Title: Gravitational Observables from Scattering Amplitudes

Speakers: Donal O'Connell

Series: Quantum Fields and Strings

Date: May 02, 2023 - 2:00 PM

URL: https://pirsa.org/23050037

Abstract: Gravity is exciting from both theoretical and observational perspectives. In this talk, I will discuss how gravitational observables, such as waveforms, can be determined from scattering amplitudes in quantum field theory. We can therefore use the full arsenal of theoretical collider physics to compute gravitational waveforms. As an example, I will describe the waveform generated in a scattering process at next-to-leading order. I will finish by discussing how amplitudes can further be used to understand non-radiative aspects of gravity, including the curvature of the Kerr metric itself. This leads to a network of "double copy" relations between classical solutions of the Maxwell and Einstein equations.

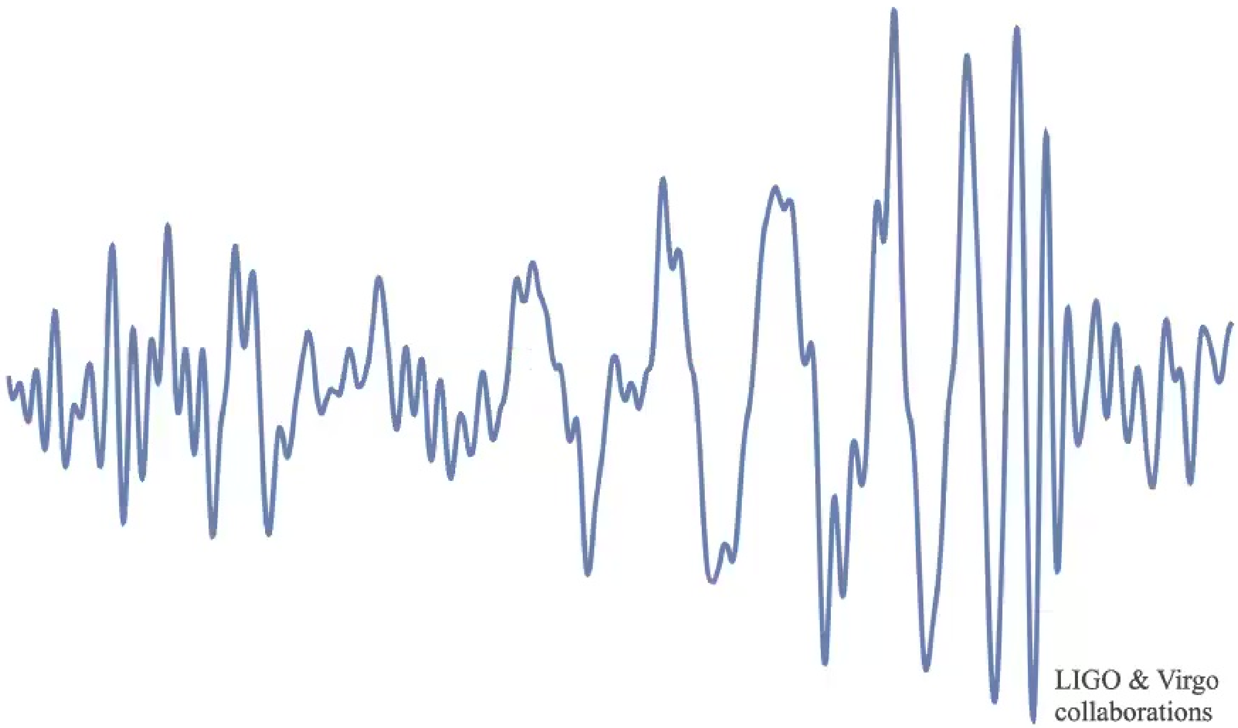
Zoom link: https://pitp.zoom.us/j/92038383246?pwd=RjQwVHo1VWR6VDl0a3VPZEU0ZXFyUT09

Perimeter Institute, May 2023

Gravitational Observables from Scattering Amplitudes

Donal O'Connell
Edinburgh

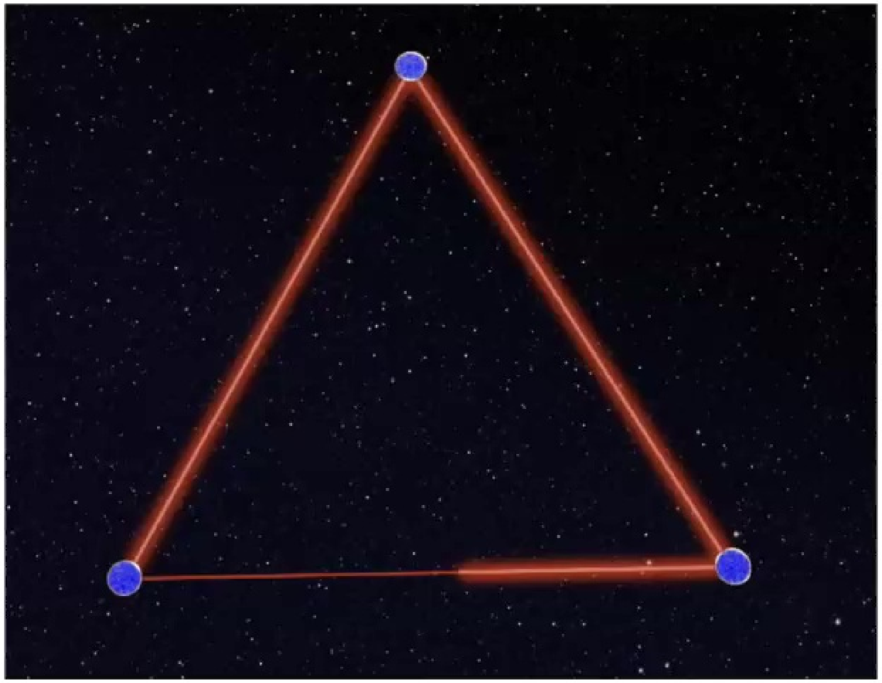
Motivation



Gravity: data rich

Motivation

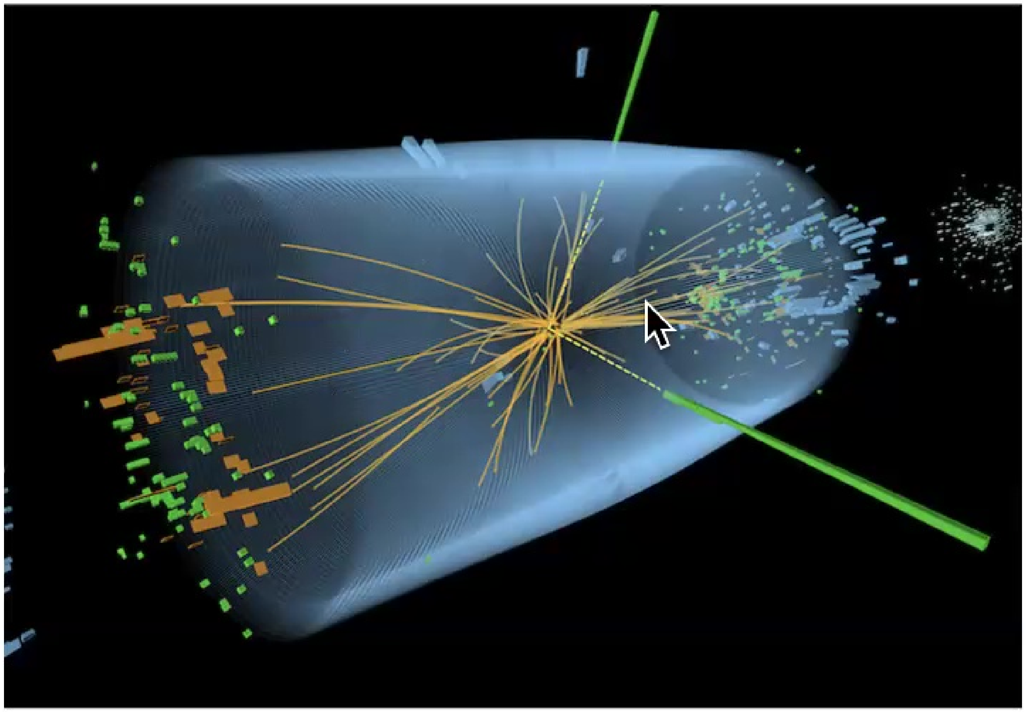
Eg LISA



High-precision gravitational wave theory required, a challenge

Motivation

Another data-rich, high-precision subject: particle physics



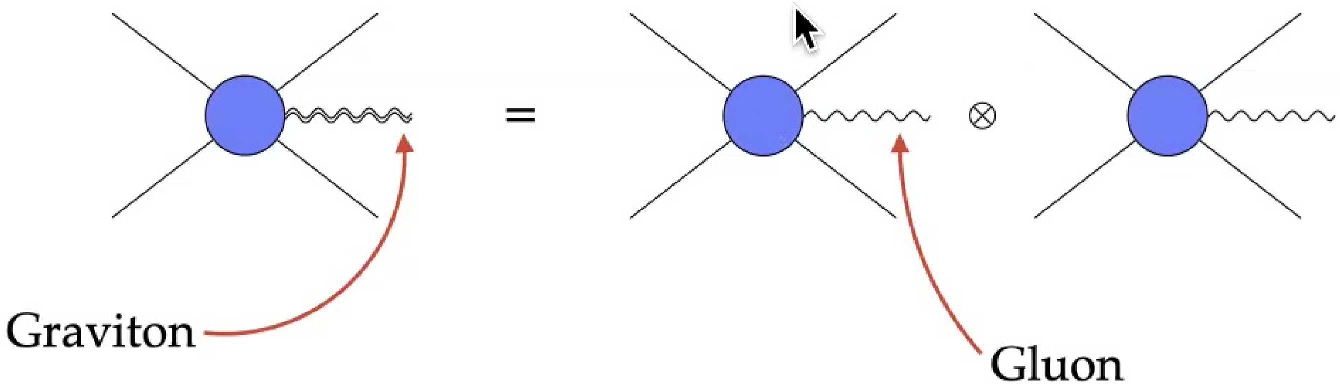
Higgs candidate event

CMS/CERN

Motivation

Gravity is *theoretically* interesting

The "double copy":



Kawai, Lewellen, Tye
Bern, Carrasco, Johansson
Cachazo, He, Yuan

Motivation

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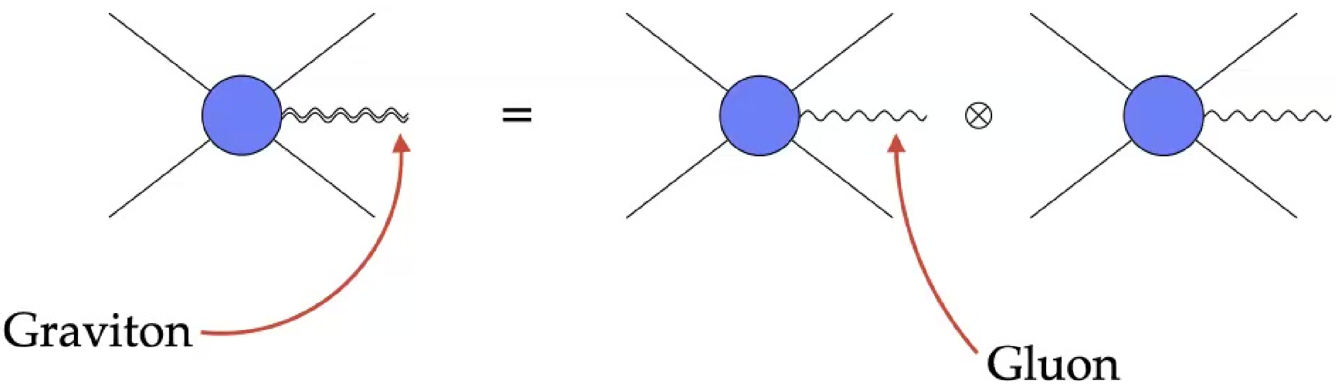

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Bern, Carrasco, Johansson
Cachazo, He, Yuan

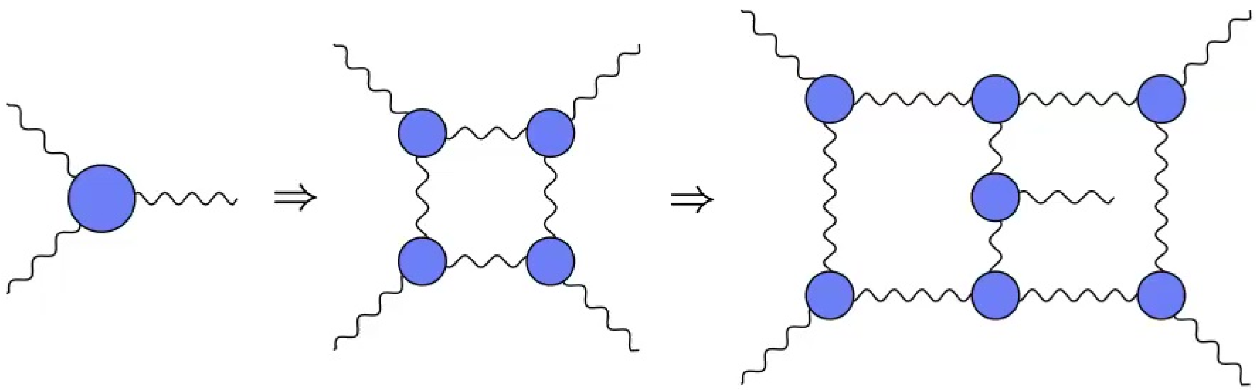
Gravity = Yang-Mills^2

Very bizarre from geometric point of view

Motivation

Gravity is *theoretically* interesting

The unitarity method:



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Motivation

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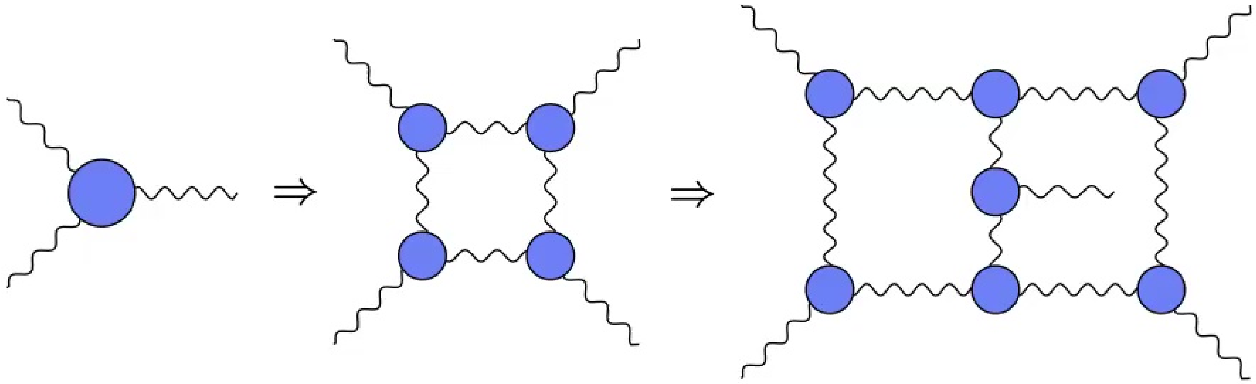

Bern, Dixon, Dunbar, Kosower
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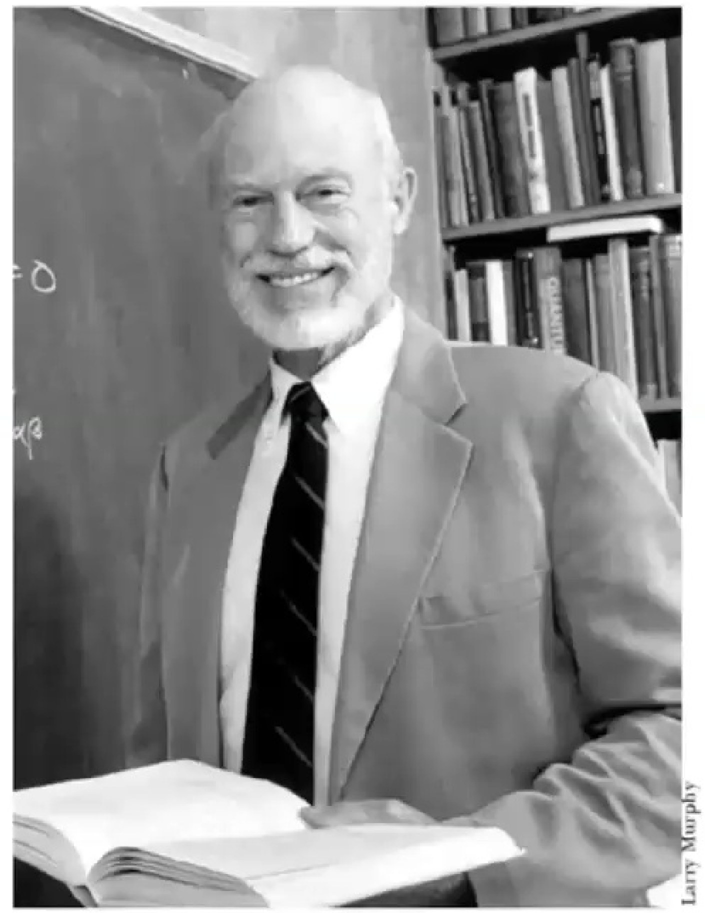
Gravitational processes determined by quantum unitarity

Very bizarre from classical point of view

Motivation

So build method for computing classical observables from amplitudes

"Only observable in (quantum) gravity in asymptotically flat space is the S-matrix"



Overview

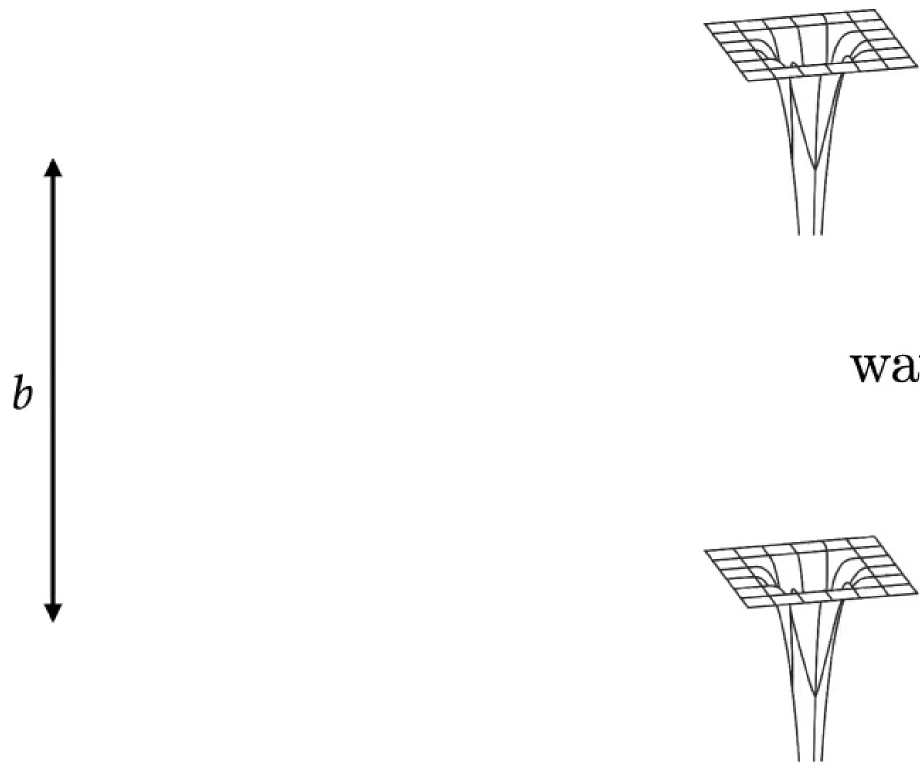
1. Observables from amplitudes, focus on gravitational waves
2. Waves at NLO
3. Spacetime curvature

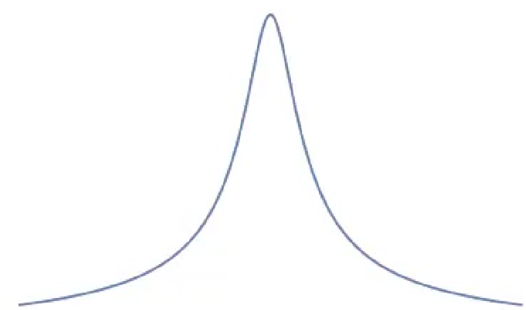
Why amplitudes?

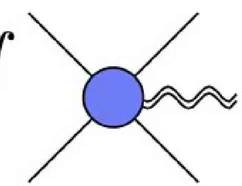
Why are amplitudes relevant to gravitational waves / scattering?

Amplitudes tell us about long-time evolution

Waves from Amplitudes



$$\text{waveform}(t) =$$ 

$$= \frac{1}{\text{distance}} \int$$  $$+ \cdots$$

Classical point particle approximation: finite size under control

Asymptotically Minkowski

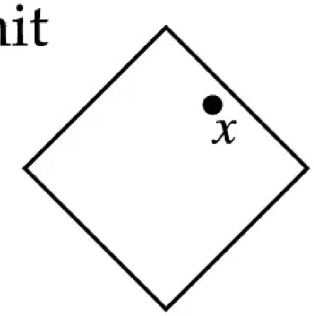
Kosower, Maybee & DOC

Cristofoli, Gonzo, Kosower & DOC

Related work: ***Bautista & Siemonsen***

Waves from Amplitudes

Measure *expectation* of curvature component in classical limit



Riemann curvature *operator*

$$\text{waveform} \equiv \langle\psi|S^\dagger \boxed{\mathbb{R}\ldots.(x)} S|\psi\rangle$$

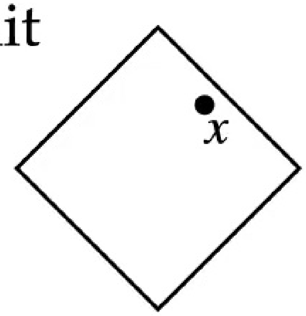
Newman-Penrose scalar Ψ_4
Dominant curvature component at large distances

$$|\psi\rangle \sim \int \bigwedge \bigwedge e^{ip_1\cdot b}|p_1\, p_2\rangle$$

Classical Cauchy data

Waves from Amplitudes

Measure *expectation* of curvature component in classical limit



Riemann curvature *operator*

$$\text{waveform} \equiv \langle\psi|S^\dagger \mathbb{R}_{....}(x) S|\psi\rangle$$

Final state Amplitudes!

Newman-Penrose scalar Ψ_4
Dominant curvature component at large distances

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Classical Cauchy data

Waves from Amplitudes

Measure *expectation* of curvature component in classical limit

$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

$$\text{waveform} = \int \widetilde{\mathrm{d}k}\, \left[k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \langle\psi|S^\dagger a(k) S|\psi\rangle + \text{c.c.}\right]$$

$$= i \int \widetilde{\mathrm{d}k}\, k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \Big[\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k) T|\psi\rangle\Big] + \text{c.c.}$$

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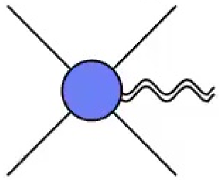
5 point amplitude



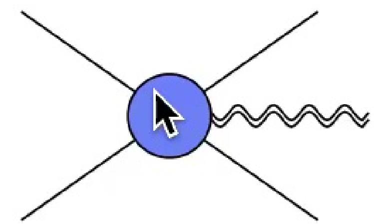
Waves from Amplitudes

Leading order:

$$\Psi_4(\omega) = \frac{1}{\text{distance}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2 \, \delta(p_1 \cdot q_1) \delta(p_2 \cdot q_2) \delta^4(k - q_1 - q_2) \, e^{ib\cdot(q_1 - q_2)}$$

Frequency space

$\times$



Cristofoli, Gonzo, Kosower & DOC

Waves from Amplitudes

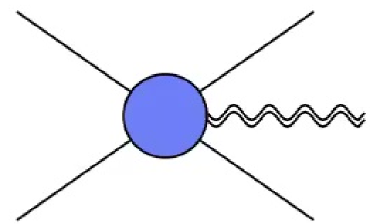
Leading order:

Spacetime: FT

Fourier integral: one bottleneck

$$\Psi_4(\omega) = \frac{1}{\text{distance}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2\, \delta(p_1 \cdot q_1) \delta(p_2 \cdot q_2) \delta^4(k - q_1 - q_2)\, e^{ib\cdot(q_1 - q_2)}$$

Frequency space

$\times$ 

Cristofoli, Gonzo, Kosower & DOC

Waves from Amplitudes

Leading order:

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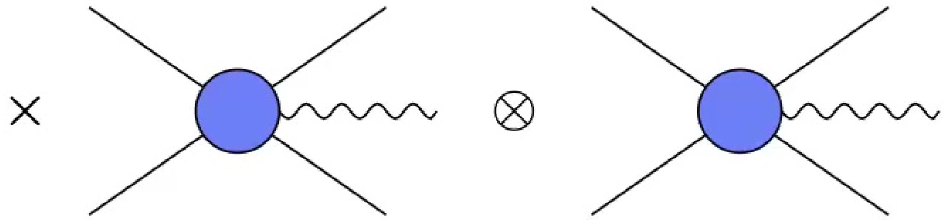
Frequency space



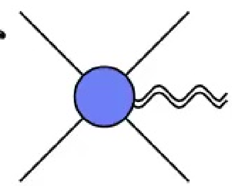
Gravitational waves from YM!

Bypasses complexity of Einstein-Hilbert Lagrangian

Eikonal connection

Seems contrary to intuition:

$$\text{waveform} = \frac{1}{\text{distance}} \int$$



$$+ \cdots$$

One graviton $\neq$ classical field

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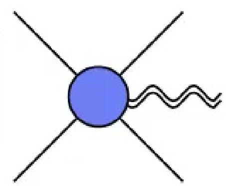
$$\text{waveform} = \frac{1}{\text{distance}} \int$$  $$+ \cdots$$

One graviton $\neq$ classical field

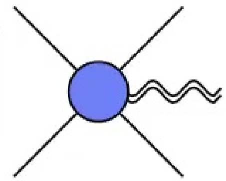
Classical field: expectation of a coherent state

$$\exp\left(\int \widetilde{\mathrm{d}k}\, \alpha(k)\, a^\dagger(k)\right) |0\rangle$$

Waveshape:
Fourier modes of classical field

Eikonal connection

Seems contrary to intuition:

$$\text{waveform} = \frac{1}{\text{distance}} \int$$  $$+ \cdots$$

Resolved if amplitude exponentiates in classical region (not proven)

$$S|\psi\rangle \sim \int \exp\left(i\tilde{\mathcal{M}}_4(x,q)\right) \exp\left(\int \widetilde{\mathrm{d}k}\ (\mathcal{M}_5(x_1,x_2,k) + \cdots)\, a^\dagger(k)\right) |p_1', p_2'\rangle$$

Generalisation of eikonal exponentiation

Ciafaloni, Colferai, Veneziano

Cristofoli, Gonzo, Moynihan, Ross, Sergola, White, DOC

Di Vecchia, Heissenberg, Russo, Veneziano

Bound binaries

Bound case?

$$\text{Gravitational wave power} = -\frac{\mathrm{d}}{\mathrm{d}t}\text{Potential}$$

Einstein quadrupole power
+ *corrections*

Newton potential
+ *corrections*

Bound binaries

Bound case?

$$\text{Gravitational wave power} = -\frac{\mathrm{d}}{\mathrm{d}t}\text{Potential}$$

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Build EFT, valid in both bound & scattering cases

Bound binaries

Bound case?

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Einstein quadrupole power
+ *corrections*

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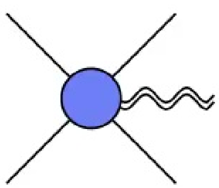
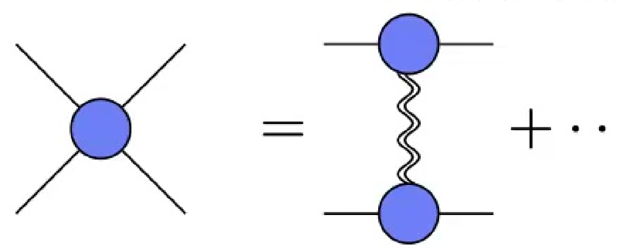




Build EFT, valid in both bound & scattering cases

Match Wilson coefficients to scattering

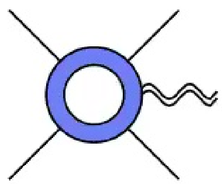
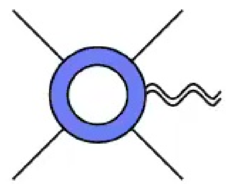
Neill, Rothstein
Cachazo, Guevara
Cheung, Rothstein, Solon
Bern, Cheung, Roiban, Shen, Solon, Zeng
Kälin, Porto

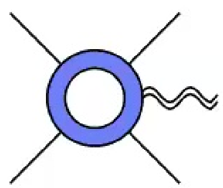
Waves at NLO

$$\text{waveform} = i \int \widetilde{dk}\, kk\, \varepsilon\varepsilon\, e^{-ik\cdot x} \left[\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k)T|\psi\rangle \right] + \text{c.c.}$$

waveshape α

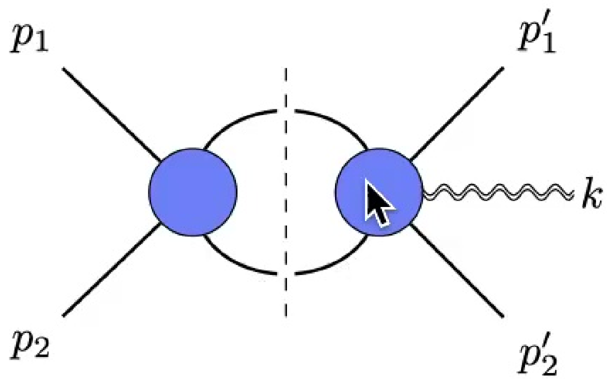
Second term in waveshape: cut aka imaginary part of one-loop 5-point

$\alpha|_{\text{NLO}} =$  $- i$ Im 

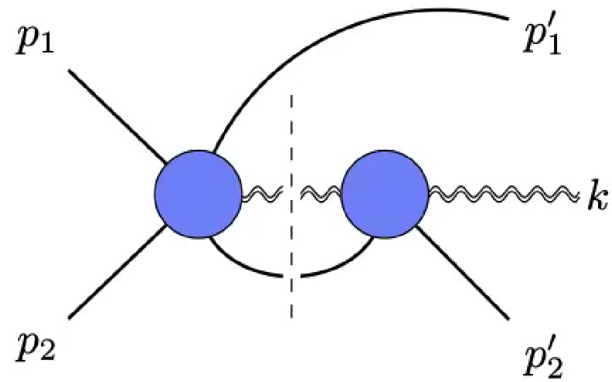
$\sim$ Re 

Waves at NLO

But more than one cut:



Removed

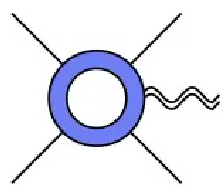
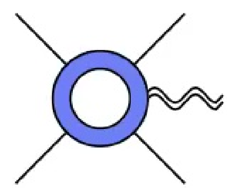


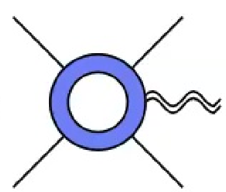
Remains

Waves at NLO

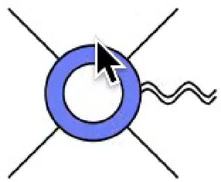
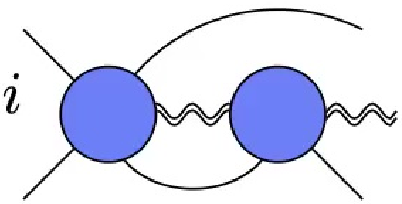
$$\text{waveform} = i \int \widetilde{dk}\, kk\, \varepsilon\varepsilon\, e^{-ik\cdot x} \Big[\underbrace{\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k)T|\psi\rangle}_{\text{waveshape } \alpha} \Big] + \text{c.c.}$$

Second term in waveshape: cut aka imaginary part of one-loop 5-point

$\alpha|_{\text{NLO}} =$  $- i\ \text{Im}$ 

$\sim \text{Re}$ 

Waves at NLO

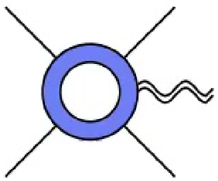
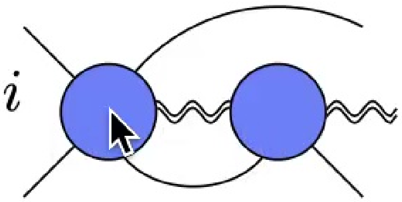
$\alpha|_{\rm NLO} = {\rm Re}$  $+\, i$  $+$ flip

Elkhidir, Sergola, Vazquez-Holm, DOC

Real part:

- One massive propagator: delta function
- Others: principle value prescription
- Force not dissipative (eg geodesic motion)

Waves at NLO

$\alpha|_{\rm NLO} = \mathrm{Re}$  $+\, i$  $+$ flip

Elkhidir, Sergola, Vazquez-Holm, DOC

Imaginary part:

- Two exposed propagators both cut
- Explicit factor of *i*: time reversal odd
- Radiation due to intrinsically dissipative force: Abraham-Lorentz-Dirac like
- Simple example of radiation reaction, connection to renormalisation

Waves at NLO

Integrated NLO waveform

Herderschee, Roiban, Teng

Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini

Georgoudis, Heissenberg, Vazquez-Holm

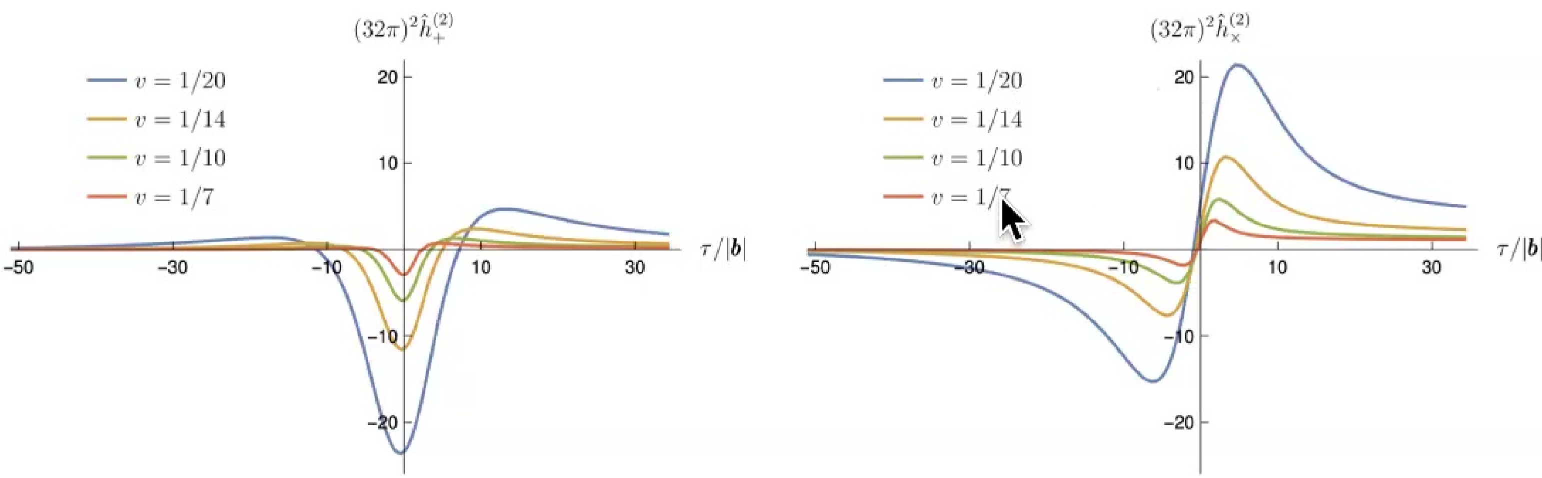
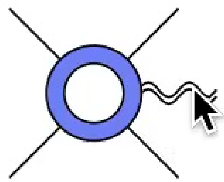
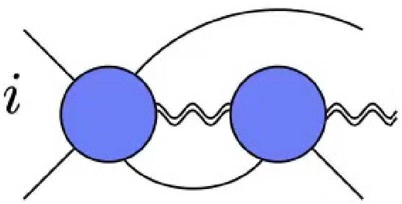


Figure from Herderschee et al

IBP, generalised unitarity, HQET, double copy, IR divergences

Waves at NLO

$\alpha|_{\mathrm{NLO}} = \mathrm{Re}$  $+\, i$  $+$ flip

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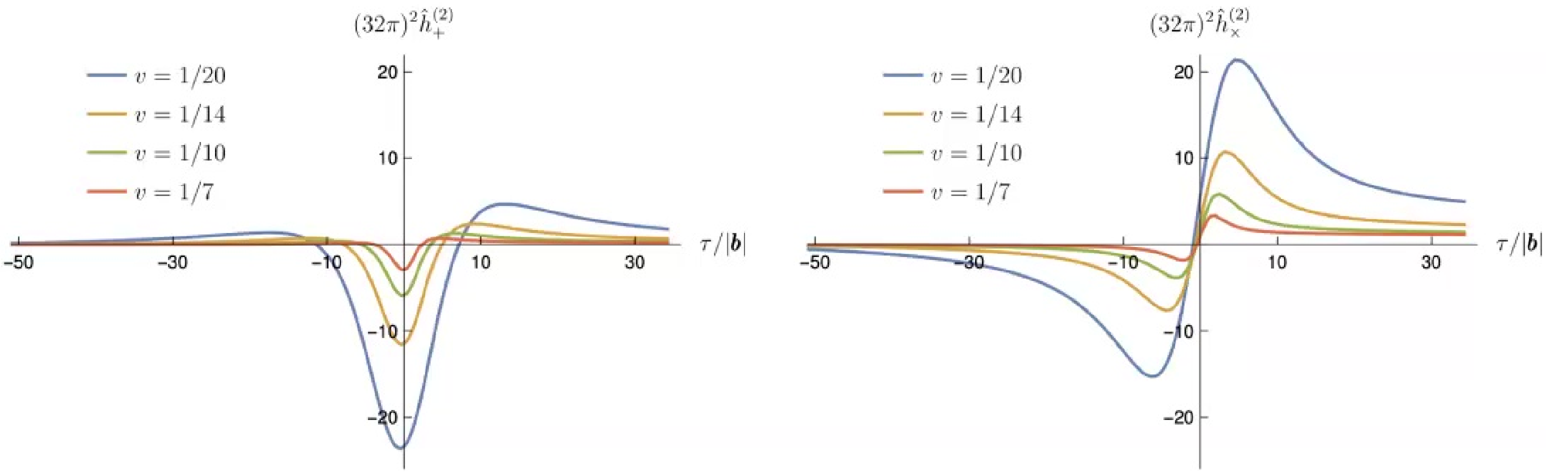
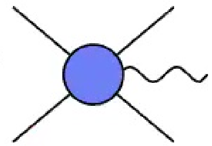


Figure from Herderschee et al

IBP, generalised unitarity, HQET, double copy, IR divergences

Curvature

$$\text{field strength} = \frac{1}{\text{distance}} \int e^{iq\cdot x}$$



$$+ \cdots$$

↕ Double copy

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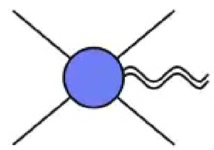

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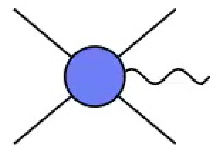

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Double copy

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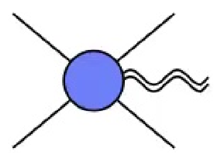
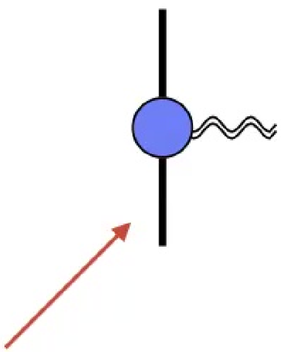


$$+ \cdots$$

What about *static* field / curvature?

Spacetime curvature

Start with simplest amplitudes

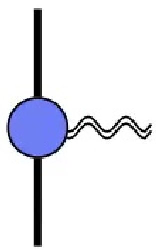
 $= \kappa\, \mathcal{M}_3$

$$\mathcal{M}_3 \sim [\epsilon(k) \cdot p]^2$$

Massive,
Spin=0

Spacetime curvature

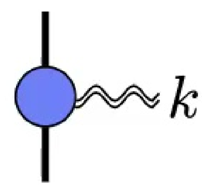
Start with simplest amplitudes



$$= \kappa\, \mathcal{M}_3 \propto |k]|k]|k]|k] \qquad \mathcal{M}_3 \sim [\epsilon(k)\cdot p]^2$$

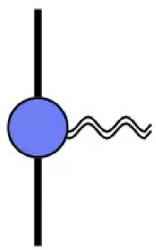
Introduce spacetime coords: Fourier transform

$$\int_{\text{on-shell}} \mathrm{d}k\; e^{-ik\cdot x} |k\rangle_\alpha |k\rangle_\beta |k\rangle_\gamma |k\rangle_\delta$$



Spacetime curvature

Start with simplest amplitudes



$$= \kappa\,\mathcal{M}_3 \propto |k]|k]|k]|k] \qquad \mathcal{M}_3 \sim [\epsilon(k)\cdot p]^2$$

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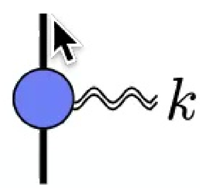
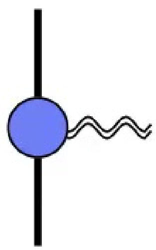


Analytic continuation
eg to metric (+,+,-,-)

Spacetime curvature

Start with simplest amplitudes



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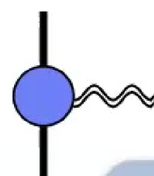


$$k \;=\; \Psi^{\text{Schw.}}_{\alpha\beta\gamma\delta}(x)$$

Analytic continuation eg to metric (+,+,-,-)

Exact Schwarzschild Weyl curvature spinor!

Monteiro, Sergola, Peinador Veiga & DOC

Spacetime curvature

1. Derive in same way as waveform
   - Also in E&M: Coulomb^2 = Schwarzschild! *Monteiro, White, DOC*
   - Derivation: linearised GR

2. With amplitudes, care about things being on-shell
   - Three-point amplitudes don't live in Minkowski
   - The continuation is important

Monteiro, Nagy, Peinador Veiga, Sergola, DOC
*Crawley, **Guevara**, Miller, Strominger*

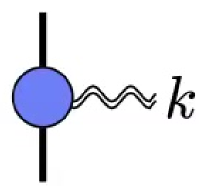
Guevara

Spacetime curvature

3. Recovered exact Weyl spinor

- "Exact": in appropriate coordinates / choice of tetrad
- Classically, metrics are (double) Kerr-Schild
- Curvature linearises in appropriate coordinates

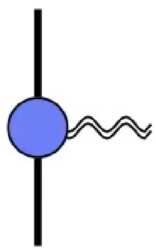
$$\int_{\text{on-shell}} \mathrm{d}k \, e^{-ik\cdot x} |k\rangle_\alpha |k\rangle_\beta |k\rangle_\gamma |k\rangle_\delta$$



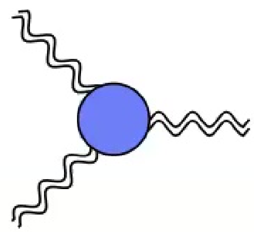
$$= \Psi^{\text{Schw.}}_{\alpha\beta\gamma\delta}(x)$$

Spacetime curvature

What about more general three-point amplitudes?



$$= \boxed{\kappa e^{i\theta}} \mathcal{M}_3$$

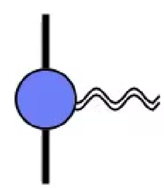


$$= \boxed{\kappa} \frac{\langle 12 \rangle^8}{\langle 12 \rangle^2 \langle 23 \rangle^2 \langle 31 \rangle^2}$$

Misalignment angle

Weyl spinor is now

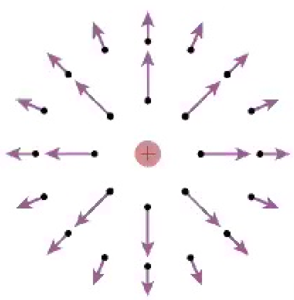
$$\int_{\text{on-shell}} \mathrm{d}k \; e^{-ik\cdot x} |k\rangle_\alpha |k\rangle_\beta |k\rangle_\gamma |k\rangle_\delta$$



$$k \; = e^{i\vartheta} \Psi^{\text{Schw.}}_{\alpha\beta\gamma\delta}(x)$$

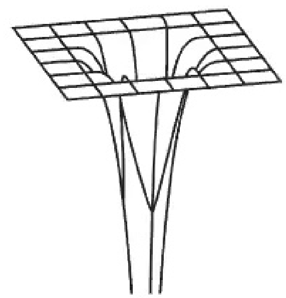
Spacetime curvature

Coulomb



Double copy →

Schwarzschild (linearised)



Monteiro, White, DOC
Luna , Monteiro, White, DOC

Conclusions

- Interesting dialogue between amplitudes and classical gravity

- The double copy: ubiquitous
  - Gravitational waves without GR
  - Geodesics via the unitarity method

- Need to understand bound states
- Connection to celestial holography?

Conclusions

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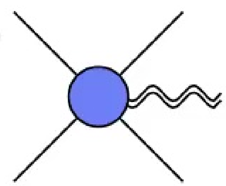
23perimeter

Eikonal connection

Seems contrary to intuition:

$$\text{waveform} = \frac{1}{\text{distance}} \int$$  $$+ \cdots$$

Resolved if amplitude exponentiates in classical region (not proven)

$$S|\psi\rangle \sim \int \exp\left(i\tilde{\mathcal{M}}_4(x,q)\right) \exp\left(\int \widetilde{\mathrm{d}k}\; (\mathcal{M}_5(x_1,x_2,k) + \cdots)\, a^\dagger(k)\right) |p_1', p_2'\rangle$$

Generalisation of eikonal exponentiation

Ciafaloni, Colferai, Veneziano

Cristofoli, Gonzo, Moynihan, Ross, Sergola, White, DOC

Di Vecchia, Heissenberg, Russo, Veneziano