambda \partial_\lambda + \Delta \right) h_{\mu\nu}, \quad \Delta = \frac{1}{2}(D-2)$

Not invariant under the standard special conformal transformations:

$$\delta^\sigma h_{\mu\nu} = \left(-2x^\sigma x^\lambda \partial_\lambda + x^2 \partial^\sigma - 2x^\sigma \Delta \right) h_{\mu\nu} - 2x_\lambda (\mathcal{J}^{\sigma\lambda})_{\mu\nu}^{\alpha\beta} h_{\alpha\beta},$$

$$\delta S = \int d^D x (D-2)(\dots)$$

Local operators in linearized gravity

Full quantum gravity is not supposed to have true local operators, but in *linearized* gravity we do have local operators:

Linearized Weyl tensor (gauge invariant, non-zero on shell)

$$W_{\mu_1 \mu_2 \mu_3 \mu_4} = -4 \mathcal{P}_{\mu_1 \mu_2 \mu_3 \mu_4}^{\nu_1 \nu_2 \nu_3 \nu_4} \partial_{\nu_1} \partial_{\nu_3} h_{\nu_2 \nu_4} , \quad \Delta = \frac{D}{2} + 1$$

Projector onto traceless 

Satisfies equations of motion:

$$\partial^{\mu_1} W_{\mu_1 \mu_2 \nu_1 \nu_2} = 0 \quad \text{conservation}$$

$$\partial_{[\mu_3} W_{\mu_1 \mu_2] \nu_1 \nu_2} = 0 \quad \text{dual conservation}$$

All other local operators are products of derivatives of W , modulo these relations

- $D = 2$: Lagrangian is a total derivative
- $D = 3$: No independent Weyl tensor, so no local operators (theory is topological)
consider only $D \geq 4$ from now on.

Local operators in linearized gravity

Weyl operator: $W_{\mu_1\mu_2\mu_3\mu_4}$, $\Delta = \frac{D}{2} + 1$

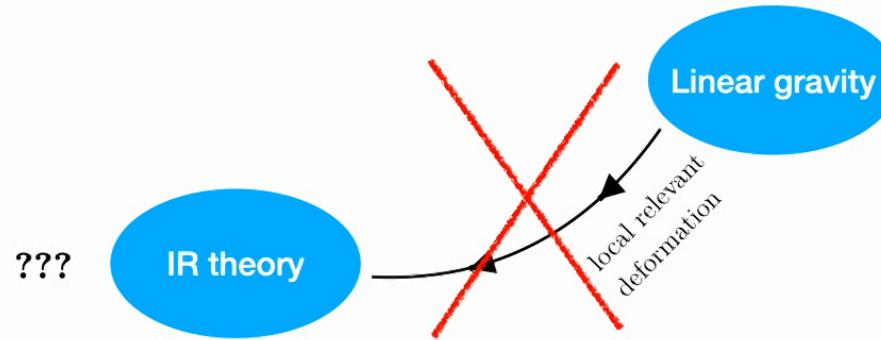
No stress tensor ($\Delta = D$ symmetric tensor):

can't make a symmetric tensor from derivatives of W
using more than one W gives $\Delta > D$

No marginal or relevant ($\Delta \leq D$) scalar operators:

Can't make a scalar from derivatives of W
using more than one W gives $\Delta > D$

Linear gravity can't be a UV fixed point, inducing RG flow via relevant local operators:



Correlators in linearized gravity

Must gauge-fix the action to compute correlators

Faddeev–Popov procedure + ignore decoupled ghosts



$$S_{\text{g.f.}} = \int d^D x \left[-\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\lambda h \partial^\lambda h - \frac{1}{\xi_1} (\partial^\nu h_{\mu\nu} - \frac{1}{2} \xi_2 \partial_\mu h)^2 \right]$$

Two gauge fixing parameters

Correlators of W independent of gauge fixing parameters

$\xi_2 = 2$ not allowed: fails to fix the gauge, residual local symmetry: $\delta h_{\mu\nu} = \partial_\mu \partial_\nu \chi$

Correlators in linearized gravity $D>4$

When $D > 4$, a useful choice for the gauge-fixing parameters is

$$\xi_1 = \frac{D+2}{D-2}, \quad \xi_2 = \frac{D+4}{D}.$$

Decompose into traceless+trace: $h_{\mu\nu} = \tilde{h}_{\mu\nu} + \phi \eta_{\mu\nu}$, $\tilde{h}_\mu^\mu = 0$,

$$S_{\text{g.f.}} = \int d^D x \left[-\frac{1}{2} \partial_\lambda \tilde{h}_{\mu\nu} \partial^\lambda \tilde{h}^{\mu\nu} + \frac{4}{D+2} \partial_\mu \tilde{h}_{\nu\lambda} \partial^\nu \tilde{h}^{\mu\lambda} + \frac{(D-4)(D-2)}{4} (\partial\phi)^2 \right]$$

This action is conformally invariant:

$\tilde{h}_{\mu\nu}$ and ϕ are primaries of weight $\Delta = \frac{1}{2}(D-2)$

Correlators in linearized gravity $D>4$

Correlators take the conformally invariant form for conformal primaries:

$$c_\phi = -\frac{\Gamma(\frac{D}{2} - 1)}{2(D-4)(D-2)\pi^{D/2}}, \quad c_h = \frac{D\Gamma(\frac{D}{2} - 1)}{4(D-4)\pi^{D/2}}.$$

$$\langle \phi(x)\phi(0) \rangle = \frac{c_\phi}{x^{2\Delta}}, \quad \langle \tilde{h}_{\mu_1\mu_2}(x)\tilde{h}^{\nu_1\nu_2}(0) \rangle = \frac{c_h}{x^{2\Delta}} \mathcal{I}_{\mu_1\mu_2}^{\nu_1\nu_2}, \quad \langle \tilde{h}_{\mu_1\mu_2}(x)\phi(0) \rangle = 0,$$

$$\Delta = \frac{1}{2}(D-2)$$

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$$\mathcal{I}_{\mu_1\mu_2}^{\nu_1\nu_2} \equiv \frac{1}{2}(I_{\mu_1}^{\nu_1} I_{\mu_2}^{\nu_2} + I_{\mu_1}^{\nu_2} I_{\mu_2}^{\nu_1}) - \frac{1}{D}\eta_{\mu_1\mu_2}\eta^{\nu_1\nu_2},$$

$$I^{\mu\nu} \equiv \eta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}$$

$\tilde{h}_{\mu\nu}$ violates spin-2 unitarity bound: $\Delta \geq D$

but $\tilde{h}_{\mu\nu}$ is not an operator in the theory (not gauge invariant)

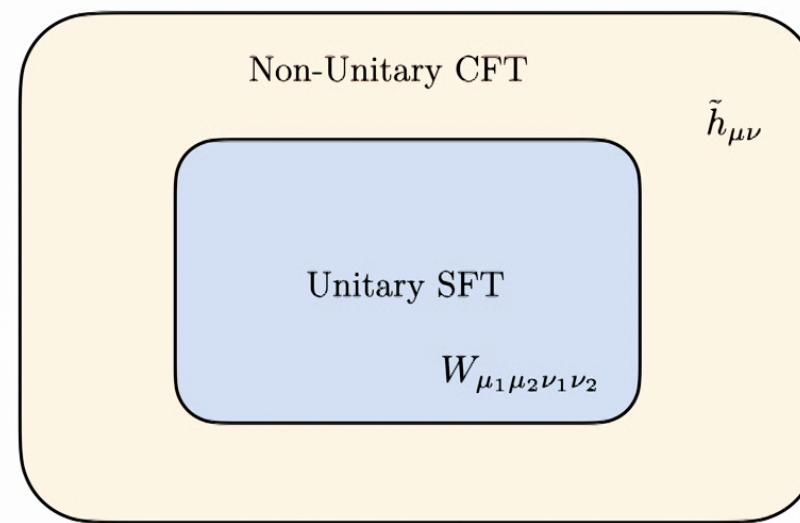
Correlators in linearized gravity $D>4$

W correlator takes the form of a descendent:

$$\langle W_{\mu_1 \mu_2 \mu_3 \mu_4}(x) W_{\nu_1 \nu_2 \nu_3 \nu_4}(0) \rangle = 16 \mathcal{P}_{\mu_1 \mu_2 \mu_3 \mu_4}^{\rho_1 \rho_2 \rho_3 \rho_4} \mathcal{P}_{\nu_1 \nu_2 \nu_3 \nu_4}^{\sigma_1 \sigma_2 \sigma_3 \sigma_4} \partial_{\rho_1} \partial_{\rho_3} \partial_{\sigma_1} \partial_{\sigma_3} \langle \tilde{h}_{\rho_2 \rho_4}(x) \tilde{h}_{\sigma_2 \sigma_4}(0) \rangle$$

Satisfies the conservation conditions:

$$\begin{aligned} \partial^{\mu_1} \langle W_{\mu_1 \mu_2 \mu_3 \mu_4}(x) W_{\nu_1 \nu_2 \nu_3 \nu_4}(0) \rangle &= 0, \\ \partial_{[\mu_5} \langle W_{\mu_1 \mu_2] \mu_3 \mu_4}(x) W_{\nu_1 \nu_2 \nu_3 \nu_4}(0) \rangle &= 0. \end{aligned}$$



Correlators in linearized gravity $D = 4$

Cannot reach the conformal gauge in $D = 4$. Using any other gauge:

$$\langle W_{\mu_1 \mu_2 \mu_3 \mu_4}(x) W^{\nu_1 \nu_2 \nu_3 \nu_4}(0) \rangle = \frac{96}{\pi^2 x^6} I_{\mu_1}^{\rho_1} I_{\mu_2}^{\rho_2} I_{\mu_3}^{\rho_3} I_{\mu_4}^{\rho_4} \mathcal{P}_{\rho_1 \rho_2 \rho_3 \rho_4}^{\nu_1 \nu_2 \nu_3 \nu_4}$$

↑
Conformally invariant form for primary $W_{\mu_1 \mu_2 \mu_3 \mu_4}$, $\Delta = 3$

Requires using dimension dependent identities for $D = 4$.

Saturates unitarity bound for  $\Delta \geq D - 1$

Conservation conditions follow from the conformal algebra

$$\partial^{\mu_1} W_{\mu_1 \mu_2 \nu_1 \nu_2} = 0 \quad , \quad \partial_{[\mu_3} W_{\mu_1 \mu_2] \nu_1 \nu_2} = 0$$

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Linear gravity is a CFT in $D = 4$

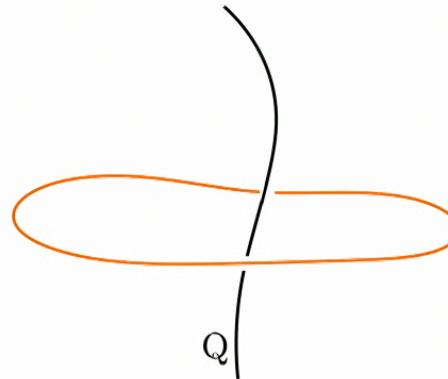
even though the action is not invariant in the usual way

1-form generalized symmetries

Gaiotto, Kapustin, Seiberg, Willett (2014)

2-form current: $\partial^\mu j_{\mu\nu} \approx 0$

Conserved charge: $Q = \oint_{\text{surface}} \tilde{j}$



Conservation of $j \leftrightarrow Q$ does not change under deformations of the 2-surface

Example: E&M in $D = 4$ has two 1-form symmetries

| | 2-form current | charge | charged solution |
|-----------|--|-------------------------------|--|
| electric: | $F_{\mu\nu}$ | $Q_E = \oint_{S_2} \tilde{F}$ | $A = -\frac{1}{4\pi r} dt$ Coulomb |
| magnetic: | $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ | $Q_M = \oint_{S_2} F$ | $A_N = \frac{g}{4\pi} (1 - \cos \theta) d\phi, \quad A_S = \frac{g}{4\pi} (-1 - \cos \theta) d\phi$ $A_N - A_S = d\Lambda$ Dirac monopole |

Generalized symmetries in linear gravity

Penrose (1986)
Jezierski (1994-present)
Casini, Magan, Olive (2021)

2-form current: $J_{\mu_1\mu_2} [\xi] = W_{\mu_1\mu_2\nu_1\nu_2} \xi^{\nu_1\nu_2}$

anti-symmetric “killing tensor”

conservation: $\partial^{\mu_2} J_{\mu_1\mu_2} [\xi] = W_{\mu_1\mu_2\nu_1\nu_2} \partial^{\mu_2} \xi^{\nu_1\nu_2} = 0$

$$\rightarrow \partial^{\mu_1} \xi^{\nu_1\nu_2} - \partial^{[\nu_1} \xi^{\nu_2]\mu_1} + \frac{3}{D-1} \eta^{\mu_1[\nu_1} \partial_\mu \xi^{\nu_2]\mu} = 0$$

Generalized conformal Killing equation: traceless mixed symmetry part vanishes 

General solution: Killing-Yano tensors

$$\xi_{\mu\nu} = K_{A_1 A_2 A_3} \frac{\partial X^{A_1}}{\partial x^\mu} \frac{\partial X^{A_1}}{\partial x^\nu} X^{A_3}$$

anti-symmetric constant coefficients 

$D+2$ dimensional conformal embedding space coords: $X^A(x) = \left(\frac{1+x^2}{2}, \frac{1-x^2}{2}, x^\mu \right)$

Generalized symmetries in linear gravity

$$K_{\mu\nu\rho} \sim c_{\mu\nu\rho}, \quad K_{-2,\mu\nu} \sim c_{\mu\nu} + c'_{\mu\nu}, \quad K_{-1,\mu\nu} \sim c_{\mu\nu} - c'_{\mu\nu}, \quad K_{-1,-2,\mu} \sim c_\mu$$

$$\xi^{\mu_1\mu_2} = c^{\mu_1\mu_2} + c^{\mu_1\mu_2\nu} x_\nu + c^{[\mu_1} x^{\mu_2]} + c'^{[\mu_1}_{\mu} x^{\mu_2]} x^\mu - \frac{1}{4} c'^{\mu_1\mu_2} x^2$$

20 independent components in $D = 4$

Dual currents in $D = 4$: $\tilde{J}_{\mu_1\mu_2}[\xi] = \tilde{W}_{\mu_1\mu_2\nu_1\nu_2}\xi^{\nu_1\nu_2}$

dual Weyl tensor
 $\tilde{W}_{\mu_1\mu_2\nu_1\nu_2} = \frac{1}{2}\epsilon_{\mu_1\mu_2\rho_1\rho_2} W^{\rho_1\rho_2}{}_{\nu_1\nu_2}$

$$\partial^{\mu_1} \tilde{W}_{\mu_1\mu_2\nu_1\nu_2} = 0 \Leftrightarrow \partial_{[\mu_3} W_{\mu_1\mu_2]\nu_1\nu_2} = 0$$

Charges:

$$Q[\xi] = \oint_{2-\text{surface}} \tilde{J}[\xi] \quad , \quad \tilde{Q}[\xi] = \oint_{2-\text{surface}} J[\xi]$$

Generalized 1-form charges in linear gravity

| Killing-Yano component | physical quantity | Charged solution |
|---|-----------------------|----------------------|
| \square | energy-momentum | Schwartzchild metric |
| $\begin{array}{ c }\hline \square \\ \hline \end{array} \sim \square$ | dual energy-momentum | Taub-NUT metric |
| $\begin{array}{ c }\hline \square \\ \hline \end{array}$ | angular momentum | Kerr metric |
| $\begin{array}{ c }\hline \square \\ \hline \end{array}$ | dual Angular momentum | C-metric |

Everything manifestly gauge invariant, charges are integrals of gauge invariant local currents (unlike ADM charges)

The charged solutions all exist non-linearly

Anomalies: simple example

Consider a spin-1 $\Delta=1$ primary operator in a $D=2$ CFT.

Conformal algebra implies: $\partial_\mu J^\mu = 0$, $\partial_{[\mu} J_{\nu]} = 0$

$$\partial_\mu \tilde{J}^\mu = 0$$

Conformal invariance fixes 2-pt. function at separated points up to normalization:

$$\langle J^\mu(x) J^\nu(0) \rangle = \frac{1}{x^2} \left(\delta^{\mu\nu} - 2 \frac{x^\mu x^\nu}{x^2} \right)$$

Satisfies both conservation conditions:

$$\partial_\mu \langle J^\mu(x) J^\nu(0) \rangle = 0 , \quad \partial_{[\mu} \langle J_{\nu]}(x) J^\rho(0) \rangle = 0$$

Anomalies: simple example

What about coincident points?

Fourier transform:

$$\langle J^\mu(p) J^\nu(-p) \rangle = 2\pi \frac{p^\mu p^\nu}{p^2} + C \delta^{\mu\nu}$$

↑ conformally invariant contact term

Conservation conditions:

$$p_\mu \langle J^\mu(p) J^\nu(-p) \rangle = (2\pi + C)p^\nu \quad \leftarrow \text{failure is local}$$
$$p_{[\mu} \langle J_{\nu]}(p) J^\rho(-p) \rangle = C p_{[\mu} \delta_{\nu]\rho} \quad \leftarrow \text{(analytic in } p)$$

Anomaly: no choice of contact term preserves both conditions. One (or both) conservation conditions must fail at co-incident points

Anomalies: effective action

$$e^{W[A]} = \langle e^{\int d^2x A_\mu J^\mu + \dots} \rangle$$

conservation condition \rightarrow gauge invariance of the effective action

conservation $\rightarrow \delta A_\mu = \partial_\mu \Lambda$ ← scalar gauge parameter

dual conservation $\rightarrow \delta A_\mu = \partial^\nu \Lambda_{\mu\nu}$ ← anti-symmetric tensor gauge parameter

Effective action is only well defined up to local terms:

$$W[A] \rightarrow W[A] + c \int d^2x \frac{1}{2} A_\mu A^\mu$$

↑ gives contact term in correlator

Cannot choose c to maintain both gauge symmetries

Imposing one, failure of the other is fixed (anomaly equation)

Anomalies: effective action (free scalar example)

Example: free massless scalar

$$S = \int d^D x - \frac{1}{2}(\partial\phi)^2$$

current: $J_\mu = \partial_\mu\phi$, $\partial_\mu J^\mu = 0$, $\partial_{[\mu} J_{\nu]} = 0$

couple to external gauge field A_μ :

$$S \rightarrow S = \int d^D x - \frac{1}{2}(\partial\phi)^2 + A^\mu \partial_\mu\phi + \frac{1}{2}c A^2$$

\uparrow \uparrow
 $J^\mu A_\mu$ coupling contact term

gauge variations:

$$\begin{aligned} \delta A_\mu &= \partial_\mu\Lambda, \quad \delta\phi = \Lambda \quad \rightarrow \quad \delta_\Lambda S = \int d^D x - (c+1)\Lambda\partial\cdot A \quad \rightarrow \quad \partial_\mu\langle J^\mu \rangle = (c+1)\partial\cdot A \\ \delta A_\mu &= \partial^\nu\Lambda_{\nu\mu}, \quad \delta\phi = 0 \quad \rightarrow \quad \delta_\Lambda S = \int d^D x - \frac{1}{2}c\Lambda^{\mu\nu}F_{\mu\nu} \quad \rightarrow \quad \partial_{[\mu}\langle J_{\nu]} \rangle = \frac{1}{2}cF_{\mu\nu} \end{aligned}$$

Can choose at most one conservation condition to hold. The other is then fixed.

Anomalies: Goldstone theorem

Delacrétaz, Hofman, Mathys (2020)

Invert the argument: assume the existence of a conserved current with given anomaly equation:

$$\partial_\mu J^\mu = 0, \quad \partial_{[\mu} J_{\nu]} = -a F_{\mu\nu}$$

General form of the 2-pt. function:

$$\langle J^\mu(p) J^\nu(-p) \rangle = f(p^2) p^\mu p^\nu + g(p^2) p^2 \delta^{\mu\nu}$$

Imposing conservation relations fixes the 2 pt. function:

conservation $\rightarrow f(p^2) = -g(p^2)$

dual (non) conservation $\rightarrow p_{[\mu} \langle J_{\nu]}(p) J^\rho(-p) \rangle = a p_{[\mu} \delta_{\nu]\rho} \rightarrow g(p^2) = -\frac{a}{p^2}$

$$\rightarrow \langle J^\mu(p) J^\nu(-p) \rangle = a \frac{p_\mu p_\nu - p^2 g_{\mu\nu}}{p^2}$$

Spectral decomposition implies the existence of a massless spin 0:

$$\langle J^\mu(p) J^\nu(-p) \rangle = \int_0^\infty ds \rho_1(s) \frac{\eta_{\mu\nu} + \frac{p_\mu p_\nu}{s}}{p^2 + s} + \rho_0(s) \frac{p_\mu p_\nu}{p^2 + s}$$

$\rho_1(s) = 0 \qquad \qquad \qquad \rho_0(s) = a\delta(s)$

Anomalies: linearized gravity

Weyl 2-pt. function in momentum space:

$$\langle W_{\mu_1 \mu_2 \mu_3 \mu_4}(p) W_{\mu_1 \mu_2 \mu_3 \mu_4}(-p) \rangle = \frac{pppp}{p^2} \eta \eta + \dots$$

non-local part

$$+ c_1 (pp\eta\eta\eta + \dots) + c_2 (pp\eta\eta\eta + \dots)$$

two possible local contact terms

Conservation equations:

$$p^{\mu_1} \langle W_{\mu_1 \mu_2 \mu_3 \mu_4}(p) W_{\mu_1 \mu_2 \mu_3 \mu_4}(-p) \rangle = () ppp\eta\eta + \dots$$

failure is local

$$p_{[\mu_5} \langle W_{\mu_1 \mu_2] \mu_3 \mu_4}(p) W_{\mu_1 \mu_2 \mu_3 \mu_4}(-p) \rangle = () pp\eta\eta\eta + \dots$$

↑
coefficients depending on c_1, c_2

If we impose conservation:

$$p^{\mu_1} \langle W_{\mu_1 \mu_2 \mu_3 \mu_4}(p) W_{\mu_1 \mu_2 \mu_3 \mu_4}(-p) \rangle = 0 \quad \rightarrow \quad \text{fixes } c_1, c_2$$

\rightarrow failure of dual conservation

$$p_{[\mu_5} \langle W_{\mu_1 \mu_2] \mu_3 \mu_4}(p) W_{\mu_1 \mu_2 \mu_3 \mu_4}(-p) \rangle = a (pp\eta\eta\eta + \dots)$$

↑
fixed anomaly coefficient

Effective action: linearized gravity

Couple linearized gravity to a background gauge field through the Weyl operator:

$$\Delta\mathcal{L} \sim W_{\mu\nu,\rho\sigma}\mathcal{A}^{\mu\nu,\rho\sigma}$$

↑
Linear Weyl operator ↑
Background gauge field
(same symmetries as Weyl)

$$S = \int d^Dx \left[-\frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \partial_\rho h^{\mu\rho}\partial^\nu h_{\nu\mu} - \partial^\nu h_{\nu\mu}\partial^\mu h + \frac{1}{2}\partial_\mu h\partial^\mu h \right] \quad \leftarrow \text{original action}$$

$$+ \partial_\mu\partial_\rho h_{\nu\sigma}\mathcal{A}^{\mu\nu,\rho\sigma} \quad \leftarrow \text{Weyl coupling}$$

$$+ c_1\partial_\lambda\mathcal{A}_{\mu\nu,\rho\sigma}\partial^\lambda\mathcal{A}^{\mu\nu,\rho\sigma} + c_2\partial_\sigma\mathcal{A}^{\mu\nu,\rho\sigma}\partial^\lambda\mathcal{A}_{\mu\nu,\rho\lambda} \quad \leftarrow \text{all possible local counterterms}$$

Conservation requires gauge symmetry:

$$\partial^{\mu_2}W_{\mu_1\mu_2\nu_1\nu_2} = 0 \quad \rightarrow \quad \delta\mathcal{A}_{\mu\nu,\rho\sigma} = \partial_\mu\Lambda_{\nu\rho,\sigma} + \dots$$

↑
 gauge parameter

graviton may also transform: $\delta h_{\mu\nu} = b_1(\partial^\rho\Lambda_{\rho\mu,\nu} + \partial^\rho\Lambda_{\rho\nu,\mu})$

Effective action: linearized gravity D>4

Imposing invariance fixes everything:

$$b_1 = \frac{D-3}{4(D-2)}, \quad c_1 = \frac{D-3}{8(D-4)}, \quad c_2 = -\frac{D-3}{2(D-4)}.$$

Dual conservation gauge invariance now broken:

$$\partial_{[\mu_3} W_{\mu_1\mu_2]\nu_1\nu_2} = 0 \quad \rightarrow \quad \delta \mathcal{A}_{\mu_1\mu_2,\nu_1\nu_2} = \partial^\mu \Lambda_{\mu\mu_1\mu_2,\nu_1\nu_2} + \dots$$

gauge parameter 

Non-invariance gives anomaly equation: $\partial_{[\mu_3} W_{\mu_1\mu_2]\nu_1\nu_2} = \partial\partial\partial\mathcal{A} + \dots$

Can turn it around:

Anomaly equation \rightarrow fixes 2-pt. function \rightarrow massless spin-2 “Goldstone”

Anomalies: linearized gravity $D = 4$

In $D = 4$ we cannot impose conservation or dual conservation

This is similar to the stress tensor in a $D = 2$ CFT: cannot impose exact conservation on a traceless symmetric tensor.

Need to include a trace:

$$T_{\mu\nu} = S_{\mu\nu} + \frac{1}{2}T\eta_{\mu\nu}$$

↑ ↑
traceless scalar

By giving T purely local correlators, $T_{\mu\nu}$ can be made conserved.

Phases of electromagnetism

| phases | 1-form symmetries | $U(1)_e$ Wilson lines | $U(1)_m$ 't Hooft lines |
|---|----------------------|--|--|
| normal phase “unbroken gauge symmetry” | | explicitly broken if there is electrically charged matter | spontaneously broken photon goldstone perimeter law for 't Hooft loop $\langle W(C) \rangle \sim e^{-\# \text{Perimeter}(C)}$ |
| superconducting phase “spontaneously broken gauge symmetry” | | explicitly broken | unbroken area law for 't Hooft loop $\langle W(C) \rangle \sim e^{-\# \text{Area}(C)}$ |
| confining phase | | unbroken area law for Wilson loop $\langle W(C) \rangle \sim e^{-\# \text{Area}(C)}$ | explicitly broken monopole condensation |

Phases of gravity?

| phases | $U(1)_e^{P_\mu, M_{\mu\nu}}$ Wilson lines | $U(1)_m^{P_\mu, M_{\mu\nu}}$ 't Hooft lines |
|---|--|---|
| normal phase “unbroken diff symmetry” | explicitly broken if there is normal matter | spontaneously broken graviton goldstone perimeter law for 't Hooft loop $\langle W(C) \rangle \sim e^{-\# \text{Perimeter}(C)}$ |
| Higgs phase of gravity ?? “spontaneously broken diff symmetry” | explicitly broken | unbroken area law for 't Hooft loop $\langle W(C) \rangle \sim e^{-\# \text{Area}(C)}$ |
| Confining phase of gravity ?? | unbroken area law for Wilson loop $\langle W(C) \rangle \sim e^{-\# \text{Area}(C)}$ | explicitly broken |

Summary and open questions

- There is still much to explore about the IR fixed point of quantum gravity
- conformal symmetry, 1-form symmetries, new anomalies
- Are any of these structures useful for saying anything beyond the fixed point? Anomaly matching?
- Are they useful for saying anything about a possible Higgs phase of gravity?