Title: Cosmology Lecture (230501)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

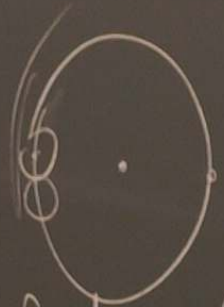
Date: May 01, 2023 - 3:15 PM

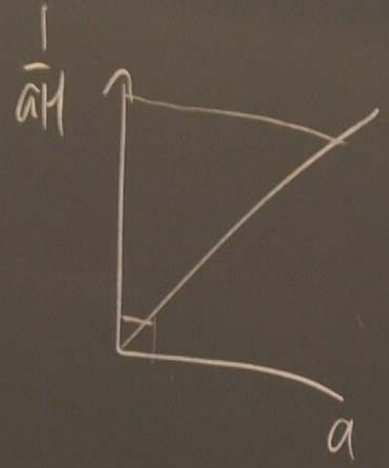
URL: https://pirsa.org/23050015

Inflation

$w \simeq -1 \rightarrow a \propto e^{Ht}$

1. Horizon problem





$$r_{Hor} = \int d\eta = \int \frac{dt}{a} = \int \frac{da}{a\dot{a}} = \int_0^1 \frac{da}{a^2 H} \sim (aH)^{-1}$$

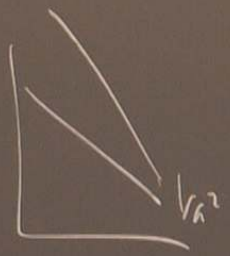
$\rho_r \propto a^{-4}$ $H \propto \sqrt{\rho} \propto a^{-2}$

$\propto a$

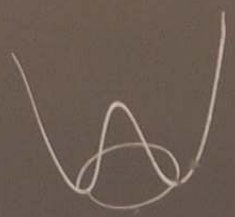
2. f

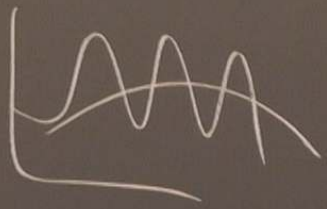
2. flatness

$\Omega_{k0} \ll \Omega_{m0}$



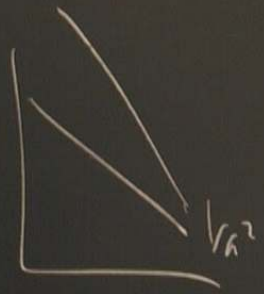
3. monopole



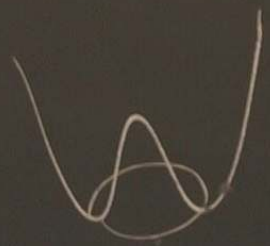


2. flatness

$\Omega_{k0} \ll \Omega_{m0}$



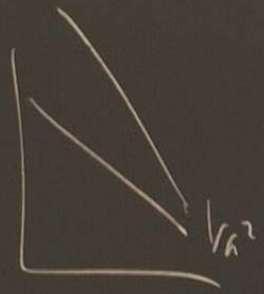
3. monopole



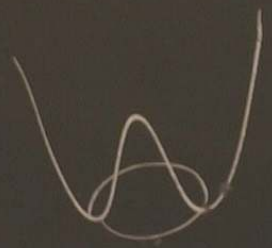
$\frac{1}{aH}$

2. flatness

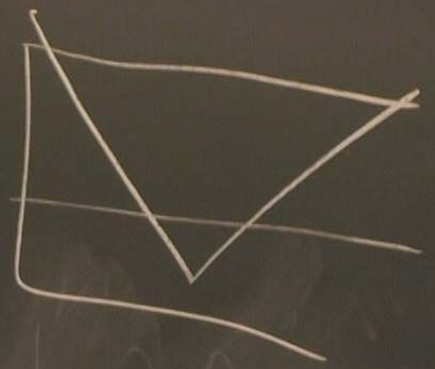
$\Omega_{k0} \ll \Omega_{m0}$



3. monopole



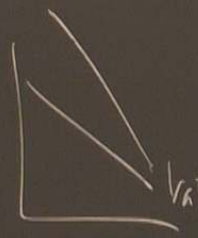
$\frac{1}{\bar{a}H}$



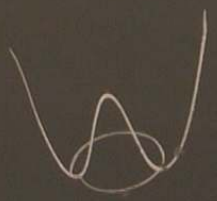
How much inflation?

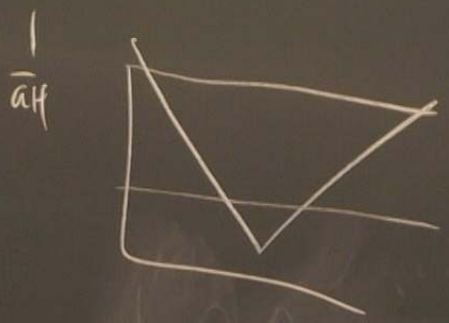
2. flatness

$\Omega_{k0} \ll \Omega_{m0}$



3. monopole





How much inflation?

$r_{LSS} = 10\,\mathrm{Gpc}$

$T_{infl} \lesssim 10^{15}\,\mathrm{GeV}$

$N \sim 60$ e-folds

$\rho \sim g_* T^4$

$H \sim \frac{\rho}{3m_p^2}$

$10^{27} \sim e^{60}$

$N \sim 60$ e-folds

$\sim g_* T^4$

$\sim \frac{\rho}{3m_p^2}$

Slow roll scalar field

ϕ: $\mathcal{L} = \frac{1}{2}|\partial\phi|^2 + V(\phi)$

$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \mathcal{L}(\phi)$

$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}|\nabla\phi|^2 + V(\phi) \approx \frac{1}{2}\dot{\phi}^2 + V$

$P = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}|\nabla\phi|^2 - V(\phi) \approx \frac{1}{2}\dot{\phi}^2 - V$

$\sim e^{60}$

~60 e-folds

$+\psi$

$\frac{\rho}{3m_p^2}$

Slow roll scalar field

ϕ: $\mathcal{L} = \frac{1}{2}|\partial\phi|^2 + V(\phi)$ $\Rightarrow \frac{1}{2}\dot{\phi}^2 \ll |V|$

$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \mathcal{L}(\phi)$

$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}|\nabla\phi|^2 + V(\phi) \approx \frac{1}{2}\dot{\phi}^2 + V$

$P = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}|\nabla\phi|^2 - V(\phi) \approx \frac{1}{2}\dot{\phi}^2 - V$

$\sim e^{60}$

~ 60 e-folds

$+\psi$

g_*

$\frac{\rho}{3m_p^2}$

Slow roll scalar field

"slow roll"

ϕ: $\mathcal{L} = \frac{1}{2}|\partial\phi|^2 + V(\phi)$ $\Rightarrow \frac{1}{2}\dot{\phi}^2 \ll |V|$

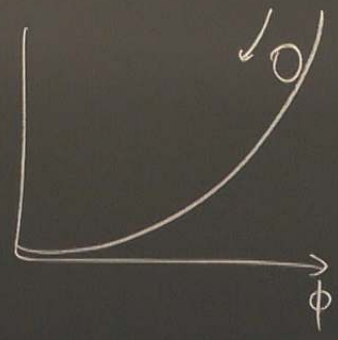
$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \mathcal{L}(\phi)$

$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}|\nabla\phi|^2 + V(\phi) \approx \frac{1}{2}\dot{\phi}^2 + V$

$P = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}|\nabla\phi|^2 - V(\phi) \approx \frac{1}{2}\dot{\phi}^2 - V$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\Rightarrow \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$



$$\varepsilon \equiv \frac{|\dot{H}|}{H^2} = \frac{d}{dt}\left(\frac{1}{H}\right) \ll 1$$

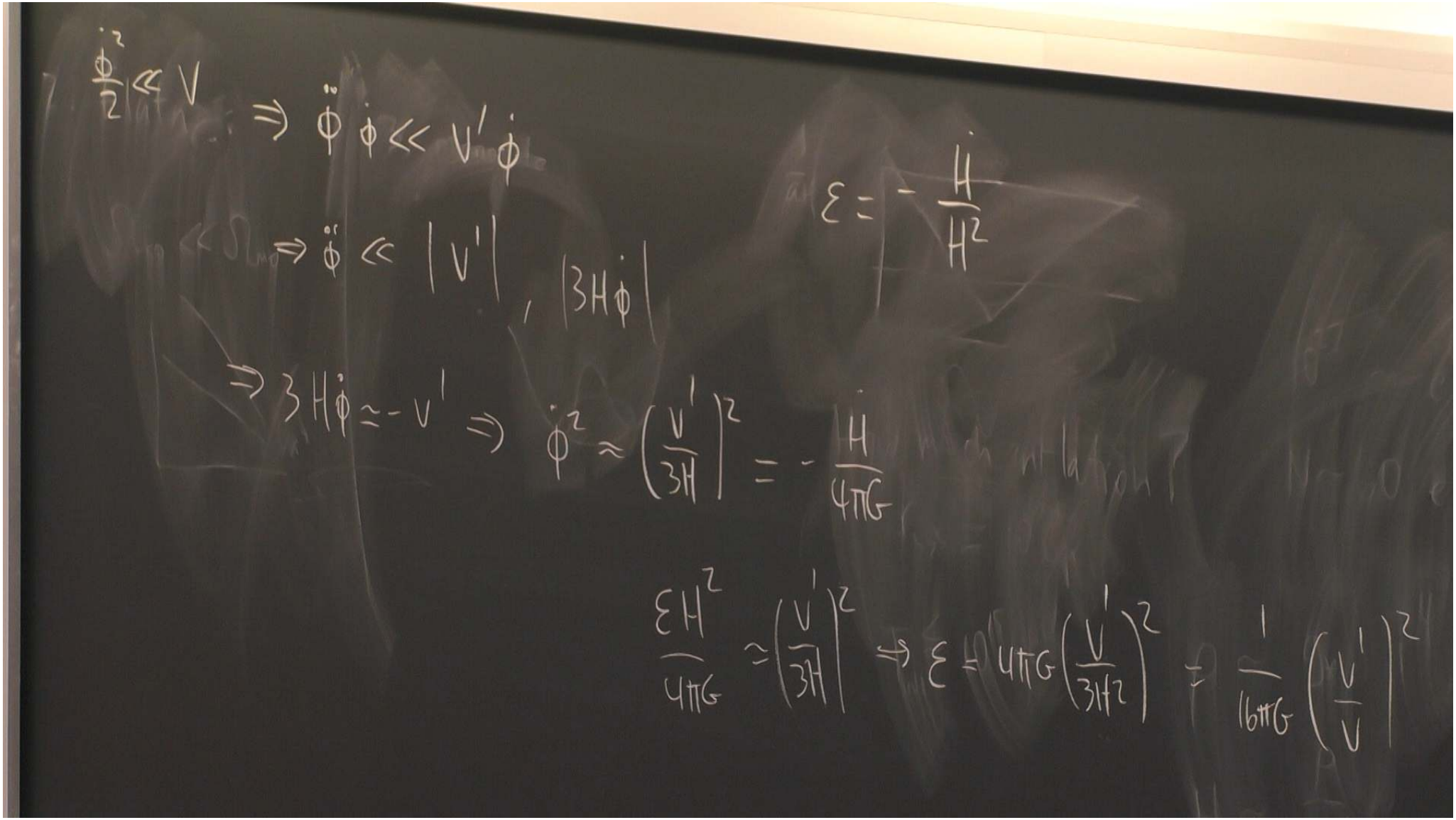
$$\varepsilon = \frac{3}{2}(1+w)$$

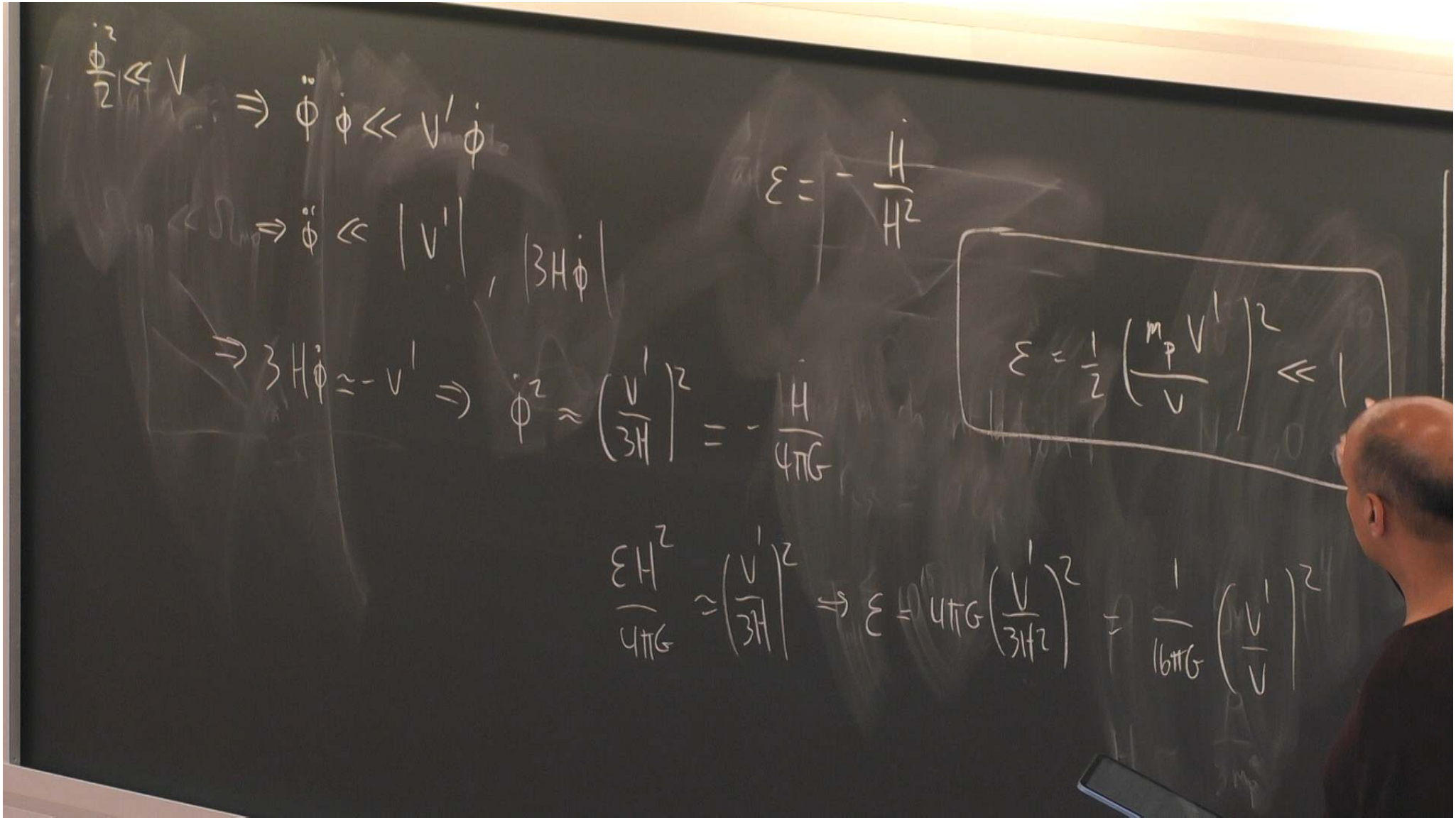
$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + V\right)$$

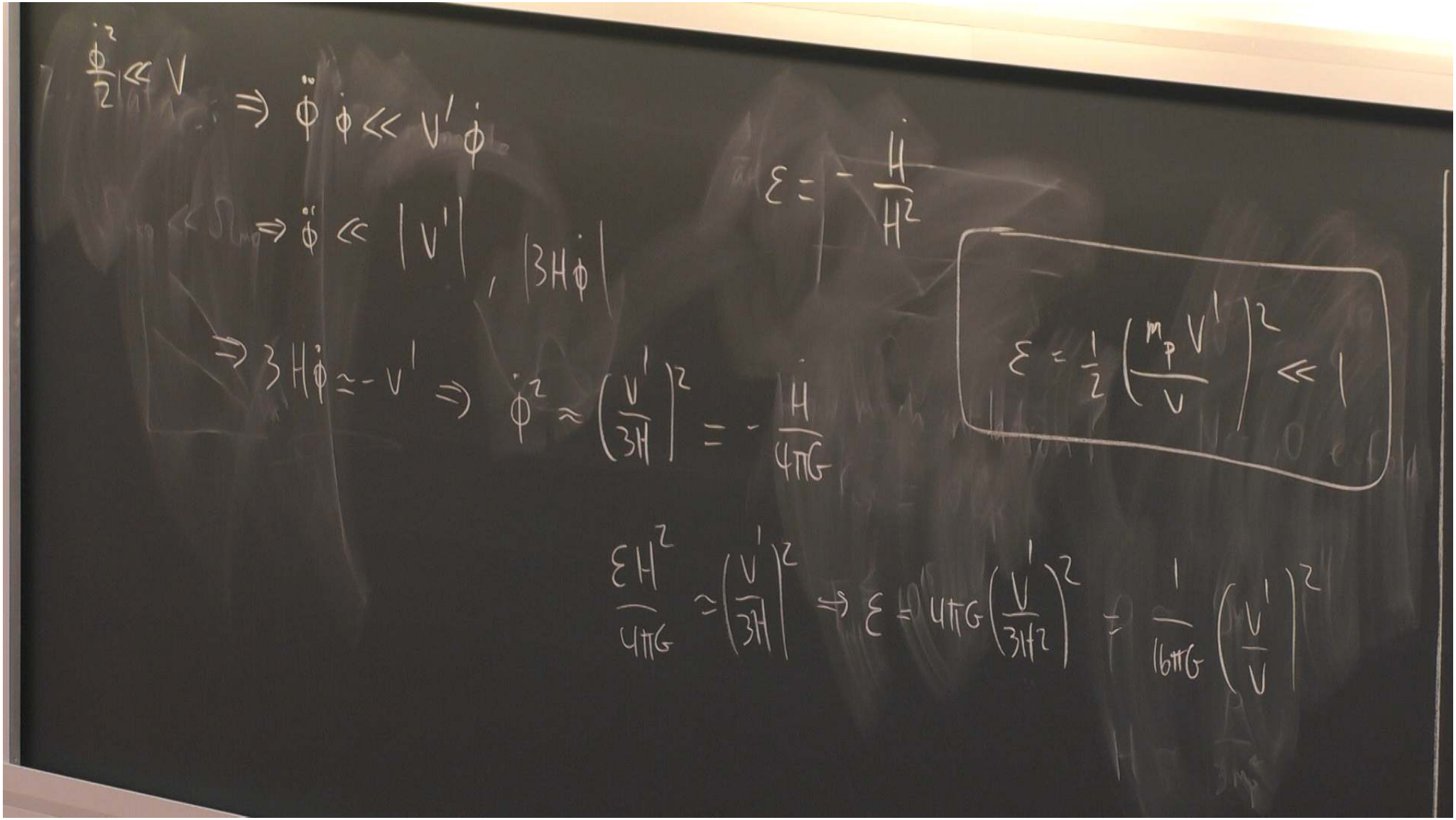
$$2H\dot{H} = \frac{8\pi G}{3}(-3H\dot{\phi})\dot{\phi} = -8\pi G H\dot{\phi}^2$$

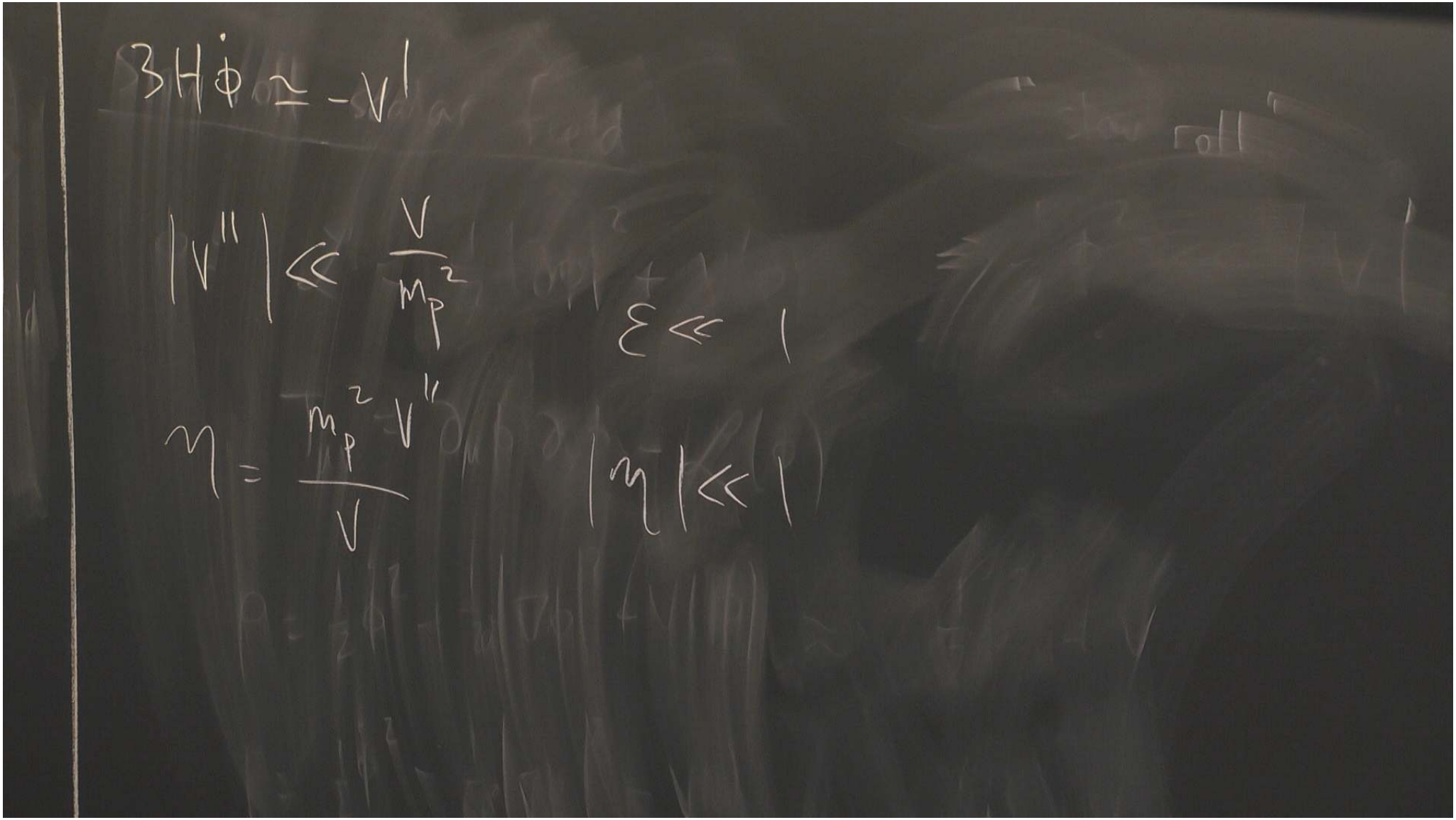
$$\dot{H} = -4\pi G\dot{\phi}^2$$

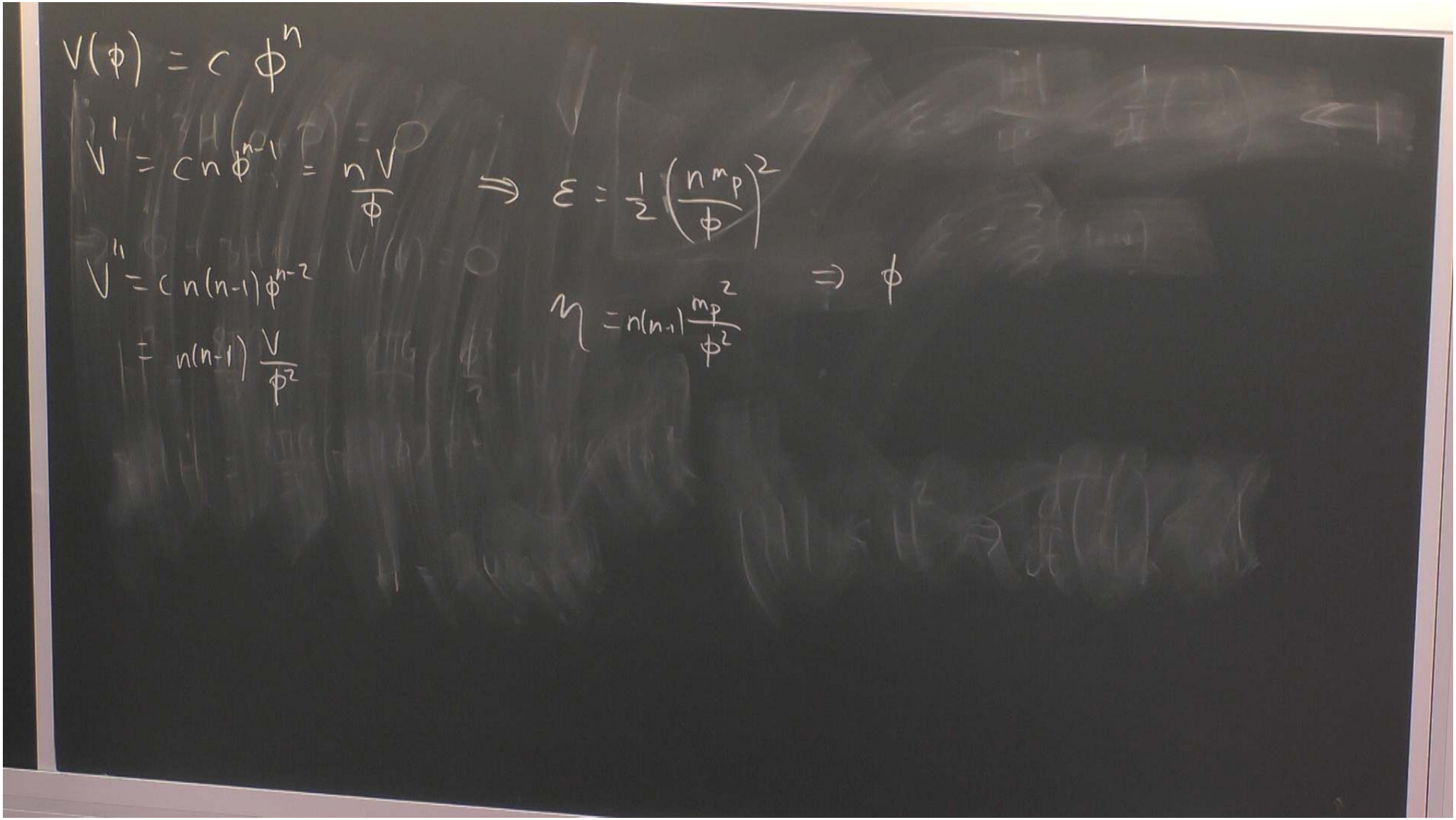
$$|\dot{H}| \ll H^2 \Rightarrow \frac{d}{dt}\left(\frac{1}{H}\right) \ll 1$$


$\frac{\dot\phi^2}{2} \ll V \Rightarrow \ddot\phi\,\dot\phi \ll V'\dot\phi$
$\Rightarrow \ddot\phi \ll |V'|, |3H\dot\phi|$
$\Rightarrow 3H\dot\phi \simeq -V' \Rightarrow \dot\phi^2 \simeq \left(\frac{V'}{3H}\right)^2 = -\frac{\dot H}{4\pi G}$
$\varepsilon = -\frac{\dot H}{H^2}$
$\frac{\varepsilon H^2}{4\pi G} \simeq \left(\frac{V'}{3H}\right)^2 \Rightarrow \varepsilon \simeq 4\pi G \left(\frac{V'}{3H^2}\right)^2 = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$


$\frac{\dot\phi^2}{2} \ll V \Rightarrow \ddot\phi\,\dot\phi \ll V'\dot\phi$
$\Rightarrow \ddot\phi \ll |V'|, |3H\dot\phi|$
$\Rightarrow 3H\dot\phi \simeq -V' \Rightarrow \dot\phi^2 \approx \left(\frac{V'}{3H}\right)^2 = -\frac{\dot H}{4\pi G}$
$\varepsilon = -\frac{\dot H}{H^2}$
$\varepsilon = \frac{1}{2}\left(\frac{m_p V'}{V}\right)^2 \ll 1$
$\frac{\varepsilon H^2}{4\pi G} \simeq \left(\frac{V'}{3H}\right)^2 \Rightarrow \varepsilon \simeq 4\pi G\left(\frac{V'}{3H^2}\right)^2 = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$


$\frac{\dot\phi^2}{2} \ll V \Rightarrow \ddot\phi\,\dot\phi \ll V'\dot\phi$
$\Rightarrow \ddot\phi \ll |V'|, |3H\dot\phi|$
$\Rightarrow 3H\dot\phi \simeq -V' \Rightarrow \dot\phi^2 \approx \left(\frac{V'}{3H}\right)^2 = -\frac{\dot H}{4\pi G}$
$\varepsilon = -\frac{\dot H}{H^2}$
$\varepsilon = \frac{1}{2}\left(\frac{m_p V'}{V}\right)^2 \ll 1$
$\frac{\varepsilon H^2}{4\pi G} \simeq \left(\frac{V'}{3H}\right)^2 \Rightarrow \varepsilon \simeq 4\pi G\left(\frac{V'}{3H^2}\right)^2 = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$

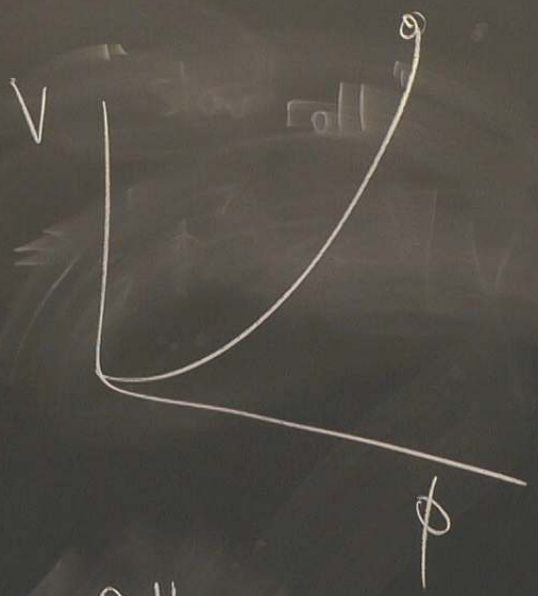

$3H\dot{\phi} \simeq -V'$
$|V''| \ll \frac{V}{M_p^2}$
$\varepsilon \ll 1$
$\eta = \frac{M_p^2 V''}{V}$
$|\eta| \ll 1$


$V(\phi) = c\,\phi^n$
$V' = cn\phi^{n-1} = \frac{nV}{\phi}$
$\Rightarrow \varepsilon = \frac{1}{2}\left(\frac{n m_p}{\phi}\right)^2$
$V'' = cn(n-1)\phi^{n-2}$
$= n(n-1)\frac{V}{\phi^2}$
$\eta = n(n-1)\frac{m_p^2}{\phi^2}$
$\Rightarrow \phi$

$3H\dot{\phi} \simeq -V'$

$|V''| \ll \frac{V}{M_p^2}$ $\quad \varepsilon \ll 1$

$\eta = \frac{M_p^2 V''}{V}$ $\quad |\eta| \ll 1$



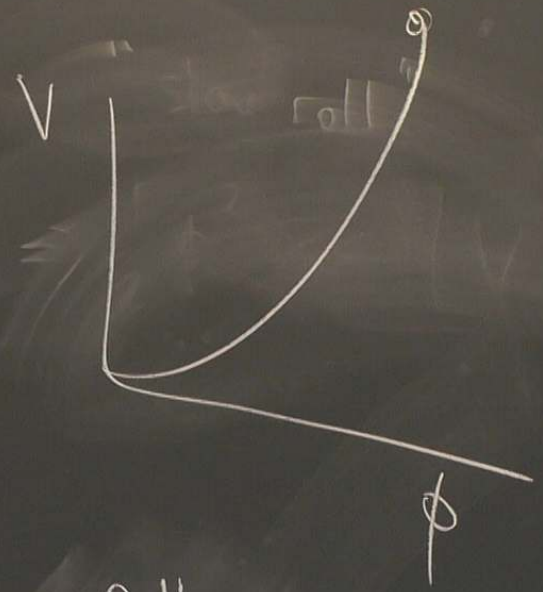
$N = \Delta(\log a) = \int \frac{d\log a}{dt} dt = \int H dt$

$= \int \frac{H}{\dot{\phi}} d\phi$

$3H\dot{\phi} \simeq -V'$

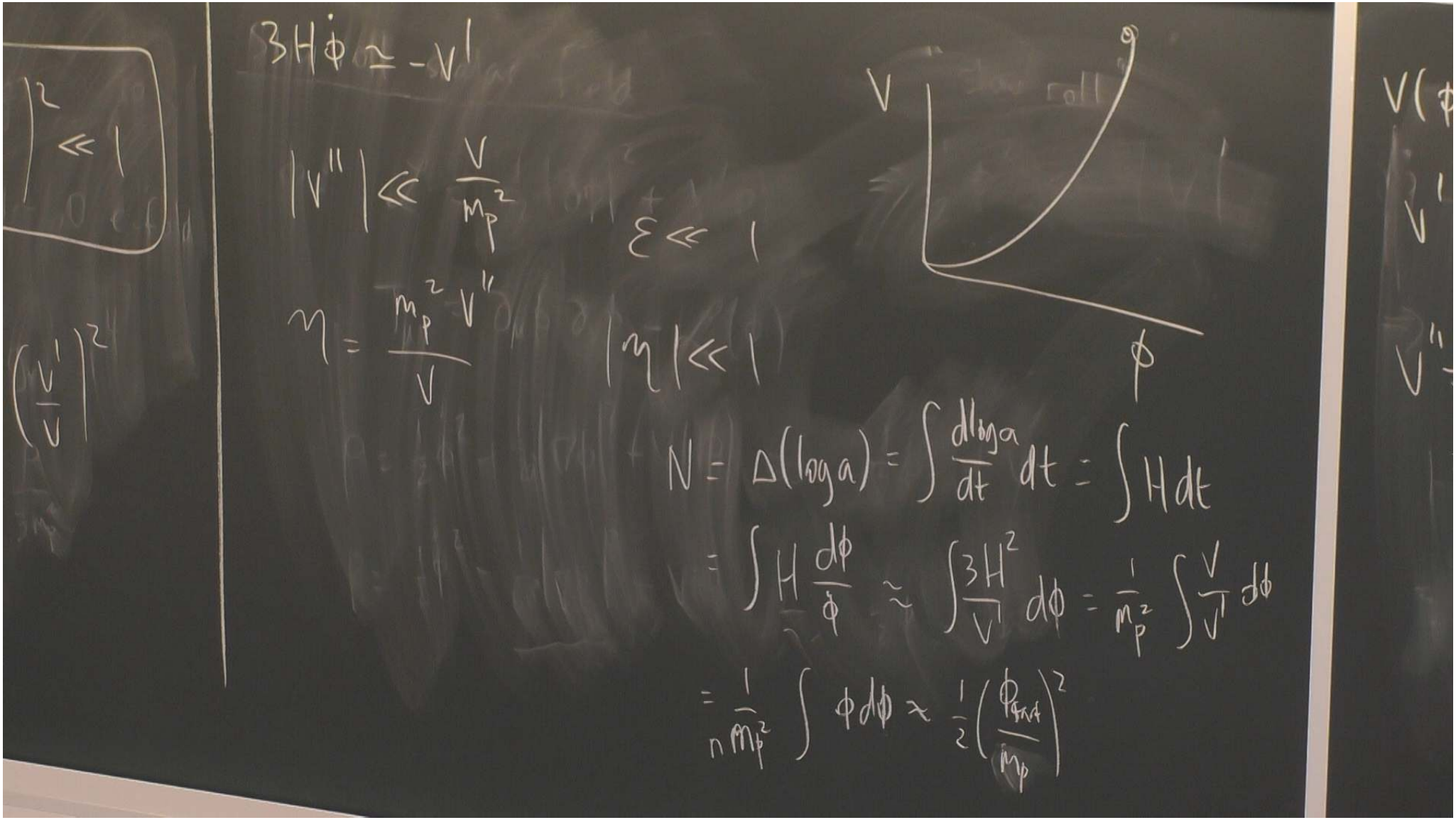
$|V''| \ll \frac{V}{M_p^2}$ $\quad \varepsilon \ll 1$

$\eta = \frac{M_p^2 V''}{V}$ $\quad |\eta| \ll 1$

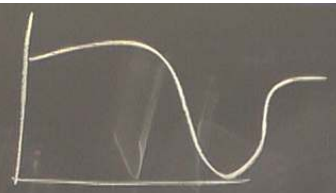


$N = \Delta(\log a) = \int \frac{d\log a}{dt} dt = \int H dt$

$= \int H \frac{d\phi}{\dot{\phi}} \simeq \int \frac{3H^2}{V'} d\phi = \frac{1}{M_p^2} \int \frac{V}{V'} d\phi$


$3H\dot{\phi} \simeq -V'$
$|V''| \ll \frac{V}{m_p^2}$
$\eta = \frac{m_p^2 V''}{V}$
$\varepsilon \ll 1$
$|\eta| \ll 1$
$\left(\frac{V'}{V}\right)^2$
$N = \Delta(\log a) = \int \frac{d\log a}{dt} dt = \int H dt$
$= \int H \frac{d\phi}{\dot{\phi}} \simeq \int \frac{3H^2}{V'} d\phi = \frac{1}{m_p^2} \int \frac{V}{V'} d\phi$
$= \frac{1}{n m_p^2} \int \phi d\phi \simeq \frac{1}{2}\left(\frac{\phi_{end}}{m_p}\right)^2$
V
roll
ϕ

$V(\phi) = c\,\phi^n$



$V' = cn\phi^{n-1} = \frac{nV}{\phi}$

$\Rightarrow \varepsilon = \frac{1}{2}\left(\frac{n m_p}{\phi}\right)^2$

$V'' = cn(n-1)\phi^{n-2}$

$= n(n-1)\frac{V}{\phi^2}$

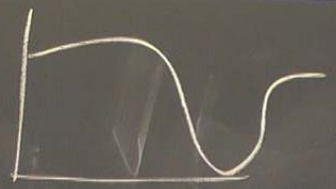
$\eta = n(n-1)\frac{m_p^2}{\phi^2}$ $\Rightarrow \phi$

$V(\phi) = c\,\phi^n$

$V' = cn\phi^{n-1} = \frac{nV}{\phi}$

$V'' = cn(n-1)\phi^{n-2}$

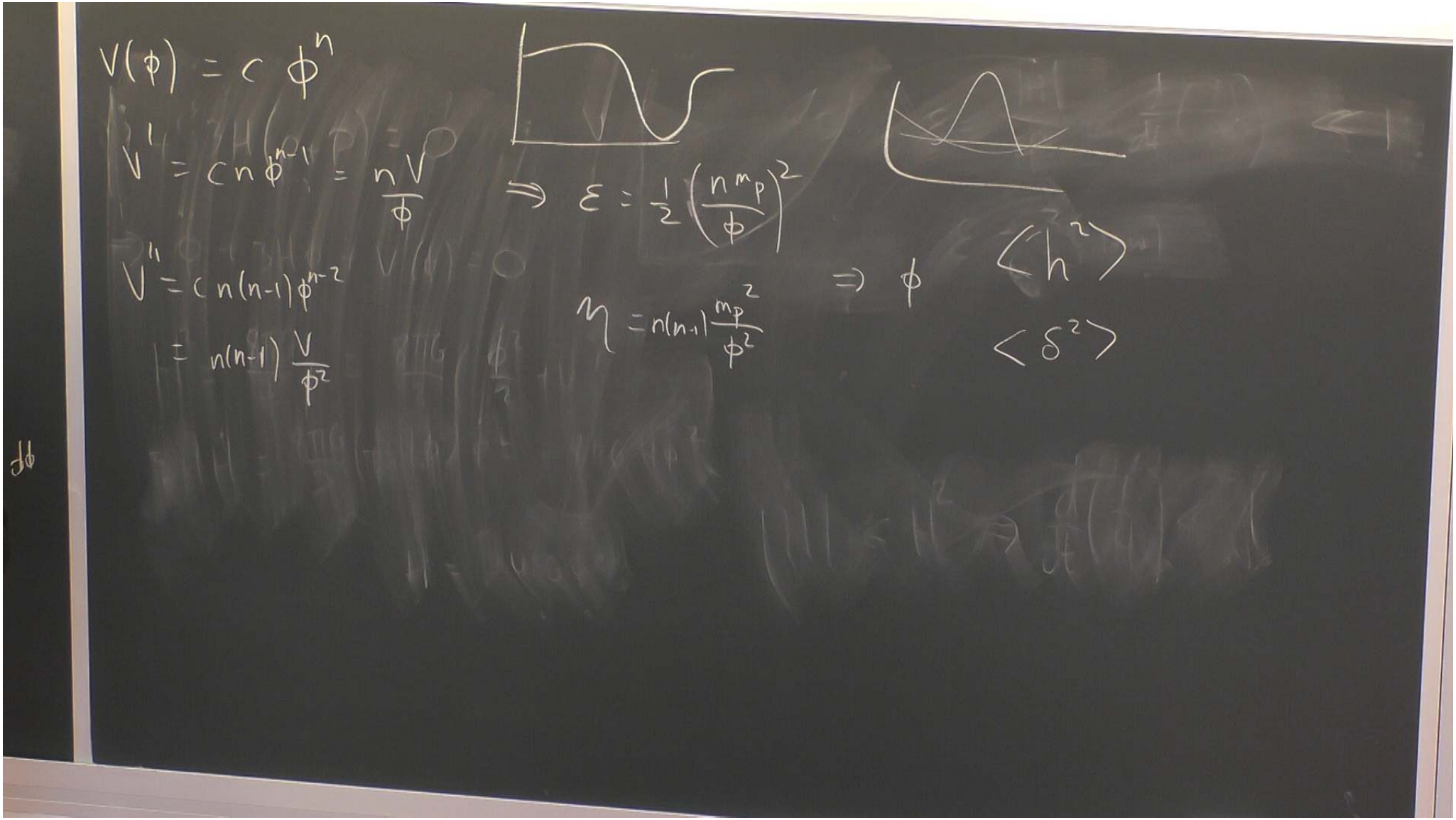
$= n(n-1)\frac{V}{\phi^2}$



$\Rightarrow \varepsilon = \frac{1}{2}\left(\frac{n m_p}{\phi}\right)^2$

$\eta = n(n-1)\frac{m_p^2}{\phi^2}$

$\Rightarrow \phi \quad \langle h^2 \rangle \quad \langle \delta^2 \rangle$


$V(\phi) = c\,\phi^n$
$V' = cn\phi^{n-1} = \frac{nV}{\phi}$
$V'' = cn(n-1)\phi^{n-2}$
$= n(n-1)\frac{V}{\phi^2}$
$\Rightarrow \varepsilon = \frac{1}{2}\left(\frac{n m_p}{\phi}\right)^2$
$\eta = n(n-1)\frac{m_p^2}{\phi^2}$
$\Rightarrow \phi$
$\langle h^2 \rangle$
$\langle \delta^2 \rangle$
$d\phi$