Title: An Asymptotic Framework for Gravitational Scattering

Speakers: Samuel Gralla

Series: Strong Gravity

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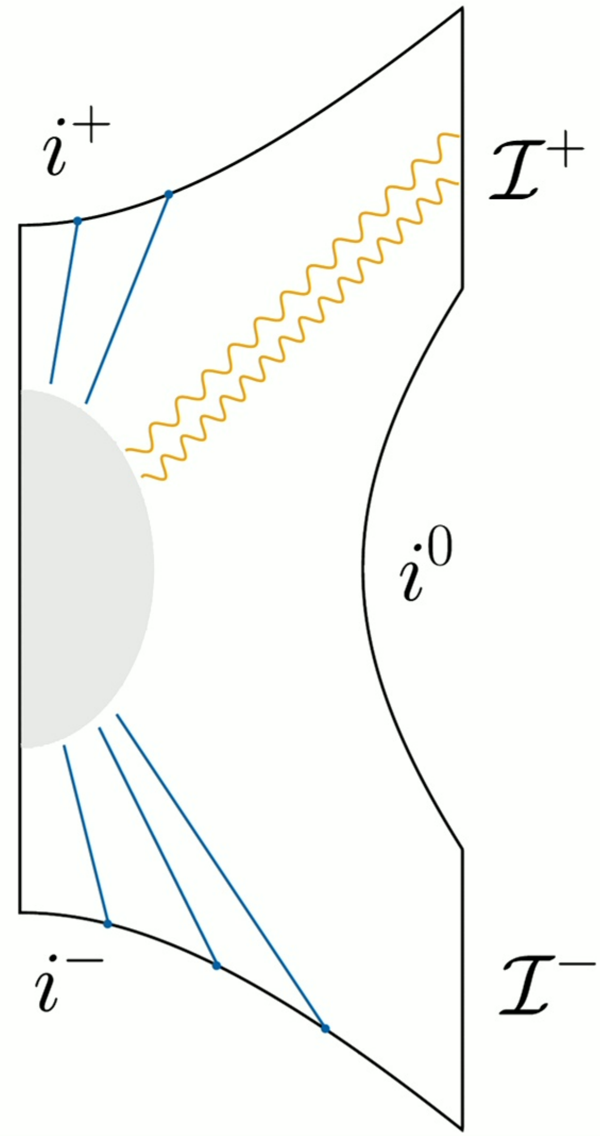
Abstract: Gravitational scattering provides valuable insight into classical dynamics and is likely of fundamental importance in quantum gravity. However, a complete framework for gravitational scattering does not yet exist. The lack of a clear framework hinders progress; for example, different groups have reported different results for the loss of angular momentum in post-Minkowskian scattering. In this talk I will report on recent work 2303.17124 with Compere and Wei constructing a general framework for classical gravitational scattering of finite-sized massive bodies in four spacetime dimensions. We formulate assumptions and definitions such that the five asymptotic regions (past/future timelike/null infinity and spatial infinity) share a single Bondi-Metzner-Sachs (BMS) group of symmetries and associated charges and derive global conservation laws stating that the total change in charge is balanced by the corresponding radiative flux. Our assumptions are compatible with all known properties of scattering spacetimes, including certain logarithmic corrections that invalidate common falloff assumptions. Among the new implications are rigorous definitions for quantities like initial/final spin, scattering angle, and impact parameter in multi-body spacetimes, without the use of any preferred background structure. We show that spin is supertranslation-invariant, while impact parameter is not. To complement these derivations I will emphasize a helpful "puzzle piece" diagram that faithfully represents all five asymptotic regions, illustrating their roles in the scattering problem.

Zoom link: https://pitp.zoom.us/j/94358515412?pwd=cUFocVNjQ1pqUmp4MDN0RmRLbjE0QT09

An asymptotic framework for gravitational scattering

Sam Gralla, University of Arizona

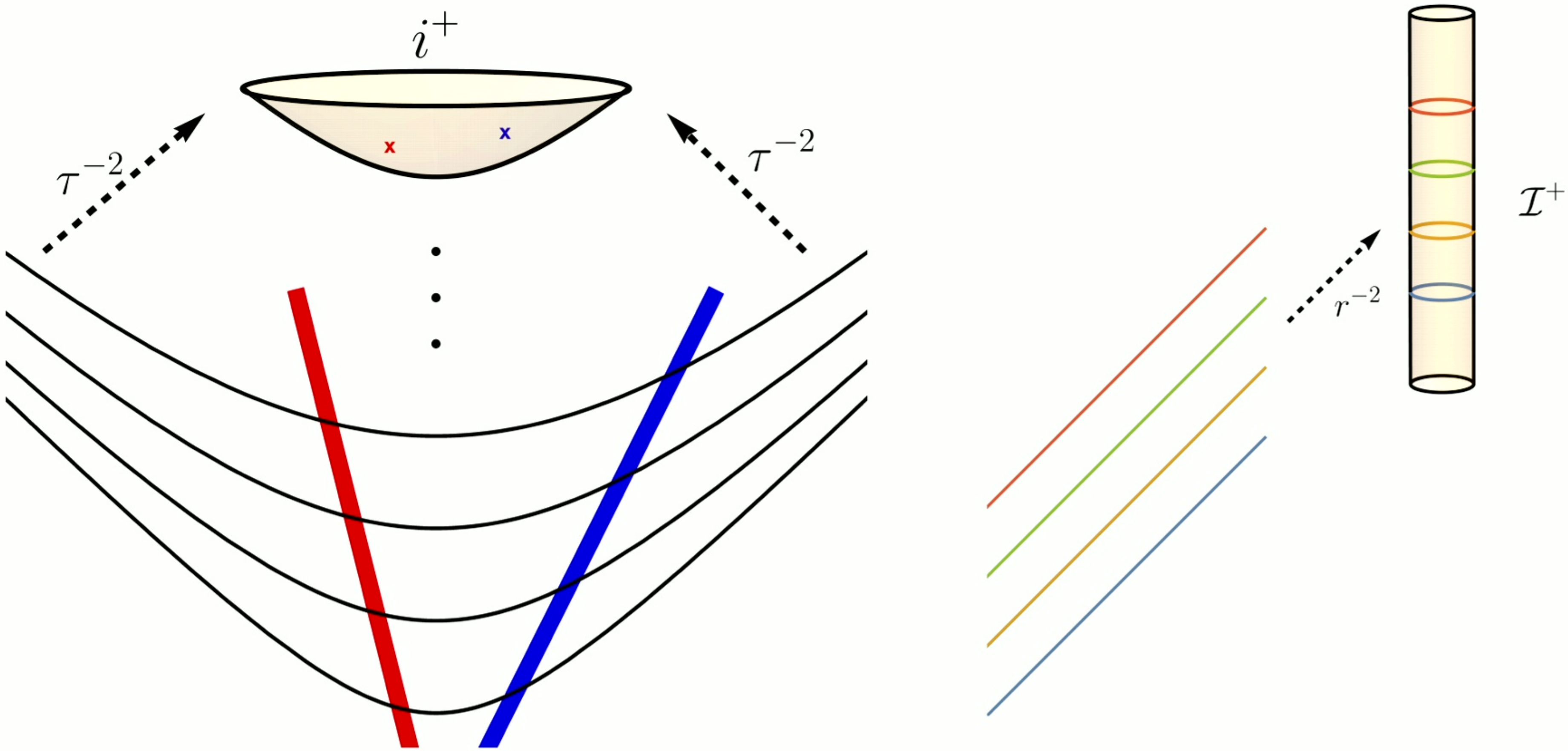
2303.17124 w/ Compere and Wei



Why a framework for scattering?

- Physical insight – conserved quantities in GR
- Quantum Gravity – symmetries of S-matrix (if it exists)
- "Practical" application – Compare different calculations
  - Need to understand "gauge-invariant observables"
  - Different groups are reporting different results for the change in angular momentum in weak-field scattering!

Need to understand "non-local observables" in gravity


i^+
τ^{-2}
τ^{-2}
r^{-2}
$\mathcal{I}^+$

Charge	name	generator
E	Energy	$T = 1$
P^i	Momentum	$T = n^i(\theta, \phi)$
$P_{\ell m}$	Supermomentum	$T = Y_{\ell m}(\theta, \phi)$
L^i	Angular Momentum	$Y^A = -\epsilon^{AB} \partial_B n^i(\theta, \phi)$
N^i	Mass Moment	$Y^A = \partial^A n^i(\theta, \phi)$

For each BMS charge (choice of T or Y):

$$\sum_{n=1}^{N^+} Q_n^{i^+} + \Delta Q^{\mathcal{I}^+} = Q^{i^0} = \sum_{n=1}^{N^-} Q_n^{i^-} + \Delta Q^{\mathcal{I}^-}$$

Charge	name	generator
E	Energy	$T = 1$
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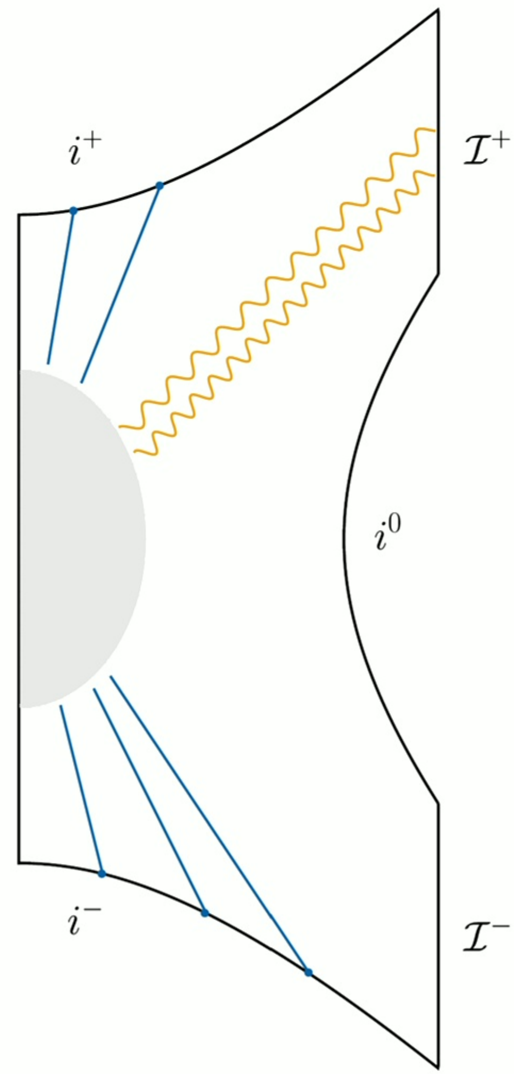
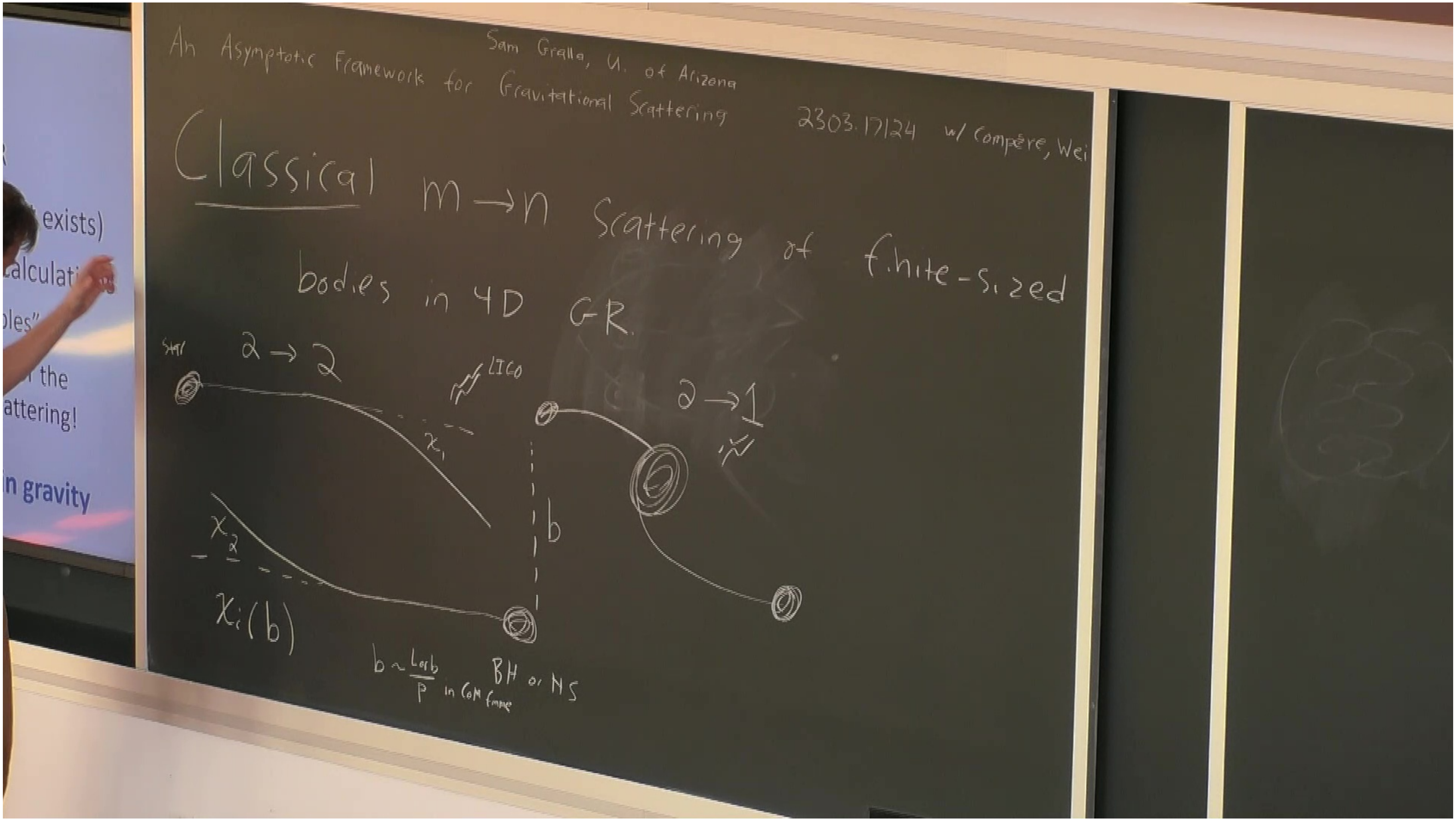
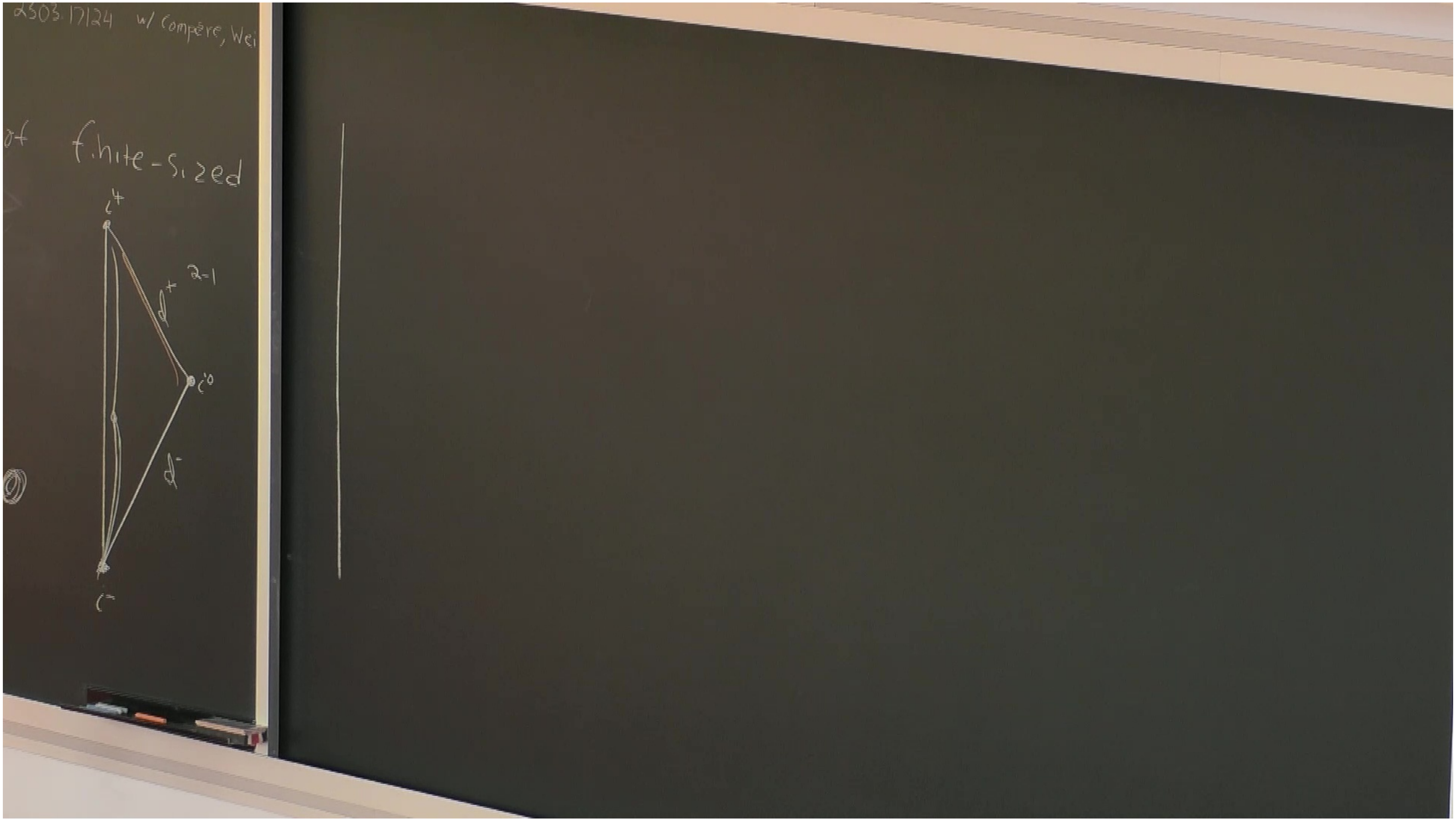
N^i	Mass Moment	$Y^A = \partial^A n^i(\theta, \phi)$

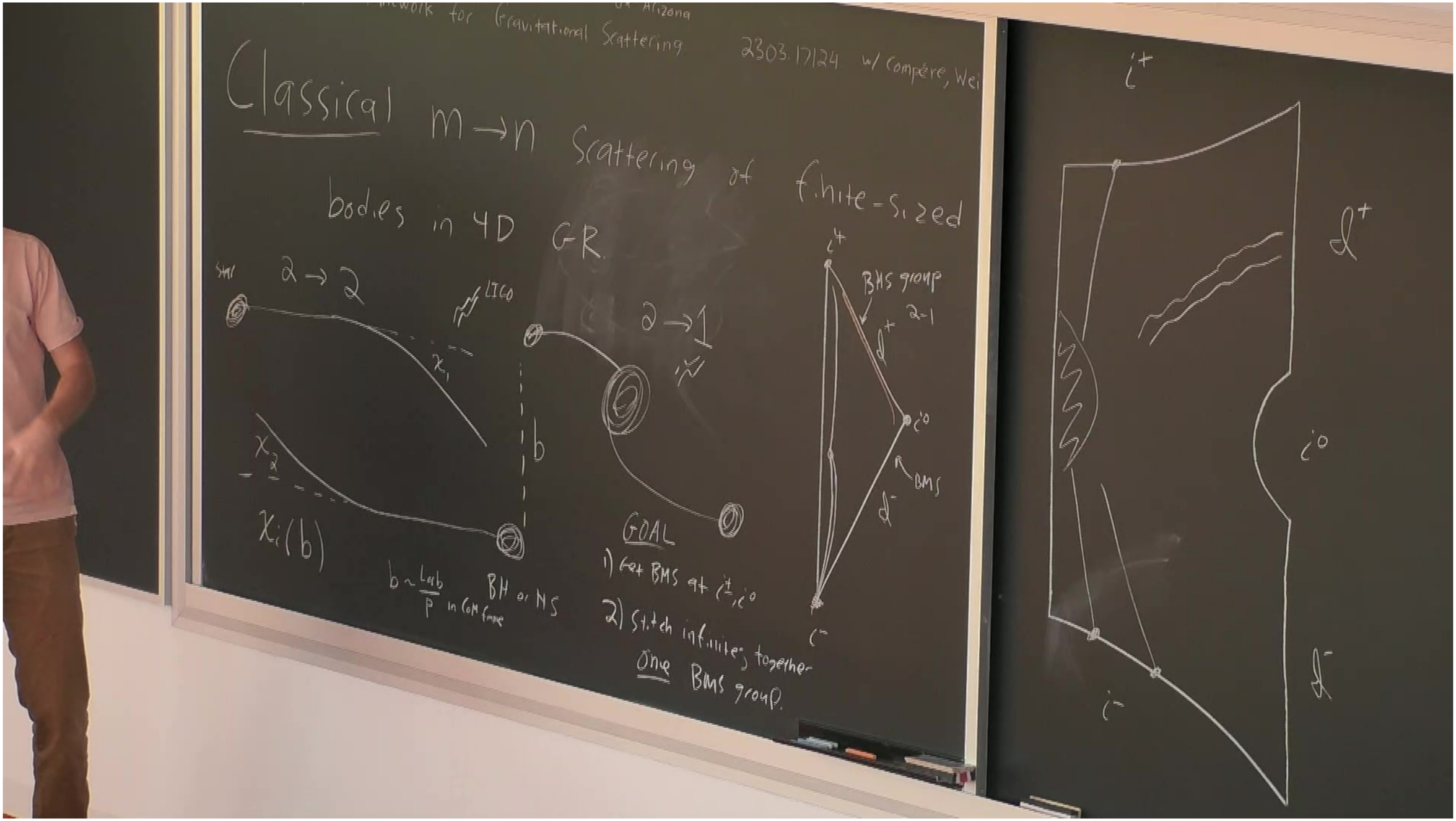
Summary

- All 5 infinities share a single BMS group
- A charge for each BMS generator
- A conservation (flux-balance) law for each charge
- Define initial and final quantities using charges

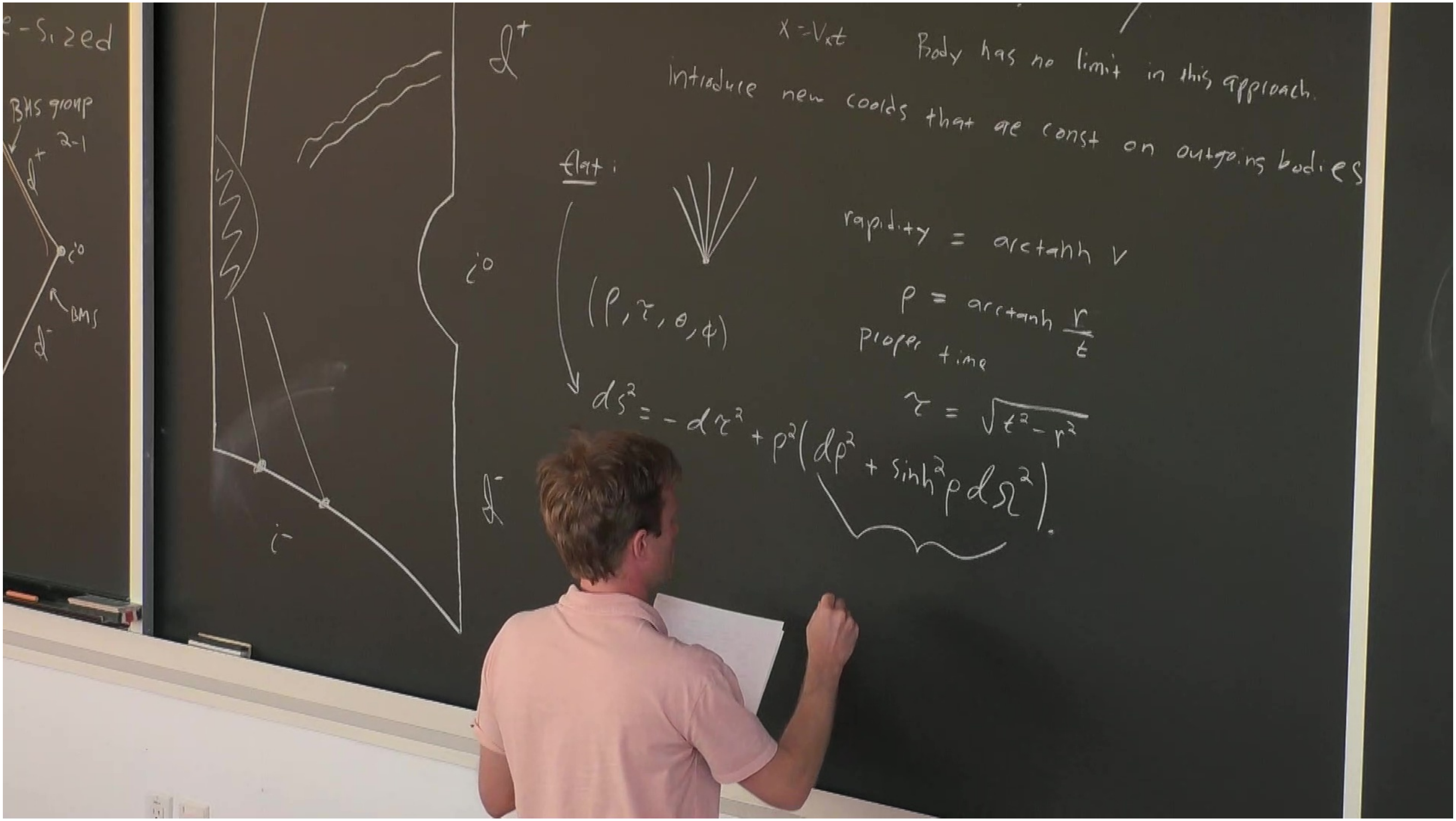


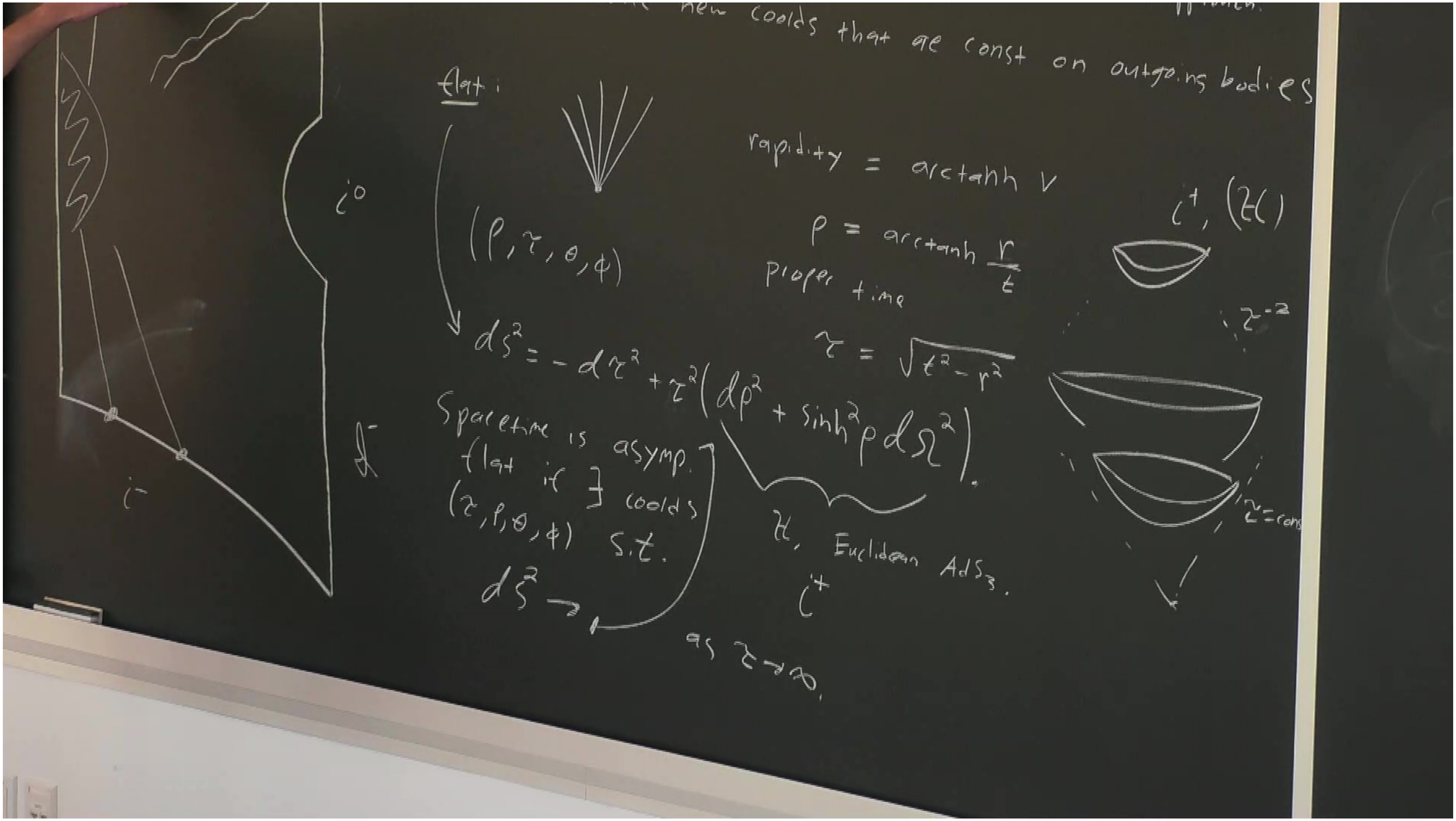

Sam Gralla, U. of Arizona
An Asymptotic Framework for Gravitational Scattering
2303.17124 w/ Compère, Wei
Classical m → n Scattering of finite-sized bodies in 4D GR.
2 → 2
LIGO
2 → 1
b
χ_1
χ_2
χ_i(b)
b ~ L_orb / p in CoM frame
BH or NS
exists)
the
attering!
in gravity

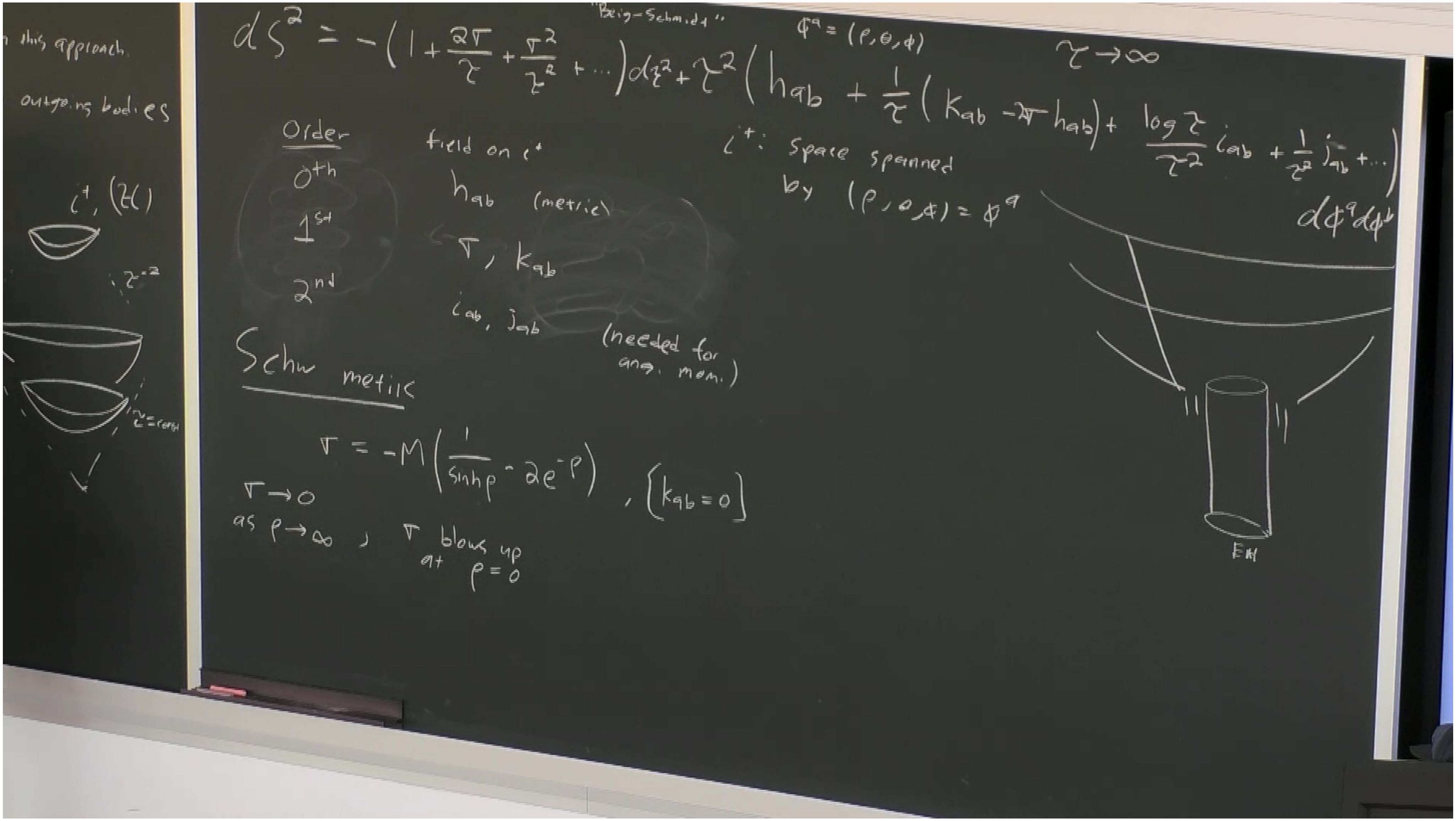

2303.17124 w/ Compère, Wei
of finite-sized
i+
i0
i−

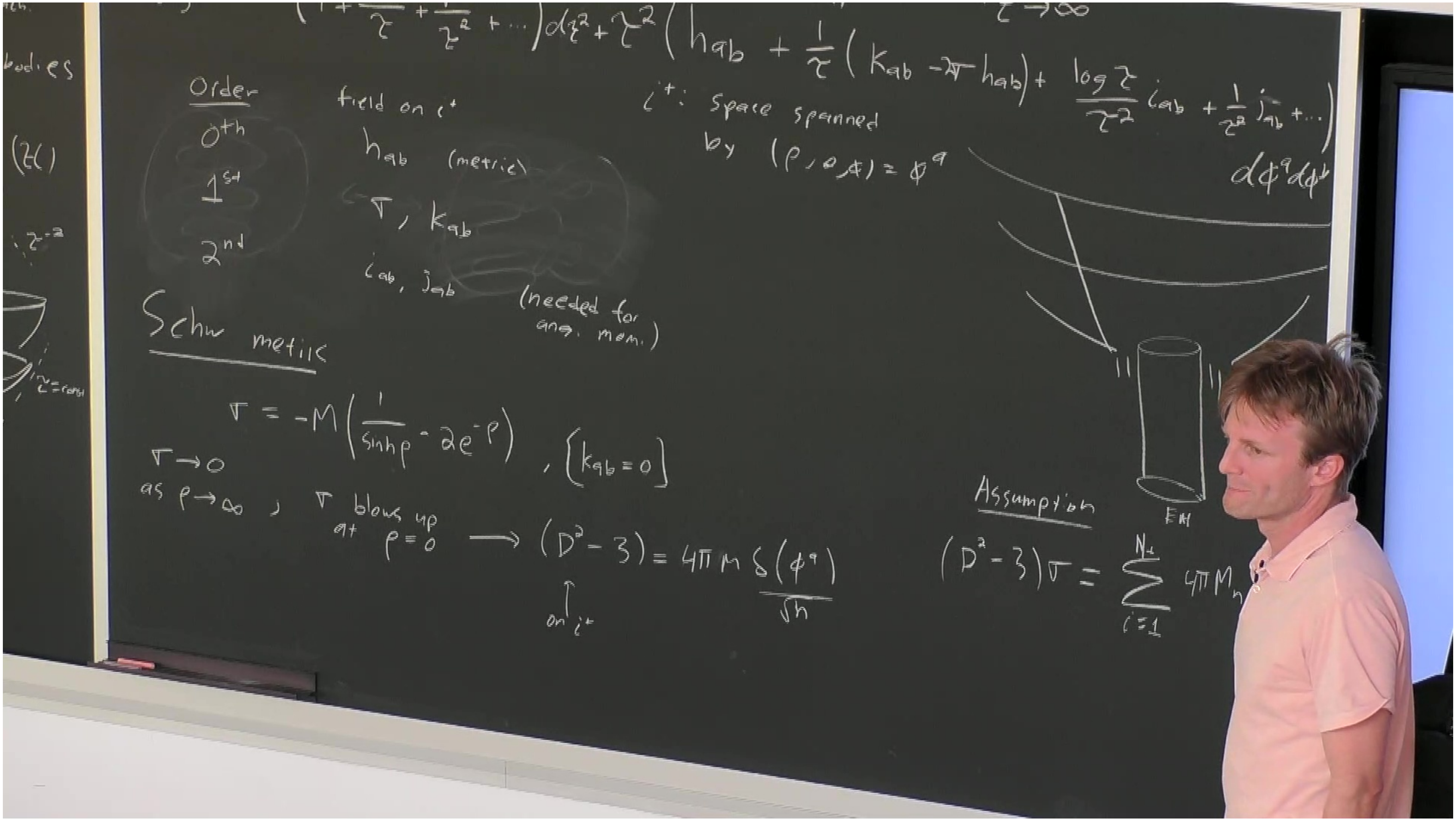


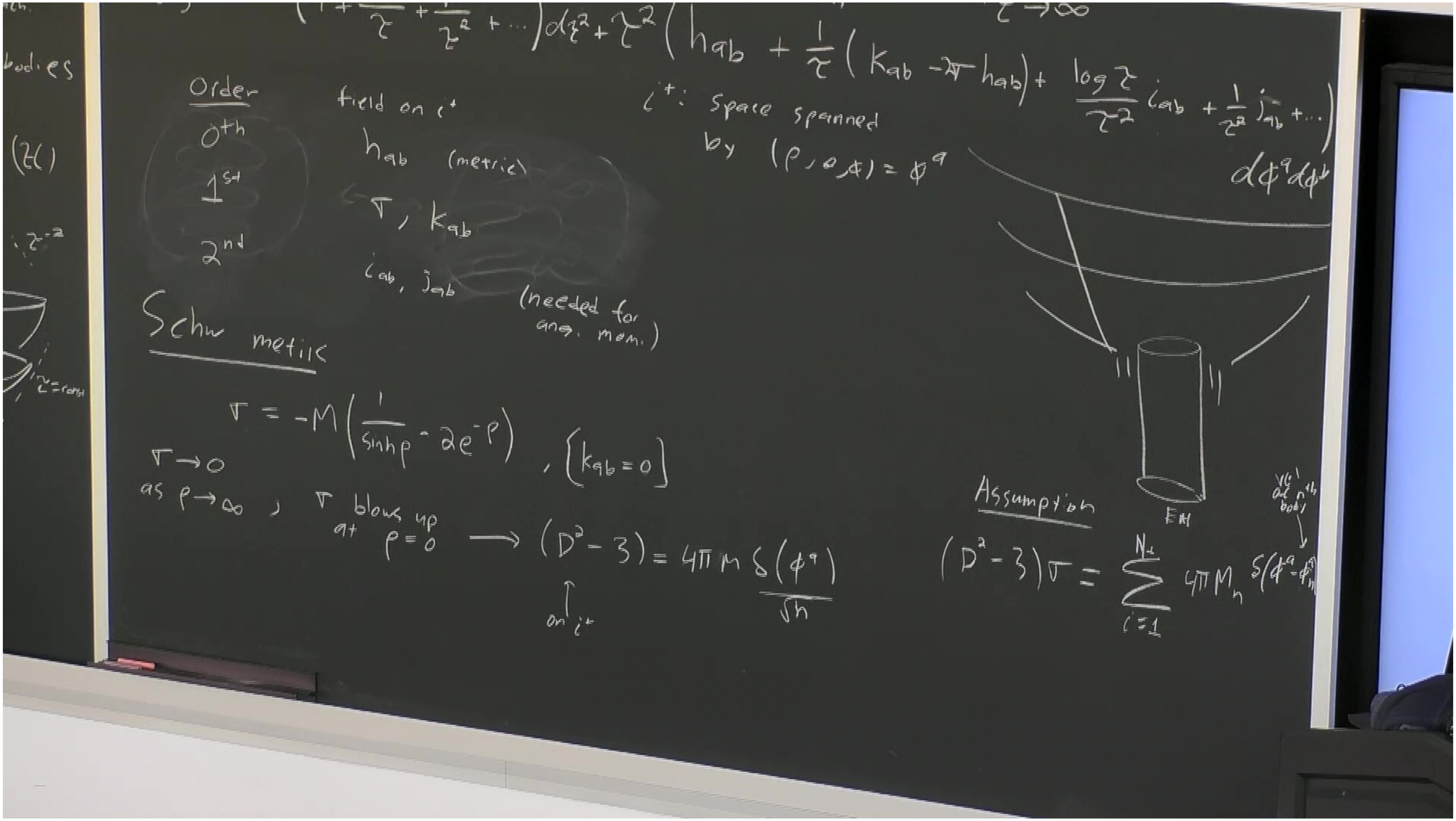
for Gravitational Scattering
2303.17124 w/ Compère, Wei
Classical m → n Scattering of finite-sized bodies in 4D GR
2 → 2
LIGO
2 → 1
b
X_1
X_2
$X_i(b)$
BH or NS
GOAL
1) Get BMS at $i^\pm$, i^0
2) Stitch infinities together
One BMS group
BMS group
BMS
i^+
i^0
i^-


-Sized
BMS group
BMS
X = V_x t
Body has no limit in this approach.
introduce new coords that are const on outgoing bodies
flat:
rapidity = arctanh v
ρ = arctanh r/t
(ρ, τ, θ, φ)
proper time
τ = √(t² − r²)
ds² = − dτ² + ρ²(dρ² + sinh²ρ dΩ²).


new coords that are const on outgoing bodies
flat:
i^0
$(\rho, \tau, \theta, \phi)$
rapidity = arctanh v
ρ = arctanh $\frac{r}{t}$
proper time
$\tau = \sqrt{t^2 - r^2}$
i^+, $\mathcal{H}$
τ^{-2}
$ds^2 = -d\tau^2 + \tau^2 (d\rho^2 + \sinh^2\rho \, d\Omega^2)$.
$\mathcal{I}^-$
i^-
Spacetime is asymp. flat if ∃ coords $(\tau, \rho, \theta, \phi)$ s.t.
$ds^2 \to$
as $\tau \to \infty$.
$\mathcal{H}$, Euclidean AdS_3.
i^+
τ = const


"Beig–Schmidt"
$\phi^a = (\rho, \theta, \phi)$
$\tau \to \infty$
$ds^2 = -\left(1 + \frac{2\sigma}{\tau} + \frac{\sigma^2}{\tau^2} + \dots\right) d\tau^2 + \tau^2 \left(h_{ab} + \frac{1}{\tau}(k_{ab} - 2\sigma h_{ab}) + \frac{\log \tau}{\tau^2} i_{ab} + \frac{1}{\tau^2} j_{ab} + \dots \right) d\phi^a d\phi^b$
this approach.
outgoing bodies
Order
field on i^+
0^{th}
h_{ab} (metric)
1^{st}
σ, k_{ab}
2^{nd}
i_{ab}, j_{ab}
(needed for ang. mom.)
i^+: Space spanned by $(\rho, \theta, \phi) = \phi^a$
Schw metric
$\sigma = -M\left(\frac{1}{\sinh \rho} - 2e^{-\rho}\right)$, $[k_{ab} = 0]$
$\sigma \to 0$ as $\rho \to \infty$, σ blows up at $\rho = 0$
EM


Order
0th
1st
2nd
field on i+
hab (metric)
σ, kab
iab, jab
(needed for ang. mom.)
i+: Space spanned by (ρ, θ, φ) = φa
Schw metric
σ = -M (1/sinh ρ - 2e^-ρ) , [kab = 0]
σ → 0 as ρ → ∞ ; σ blows up at ρ = 0
(D² - 3) = 4π M δ(φa)/√h
on i+
Assumption
(D² - 3) σ = Σ 4π Mn
EM


Order
field on i^+
0^{th} h_{ab} (metric)
1^{st} σ, k_{ab}
2^{nd} i_{ab}, j_{ab} (needed for ang. mom.)
i^+: Space spanned by $(\rho, \theta, \phi) = \phi^a$
$\tau^2 \left(h_{ab} + \frac{1}{\tau}(k_{ab} - 2\sigma h_{ab}) + \frac{\log \tau}{\tau^2} i_{ab} + \frac{1}{\tau^2} j_{ab} + \dots \right) d\phi^a d\phi^b$
$\tau \to \infty$
Schw metric
$\sigma = -M\left(\frac{1}{\sinh\rho} - 2e^{-\rho}\right)$, $[k_{ab} = 0]$
$\sigma \to 0$ as $\rho \to \infty$, σ blows up at $\rho = 0$
$\to (D^2 - 3) = 4\pi M \frac{\delta(\phi^a)}{\sqrt{h}}$
on i^+
Assumption
BH
$(D^2 - 3)\sigma = \sum_{i=1}^{N_b} 4\pi M_n \delta(\phi^a - \phi^a_n)$