Title: Cosmology Lecture (230501)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

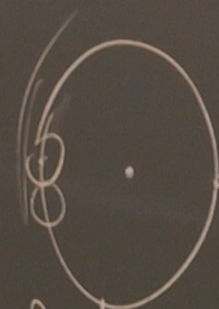
Date: May 01, 2023 - 3:15 PM

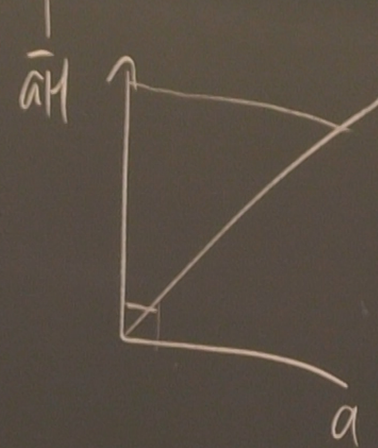
URL: https://pirsa.org/23050015

Inflation

$w \simeq -1 \to a \propto e^{Ht}$

1. Horizon problem

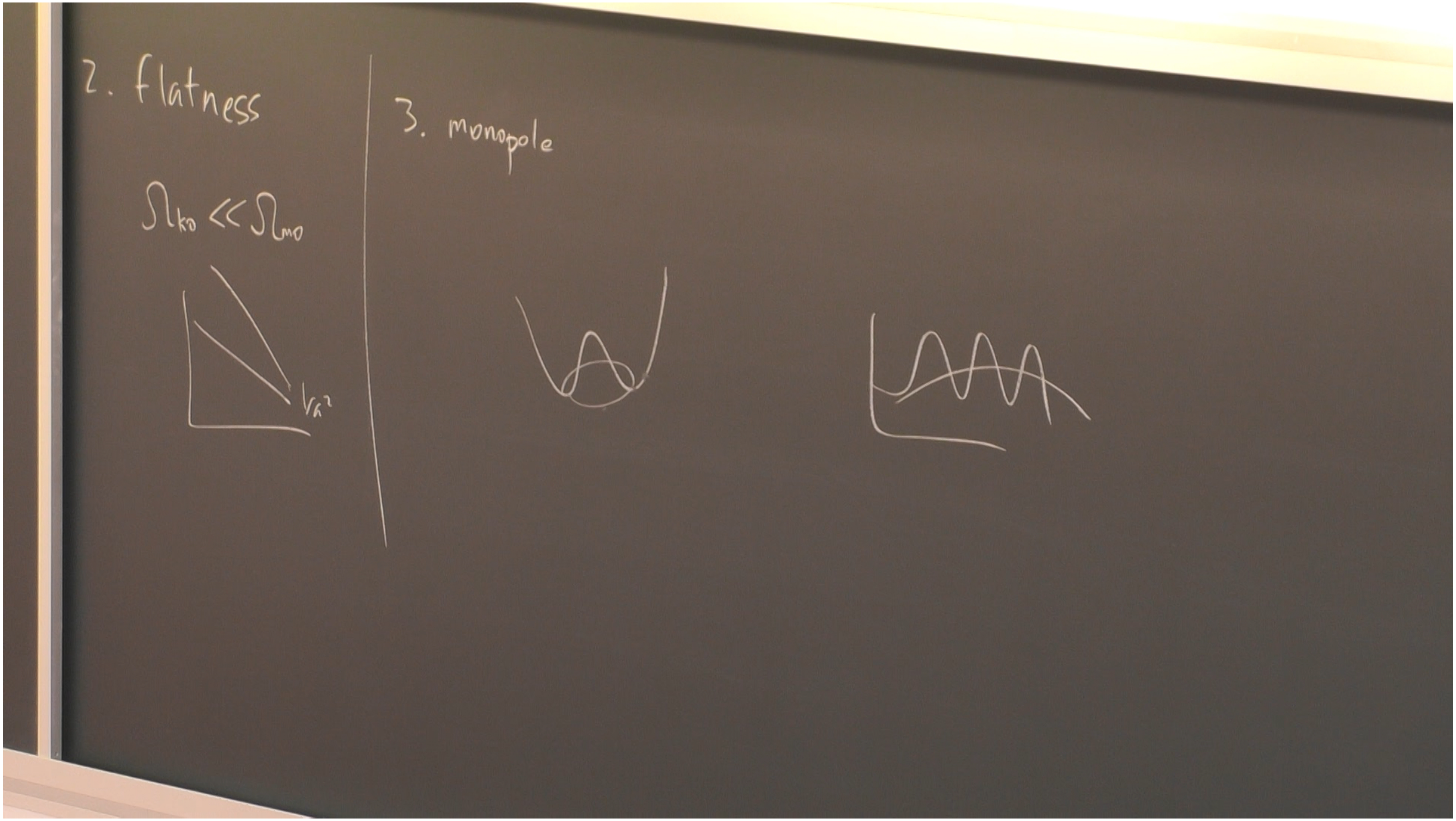


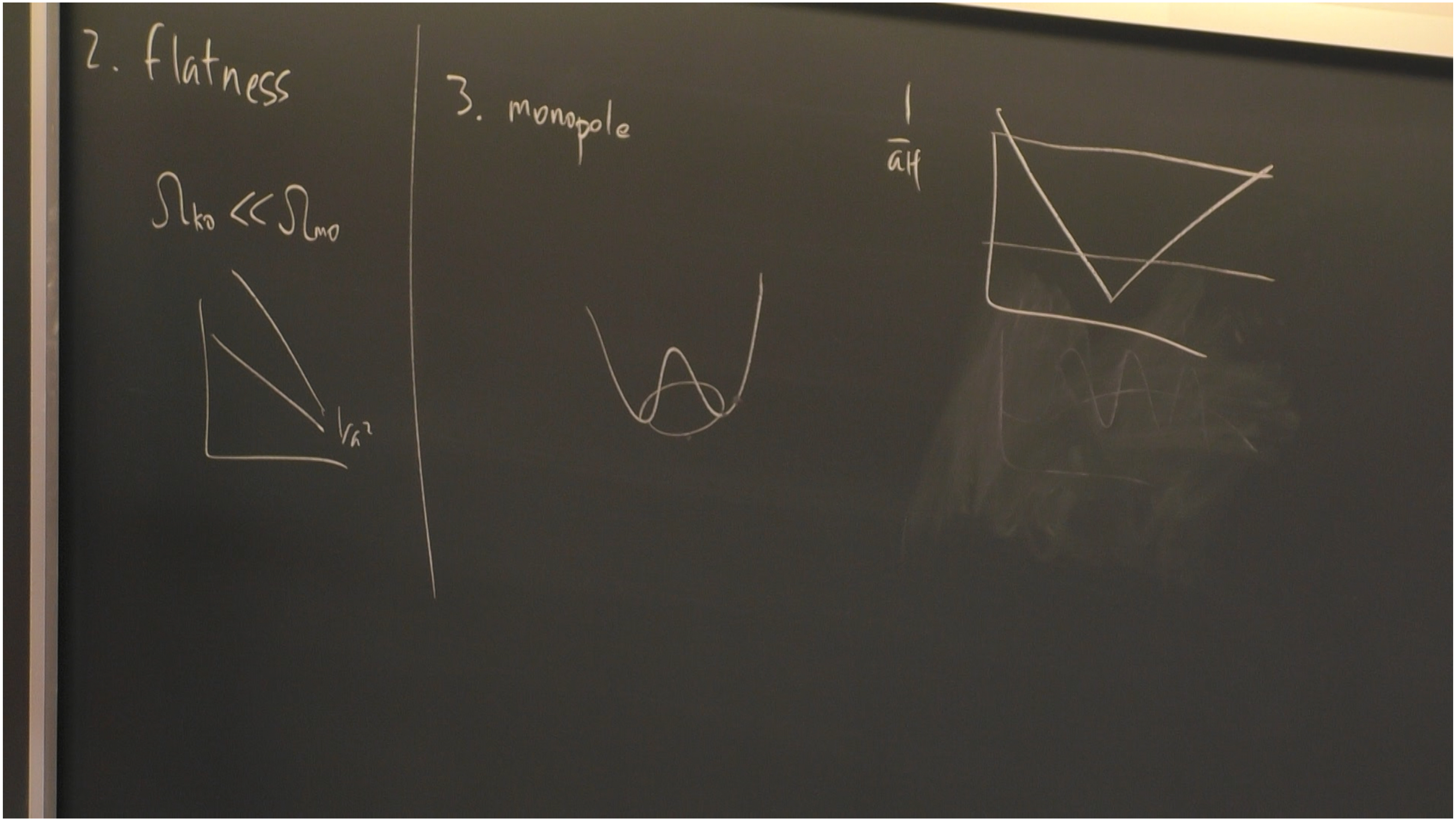


$$r_{Hor} = \int d\eta = \int \frac{dt}{a} = \int \frac{da}{a\,\dot{a}} = \int_0^1 \frac{da}{a^2 H} \sim (aH)^{-1}$$

$\rho_r \propto a^{-4}$ $\quad H \propto \sqrt{\rho} \propto a^{-2}$ $\quad \propto a$

2. f


2. flatness
$\Omega_{k0} \ll \Omega_{m0}$
$1/a^2$
3. monopole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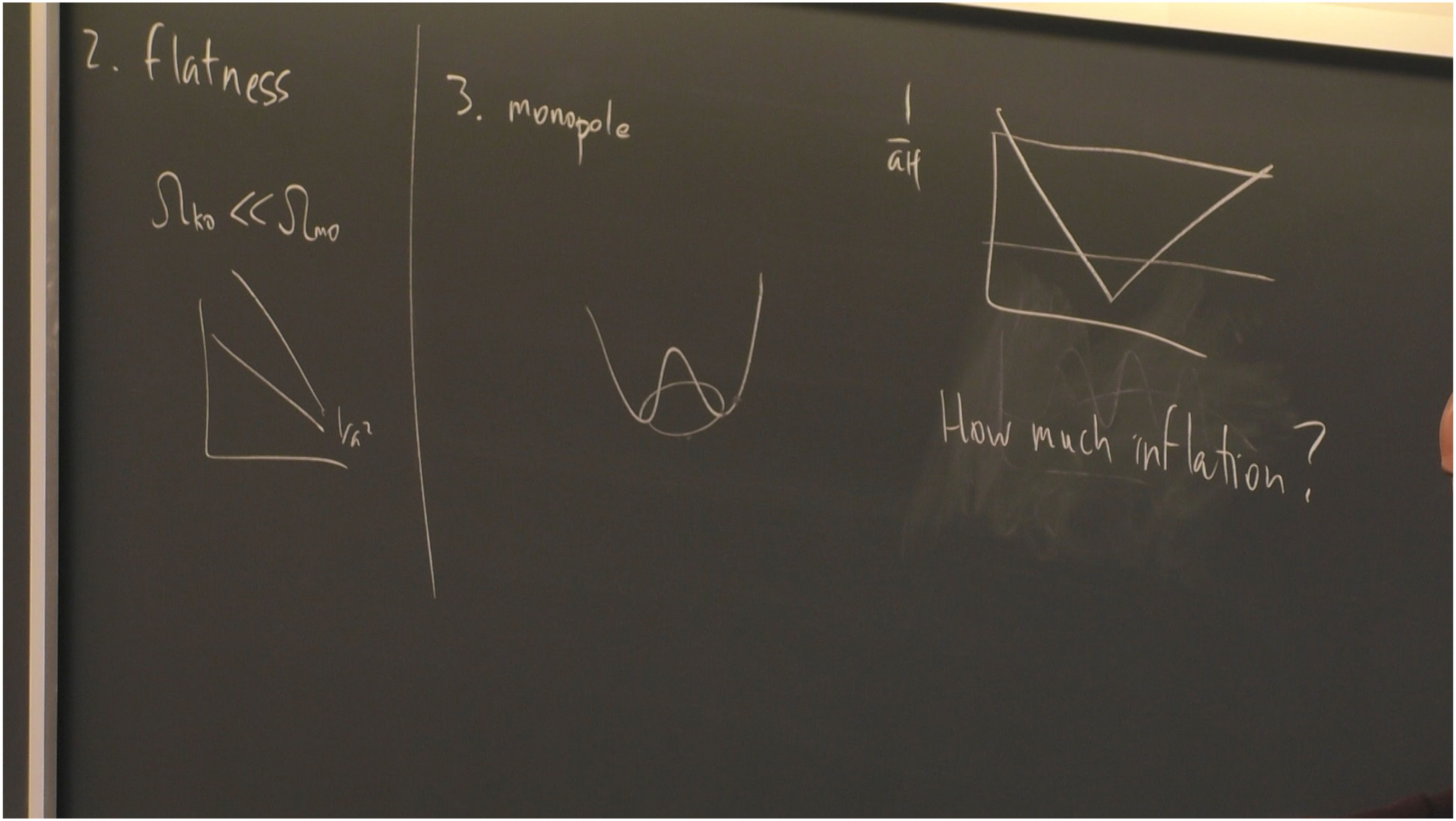
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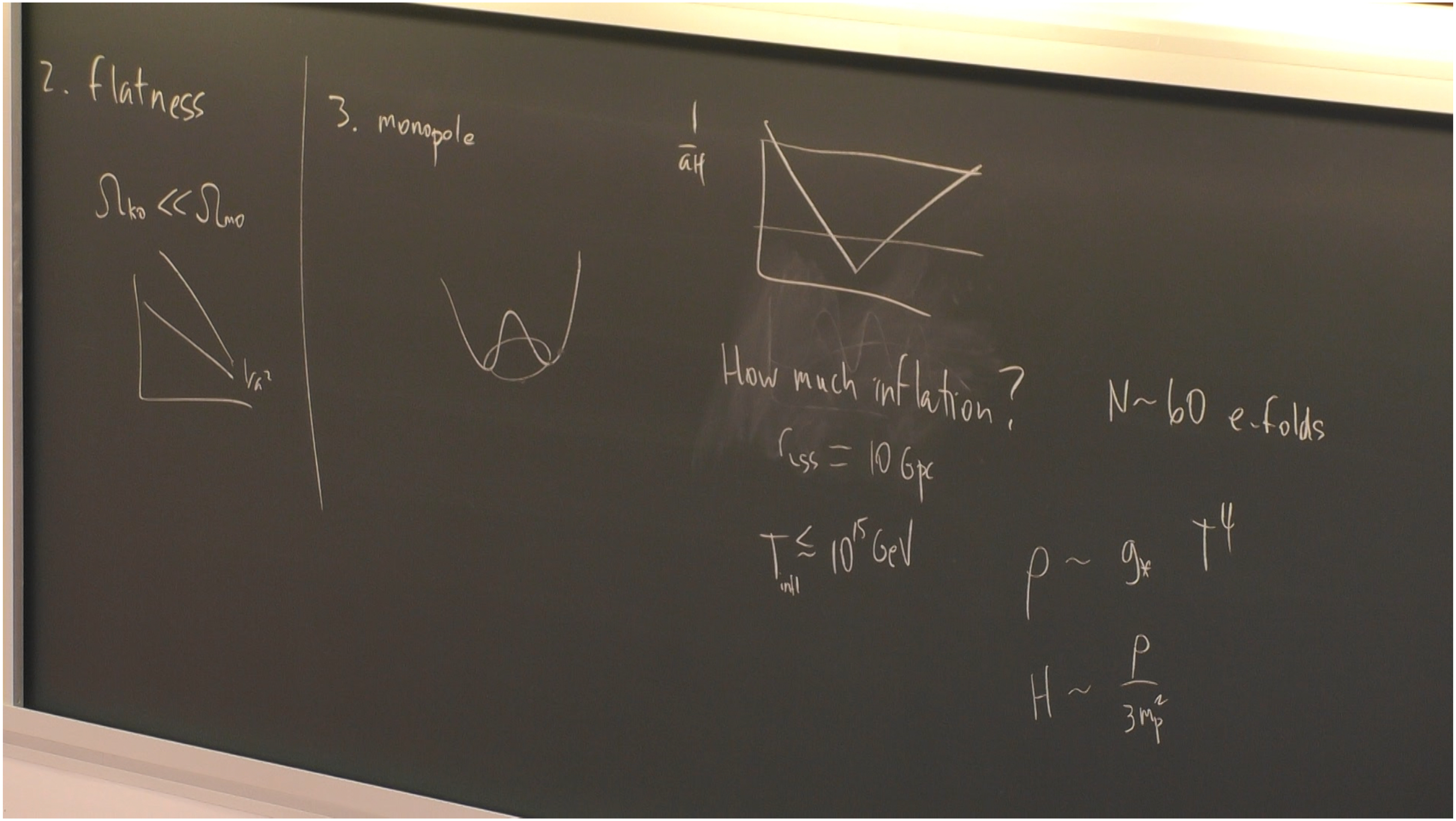
How much inflation?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$\frac{1}{aH}$
How much inflation?
$N \sim 60$ e-folds
$r_{LSS} = 10$ Gpc
$T_{infl} \lesssim 10^{15}$ GeV
$\rho \sim g_* T^4$
$H \sim \frac{\rho}{3 m_p^2}$

$10^{27} \sim e^{60}$

$N \sim 60$ e-folds

$\sim g_* \, T^4$

$\sim \dfrac{\rho}{3 m_p^2}$

Slow roll scalar field

$$\phi : \quad \mathcal{L} = \frac{1}{2} |\partial\phi|^2 + V(\phi)$$

$$T_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \mathcal{L}(\phi)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2} |\nabla\phi|^2 + V(\phi) \approx \frac{1}{2}\dot{\phi}^2 + V$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2} |\nabla\phi|^2 - V(\phi) \approx \frac{1}{2}\dot{\phi}^2 - V$$

$\sim e^{60}$

~ 60 e-folds

T^4

$\frac{\rho}{3m_p^2}$

Slow roll scalar field

$$\phi: \quad \mathcal{L} = \frac{1}{2}|\partial\phi|^2 + V(\phi) \qquad \Rightarrow \frac{1}{2}\dot{\phi}^2 \ll |V|$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \mathcal{L}(\phi)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}|\nabla\phi|^2 + V(\phi) \approx \frac{1}{2}\dot{\phi}^2 + V$$

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$27 \sim e^{60}$

~ 60 e-folds

$g_* \; T^4$

$\frac{\rho}{3 m_p^2}$

Slow roll scalar field

"slow roll"

$$\phi: \quad \mathcal{L} = \frac{1}{2}|\partial\phi|^2 + V(\phi) \qquad \Rightarrow \frac{1}{2}\dot{\phi}^2 \ll |V|$$

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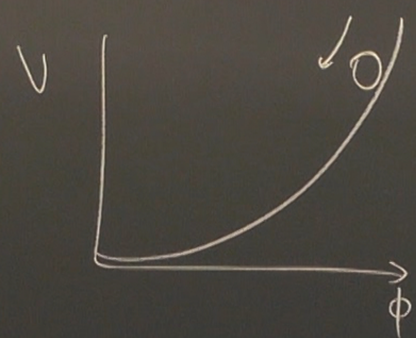
$$\dot{\rho} + 3H(\rho + P) = 0$$

$$\Rightarrow \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + V\right)$$

$$2H\dot{H} = \frac{8\pi G}{3}(-3H\dot{\phi})\dot{\phi} = -8\pi G H\dot{\phi}^2$$

$$\dot{H} = -4\pi G\dot{\phi}^2$$



$$\varepsilon \equiv \frac{|\dot{H}|}{H^2} = \frac{d}{dt}\left(\frac{1}{H}\right) \ll 1$$

$$\varepsilon = \frac{3}{2}(1+w)$$

$$|\dot{H}| \ll H^2 \Rightarrow \frac{d}{dt}\left(\frac{1}{H}\right) \ll 1$$

$$\frac{\dot\phi^2}{2} \ll V \Rightarrow \ddot\phi\,\dot\phi \ll V'\dot\phi$$

$$\Rightarrow \ddot\phi \ll |V'|\,,\ |3H\dot\phi|$$

$$\epsilon = -\frac{\dot H}{H^2}$$

$$\Rightarrow 3H\dot\phi \simeq -V' \Rightarrow \dot\phi^2 \approx \left(\frac{V'}{3H}\right)^2 = -\frac{\dot H}{4\pi G}$$

$$\frac{\epsilon H^2}{4\pi G} \simeq \left(\frac{V'}{3H}\right)^2 \Rightarrow \epsilon = 4\pi G\left(\frac{V'}{3H^2}\right)^2 = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$$

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$$\varepsilon = -\frac{\dot H}{H^2}$$

$$\boxed{\varepsilon = \frac{1}{2}\left(\frac{m_p V'}{V}\right)^2 \ll 1}$$

$$\frac{\varepsilon H^2}{4\pi G} \simeq \left(\frac{V'}{3H}\right)^2 \Rightarrow \varepsilon = 4\pi G\left(\frac{V'}{3H^2}\right)^2 = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$$

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$$3H\dot{\phi} \simeq -V'$$

$$|V''| \ll \frac{V}{m_p^2}$$

$$\epsilon \ll 1$$

$$\eta = \frac{m_p^2 V''}{V}$$

$$|\eta| \ll 1$$

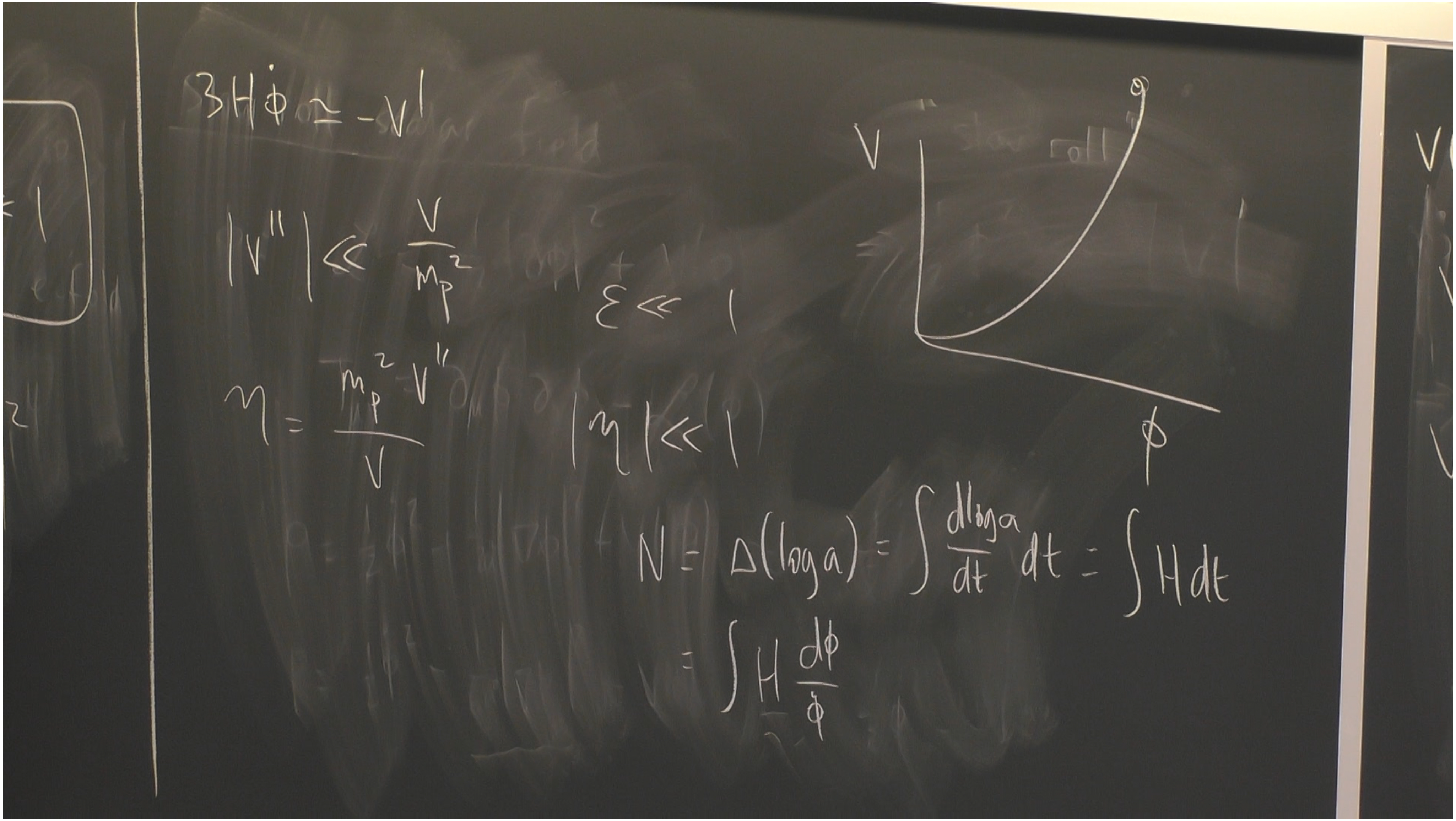
$$V(\phi) = c\,\phi^n$$

$$V' = c\,n\,\phi^{n-1} = \frac{nV}{\phi} \Rightarrow \epsilon = \frac{1}{2}\left(\frac{n\,m_p}{\phi}\right)^2$$

$$V'' = c\,n(n-1)\,\phi^{n-2}$$

$$= n(n-1)\frac{V}{\phi^2}$$

$$\eta = n(n-1)\frac{m_p^2}{\phi^2} \Rightarrow \phi$$


$3H\dot{\phi} \simeq -V'$
$|V''| \ll \frac{V}{M_p^2}$
$\varepsilon \ll 1$
$\eta = \frac{M_p^2 V''}{V}$
$|\eta| \ll 1$
V
ϕ
$N = \Delta(\log a) = \int \frac{d\log a}{dt} dt = \int H dt$
$= \int H \frac{d\phi}{\dot{\phi}}$

$$3H\dot{\phi} \simeq -V'$$

$$|V''| \ll \frac{V}{m_p^2} \qquad \varepsilon \ll 1$$

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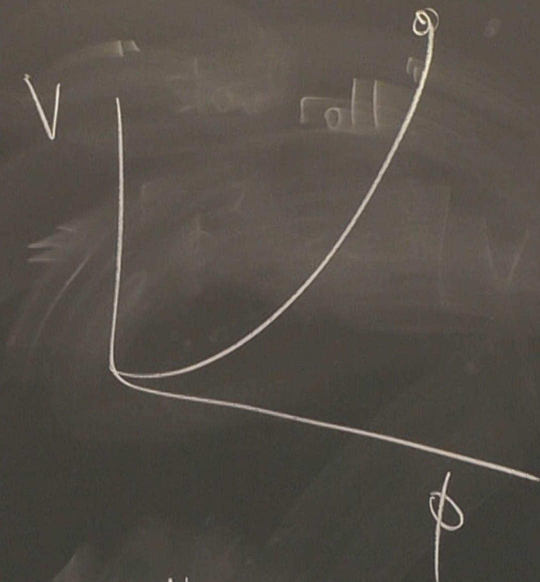

$$N = \Delta(\log a) = \int \frac{d\log a}{dt}\, dt = \int H\, dt$$

$$= \int H \frac{d\phi}{\dot{\phi}} \simeq \int \frac{3H^2}{V'}\, d\phi = \frac{1}{m_p^2} \int \frac{V}{V'}\, d\phi$$

$\left(\; \right)^2 \ll 1$

$\left(\frac{V'}{V}\right)^2$

$3H\dot{\phi} \simeq -V'$

$|V''| \ll \frac{V}{M_p^2}$

$\varepsilon \ll 1$

$\eta = \frac{M_p^2 V''}{V}$

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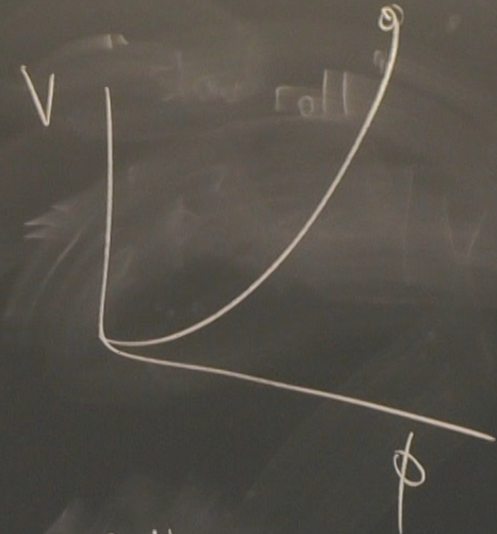

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$$= \int H \frac{d\phi}{\dot{\phi}} \simeq \int \frac{3H^2}{V'}\,d\phi = \frac{1}{M_p^2}\int \frac{V}{V'}\,d\phi$$

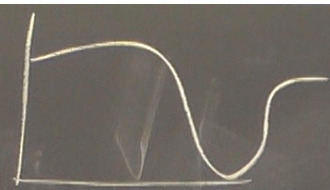
$$= \frac{1}{n M_p^2}\int \phi\,d\phi \simeq \frac{1}{2}\left(\frac{\phi_{\text{start}}}{M_p}\right)^2$$

$V(\phi$

V'

V''

$$V(\phi) = c\,\phi^n$$



$$V' = c\,n\,\phi^{n-1} = \frac{nV}{\phi} \Rightarrow \varepsilon = \frac{1}{2}\left(\frac{n\,m_p}{\phi}\right)^2$$

$$V'' = c\,n(n-1)\,\phi^{n-2}$$
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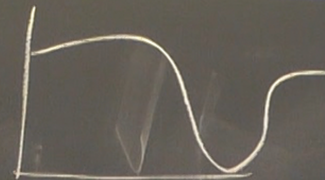
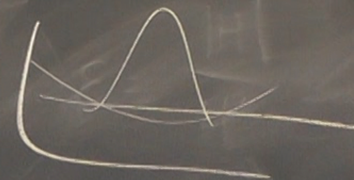


$$\Rightarrow \varepsilon = \frac{1}{2}\left(\frac{n\,m_p}{\phi}\right)^2$$



$$V'' = c\,n(n-1)\,\phi^{n-2} = n(n-1)\frac{V}{\phi^2}$$

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$$\Rightarrow \phi$$

$$\langle h^2 \rangle$$

$$\langle \delta^2 \rangle$$

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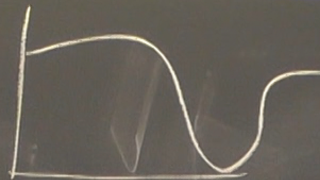
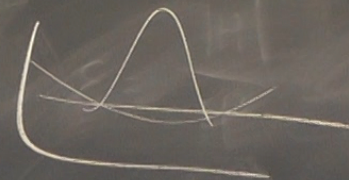


$$\Rightarrow \quad \varepsilon = \frac{1}{2}\left(\frac{n\,m_p}{\phi}\right)^2$$



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