Title: Lieb-Schultz-Mattis, â€™t Hooft and Luttinger: anomalies in lattice systems

Speakers: Meng Cheng

Series: Quantum Matter

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Abstract: Macroscopic physics of a quantum many-body systems on a lattice is commonly captured by a continuum field theory. We will discuss the interplay between lattice effects and continuum theory from the perspective of symmetry and 't Hooft anomalies. In the first part of the talk, using the example of a spin-1/2 XXZ chain, we will show how the continuum limit of a lattice model is properly described in terms of a field theory with topological defects. In particular, anomaly explains a curious size dependence of the ground state momentum in the XXZ chain. In the second part, we will examine U(1) filling anomaly for subsystem symmetries. With a generalized flux-insertion argument, we derive nontrivial constraints on the mobility of excitations in a symmetry-preserving gapped phase.

Zoom link: https://pitp.zoom.us/j/96117447396?pwd=QVNaSHdHeDh1RENvenRjamVlVGNudz09

The problem of emergence in condensed matter

Lattice models with local interactions

Assumption: lattice systems have a nice continuum QFT description of the low-energy physics

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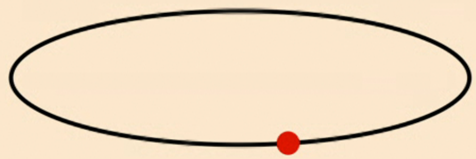
No-go (“LSM”) theorem: a system with ’t Hooft anomaly can not have a trivial low-energy theory

Anomaly matching: Low-energy theory must reproduce the anomalous response

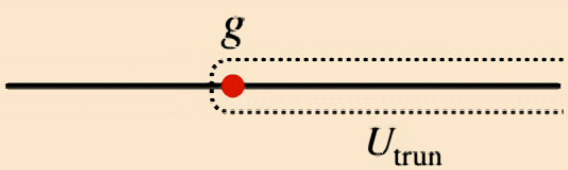
Symmetry defects



A defect is a localized change of the Hamiltonian

Topological = defects can be moved with a unitary

Does not affect local physics



Construct a defect from a global symmetry U_g for $g \in G$

Truncate U_g to a finite interval

$U_{\text{trun}} H U_{\text{trun}}^{-1}$ only differs from H at the end → a defect

Results do not depend on the details of the Hamiltonian.

Probing anomaly using symmetry defects

For internal symmetries,'t Hooft anomaly can be extracted by a "local" computation, complete for bosonic systems in 1+1d (see Else and Nayak; Kawagoe and Levin).

However, it is not clear how to apply them to spatial symmetries.

The continuum limit

It is convenient to consider a finite-volume system, and then take $L \to \infty$

Sometimes the limit is smooth in system size

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Sometimes the limit is not smooth in system size

Many examples in spin-1/2 systems with anti-ferromagnetic interactions

There is a well-defined QFT in continuum, but $L \to \infty$ is not smooth

Different limits correspond to QFT with topological defects inserted

Non-smooth limits are often related to anomaly of **emanant** symmetry

Non-smooth continuum limit: spin-1/2 Heisenberg chain

$$H = \sum_{j=1}^{L} \mathbf{S}_j \cdot \mathbf{S}_{j+1} + \cdots$$

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However, for even L there are still two limits

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Eigenstates labeled by lattice momentum $T = e^{iK}$

L	G.S. K
$L = 4k$	0
$L = 4k + 2$	π
Odd L	$\pm\pi/2 + O(L^{-1})$

Emergent vs Emanant symmetry

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UV symmetry = $\mathrm{SO}(3) \times \text{translations}$

Emergent symmetry in the IR:
$\mathrm{SO}(4) = \left[\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R\right]/\mathbb{Z}_2$

Usually emergent symmetries, e.g. $\mathrm{SU}(2)_L$, are broken by irrelevant operators

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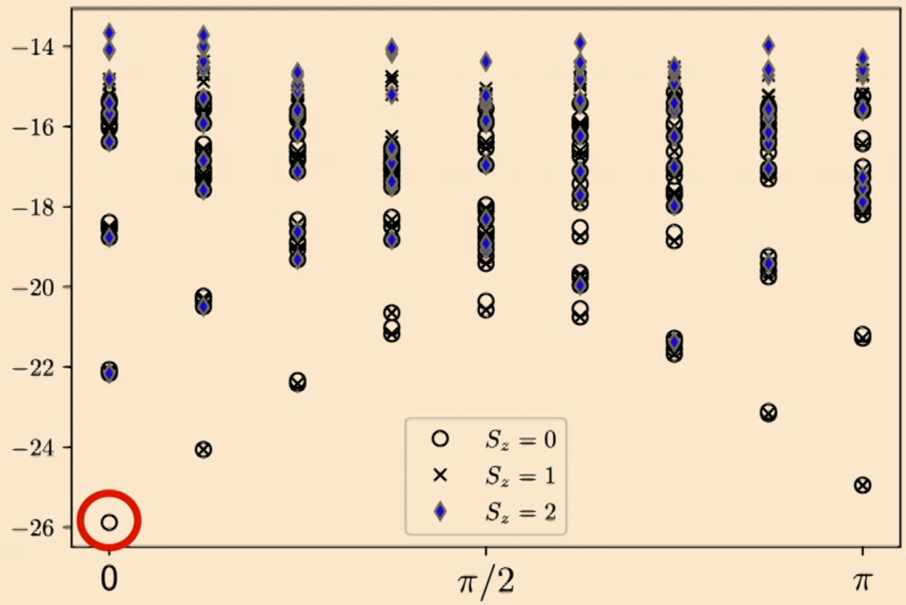
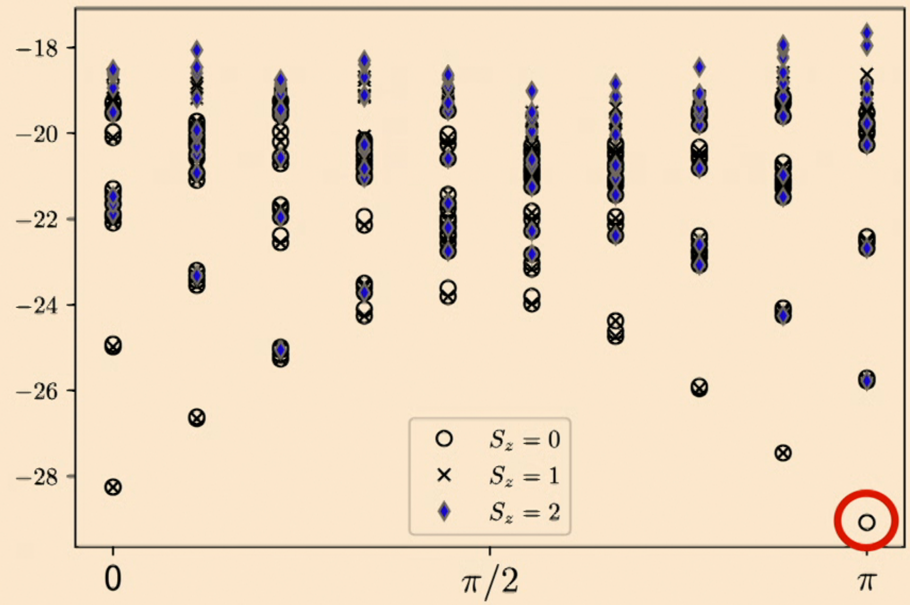
C is an example of an **emanant** symmetry

Finite-size spectra of spin-1/2 AFM Heisenberg chain

Eigenstates labeled by lattice momentum K (so $T = e^{iK}$)

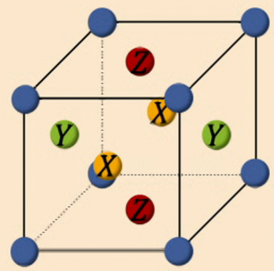


$L = 16$

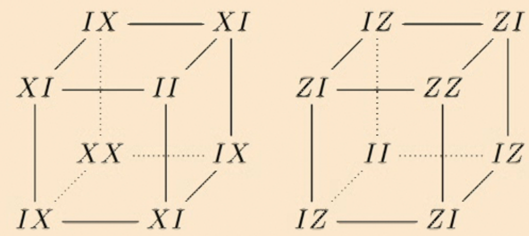


$L = 18$

Non-smooth continuum limit: fracton



Chamon's model
Also Bravyi et al



Haah's code
1 out of 18

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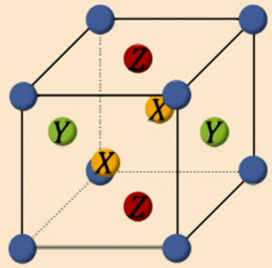

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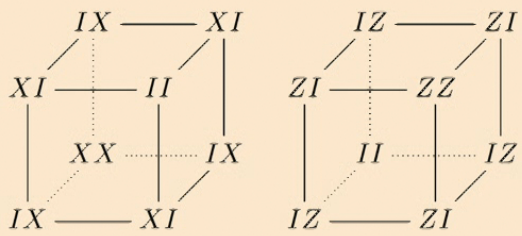

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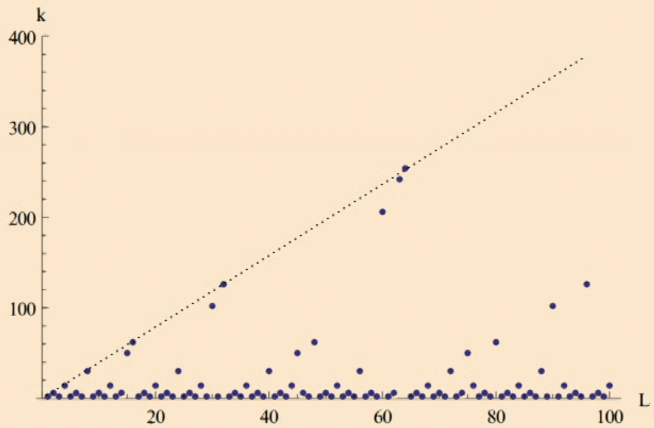

$k = \log_2 \text{GSD}$

Topological ground state degeneracy on L^3 torus depends on L

Finite-size spectra of spin-1/2 AFM Heisenberg chain

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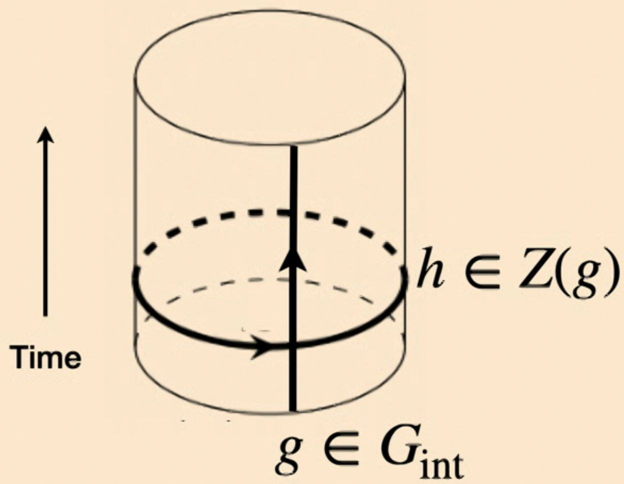
Explain the universal aspects of the finite-L low-energy spectra using the continuum theory?

Defects in spin chains



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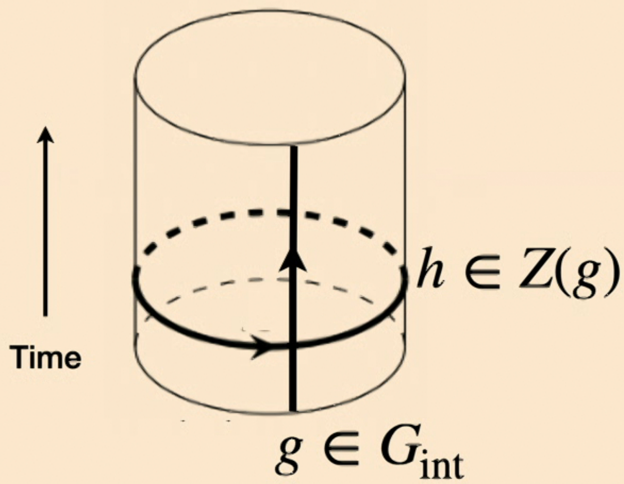

G_{int} is the group of internal symmetry

(Spatial) defect labeled by $g \in G_{\text{int}}$

Symmetry of the system with a g-defect is modified:

$$Z(g) = \{h \in G \,|\, hg = gh\}$$

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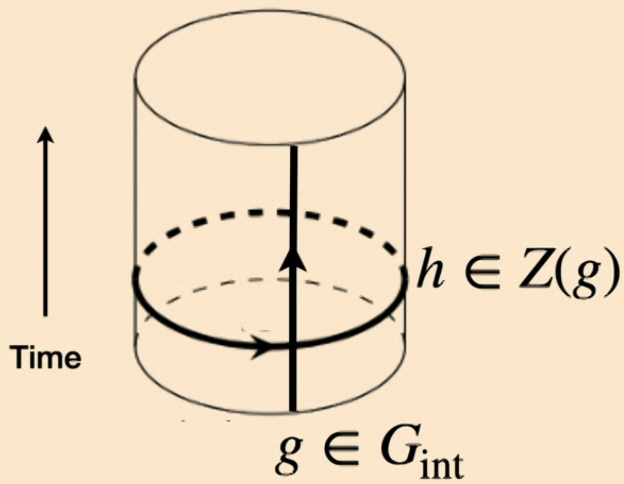

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SO(3) defects in spin-1/2 Heisenberg chain

Symmetry: $G_{\text{int}} = \text{SO}(3), G_{\text{lat}} = \mathbb{Z}_L = \{T \,|\, T^L = 1\}$

Up to conjugacy $g = e^{i\sigma S_z}, \sigma \sim \sigma + 2\pi \sim -\sigma$

Remaining symmetry $G_{\text{int}}(\sigma) = \begin{cases} \text{SO}(3) & \sigma = 0 \\ \text{O}(2) & \sigma = \pi \\ \text{U}(1) & \sigma \neq 0,\pi \end{cases}$

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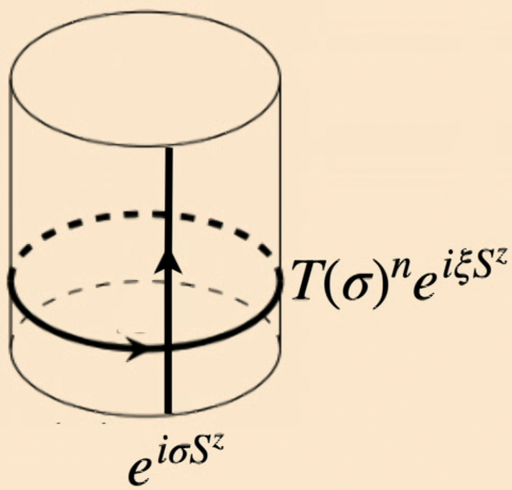
$e^{i\sigma S^z}$ defect at link $(J, J+1)$: $H[\sigma] = \sum_{j \neq J} \mathbf{S}_j \cdot \mathbf{S}_{j+1} + \frac{1}{2}(e^{-i\sigma} S_J^+ S_{J+1}^- + e^{i\sigma} S_J^- S_{J+1}^+) + S_J^z S_{J+1}^z$

The original translation operator does not commute with $H[\sigma]$

Need to modify it to $T(\sigma) = e^{i\sigma S_{J+1}^z} T$

LSM anomaly in spin-1/2 chain



Oshikawa 2000

Defect partition function: $Z(\beta, L, \sigma, n, \xi) = \mathrm{Tr}[e^{-\beta H[\sigma]} T(\sigma)^n e^{i\xi S^z}]$

(σ, ξ) the most general background SO(3) gauge field on a spacetime torus

- Additional possibility when σ or ξ equal to π, next slide

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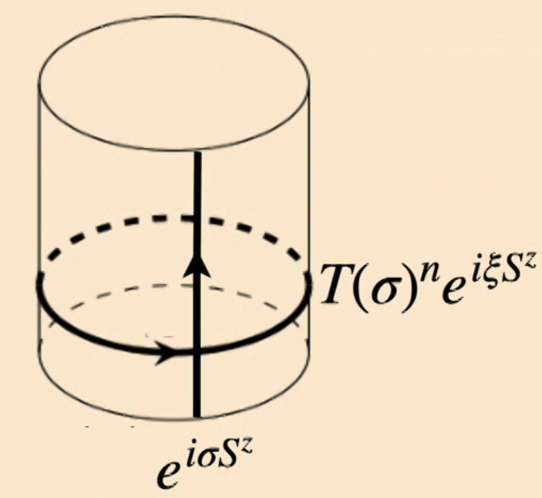

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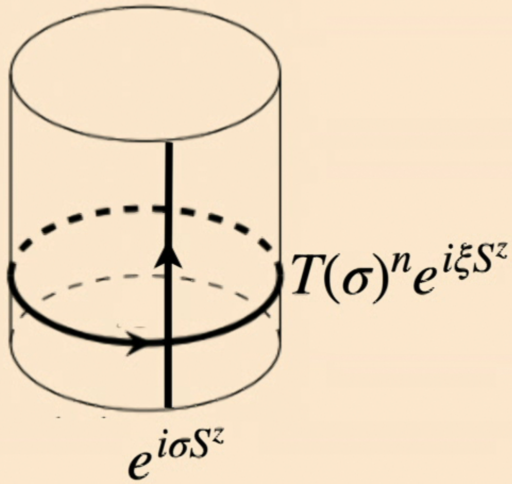

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$T(\sigma)^n$ is a temporal gauge field of translation

$Z(\beta, L, \sigma, n + L, \xi) = Z(\beta, L, \sigma, n, \xi + \sigma)$ follows from $T(\sigma)^L = e^{i\sigma S^z}$

$Z(\beta, L, -\sigma, n, -\xi) = Z(\beta, L, \sigma, n, \xi)$ follows from $e^{i\pi S^x}$ symmetry

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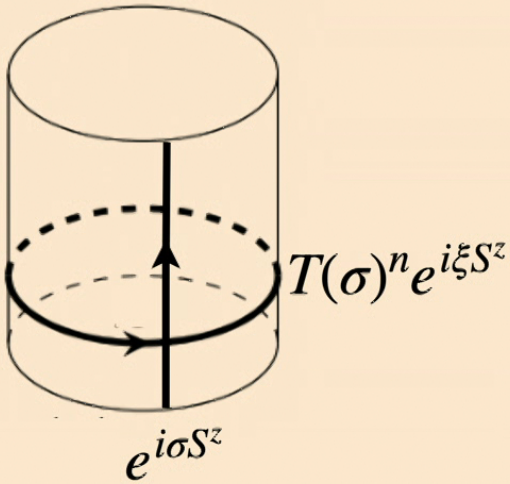

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These phases can be changed by redefining operators, but can not be all eliminated

The continuum theory of the XXZ chain

$$H = \sum_{j=1}^{L} (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \lambda_z S_j^z S_{j+1}^z)$$

Global symmetry: spin O(2) and lattice translation $\mathbb{Z}_L$

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$$S^z = Q^m$$

$$T = C^{-1} e^{\frac{2\pi i}{L}P}, \; C = e^{i\pi(Q^m + Q^w)}$$

$$C : \phi \to \phi + \pi, \theta \to \theta + \pi$$

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$\sim$ the continuum limit of the LSM anomaly

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Because $C^2 = 1$, L and $L + 2$ have identical low-energy spectrum (up to rescaling)

So naively, the continuum limit depends on L mod 2

’t Hooft anomaly of $\mathbb{Z}_2^C$

$$Z_{2+1}(a) = \exp\left(i\pi \int \left(w_2^{\mathrm{O}(2)} \cup a + a \cup a \cup a\right)\right)$$

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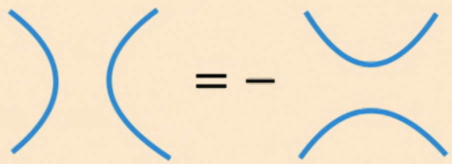
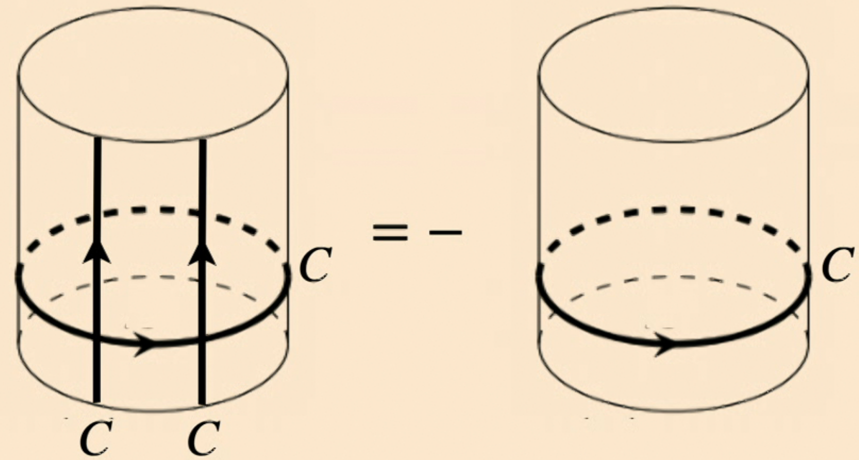
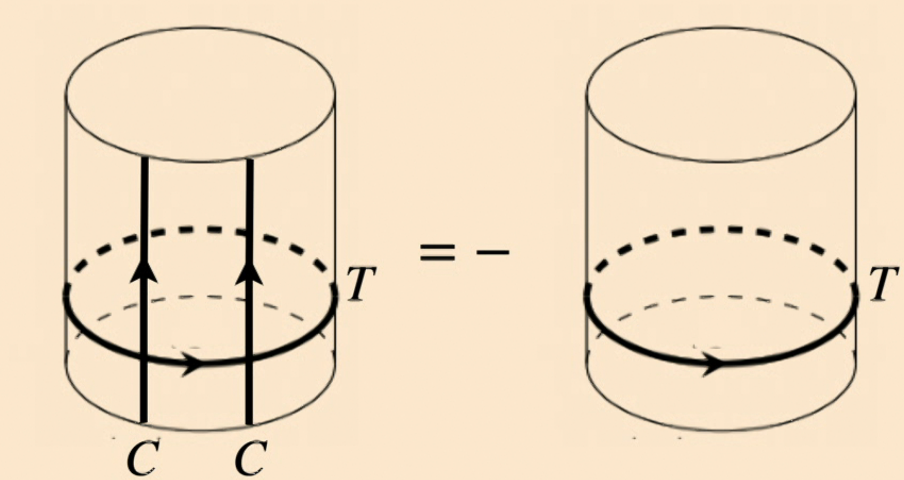
For $\mathbb{Z}_2$ symmetry, 1+1 anomaly from crossing relation:

Chang et al 2018; Lin and Shao 2019



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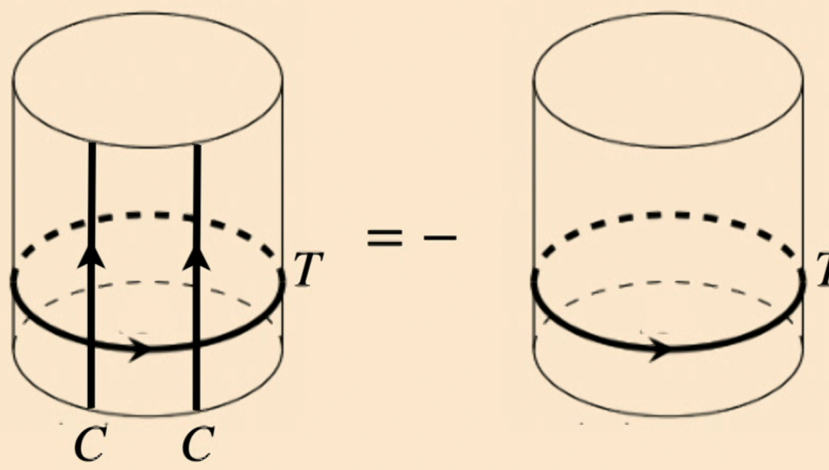

Lattice interpretation: the lattice momentum $T(L+2) = -\,T(L)$

Therefore the continuum limit depends on L mod 4

Lattice momentum of the ground state

	$(Q, \overline{Q})$	T
Even L	$(0,0)$	i^L
Odd L	$\left(\pm\frac{1}{2},0\right)$	$-i^L e^{\frac{i\pi}{2L}}$
	$\left(0,\pm\frac{1}{2}\right)$	$i^L e^{-\frac{i\pi}{2L}}$

Works even for $L = 2,3$ **!**

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Filling anomaly

Translation-invariant lattice models with U(1) symmetry

Total charge $Q = \sum_x n_x, n_x \in \mathbb{Z}$

Thermodynamic limit with a fixed filling factor $\nu = \frac{Q}{N_s}$

Assume that ν is a rational number $\nu = \frac{p}{q}$

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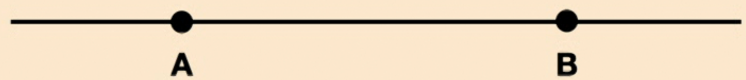
What kinds of LRE states are allowed?

U(1) defect operator



$$Q_{AB} = \sum_{x_A < x < x_B} n_x$$

For a gapped GS $|\psi\rangle$: $Q_{AB}|\psi\rangle = (K_B - K_A)|\psi\rangle$

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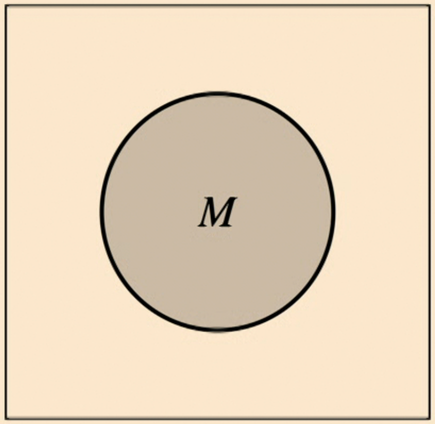
$$e^{2\pi i Q_{AB}}|\psi\rangle = e^{2\pi i K_B} e^{-2\pi i K_A}|\psi\rangle$$

$|\psi\rangle$ has a uniform charge density ν: $\langle\psi| e^{2\pi i K_B} e^{-2\pi i K_A} |\psi\rangle = e^{2\pi i \nu |x_B - x_A|}$

If $\nu \in \mathbb{Z}$, translation symmetry breaking in the ground state

LSM from U(1) defect operator



$$e^{i\theta Q_M}\,|\psi\rangle = U_{\partial M}(\theta)\,|\psi\rangle$$

$$\langle U_{\partial M}(2\pi)\rangle = e^{2\pi i \nu N_M}$$

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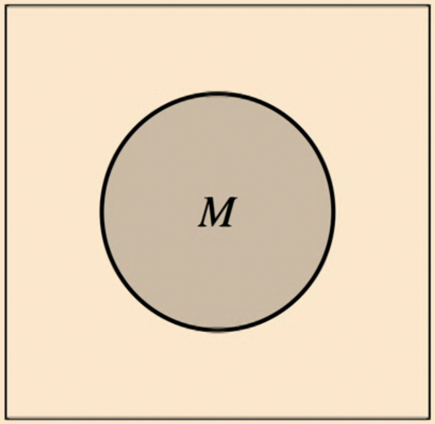

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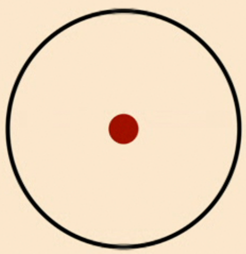
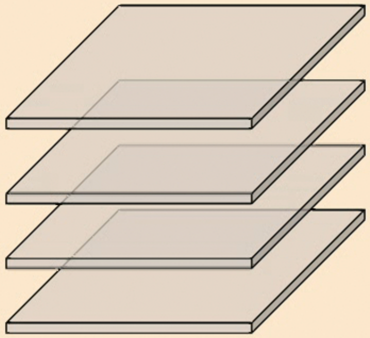


"Vison" (2π flux insertion) braids around the background anyon

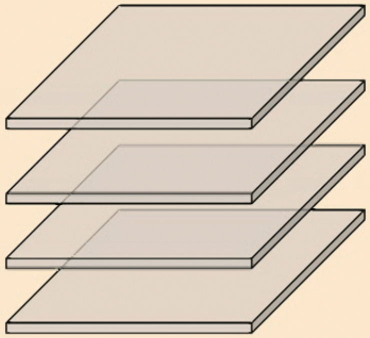
Subsystem symmetry



Z_2 symmetry on each plane

Charged particle are planons

Subsystem symmetry



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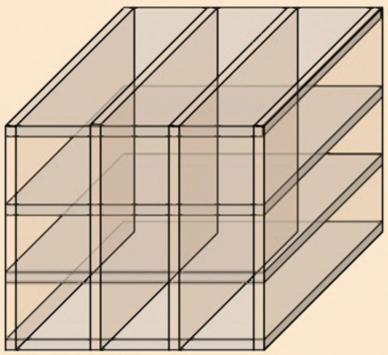
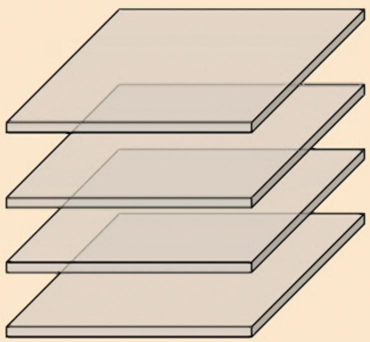


Z_2 symmetries on two sets of planes

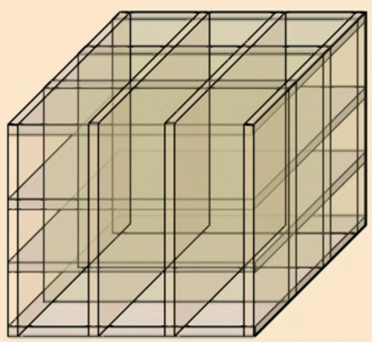
Charged particle are lineons

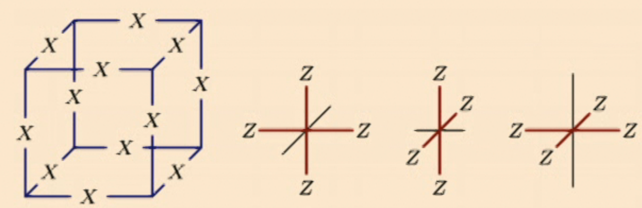
Fractons from gauging subsystem symmetry



Gauging subsystem symmetries

Stack of $\mathbb{Z}_2$ toric codes





X-cube model with fractons

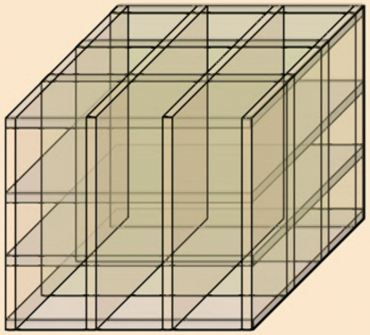
Vijay, Haah and Fu, PRB 2016

Subsystem U(1) LSM theorem

Translation-invariant lattice models with U(1) symmetry

Total charge $Q = \sum_{\mathbf{r}} n_{\mathbf{r}}, n_{\mathbf{r}} \in \mathbb{Z}$

Thermodynamic limit with a fixed filling factor $\nu = \frac{Q}{N_s} = \frac{p}{q}$



3-foliated subsystem U(1) symmetry:

$$Q_{xy}(z) = \sum_{x,y} n_{x,y,z},$$

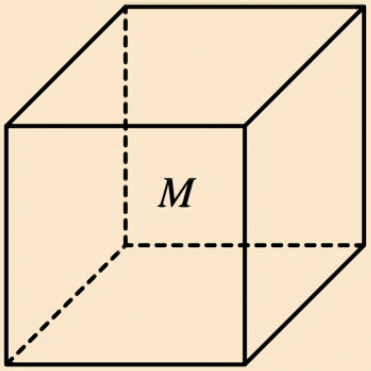
$$Q_{xz}(y) = \sum_{x,z} n_{x,y,z},$$

$$Q_{yz}(x) = \sum_{y,z} n_{x,y,z}$$

He, You and Prem 2020

U(1) subsystem LSM in 3D

The (global) U(1) LSM implies that a gapped symmetric state must be LRE



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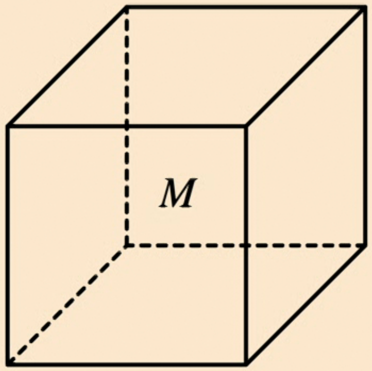

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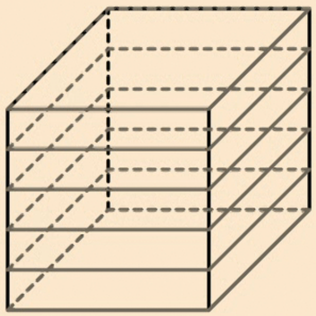
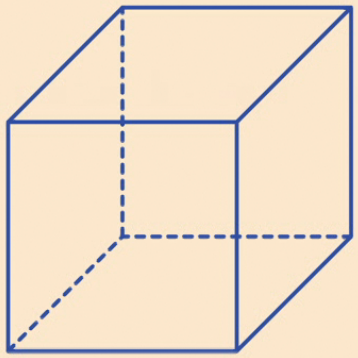


$$Q_M = \sum_z Q_{xy}(z; M)$$

$U_{\partial M}(2\pi)$ has no support on the top and bottom surfaces!

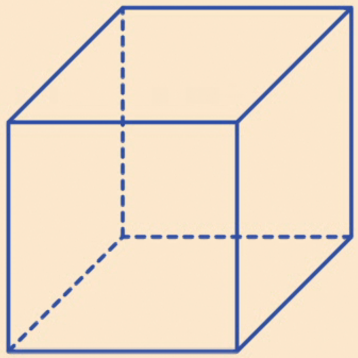
U(1) subsystem LSM in 3D



$U_{\partial M}(2\pi)$ is only supported on the "cage"

$$\langle U_{\partial M}(2\pi) \rangle = e^{2\pi i \nu N_M}$$

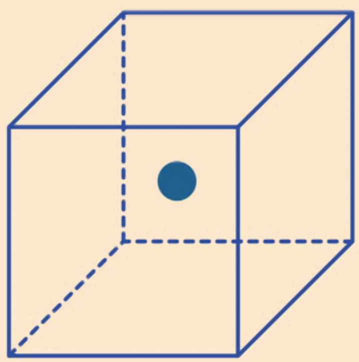
U(1) subsystem LSM in 3D



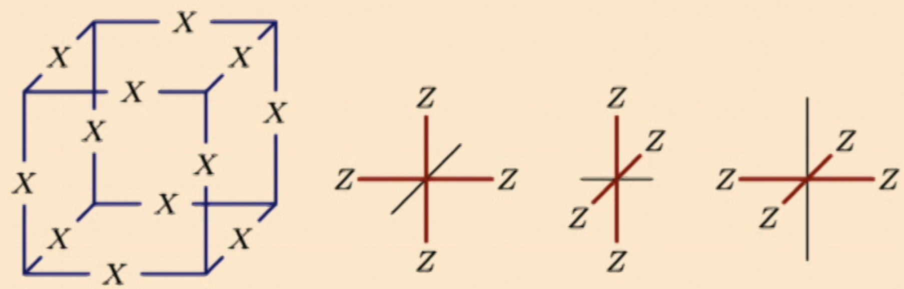
$U_{\partial M}(2\pi)$ is only supported on the "cage"

$$\langle U_{\partial M}(2\pi)\rangle = e^{2\pi i \nu N_M}$$

Suggest: a particle with an unusual braiding with a background particle?

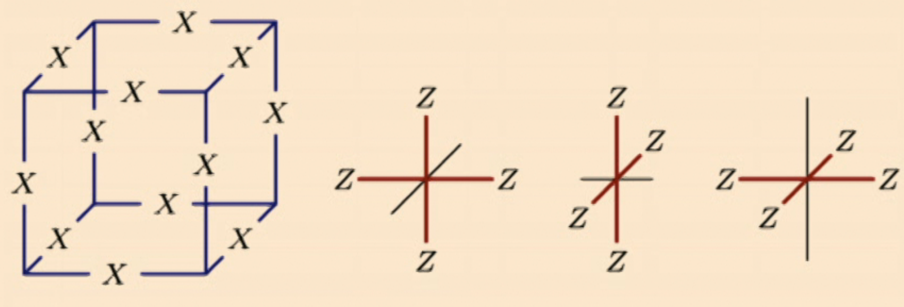


Lineon-fracton braiding in X-cube



Vijay, Haah and Fu, PRB 2016

Lineon-fracton braiding in X-cube



Vijay, Haah and Fu, PRB 2016

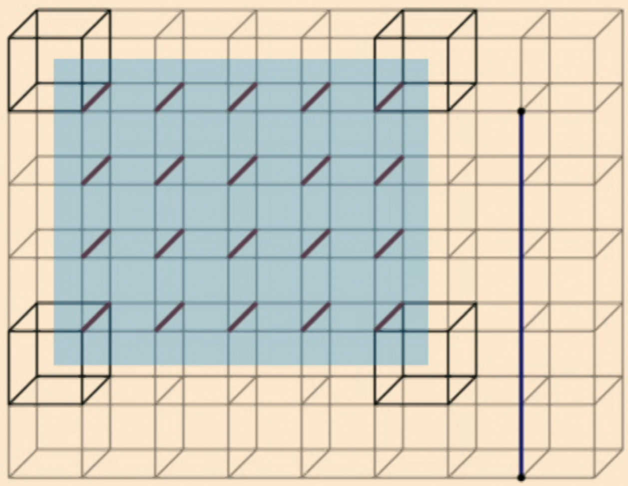
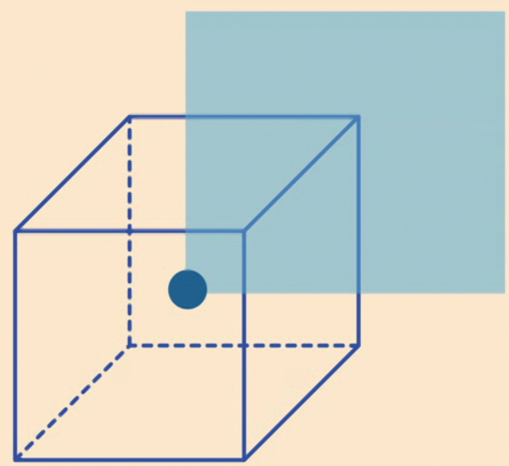
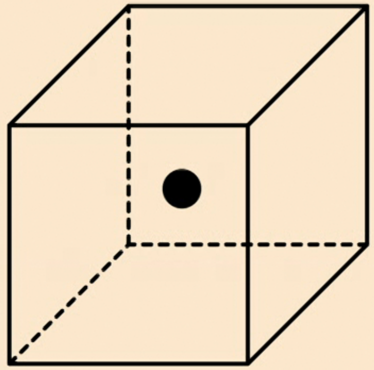


Figure from Shirley et al PRX 2018



X-cube model at half filling



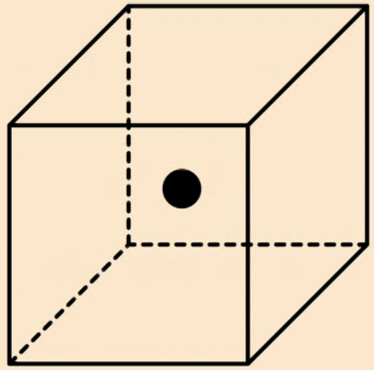
The fracton has charge 1/2 under U(1)

There is a background fracton per cube

A physical model can be constructed using partons

Pretko et al 2020

X-cube model at half filling



The fracton has charge 1/2 under U(1)

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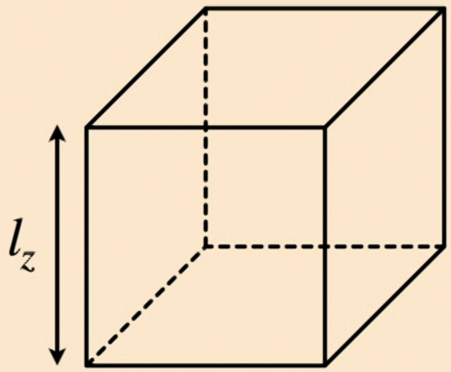
A physical model can be constructed using partons

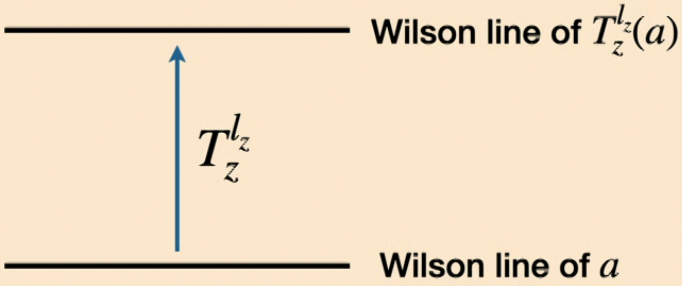
Pretko et al 2020

Reduce the theory further: condense certain lineon dipoles

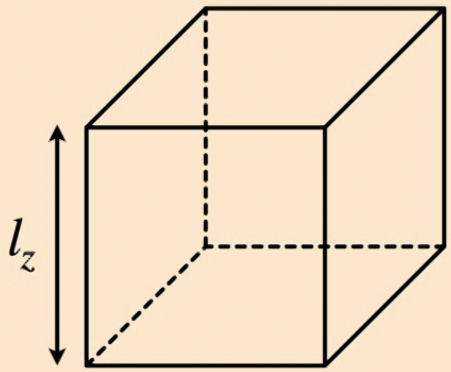
e.g $\mathbb{Z}_2^{\otimes 4}$ toric code in 3D (Lake and Hermele 2021)

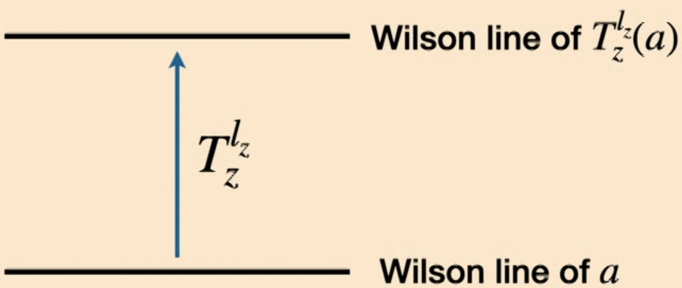
Mobility constraint





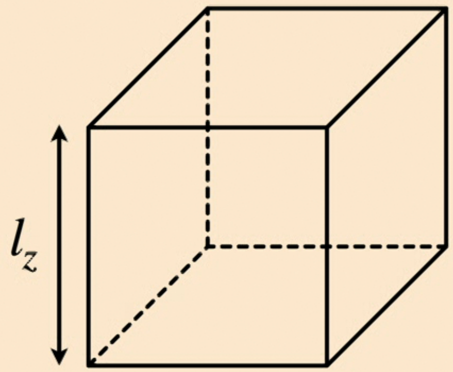
Mobility constraint

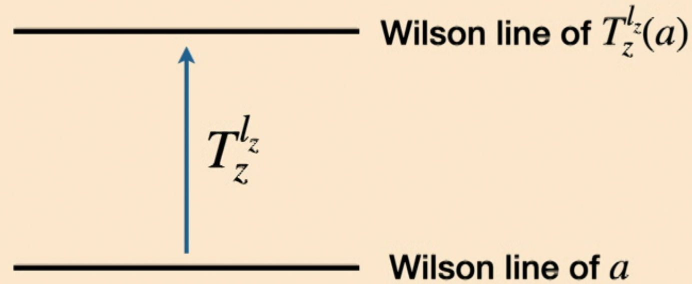




$T_z^{l_z}(a) \neq a$ if $l_z \neq 0$ mod q, otherwise the phase factor is trivial

Mobility constraint





$T_z^{l_z}(a) \neq a$ if $l_z \neq 0$ mod q, otherwise the phase factor is trivial

Translation by $l_z \neq 0$ mod q must change the "superselection sector"

No local operator can hop a by l_z if $l_z \neq 0$ mod q

Summary

- Lattice effects can enter low-energy theory via emanant symmetries, their defect and anomaly
- Anomalies in emanant symmetry have consequence in UV (e.g. subtle system size dependence)
- Lieb-Schultz-Mattis anomaly for subsystem U(1) leads to mobility constraints similar to fracton systems.