Title: Burns space and holography

Speakers: Atul Sharma

Series: Quantum Fields and Strings

Date: May 09, 2023 - 2:00 PM

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Abstract: I will construct a top-down example of celestial holography, based on recent work with Costello and Paquette. Our duality relates certain models of self-dual gauge theory and conformal gravity, placed on an asymptotically flat four dimensional spacetime called Burns space, to a two dimensional chiral algebra living on D1-branes in a topological string theory on twistor space.

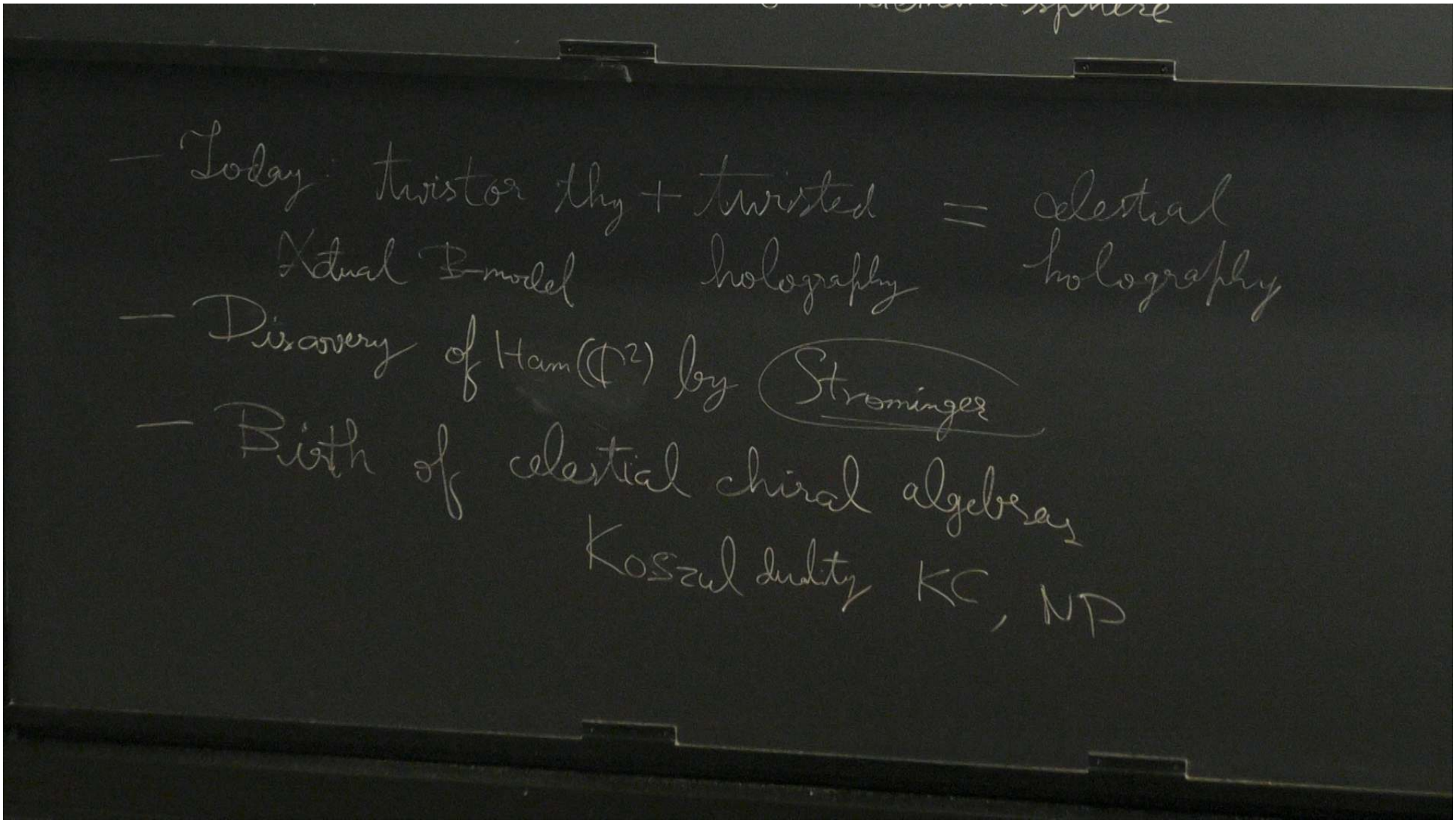
Zoom link: https://pitp.zoom.us/j/92898535744?pwd=T2w2SUp0VnU5NWtPSm50VENvWlRYQT09

Burns Space & Holography

Based on Costello, Paquette, AS
[PRL 130, 061602] + wip

1. The story so far

Q.G. in $\Lambda=0$ 4d spacetimes $=$ 2d CFT living on Riemann sphere


— Today: twistor thy + twisted = celestial
Actual B-model holography holography
— Discovery of Ham($\mathbb{C}^2$) by Stromingers
— Birth of celestial chiral algebras
Koszul duality KC, NP

& Holography

lo, Paquette, AS

130, 061602] + wip

so far

$\Lambda=0$ = 2d CFT living on Riemann sphere

ristor Hhy + twisted = celestial

D-model holography holography

2. Twistorial holography

$(M, g) \longrightarrow S^2 \hookrightarrow Z \longrightarrow M$

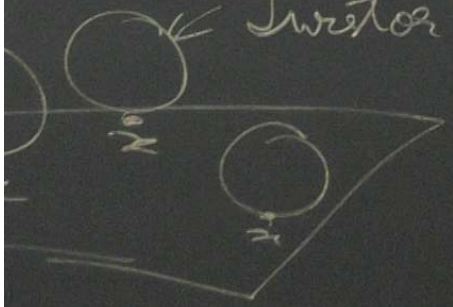
↑ 4d spacetime

Twistorial holography

$(M, g) \longrightarrow S^2 \longrightarrow Z \longrightarrow M$

spacetime $\qquad Z =$ projective 2-spinor bundle over M

Twistor lines

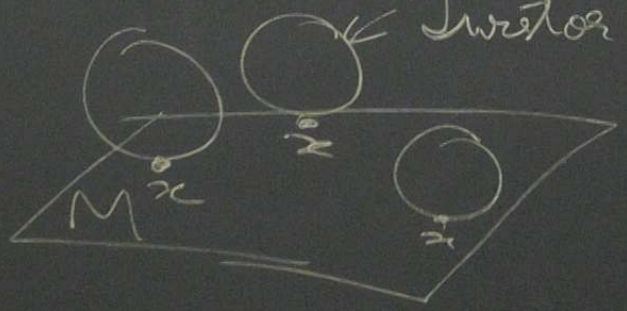


2. Twistorial holography

$(M, g) \longrightarrow S^2 \hookrightarrow Z \longrightarrow M$

↑ 4d spacetime

Z = projective 2-spinor bundle o



Penrose

Atiyah-Hitchin-Singer

Z is a complex manifold iff

l holography

$S^2 \to Z \longrightarrow M$

$=$

projective 2-spinor bundle over M

hor lines

Penrose

Atiyah-Hitchin-Singer:

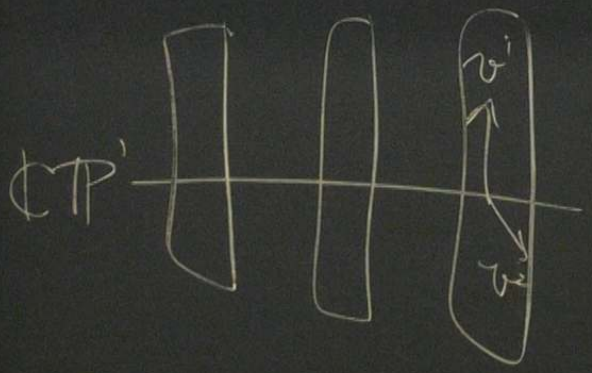
Z is a complex manifold iff $W_3 = 0$

M^{2n}

Z is a complex manifold iff $W_g =$

Start with $\mathbb{P}\Pi = \mathcal{O}(1) \oplus \mathcal{O}(1) \longrightarrow \mathbb{CP}^1_Z$

$v^\alpha = (v^1, v^2)$

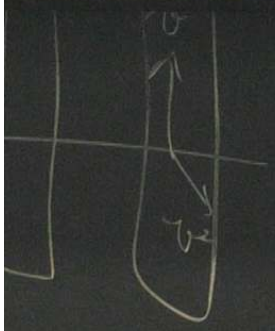


$(z, v^\alpha) \longrightarrow \left(\frac{1}{z}, \frac{v^\alpha}{z}\right)$

$\mathbb{P}\Pi = \mathbb{R}^4 \times \mathbb{CP}^1$

Atiyah-Hitchin-Singer:

Z is a complex manifold iff $W_g = 0$



$$(z, v^\alpha) \longrightarrow \left(z, \frac{v^\alpha}{z}\right)$$

$$\mathbb{P}\mathrm{II} \simeq \mathbb{R}^4 \times \mathbb{RP}^1$$

$$v^{\dot\alpha} = u^{\dot\alpha} + z\hat{u}^{\dot\alpha},$$

$$(v, z) \longrightarrow (u, \hat{u}, z)$$

$$x^{1\dot\alpha}, \quad x^{1\dot\alpha} = u^{\dot\alpha}$$

$$x^{2\dot\alpha} = \hat{u}^{\dot\alpha}$$

$$= (-\overline{u^2}, \overline{u^1})$$

There is no global (3,0)-volume form

$$dz\wedge dw_1\wedge dw_2 \xrightarrow{(z,w)\to\left(\frac{1}{2}z,\frac{1}{2}w\right)} -\frac{dz\wedge dw_1\wedge dw_2}{2^4}$$

Ponteconvoé thm

- g is $\partial,d+$ Kähler $\Rightarrow \check{\omega}\in H^0($

al (3,0) - volume form

$dv^1 \wedge dv^2 \xrightarrow{(v^1, v^2) \to \left(\frac{1}{z}, \frac{v}{z}\right)} - \frac{dz \wedge dv^1 \wedge dv^2}{z^4}$

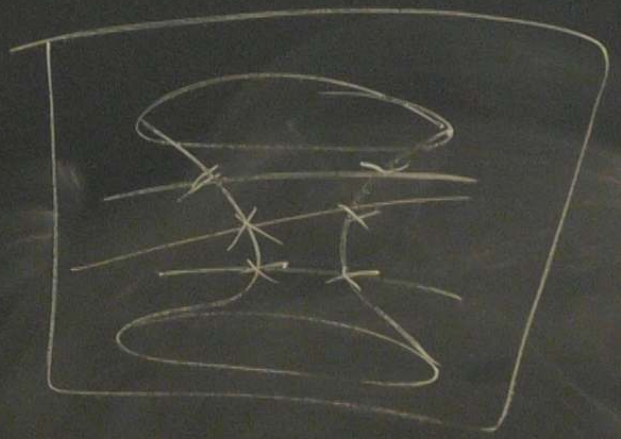
thm

$K_Z \to (3,0)$-forms. $K_Z^{1/2}$ exists [Hitchin]

+ Kähler $\Rightarrow \Omega \in H^0(Z, K_Z^{-1/2})$

(3,0) - volume form

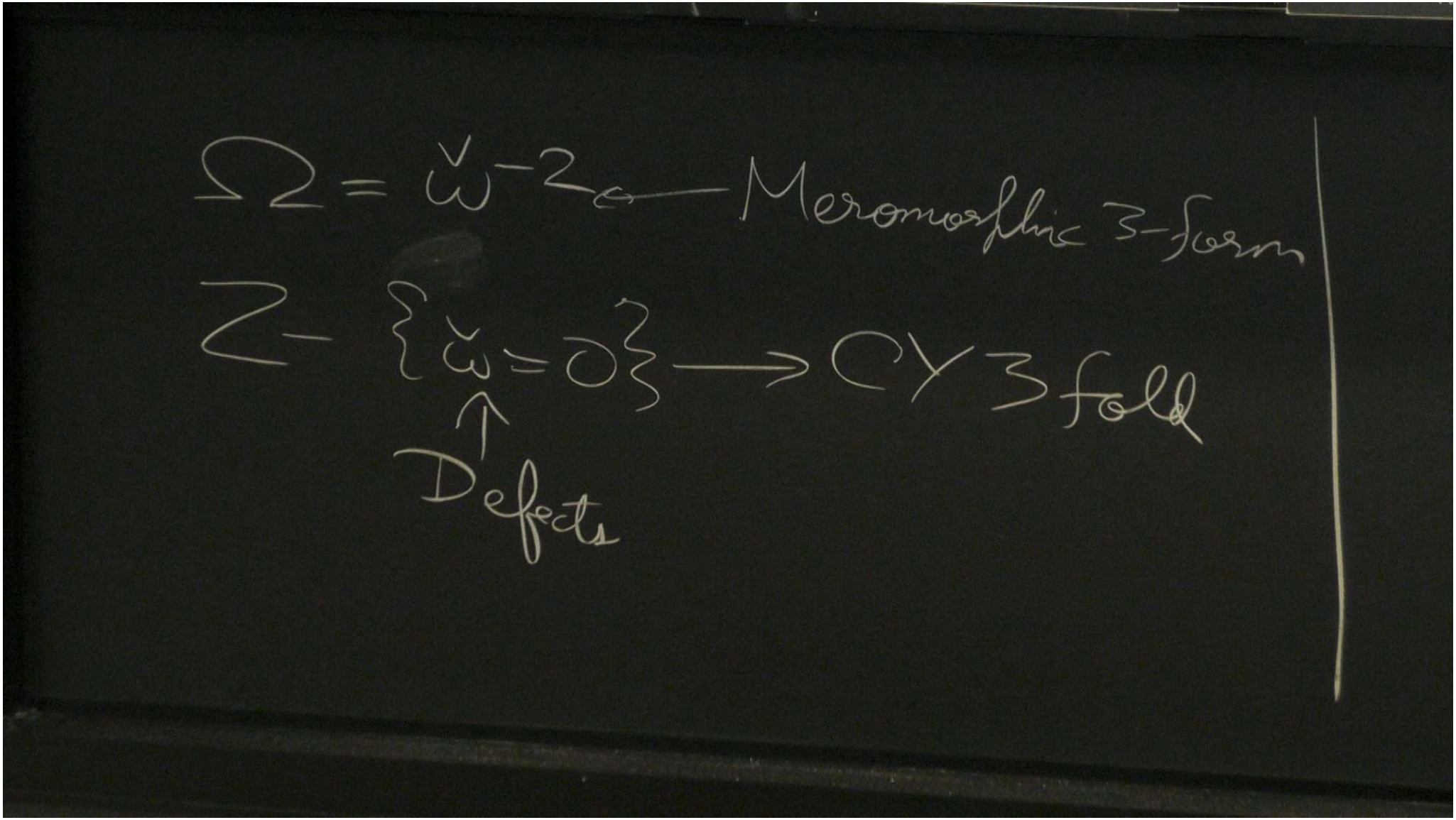
$\Omega \, dv^2 \; (v \to (\frac{1}{z} - \frac{v}{z})) \longrightarrow - \frac{dz \wedge dv_1 \wedge dv^2}{z^4}$

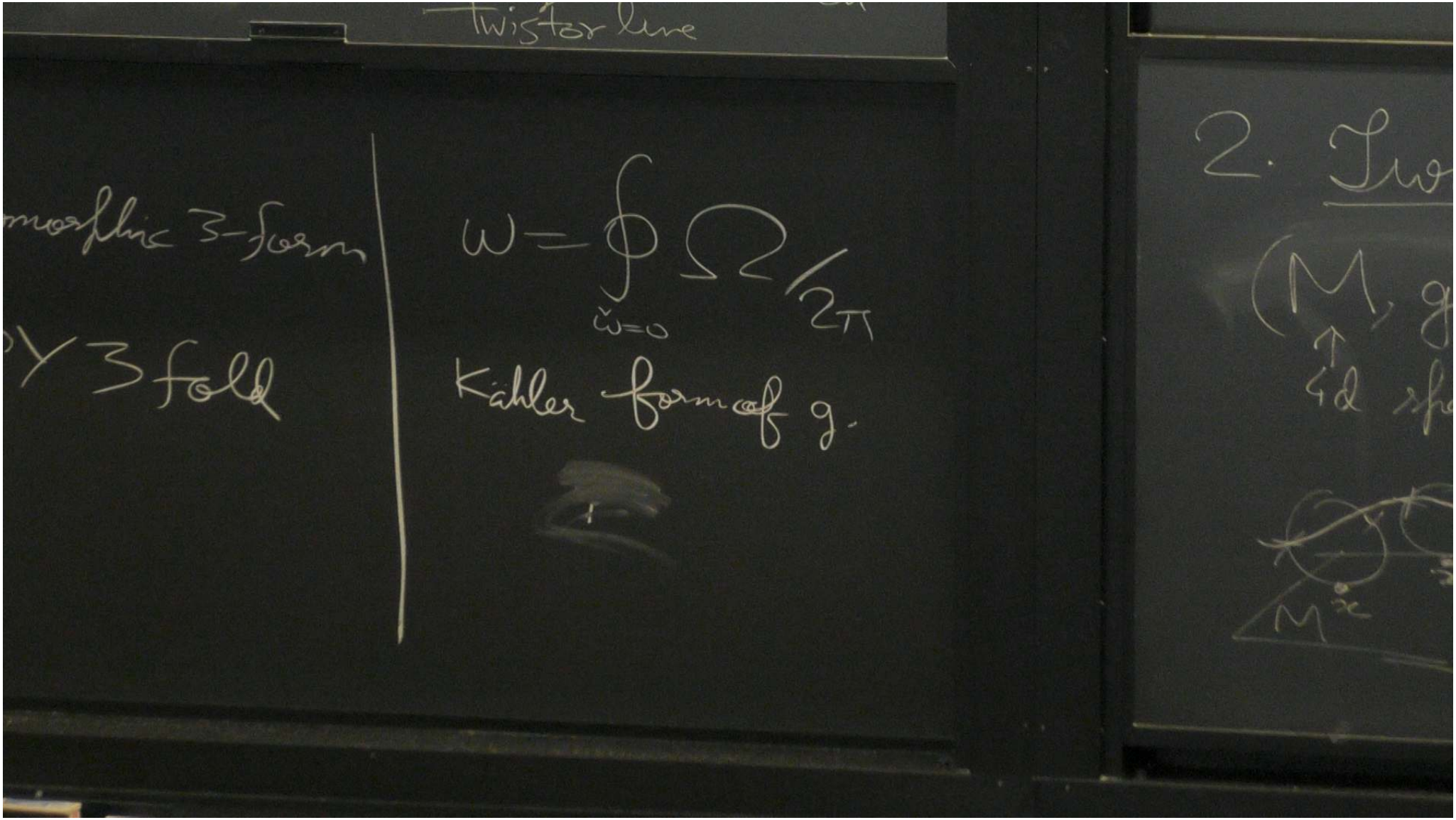


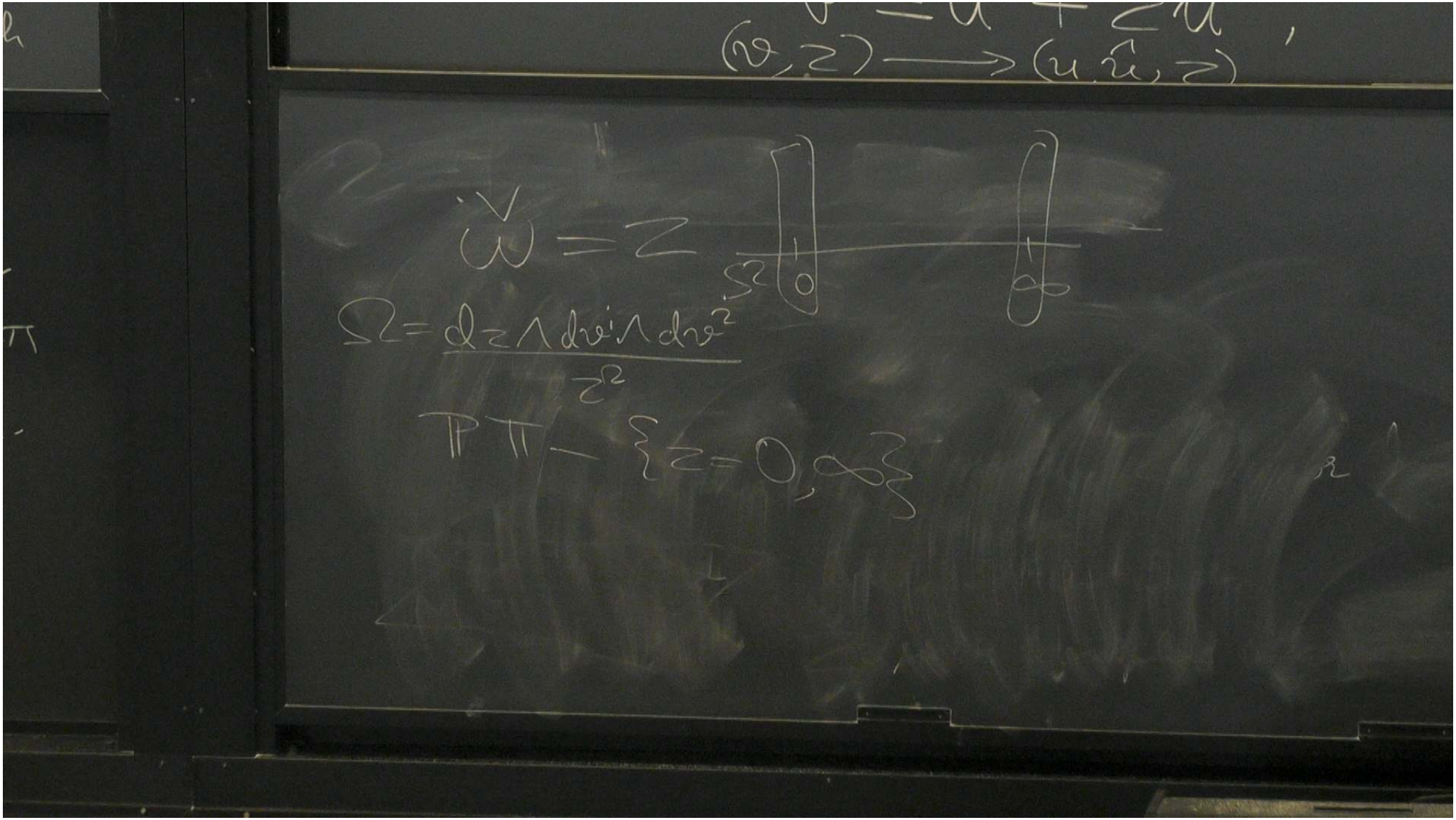
$K_Z \to (3,0)$-forms. $K_Z^{1/2}$ exists [Hitchin]

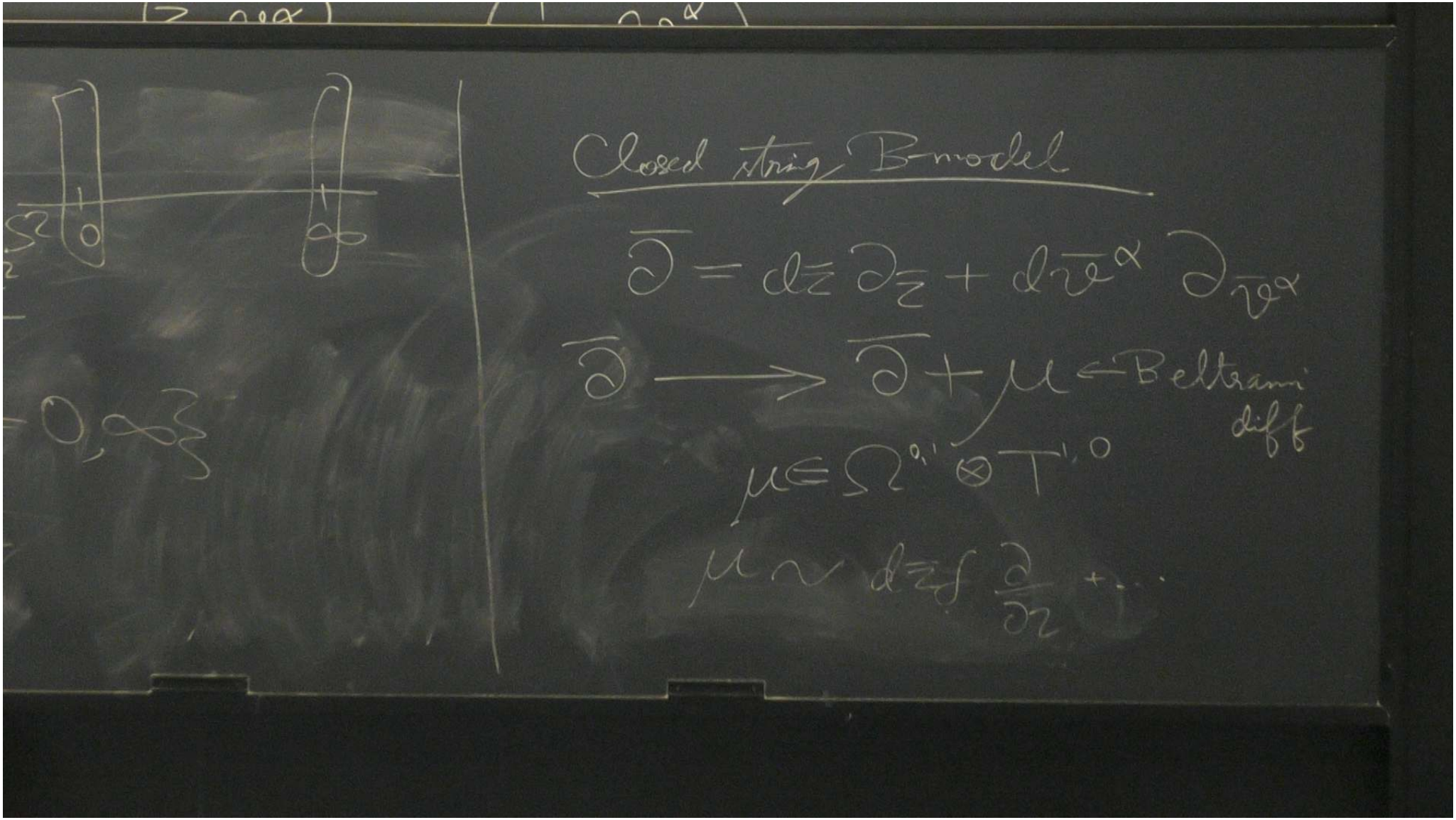
Kähler $\Rightarrow \exists \omega \in H^0(Z, K_Z^{-1/2})$

s.t. $\omega = 0$ at exactly 2 pts on each twistor line


$\Omega = \check{\omega}^{-2}\sigma$ — Meromorphic 3-form
$\Sigma = \{\check{\omega} = \partial\xi \longrightarrow$ CY 3 fold
Defects


Twistor line
morphic 3-form
CY 3 fold
$\omega = \oint_{\tilde{\omega}=0} \Omega / 2\pi$
Kähler form of g.
2. Tw
(M, g
4d
M

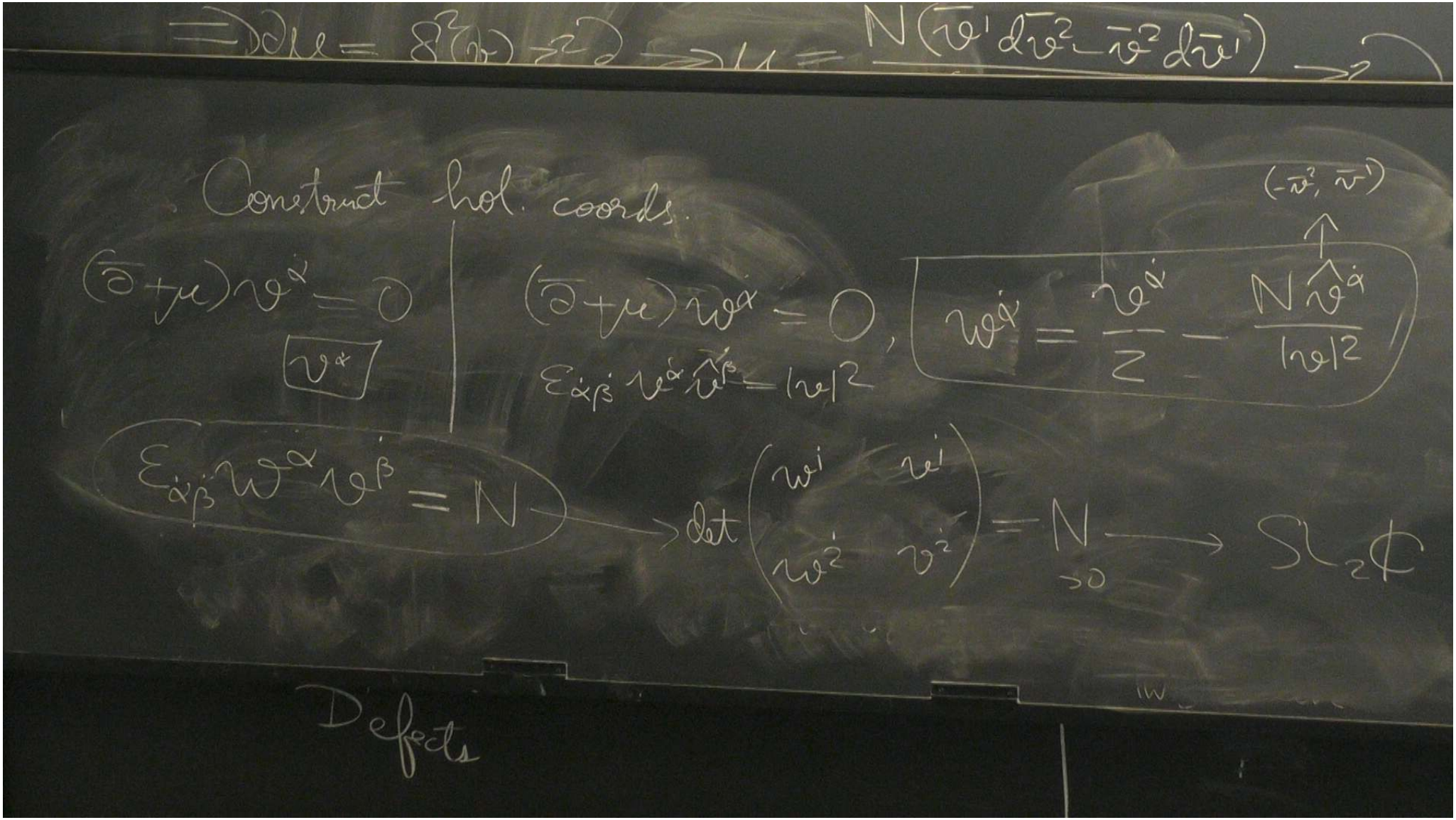

$(v, z) \longrightarrow (u, \tilde{u}, z)$
$\omega = z$
S^2
0
∞
$\Omega = \frac{dz \wedge dv \wedge dv^2}{z^2}$
$\mathbb{P}\Pi - \{z = 0, \infty\}$


Closed string B-model
$\bar{\partial} = d\bar{z}\,\partial_{\bar{z}} + d\bar{v}^{\alpha}\,\partial_{\bar{v}^{\alpha}}$
$\bar{\partial} \longrightarrow \bar{\partial} + \mu$ ← Beltrami diff
$\mu \in \Omega^{0,1} \otimes T^{1,0}$
$\mu \sim d\bar{z}\,\frac{\partial}{\partial z} + \ldots$

$\bar{\partial} + \mu$ is a complex struc if $\bar{\partial}\mu + \frac{1}{2}[\mu, \mu] = 0$

$(\bar{\partial} + \mu) w^i = 0$

Work N D1's along the locus $v^\alpha = 0$


Construct hol. coords.
$(\bar\partial+\mu)v^\alpha=0$
v^α
$(\bar\partial+\mu)w^\alpha=0,$
$\varepsilon_{\alpha\beta}w^\alpha\bar v^\beta=|v|^2$
$w^\alpha=\frac{v^\alpha}{z}-\frac{N\hat v^\alpha}{|v|^2}$
$(-\bar v^2,\bar v^1)$
$\varepsilon_{\alpha\beta}w^\alpha v^\beta=N$
$\det\begin{pmatrix} w^1 & v^1 \\ w^2 & v^2\end{pmatrix}=N>0$
$SL_2\mathbb{C}$
Defects

Why is $SL_2\mathbb{C}$ a twistor space?

[Ward] $(\mathbb{C}^2 - 0) \times (S^2 - 0, \infty) \longrightarrow SL_2\mathbb{C}$

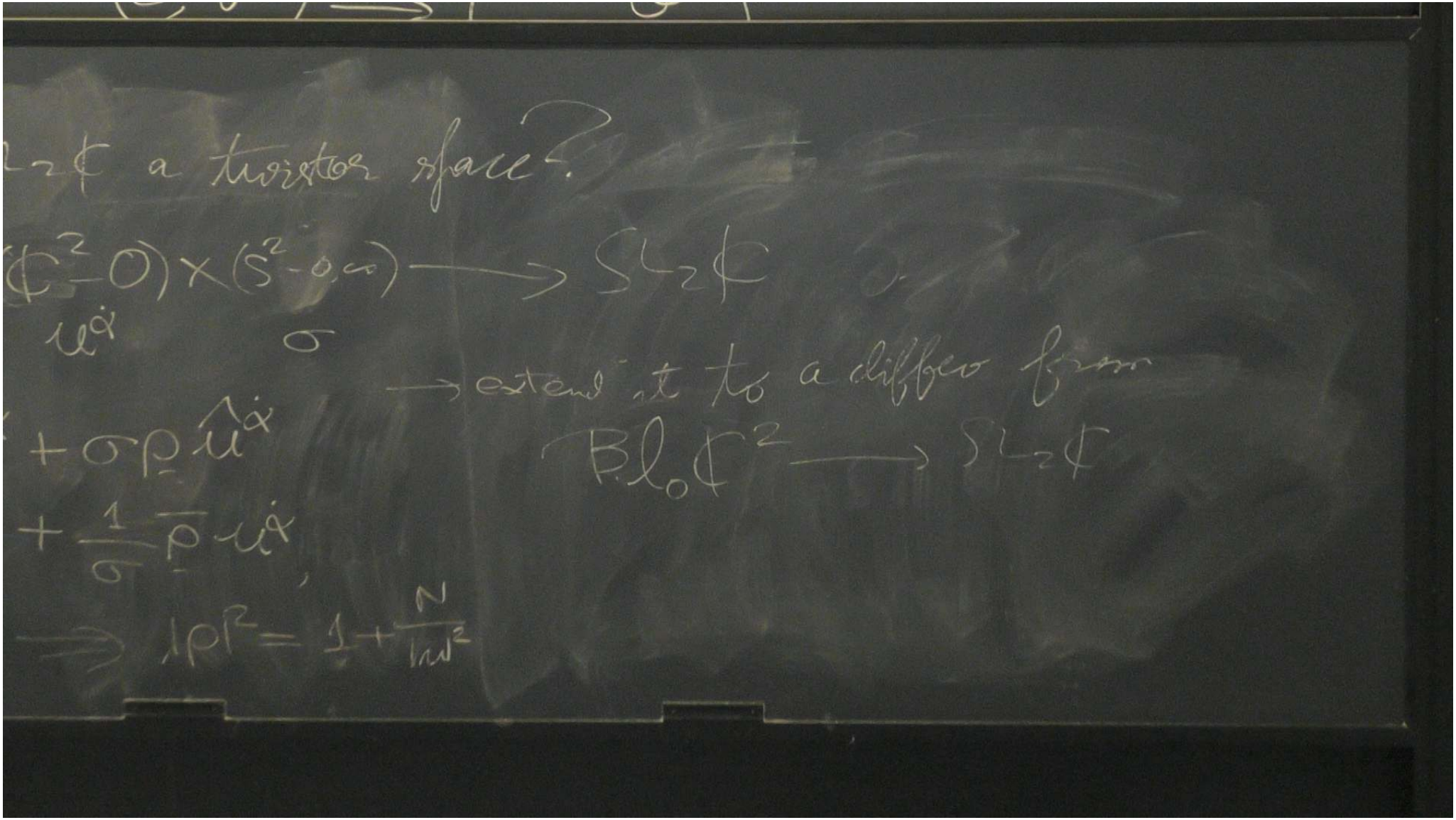
$u^{\dot\alpha}$ $\quad\quad$ σ

$$v^{\dot\alpha} = u^{\dot\alpha} + \sigma \rho \hat{u}^{\dot\alpha}$$

$$w^\alpha = \hat{u}^\alpha + \frac{1}{\sigma} \bar\rho \, u^\alpha$$

$$\det(w, v) = N \Longrightarrow |\rho|^2 = 1 + \frac{N}{|w|^2}$$

$$(\bar\partial + \mu) w^1 = 0$$


$L_2\mathbb{C}$ a twistor space?
$(\mathbb{C}^2-0)\times(S^2-0,\infty) \longrightarrow SL_2\mathbb{C}$
$u^{\dot\alpha}$ σ
$+\sigma\rho\,\hat{u}^{\dot\alpha}$
$+\frac{1}{\sigma}\bar\rho\,u^{\dot\alpha}$,
$\Rightarrow |\rho|^2 = 1+\frac{N}{|w|^2}$
$\longrightarrow$ extend it to a diffeo from
$Bl_0\mathbb{C}^2 \longrightarrow SL_2\mathbb{C}$

Why is $SL_2\mathbb{C}$ a twistor space?

[Ward] $(\mathbb{C}^2 - 0) \times (S^2 - 0, \infty) \longrightarrow SL_2\mathbb{C}$

$u^{\dot\alpha}$ $\quad$ σ

$\longrightarrow$ extend it to a diffeo from

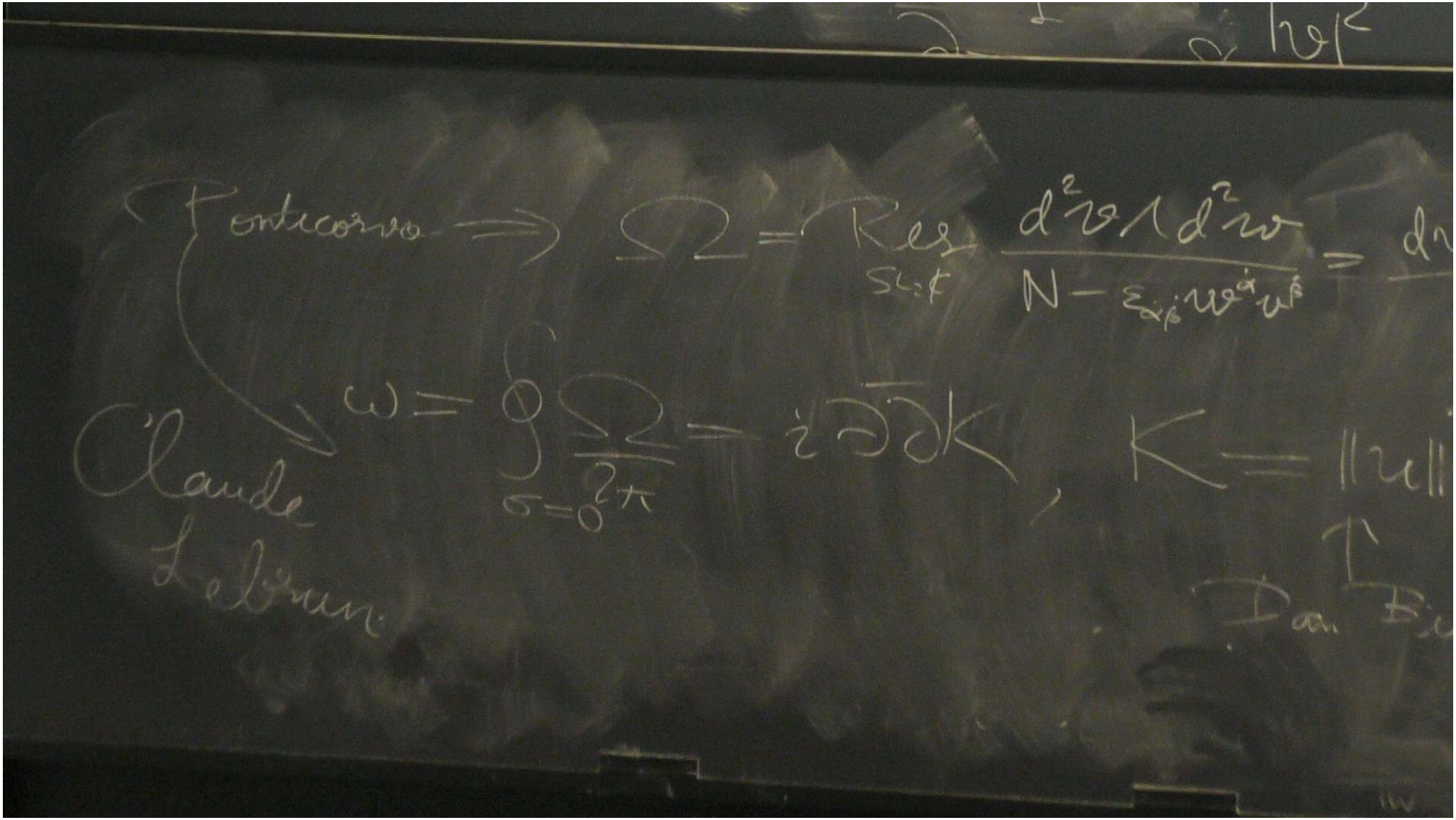
$Bl_0\mathbb{C}^2 \longrightarrow SL_2\mathbb{C}$

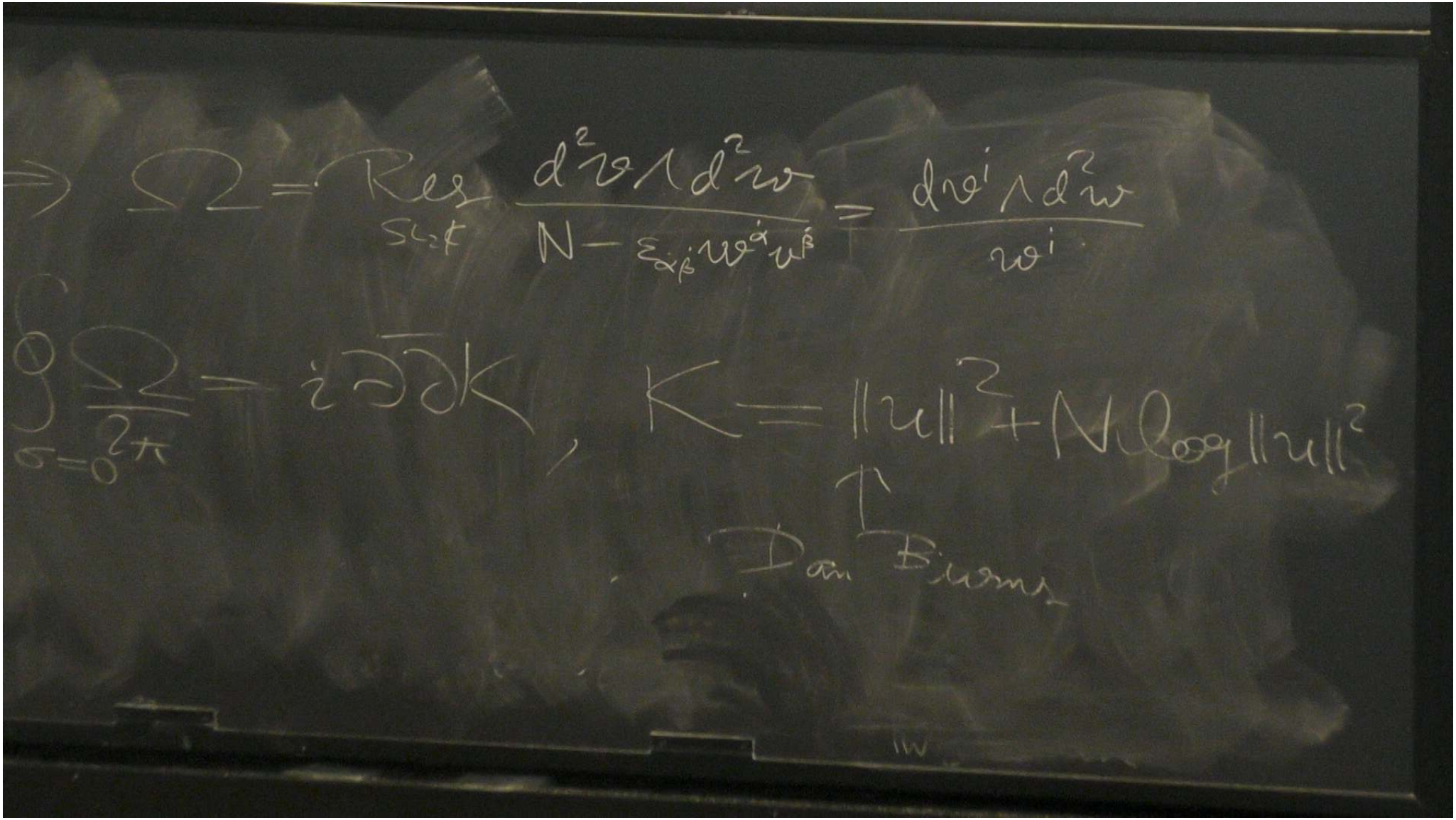
$v^{\dot\alpha} = u^{\dot\alpha} + \sigma \rho \hat{u}^{\dot\alpha}$

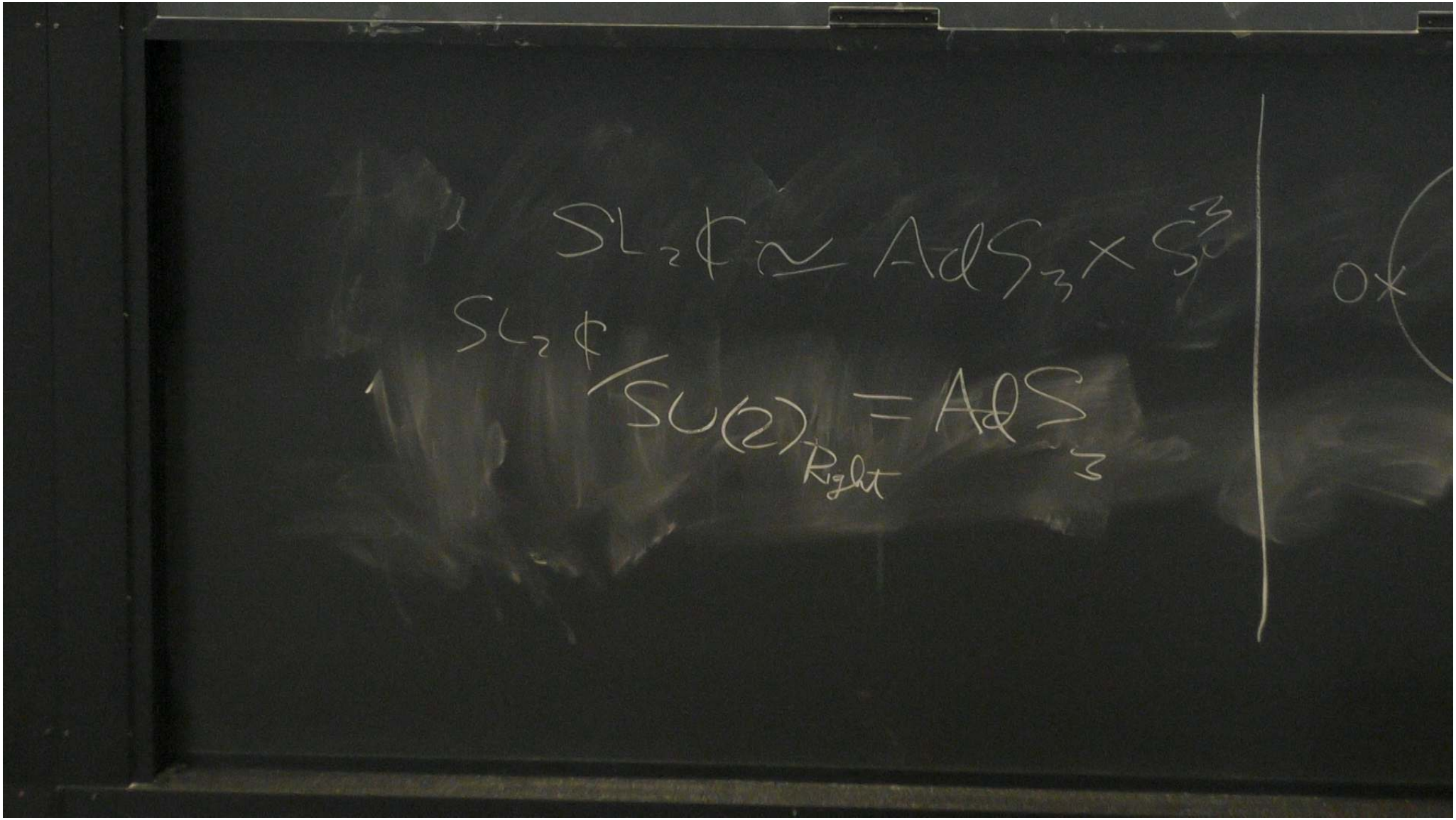
$w^{\alpha} = \hat{u}^{\dot\alpha} + \frac{1}{\sigma} \bar\rho u^{\dot\alpha}$,

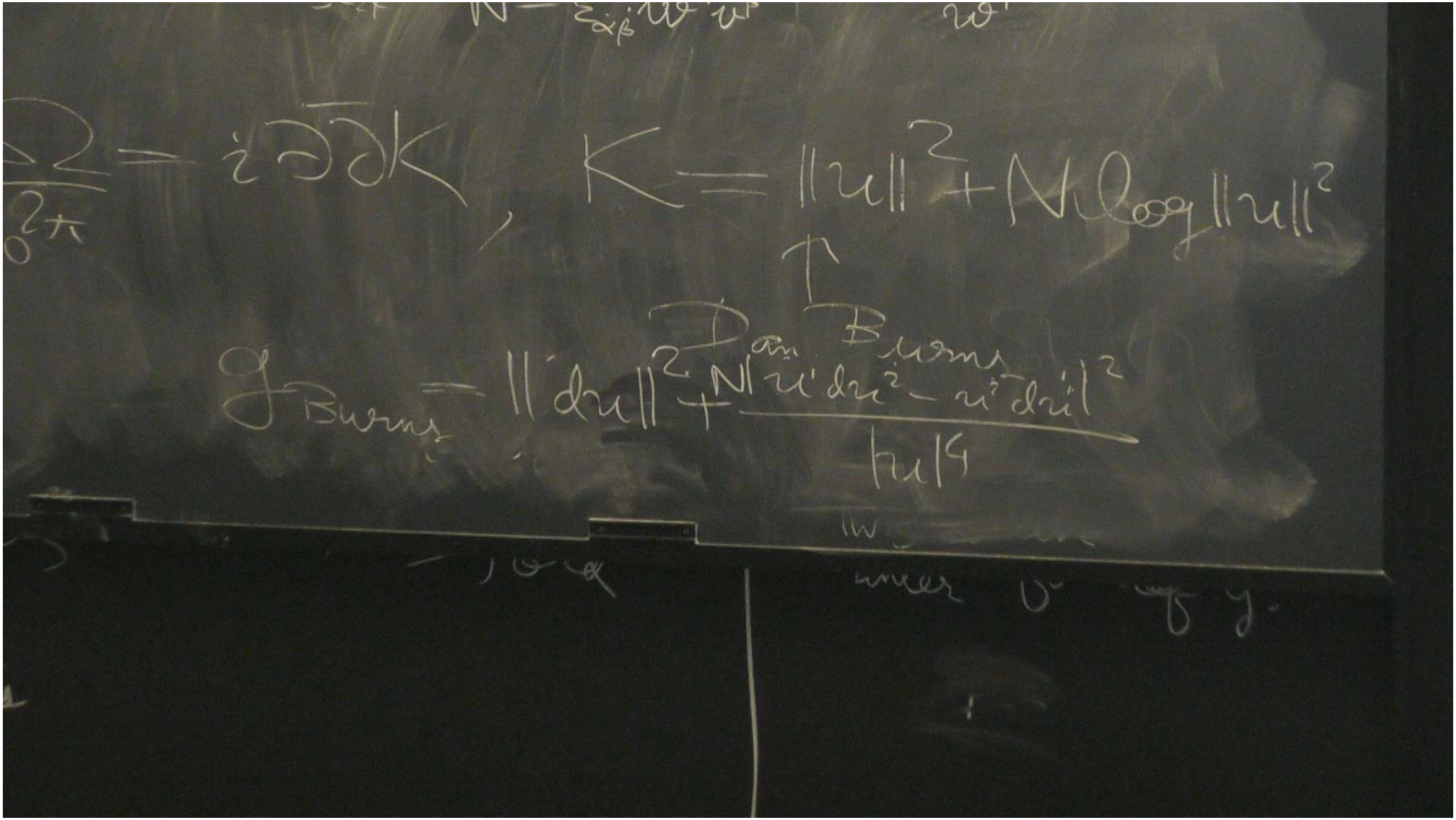
$(\mathbb{C}^2 - 0) \cup \mathbb{CP}^1$

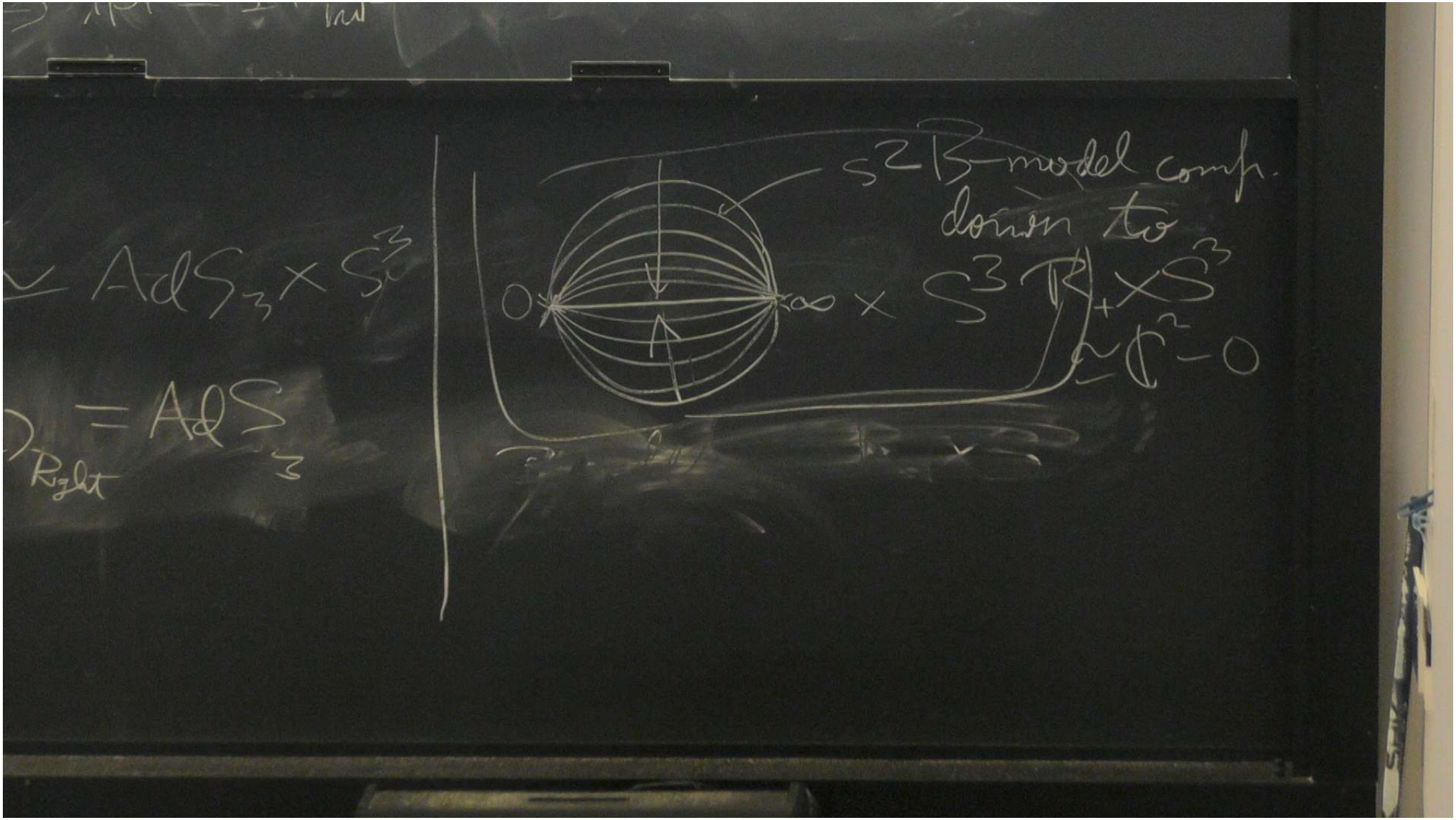
$\det(w, v) = N \Longrightarrow |\rho|^2 = 1 + \frac{N}{|u|^2}$

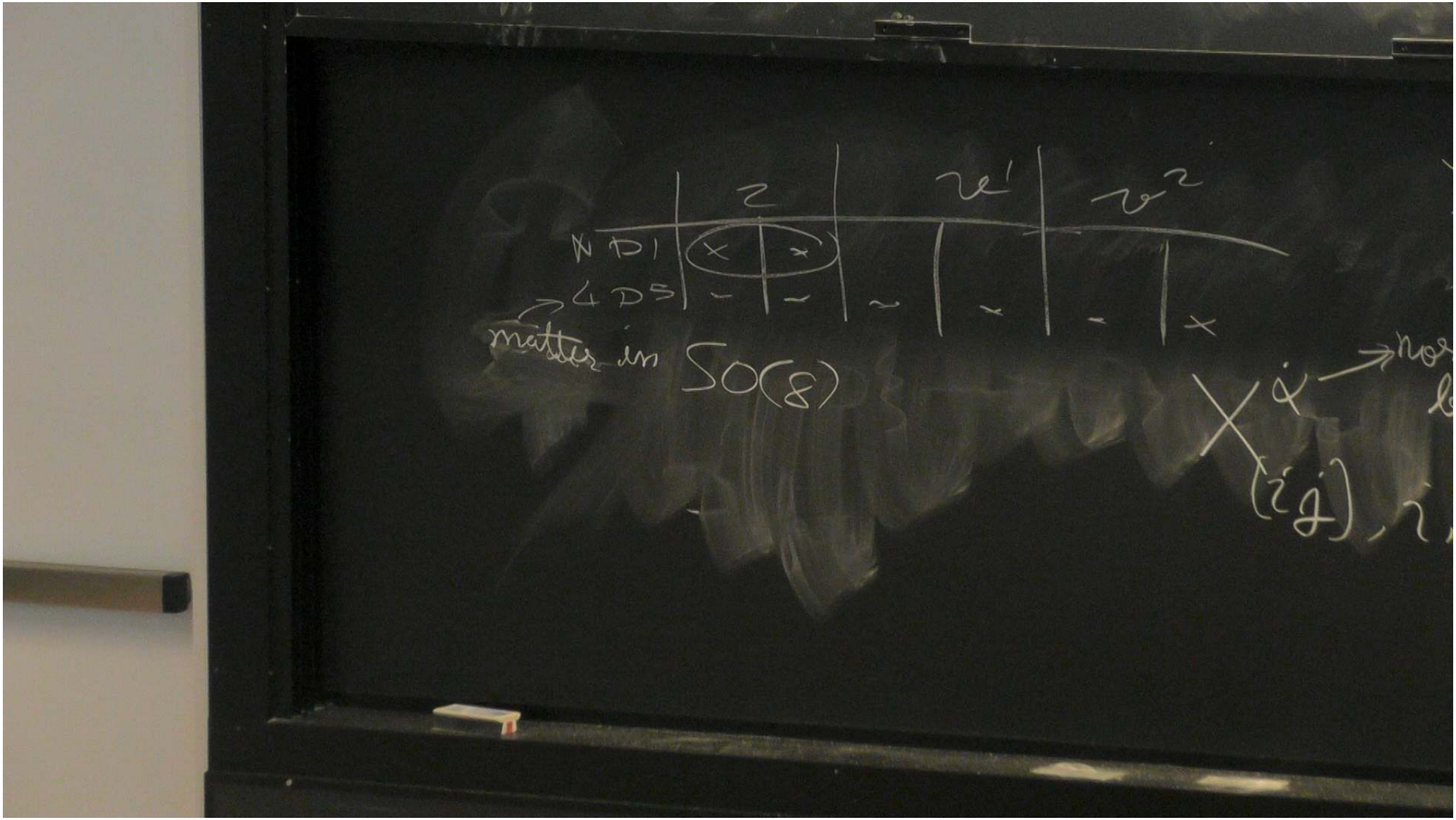

Fonticoso
Claude
Lebrun
Dan Bi

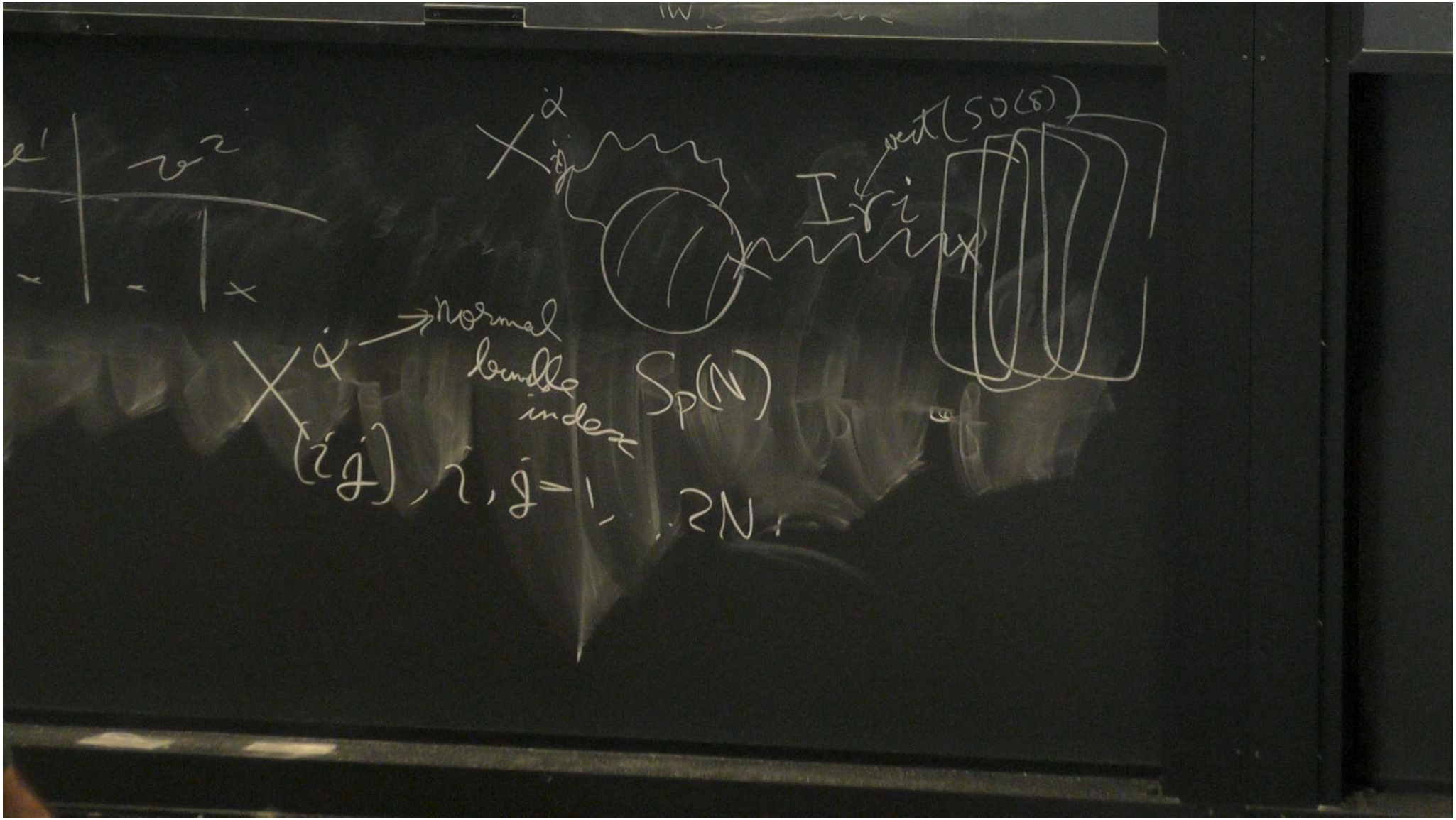

$\Rightarrow \Omega = \operatorname{Res}_{S_L=t} \frac{d^2w \wedge d^2w}{N - \varepsilon_{\alpha\beta} w^\alpha w^\beta} = \frac{dw^i \wedge d^2w}{w^i}$
$\frac{\partial}{\partial t}\frac{\Omega}{2\pi}\Big|_{t=0} = i\partial\bar{\partial}K, \quad K = \|u\|^2 + N\log\|u\|^2$
Dan Birmajer

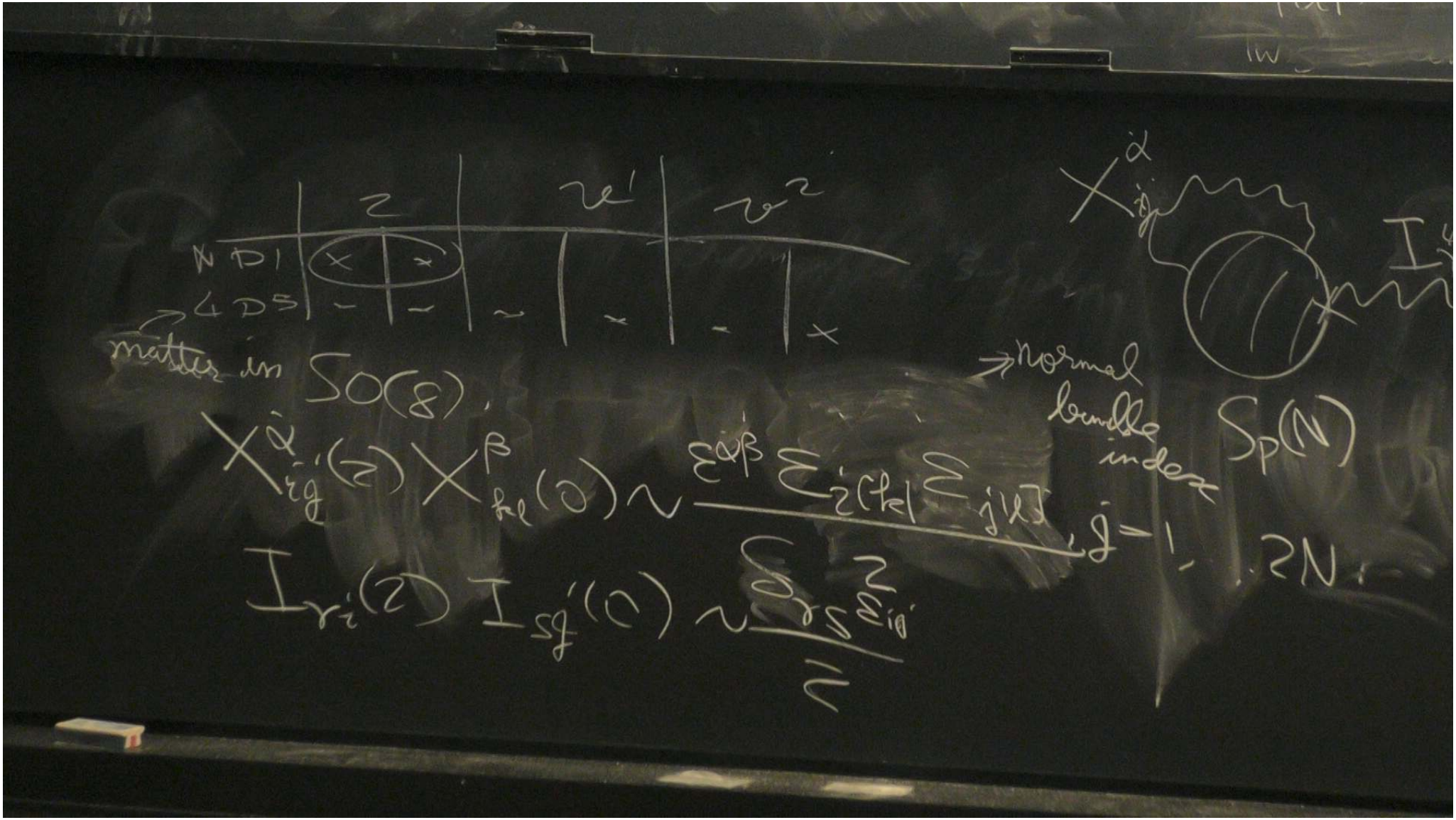

$SL_2\mathbb{C} \simeq AdS_3 \times S^3$
$SL_2\mathbb{C}/SU(2)_{Right} = AdS_3$

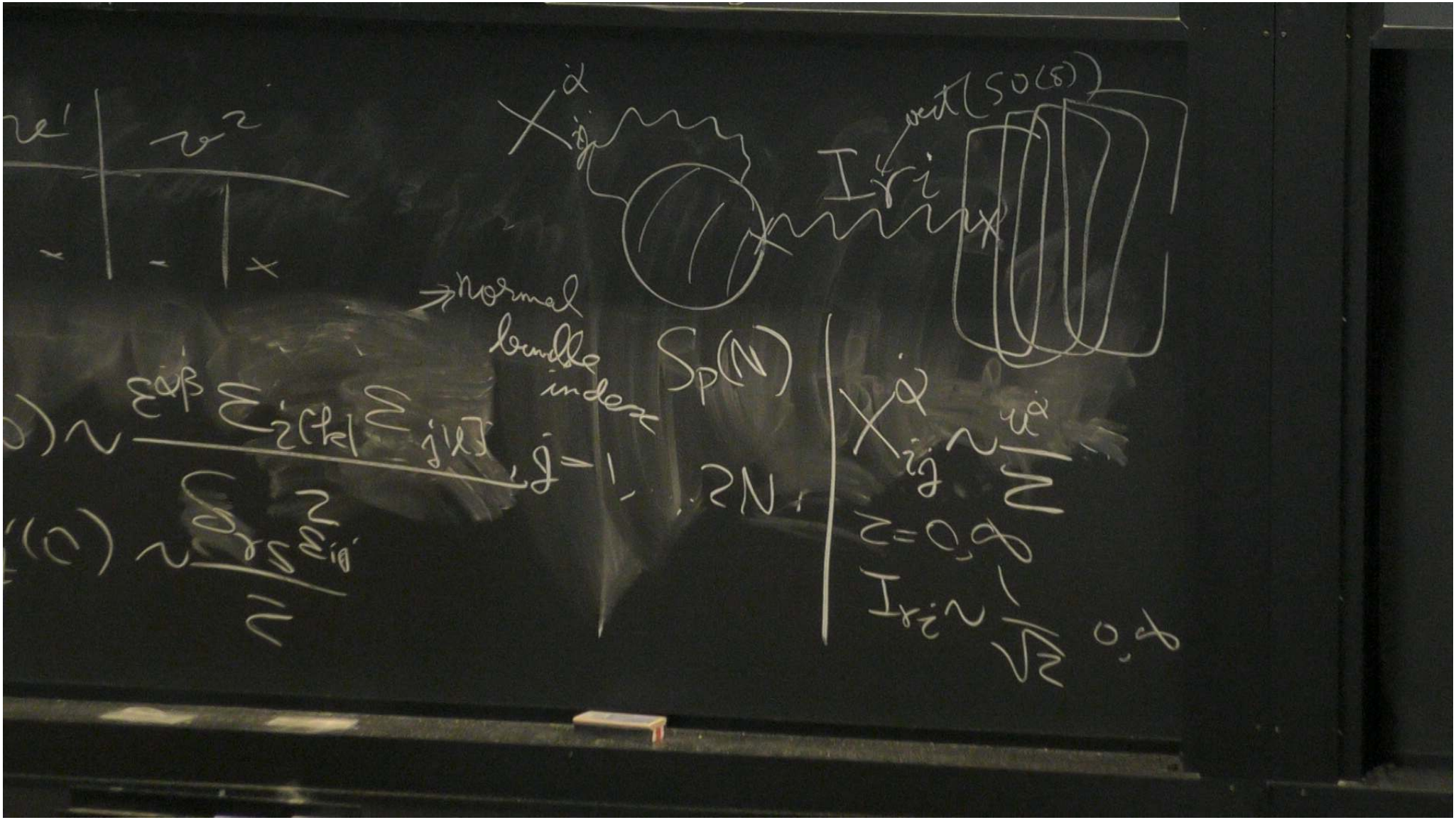

$\frac{\Omega}{2\pi} = i\bar{\partial}\partial K$, $K = \|u\|^2 + N\log\|u\|^2$
$g_{Burns} = \|du\|^2 + N\frac{|u'du^2 - u^2du'|^2}{|u|^4}$
Pour Burns

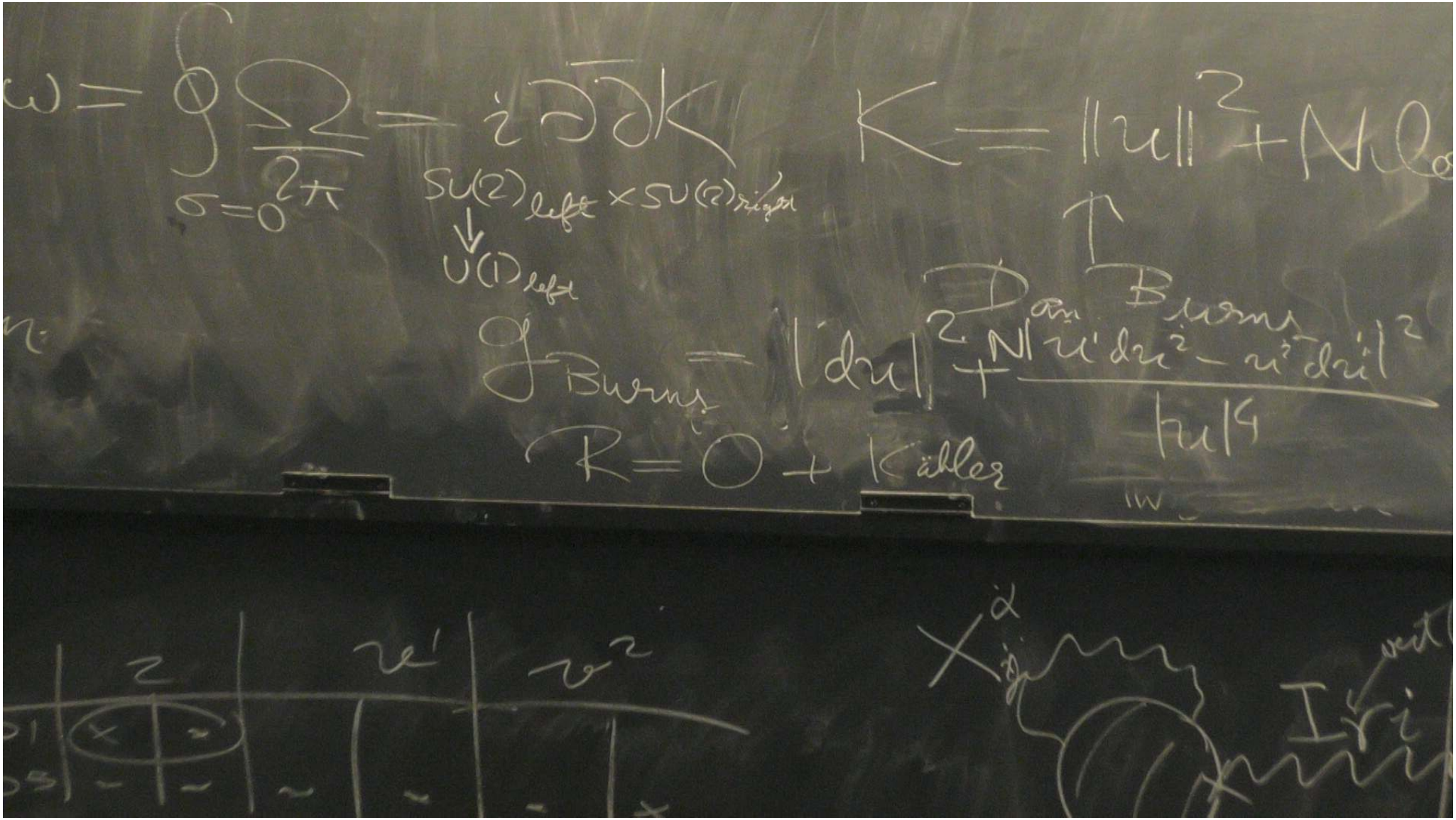

$\sim AdS_3 \times S^3$
$= AdS_3$
D_{Right}
S^2 B-model comp.
down to
$0 \times \infty \times S^3$
$\mathbb{R}_+ \times S^3$
$\sim \mathbb{C}^2 - 0$


z
z'
v^2
N D1
4 D5
matter in SO(8)


Normal
bundle
index
$Sp(N)$
(ij), $i, j = 1, \ldots, 2N$


z
z'
v²
N D1
4 D5
matter in SO(8).
Normal bundle index
Sp(N)


vect(SO(8))
Normal bundle index
$Sp(N)$


$\omega = \oint_{\sigma=0} \frac{\Omega}{2\pi} = i\bar{\partial}\partial K$
$K = \|u\|^2 + N \log$
$SU(2)_{left} \times SU(2)_{right}$
$U(1)_{left}$
$g_{Burns} = |du|^2 + N \frac{|u' du^2 - u^2 du|^2}{|u|^4}$
Dan Burns
$R = 0$ + Kähler
IW