Title: Discrete holography in dual-unitary circuits

Speakers: Lluis Masanes

Series: Perimeter Institute Quantum Discussions

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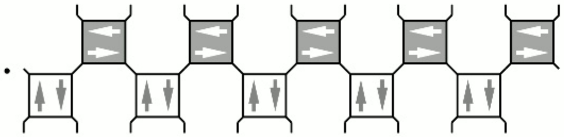
Abstract: I will introduce a family of dual-unitary circuits in 1+1 dimensions which are invariant under the joint action of Lorentz and scale transformations. With the same dual unitaries I will construct tensor-network states for this 1+1 model and interpret them as spatial slices of curved 2+1 discrete geometries. These tensor-network states satisfy the Ryu-Takayanagi conjecture outside event horizons, but the geometry of the network is also well defined inside the horizon. The dynamics of the circuit induces a natural dynamics on these geometries which reproduces GR phenomena like the gravitational redshift, the formation of black holes and the growth of their throat.

Zoom link: https://pitp.zoom.us/j/92034586204?pwd=eEo0NzVjeFkxWVJnY0hjOUhodXNWdz09

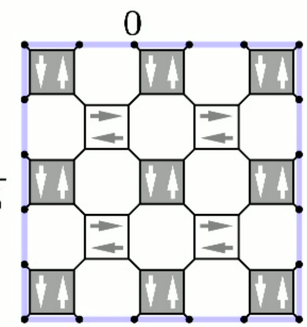
Discrete holography in dual-unitary circuits

Lluís Masanes UCL

Two ideas

- **Conformal quantum cellular automata** $T = \cdots$  $\cdots$
  - Invariant under discrete Lorentz+scale transformations
- **Real-time tensor-network states**
  - Dynamics of discrete geometries
  - Finite-radius holography (AdS, flat space, …)
  - Satisfy Ryu-Takayanagi entanglement/geometry relation
  - Gravitational time dilation, BH formation, growth of BH throat

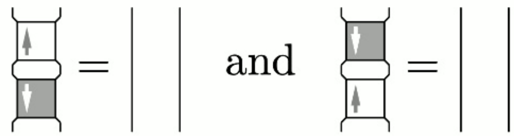
$$|\Psi\rangle = \frac{1}{q^5}$$



Conformal QCAs

Dual unitaries

- Unitarity in the time direction:



- Unitarity in the space direction:

- …equivalent to: $\mathtt{u}(\mathtt{a} \otimes \mathbb{1})\mathtt{u}^\dagger = \cancel{\mathtt{b} \otimes \mathbb{1}} + \mathbb{1} \otimes \mathtt{c} + \sum_k \mathtt{d}_k \otimes \mathtt{e}_k$

B. Bertini, P. Kos, and T. Prosen, Phys. Rev. Lett. 123, 210601 (2019)

Dual unitaries

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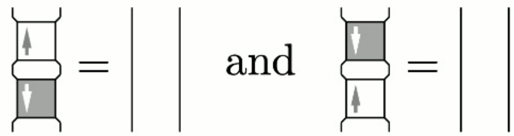
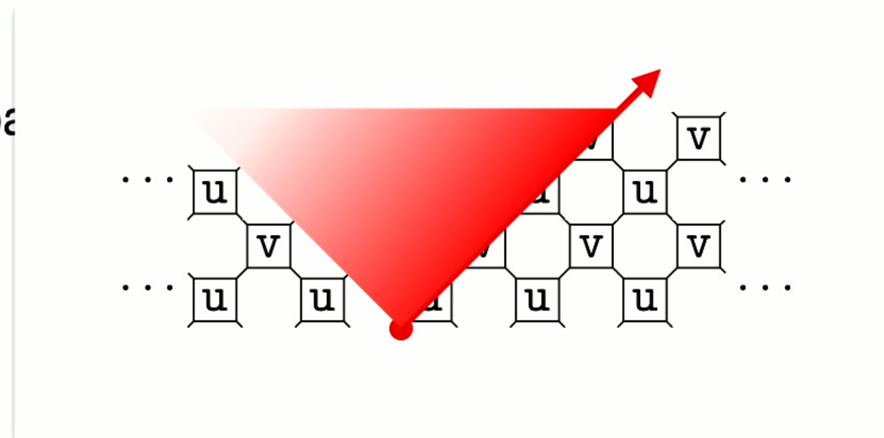
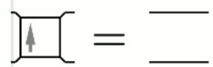


- Unitarity in the spa





- …equivalent to:

$\otimes e_k$

B. Bertini, P. Kos, and T. Prosen, Phys. Rev. Lett. 123, 210601 (2019)

Dual unitaries

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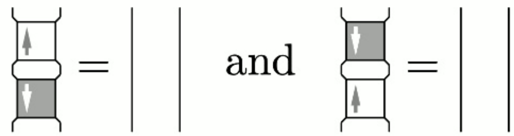
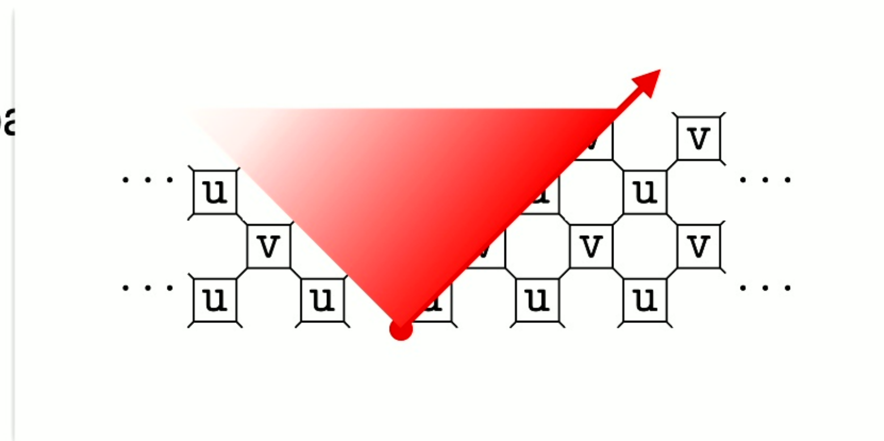
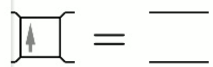


- Unitarity in the spa





- …equivalent to:

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B. Bertini, P. Kos, and T. Prosen, Phys. Rev. Lett. 123, 210601 (2019)

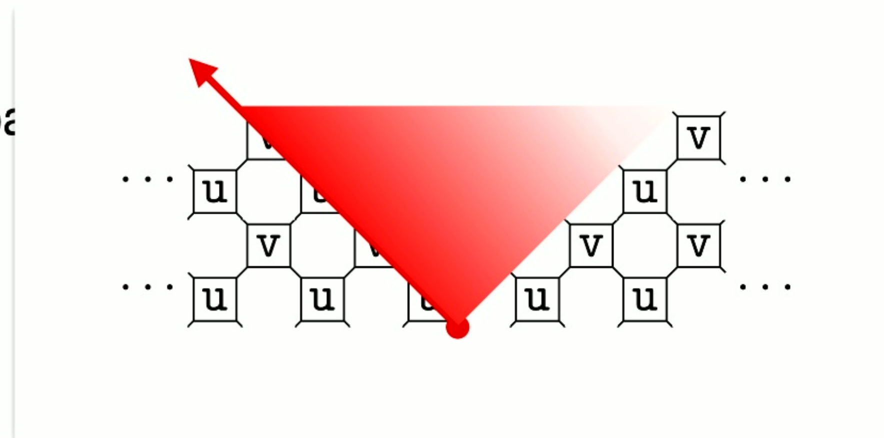
Dual unitaries

$\mathrm{u} =$ $\qquad \mathrm{u}^T =$ $\qquad \mathrm{sus}^\dagger =$ $\qquad \mathrm{u}^* =$

- Unitarity in the time direction: $=$ and $=$

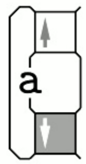
- Unitarity in the spa $=$

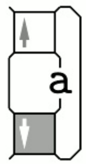
- …equivalent to: $\otimes\, \mathrm{e}_k$



B. Bertini, P. Kos, and T. Prosen, Phys. Rev. Lett. 123, 210601 (2019)

Free particles and quantum chaos

$$\Omega_+(\mathtt{a}_0) = \frac{1}{q}\,\mathrm{tr}_0(\mathtt{u}_0\mathtt{a}_0\mathtt{u}_0^\dagger) = \frac{1}{q}$$ 

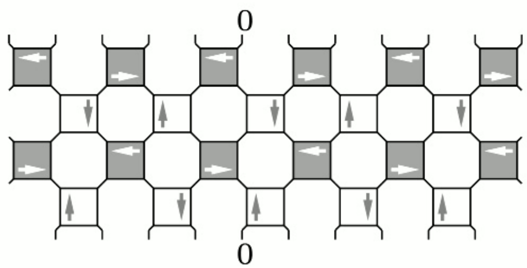
$$\Omega_-(\mathtt{a}_1) = \frac{1}{q}\,\mathrm{tr}_1(\mathtt{u}_0\mathtt{a}_1\mathtt{u}_0^\dagger) = \frac{1}{q}$$ 

$$\Omega_\pm(\mathsf{e}) = e^{im}\mathsf{e}$$ free particle

$$T\mathsf{e}_x T^\dagger = e^{im}\mathsf{e}_{x\pm 2}$$

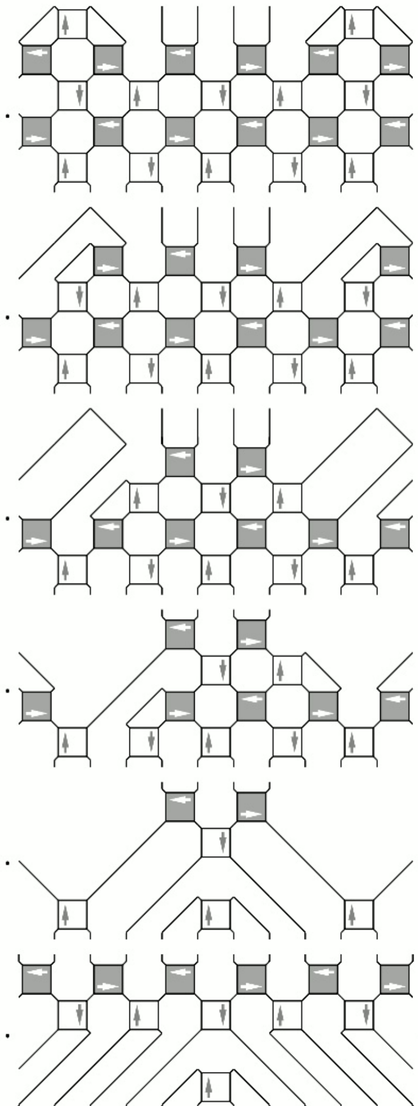
$$\Omega_\pm(\mathsf{e}) = \lambda\mathsf{e} \quad |\lambda| < 1$$ quantum chaos

Conformal QCAs

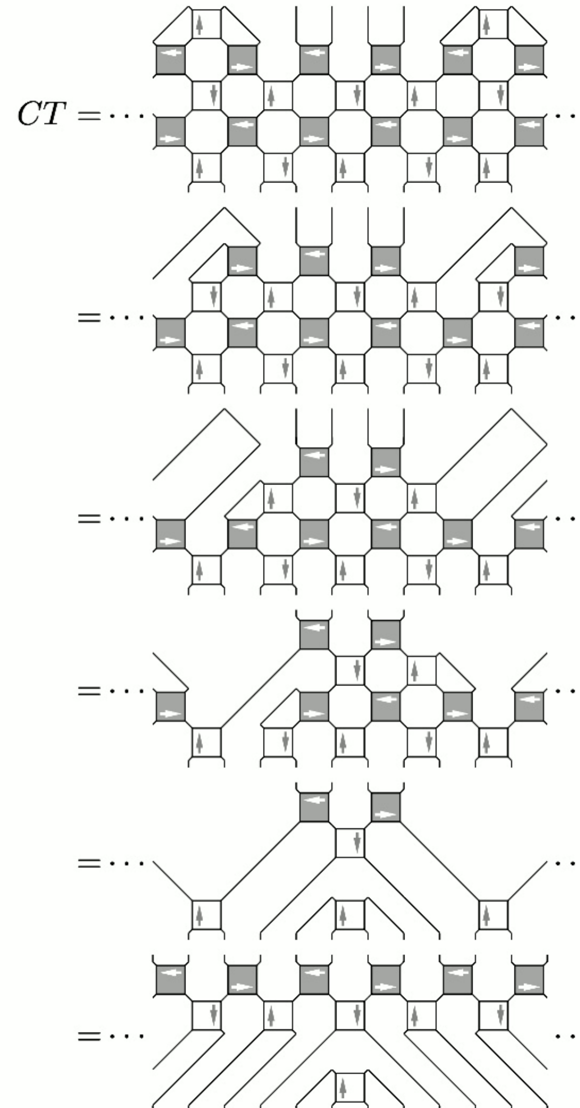
- Dynamics: $T = \cdots$  $\cdots$

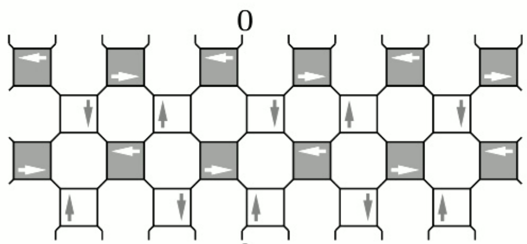
- Contraction isometry: $C : (\mathbb{C}^q)^{2n} \to (\mathbb{C}^q)^n$

$C = \cdots$  $\cdots$

$CT = \cdots$  $\cdots$

Conformal QCAs



- Dynamics: $T = \cdots$ 

$\cdots$

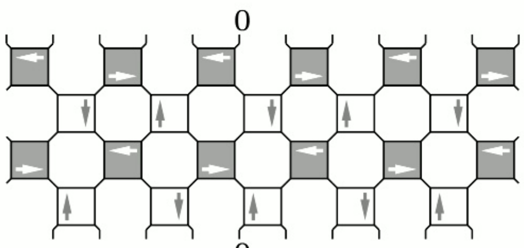
- Contraction isometry: $C : (\mathbb{C}^q)^{2n} \to (\mathbb{C}^q)^n$

$C = \cdots$ 

$\cdots$

- Scale invariance: $CT_{2n}^2 = T_n C$

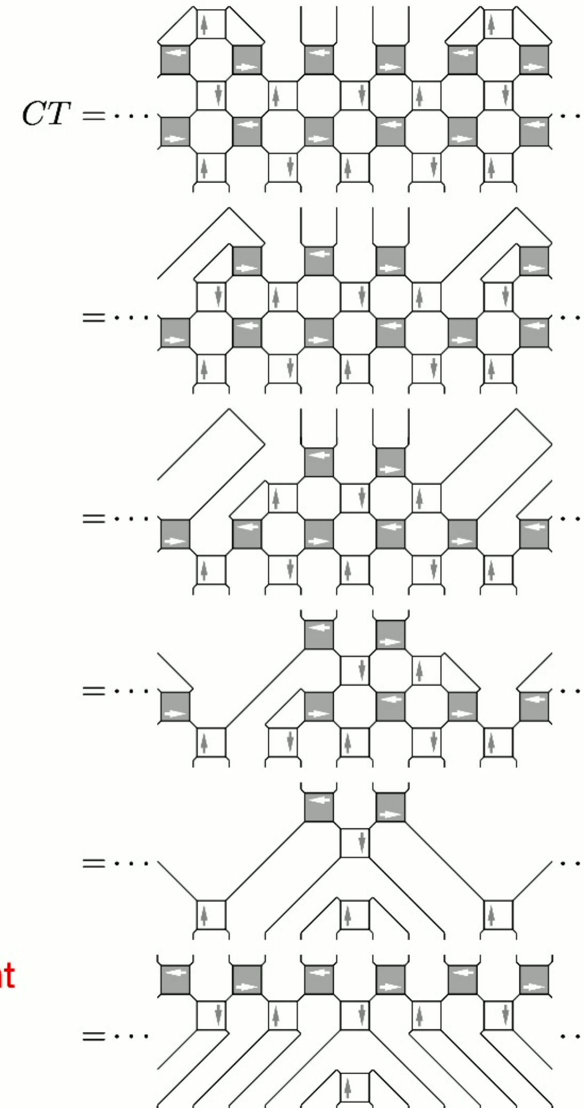
Conformal QCAs

- Dynamics: $T = \cdots$  $\cdots$

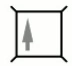
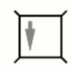
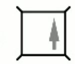
- Contraction isometry: $C : (\mathbb{C}^q)^{2n} \to (\mathbb{C}^q)^n$

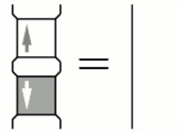
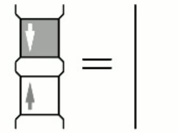
$C = \cdots$  $\cdots$

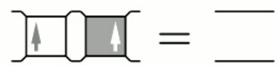
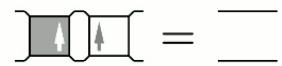
- Scale invariance: $CT_{2n}^2 C^\dagger = T_n$ exact RG fixed point



Dual unitaries

$\mathtt{u} =$  $\mathtt{u}^T =$  $\mathtt{sus}^\dagger =$  $\mathtt{u}^* =$ 

- Unitarity in the time direction:  and 

- Unitarity in the space direction:  and 

- …equivalent to: $$\mathtt{u}(\mathtt{a}\otimes\mathbb{1})\mathtt{u}^\dagger = \cancel{\mathtt{b}\otimes\mathbb{1}} + \mathbb{1}\otimes\mathtt{c} + \sum_k \mathtt{d}_k\otimes\mathtt{e}_k$$

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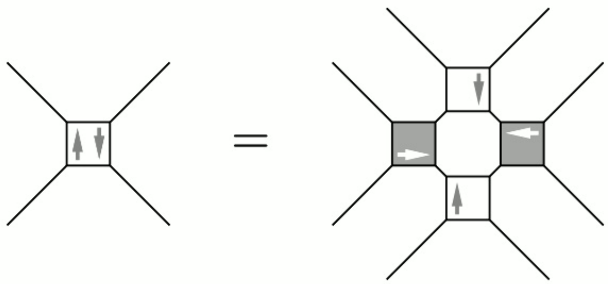
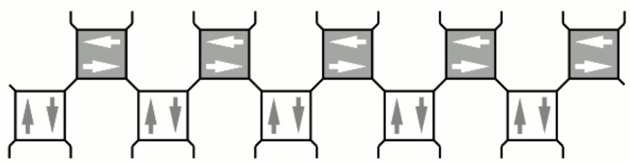
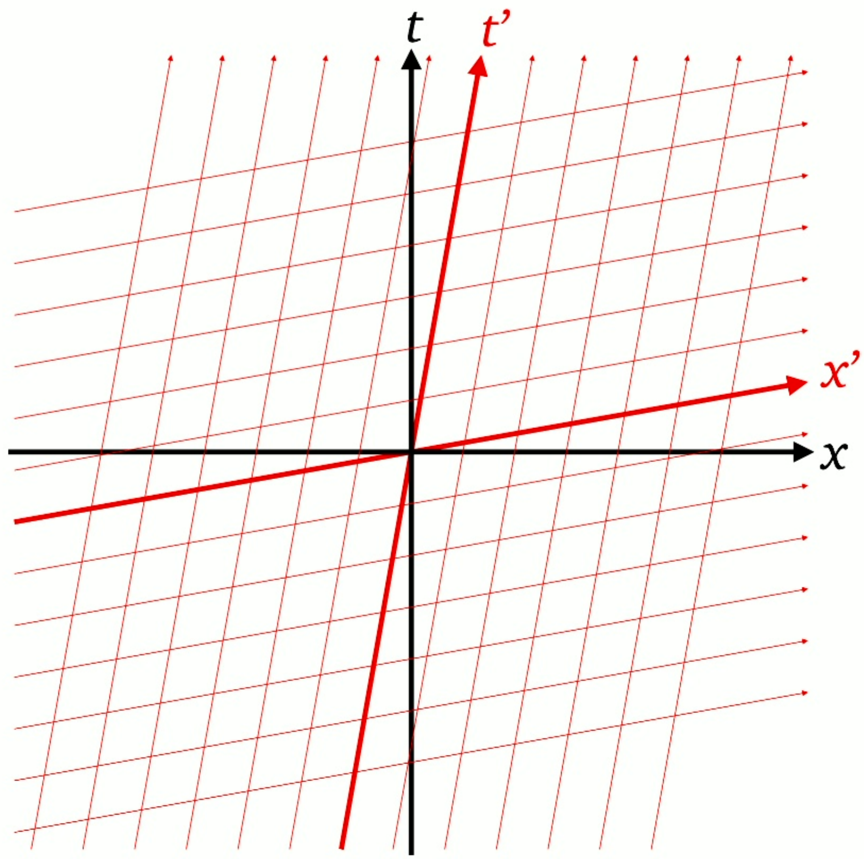
CPT-symmetric conformal QCA

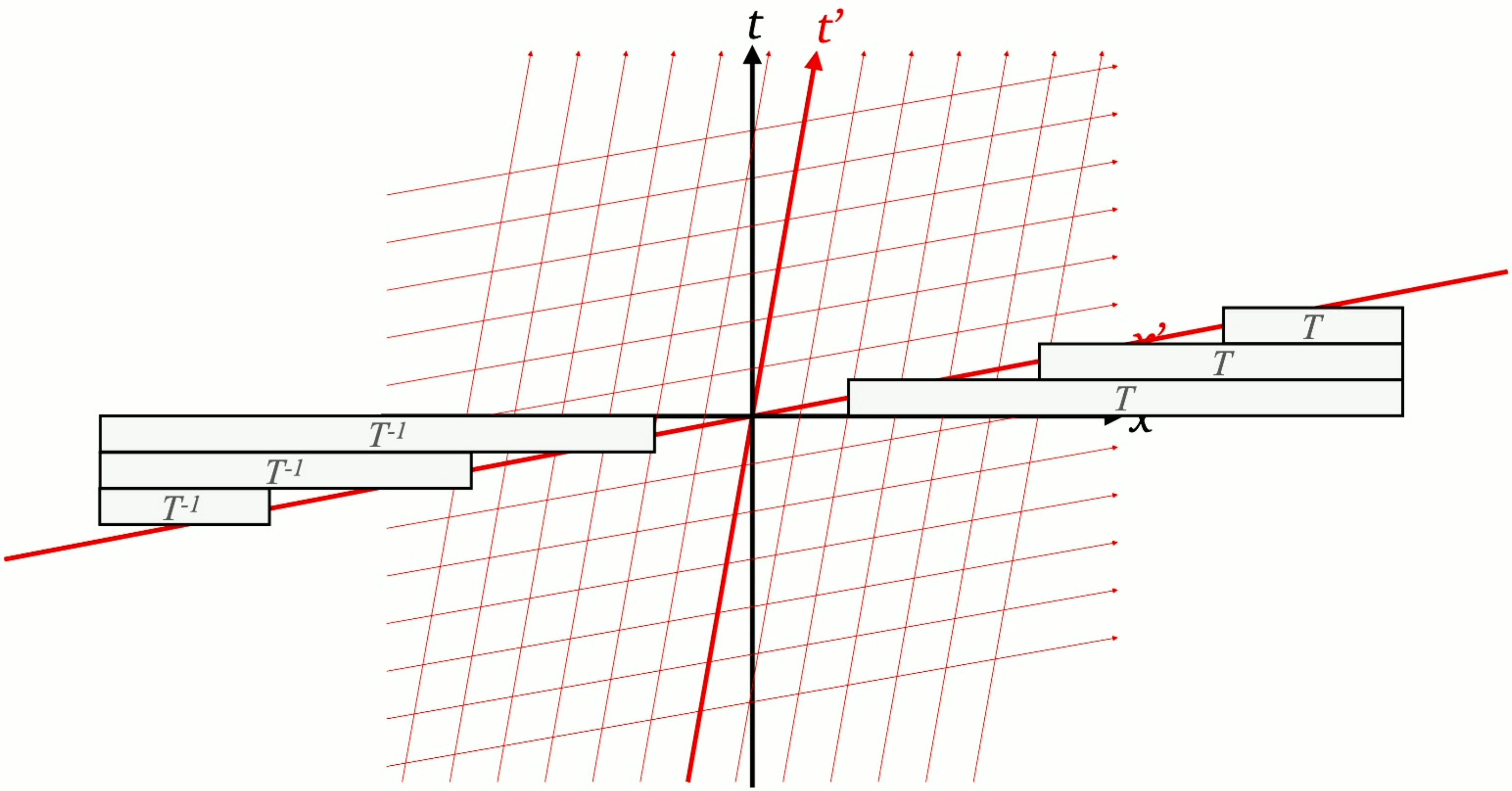
$\mathtt{u} =$  dual unitary with CPT symmetry $\mathtt{u}^T = \mathtt{sus}^\dagger$

$T = \cdots$  $\cdots$

Lorentz transformations

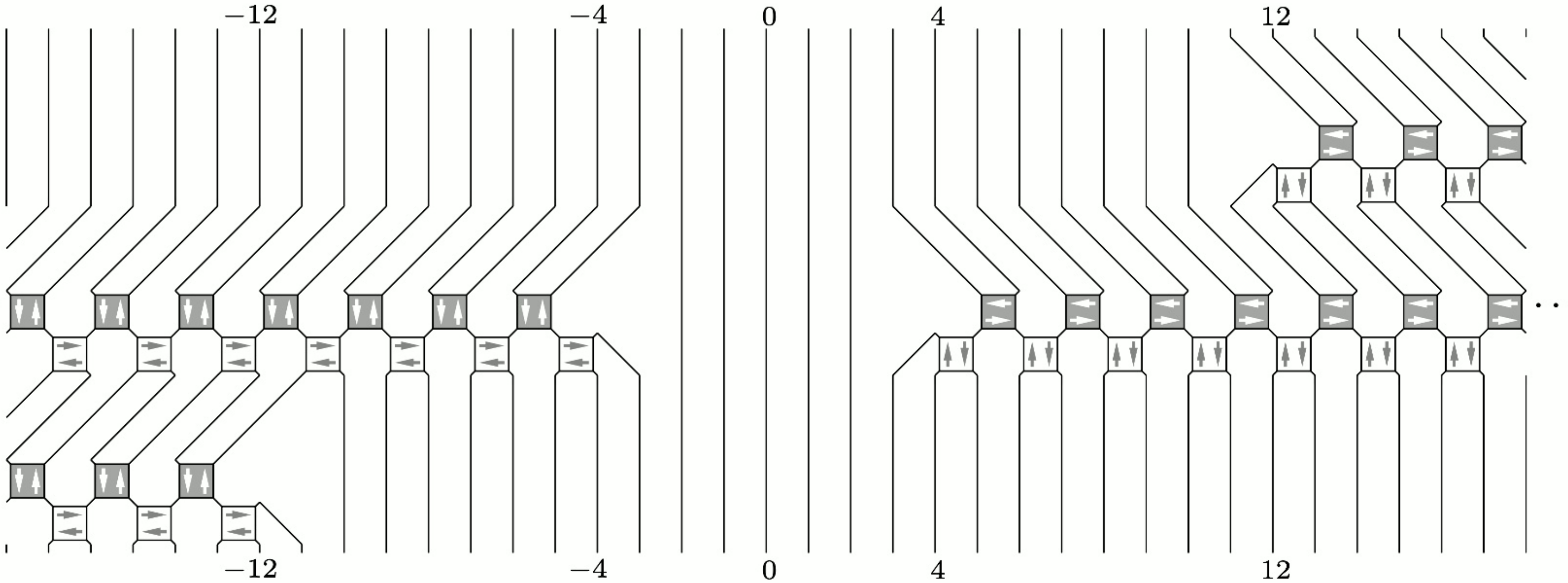


Lorentz transformations



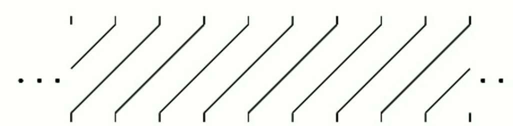
Lorentz+scale transformation

$R_4 = \cdots$



$$R_l(ST) = (ST)R_l$$

$S = \cdots$



Lorentz+scale transformation

$$\mathtt{a}(x,t) := T^t \mathtt{a}_x T^{-t}$$

$$R_l \mathtt{a}(x,t) R_l^\dagger = \begin{cases} \mathtt{a}(x',t') & \text{if } (x-2t) \notin l\mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

$$\left.\begin{array}{l} x' = x - 2f_l(x-2t) \\ t' = t + f_l(x-2t) \end{array}\right\}$$

$$f_l(x) = \left\lfloor \frac{x-l}{2l} \right\rfloor + 1$$

Lorentz+scale transformation

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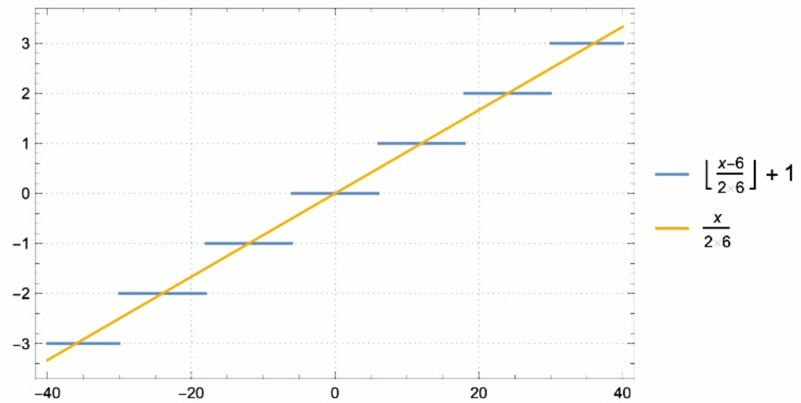
$$f_l(x) = \left\lfloor \frac{x-l}{2l} \right\rfloor + 1 \approx \frac{x}{2l}$$



Lorentz+scale transformation

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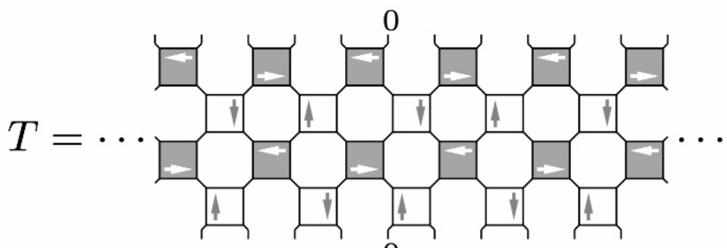
$$\left.\begin{aligned} x' &= \left(1-\tfrac{1}{l}\right)x + \tfrac{2}{l}t \\ t' &= \left(1-\tfrac{1}{l}\right)t + \tfrac{1}{2l}x \end{aligned}\right\} \text{ Lorentz+scale transformation}$$

Preserves space-time interval up to scale factor: $(2t')^2 - x'^2 = \left(1-\dfrac{2}{l}\right)\left[(2t)^2 - x^2\right]$

Lorentz boost velocity: $v = \dfrac{-2}{\sqrt{4-2l+l^2}}$

Conformal QCAs

- Dynamics:



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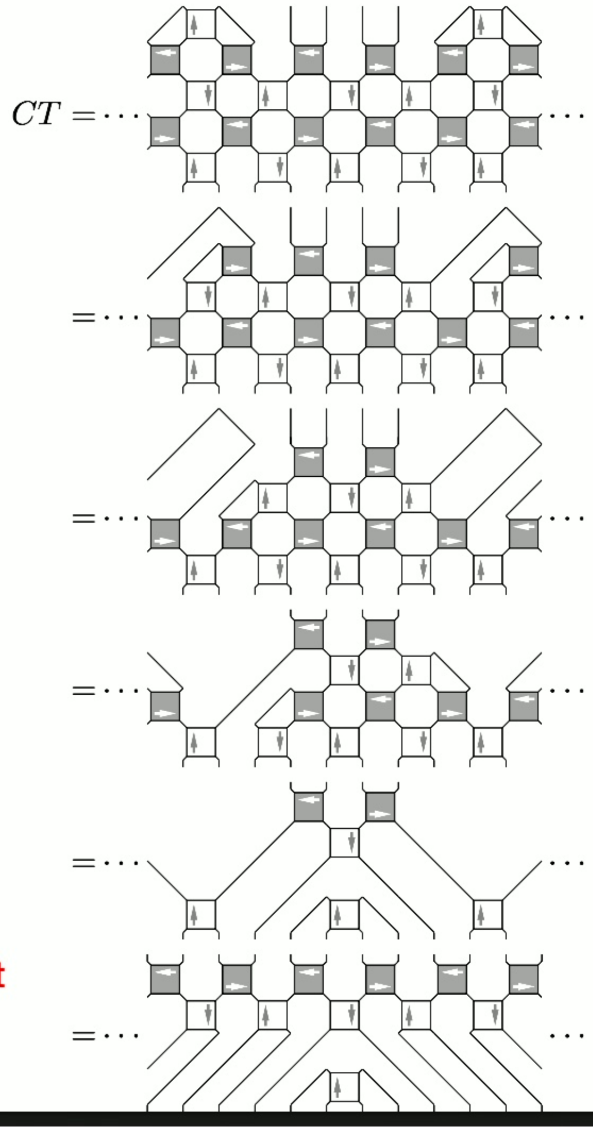
- Scale invariance: $CT_{2n}^2 C^\dagger = T_n$ exact RG fixed point



Lorentz+scale transformation

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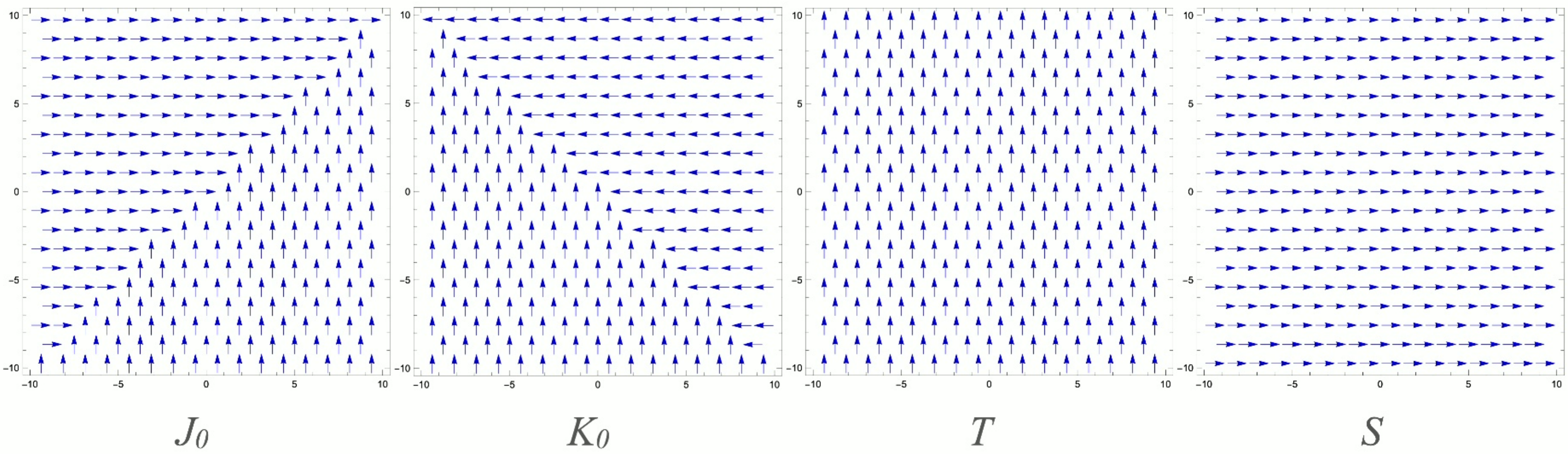
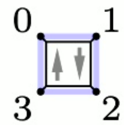
Smeared field operators: $\Phi(x,t) = \sum_{y\in\mathbb{Z}_n} \varphi(y-x)\, T^t \mathsf{a}_y T^{-t}$

$$R_l \Phi(x,t) R_l^\dagger \approx \left(1-\frac{1}{l}\right)\Phi(x',t')$$

Complete set of generators (n=∞)

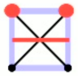


Geometry and entanglement

- Tensor-network geometry: $|\Psi\rangle = \frac{1}{q}$ 

$=$ 

- This geometry satisfies Ryu-Takayanagi

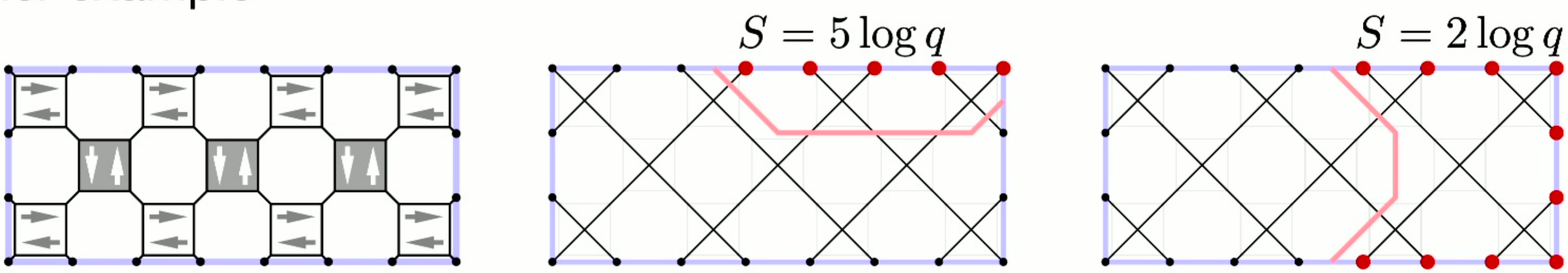
 $S = 2\log q$  $S = \log q$

- Maximally entangled with respect to partitions 01|23 and 03|12

Entanglement and geometry

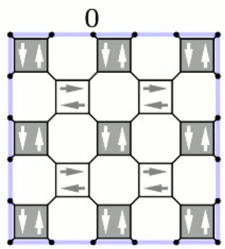
- Another example

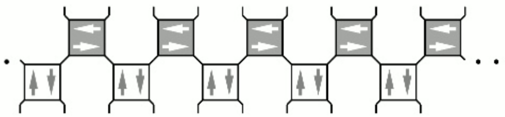
$|\Psi\rangle =$

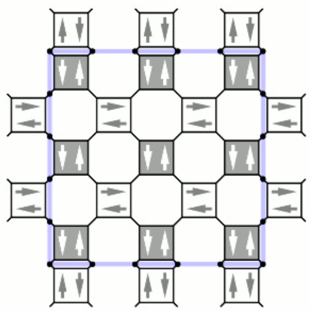
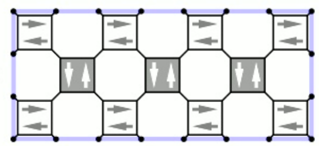


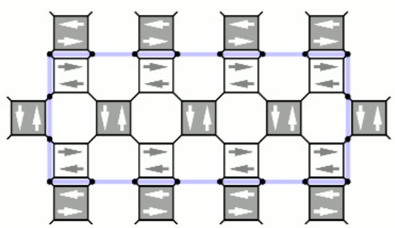
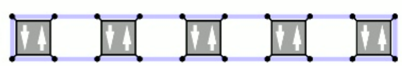
- The TN geometry satisfies RT but provides much more information than RT (e.g. length of BH throat).

Evolution of real-time tensor-network states

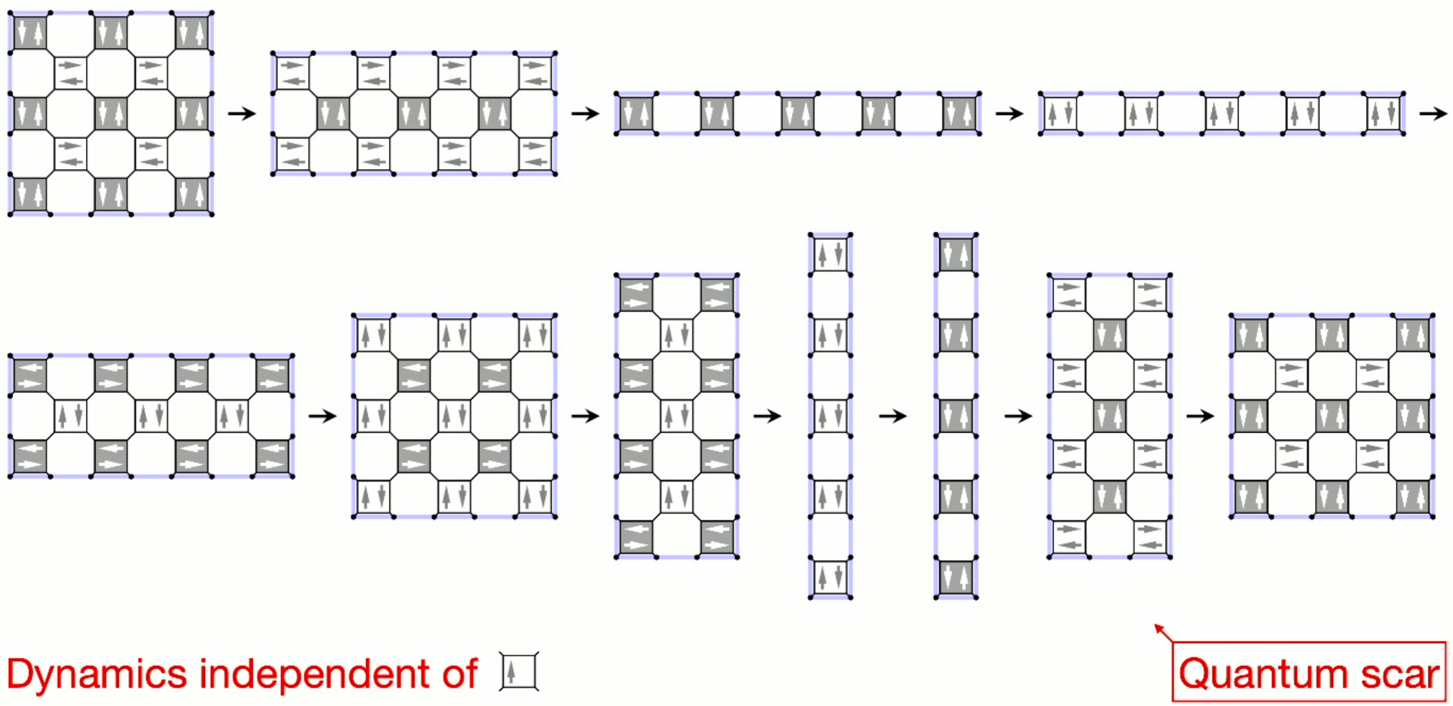
$|\Psi\rangle = \frac{1}{q^5}$ 

$T = \cdots$  $\cdots$

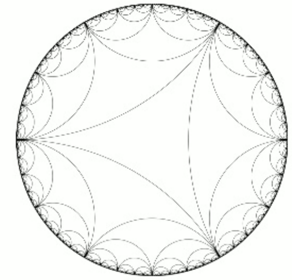
$T_{\text{even}} |\Psi\rangle =$  $=$  ,

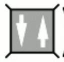
$T_{\text{odd}} T_{\text{even}} |\Psi\rangle =$  $=$ 

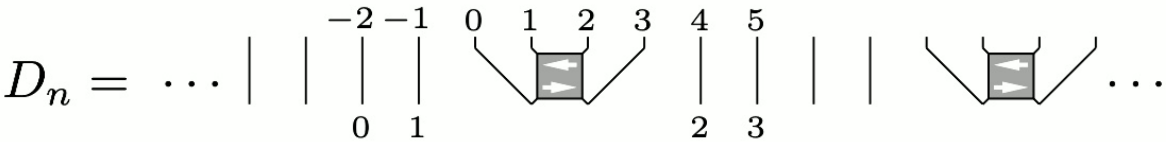
Evolution of real-time tensor-network states

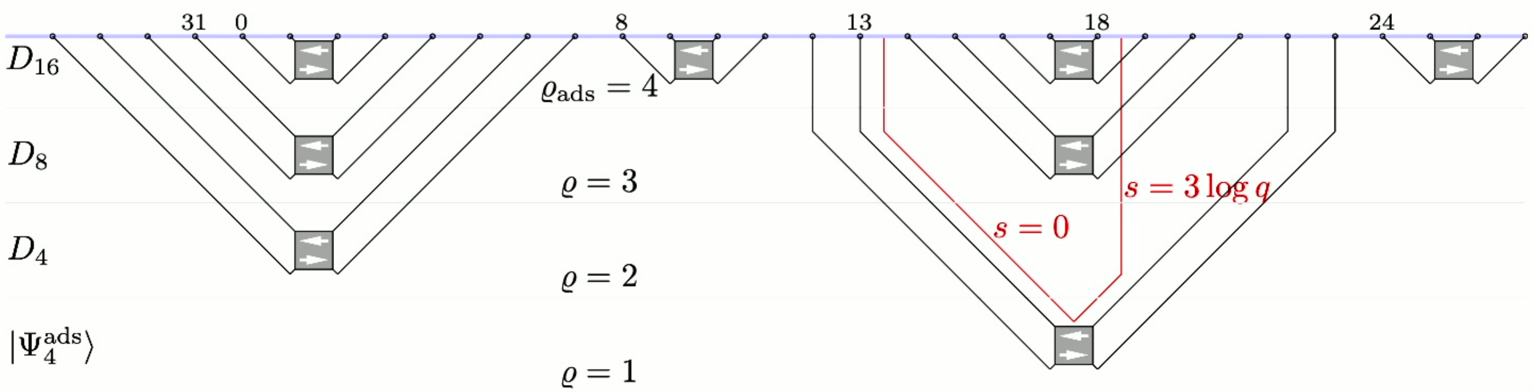


Toy anti-de Sitter state



$|\Psi_{32}^{\text{ads}}\rangle = D_{16} D_8 D_4 \, |$  $\rangle$





Anti-de Sitter eigen-state

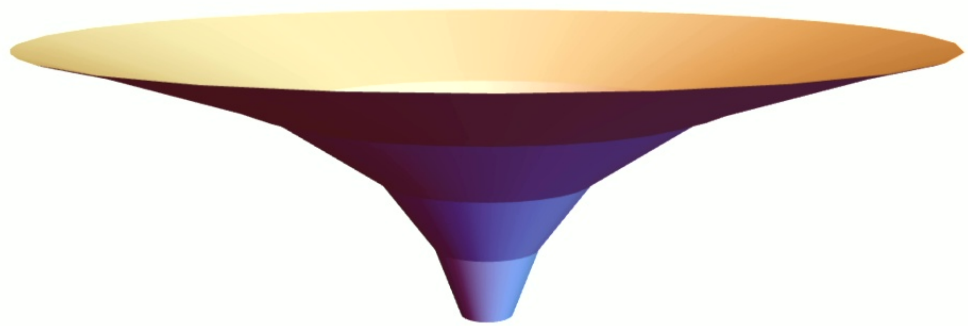
$$|\Psi_n^{\mathrm{eigen-ads}}\rangle = \sum_{t=0}^{\frac{n}{2}-1} T^t \, |\Psi_n^{\mathrm{ads}}\rangle$$

$$\Delta s^2_{\Psi^{\mathrm{eigen-ads}}} = \log^2 q \left(-2^{2\varrho} \Delta\tau^2 + \Delta\varrho^2 + 2^{2\varrho} \frac{\Delta\theta^2}{\pi^2} \right)$$ large q

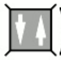
$$ds^2_{\mathrm{ads}} = \alpha^2 \left(-\cosh^2\varrho \, d\tau^2 + d\varrho^2 + \sinh^2\varrho \, d\theta^2 \right)$$

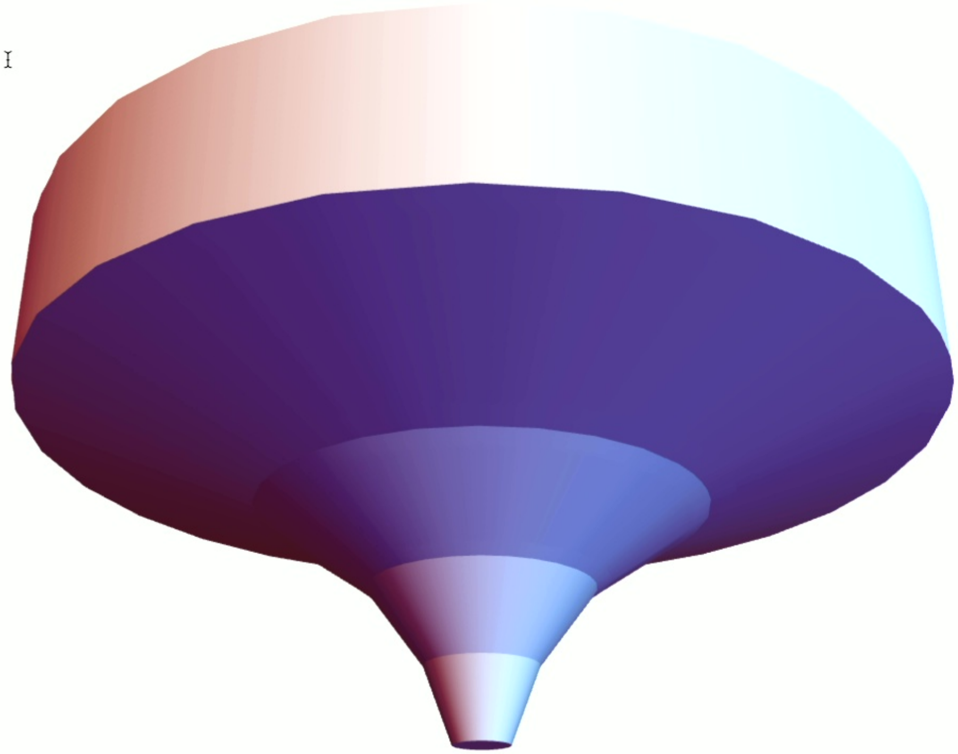
Toy anti-de Sitter state

$$|\Psi_{64}^{\mathrm{ads}}\rangle = DDDD\,|\boxed{\downarrow\uparrow}\rangle$$



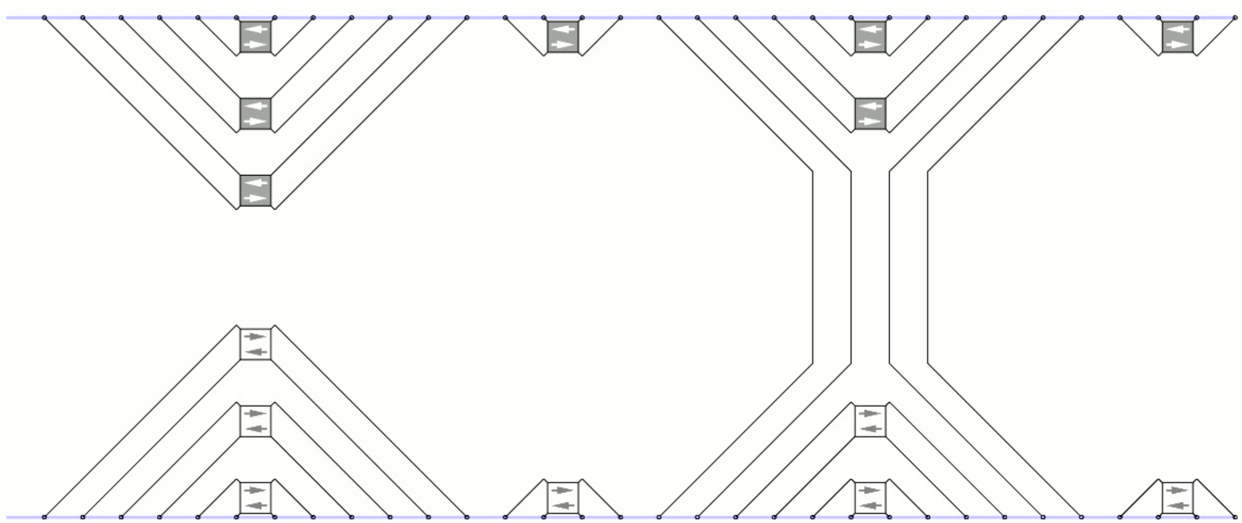
Toy anti-de Sitter state

$T^{32}\,|\Psi^{\mathrm{ads}}_{64}\rangle = T^{32}DDDD\,|$  $\rangle$



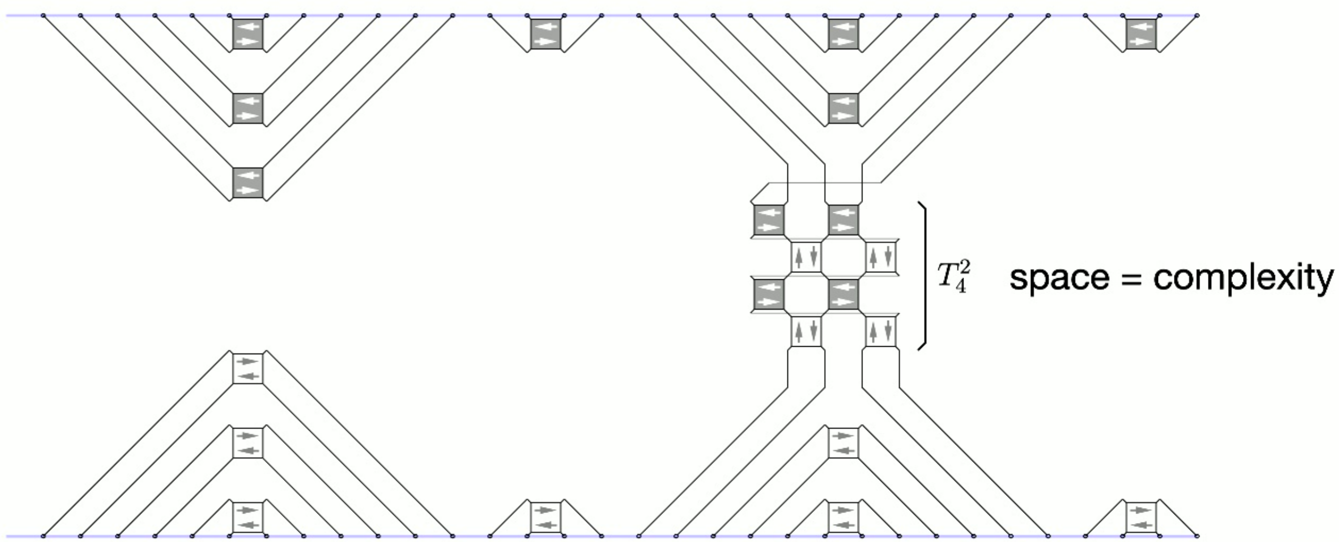
Double-sided black hole

$$|\Psi^{\text{tfd}}_{n,s}\rangle = \underbrace{(D_{\frac{n}{2}} \cdots D_{2s} D_s)}_{\varrho_{\text{ads}} - \varrho_{\text{h}}} \otimes \underbrace{(D_{\frac{n}{2}} \cdots D_{2s} D_s)^*}_{\varrho_{\text{ads}} - \varrho_{\text{h}}} |\Psi^{\text{tfd}}_{s,s}\rangle$$



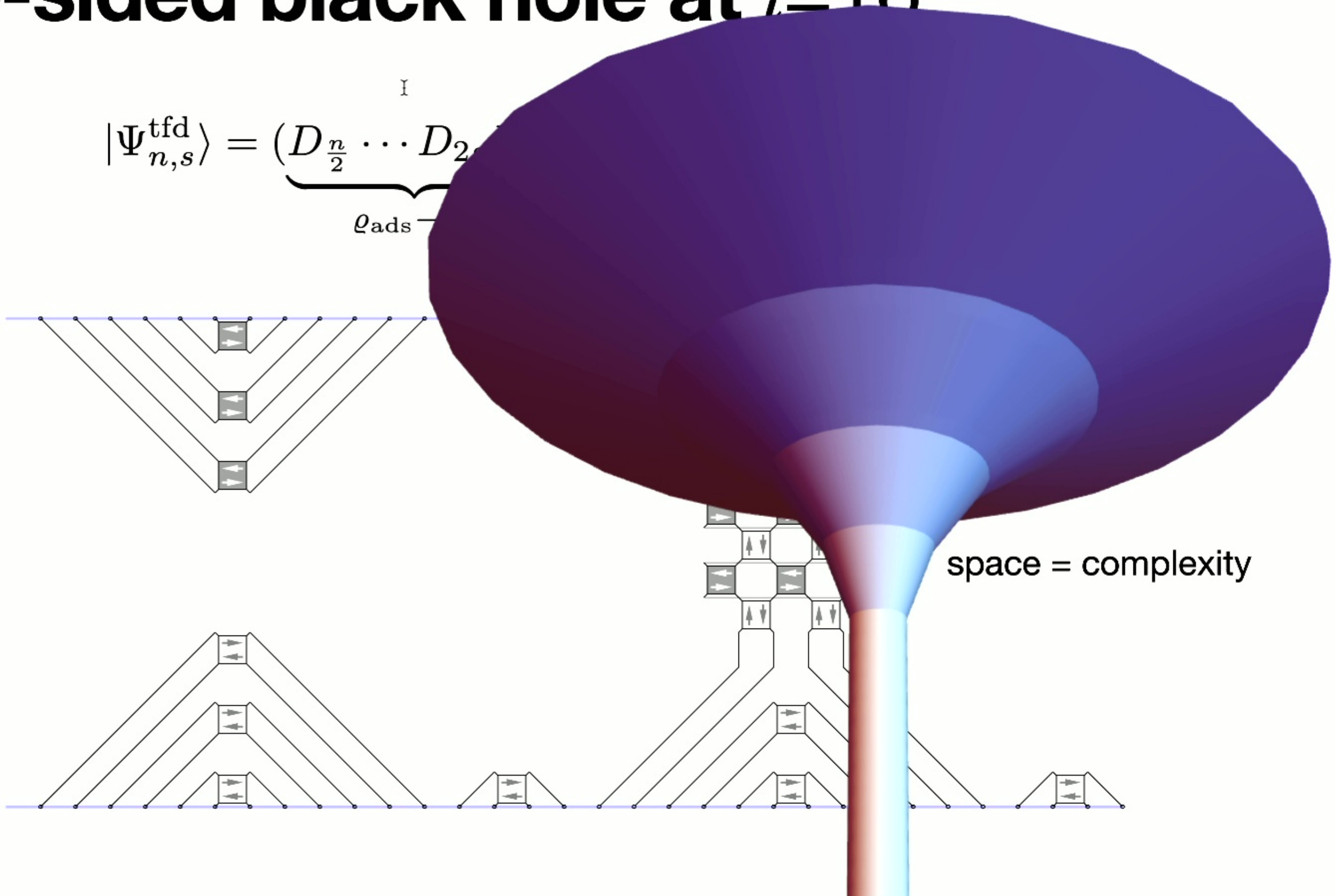
Double-sided black hole at t=16

$$|\Psi^{\text{tfd}}_{n,s}\rangle = \underbrace{(D_{\frac{n}{2}} \cdots D_{2s} D_s)}_{\varrho_{\text{ads}} - \varrho_{\text{h}}} \otimes \underbrace{(D_{\frac{n}{2}} \cdots D_{2s} D_s)}_{\varrho_{\text{ads}} - \varrho_{\text{h}}}^{*} |\Psi^{\text{tfd}}_{s,s}\rangle$$



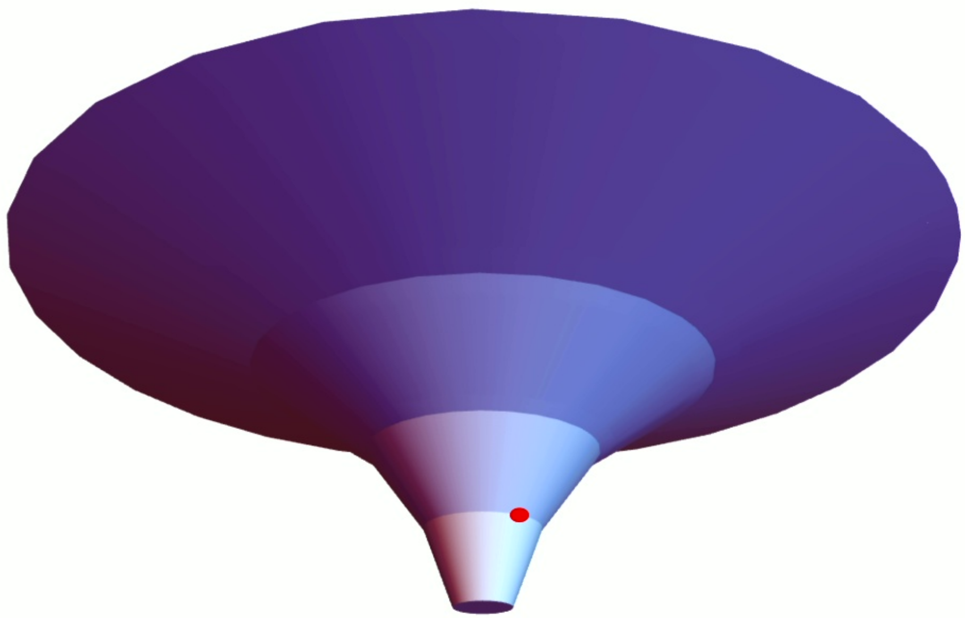
Double-sided black hole at t=16

$$|\Psi^{\text{tfd}}_{n,s}\rangle = (D_{\frac{n}{2}} \cdots D_{2} \ldots$$



Gravitational time dilation

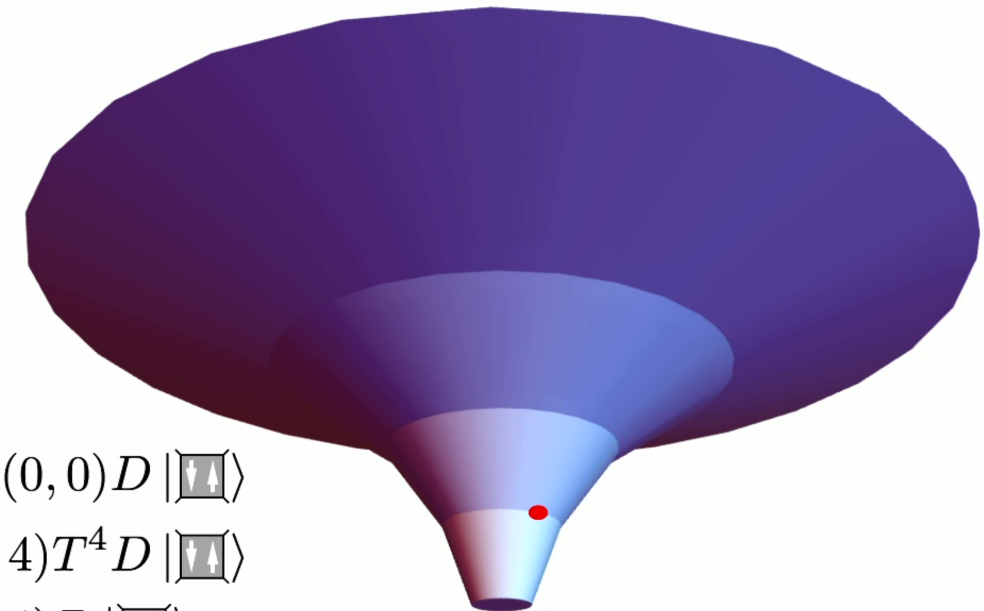
$$|\Psi_{64}^{\text{ads+1 part}}\rangle = DDD\,\mathtt{a}_0 D\,|\,\rangle$$



Gravitational time dilation

$$|\Psi_{64}^{\text{ads+1 part}}\rangle = D D D\, \mathtt{a}_0 D\, |\boxed{\downarrow\uparrow}\rangle$$

$$\begin{aligned} T^{32}\, |\Psi_{64}^{\text{ads+1 part}}\rangle &= D D D T^4 \mathtt{a}(0,0) D\, |\boxed{\downarrow\uparrow}\rangle \\ &= D D D \mathtt{a}(0,4) T^4 D\, |\boxed{\downarrow\uparrow}\rangle \\ &= D D D \mathtt{a}(0,4) D\, |\boxed{\downarrow\uparrow}\rangle \end{aligned}$$



Anti-de Sitter eigen-state

$$|\Psi_n^{\text{eigen}-\text{ads}}\rangle = \sum_{t=0}^{\frac{n}{2}-1} T^t \, |\Psi_n^{\text{ads}}\rangle$$

$$\Delta s^2_{\Psi^{\text{eigen}-\text{ads}}} = \log^2 q \left(-2^{2\varrho} \Delta\tau^2 + \Delta\varrho^2 + 2^{2\varrho} \frac{\Delta\theta^2}{\pi^2} \right) \qquad \text{large } q$$

$$ds^2_{\text{ads}} = \alpha^2 \left(-\cosh^2\varrho \, d\tau^2 + d\varrho^2 + \sinh^2\varrho \, d\theta^2 \right)$$

Gravitational time dilation

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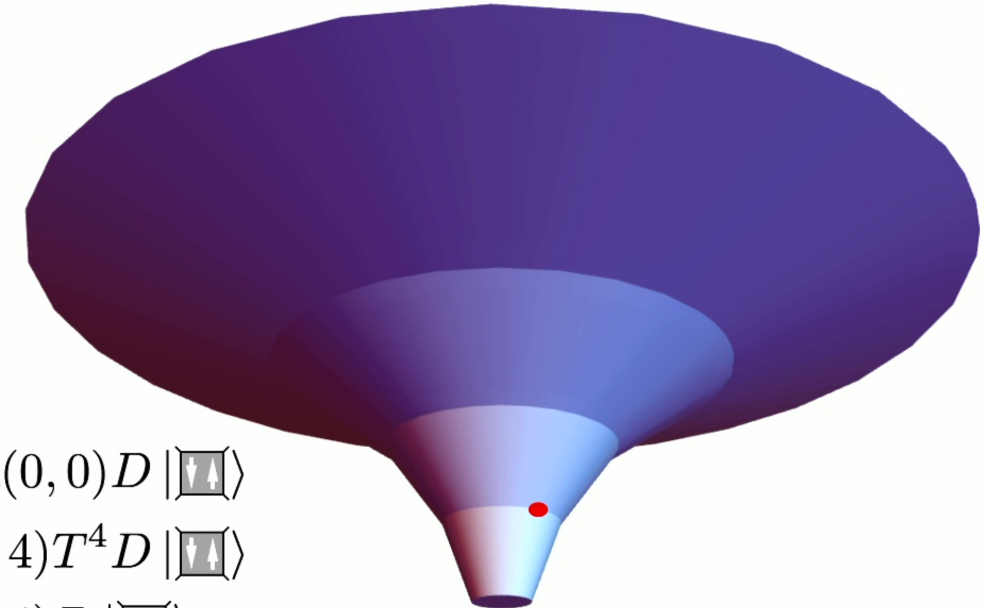
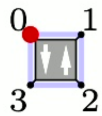
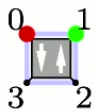
$$\begin{aligned} T^{32}\,|\Psi_{64}^{\text{ads+1 part}}\rangle &= DDDT^4\mathtt{a}(0,0)D\,|\boxed{\downarrow\uparrow}\rangle \\ &= DDD\mathtt{a}(0,4)T^4D\,|\boxed{\downarrow\uparrow}\rangle \\ &= DDD\mathtt{a}(0,4)D\,|\boxed{\downarrow\uparrow}\rangle \end{aligned}$$



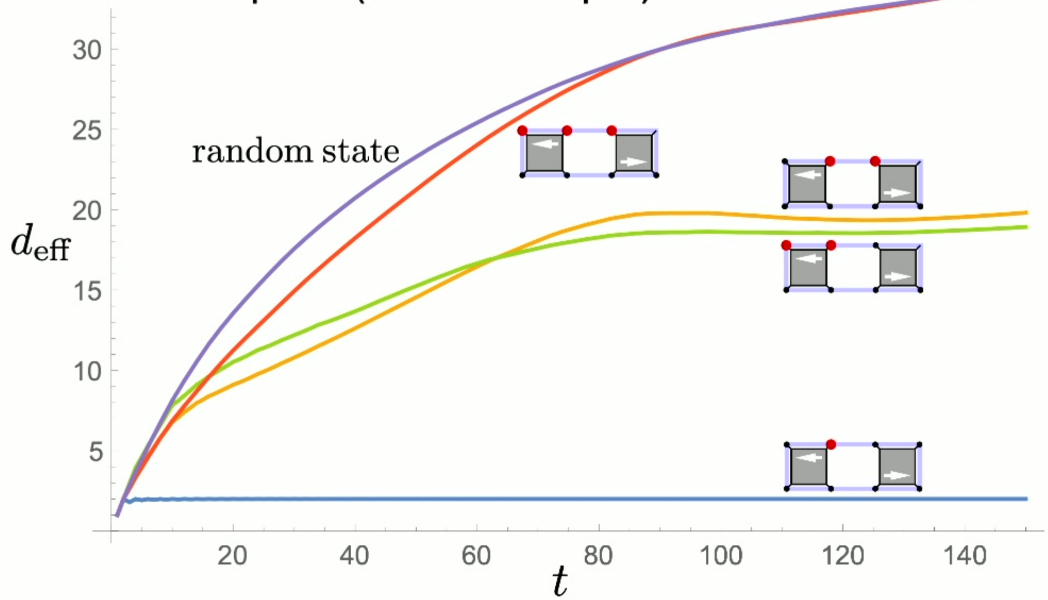
1 particle = scar, 2 particles = chaos

$$\mathsf{a}_0\,|\Psi_4^{\mathrm{ads}}\rangle = \frac{1}{q}$$ 

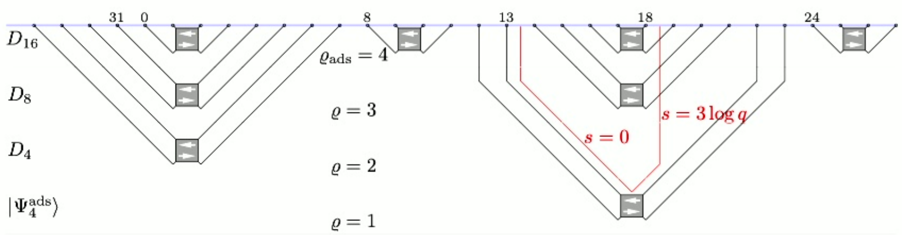
Explores a sub-space of dimension 2

$$\mathsf{a}_0\mathsf{b}_1\,|\Psi_4^{\mathrm{ads}}\rangle = \frac{1}{q}$$ 

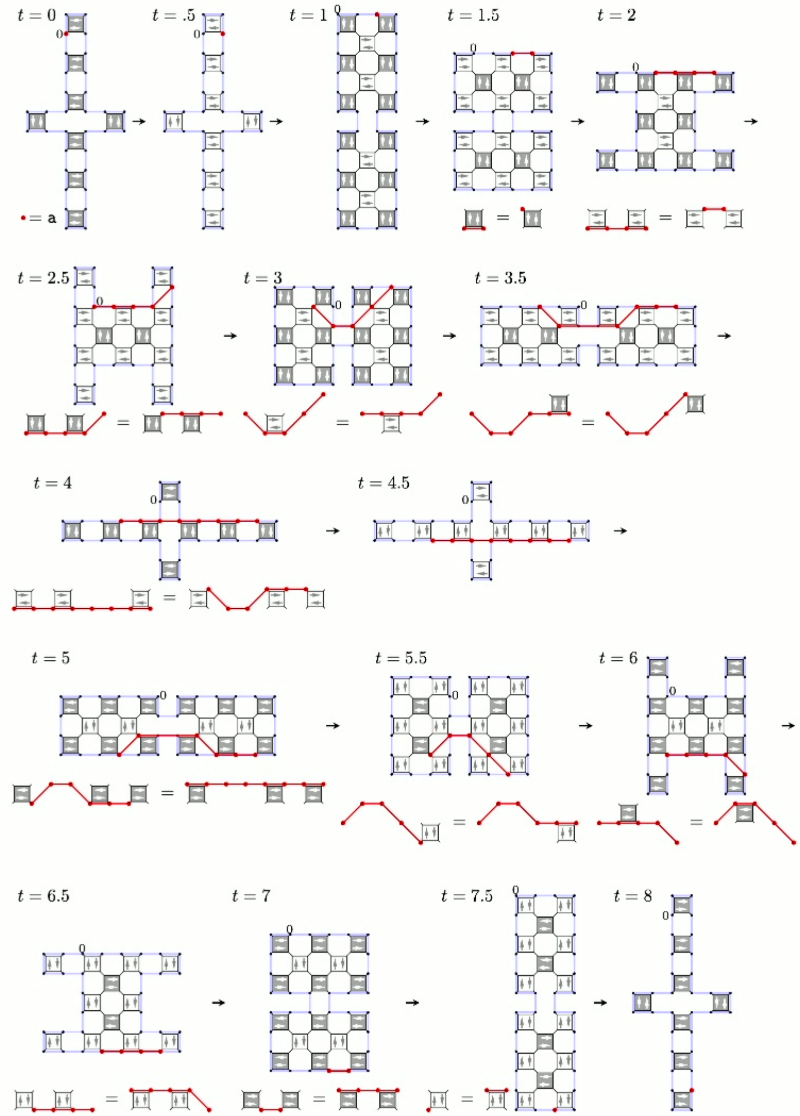
Explores a sub-space (dimension $q^4/2$)



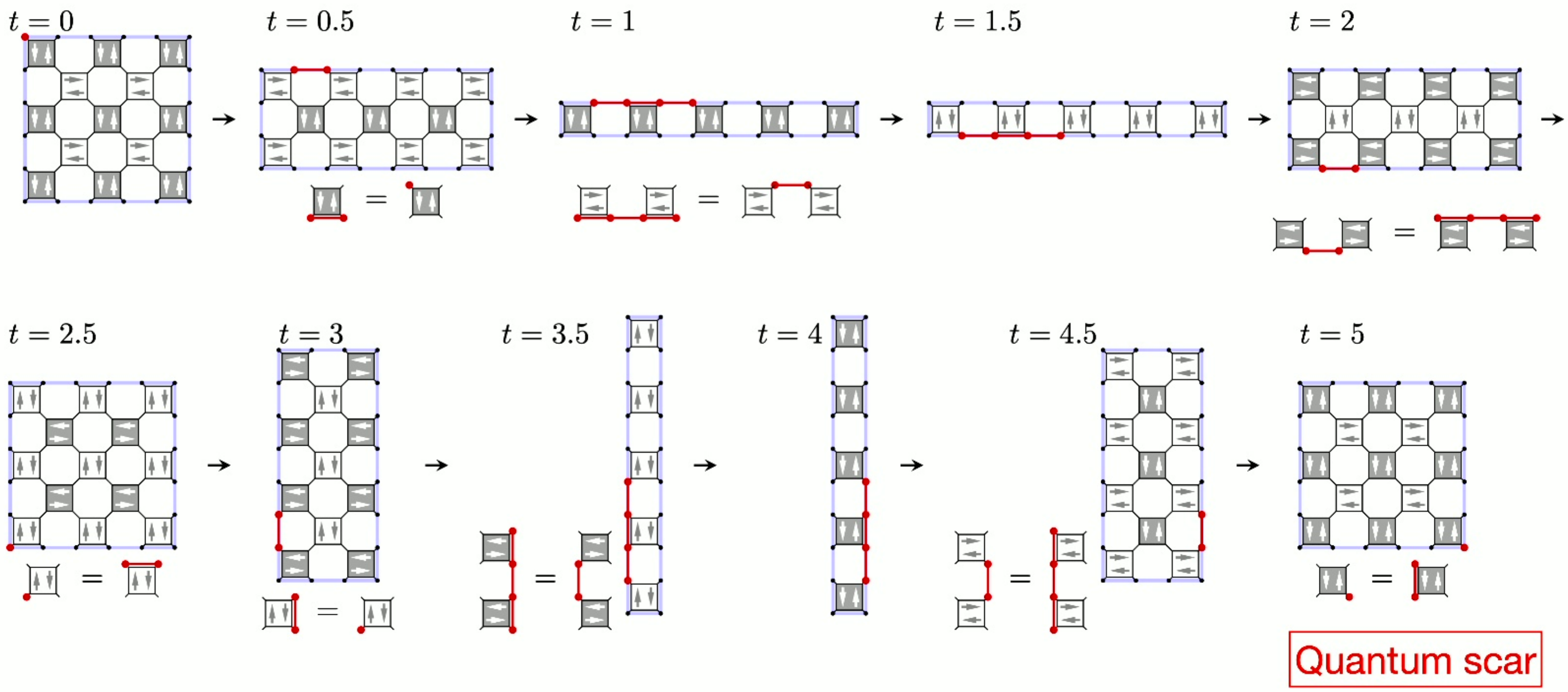
Toy AdS with 1 particle



Quantum scar

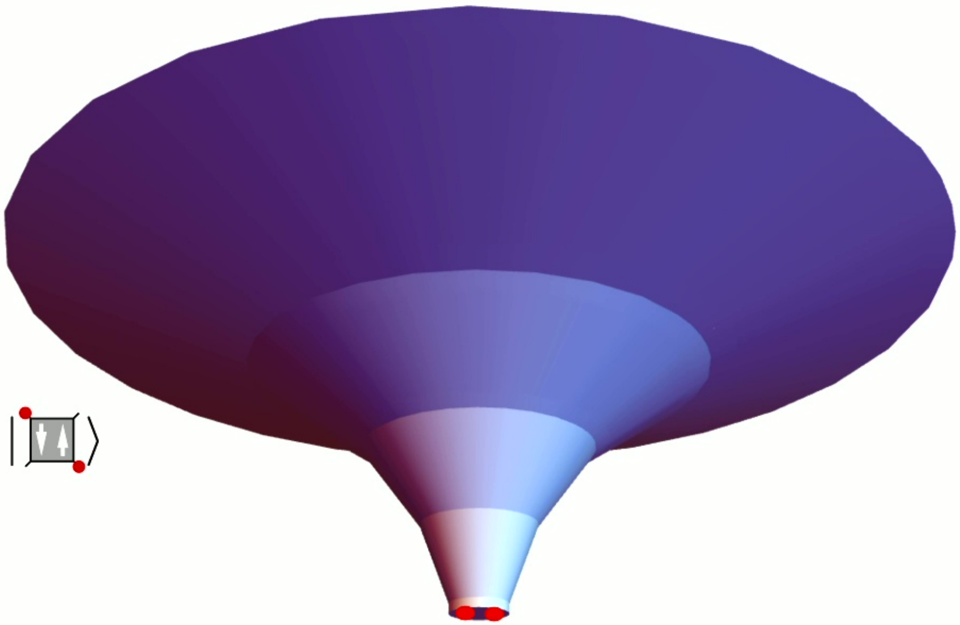


Flat space with 1 particle



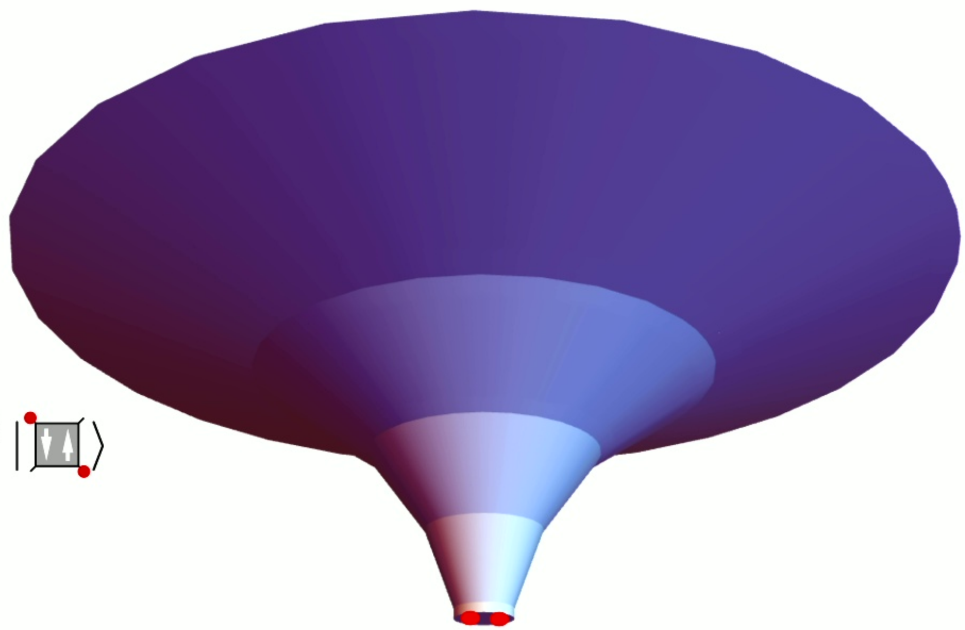
2 particles: yes black hole

$$T^{32}\,|\Psi_{64}^{\text{ads+2 particles}}\rangle = DDDDT^{2}\,|\,\rangle$$



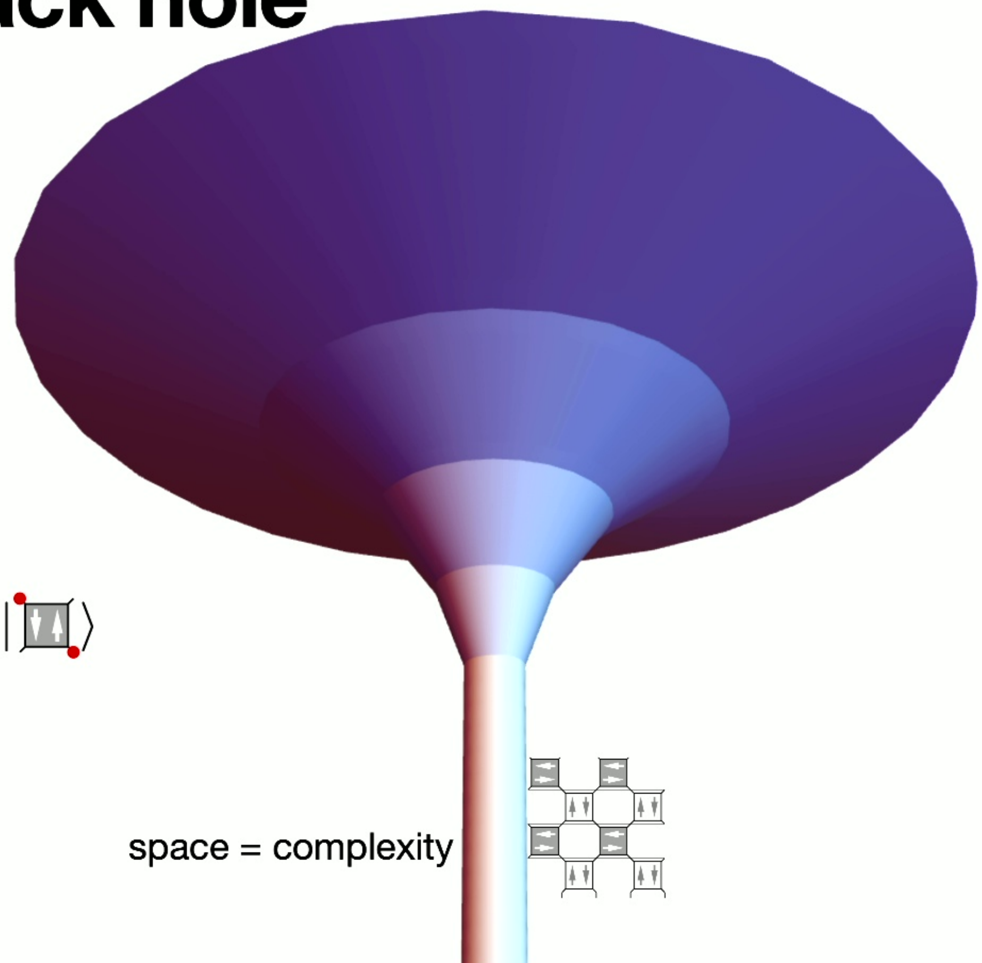
2 particles: yes black hole

$$T^{32}\,|\Psi_{64}^{\text{ads+2 particles}}\rangle = DDDDT^2\,|\;\;\rangle$$



2 particles: yes black hole

$$T^{32}\,|\Psi_{64}^{\text{ads+2 particles}}\rangle = DDDDT^2\,|\quad\rangle$$



Conclusions

- Conformal QCAs are invariant under discrete Lorentz+scale transformations.
- Induced dynamics on real-time tensor-network states (discrete geometries).
- Finte-radius discrete holography for variety of spaces different than AdS.
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- Do these dynamical geometries obey a discrete analog of Einstein's eqs?

Thank you