Title: Fermi Surface Anomaly and Symmetric Mass Generation

Speakers: Yi-Zhuang You

Series: Quantum Matter

Date: May 18, 2023 - 3:30 PM

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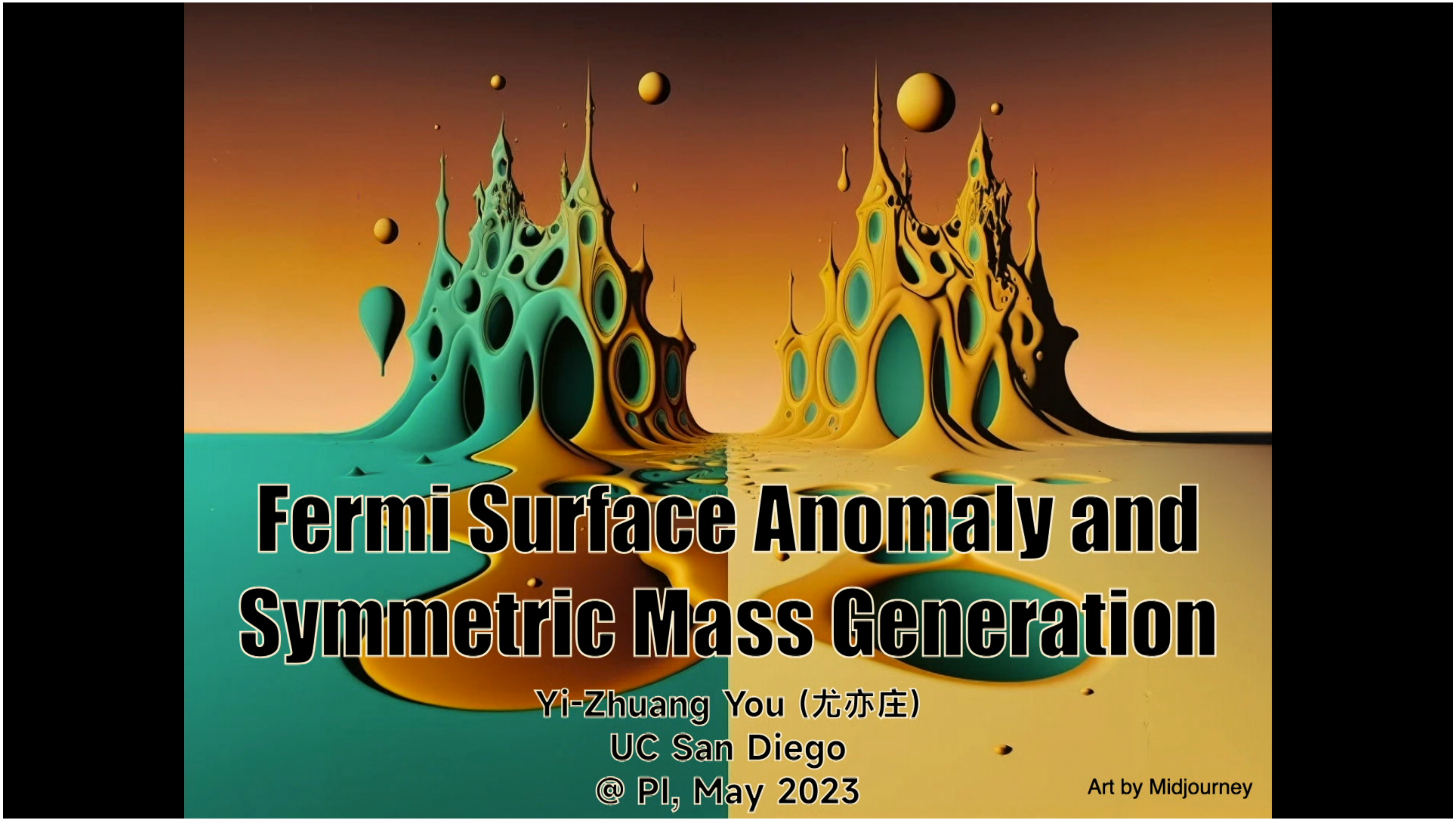
Abstract: Fermi liquids are gapless quantum many-body states of fermions, which describes electrons in the normal state of most metals at low temperature. Despite its long history of study, there has been renewed interest in understanding the stability of Fermi liquid from the perspectives of emergent symmetry and quantum anomaly. In this talk, I will introduce the concept of Fermi surface anomaly and propose a possible scheme to classify it. The classification scheme is based on viewing the Fermi surface as the boundary of a Chern insulator in the phase space, with an unusual dimension counting arising from the non-commutative phase space geometry. This enables us to extend the notion of Fermi surface anomaly to the non-perturbative cases and discuss symmetric mass generation on the Fermi surface when the anomaly is canceled. I will provide examples of lattice models that demonstrate Fermi surface symmetric mass generation and make connections to the recent progress in understanding the pseudo-gap transition in cuprate materials.

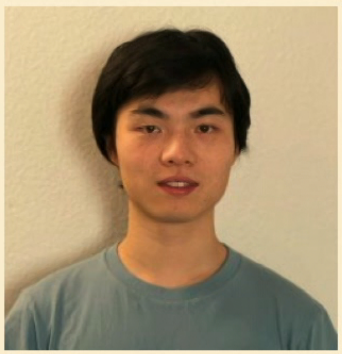
Zoom link: https://pitp.zoom.us/j/97223165997?pwd=SkhJZEt1ejhQRm0yK2tKS3NhM2o2Zz09


Fermi Surface Anomaly and Symmetric Mass Generation
Yi-Zhuang You (尤亦庄)
UC San Diego
@ PI, May 2023
Art by Midjourney

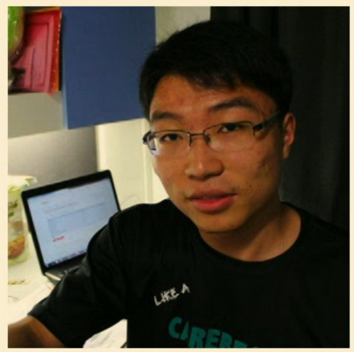
Acknowledgements

[1] DC Lu, M Zeng, J Wang, YZ You. *Fermi Surface Symmetric Mass Generation* (arXiv:2210.16304)

[2] DC Lu, J Wang, YZ You. *Definition and Classification of Fermi Surface Anomaly* (arXiv:2302.12731)



Da-Chuan Lu



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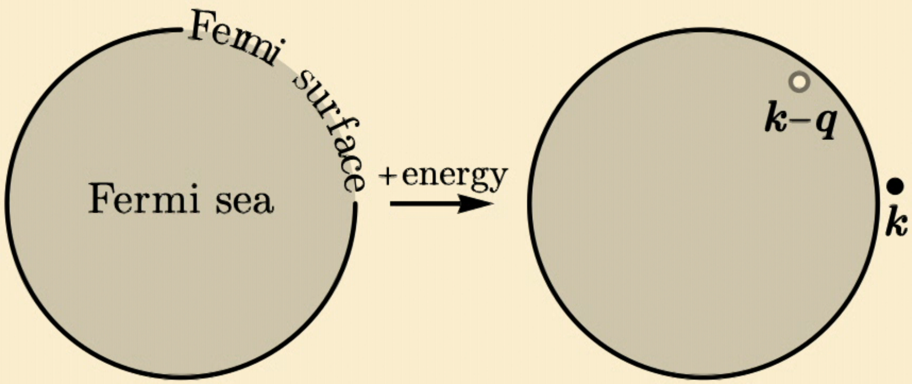
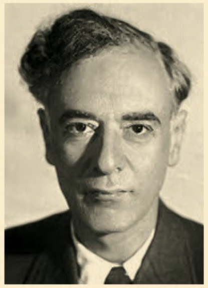
Fermi Liquids

- Fermi liquids are gapless quantum-many body phases of fermions with Fermi surfaces and well-defined quasi-particle excitations at low energy.



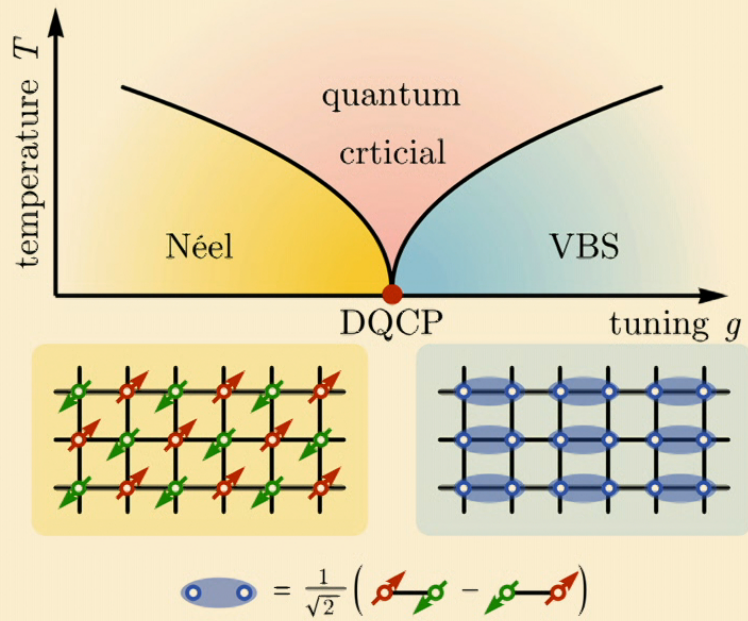


- Describe electrons in the normal state of most metals at low temperatures.
- Despite its long history, there has been renewed interest — what is protecting these gapless fermions?

Gapless Phases and Anomaly

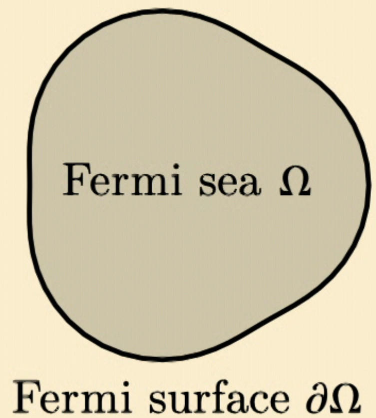
- A recent trend: understand gapless quantum phases of matter from emergent symmetry and quantum anomaly.
  - Decofined quantum criticality - $\mathrm{SO}(5)$

Senthil, Vishwanath, Balents, Sachdev, Fisher 2003



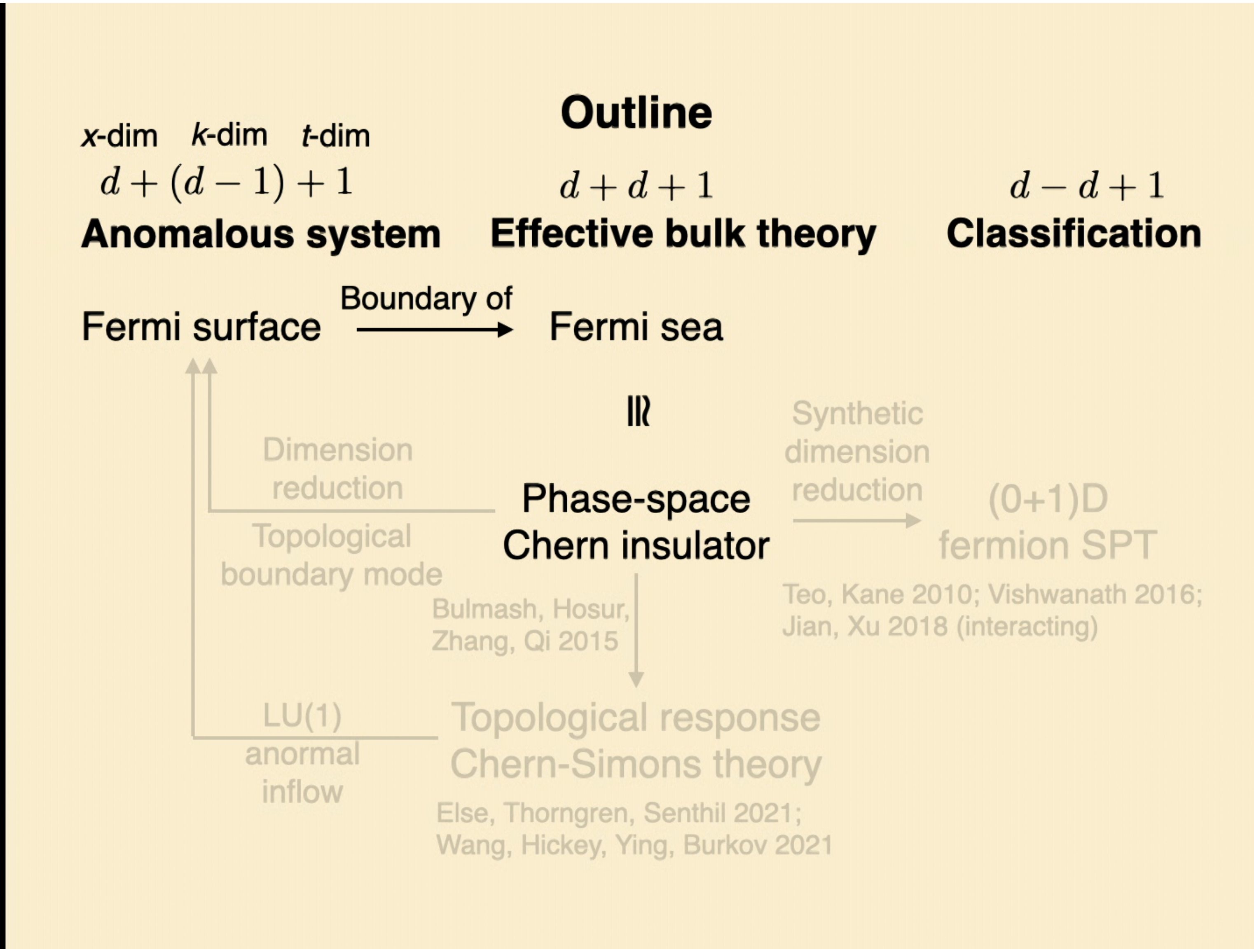
Bulk-Boundary Correspondence

- Boundary: quantum anomaly of symmetry →
  Bulk: symmetry-protected topological (SPT) order
- What is the "bulk" of a Fermi surface? - Fermi sea.



- What is "topological" about the Fermi sea? - Fermi sea can be viewed as a Chern insulator in the phase space.

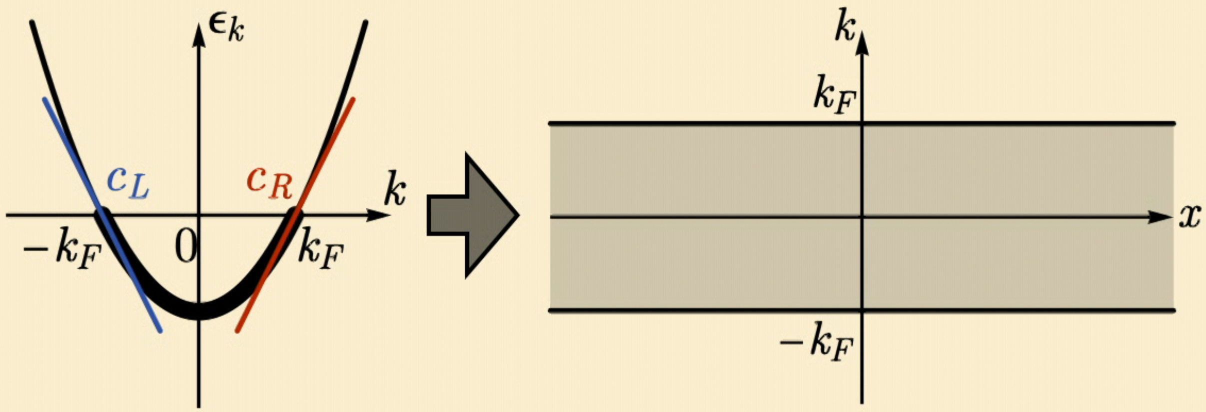
Bulmash, Hosur, Zhang, Qi 2015


Outline
x-dim k-dim t-dim
$d + (d-1) + 1$
Anomalous system
$d + d + 1$
Effective bulk theory
$d - d + 1$
Classification
Fermi surface
Boundary of
Fermi sea
Dimension reduction
Topological boundary mode
Phase-space Chern insulator
Synthetic dimension reduction
(0+1)D fermion SPT
Teo, Kane 2010; Vishwanath 2016; Jian, Xu 2018 (interacting)
Bulmash, Hosur, Zhang, Qi 2015
LU(1) anormal inflow
Topological response Chern-Simons theory
Else, Thorngren, Senthil 2021; Wang, Hickey, Ying, Burkov 2021

Toy Example: (1+1)D Fermi Liquid

- Consider a free fermion system in (1+1)D

$$H = \sum_k \epsilon_k c_k^\dagger c_k$$



Fermi liquid in momentum-energy space

Fermi liquid in phase space (position-momentum space)

- Fermi liquid is a Chern insulator in the phase space.

Toy Example: (1+1)D Fermi Liquid

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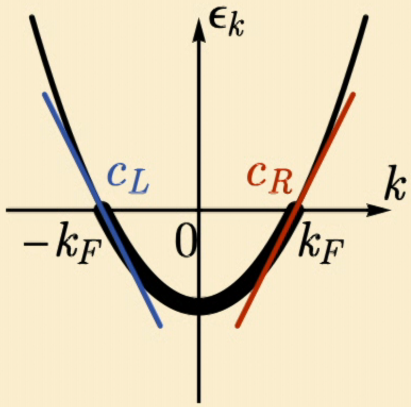
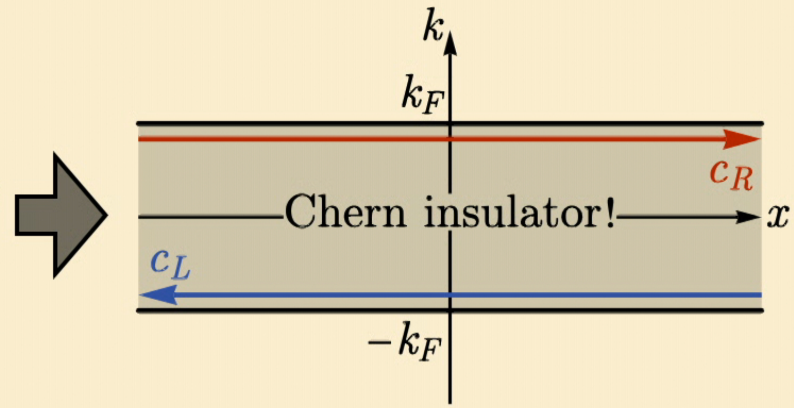
$$H = \sum_k \epsilon_k c_k^\dagger c_k$$



Fermi liquid in momentum-energy space



Fermi liquid in phase space (position-momentum space)

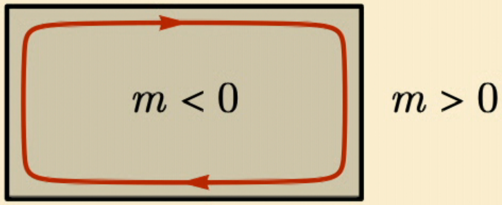
- Fermi liquid is a Chern insulator in the phase space.

Phase-Space Chern-Insulator

- Chern-insulator in the real space

$$H = \int \mathrm{d}x\mathrm{d}y\, \psi^\dagger (\mathrm{i}\partial_x \sigma^x + \mathrm{i}\partial_y \sigma^y + m\sigma^z)\psi$$

  - $\psi(x, y)$ - fermion field at each point
  - $m(x, y)$ - Dirac mass profile



Outline

x-dim *k*-dim *t*-dim

$d+(d-1)+1$

Anomalous system

$d+d+1$

Effective bulk theory

$d-d+1$

Classification

Fermi surface —Boundary of→ Fermi sea

≅

Phase-space Chern insulator

Dimension reduction

Topological boundary mode

Synthetic dimension reduction →

(0+1)D fermion SPT

Teo, Kane 2010; Vishwanath 2016;
Jian, Xu 2018 (interacting)

Bulmash, Hosur,
Zhang, Qi 2015

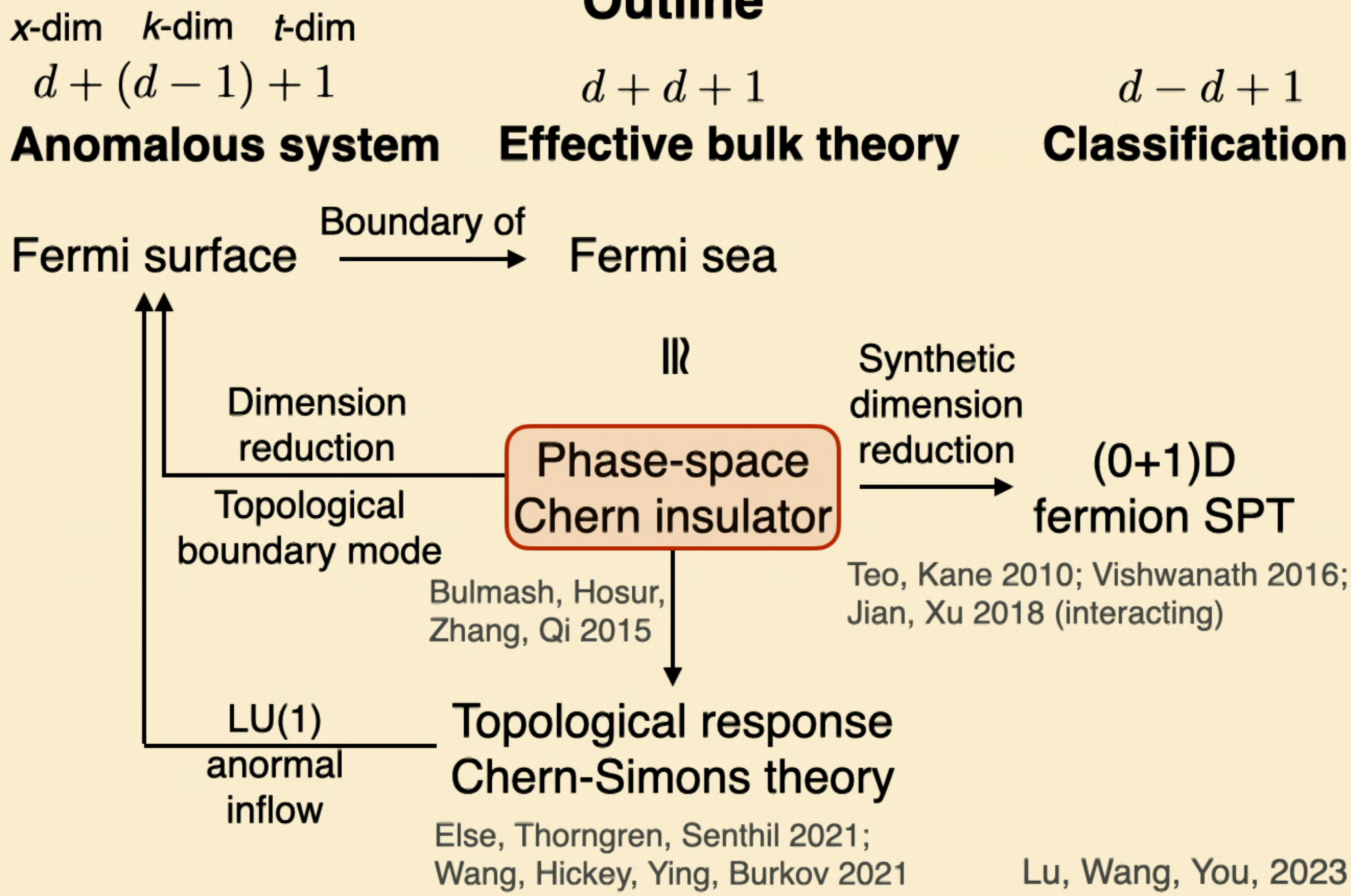
Topological response Chern-Simons theory

LU(1) anormal inflow

Else, Thorngren, Senthil 2021;
Wang, Hickey, Ying, Burkov 2021

Lu, Wang, You, 2023

Outline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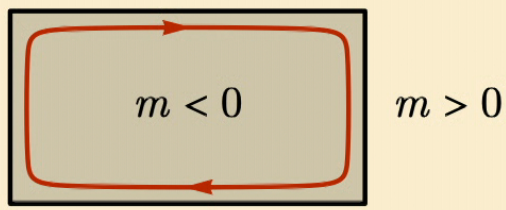
Phase-Space Chern-Insulator

- Chern-insulator in the real space

$$H = \int \mathrm{d}x\mathrm{d}y\ \psi^\dagger(\mathrm{i}\partial_x\sigma^x + \mathrm{i}\partial_y\sigma^y + m\sigma^z)\psi$$

  - $\psi(x, y)$ - fermion field at each point
  - $m(x, y)$ - Dirac mass profile



- Chern-insulator in the phase space   Bulmash, Hosur, Zhang, Qi 2015

$$H = \int \mathrm{d}x\mathrm{d}k\ \psi^\dagger(\mathrm{i}\partial_x\sigma^x + \mathrm{i}\partial_k\sigma^y + m\sigma^z)\psi$$

  - $\psi(x, k)$ - fermion field at each point … Hey, wait!

Non-Commutative Phase Space Geometry

- Quantum mechanics: phase space coordinates (position and momentum) do not commute

$$[x, k] = \mathrm{i} \qquad \text{(set } \hbar = 1\text{)}$$

- The phase-space Dirac fermion is a non-commutative field theory (a field theory defined on non-commutative manifold)

$$H = \int \mathrm{d}x\mathrm{d}k\, \psi^\dagger(\mathrm{i}\partial_x\sigma^x + \mathrm{i}\partial_k\sigma^y + m\sigma^z)\psi$$

- Perturbative treatment: expand around commutative limit

$$[x^i, x^j] = \mathrm{i}\theta^{ij}$$

$$f(x) * g(x) = fg + \tfrac{\mathrm{i}}{2}\theta^{ij}\partial_i f\partial_j g + \mathcal{O}(\theta^2)$$

Seiberg, Witten 1999; Dong, Senthil 2020

Non-Commutative Phase Space Geometry

- Quantum mechanics: phase space coordinates (position and momentum) do not commute

$$[x, k] = \mathrm{i} \qquad \text{(set } \hbar = 1\text{)}$$

- Non-perturbative treatment — two ways to resolve the non-commutative geometry:
  - **Phase-space Berry curvature** - a uniform "magnetic field" through the (x, k)-plane.

    $$F = \mathrm{d}A = \mathrm{d}x \wedge \mathrm{d}k$$

    Wen 2021; Delacrétaz, Du, Mehta, Son 2022 …

  - **Canonical quantization** - represented as gradient operator with respect to each other.

    $$x = \mathrm{i}\partial_k \quad \Leftrightarrow \quad k = -\mathrm{i}\partial_x$$

Outline

x-dim *k*-dim *t*-dim

$d+(d-1)+1$

Anomalous system

$d+d+1$

Effective bulk theory

$d-d+1$

Classification

Fermi surface → (Boundary of) Fermi sea

$$H=\int \mathrm{d}x\mathrm{d}k\ \psi^\dagger(\mathrm{i}\partial_x\sigma^x+\mathrm{i}\partial_k\sigma^y+m\sigma^z)\psi$$

D reduction

Topological boundary mode

Phase-space Chern insulator

reduction → (0+1)D fermion SPT

Teo, Kane 2010; Vishwanath 2016; Jian, Xu 2018 (interacting)

Bulmash, Hosur, Zhang, Qi 2015

Topological response Chern-Simons theory

LU(1) anormal inflow

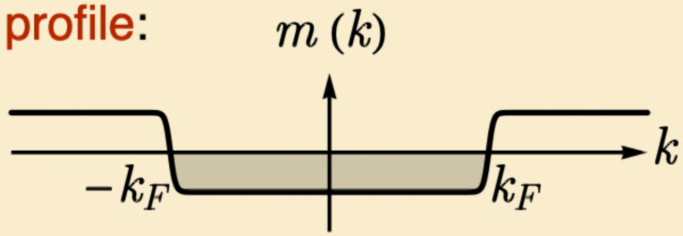
Else, Thorngren, Senthil 2021; Wang, Hickey, Ying, Burkov 2021

Bulk Response Theory

- Take the "phase-space Berry curvature" approach by introducing the $\mathrm{U}(1)$ symmetry background field A.

$$H = \int \mathrm{d}x\mathrm{d}k\ \psi^\dagger(\mathrm{i}D_x\sigma^x + \mathrm{i}D_k\sigma^y + m\sigma^z)\psi$$

  - Covariant derivative $D = \partial + \mathrm{i}A$ in the phase space,
  - with a background curvature $F = \mathrm{d}A = \mathrm{d}x \wedge \mathrm{d}k$,
  - and a mass profile:



- With the background curvature, x and k can now be treated as ordinary commuting coordinates.

Bulk Response Theory

- Integrate out the fermion field → Chern-Simons theory in the phase spacetime.

$$S = \frac{1}{4\pi}\int_{(t,x,k)} \frac{1-\operatorname{sgn} m(k)}{2} A \wedge \mathrm{d}A$$

  - The Chern-Simon term is effective only inside the Fermi sea (i.e., $k \in \Omega = [-k_F, k_F]$ as $m(k) < 0$)
  - Recall: the gauge field has a uniform background curvature through the (x, k)-plane: $F = \mathrm{d}A = \mathrm{d}x \wedge \mathrm{d}k$
  - Fermion filling (charge density) is a topological response

$$\nu = \frac{\delta S}{\delta A_0} = \int_{-k_F}^{k_F} \frac{F_{xk}}{2\pi} = \frac{\operatorname{vol}\Omega}{2\pi} \qquad \text{(Luttinger theorem)}$$

Outline

x-dim *k*-dim *t*-dim

$d + (d-1) + 1$ **Anomalous system**

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Fermi surface —Boundary of→ Fermi sea

$\cong$

Dimension reduction

Topological boundary mode

Phase-space Chern insulator —Synthetic dimension reduction→ (0+1)D fermion SPT

Teo, Kane 2010; Vishwanath 2016; …teracting)

$$H = \int \mathrm{d}x\mathrm{d}k\ \psi^\dagger (\mathrm{i}\partial_x \sigma^x + \mathrm{i}\partial_k \sigma^y + m\sigma^z)\psi$$

LU(1) anormal inflow

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Else, Thorngren, Senthil 2021;
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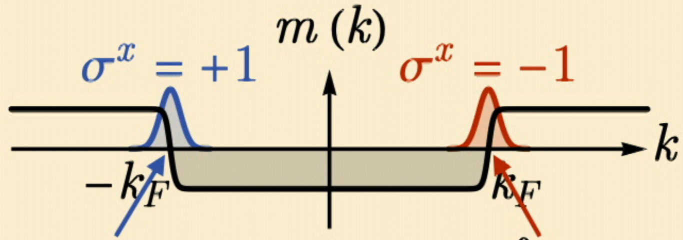
Topological Boundary Mode

- Given the correct bulk response, what about the topological boundary mode?

$$H = \int \mathrm{d}x\mathrm{d}k\ \psi^\dagger \left(\boxed{\mathrm{i}\partial_x \sigma^x} + \mathrm{i}\partial_k \sigma^y + m(k)\sigma^z\right)\psi$$



$$H_{\text{eff}} = \int \mathrm{d}x\ \psi^\dagger(+\mathrm{i}\partial_x)\psi$$

Left-moving chiral fermions at $-k_F$

$$H_{\text{eff}} = \int \mathrm{d}x\ \psi^\dagger(-\mathrm{i}\partial_x)\psi$$

Right-moving chiral fermions at $+k_F$

- Matching a (1+1)D Fermi liquid's low-energy gapless fermions modes (at both Fermi points).

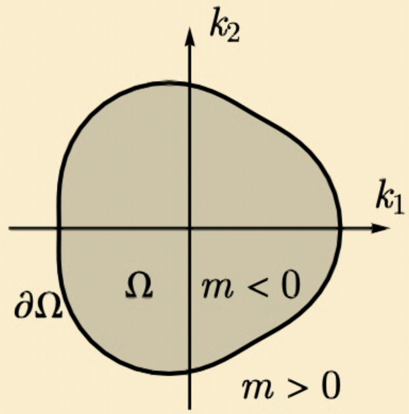
Higher Dimension Generalizations

- **Bulk**: Fermi sea = Phase-space Chern insulator

$$H_{\mathrm{blk}} = \int \mathrm{d}^d\boldsymbol{x}\mathrm{d}^d\boldsymbol{k}\ \psi^\dagger(\mathrm{i}\partial_{\boldsymbol{x}}\cdot\boldsymbol{\Gamma}_x + \mathrm{i}\partial_{\boldsymbol{k}}\cdot\boldsymbol{\Gamma}_k + m(\boldsymbol{k})\Gamma^0)\psi$$

- Dirac mass profile

$$m(\boldsymbol{k}) \begin{cases} \leq 0 & \text{if } \boldsymbol{k}\in\Omega, \\ > 0 & \text{if } \boldsymbol{k}\notin\Omega. \end{cases}$$



- Bulk topological response:
  phase-space Chern-Simons theory

$$S = \frac{1}{(d+1)!(2\pi)^d}\int_{\mathbb{R}^d\times\Omega\times\mathbb{R}} A\wedge(\mathrm{d}A)^{\wedge d}$$

$$F := \mathrm{d}A = \sum_{i=1}^{d}\mathrm{d}x_i\wedge\mathrm{d}k_i$$

(with a background Berry flux through every (x_i, k_i)-plane)

Higher Dimension Generalizations

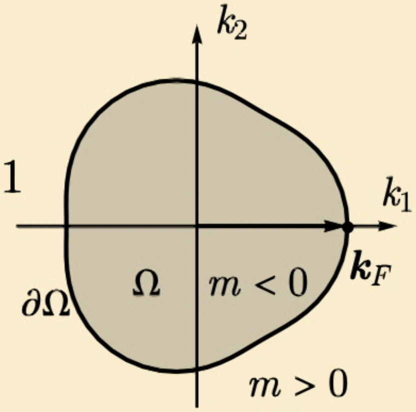
- **Boundary**: gapless Fermi surface = phase-space topological boundary modes
  - Pick a point $\boldsymbol{k}_F \in \partial\Omega$ on the Fermi surface (say along k_1)

$$H = \int \psi^\dagger (\mathrm{i}\partial_{x_1}\Gamma^1 +$$

$$\sout{\mathrm{i}\partial_{k_1}\Gamma^2 + m(\boldsymbol{k})\Gamma^0 +} \rightarrow \mathrm{i}\Gamma^2\Gamma^0 = 1$$

$$\mathrm{i}\partial_{x_2}\Gamma^3 + x_2\Gamma^4 +$$

$$\cdots)\psi$$



  - Use canonical quantization $\mathrm{i}\partial_{\boldsymbol{k}} \rightarrow \boldsymbol{x}$ to resolve the non-commutativity in the remaining dimensions.
  - Only one mode survives projections: $H = \int \psi^\dagger (\mathrm{i}\partial_{x_1})\psi$

Outline

x-dim *k*-dim *t*-dim

$d + (d-1) + 1$ **Anomalous system**

$d + d + 1$ **Effective bulk theory**

$d - d + 1$ **Classification**

Fermi surface —Boundary of→ Fermi sea

≅

Phase-space Chern insulator

Dimension reduction

Topological boundary mode

Bulmash, Hosur, Zhang, Qi 2015

Topological response Chern-Simons theory

LU(1) anormal inflow

Else, Thorngren, Senthil 2021; Wang, Hickey, Ying, Burkov 2021

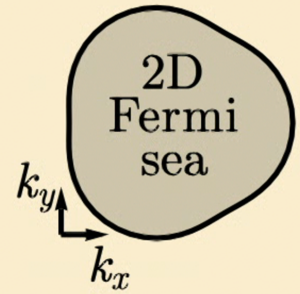
Synthetic dimension reduction → (0+1)D fermion SPT

Teo, Kane 2010; Vishwanath 2016; Jian, Xu 2018 (interacting)

Beyond U(1) Symmetry

- Consider a 2D Fermi sea with 1D Fermi surface, naively:
  - Phase-spacetime dimension is 2+2+1 = 5
  - Bulk: like a (4+1)D Chern insulator

$$S = \frac{1}{24\pi^2}\int A \wedge \mathrm{d}A \wedge \mathrm{d}A$$

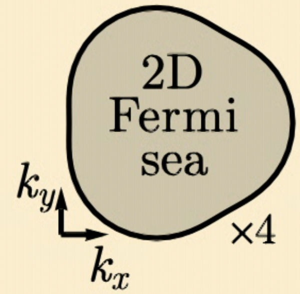


  - Boundary: like a (3+1)D Weyl fermion
  - Fermi surface anomaly: like the chiral anomaly of Weyl fermions, which is $\mathbb{Z}$ classified under $\mathrm{U}(1)$ symmetry.
- What if the $\mathrm{U}(1)$ symmetry is broken to its $\mathbb{Z}_4$ subgroup?
- $\mathbb{Z}_4$-symmetric Weyl fermions have a non-perturbative anomaly that is $\mathbb{Z}_{16}$ classified → Weyl fermions can only be trivialized by interaction in multiple of 16.

Weń 2013, You, Xu 2014, Wan, Wang 2018

Beyond U(1) Symmetry

- Does this indicate that the $\mathbb{Z}_4$-symmetric Fermi surface in a (2+1)D Fermi liquid can only be symmetrically gapped into product states in multiple of ~~16~~



- But we know this can not be correct because here is an explicit counter-example:

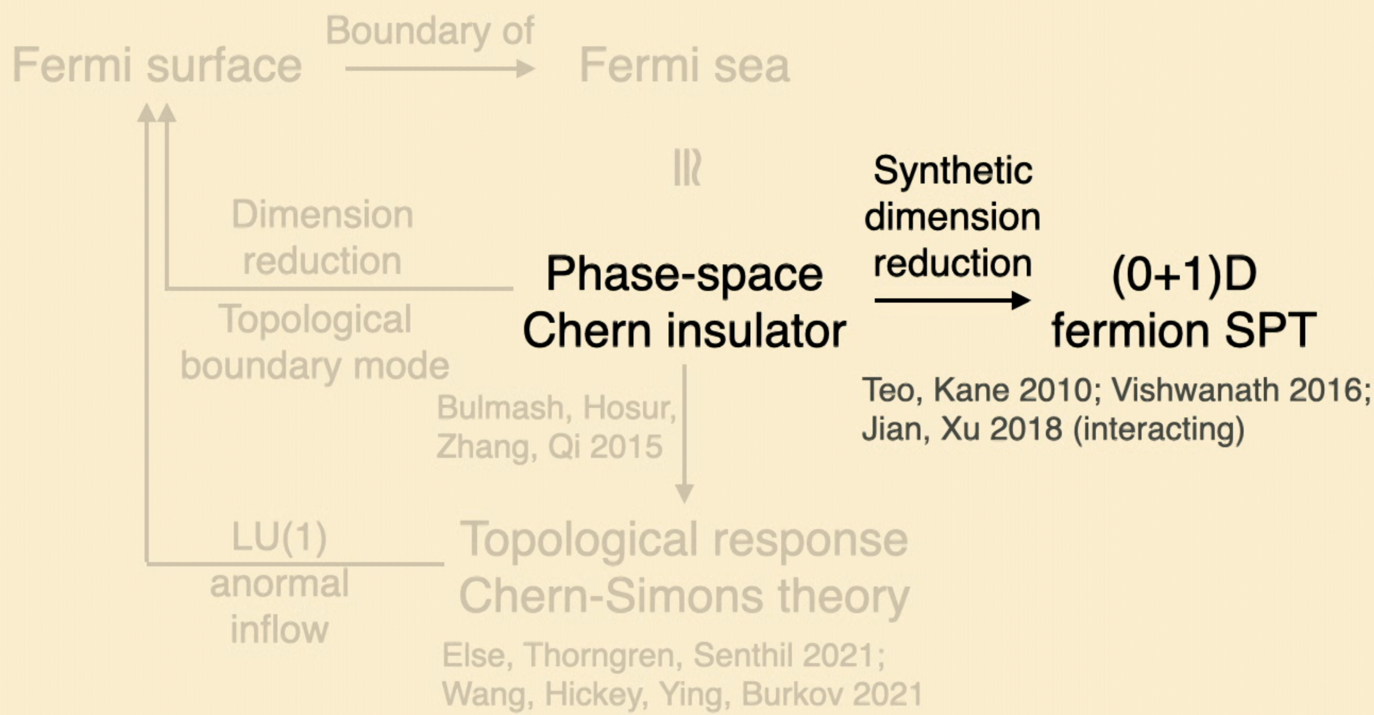
$$H = -\sum_{a=1}^{4}\sum_{ij} t_{ij}\psi_{ia}^{\dagger}\psi_{ja} - g\sum_{i}\psi_{i1}\psi_{i2}\psi_{i3}\psi_{i4} + \text{h.c.}.$$

  - By anomaly matching, the Fermi surface anomaly must already vanish at multiplicity 4, not 16.
- What is going wrong with the phase-space Chern insulator picture?

Outline

x-dim *k*-dim *t*-dim

$d + (d-1) + 1$	$d + d + 1$	$d - d + 1$
Anomalous system	**Effective bulk theory**	**Classification**



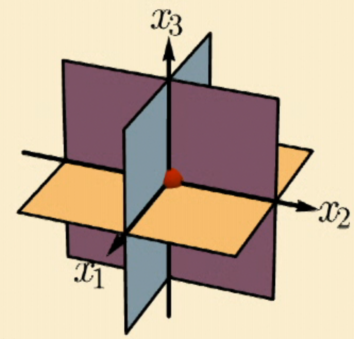
Domain-Wall Dimension Reduction

- The bulk theory describes a set of perpendicular domain walls intersecting at the origin → trapping a single fermion zero mode

$$H_{\text{blk}} = \int \mathrm{d}^d\boldsymbol{x}\,\psi^\dagger(\mathrm{i}\partial_{\boldsymbol{x}}\cdot\boldsymbol{\Gamma}_x + \boldsymbol{x}\cdot\boldsymbol{\Gamma}_k + m\Gamma^0)\psi$$

$$\downarrow$$

$$H_{\text{eff}} = m\psi^\dagger\psi \quad \text{(0+1)D}$$



  - m tunes the topological transition in the bulk
  - The bulk state is trivial if the gapless critical fermion modes at $m = 0$ can be symmetrically gapped by interaction.
  - For $\mathbb{Z}_4$-symmetric fermions, trivialization can be achieved by $(\psi_1\psi_2\psi_3\psi_4 + \text{h.c.})$ at multiplicity 4 → $\mathbb{Z}_4$ classified!

Lu, Wang, You 2023

Synthetic Dimension Reduction

- Momentum space dimension = negative dimension

$$d_{\text{eff}} = d - \delta$$

Teo, Kane 2010; Vishwanath 2016
Jian, Xu 2018 (interacting)

- d_{eff}: Effective spatial dimension (for SPT classification)
- d: Real space dimension
- δ: Momentum/ parameter space dimension, synthetic dimension

- In $d = 3$ dimensional real space, assuming $\mathrm{U}(1)$ symmetry

	Fermi surface codim	Fermi surface dim	Fermi sea dim δ	$d_{\text{eff}}+1$	Classification
Weyl points	3	0	1	2+1	$\mathbb{Z} \times \mathbb{Z}$
Fermi rings	2	1	2	1+1	0
Fermi surfaces	1	2	3	0+1	$\mathbb{Z}$

Fermi Surface Symmetric Mass Generation

- **Fermi surface SMG**: gap out the Fermi surface by interaction effects without spontaneous symmetry breaking or topological ordering. Lu, Zeng, Wang, You 2022
  - Necessary condition: Fermi surface anomaly vanishes.
- Fermi surface anomaly index

Fermi volume

$$\nu = \sum_\alpha k_\alpha \frac{\mathrm{vol}\,\Omega_\alpha}{(2\pi)^d} \mod 1$$

Sum over all Fermi surfaces

Level $k_\alpha = \pm q_\alpha N_\alpha$ ← Multiplicity (degeneracy)

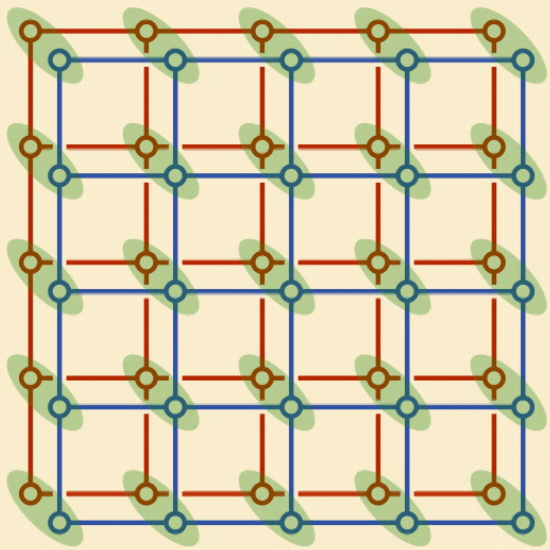
Electron/ hole-like

Charge

Lu, Wang, You 2023

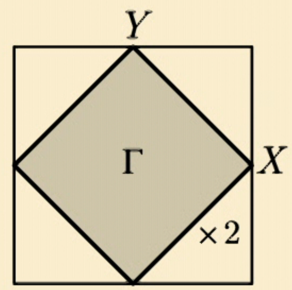
Example: Bilayer Square Lattice Model

- Spin-1/2 fermions on a bilayer square lattice at half-filling with $\mathrm{U}(1)^2 \times \mathrm{SU}(2)$ and translation symmetries.

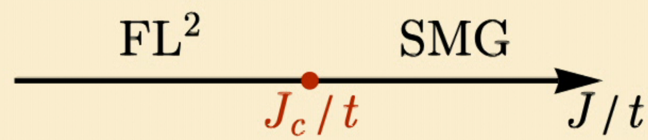


$$H = -t \sum_{\langle ij \rangle} \sum_{l=1,2} (c_{il}^\dagger c_{jl} + \text{h.c.}) + J \sum_i \boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2}$$

$$\boldsymbol{S}_{il} = \tfrac{1}{2} c_{il}^\dagger \boldsymbol{\sigma} c_{il}$$

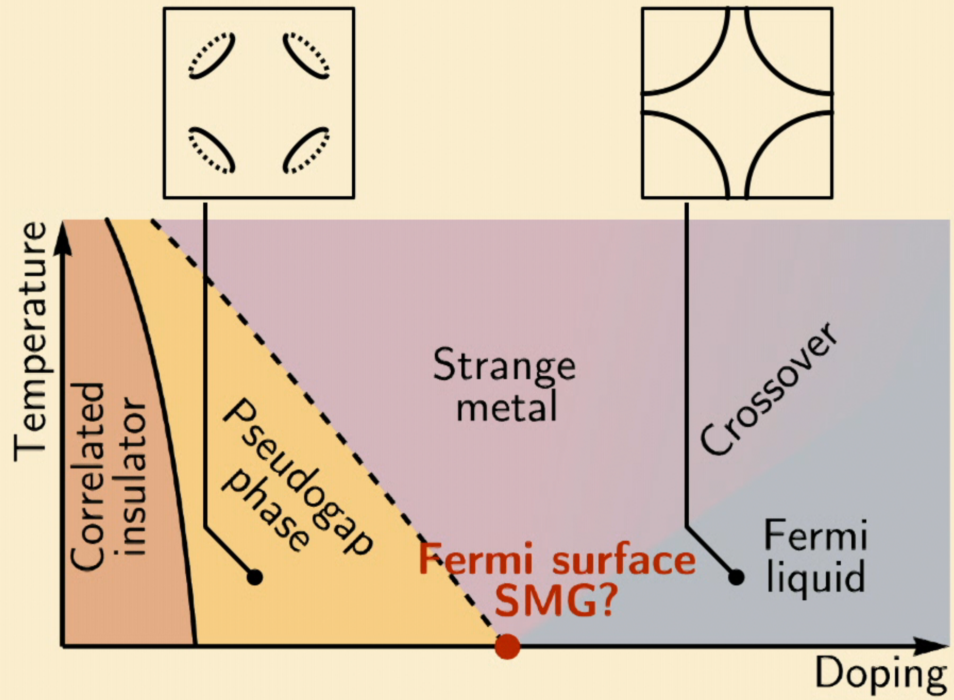


Zhang, Sachdev 2020
Zou, Chowdhury 2020



Pseudogap Physics

- Pseudogap: a (partial) gap of electron Fermi surface *without* symmetry breaking — Fermi surface SMG?



Zhang, Sachdev 2020

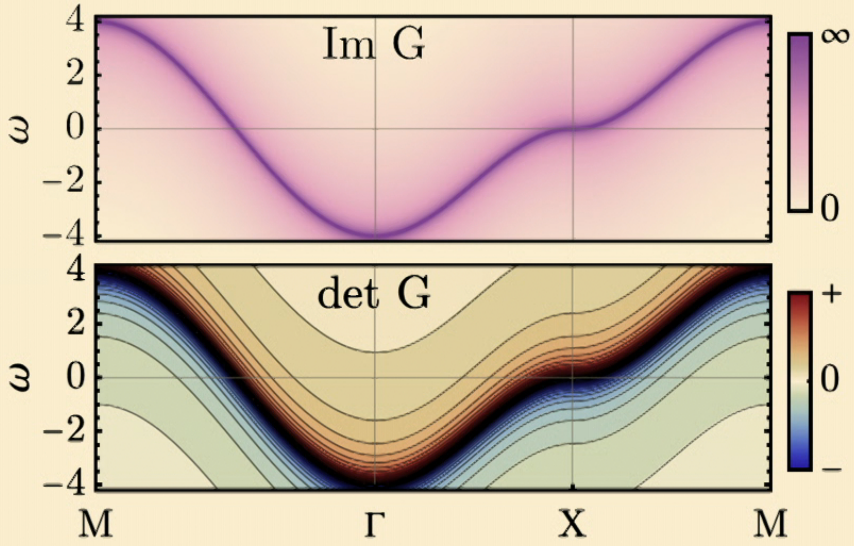
Green's Function Zeros

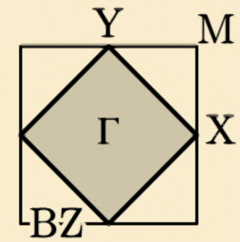
- Fermion Green's function (two-point correlation function)

$$G(\omega, \boldsymbol{k}) = -\langle c_{\omega,\boldsymbol{k}} c^{\dagger}_{\omega,\boldsymbol{k}} \rangle$$

$$A(\omega, \boldsymbol{k}) = -2\,\mathrm{Im}\,G(\omega + \mathrm{i}0_{+}, \boldsymbol{k}) \quad \text{(Spectral function)}$$

- Poles ($\det G \to \infty$) v.s. zeros ($\det G \to 0$).





Fermi liquid

DC Lu … (to appear)

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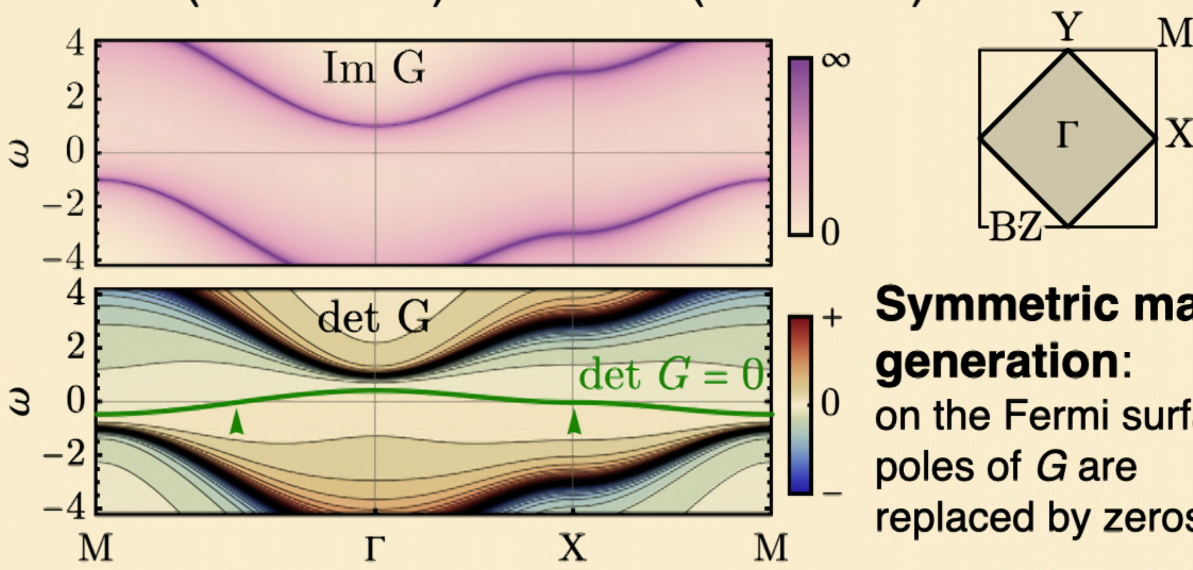
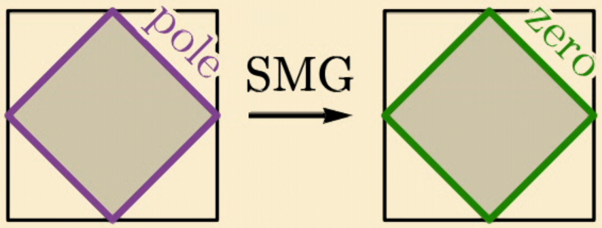


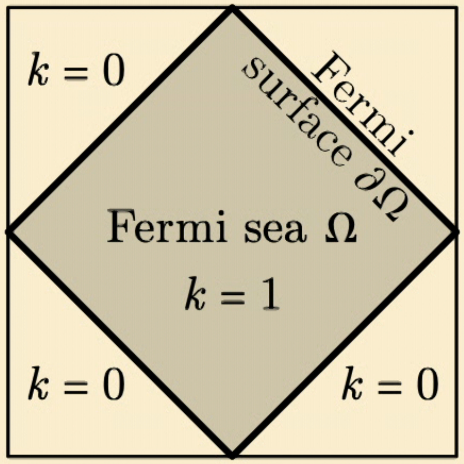
Symmetric mass generation: on the Fermi surface, poles of G are replaced by zeros

DC Lu … (to appear)

Green's Function Zeros

- SMG: pole-to-zero transition
  - Fermi sea = Phase-space Chern insulator
  - Gapless Fermi surface = Topological boundary modes
  - Topological response (2+2+1)D





$$S = \frac{k}{24\pi^2}\int_{\mathcal{M}_5} \mathcal{A}\wedge \mathrm{d}\mathcal{A}\wedge \mathrm{d}\mathcal{A}$$

$$k = \frac{1}{480\pi^3}\int_{\hat{\mathcal{M}}_5} \mathrm{Tr}\left[(G^{-1}\mathrm{d}G)^{\wedge 5}\right]$$

Bulmash, Hosur, Zhang, Qi 2015,
Else, Thorngren, Senthil 2020 …

- Jump of k: either through $\det G \to \infty$ (poles)
  or through $\det G \to 0$ (zeros)

You, Wang, Oon, Xu 2014; Y Xu, C Xu 2021

Summary

- Classification of Fermi surface anomaly

> Codimension-p Fermi surface anomaly of symmetry group G is classified by G-symmetric interacting fermionic SPT states in p-dimensional spacetime.

- Fermi surface symmetric mass generation can happen when the Fermi surface anomaly vanishes.
  - Potential applications to pseudo-gap physics
  - Spectral signatures: Green's function zeros on the Fermi surface
  - Matching Fermi surface anomaly with topological order?