Title: A model of the cuprates: from the pseudogap metal to d-wave superconductivity and charge order

Speakers: Subir Sachdev

Series: Quantum Matter

Date: May 17, 2023 - 3:30 PM

URL: https://pirsa.org/23050038

Abstract: Soon after the discovery of high temperature superconductivity in the cuprates, Anderson proposed a connection to quantum spin liquids. But observations since then have shown that the low temperature phase diagram is dominated by conventional states, with a competition between superconductivity and charge-ordered states which break translational symmetry. We employ the "pseudogap metal" phase, found at intermediate temperatures and low hole doping, as the parent to the phases found at lower temperatures. The pseudogap metal is described as a fractionalized phase of a single-band model, with small pocket Fermi surfaces of electron-like quasiparticles whose enclosed area is not equal to the free electron value, and an underlying pi-flux spin liquid with an emergent SU(2) gauge field. This pi-flux spin liquid is now known to be unstable to confinement at sufficiently low energies. We develop a theory of the different routes to confinement of the pi-flux spin liquid, and show that d-wave superconductivity, antiferromagnetism, and charge order are natural outcomes. We are argue that this theory provides routes to resolving a number of open puzzles on the cuprate phase diagram.

As a side result, at half-filling, we propose a deconfined quantum critical point between an antiferromagnet and a d-wave superconductor described by a conformal gauge theory of 2 flavors of massless Dirac fermions and 2 flavors of complex scalars coupled as fundamentals to a SU(2) gauge field.

This talk is based on Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias S. Scheurer, and S. S., arXiv:2302.07885

Zoom link: https://pitp.zoom.us/j/97370076705?pwd=Q1MwQmNaSFkxaWFEdUl5NFZDS0E4Zz09

# Spin liquids and the phases of the cuprates

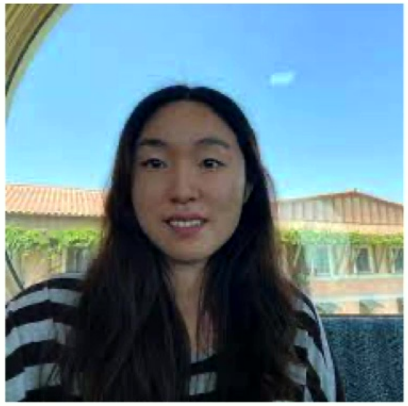
Perimeter Institute
May 17, 2023
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. S., PNAS **120**, e2302701120 (2023)
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.S., arXiv:2211.10452

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD

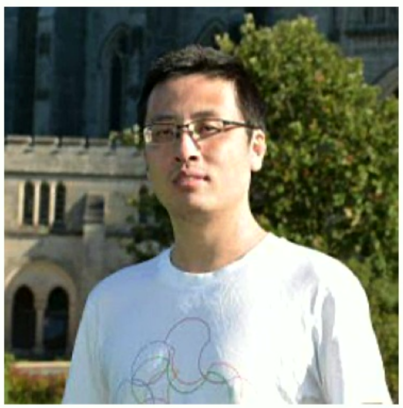
Maine Christos



Zhu-Xi Luo
→GA Tech

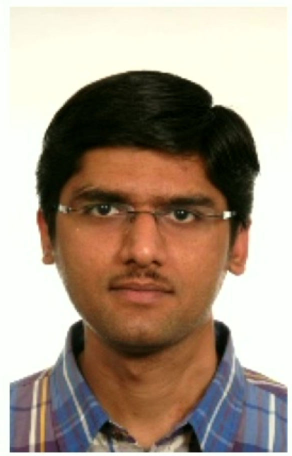
Henry
Shackleton

Mathias Scheurer
Innsbruck→Stuttgart

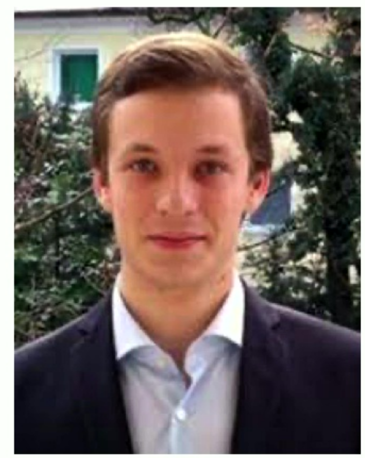


Ya-Hui Zhang
Johns Hopkins

Alexander
Nikolaenko

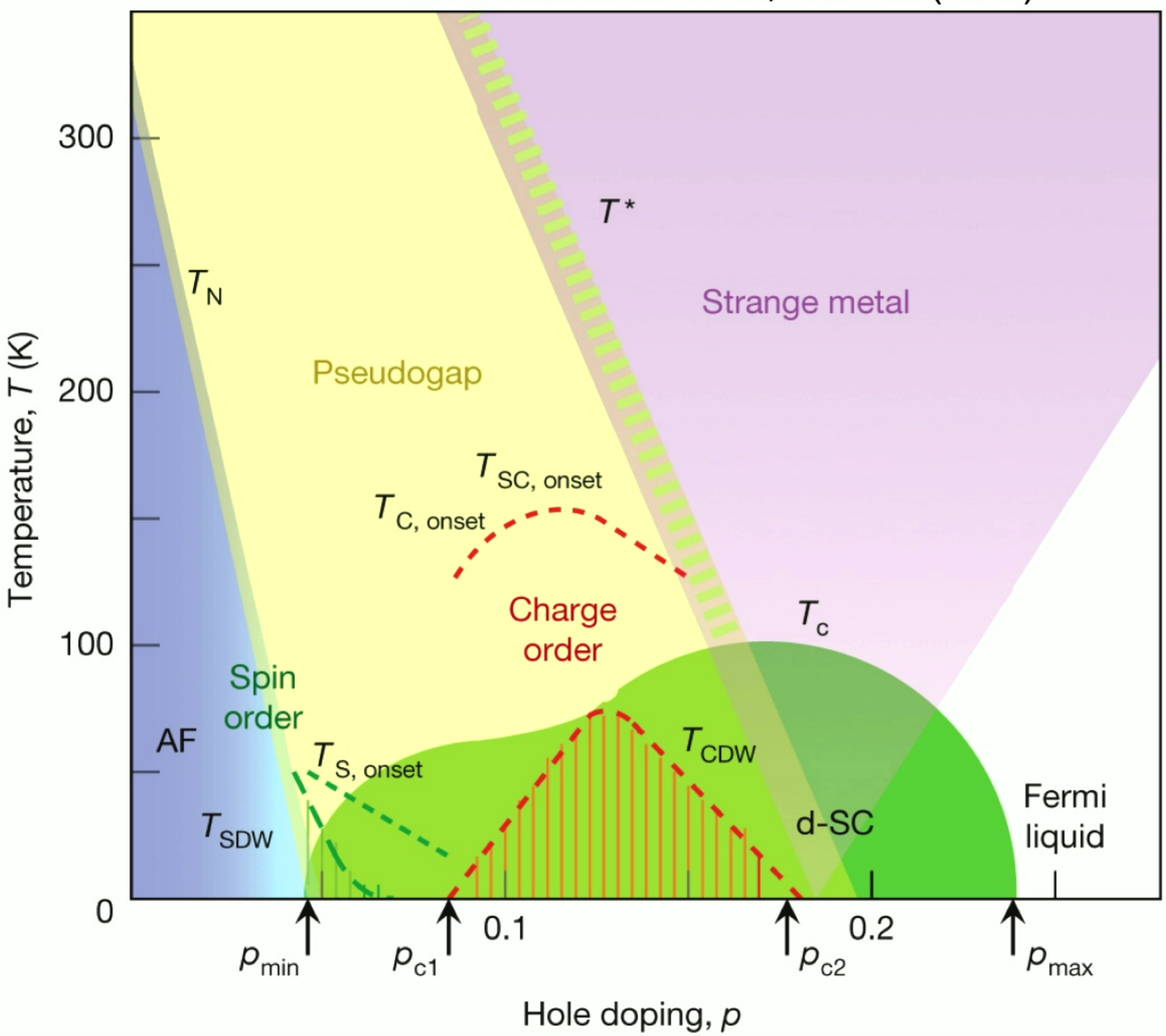


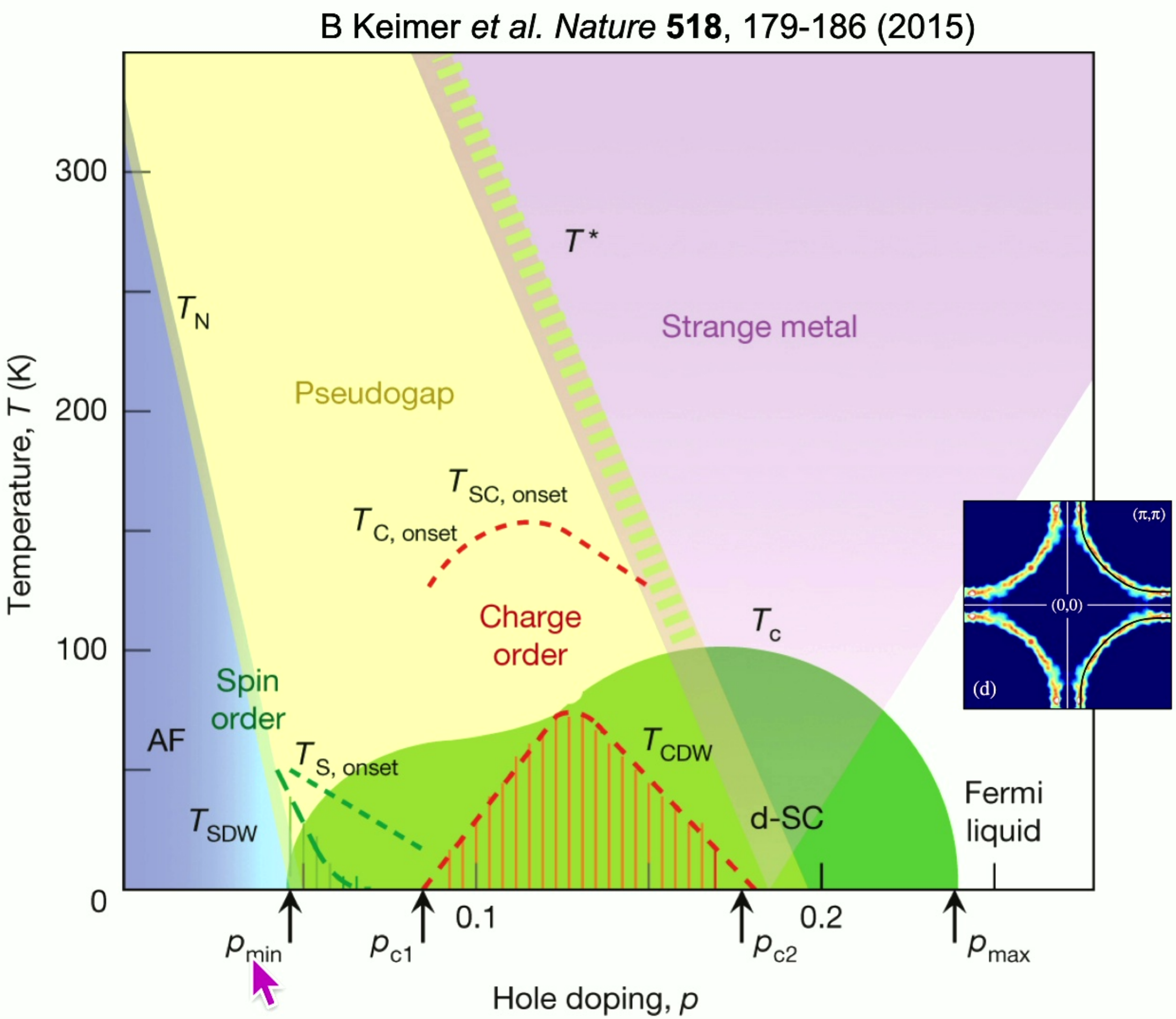
Darshan Joshi
TIFR Hyderabad



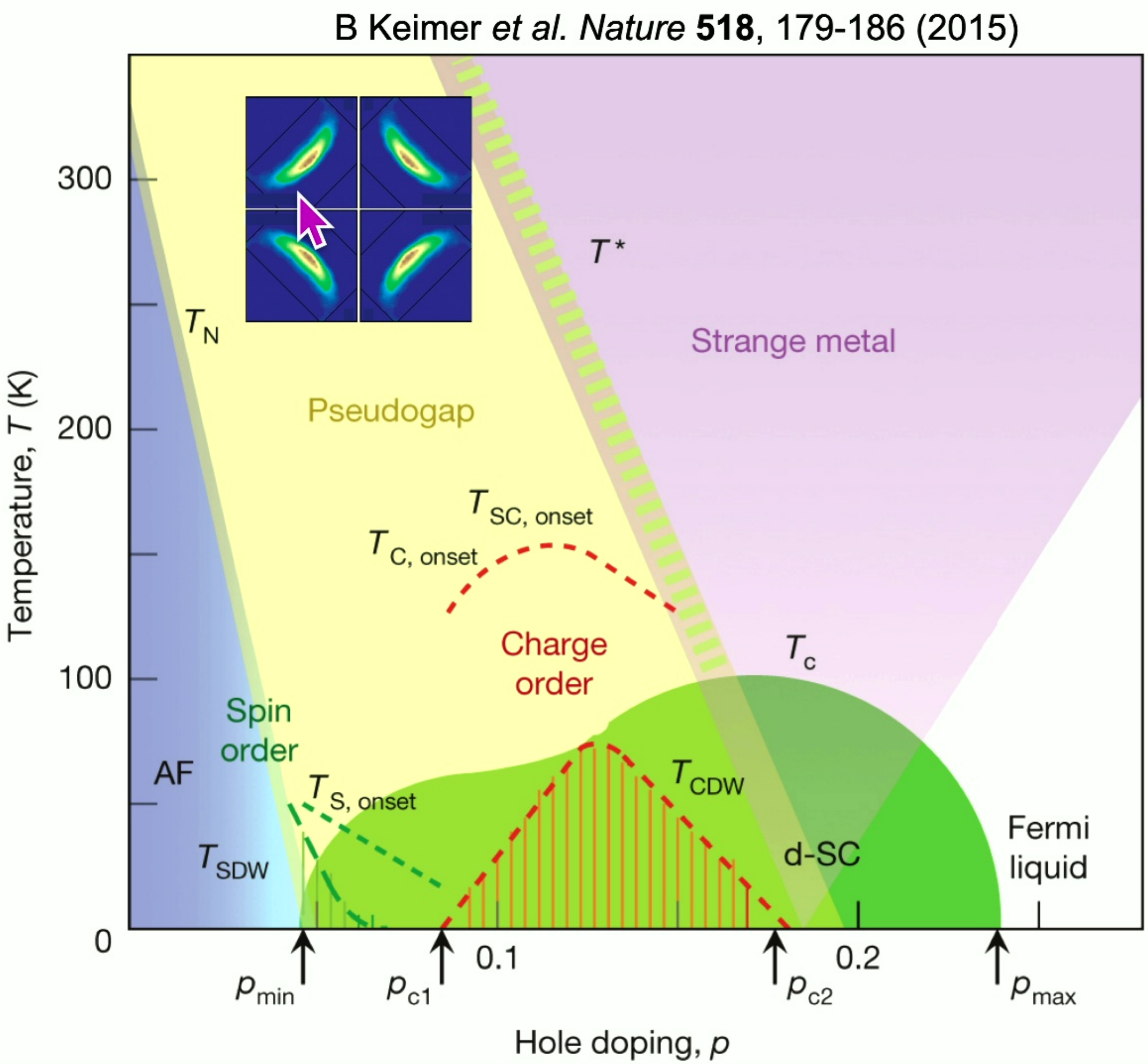
Jonas von Milczewski

B Keimer *et al.* *Nature* **518**, 179-186 (2015)

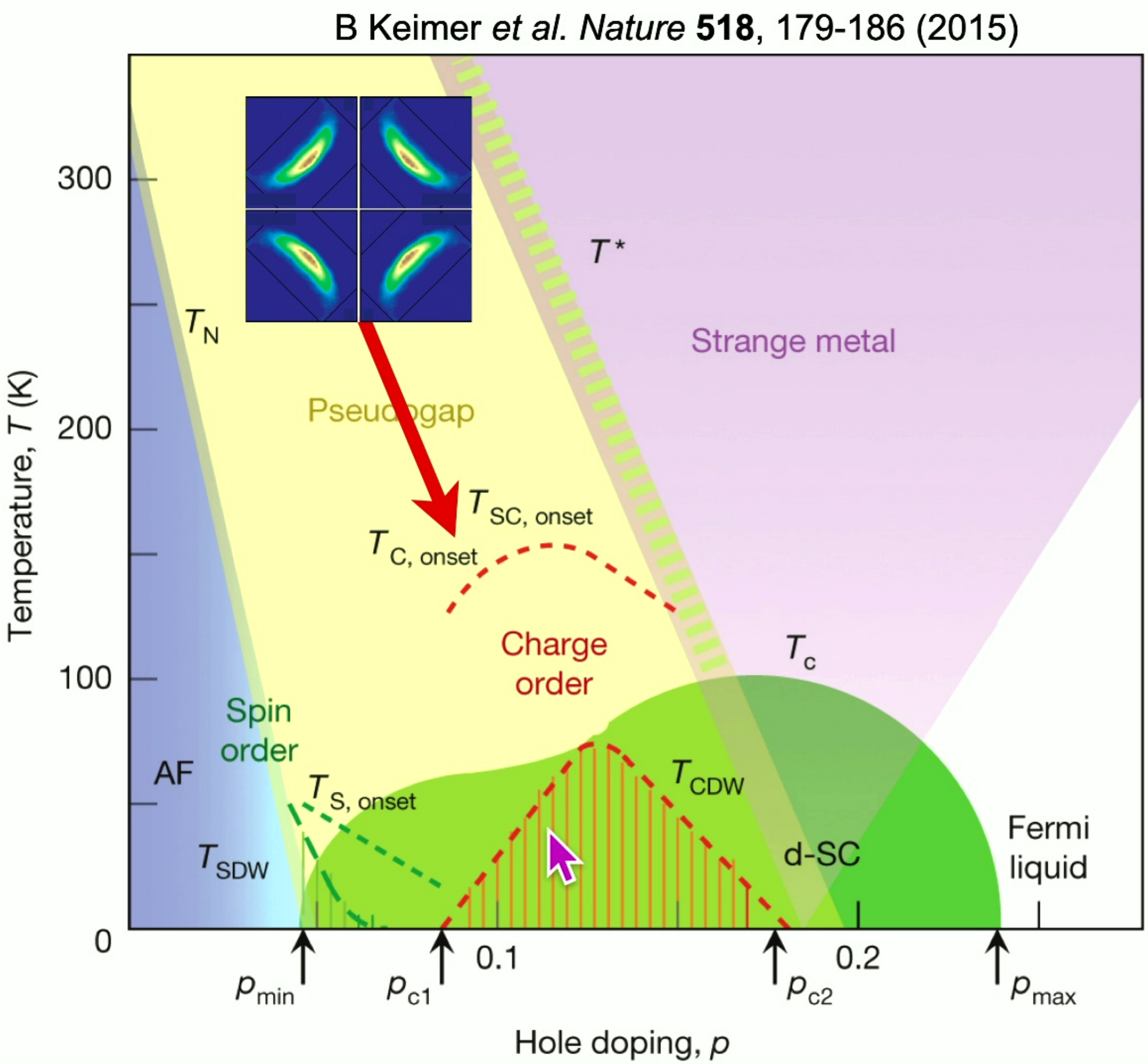





# Fermi liquid in the overdoped metal



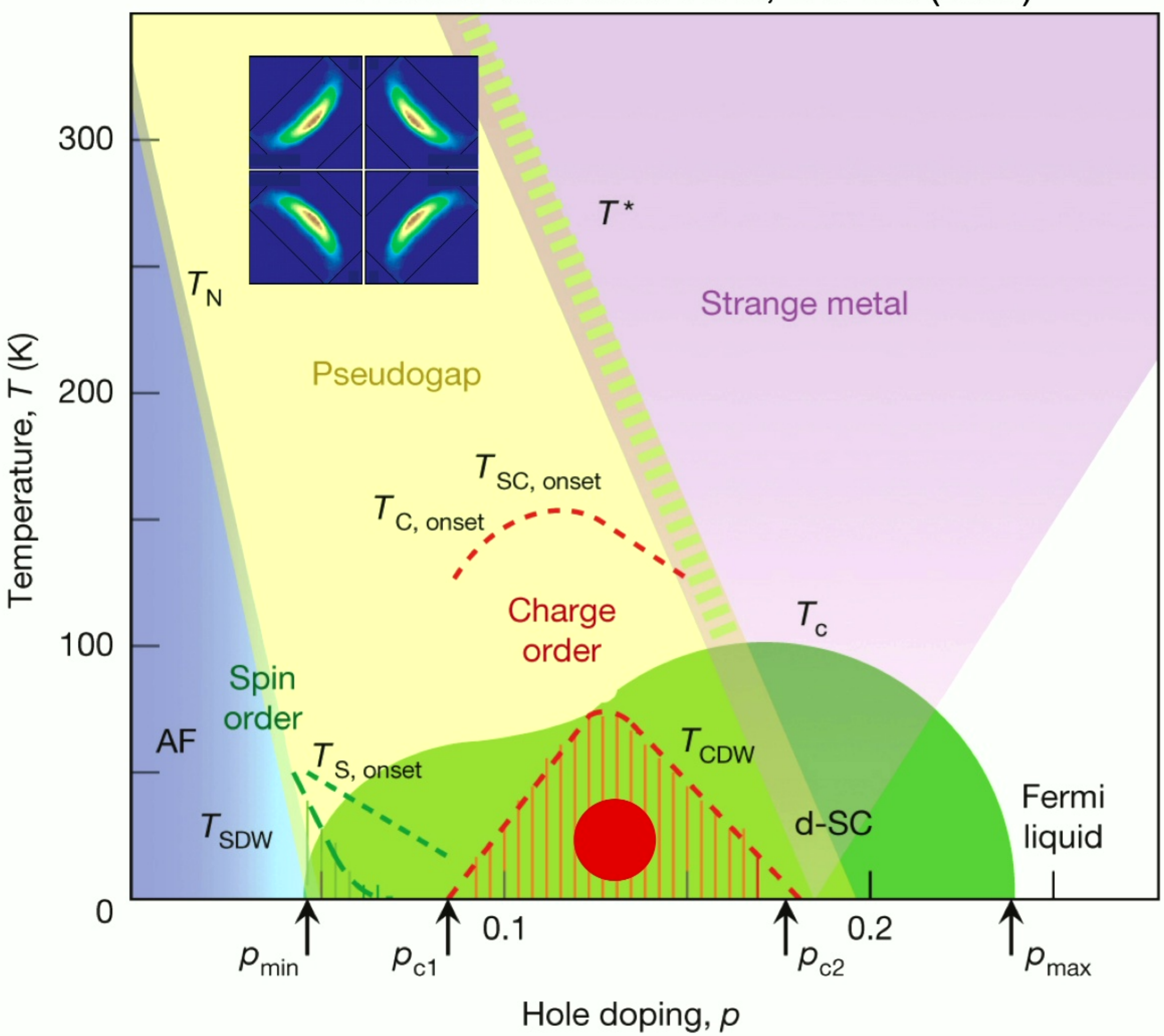

# Theory for "pseudogap metal" with "Fermi arcs"?



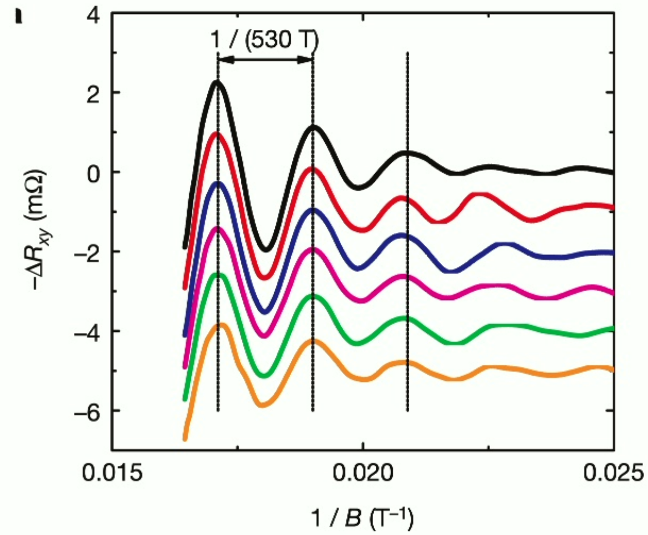

Needed: a theory for the onset of charge order and $d$-wave superconductivity from the pseudogap metal.

Why are $T_c$ and $T_{\rm CDW}$ about the same?

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



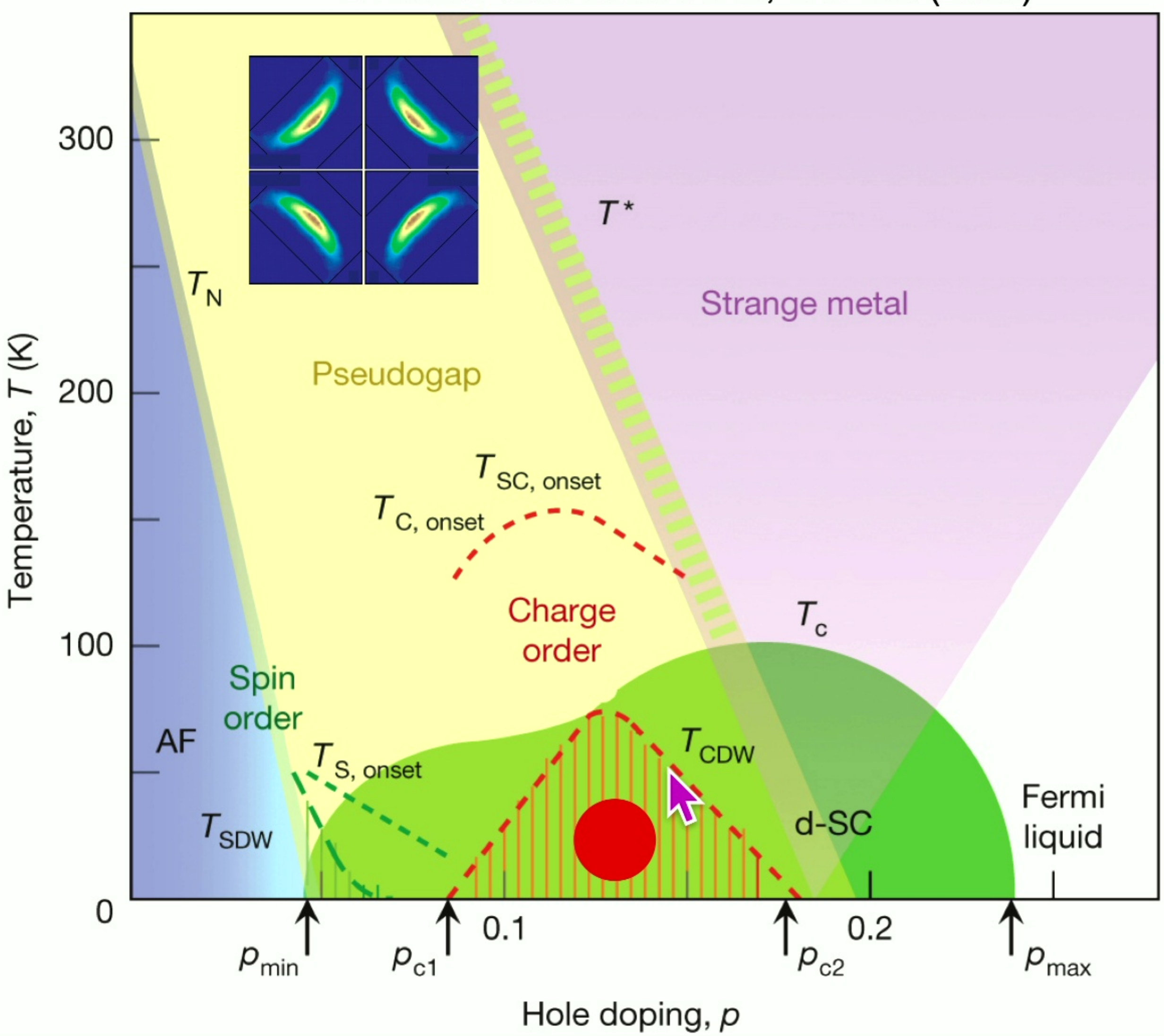

N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.B. Bonnemaison, R. Liang, D.A. Bonn, W.N. Hardy, L. Taillefer, Nature **447**, 565 (2007)



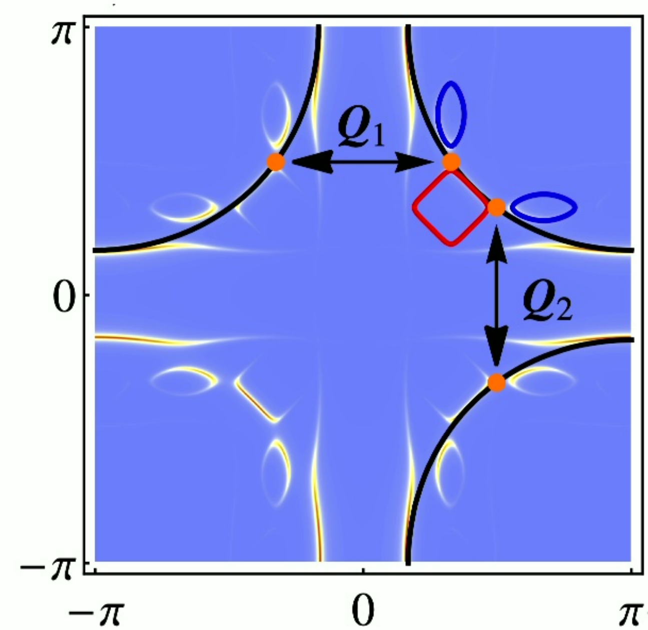

Quantum oscillations in the CDW phase at low $T$ show only a single electron pocket of size $p$.

This cannot be obtained in the theory of CDWs in a Fermi liquid.

B Keimer *et al. Nature* **518**, 179-186 (2015)




A. Allais, D. Chowdhury, and S. Sachdev, Nature Communications **5**, 5771 (2014)

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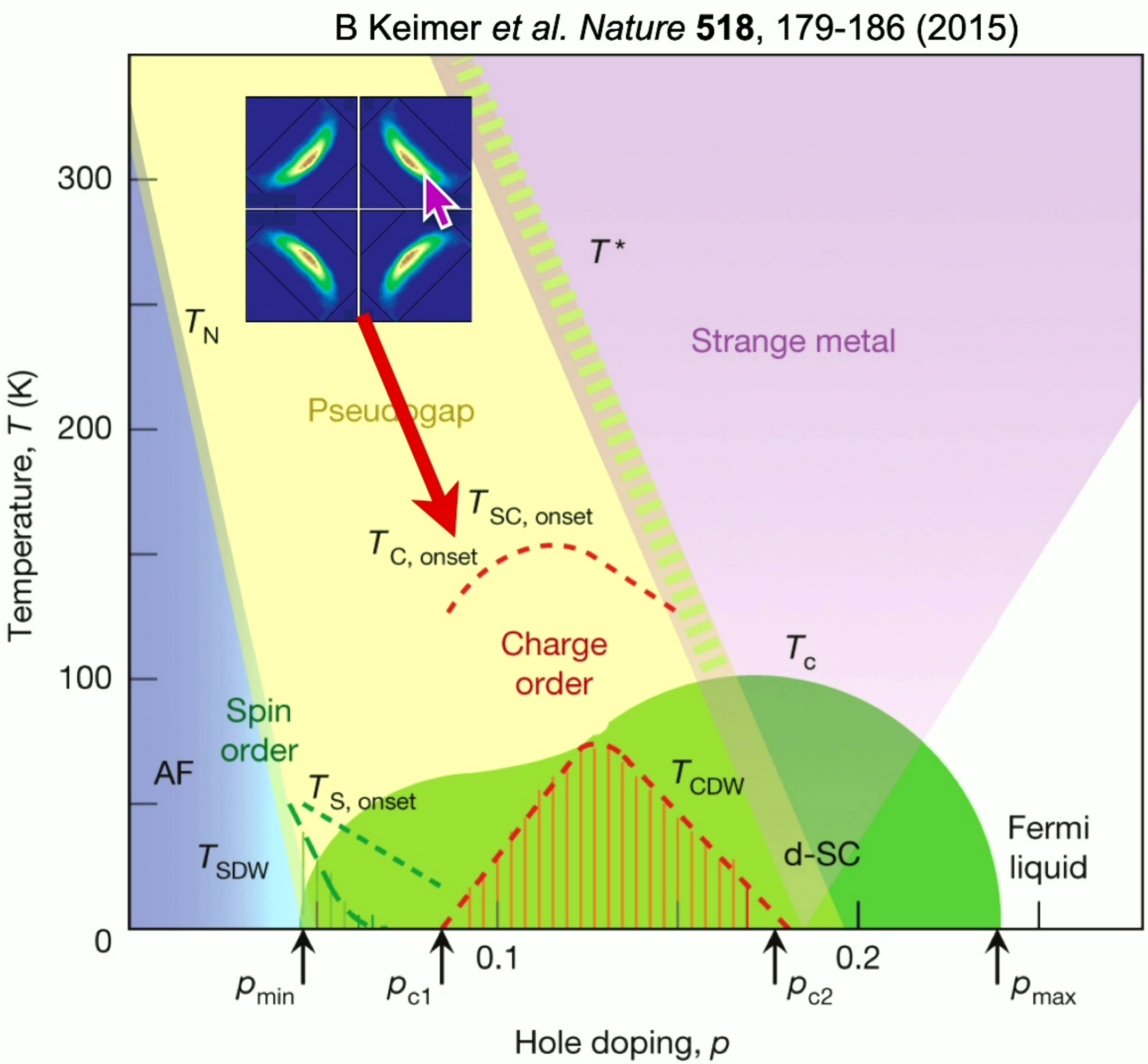
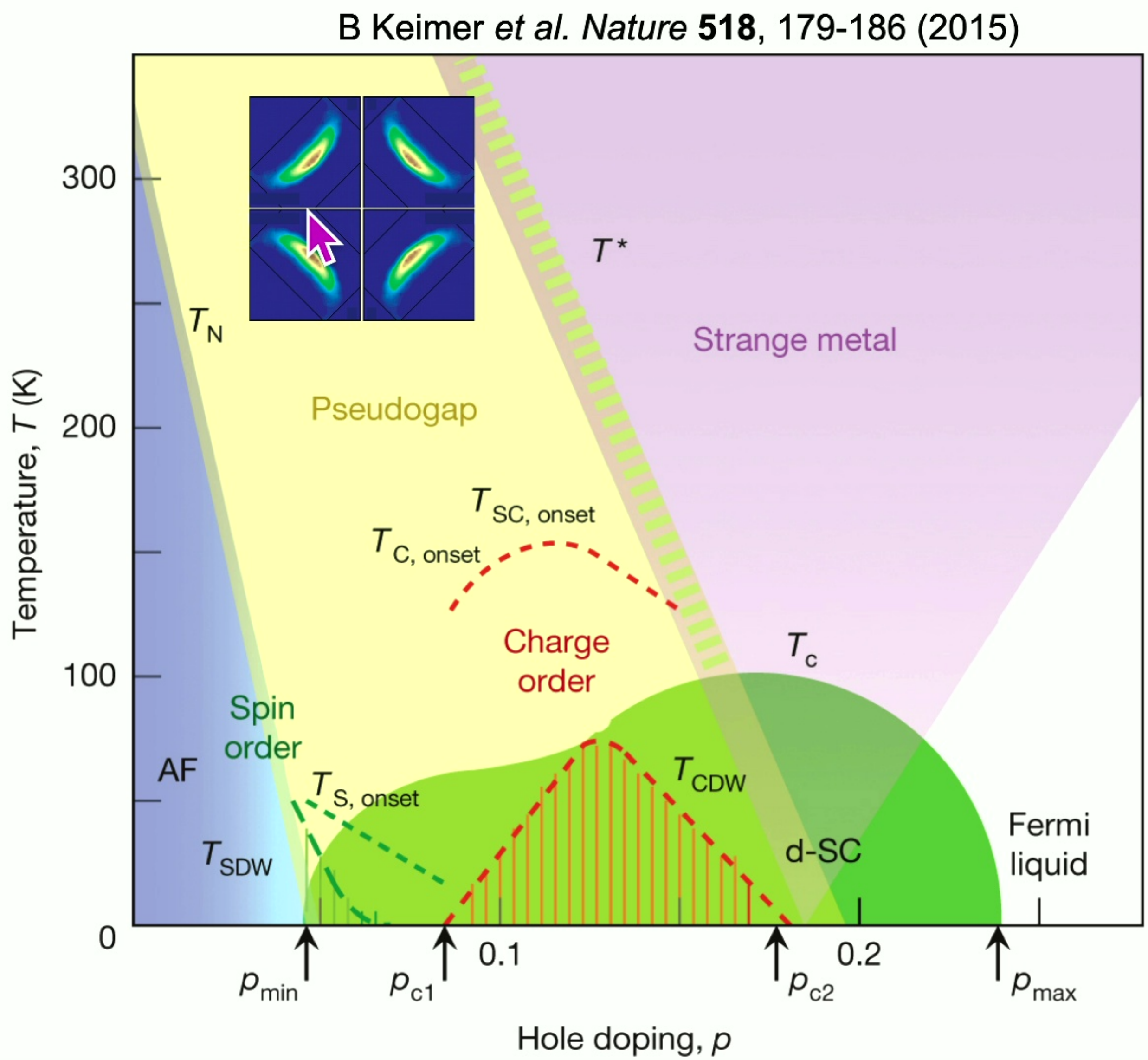
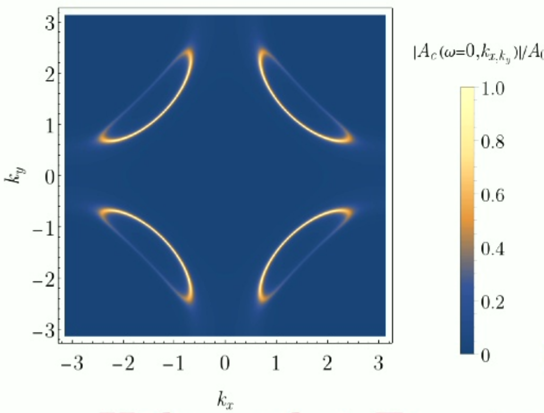



Theory for
"pseudogap metal"
with "Fermi arcs"?

Use the pseudogap metal
in place of the Fermi liquid
as the 'parent' to
*conventional*
$d$-wave superconductor,
charge density wave,
spin density wave,
pair density wave
...




Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, PRB **105**, 075146 (2022)

# Hole pocket Fermi surfaces of size $p$ with charge $e$, spin-1/2 quasiparticles

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C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, PRL **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009).
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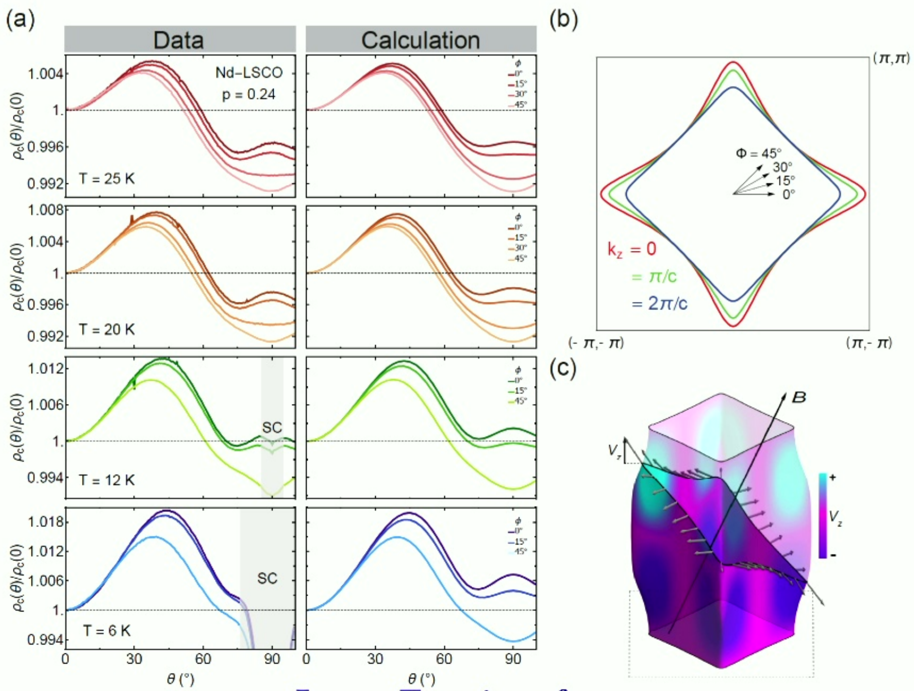
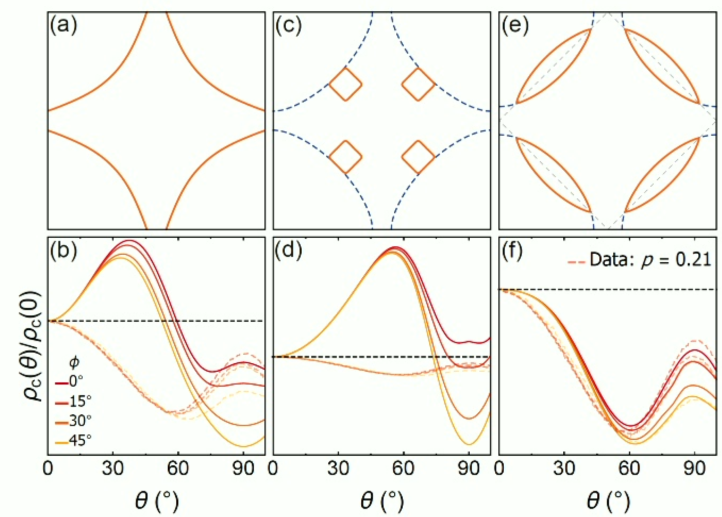
## Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics **18**, 558 (2022)

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate Nd-LSCO. Above the critical doping p⋆ we extract a Fermi surface geometry that is in quantitative agreement with angle-resolved photoemission. Below p⋆ the ADMR is qualitatively different, revealing a clear transformation of the Fermi surface. We find that our data are most consistent with a reconstruction of the Fermi surface by a Q = (π, π) wavevector.

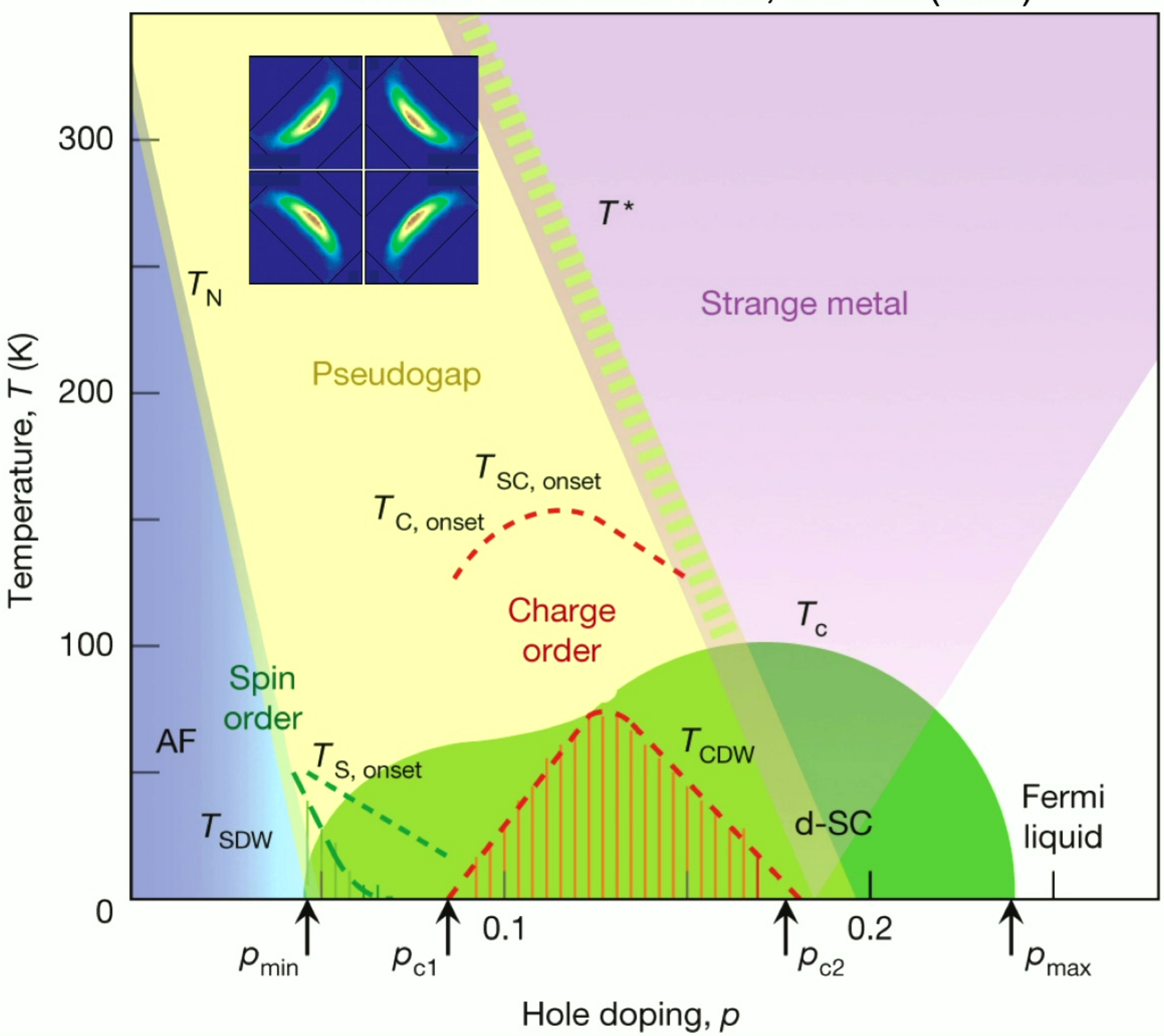


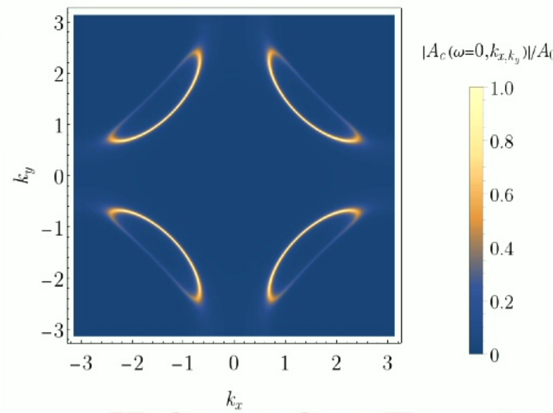

$p > p_c$ Large Fermi surface




$p < p_c$ Reconstructed Fermi surface

B Keimer *et al.* *Nature* **518**, 179-186 (2015)

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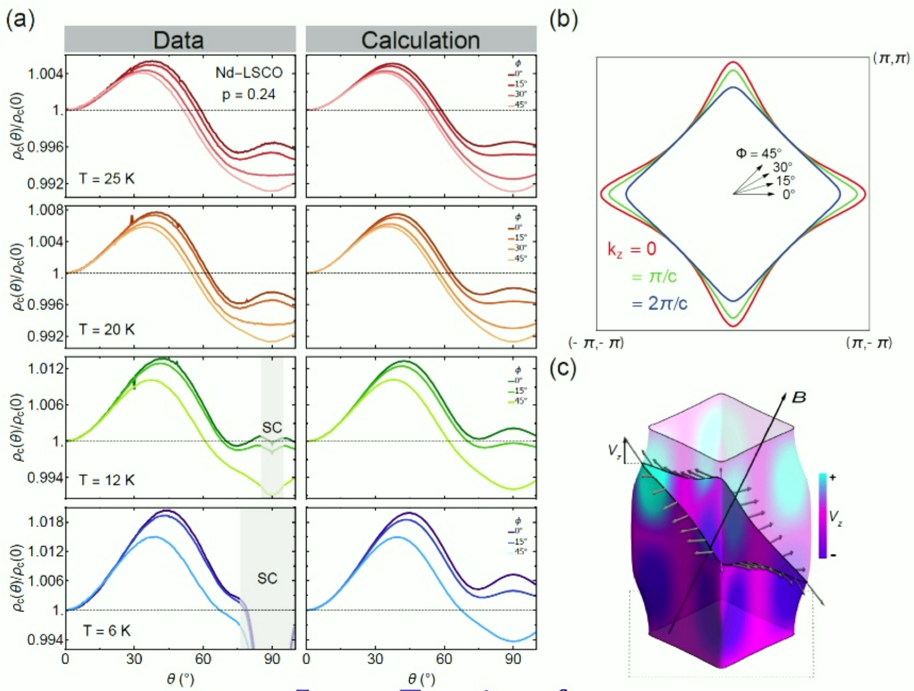
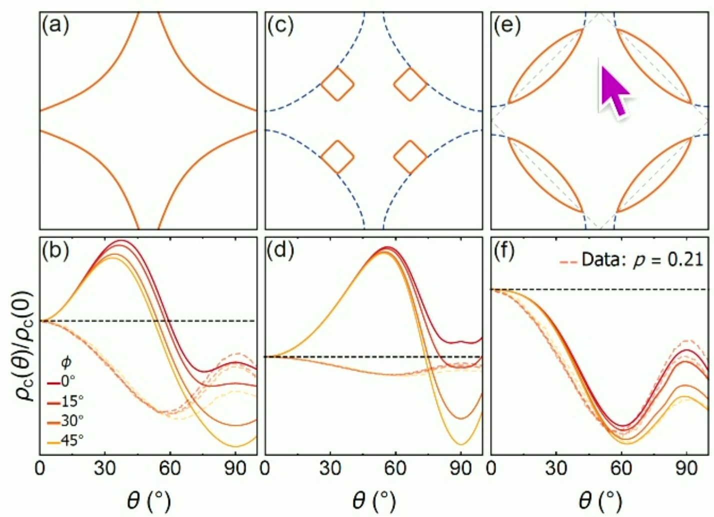



$p > p_c$ Large Fermi surface

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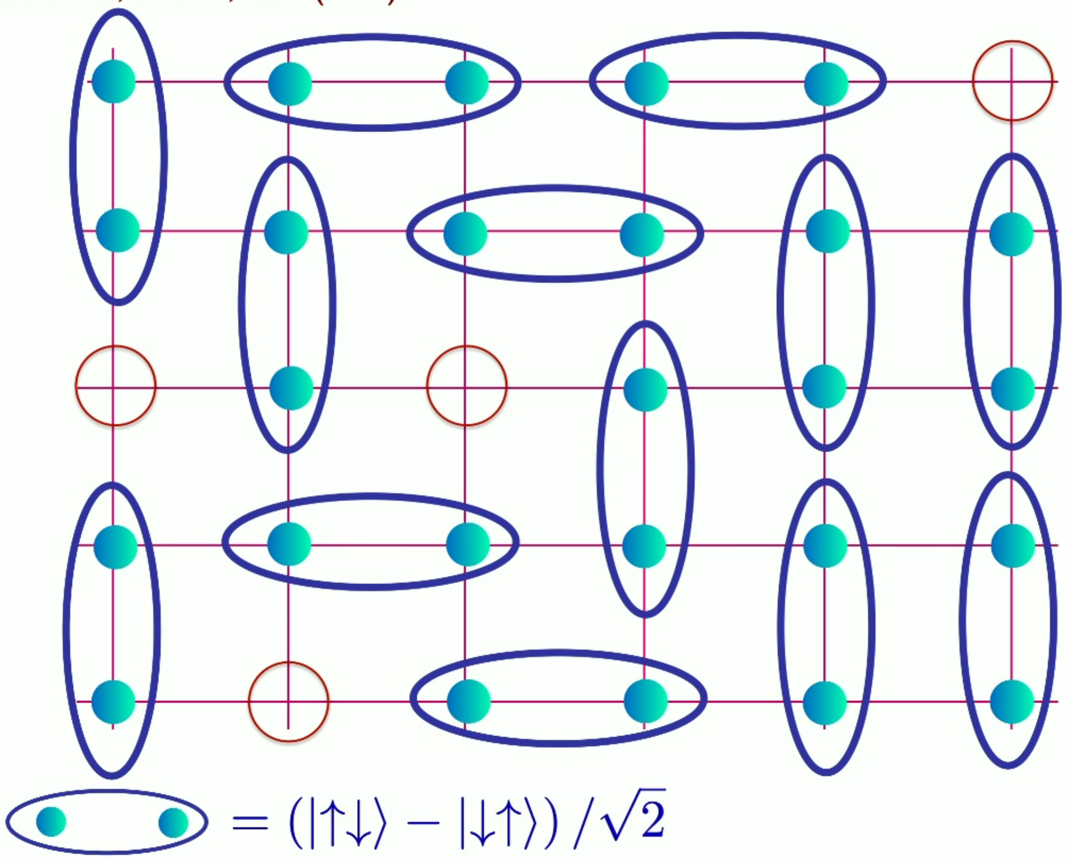
# Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid with density $p$ of spinless, charge +e "holons".

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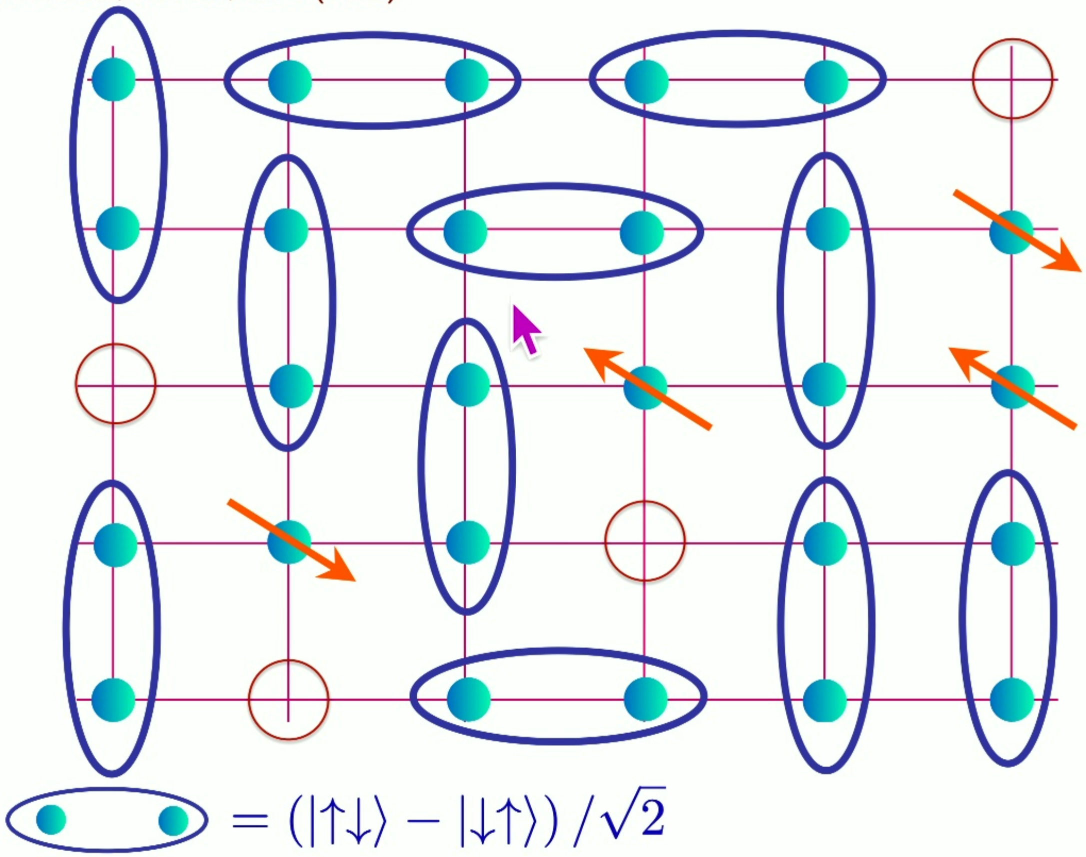
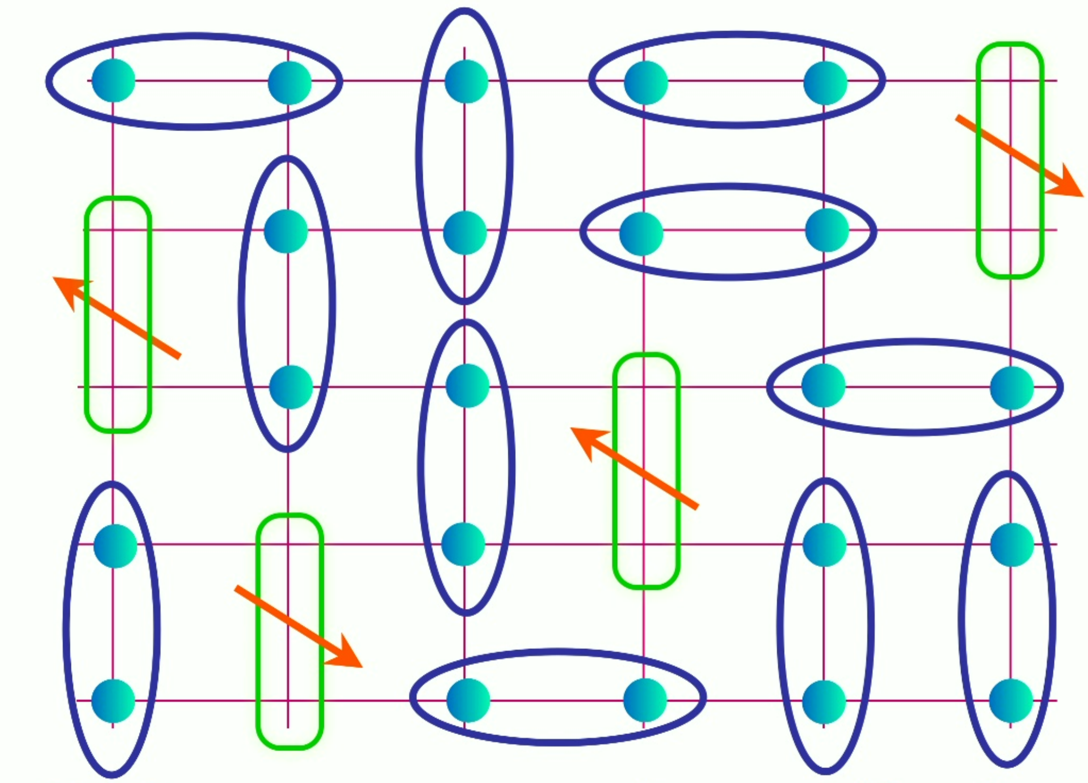


$= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Spin liquid with density *p* of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

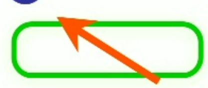
# FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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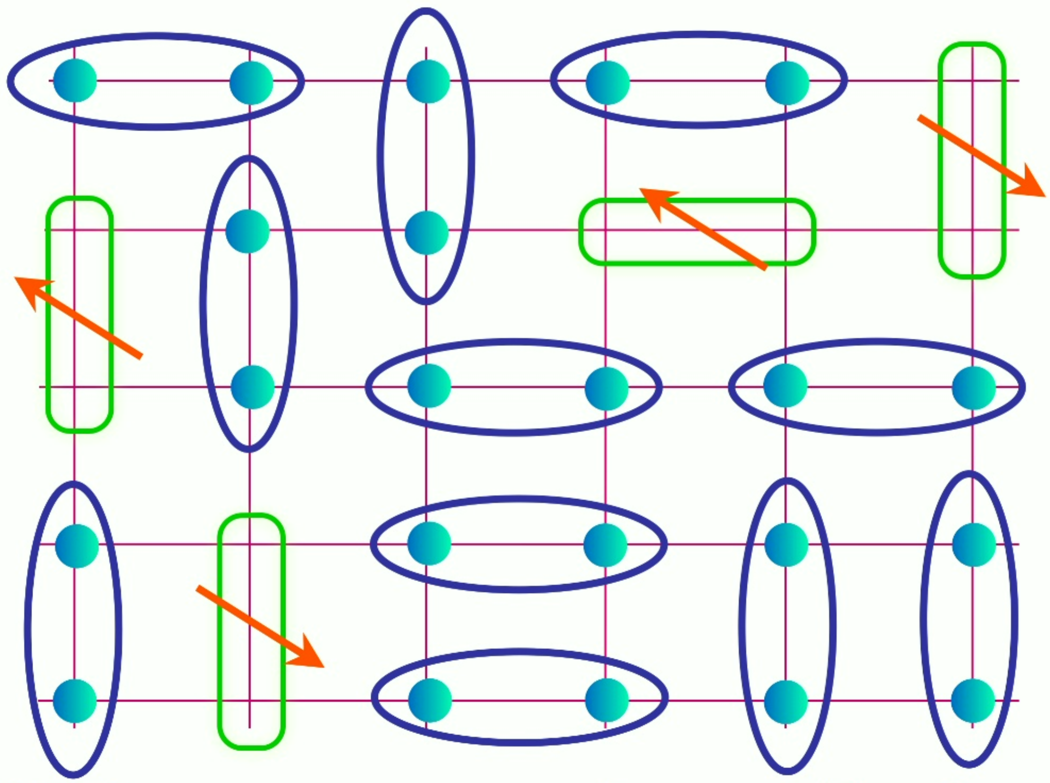
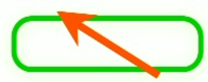


Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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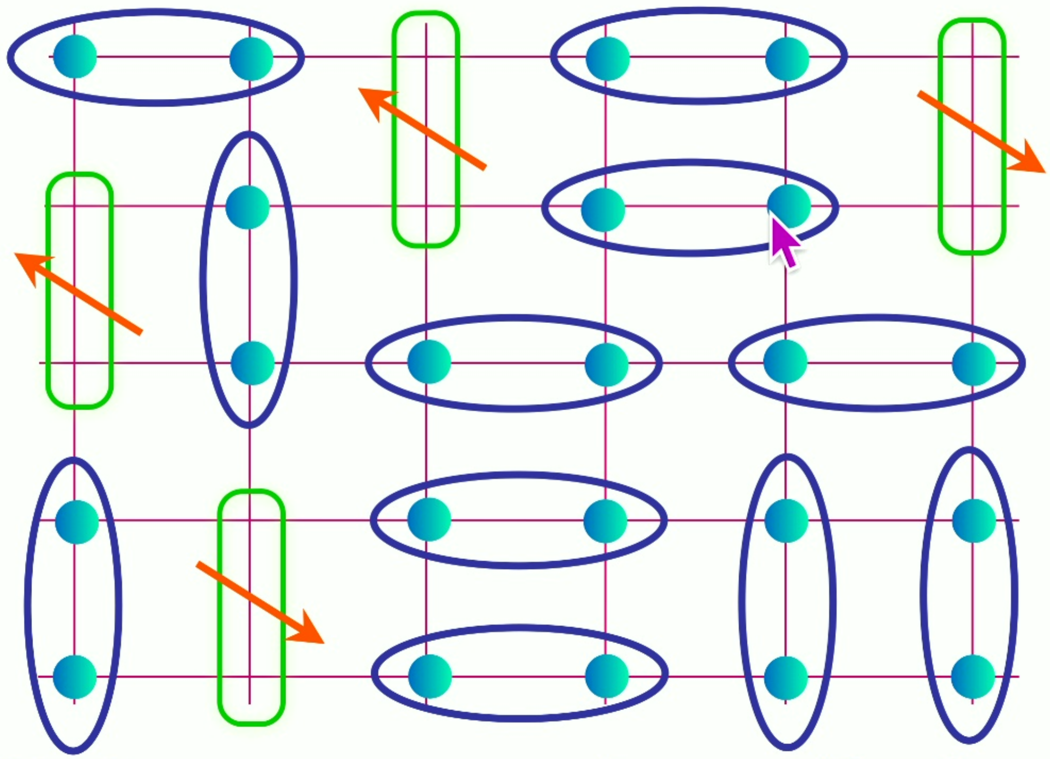

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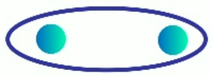
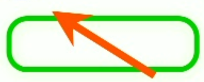


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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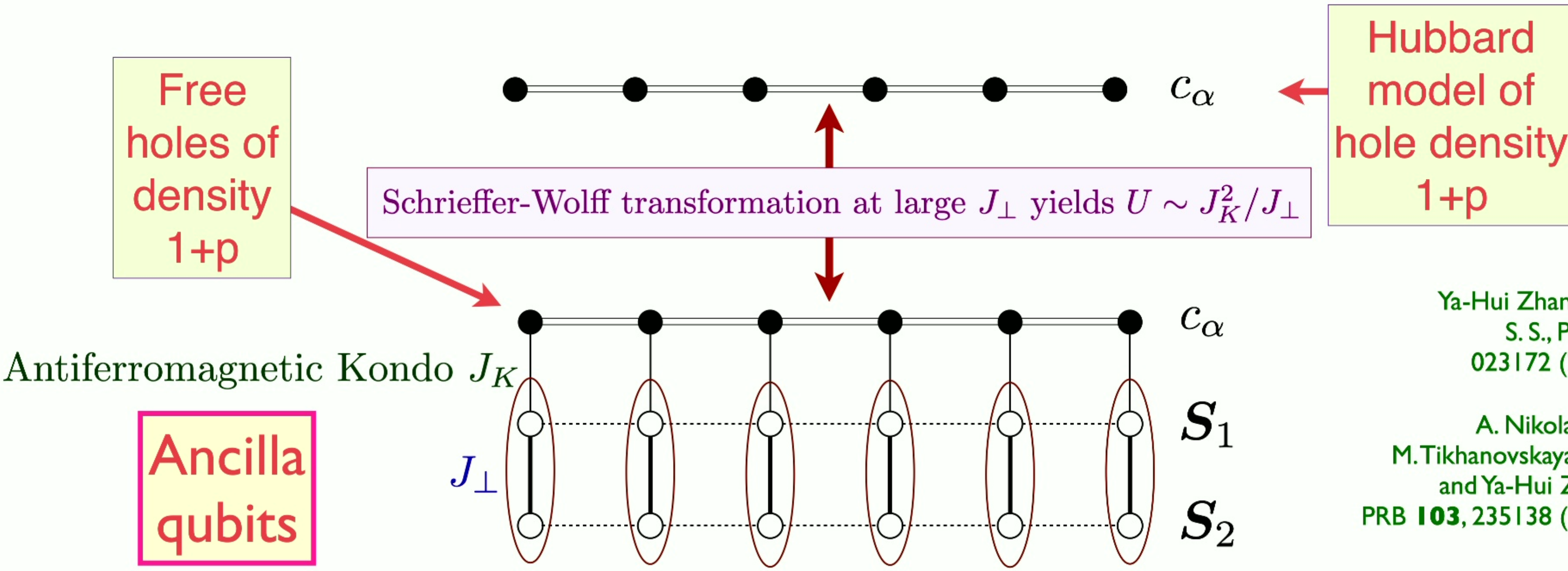
# Ancilla theory of the Hubbard model

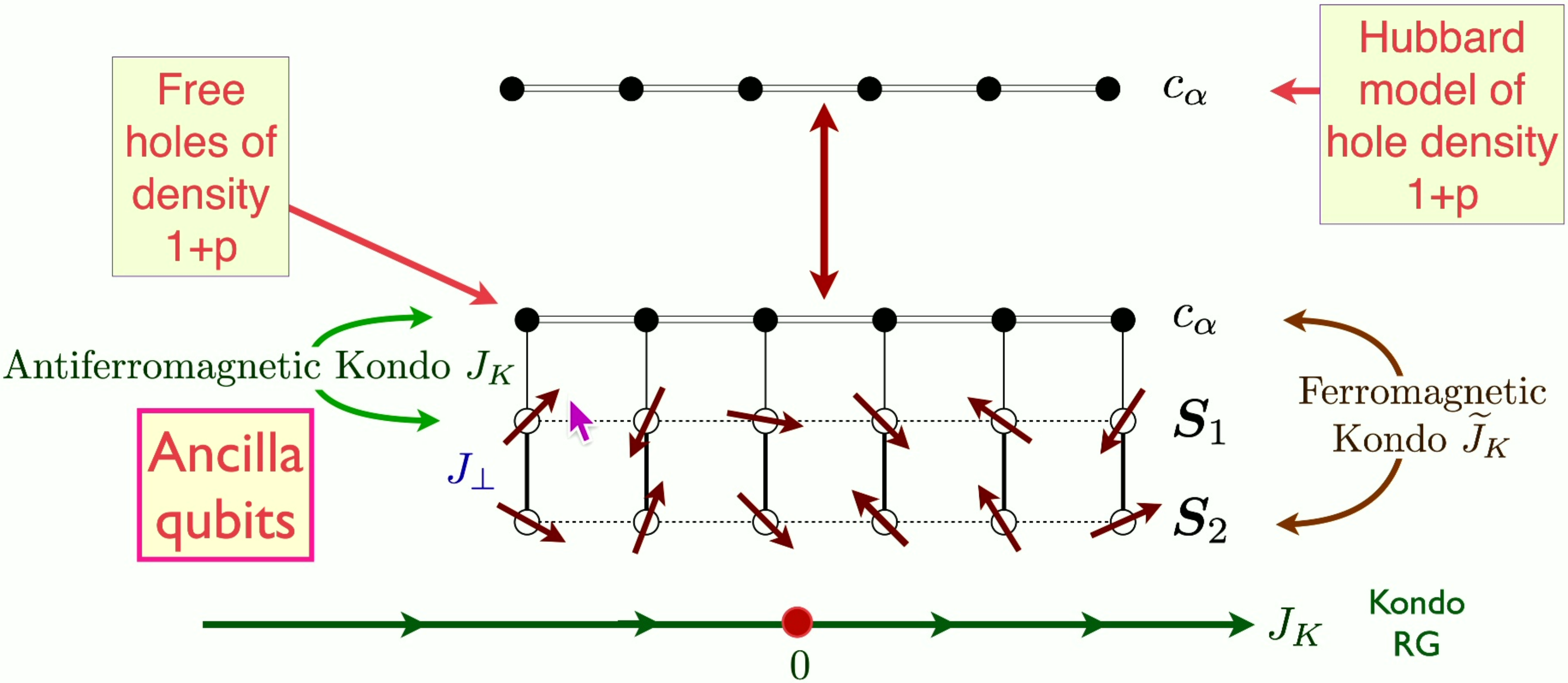



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

# Ancilla theory of the Hubbard model




$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

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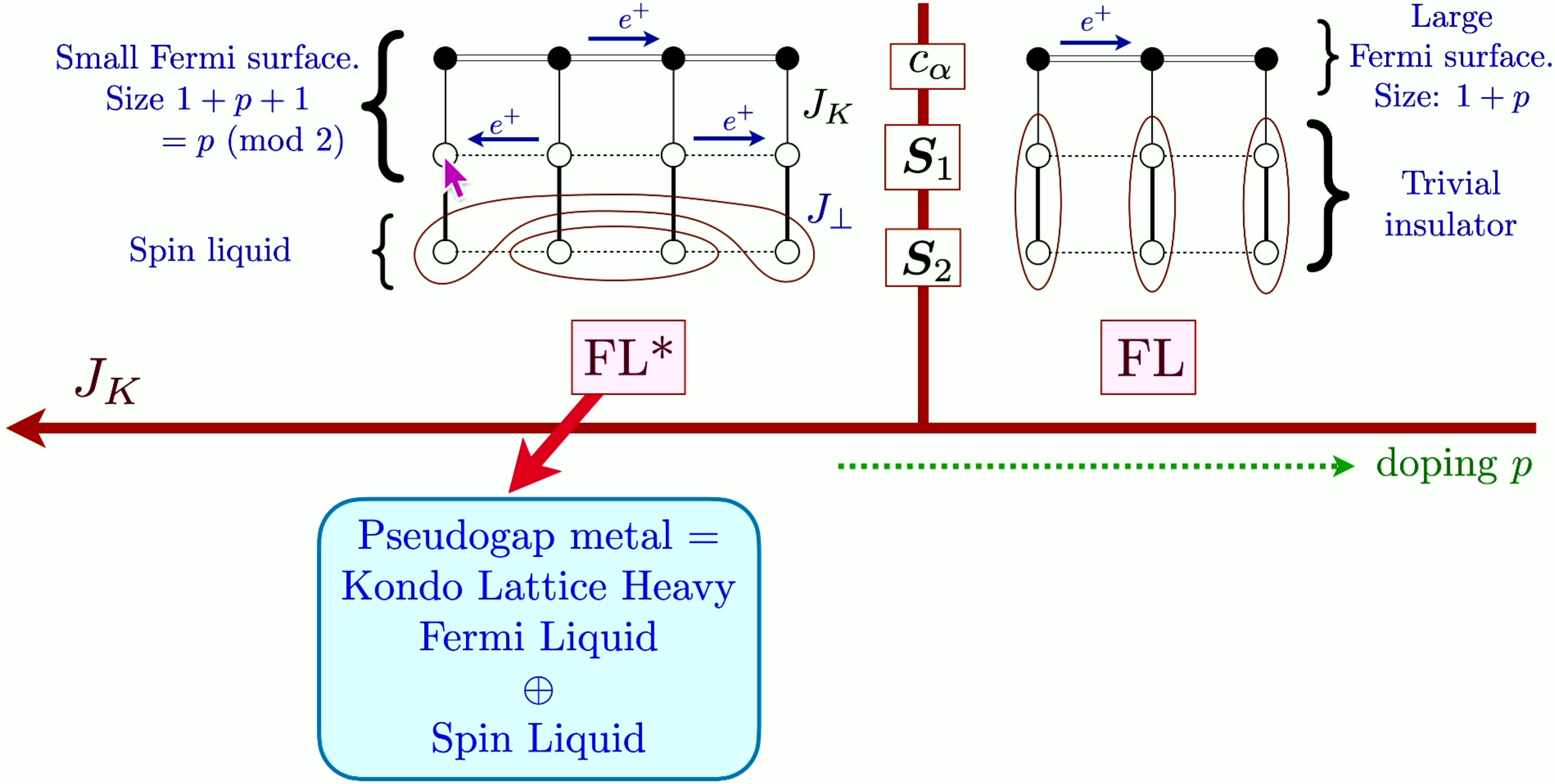

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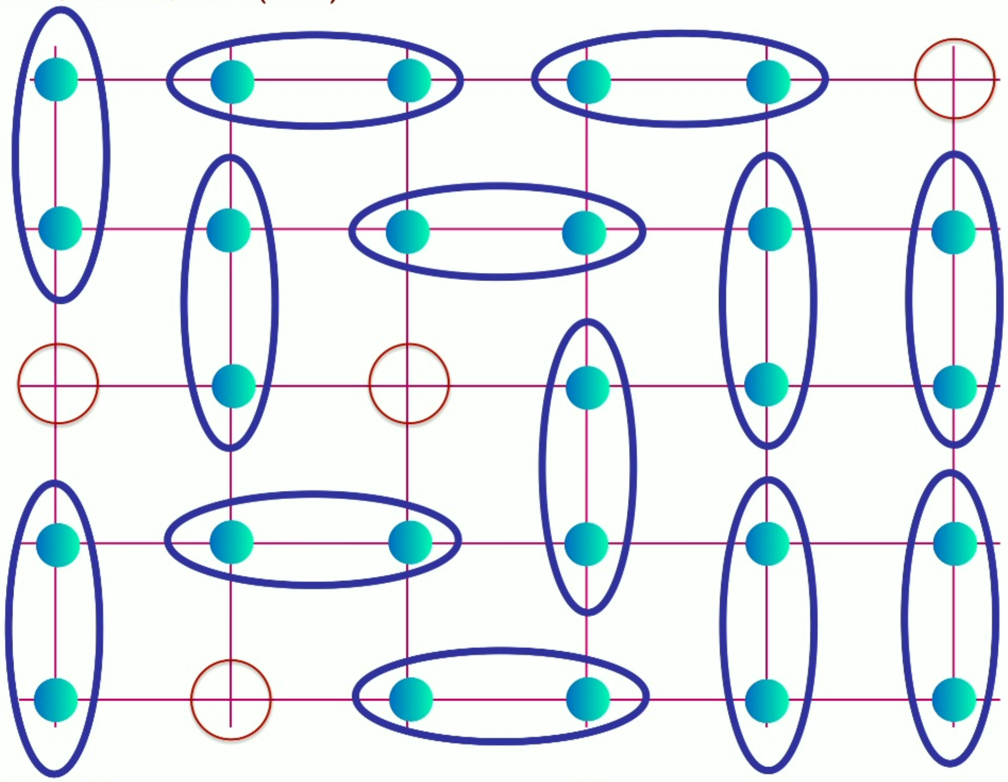
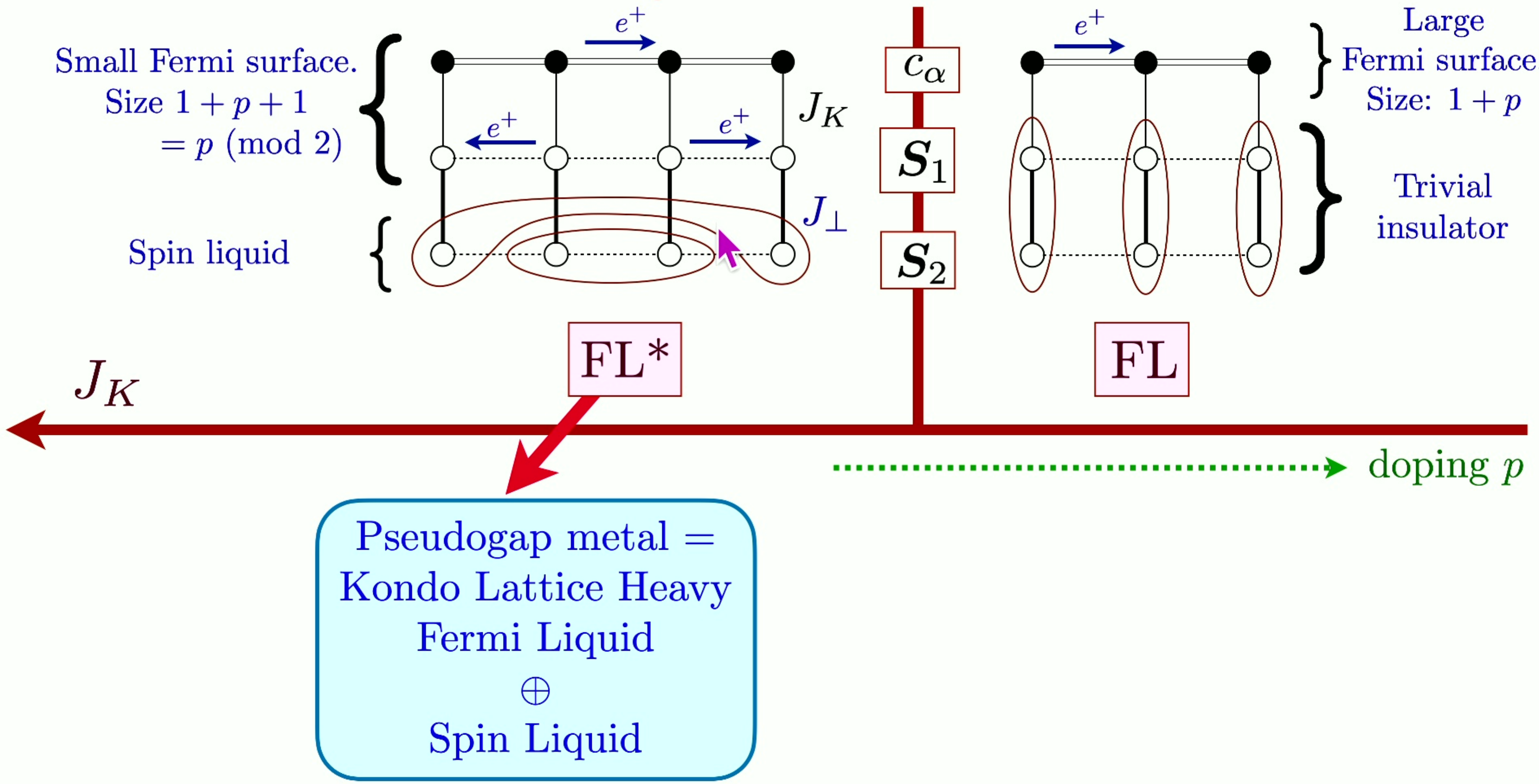


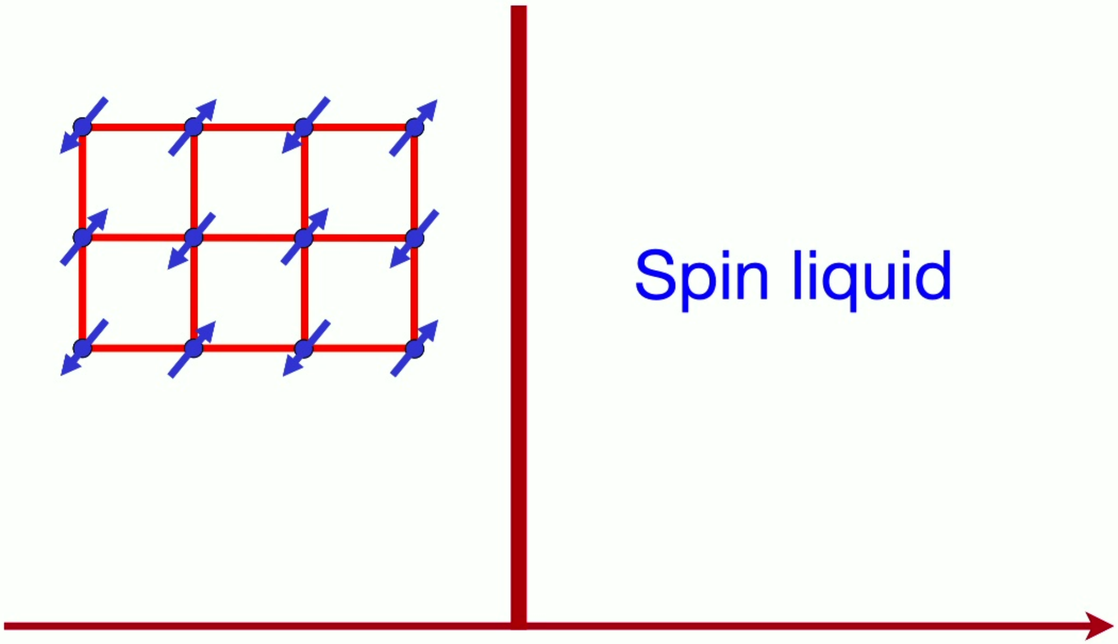
Spin liquid with density *p* of spinless, charge +e "holons".

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

# Ancilla theory of the Hubbard model



# Insulating $S$=1/2 antiferromagnet



Spin liquid

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

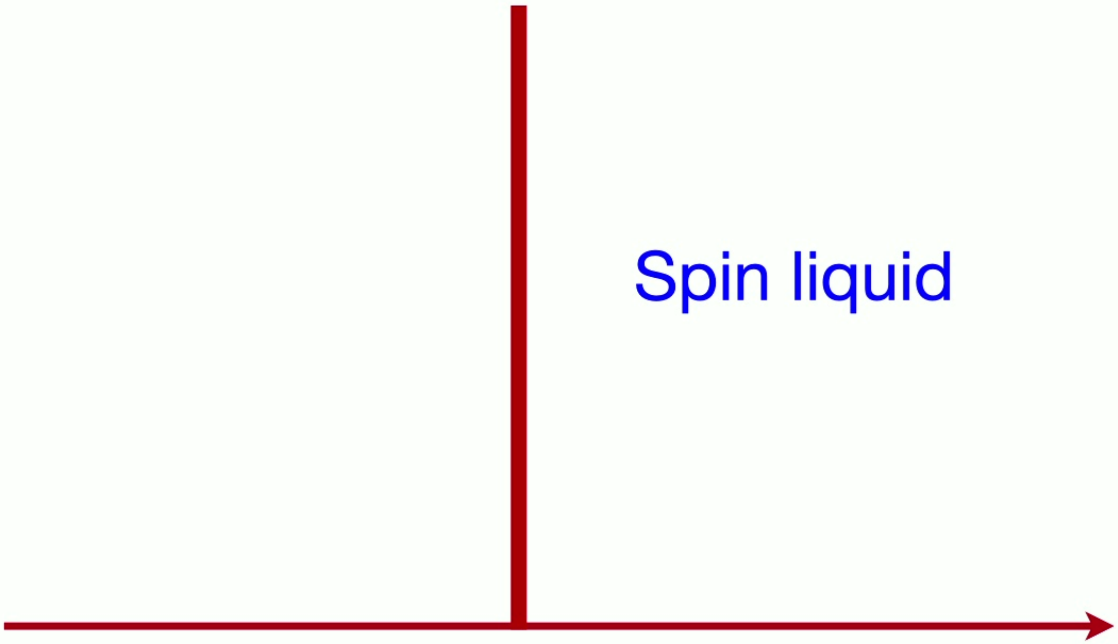
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, PRB **38**, 316 (1988)

# Insulating S=1/2 antiferromagnet




$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

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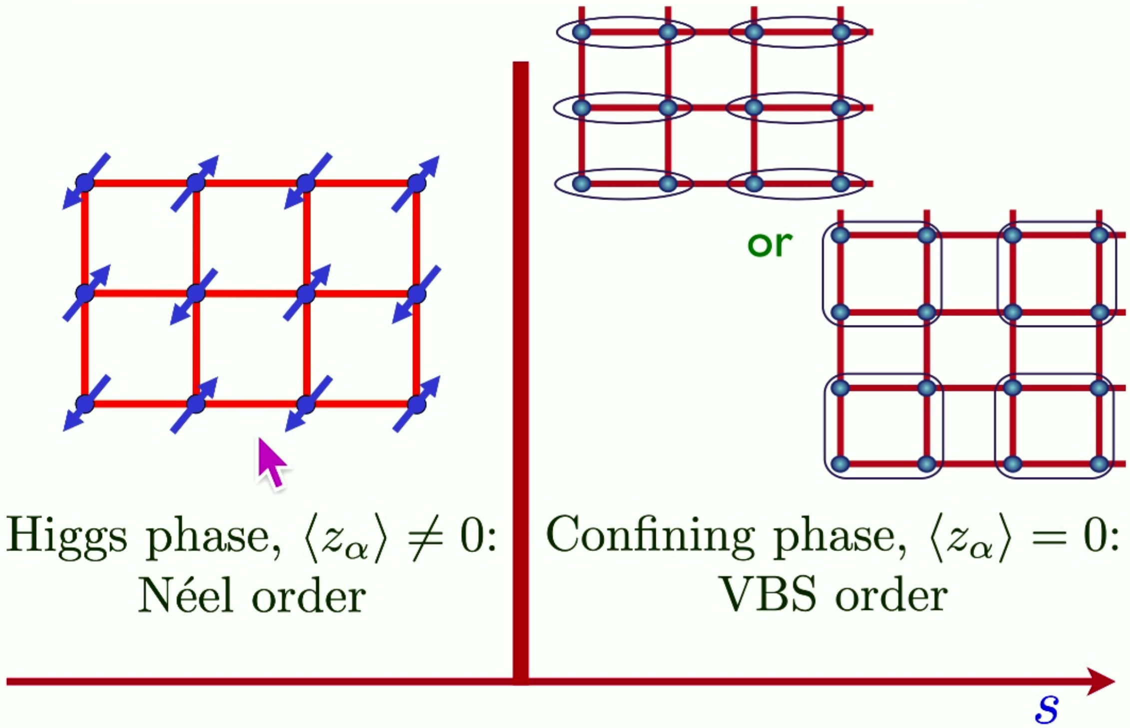
$\pi$-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

# Insulating S=1/2 antiferromagnet

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Mean-field spin liquid
with gapped bosonic spinons.

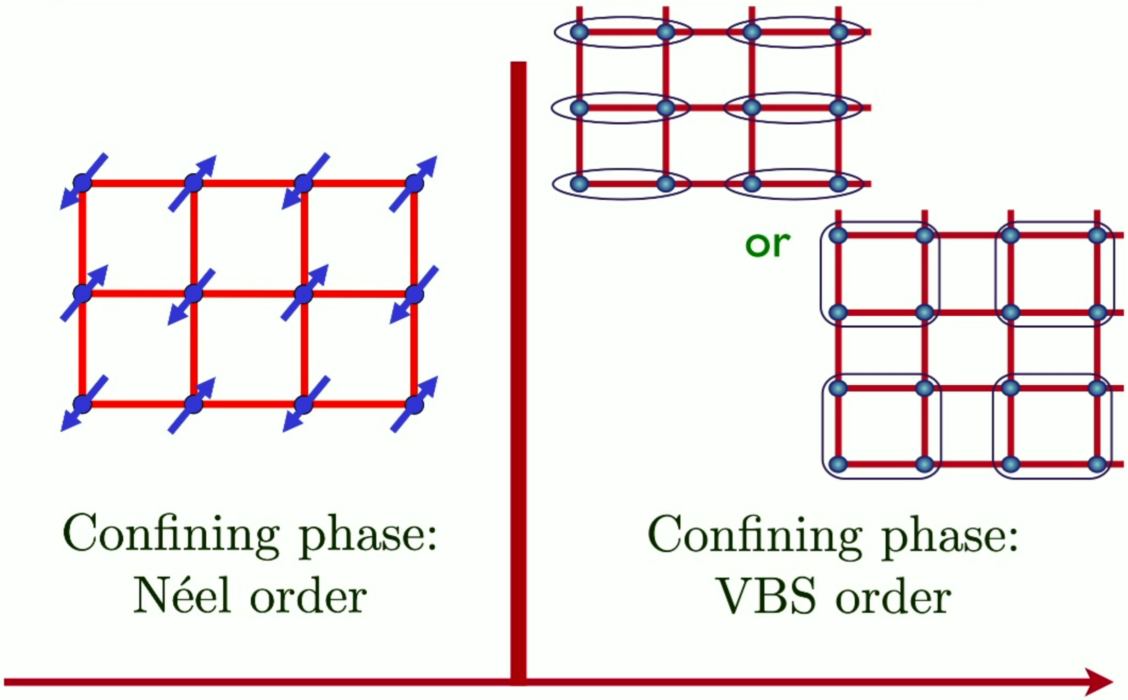
Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\rm monopole}$$

N. Read and S. S., PRL **62**, 1694 (1989)

# Insulating S=1/2 antiferromagnet




$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$$

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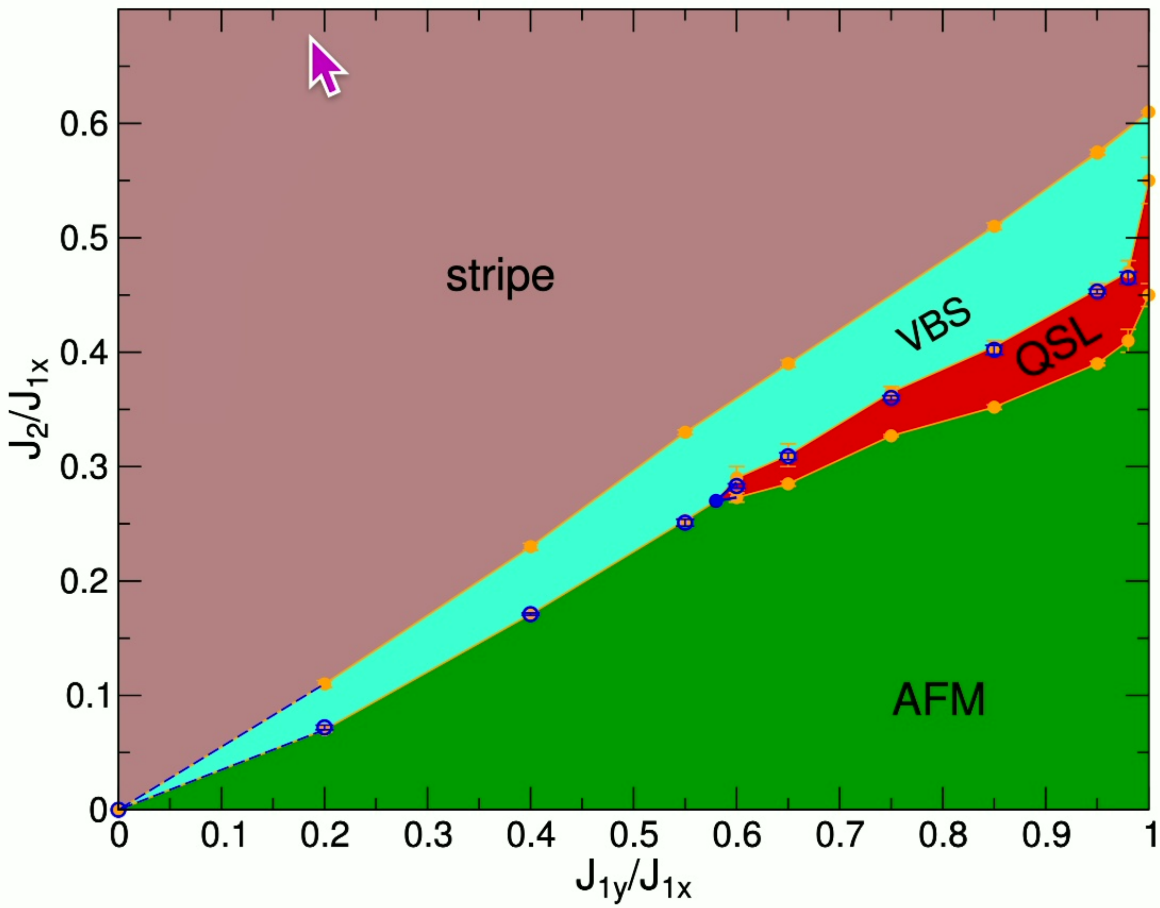
**Dual to $\mathbb{CP}^1$ U(1) gauge theory.**

A. Tanaka and X. Hu, PRL **95**, 036402 (2005); T. Senthil and M.P.A. Fisher PRB **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Physical Review X **7**, 031051 (2017)

$$H = J_{1x} \sum_{\langle i,j \rangle_x} \mathbf{S_i} \cdot \mathbf{S_j} + J_{1y} \sum_{\langle i,j \rangle_y} \mathbf{S_i} \cdot \mathbf{S_j} + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S_i} \cdot \mathbf{S_j}.$$




Wen-Yuan Liu,
Shou-Shu Gong,
Wei-Qiang Chen,
and Zheng-Cheng Gu,
arXiv:2212.00707

# High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

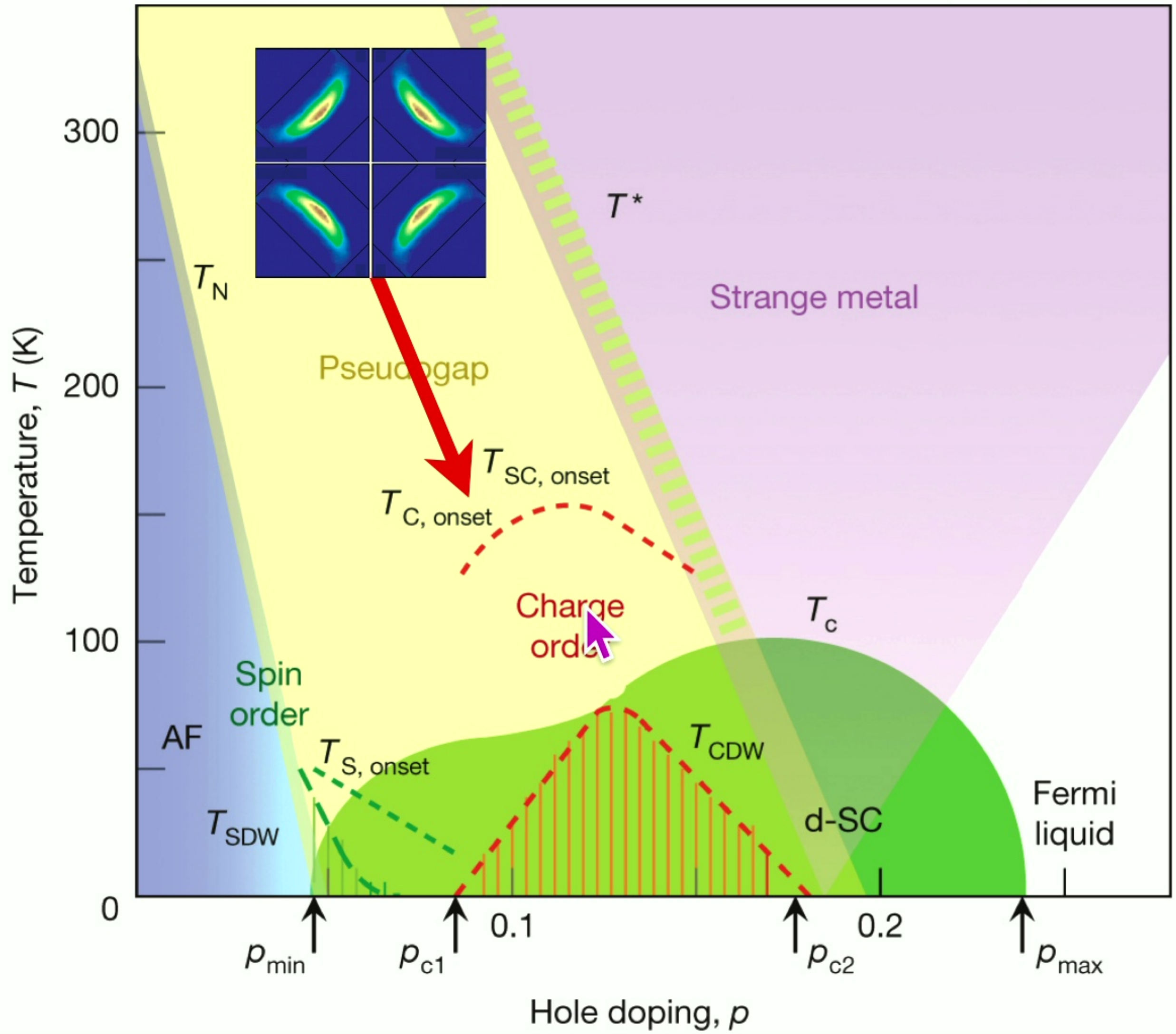
We have performed density-matrix renormalization group studies of a square lattice $t$-$J$ model with small hole doping, $\delta \ll 1$, on long four and six-leg cylinders. We include frustration in the form of a second-neighbor exchange coupling, $J_2 = J_1/2$, such that the undoped ($\delta = 0$) "parent" state is a quantum spin liquid. In contrast to the relatively short range superconducting (SC) correlations that have been observed in recent studies of the six-leg cylinder in the absence of frustration, we find power-law SC correlations with a Luttinger exponent, $K_{SC} \approx 1$, consistent with a strongly diverging SC susceptibility, $\chi \sim T^{-(2-K_{SC})}$ as the temperature $T \to 0$. The spin-spin correlations—as in the undoped state—fall exponentially suggesting that the SC "pairing" correlations evolve smoothly from the insulating parent state.

DOI: 10.1103/PhysRevLett.127.097002

**Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders**
**Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, arXiv:2303.11633**

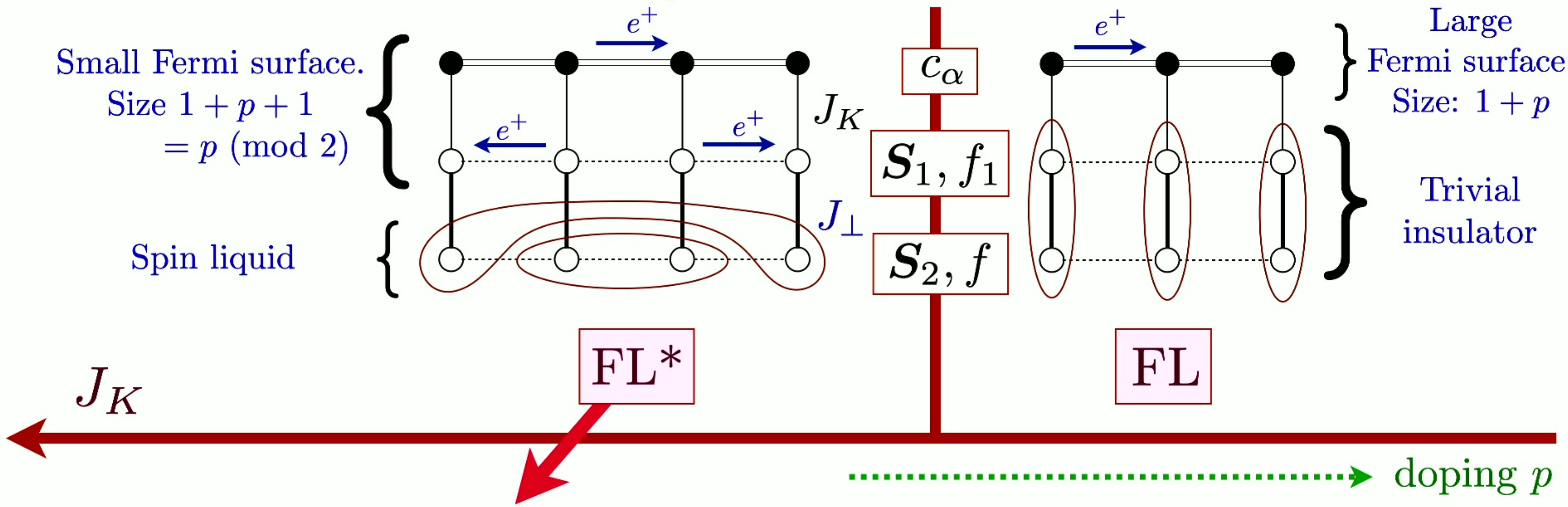
Superconductivity in doped quantum paramagnets has been a subject of long theoretical inquiry. In this work we report a density matrix renormalization group study of lightly doped $t$-$J$ models on the square lattice (doped hole densities $\delta = 1/12$ and $1/8$) with parameters for which previous studies have suggested that the undoped system in 2D is either a quantum spin liquid or a valence bond crystal. Our studies are performed on cylinders with width up to 8. Ground-state correlations are found to be nearly identical for the "doped quantum spin liquid" and "doped valence bond crystal". Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

B Keimer *et al.* *Nature* **518**, 179-186 (2015)




A theory for the confinement of fractionalized excitations in the Arovas-Auerbach-Affleck-Marston $\pi$-flux spin liquid (which is dual to the $\mathbb{CP}^1$ spin liquid) from electrically charged excitations.

# Ancilla theory of the Hubbard model

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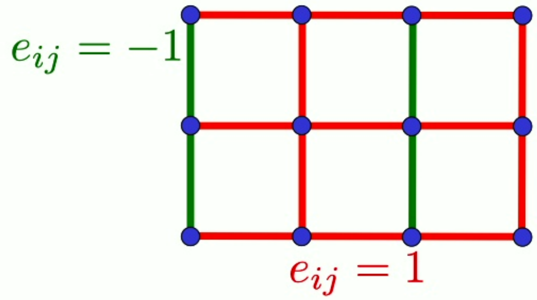
Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

## Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the $\pi$-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left( f^\dagger_{i\alpha} f_{j\alpha} - f^\dagger_{j\alpha} f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left( \Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right); \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f^\dagger_{i\downarrow} \end{pmatrix}$$

$H_f$ is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; $U_{ij}$ is the SU(2)$_N$ gauge field.




- The nearest-neighbor effective Hamiltonian for charge $e$, SU(2)$_N$ fundamental boson $B_i$ is constrained by the fact that the composite of $B_i$ and $\Psi_i$ is an electron:

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left( B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \ldots$$

# Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} |\Delta_{\boldsymbol{i}\boldsymbol{j}}|^2$$
$$+ J_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} Q_{\boldsymbol{i}\boldsymbol{j}}^2 + K_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} J_{\boldsymbol{i}\boldsymbol{j}}^2 .$$

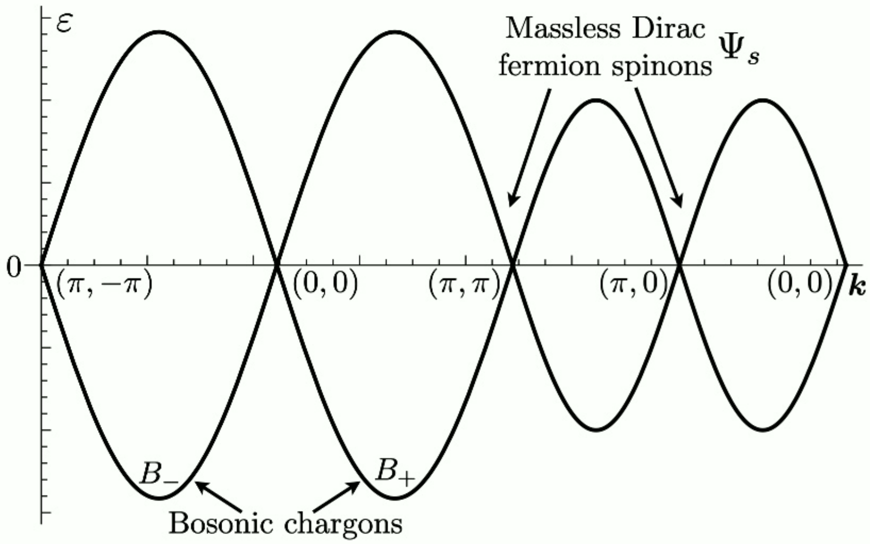
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

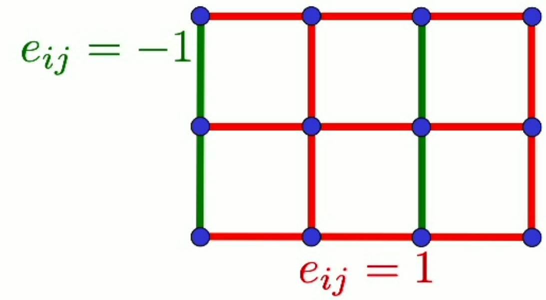
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{i}\boldsymbol{j}} = Q_{\boldsymbol{j}\boldsymbol{i}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}}\right)$

bond current: $i\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{i}\boldsymbol{j}} = -J_{\boldsymbol{j}\boldsymbol{i}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}}\right)$

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{i}\boldsymbol{j}} = \Delta_{\boldsymbol{j}\boldsymbol{i}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{b\boldsymbol{j}}$ .

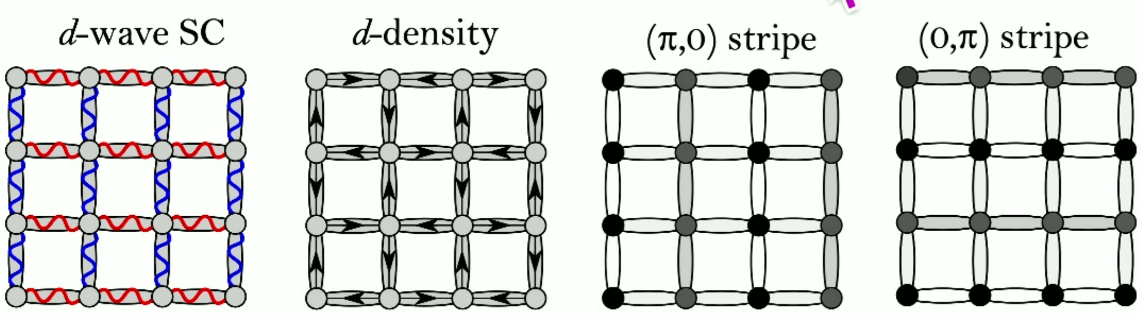
# Confinement of SU(2)$_N$ gauge theory by charge fluctuations




SU(2)$_N$ gauge-invariant and SU(2) spin invariant order parameters of Higgs phases:

$$
\begin{aligned}
x\text{-CDW}: &\quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: &\quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: &\quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: &\quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}
$$




The $\mathcal{O}(B^2_{a\pm})$ terms in the energy have a SO(5)$_b$ rotation symmetry between these orders.

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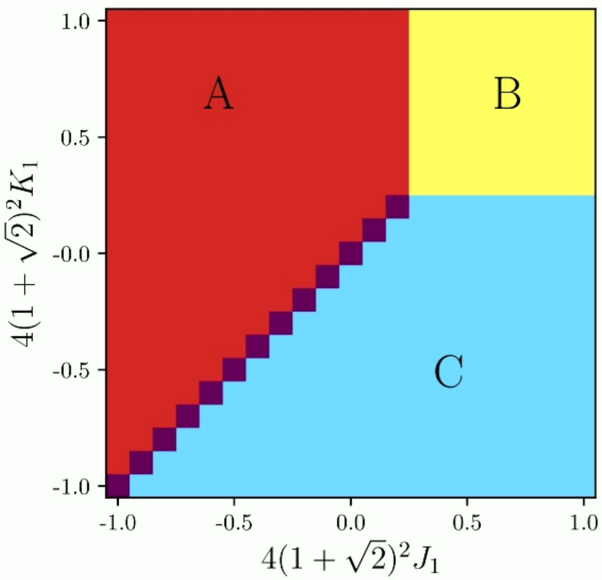
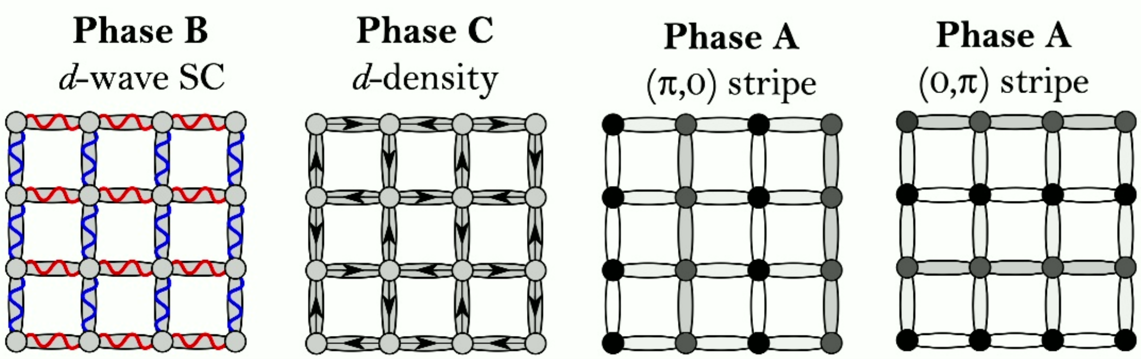
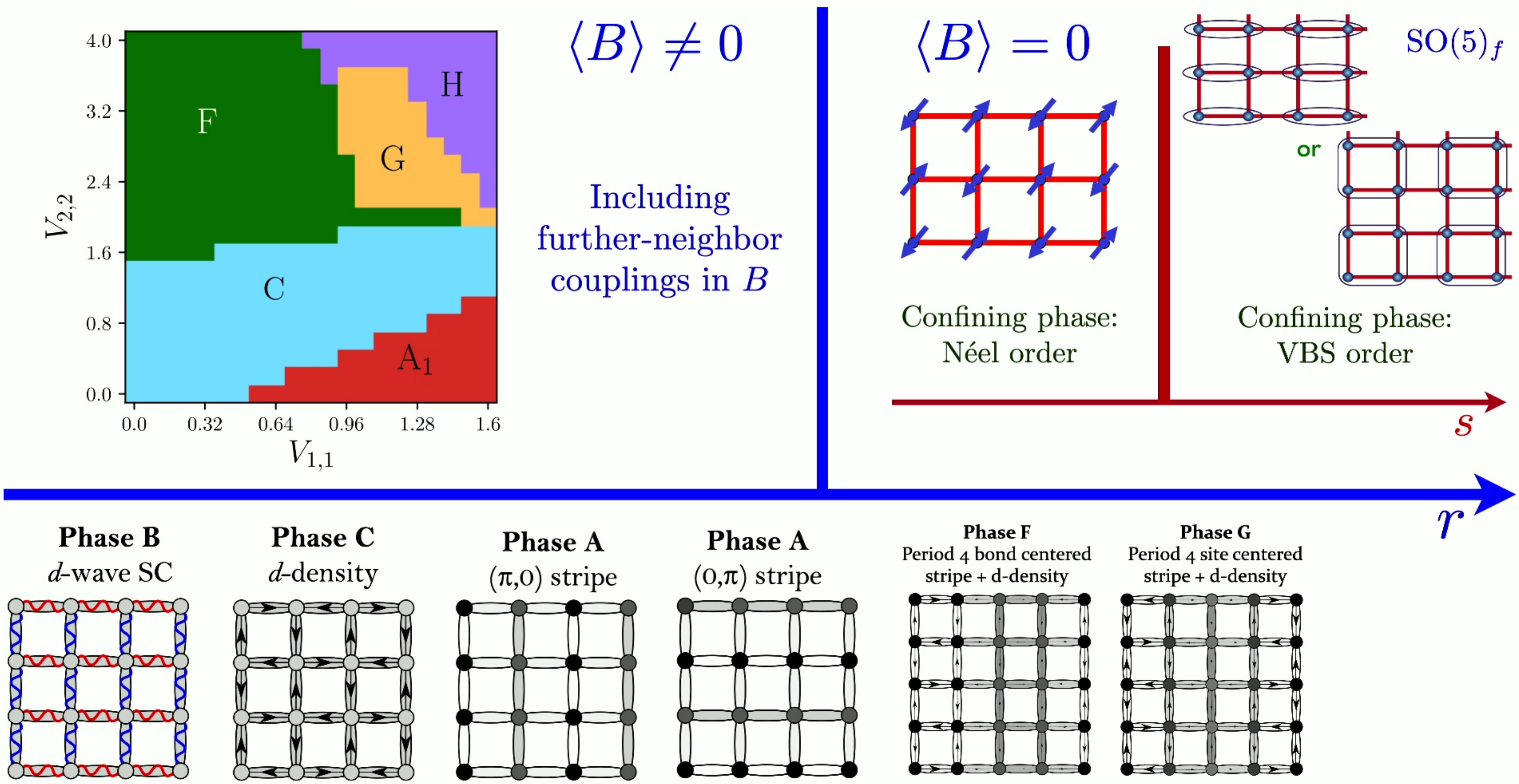



$$\langle B \rangle \neq 0$$



# Global phase diagram of SU(2)$_N$ gauge theory



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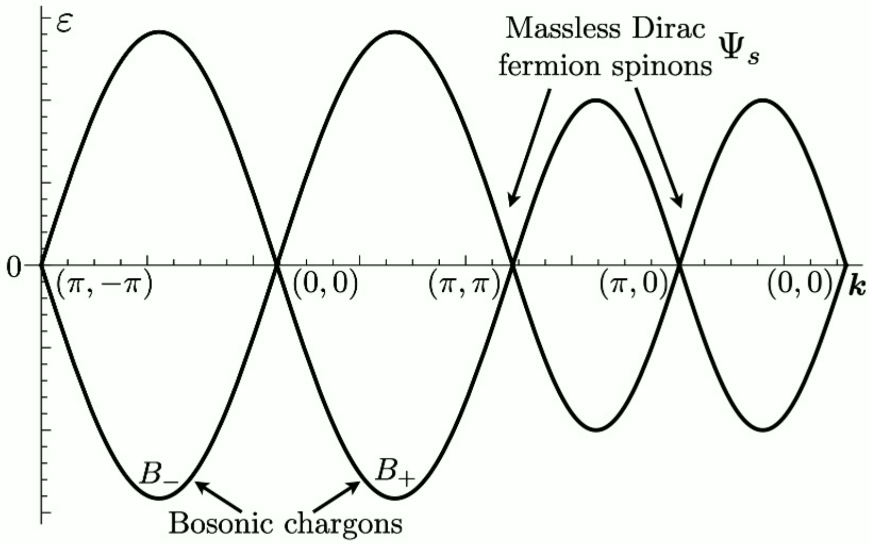


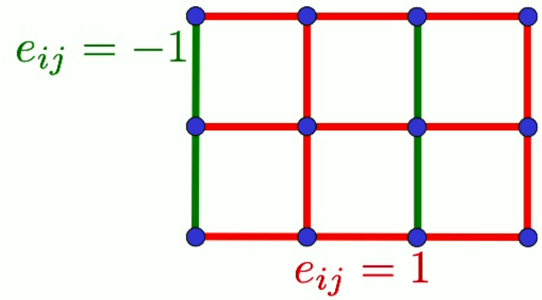


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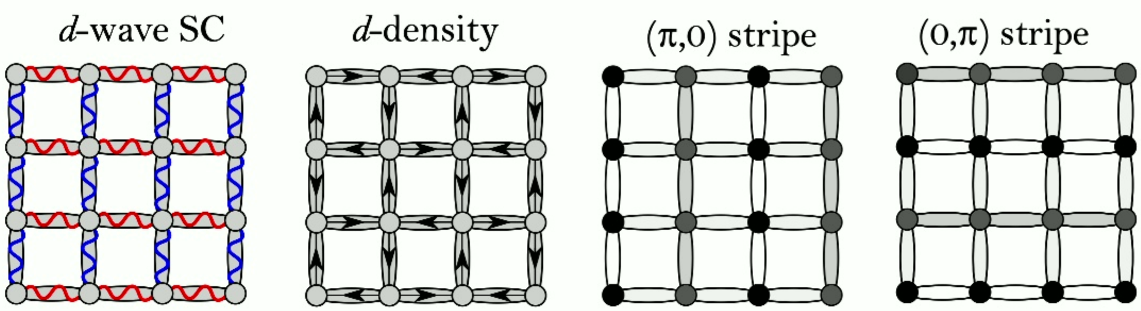


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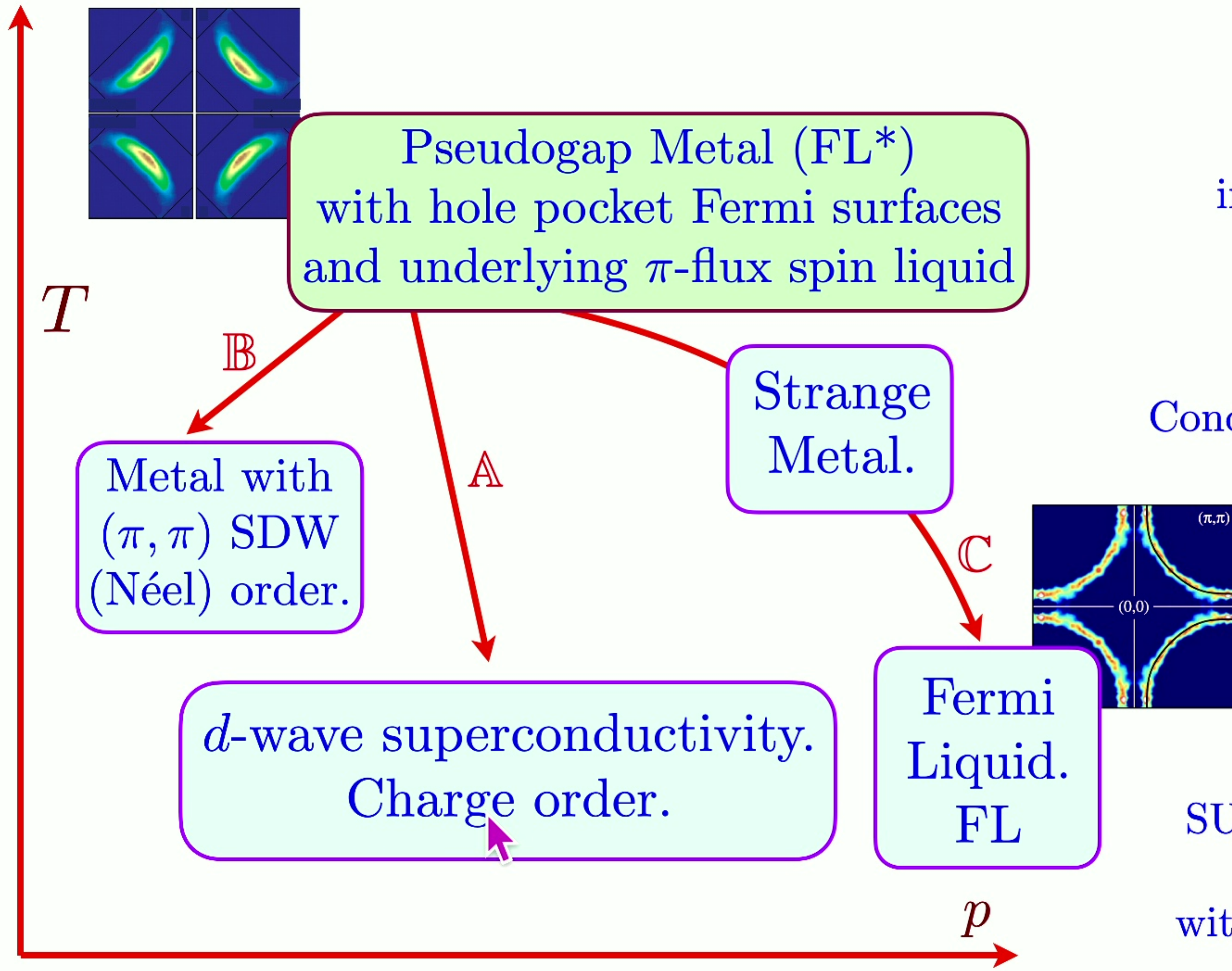


<u>Arrow $\mathbb{A}$</u>

Condensation of B
in $SU(2)_N$ gauge theory.

<u>Arrow $\mathbb{B}$</u>

Condensation of $z_\alpha$ in dual $\mathbb{CP}^1$
U(1) gauge theory.

<u>Arrow $\mathbb{C}$</u>

$SU(2)_s \times U(1)_a$ gauge theory
in ancilla model
with Higgs field $H_{\alpha a} \sim c_\alpha^\dagger f_{1a}$

## Can the underlying spin liquid help understand experimental observations of spin dynamics ?

# Spin fluctuations associated with the collapse of the pseudogap in a cuprate superconductor

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M. Zhu[1], D. J. Voneshen[2,3], S. Raymond[4], O. J. Lipscombe[1], C. C. Tam[1,5] & S. M. Hayden[1]

Theories of the origin of superconductivity in cuprates depend on an understanding of their normal state, which exhibits various competing orders. Transport and thermodynamic measurements on $La_{2-x}Sr_xCuO_4$ show signatures of a quantum critical point and the associated fluctuations, including a peak in the electronic specific heat versus doping, near the doping $p^*$ where the pseudogap collapses. The fundamental nature of these quantum fluctuations is unclear. Here we use inelastic neutron scattering to show that, close to the superconducting critical temperature and near $p^*$, there are very-low-energy collective spin excitations with characteristic energies of ~5 meV. Cooling and applying a magnetic field creates a mixed state with a stronger magnetic response below 10 meV. We conclude that the low-energy spin fluctuations are due to the collapse of the pseudogap combined with an underlying tendency to magnetic order. We show that the large specific heat near $p^*$ can be understood in terms of collective spin fluctuations. The spin fluctuations we measure exist across the superconducting phase diagram and may be related to the strange metal behaviour observed in overdoped cuprates.

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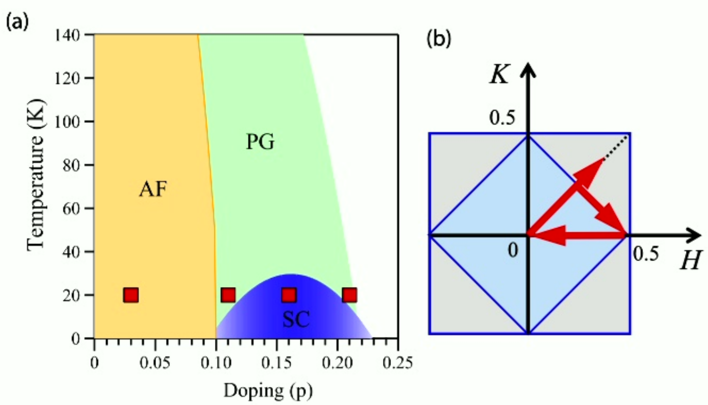



FIG. 1. (a) Schematic temperature-doping phase diagram of $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$. It shows the antiferromagnetic (AF), superconducting (SC), and the pseudogap (PG) regions. Here we study four doping levels as indicated by the solid red squares. (b) 2D reciprocal lattice for the pseudotetragonal structure and the first Brillouin zones (structural in light grey, magnetic in light blue). Coordinates $H$ and $K$ are in r.l.u.. The path followed for the measurements is indicated by the red arrows, starting at (0.25,0.25) and ending around (0.30,0.30) via (0.5,0) and (0,0).

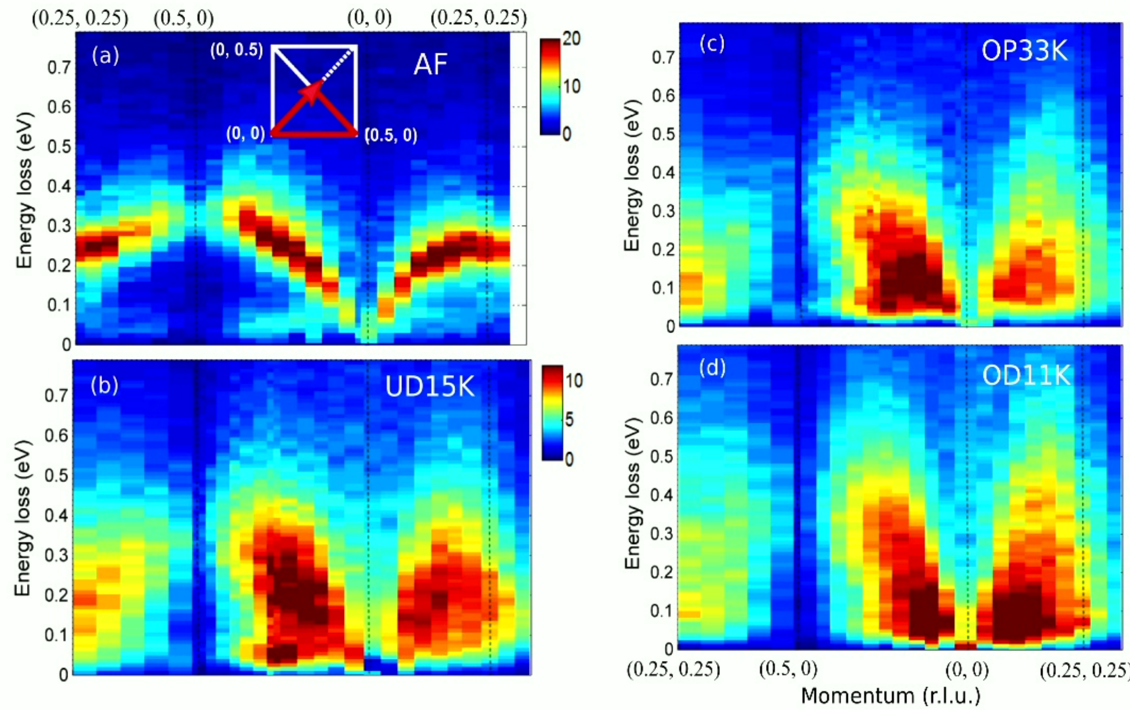
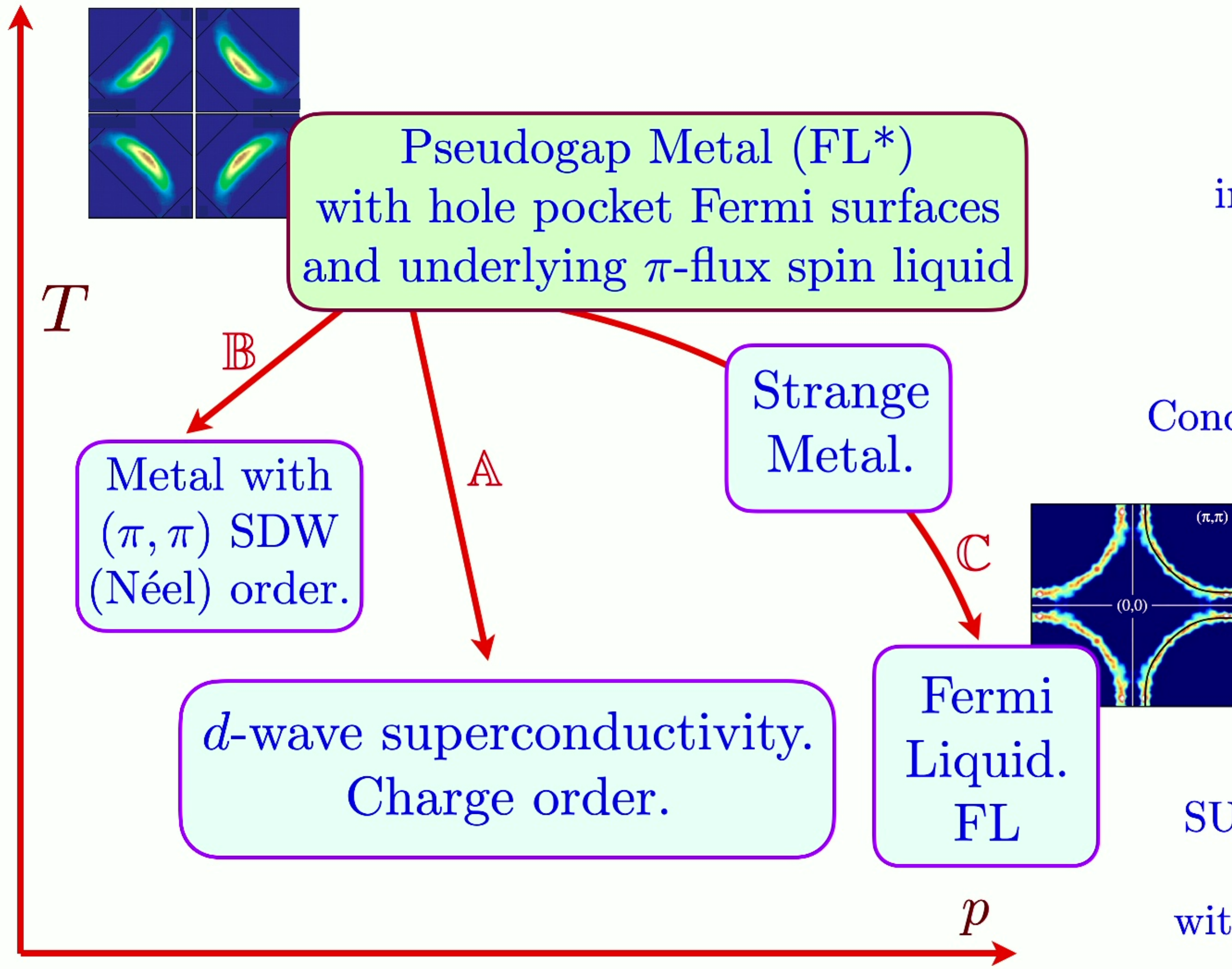



FIG. 2. Energy/momentum intensity maps of RIXS spectra for (a) AF ($p \simeq 0.03$), (b) UD15K ($p \simeq 0.11$), (c) OP33K ($p \simeq 0.16$), and (d) OD11K ($p \simeq 0.21$) along the high-symmetry momentum trajectory indicated in Fig. 1(b) and in the inset of (a). The intensity is in unit of photons/s/eV. Data were taken with $\pi$-polarized incident light at 20 K. Elastic peaks were subtracted for a better visualization of the low energy features.

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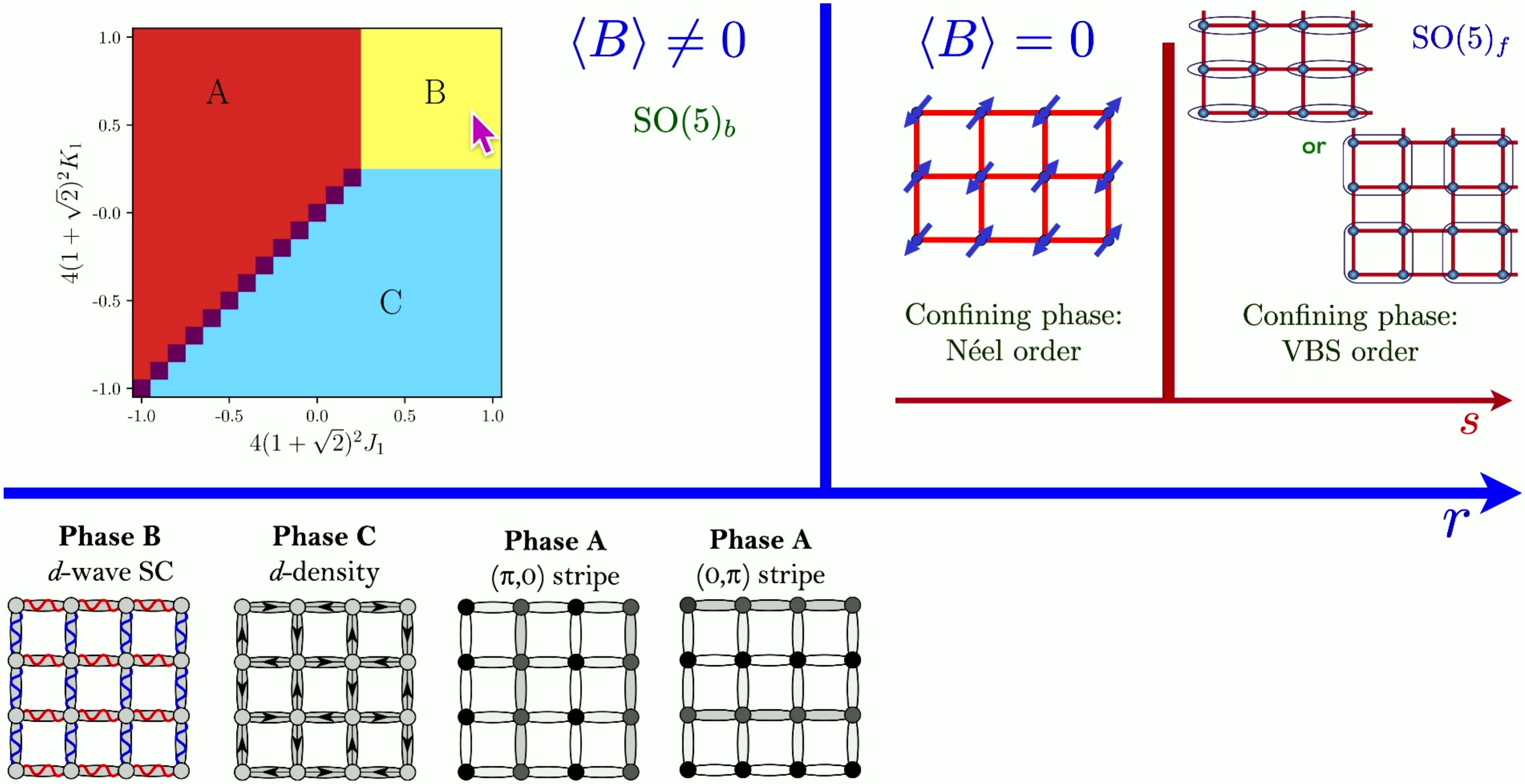

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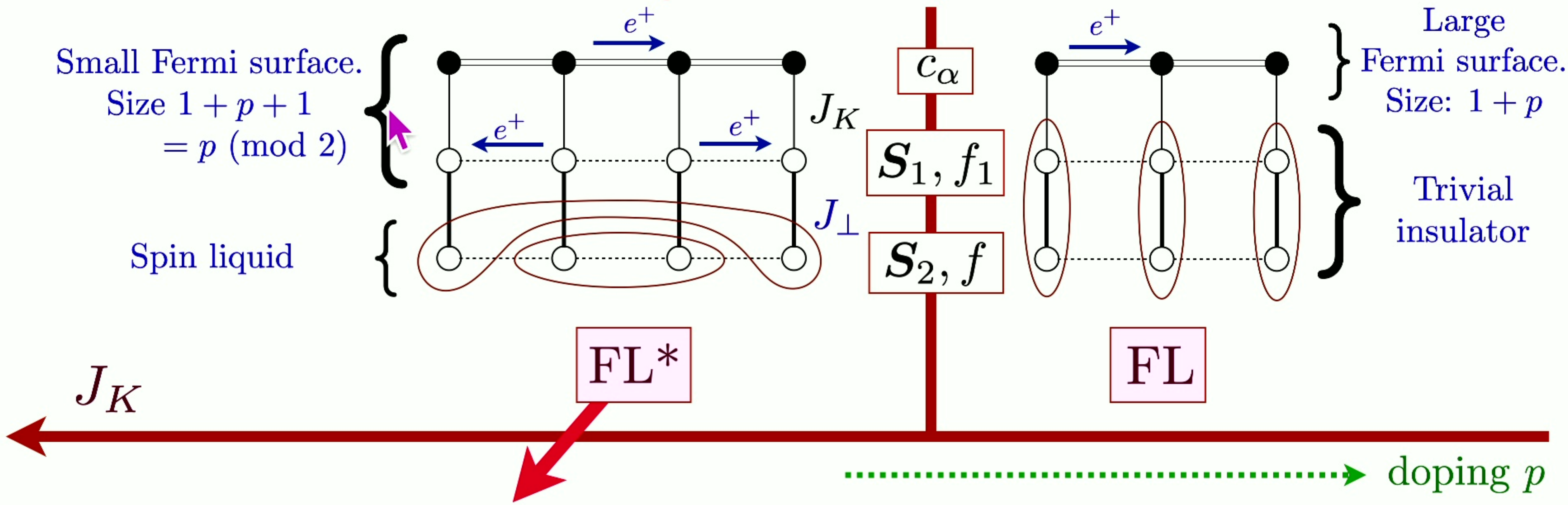


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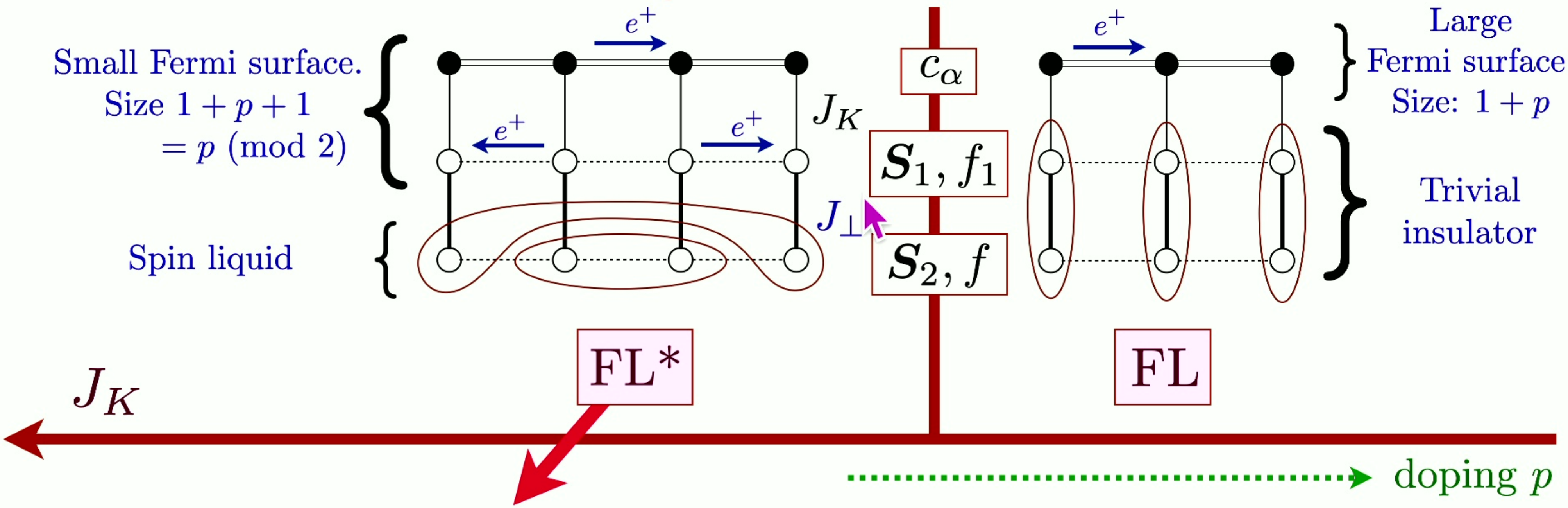


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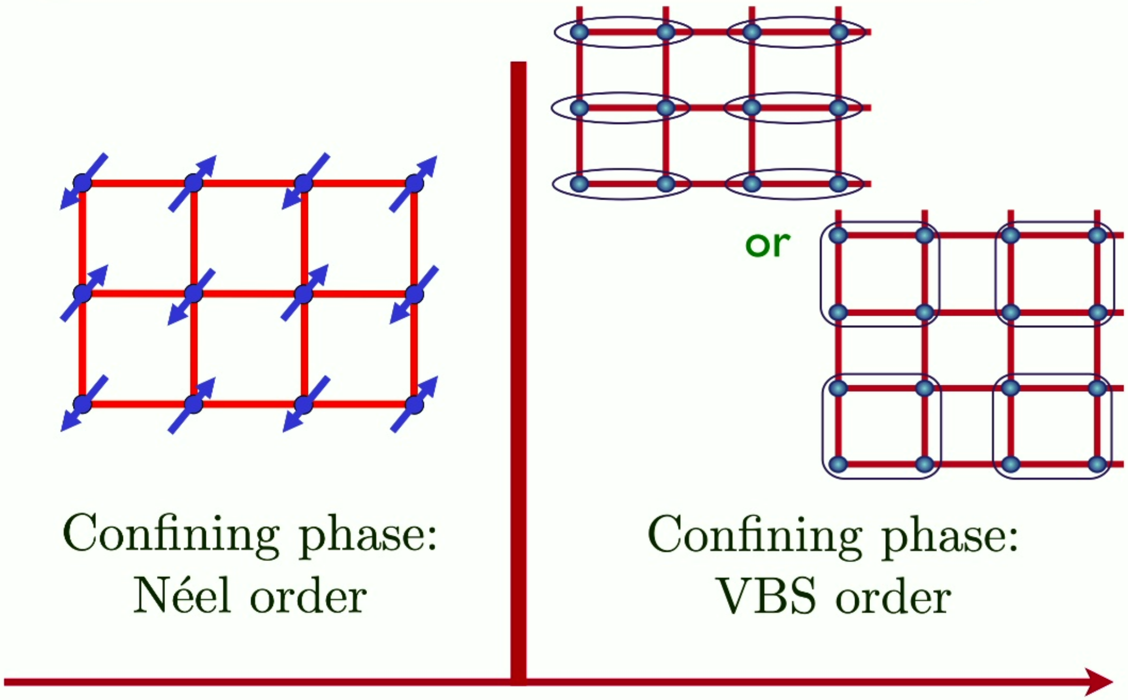
# Insulating S=1/2 antiferromagnet

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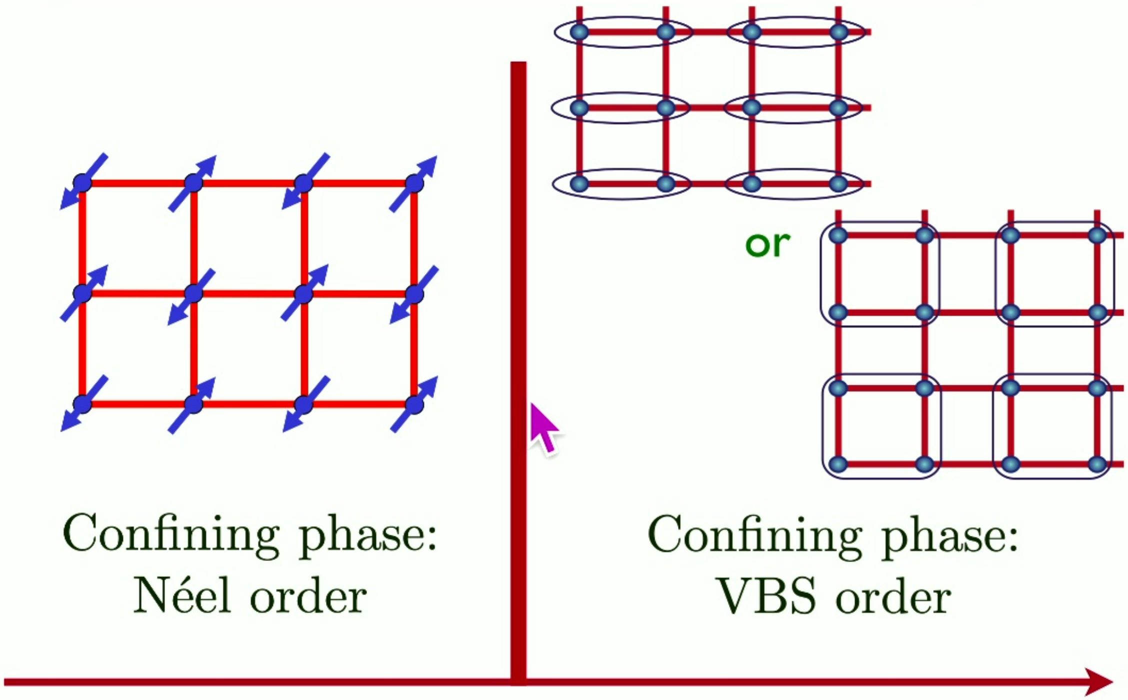


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