Title: Burns space and holography

Speakers: Atul Sharma

Series: Quantum Fields and Strings

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Abstract: I will construct a top-down example of celestial holography, based on recent work with Costello and Paquette. Our duality relates certain models of self-dual gauge theory and conformal gravity, placed on an asymptotically flat four dimensional spacetime called Burns space, to a two dimensional chiral algebra living on D1-branes in a topological string theory on twistor space.

Zoom link: https://pitp.zoom.us/j/92898535744?pwd=T2w2SUp0VnU5NWtPSm50VENvWlRYQT09

Burns Space & Holography

Based on Costello, Paquette, AS
[PRL 130, 061602] + wip

1. The story so far

Q.G. in $\Lambda=0$ 4d spacetimes $=$ 2d CFT living on Riemann sphere

- Today: twistor thy + twisted = celestial
  Actual B-model holography holography
- Discovery of $\mathrm{Ham}(\mathbb{C}^2)$ by Strominger
- Birth of celestial chiral algebras
  Koszul duality KC, NP

& Holography

lo, Paquette, AS

130, 061602] + wip

so far

$\Lambda=0$ = 2d CFT living on Riemann sphere



twisted = celestial holography

2. Twistorial holography

$$(M, g) \longrightarrow S^2 \hookrightarrow Z \longrightarrow M$$

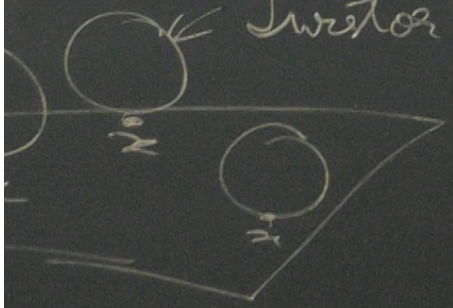
↑ 4d spacetime

Twistorial holography

$(\ldots, g) \longrightarrow S^2 \hookrightarrow Z \longrightarrow M$

spacetime

Z = projective 2-spinor bundle over M

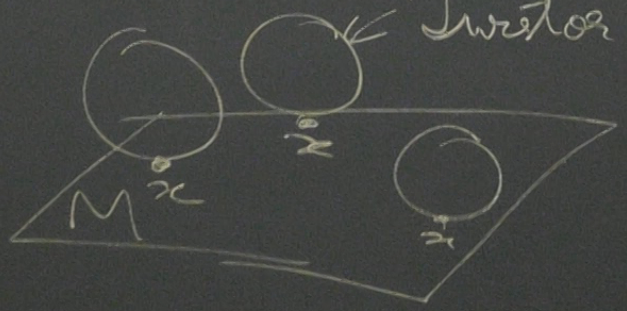


2. Twistorial holography

$(M, g) \longrightarrow S^2 \longrightarrow Z \longrightarrow M$

(M, g): 4d spacetime

Z = projective 2-spinor bundle o



Penrose

Atiyah-Hitchin-Singer

Z is a complex manifold iff

l holography

$S^2 \to Z \longrightarrow M$

$=$

projective 2-spinor bundle over M

hor lines

Penrose

Atiyah-Hitchin-Singer:

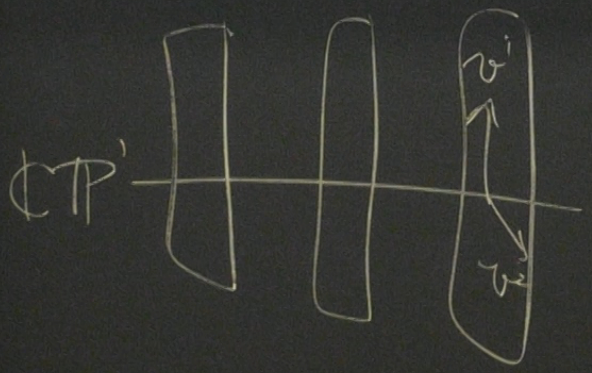
Z is a complex manifold iff $W_3 = 0$

M^2 Σ

Σ is a complex manifold iff $W_g =$

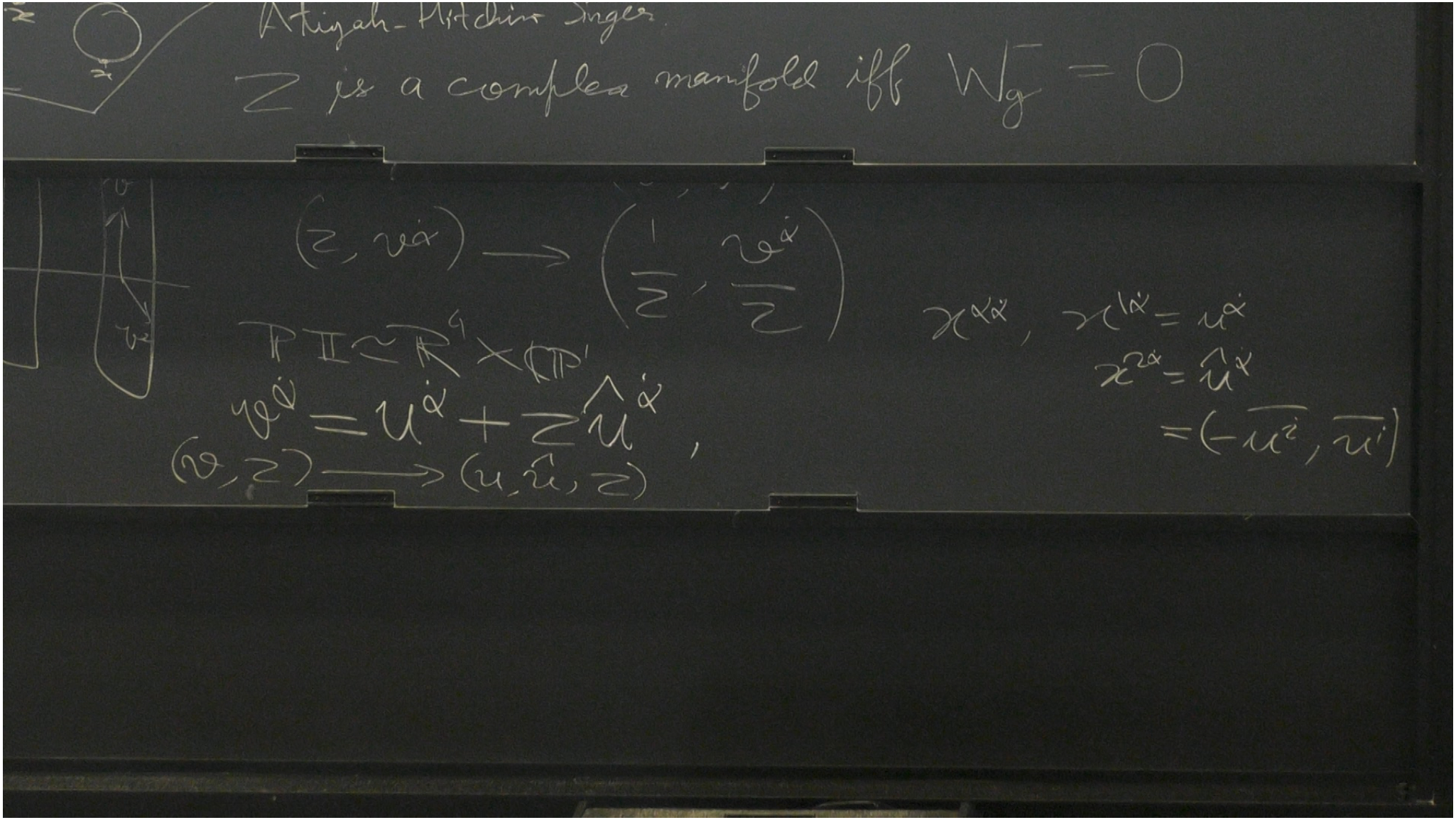
Start with $\mathbb{P}\Pi = \mathcal{O}(1) \oplus \mathcal{O}(1) \longrightarrow \mathbb{CP}^1_\Sigma$

$v^a = (v^1, v^2)$



$(z, v^a) \longrightarrow \left(\frac{1}{z}, \frac{v^a}{z}\right)$

$\mathbb{P}\Pi = \mathbb{R}^4 \times \mathbb{CP}^1$


Atiyah-Hitchin-Singer
Z is a complex manifold iff $W_3 = 0$
$(z, v^\alpha) \longrightarrow \left(z, \frac{v^\alpha}{z}\right)$
$TP^1 \simeq \mathbb{R}^4 \times \mathbb{RP}^1$
$v^\alpha = u^\alpha + z\hat{u}^\alpha$,
$(v, z) \longrightarrow (u, \hat{u}, z)$
$x^{1\alpha}, \; x^{1\alpha} = u^\alpha$
$x^{2\alpha} = \hat{u}^\alpha$
$= (-\bar{u}^2, \bar{u}^1)$

There is no global $(3,0)$-volume form

$$dz \wedge dw_1 \wedge dw_2 \xrightarrow{(z,w) \to \left(\frac{1}{2}z, \frac{w}{2}\right)} -\frac{dz \wedge dw_1 \wedge dw_2}{2^4}$$

Pontecorvo's thm

- g is $\partial\bar{\partial}$-Kähler $\Rightarrow \forall \omega \in H^0($

al (3,0) - volume form

$dv^1 \wedge dv^2 \xrightarrow{(v^1 \to (\frac{1}{2}, \frac{v}{2}))} - \frac{dz \wedge dv^1 \wedge dv^2}{2^4}$

thm

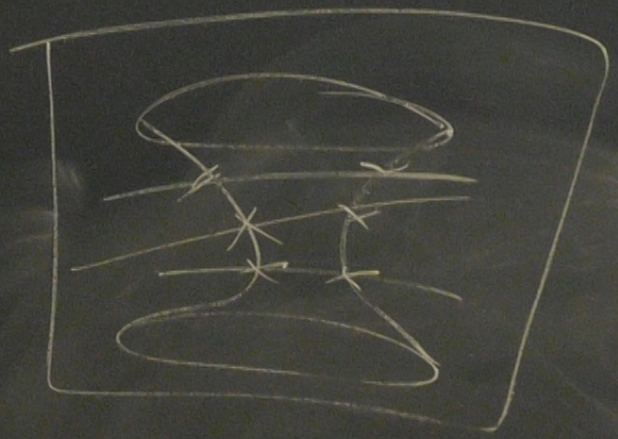
$K_Z \to (3,0)$-forms. $K_Z^{1/2}$ exists [Hitchin]

+ Kähler $\Rightarrow \Omega \in H^0(Z, K_Z^{-1/2})$

(3,0) - volume form

$\Lambda dv^2 \langle v \rangle \to \left(\frac{1}{z}, \frac{v}{z}\right)$

$\longrightarrow \frac{dz \wedge dv_1 \wedge dv^2}{z^4}$



$K_Z \to (3,0)$-forms. $K_Z^{1/2}$ exists [Hitchin]

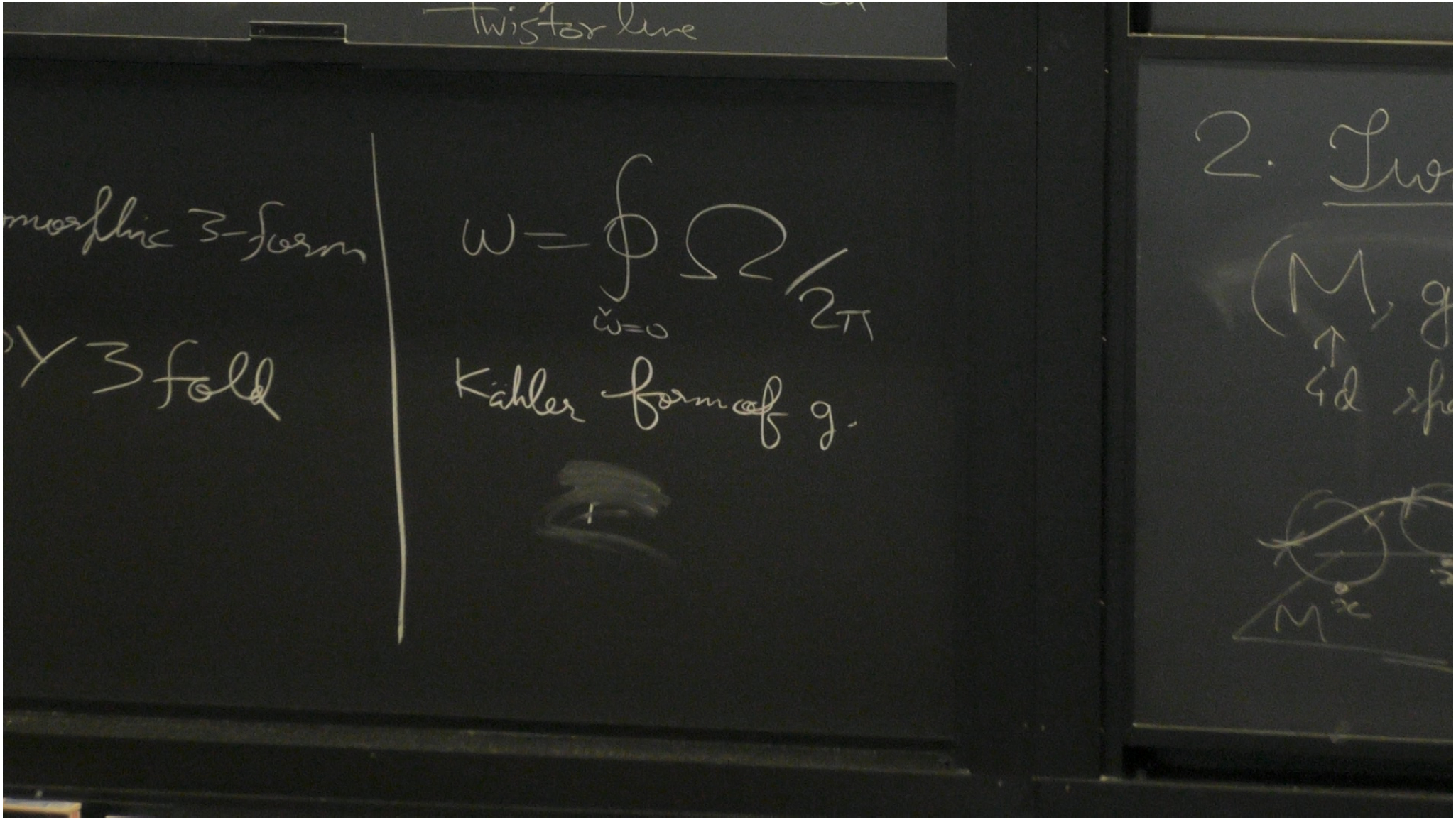
Kähler $\Rightarrow \exists \omega \in H^0(Z, K_Z^{-1/2})$

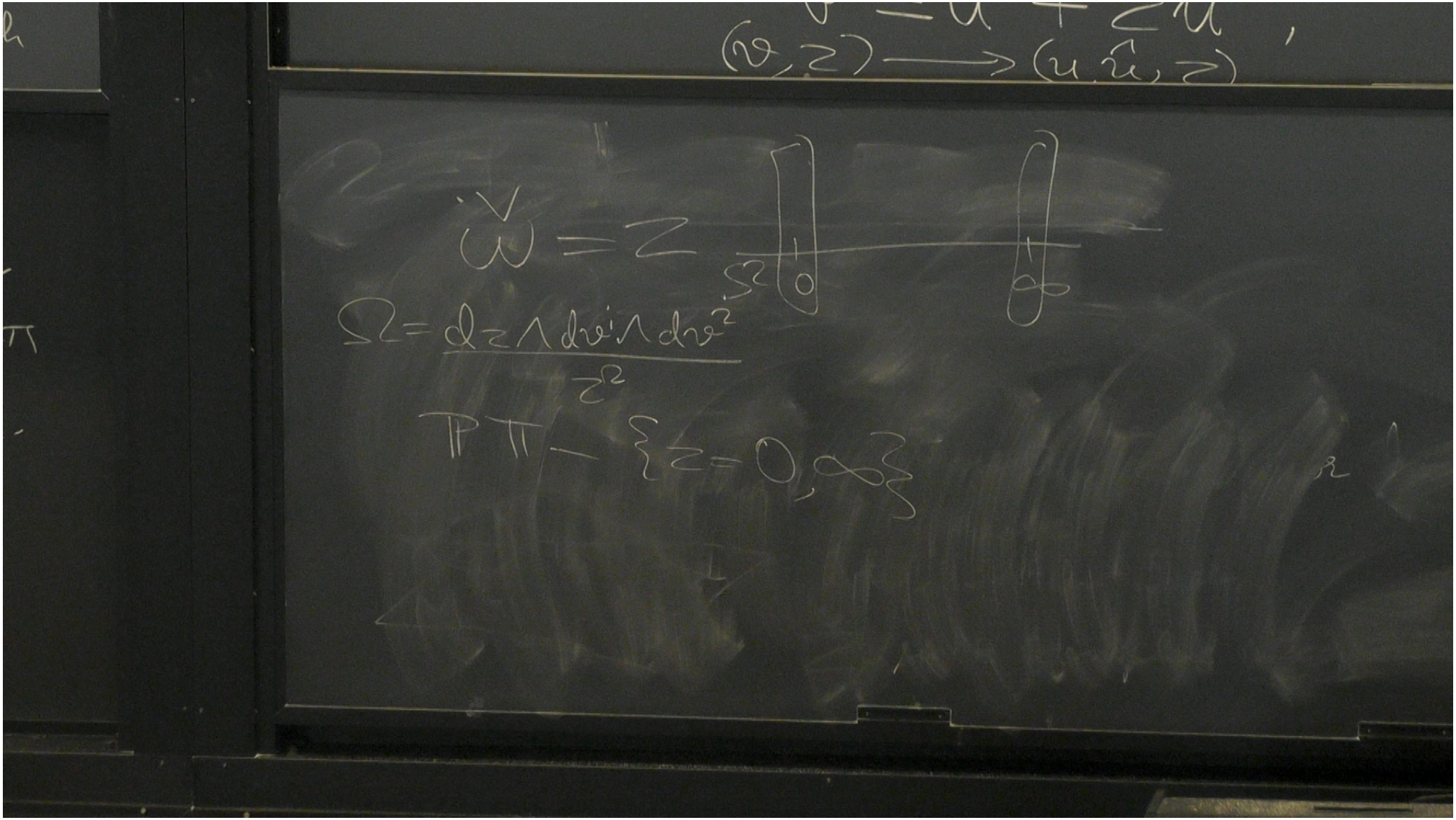
s.t. $\omega = 0$ at exactly 2 pts on each twistor line

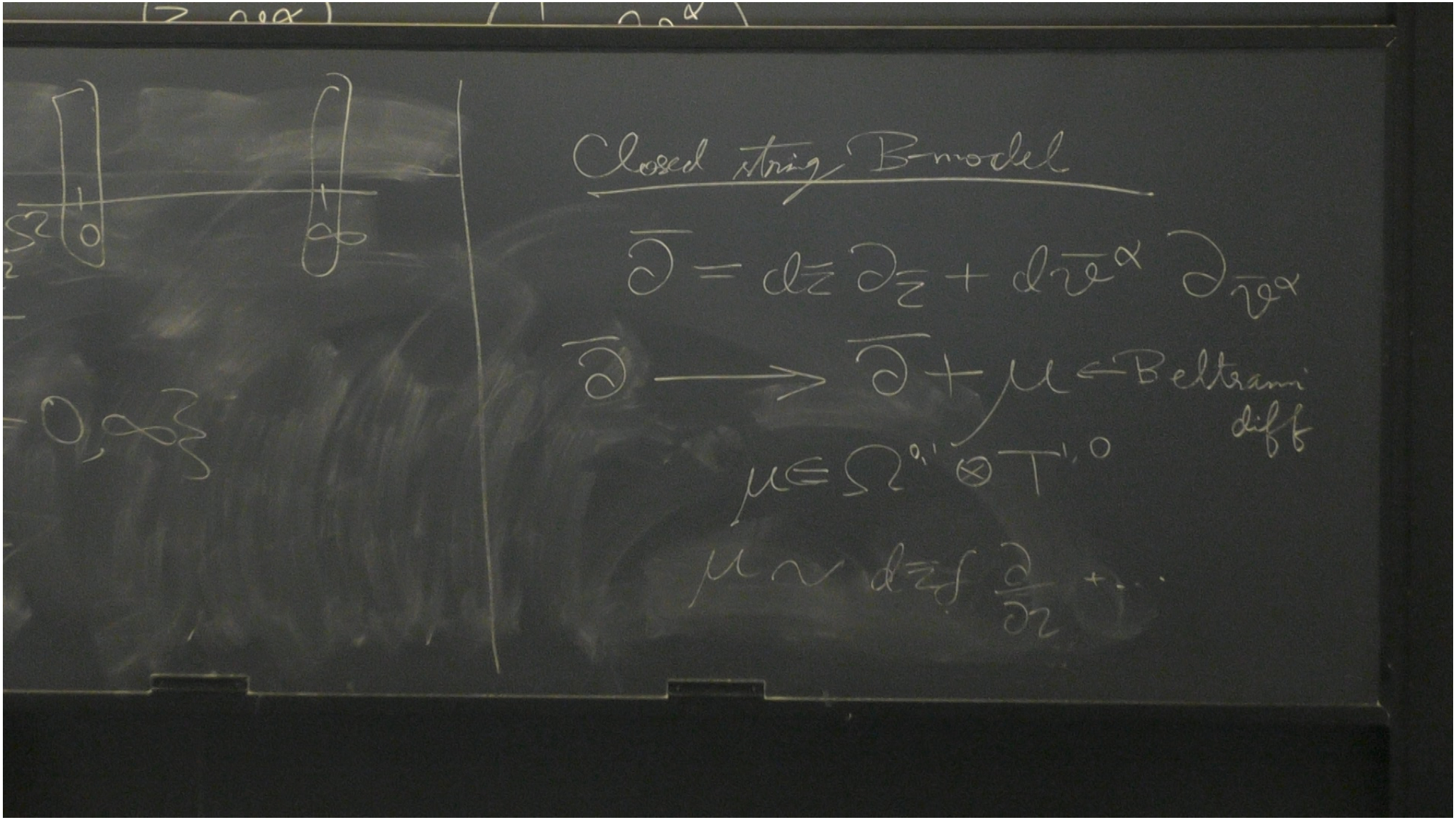
$\Omega = \check{\omega}^{-2}\sigma$ — Meromorphic 3-form

$\Sigma = \{\check{\omega} = 0\} \longrightarrow$ CY 3 fold

↑ Defects


Twistor line
morphic 3-form
Y 3 fold
$\omega = \oint_{\check{\omega}=0} \Omega / 2\pi$
Kähler form of g.
2. Tw
(M, g
4d

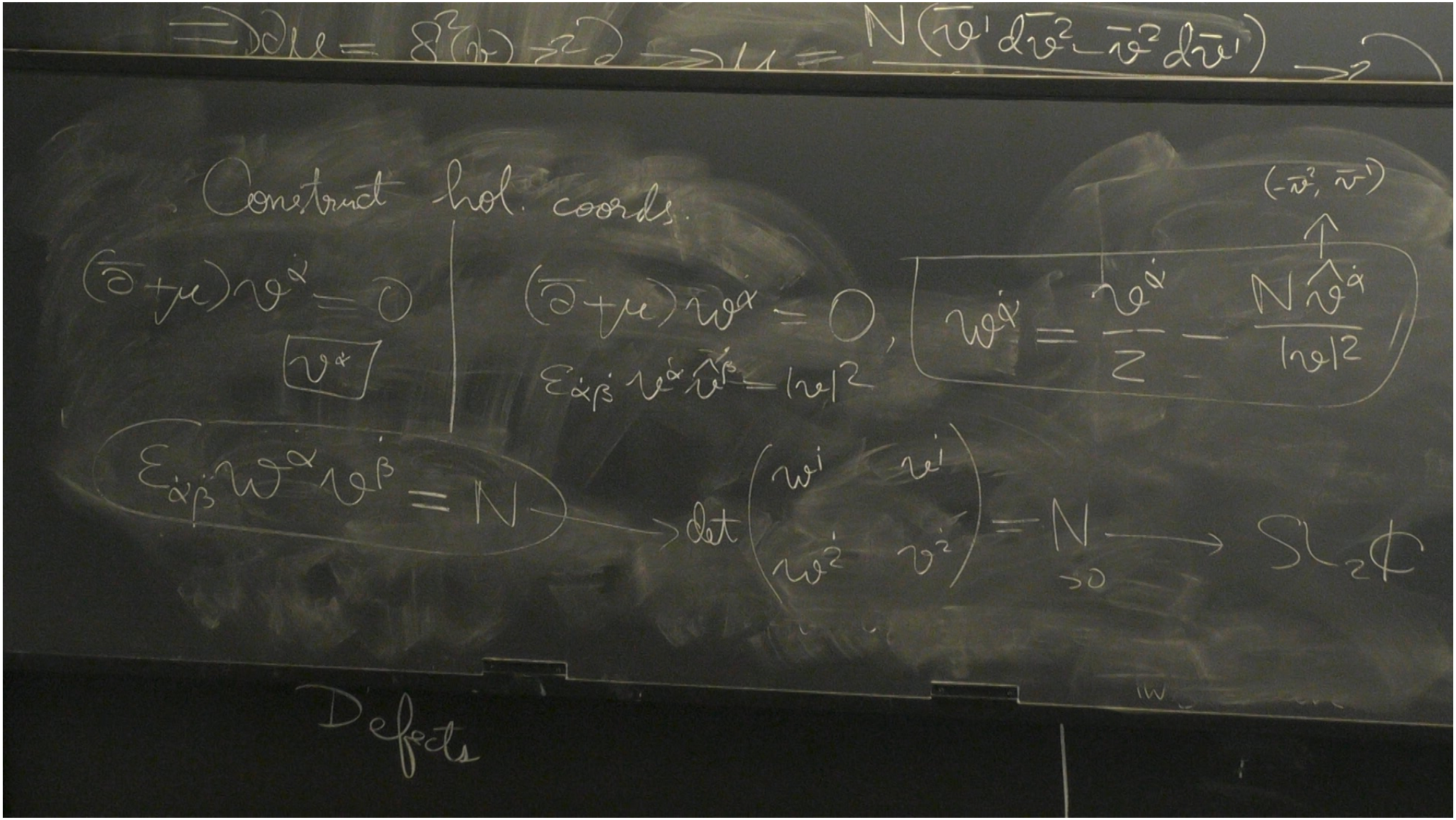

(v, z) → (u, ũ, z)
ω = z
S²
0
∞
Ω = dz ∧ dv ∧ dv² / z²
PT − {z = 0, ∞}


Closed string B-model
$\bar{\partial} = d\bar{z}\,\partial_{\bar{z}} + d\bar{v}^{\alpha}\,\partial_{\bar{v}^{\alpha}}$
$\bar{\partial} \longrightarrow \bar{\partial} + \mu$
$\mu \in$ Beltrami diff
$\mu \in \Omega^{0,1} \otimes T^{1,0}$
$\mu \sim d\bar{z}\,\frac{\partial}{\partial z} + \ldots$

$\bar{\partial}+\mu$ is a complex struc if $\bar{\partial}\mu+\frac{1}{2}[\mu,\mu]=0$

$(\bar{\partial}+\mu)w^i=0$

Work N Dl's along the lower $v^{\dot{\alpha}}=0$


Construct hol. coords.
$(\bar\partial + \mu) v^\alpha = 0$
v^α
$(\bar\partial + \mu) w^\alpha = 0$,
$\varepsilon_{\alpha\beta} w^\alpha \bar v^\beta = |v|^2$
$w^\alpha = \frac{v^\alpha}{z} - \frac{N \bar v^\alpha}{|v|^2}$
$(-\bar v^2, \bar v^1)$
$\varepsilon_{\alpha\beta} w^\alpha v^\beta = N$
$\det \begin{pmatrix} w^1 & v^1 \\ w^2 & v^2 \end{pmatrix} = N > 0 \longrightarrow SL_2 \mathbb{C}$
Defects

Why is $SL_2\mathbb{C}$ a twistor space?

[Ward] $(\mathbb{C}^2 - 0) \times (S^2 - 0, \infty) \longrightarrow SL_2\mathbb{C}$

$u^{\dot\alpha}$ $\quad$ σ

$$v^{\dot\alpha} = u^{\dot\alpha} + \sigma \rho \hat{u}^{\dot\alpha}$$

$$w^{\alpha} = \hat{u}^{\dot\alpha} + \frac{1}{\sigma} \bar\rho \, u^{\dot\alpha},$$

$$\det(w, v) = N \Longrightarrow |\rho|^2 = 1 + \frac{N}{|w|^2}$$

$$(\bar\partial + \mu) w^{\dot\alpha} = 0$$

$L_2\mathbb{C}$ a twistor space?

$(\mathbb{C}^2 - 0) \times (S^2 - \text{pt}) \longrightarrow SL_2\mathbb{C}$

$u^{\dot\alpha}$ $\quad\quad$ σ

$\longrightarrow$ extend it to a diffeo from

$+ \sigma \rho \hat{u}^{\dot\alpha}$ $\quad\quad$ $Bl_0\mathbb{C}^2 \longrightarrow SL_2\mathbb{C}$

$+ \frac{1}{\sigma} \bar\rho u^{\dot\alpha}$

$\Rightarrow |\rho|^2 = 1 + \frac{N}{|w|^2}$

Why is $SL_2\mathbb{C}$ a twistor space?

[Ward] $(\mathbb{C}^2 - 0) \times (S^2 - 0, \infty) \longrightarrow SL_2\mathbb{C}$

$u^{\dot\alpha}$ $\quad$ σ

$\longrightarrow$ extend it to a diffeo from

$$v^{\dot\alpha} = u^{\dot\alpha} + \sigma \rho \hat{u}^{\dot\alpha}$$

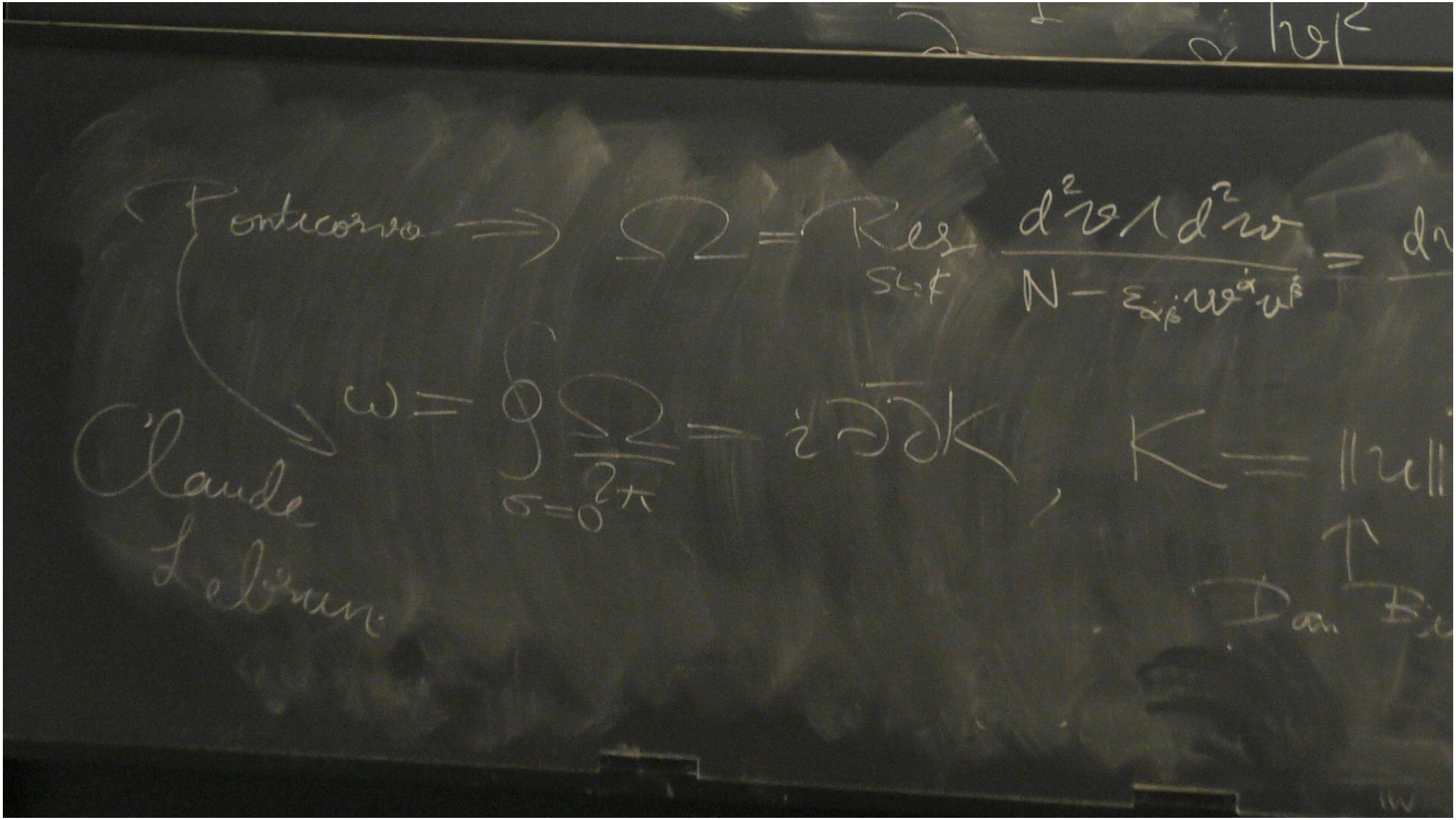
$$w^{\alpha} = \hat{u}^{\alpha} + \frac{1}{\sigma} \bar{\rho} u^{\alpha}$$

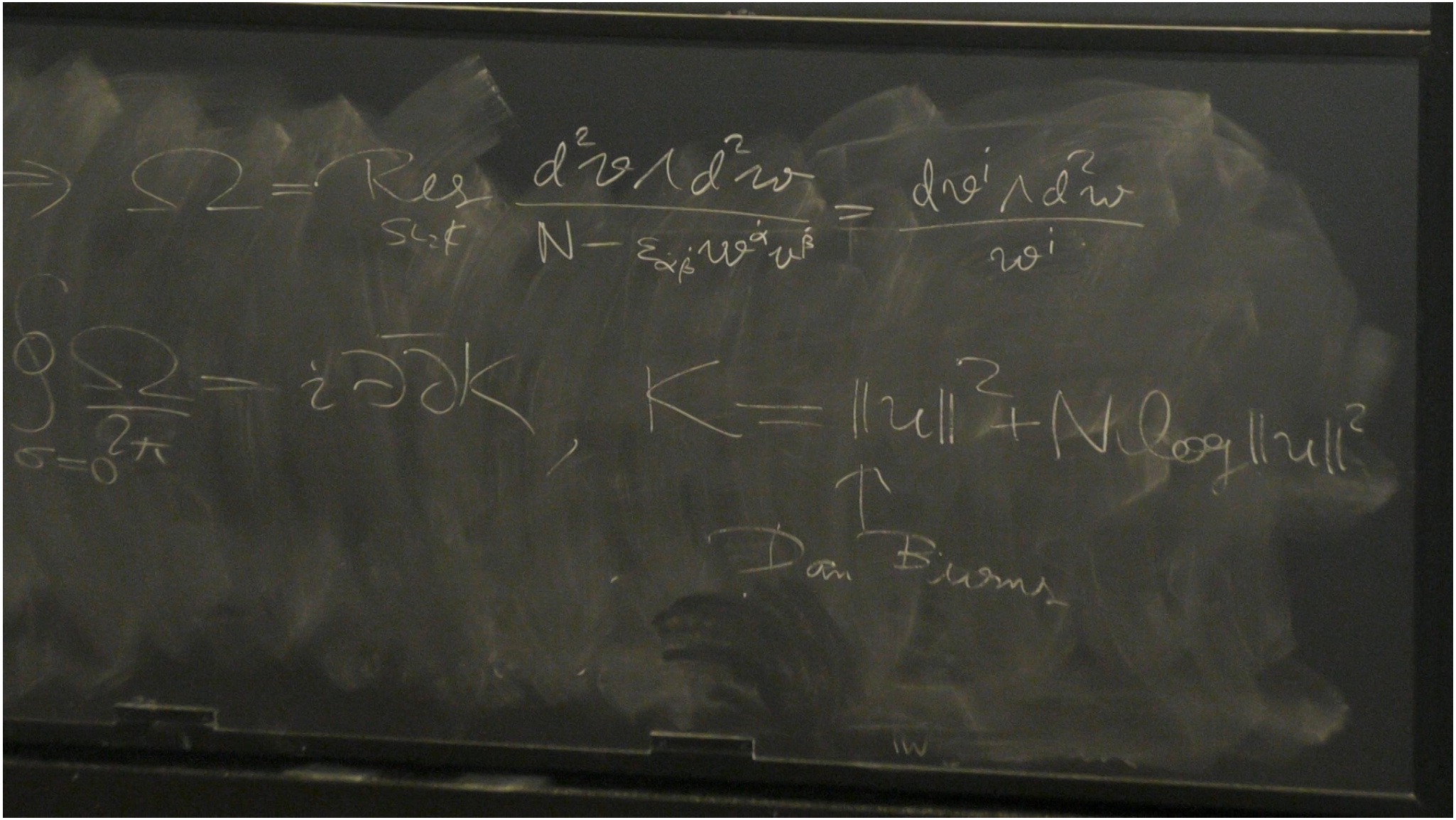
$$\det(w, v) = N \Longrightarrow |\rho|^2 = 1 + \frac{N}{|u|^2}$$

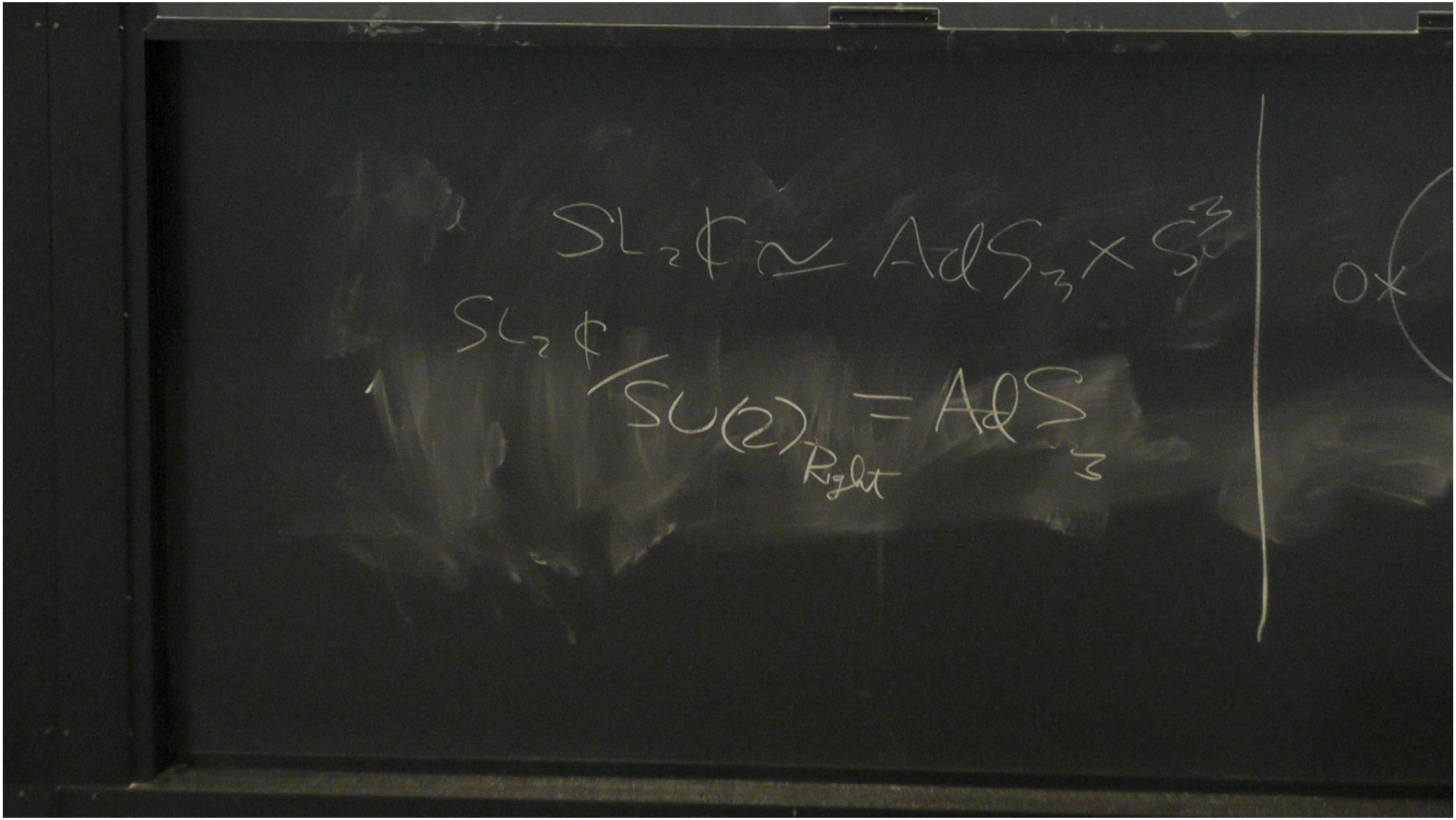
$$Bl_0\mathbb{C}^2 \longrightarrow SL_2\mathbb{C}$$

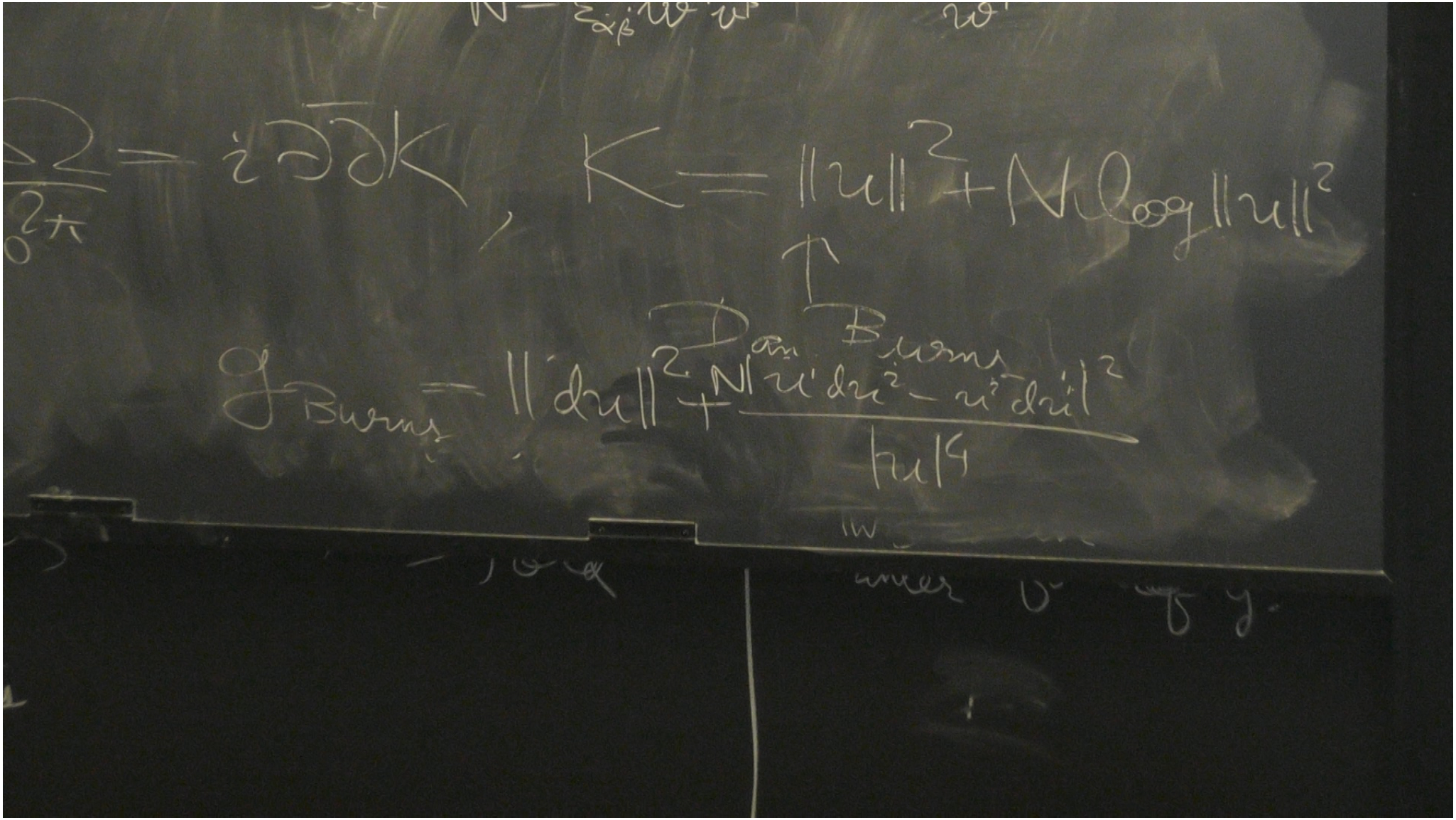
$$\downarrow$$

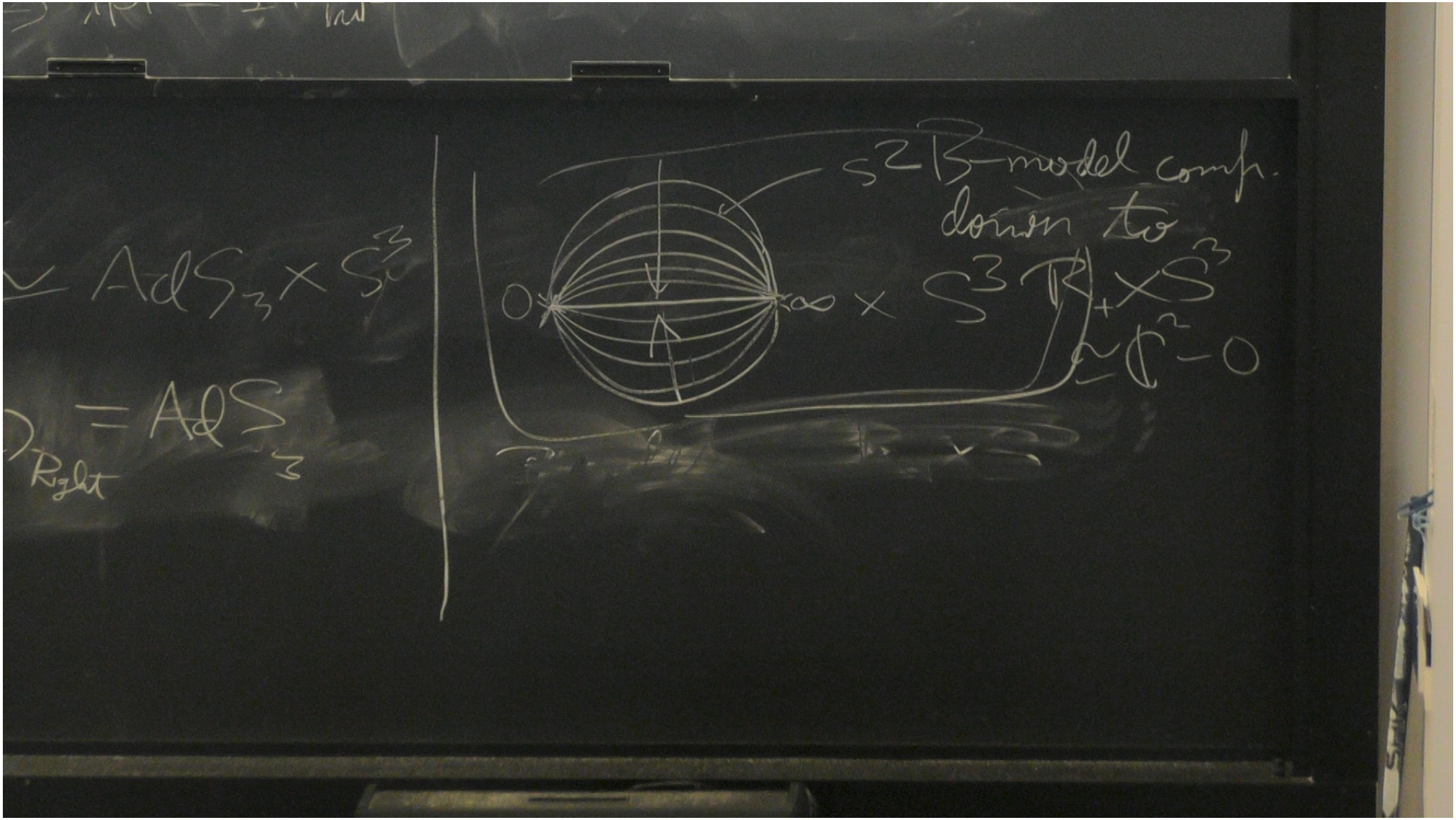
$$(\mathbb{C}^2 - 0) \cup \mathbb{C}P^1$$

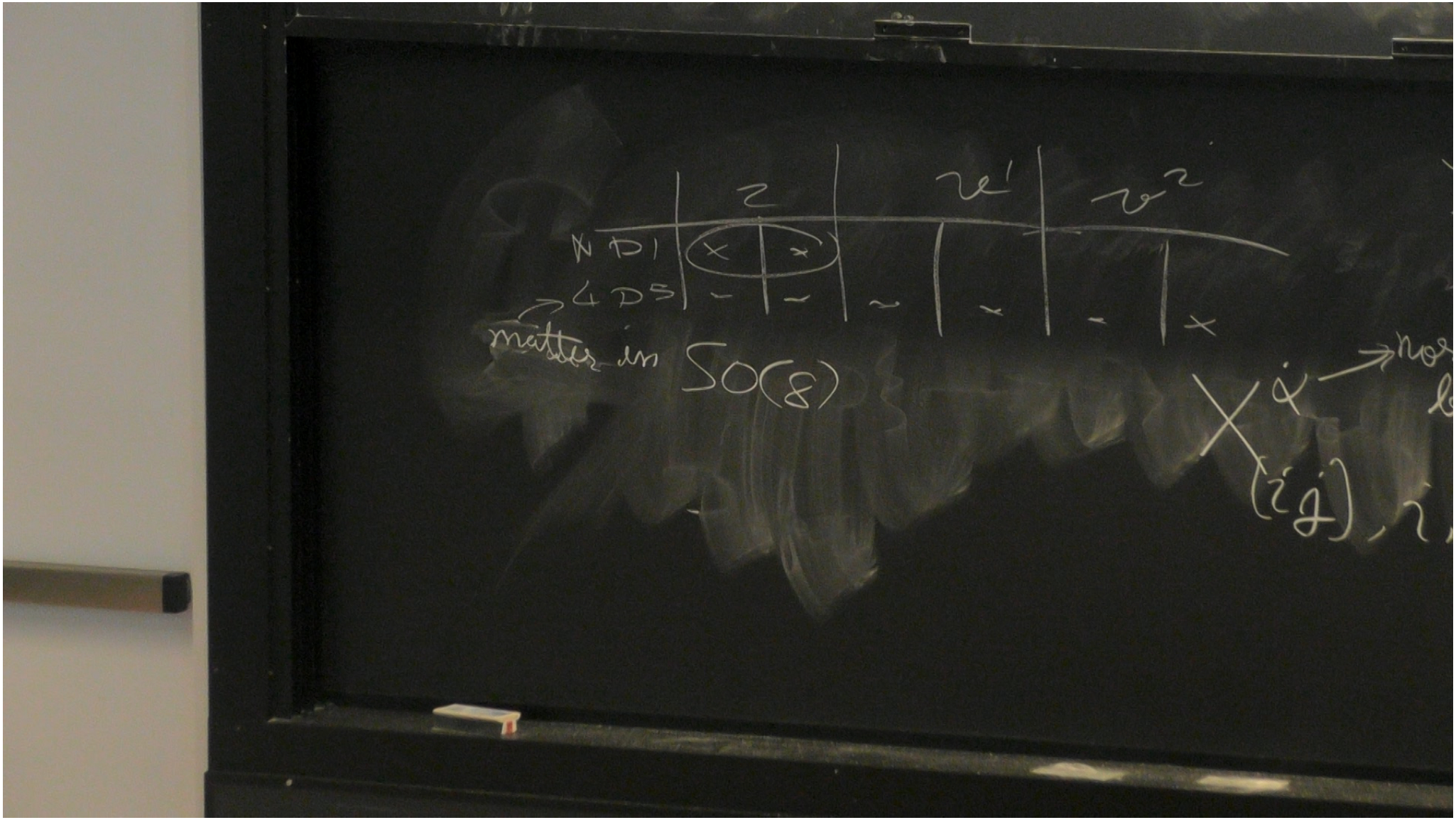

Fonticoso
Claude
Lebrun
Dan Bi

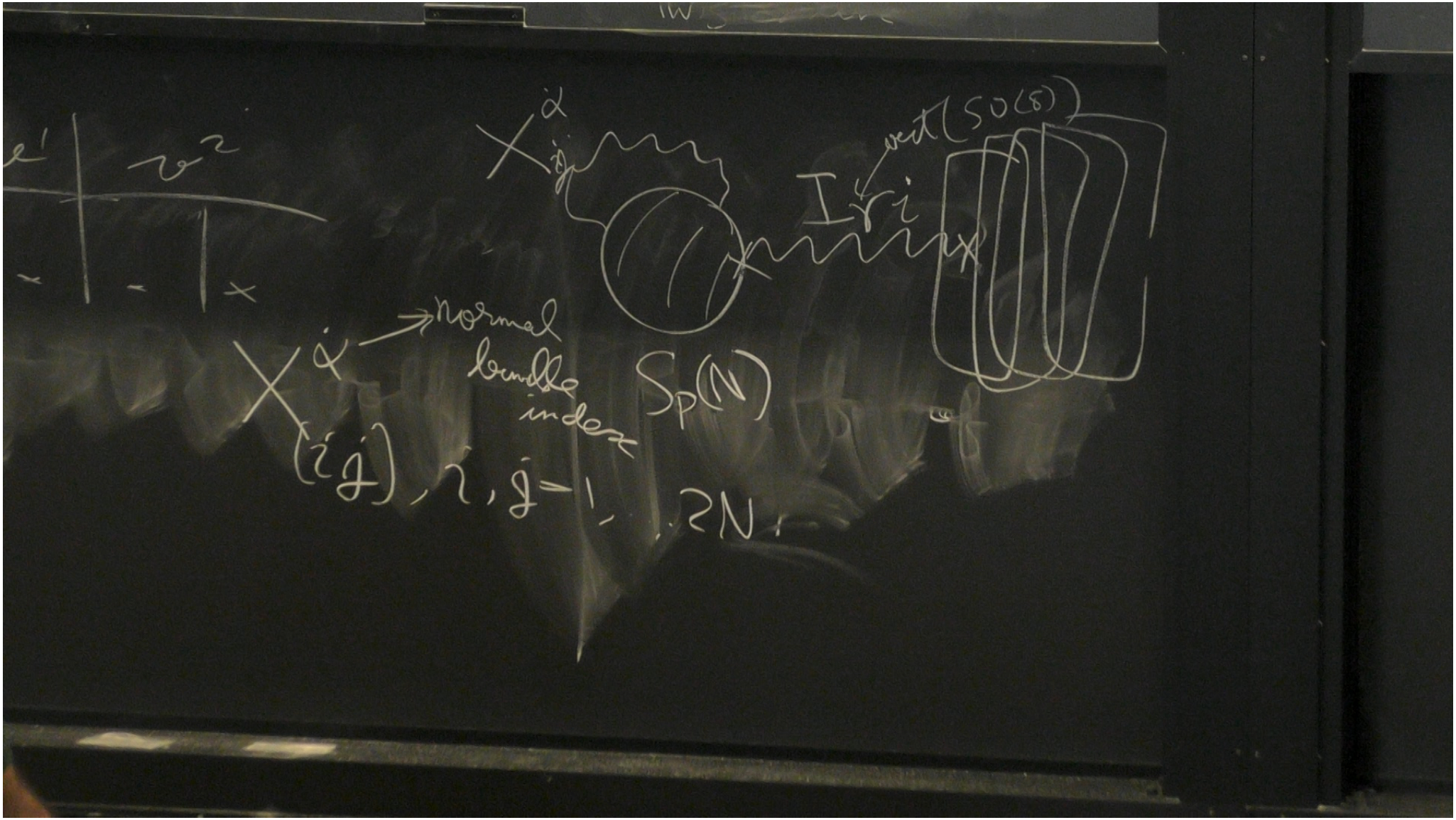

$\Rightarrow \Omega = \operatorname{Res}_{s=t} \frac{d^2w \wedge d^2w}{N - \varepsilon_{\alpha\beta} w^\alpha w^\beta} = \frac{dw^i \wedge d^2w}{w^i}$
$\frac{\partial \Omega}{\partial t}\Big|_{t=0} = \frac{i}{2\pi} \bar{\partial}\partial K$, $K = \|u\|^2 + N \log \|u\|^2$
Dan Burns

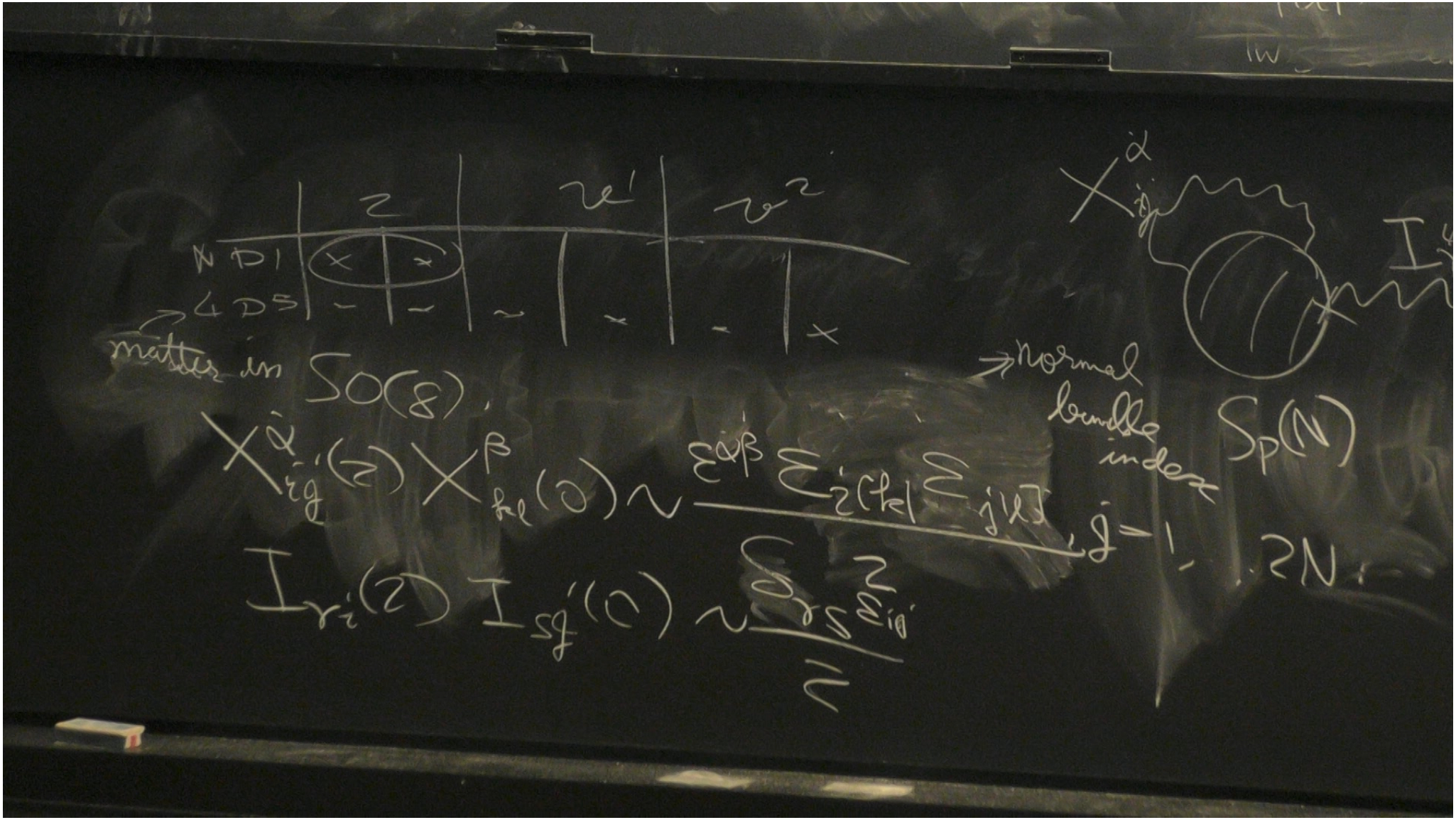

$SL_2\mathbb{C} \simeq AdS_3 \times S^3$
$SL_2\mathbb{C}/SU(2)_{Right} = AdS_3$

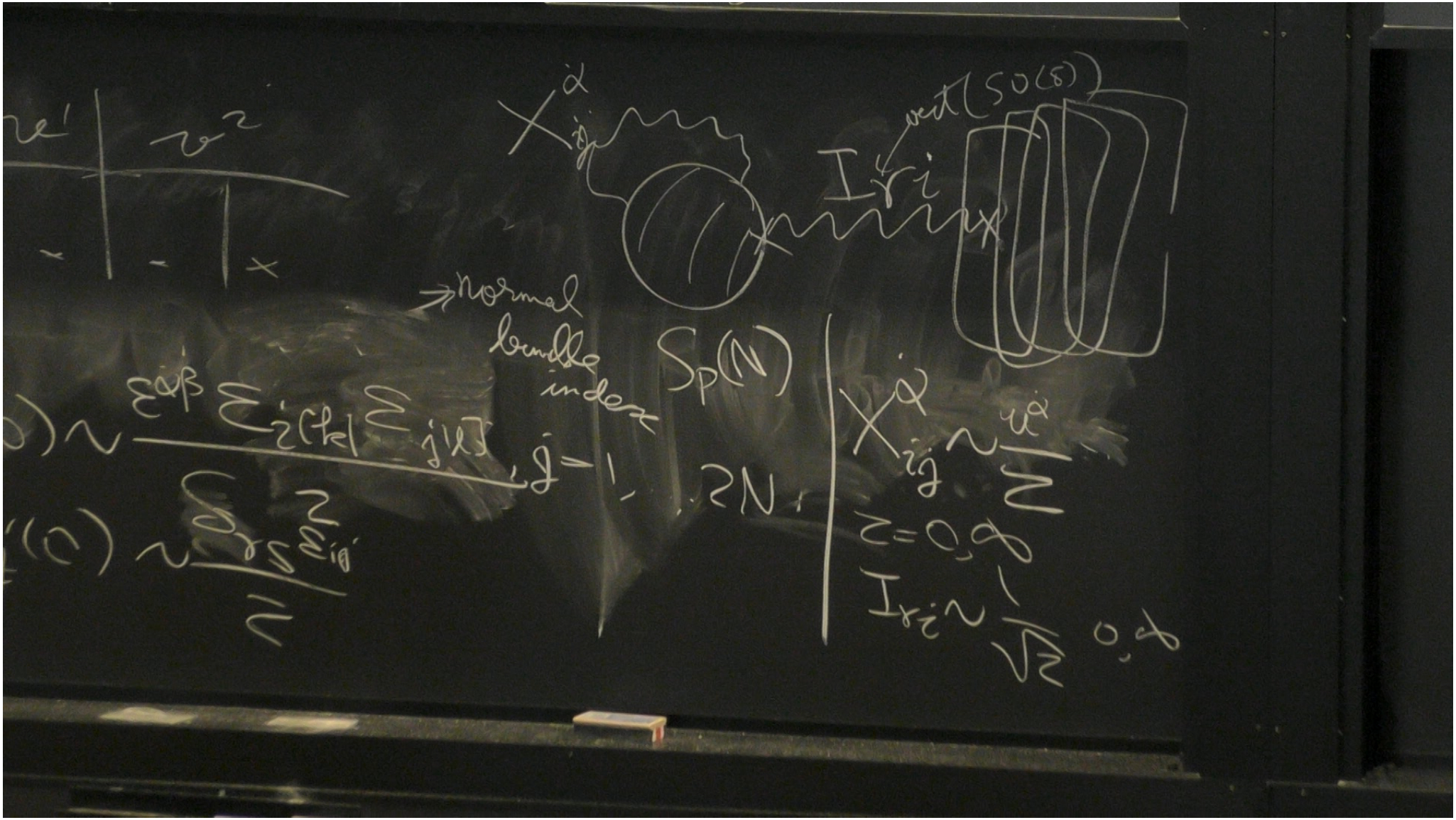

$\frac{\Omega}{2\pi} = i\bar{\partial}\partial K$, $K = \|u\|^2 + N\log\|u\|^2$
$g_{Burns} = \|du\|^2 + N\frac{|u|^2|du|^2 - |u^\dagger du|^2}{|u|^4}$
Pour Burns

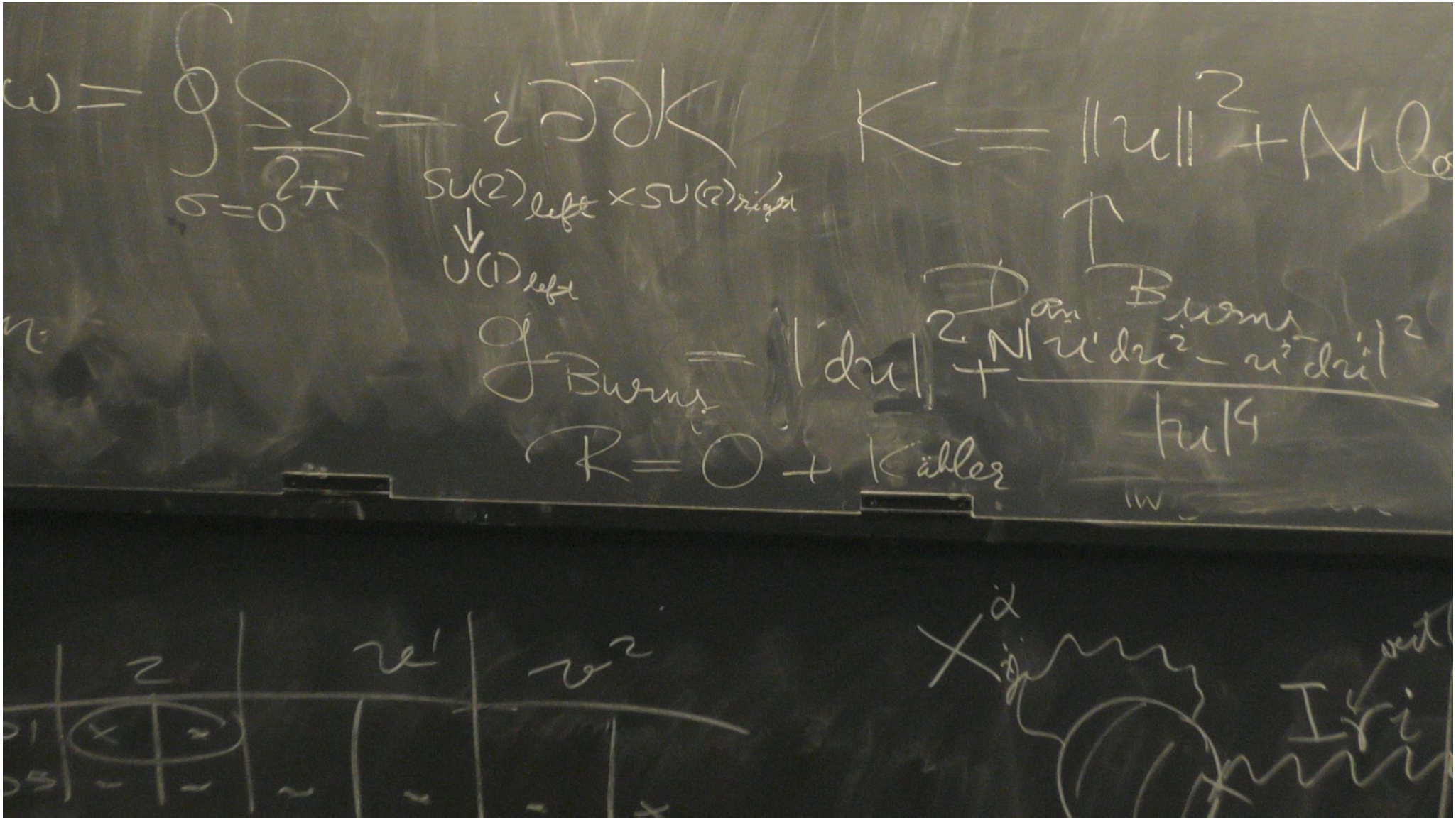

$\sim AdS_3 \times S^3$
$= AdS_3$
Right
S^2 B-model comp.
down to
$\mathbb{R}_+ \times S^3$
$\sim \mathbb{C}^2 - 0$


z
z'
v²
N D1
4 D5
matter in SO(8)


normal
bundle
index
$Sp(N)$
(ij), $i, j = 1, \ldots 2N$


z
z'
v²
N D1
4 D5
matter in SO(8)
Normal bundle index
Sp(N)
I


vect(SO(8))
normal bundle index
Sp(N)


$\omega = \int_{\sigma=0}^{2\pi} \frac{\Omega}{2\pi} = i\bar{\partial}\partial K$
$K = \|u\|^2 + N \log$
$SU(2)_{left} \times SU(2)_{right}$
$U(1)_{left}$
Dan Burns
$g_{Burns} = |dw|^2 + N \frac{|u' du^2 - u^2 du|^2}{|u|^4}$
$R = 0 +$ Kähler
IW
z
z'
v^2
α
i
j
I_{ri}