Title: From Gaussian thermal partition functions to multi loop conformal graphs

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Series: Quantum Gravity

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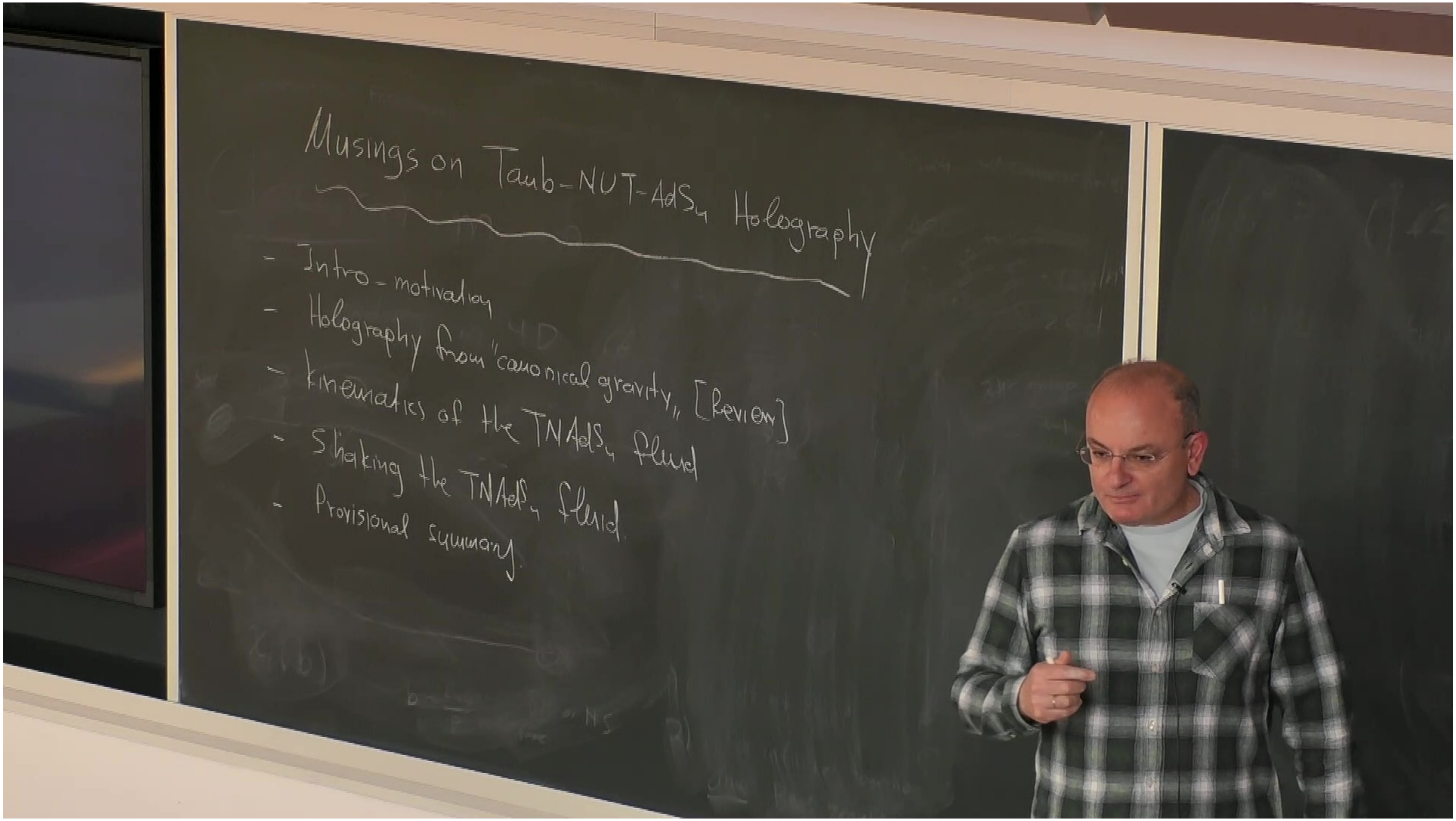
Abstract:

I will discuss a recently observed connection between Gaussian thermal partition functions in d=2L+1 dimensions and L-loop conformal graphs in D=4. While the origin of such a connection is still unclear, I will argue that it offers some interesting perspectives in quantum field theory and in mathematics.

Zoom link: https://pitp.zoom.us/j/98795029694?pwd=aGdSdWtDQ1VqbzFvSWE4SkM5c3lJUT09



Musings on Taub-NUT-AdS4 Holography
- Intro - motivation
- Holography from "canonical gravity" [Review]
- Kinematics of the TNAdS4 fluid
- Shaking the TNAdS4 fluid
- Provisional summary.

• $AlAdS_4$ spacetimes $\longleftrightarrow$ relativistic fluids 3d.

$\downarrow$ bulk reconstruction $\longleftarrow$

exact solutions

vacuum. $\longleftrightarrow$

[Plebanski-Demianski] $\longleftrightarrow$ { perfect relativistic fluid or a "generic" stationary background } hydro variabe[s] $T, P, \underline{\underline{u^\mu}}$, ζ^μ

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• Schw. AdS_4

• Kerr AdS_4

• $AlAdS_4$ spacetimes $\longleftrightarrow$ relativistic fluids 3d.

$\downarrow$ bulk reconstruction $\longleftarrow$

exact solutions

vacuum $\longleftrightarrow$

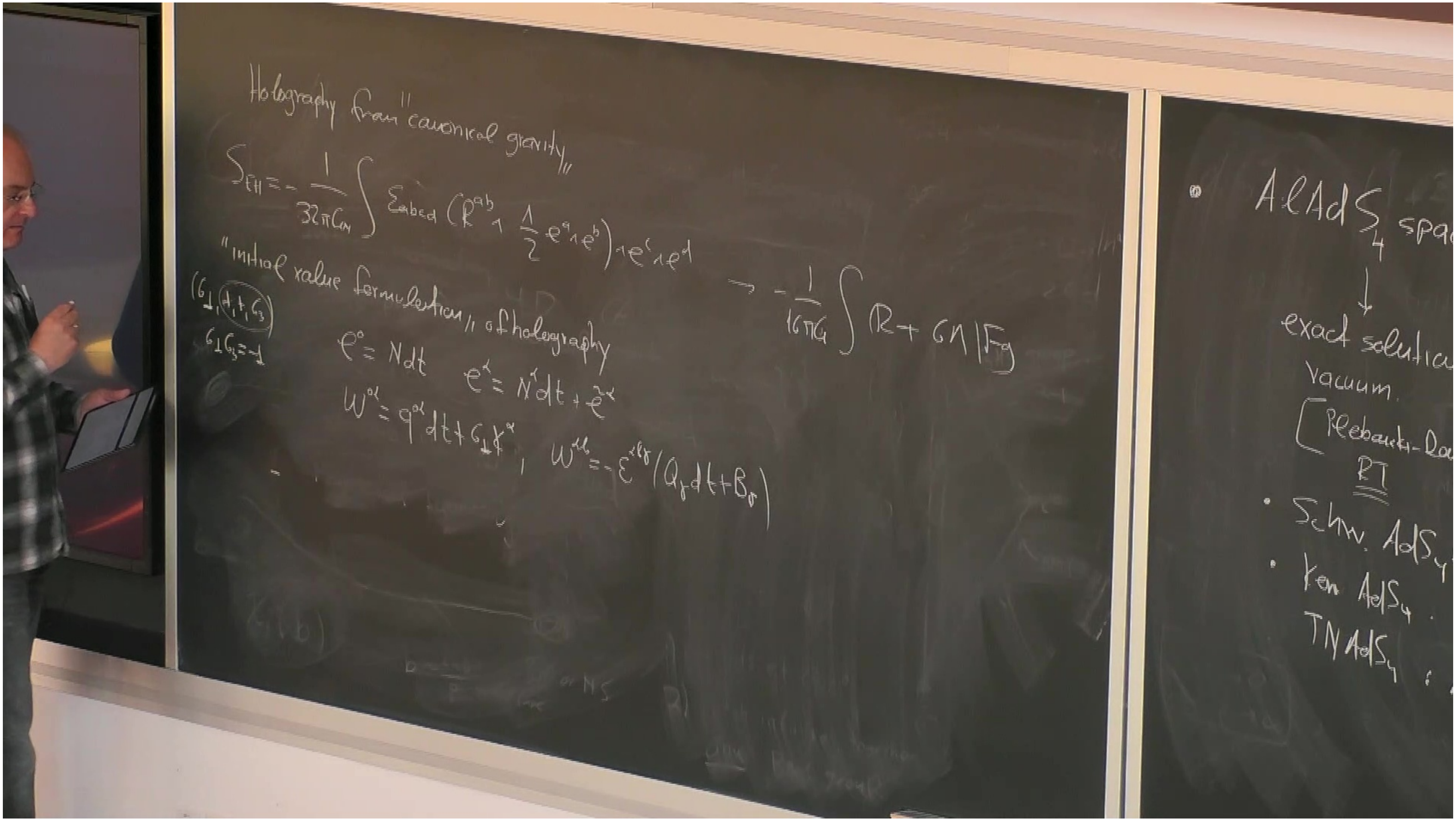
$\gg$

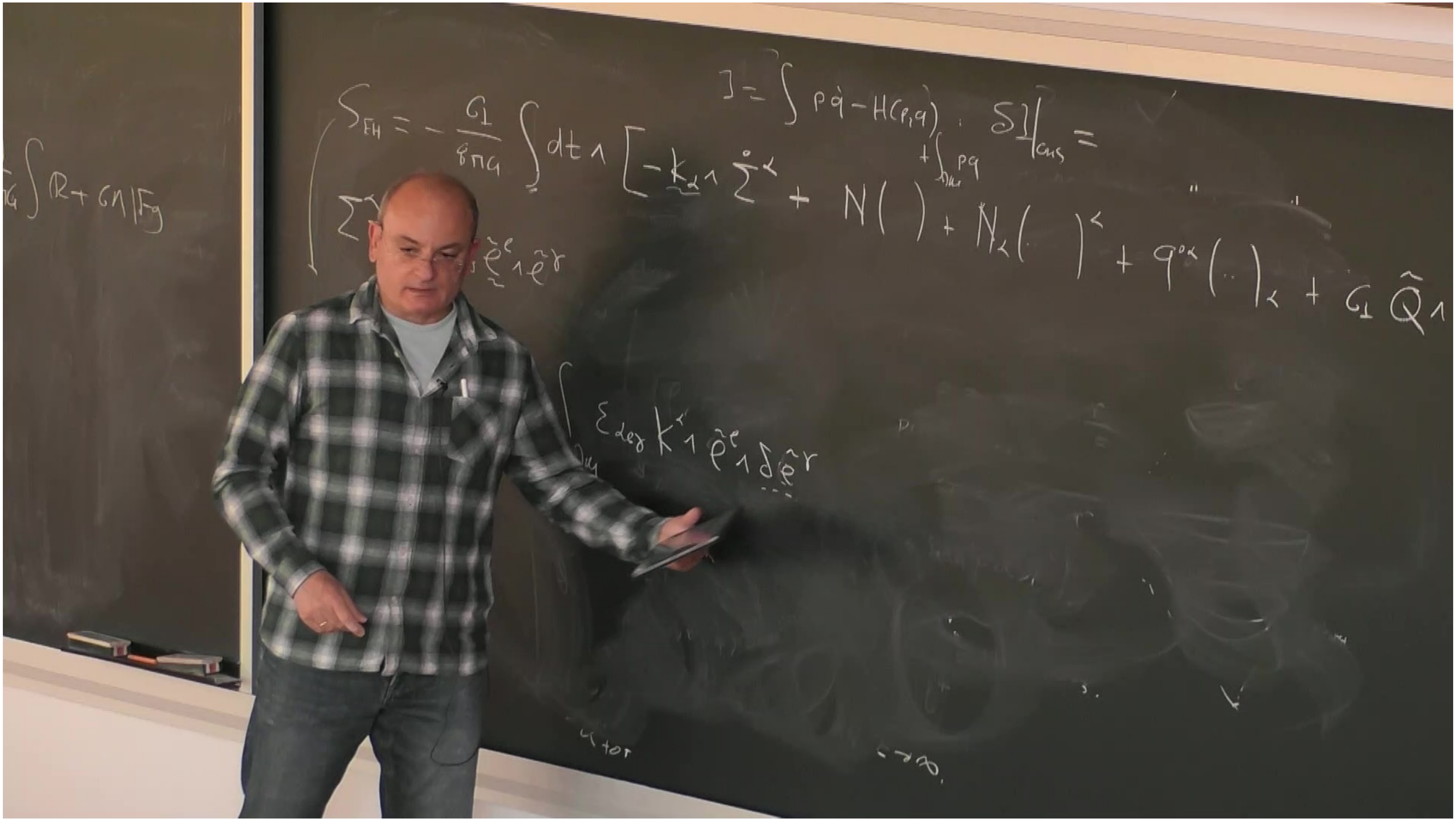
[Plebanski-Demianski] $\longleftrightarrow$ $\left\{ \begin{array}{l} \text{perfect relativistic fluid} \\ \text{or} \\ \text{a "generic" stationary background} \end{array} \right\}$ hydro variables T, p, u^μ ξ^μ

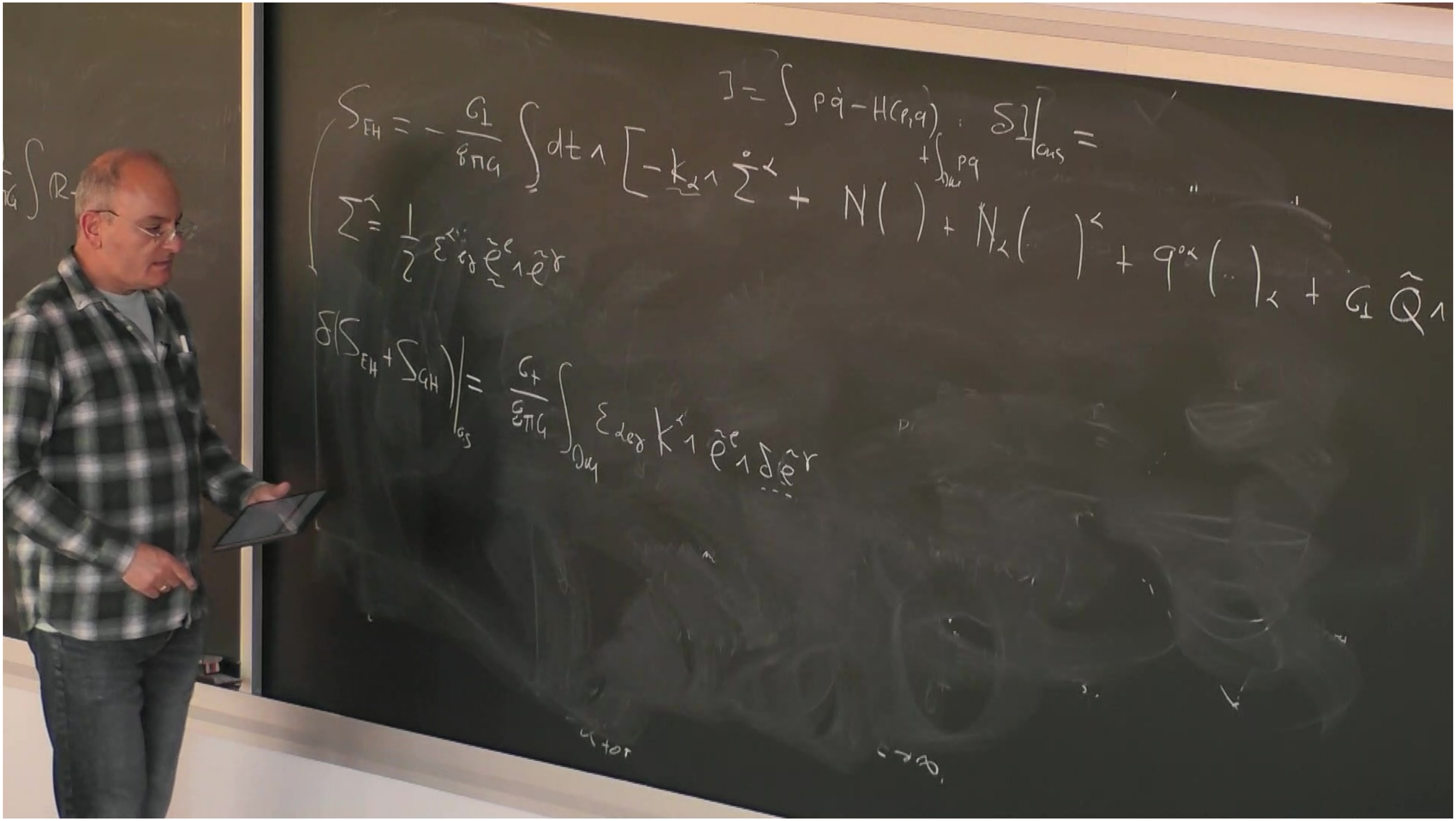
• Schw. AdS_4 : M $\longrightarrow$ T, p, ε

• Kerr AdS_4 : M, a $\longrightarrow$ $T, p, \varepsilon ; a$

TN AdS_4 : M, n $\longrightarrow$ $T, p, \varepsilon ; n$.


Holography from "canonical gravity"
S_EH = - 1/(32πG_N) ∫ ε_abcd (R^ab ∧ Λ/2 e^a ∧ e^b) ∧ e^c ∧ e^d
→ - 1/(16πG) ∫ R + 6Λ |F_g
"initial value formulation" of holography
e^0 = N dt   e^α = N^α dt + ẽ^α
W^0α = q^α dt + σ_1 γ^α ,   W^αβ = - ε^αβγ (Q_γ dt + B_γ)
AlAdS_4 spa
↓
exact solutio
vacuum.
Robinson-Tra
RT
• Schw. AdS_4
• Kerr AdS_4
TN AdS_4 :




$S = \int p\dot{q} - H(p,q)$ $+\int_{\partial M} pq$: $\delta S|_{\text{cus}} =$
$S_{EH} = -\frac{G_1}{8\pi G}\int dt \wedge \left[-k_\alpha \wedge \dot{\Sigma}^\alpha + N(\ ) + N_\alpha(\ )^\alpha + q^{0\alpha}(\ )_\alpha + G_1 \hat{Q} \wedge \right.$
$\Sigma^\alpha = \frac{1}{2}\epsilon^\alpha{}_{\beta\gamma}\tilde{e}^\beta \wedge \tilde{e}^\gamma$
$\delta(S_{EH} + S_{GH})|_{os} = \frac{G_+}{8\pi G}\int_{\partial M} \epsilon_{\alpha\beta\gamma} k^\alpha \wedge \tilde{e}^\beta \wedge \delta\tilde{e}^\gamma$

$$I = \int p\dot{q} - H(p,q) + \int_{\partial M} pq \qquad \delta I\big|_{cons} =$$

$$S_{EH} = -\frac{G_1}{8\pi G}\int dt \wedge \left[-k_\alpha \wedge \hat{\Sigma}^\alpha + N(\ ) + N_\alpha(\ )^\alpha + q^{o\alpha}(\ )_\alpha + G_1\, \hat{Q} \wedge k_\sigma \wedge \tilde{e}^\sigma \right] +$$

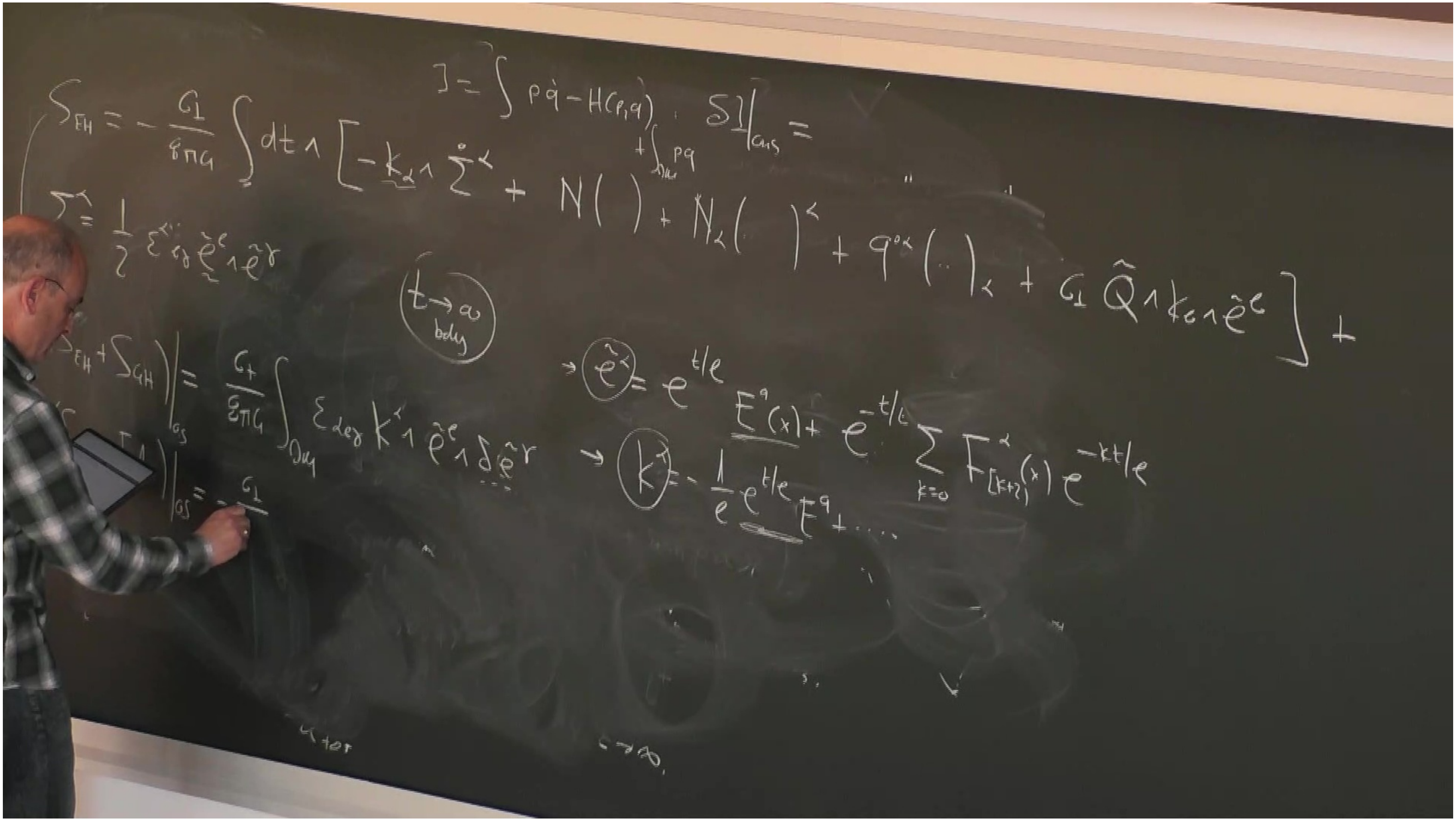
$$\hat{\Sigma} = \frac{1}{2}\varepsilon^\alpha{}_{\beta\gamma}\, \tilde{e}^\beta \wedge \tilde{e}^\gamma$$

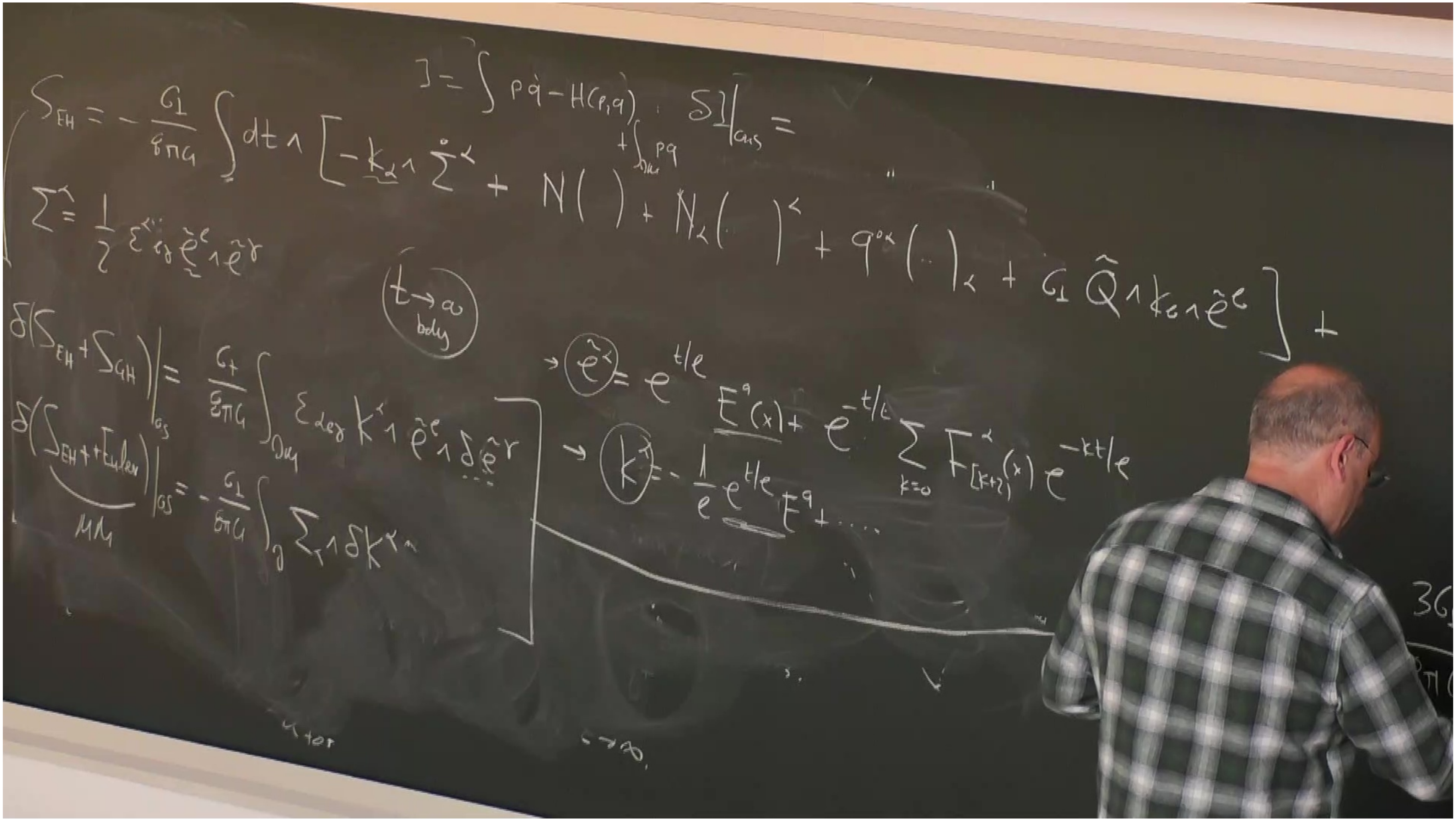
$t \to \infty$
bdy

$$\delta(S_{EH} + S_{GH})\big|_{os} = \frac{G_+}{8\pi G}\int_{\partial M} \varepsilon_{\alpha\beta\gamma}\, k^\alpha \wedge \tilde{e}^\beta \wedge \delta\tilde{e}^\gamma$$

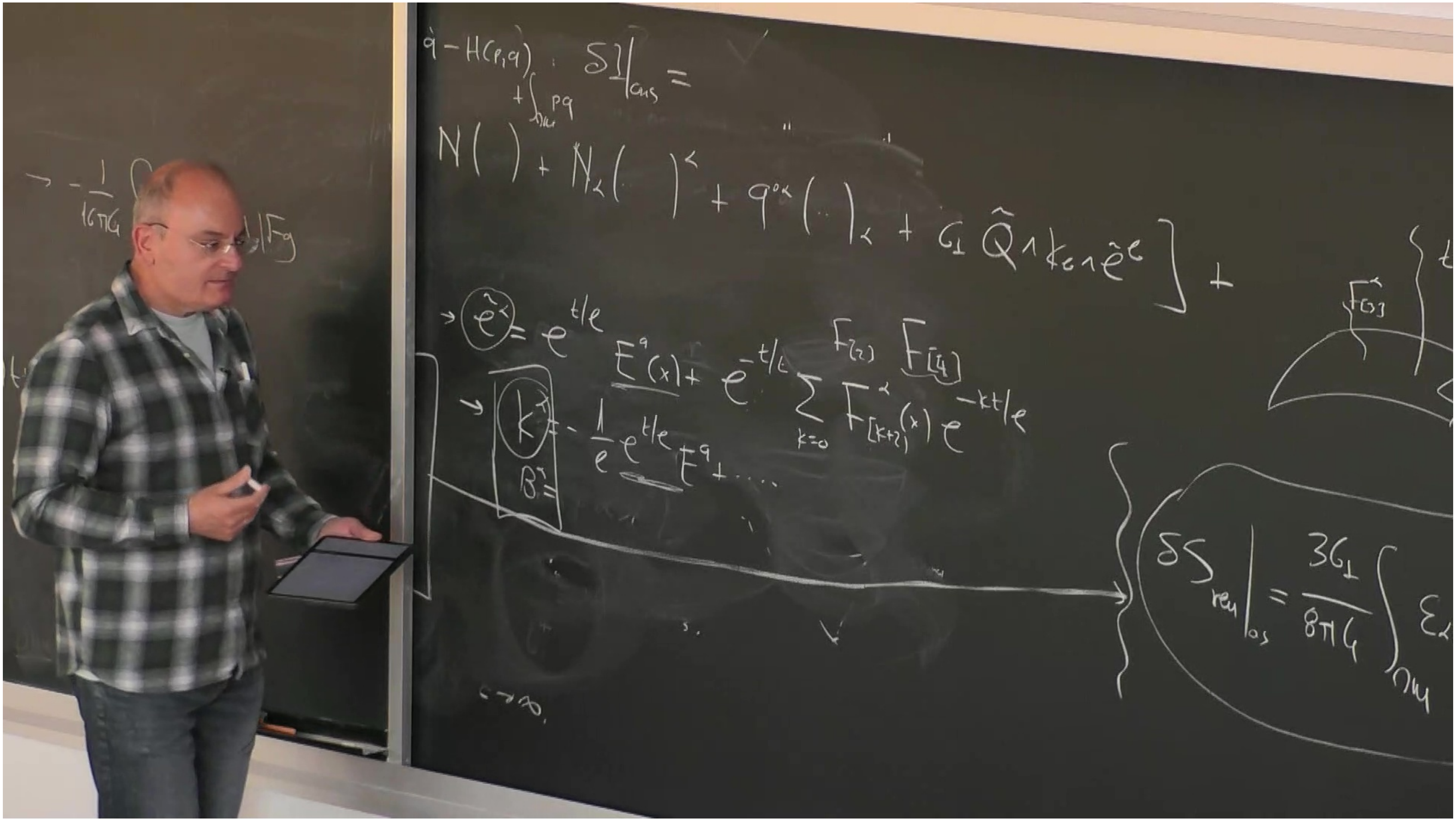
$$\tilde{e}^\alpha = e^{t/\ell} E^\alpha(x) + e^{-t/\ell} \sum_{k=0} F^\alpha_{[k+2]}(x)\, e^{-kt/\ell}$$

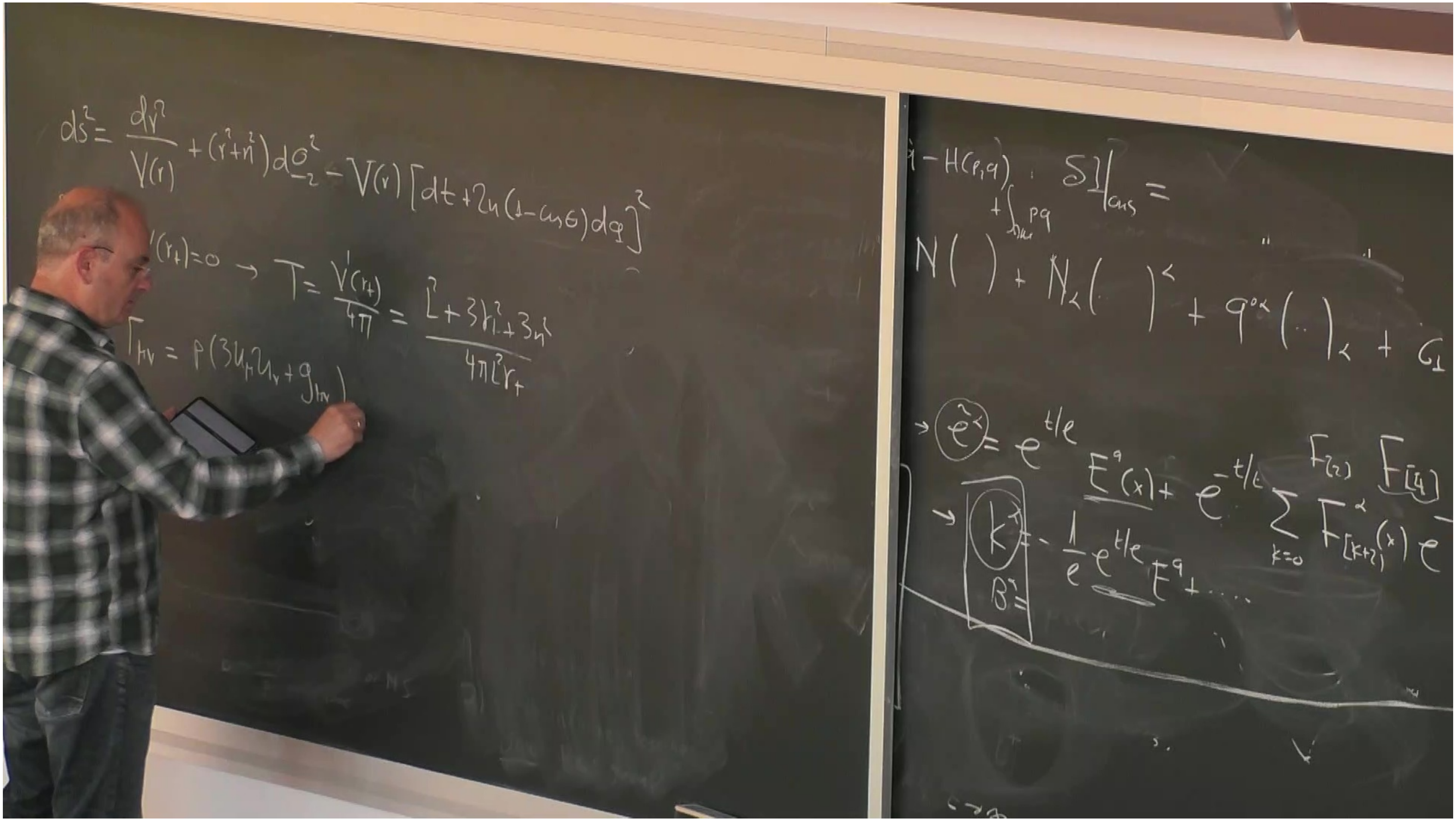
$$k^\alpha = -\frac{1}{\ell} e^{t/\ell} E^\alpha + \dots$$

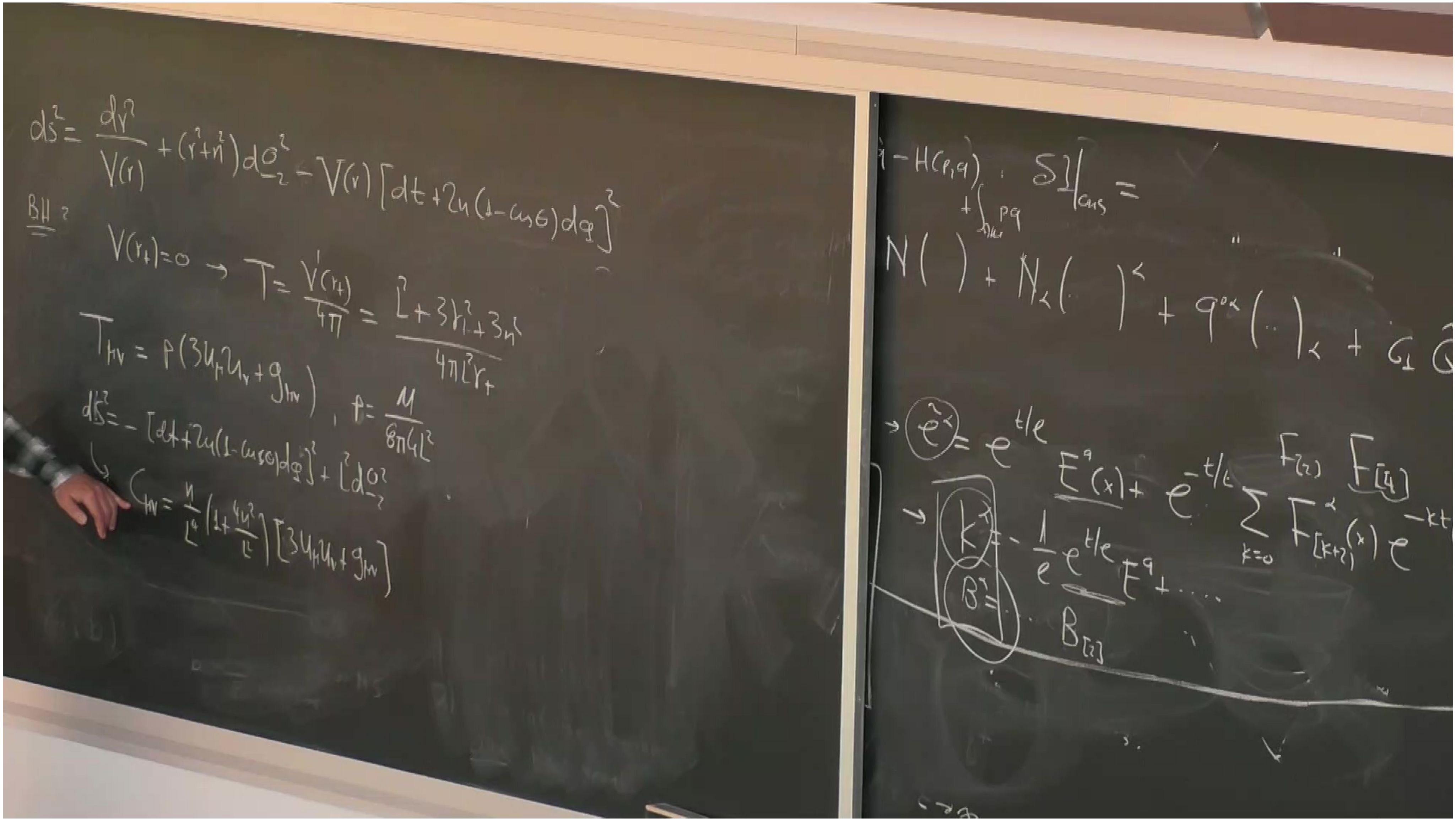


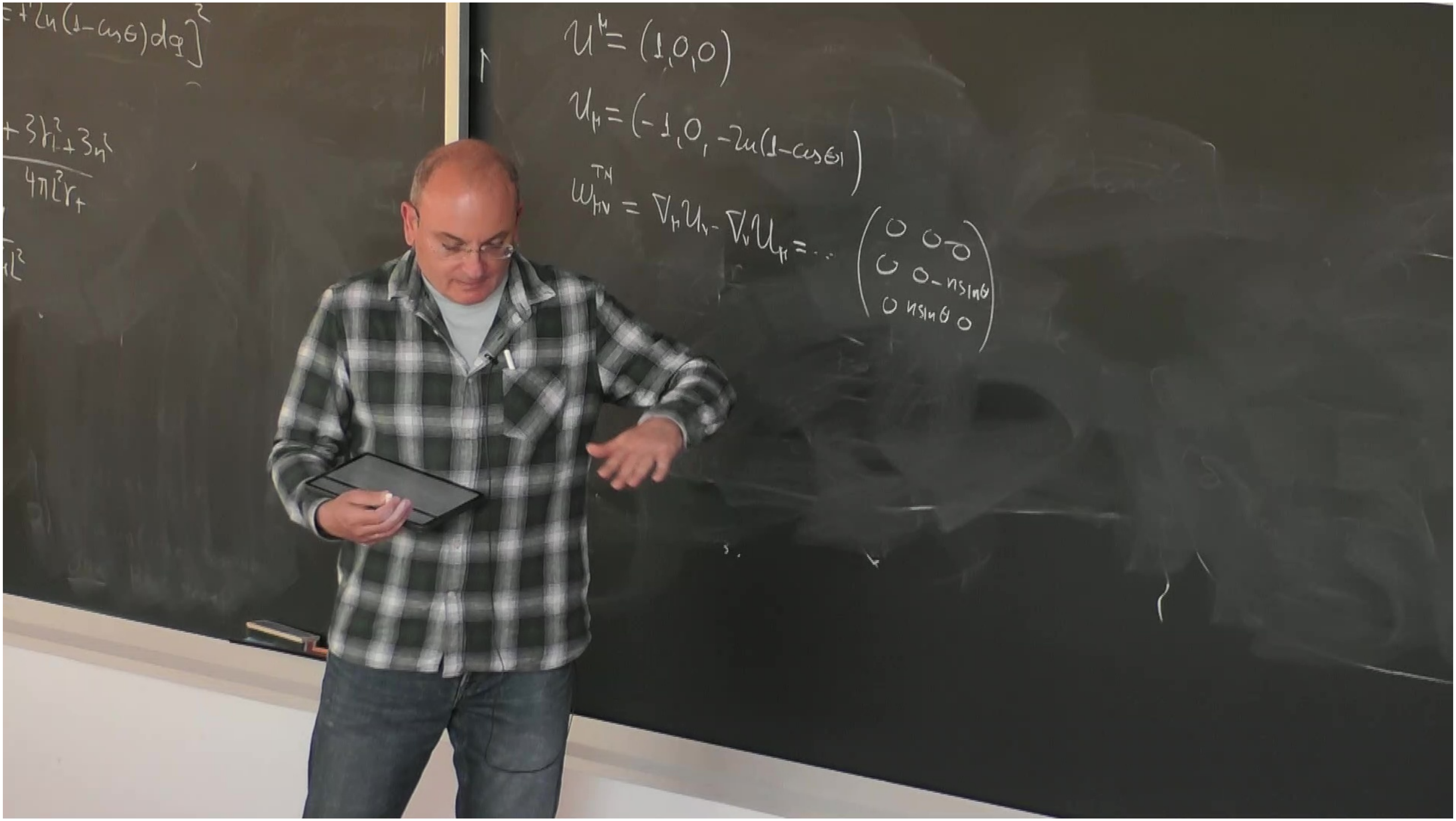


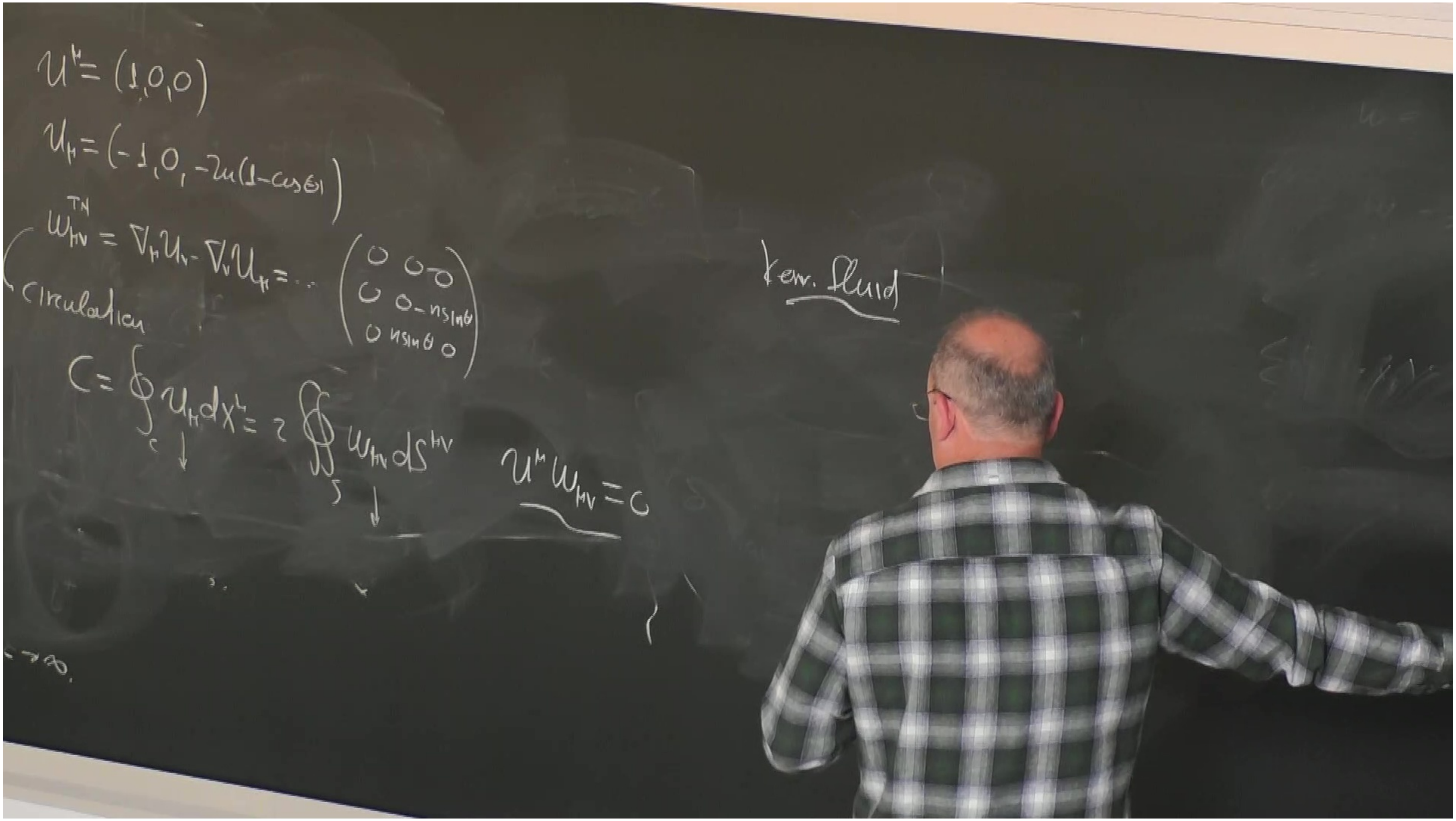
$S_{EH} = -\frac{G_1}{8\pi G} \int dt \wedge [-k_\alpha \wedge \dot{\Sigma}^\alpha + N(\ ) + N_\alpha(\ )^\alpha + q^{o\alpha}(\ )_\alpha + G_1 \hat{Q} \wedge k_\sigma \wedge \tilde{e}^\sigma] +$
$I = \int p\dot{q} - H(p,q)$
$+\int_{\partial M} pq$
$\delta I|_{cs} =$
$\Sigma^\alpha = \frac{1}{2} \varepsilon^{\alpha}{}_{\beta\gamma} \tilde{e}^\beta \wedge \tilde{e}^\gamma$
$t \to \infty$
bdy
$\delta(S_{EH} + S_{GH})|_{cs} = \frac{G_+}{8\pi G} \int_{\partial M} \varepsilon_{\alpha\beta\gamma} k^\alpha \wedge \tilde{e}^\beta \wedge \delta\tilde{e}^\gamma$
$\delta(S_{EH} + S_{Euler})|_{cs} = -\frac{G_1}{8\pi G} \int_\partial \Sigma_\gamma \wedge \delta k^\gamma$
MM
$\tilde{e}^\alpha = e^{t/\ell} E^a(x) + e^{-t/\ell} \sum_{k=0} F^\alpha_{[k+2]}(x) e^{-kt/\ell}$
$k^\alpha = -\frac{1}{\ell} e^{t/\ell} E^a + \dots$
$t \to \infty$

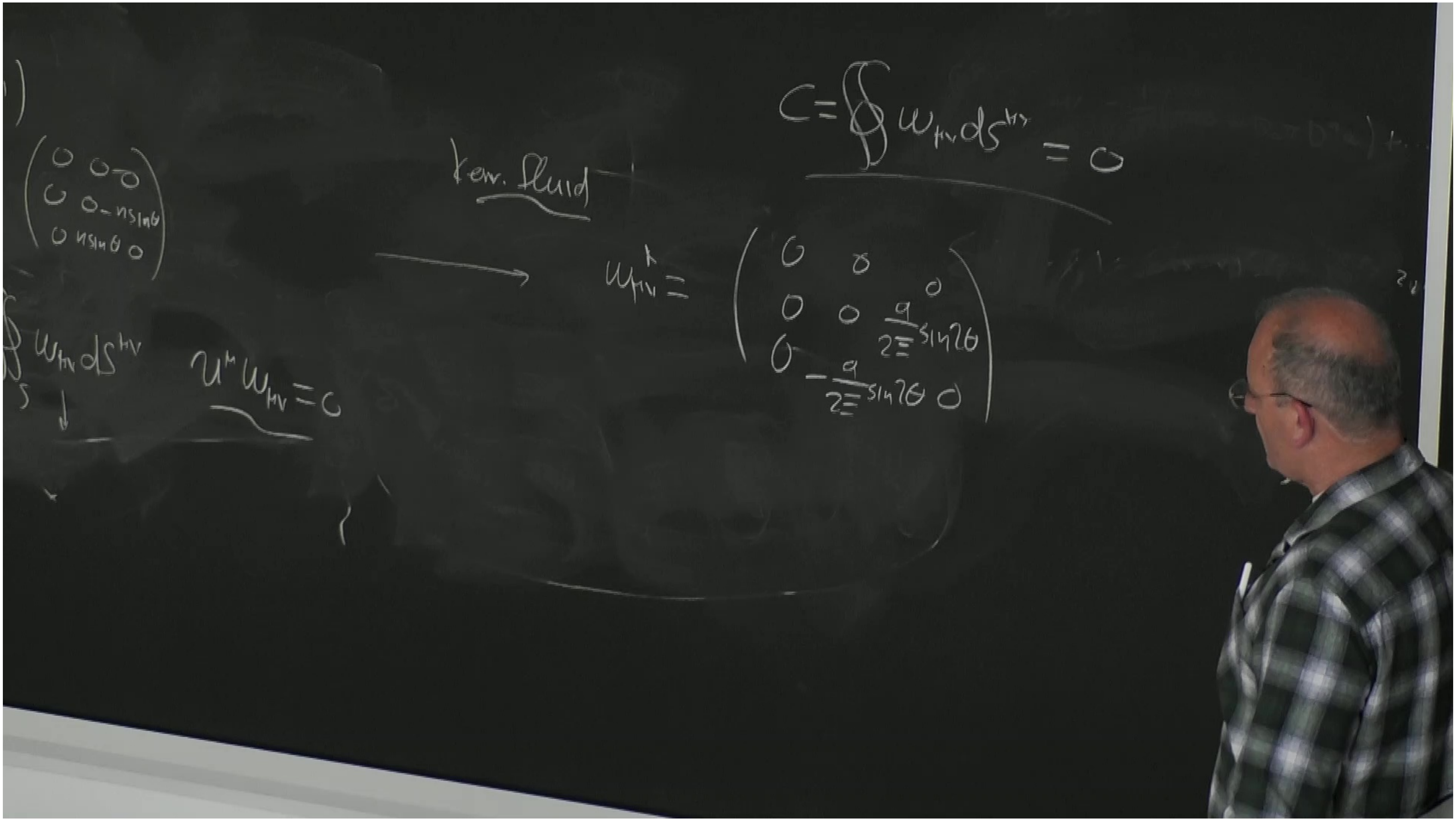

δS_ren|_os = 3G_1/8πG ∫_∂M ε_α
F_[2]
F_[4]
k=0

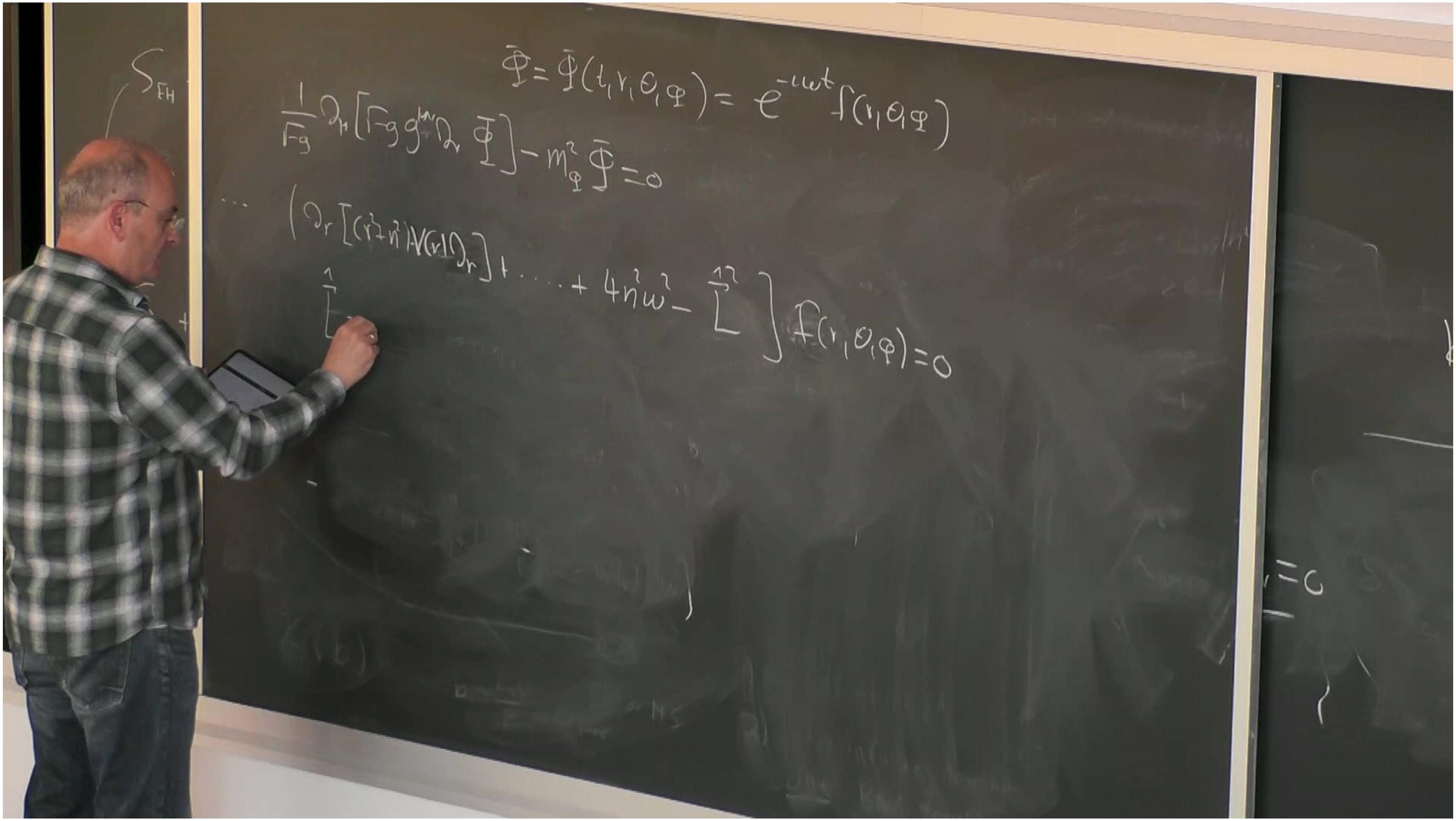



ds² = dr²/V(r) + (r²+n²) dΩ₂² − V(r) [dt + 2n(1 − cos θ) dφ]²
BH:
V(r₊) = 0 → T = V′(r₊)/4π = (L² + 3r₊² + 3n²)/(4π L² r₊)
T_μν = p(3u_μ u_ν + g_μν), p = M/(8πG L²)
ds² = − [dt + 2n(1 − cos θ) dφ]² + L² dΩ₂²
T_μν = M/L⁴ (1 + 4n²/L²) [3u_μ u_ν + g_μν]
S]|ons =
e^(t/l) E^a(x) + e^(−t/l) Σ_(k=0) F_[k+2](x) e^(−kt)
F_[2] F_[4]
k = − 1/l e^(t/l) E^a + ...
B_[2]

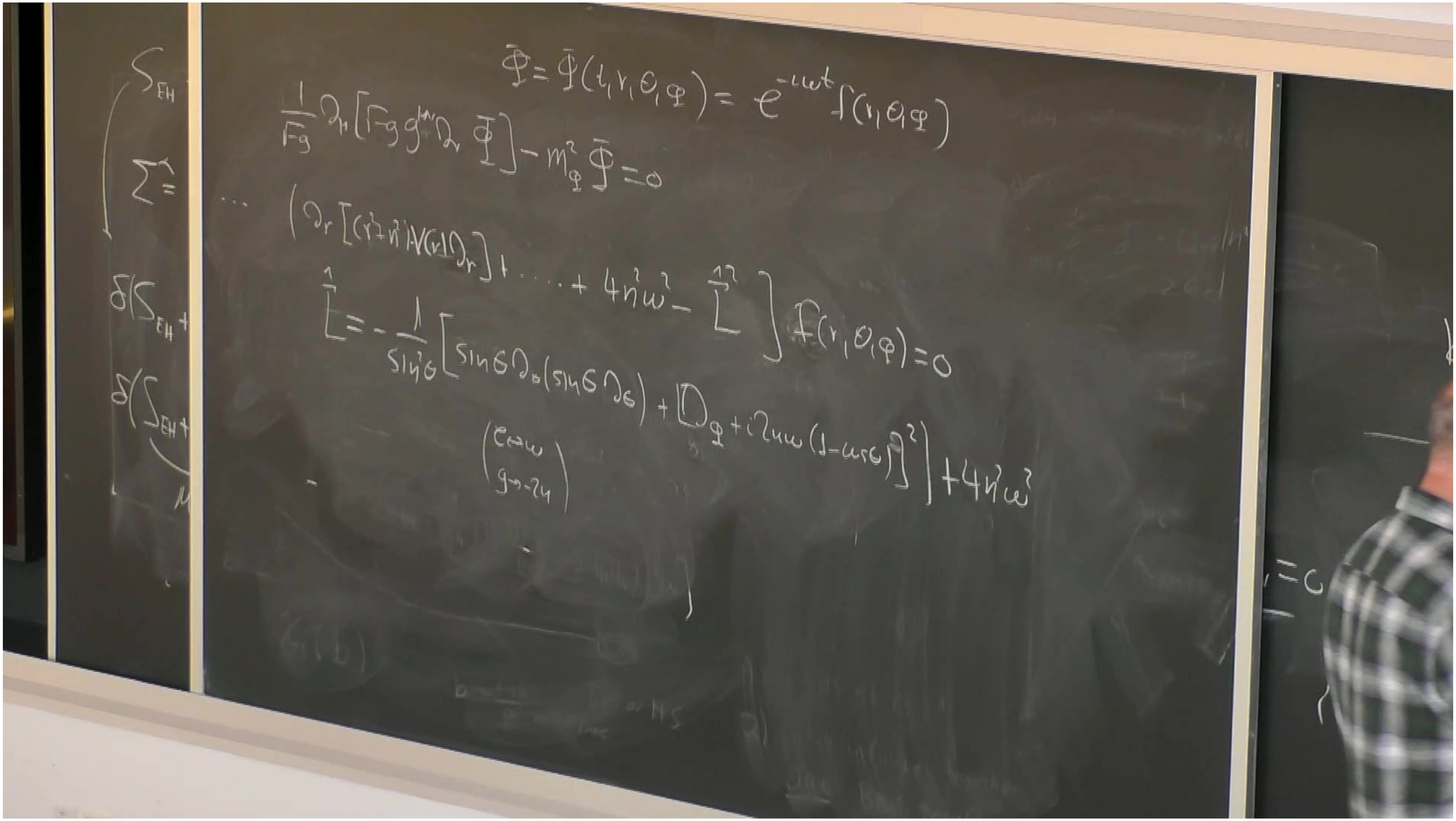

$U^\mu = (1,0,0)$
$U_\mu = (-1,0,-2n(1-\cos\theta))$
$\omega^{TN}_{\mu\nu} = \nabla_\mu U_\nu - \nabla_\nu U_\mu = \dots \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -n\sin\theta \\ 0 & n\sin\theta & 0 \end{pmatrix}$

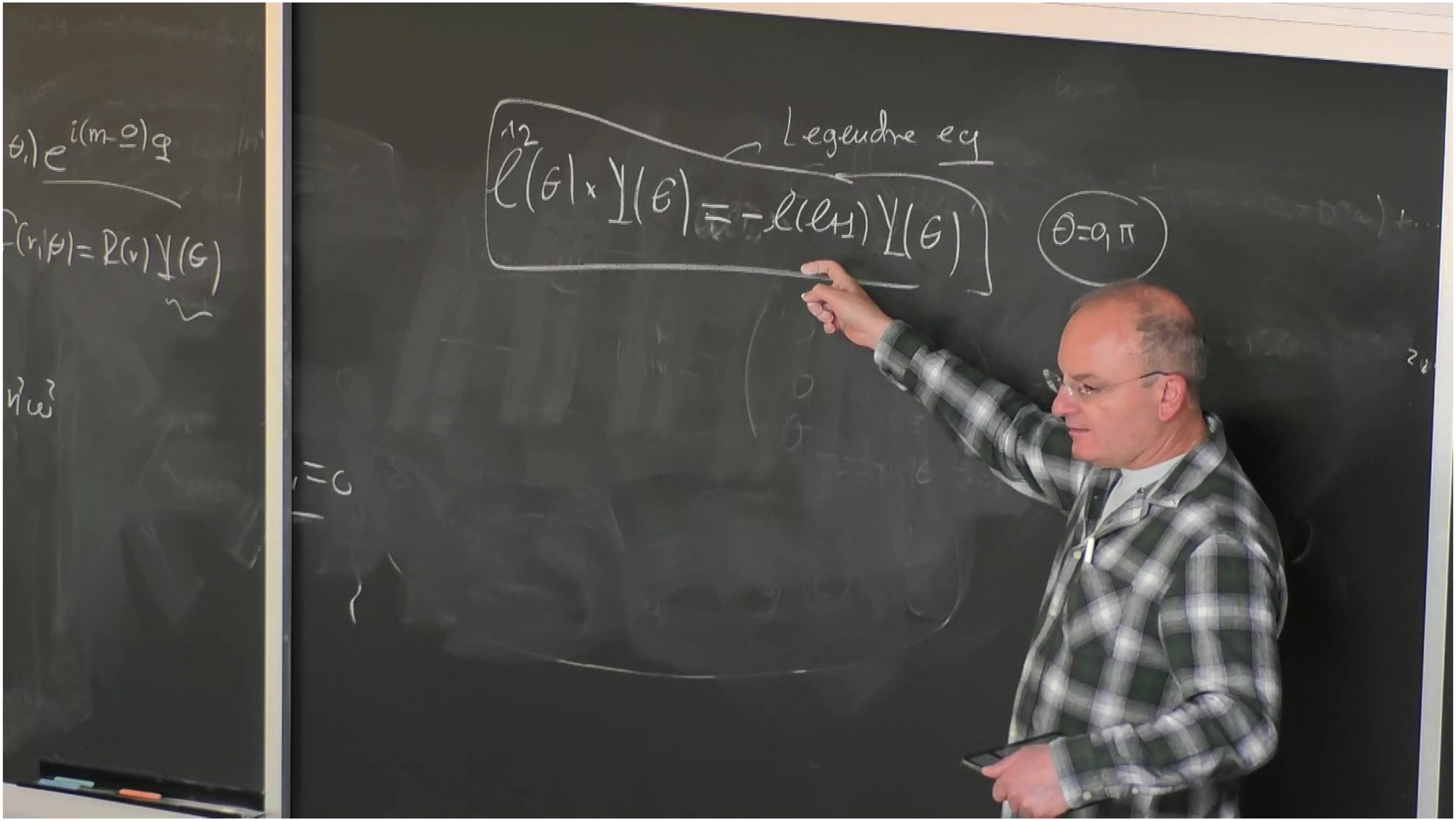

$u^\mu = (1,0,0)$
$u_\mu = (-1, 0, -2n(1-\cos\theta))$
$\omega^{TN}_{\mu\nu} = \nabla_\mu u_\nu - \nabla_\nu u_\mu = \ldots$
circulation
$C = \oint_c u_\mu dx^\mu = 2 \iint_S \omega_{\mu\nu} dS^{\mu\nu}$
$u^\mu \omega_{\mu\nu} = 0$
Kerr. fluid

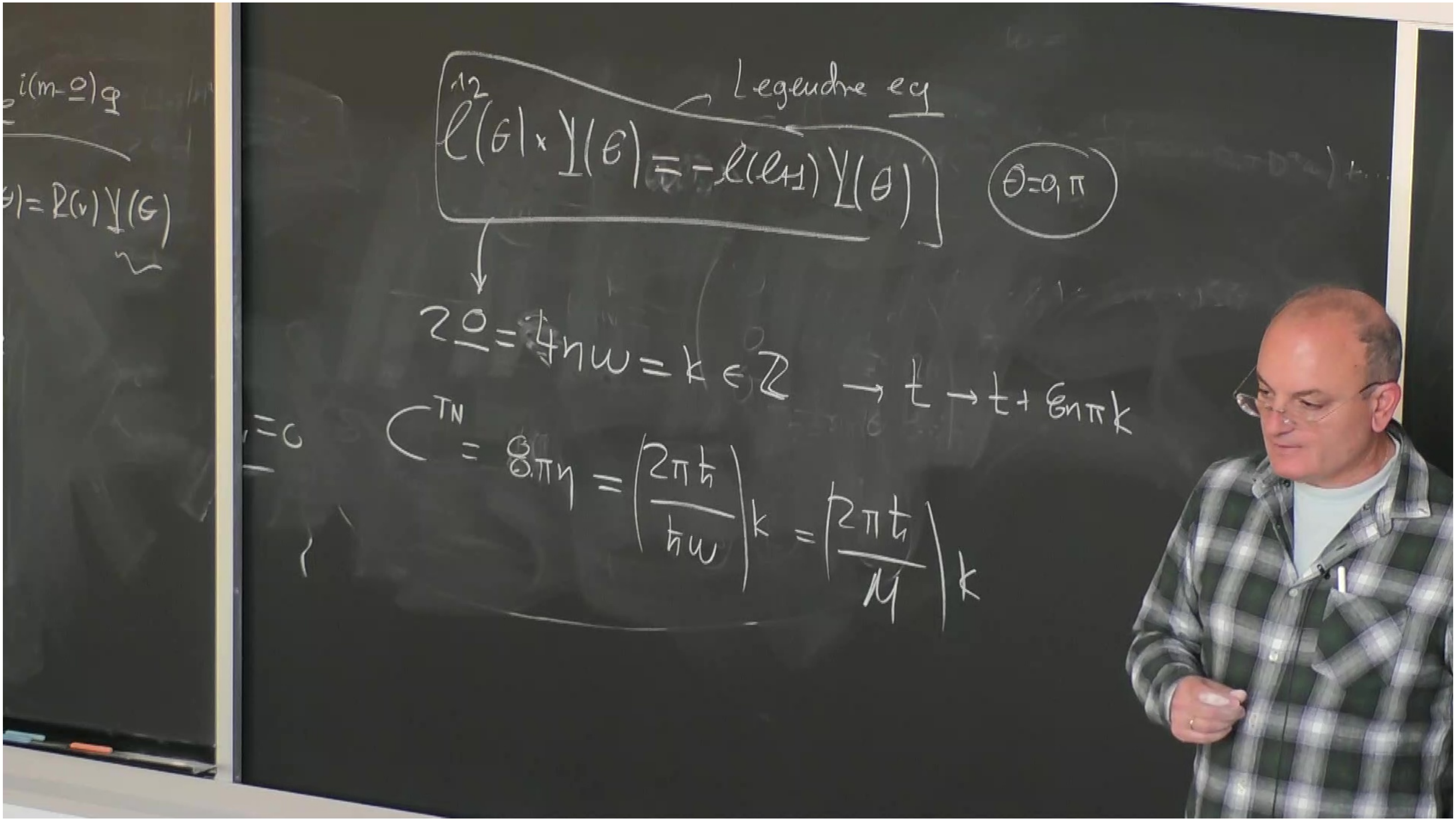

$C = \oint \omega_{\mu\nu} dS^{\mu\nu} = 0$
Kerr. fluid
$\omega_{\mu\nu} =$
$u^\mu \omega_{\mu\nu} = 0$
$\oint_S \omega_{\mu\nu} dS^{\mu\nu}$



S_{EH}
$\bar{\Phi} = \bar{\Phi}(t, r, \theta, \varphi) = e^{-i\omega t} f(r, \theta, \varphi)$
$\frac{1}{\sqrt{-g}} \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu} \partial_\nu \bar{\Phi} \right] - m_\Phi^2 \bar{\Phi} = 0$
$\ldots \left(\partial_r \left[(r^2 + n^2) V(r) \partial_r \right] + \ldots + 4 n^2 \omega^2 - \hat{L}^2 \right] f(r, \theta, \varphi) = 0$


Φ = Φ(t, r, θ, φ) = e^{-iωt} f(r, θ, φ)
1/√-g ∂_μ [√-g g^{μν} ∂_ν Φ] − m_Φ^2 Φ = 0
(∂_r [(r^2+a^2)Δ(r) ∂_r] + … + 4a^2ω^2 − L^2] f(r, θ, φ) = 0
L^2 = − 1/sin^2θ [sinθ ∂_θ (sinθ ∂_θ) + ∂_φ + i2aω (1−cosθ)]^2] + 4a^2ω^2


Legendre eq
ℓ²(θ) × Y(θ) = −ℓ(ℓ+1) Y(θ)
θ=0, π
f(r,θ) = R(r) Y(θ)


Legendre eq
θ=0, π