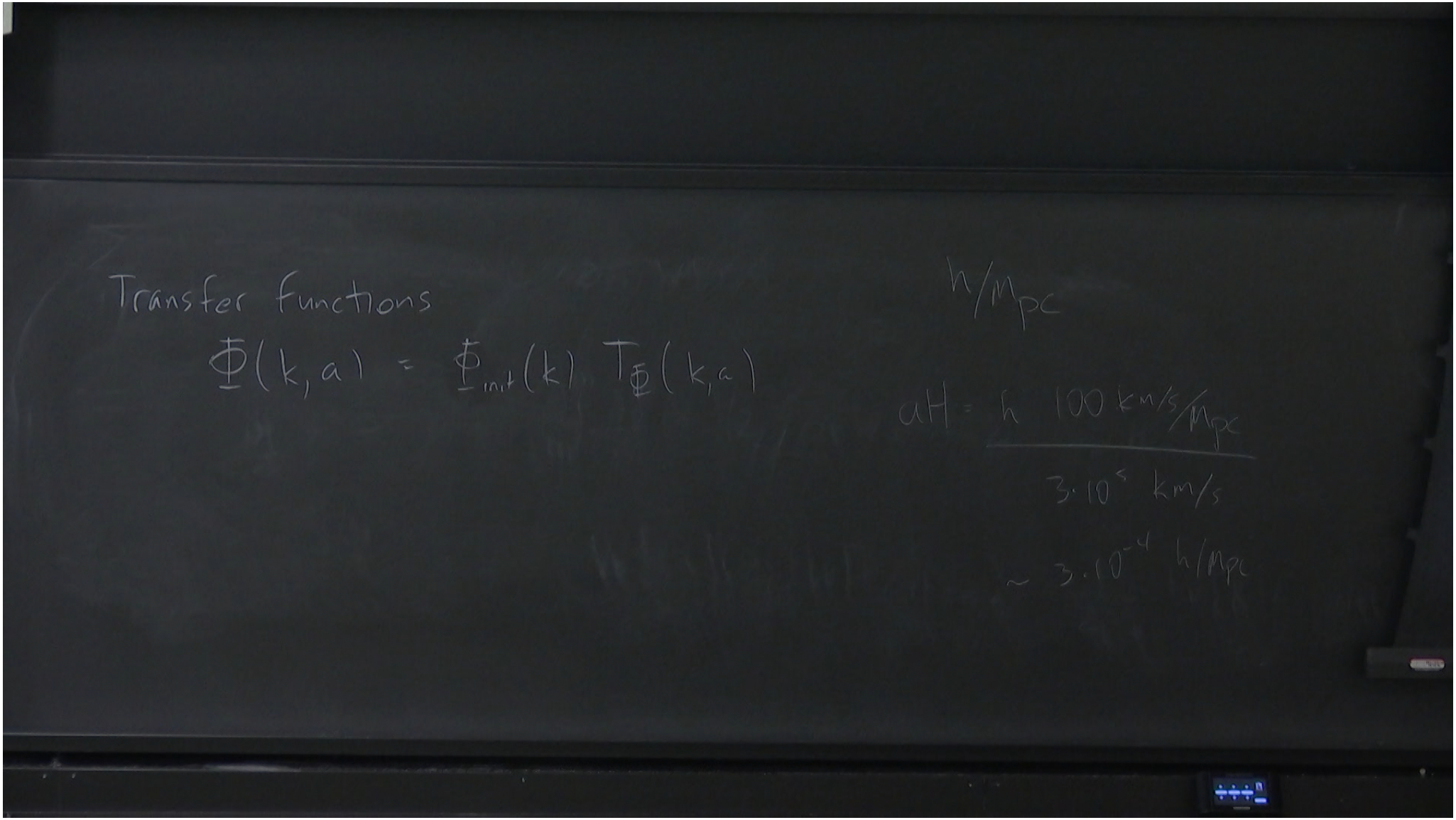
Title: Cosmology Lecture (230505)

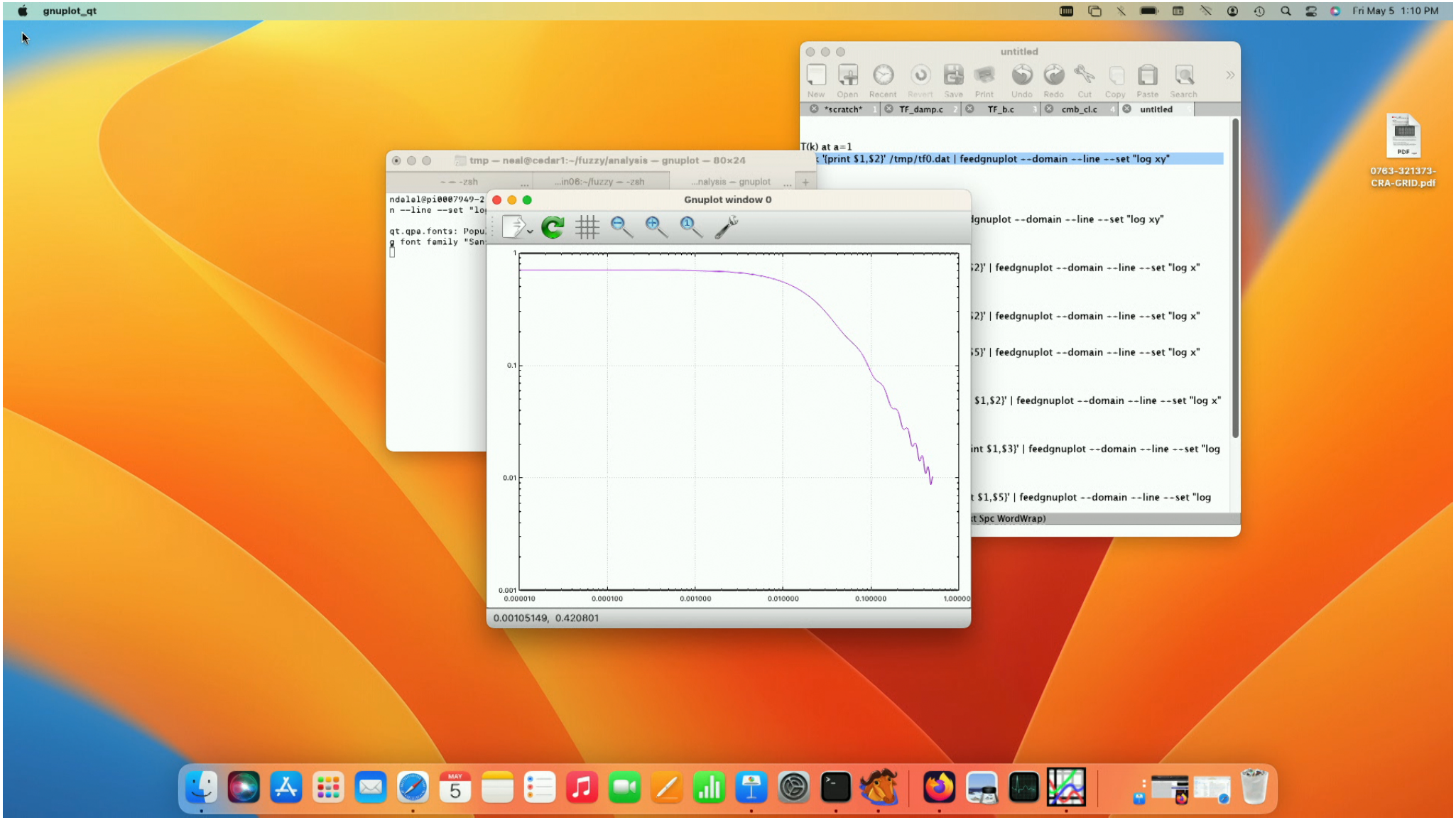
Speakers: Neal Dalal

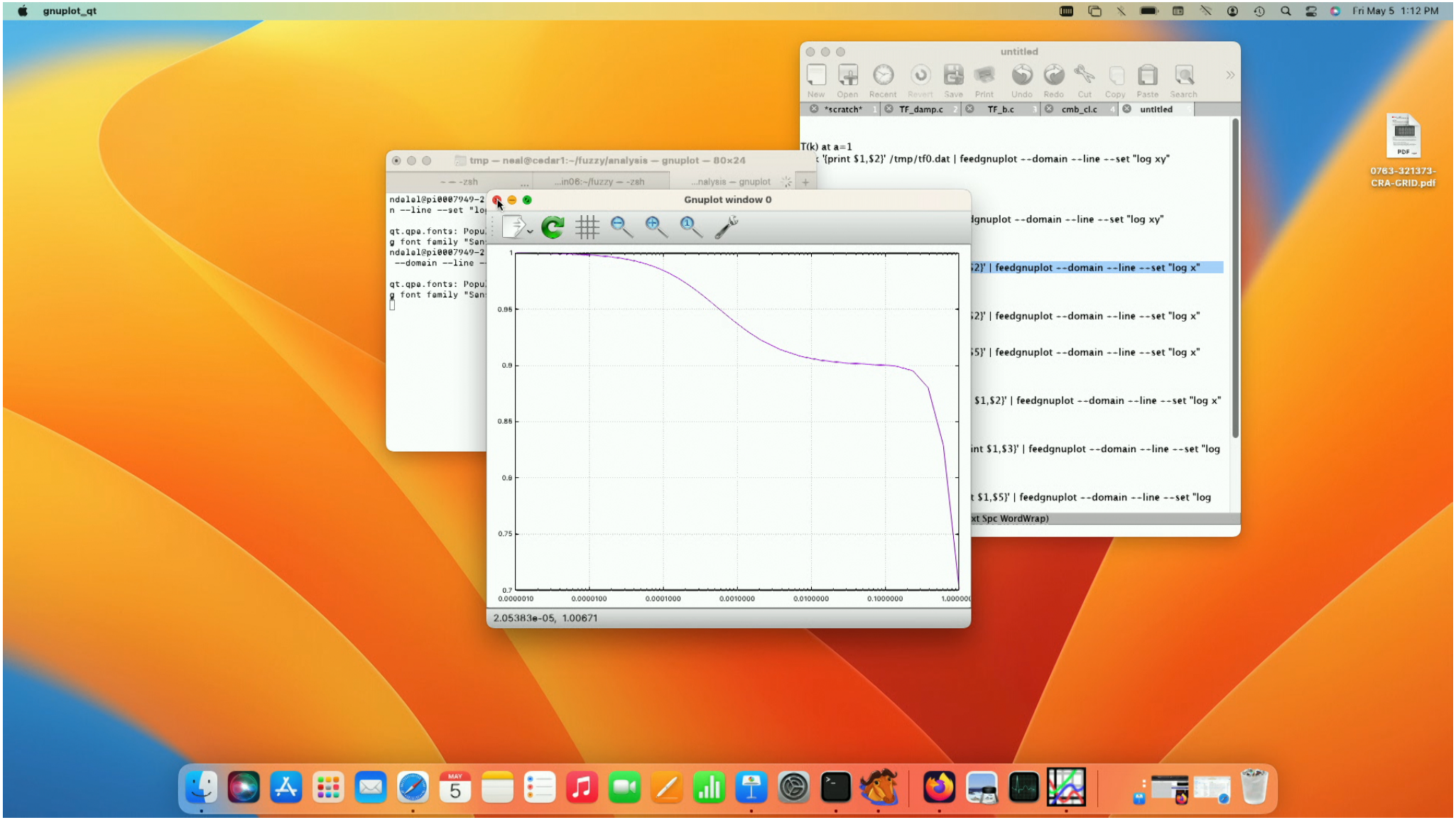
Collection: Cosmology (2022/2023)

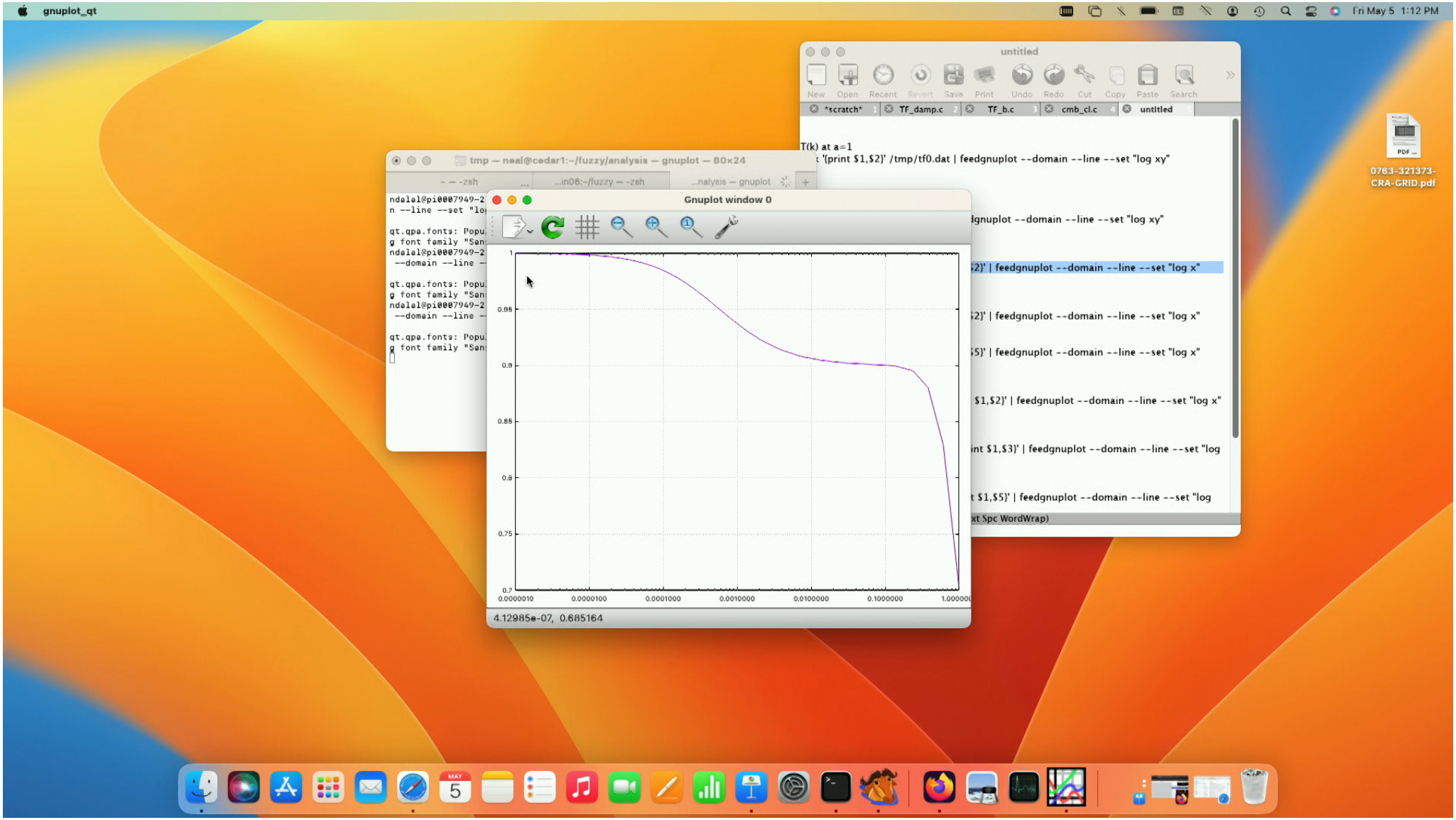
Date: May 05, 2023 - 1:00 PM

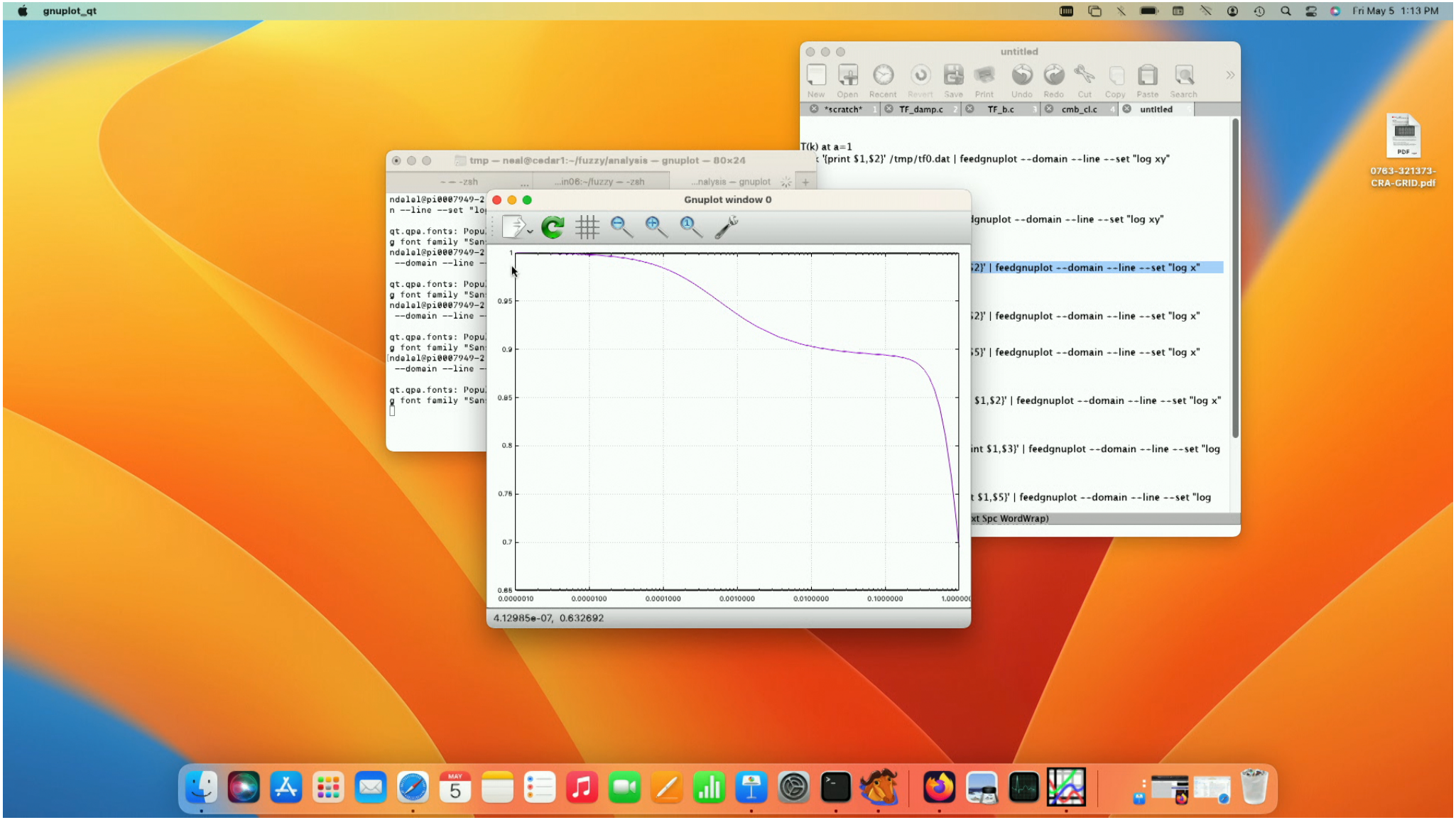
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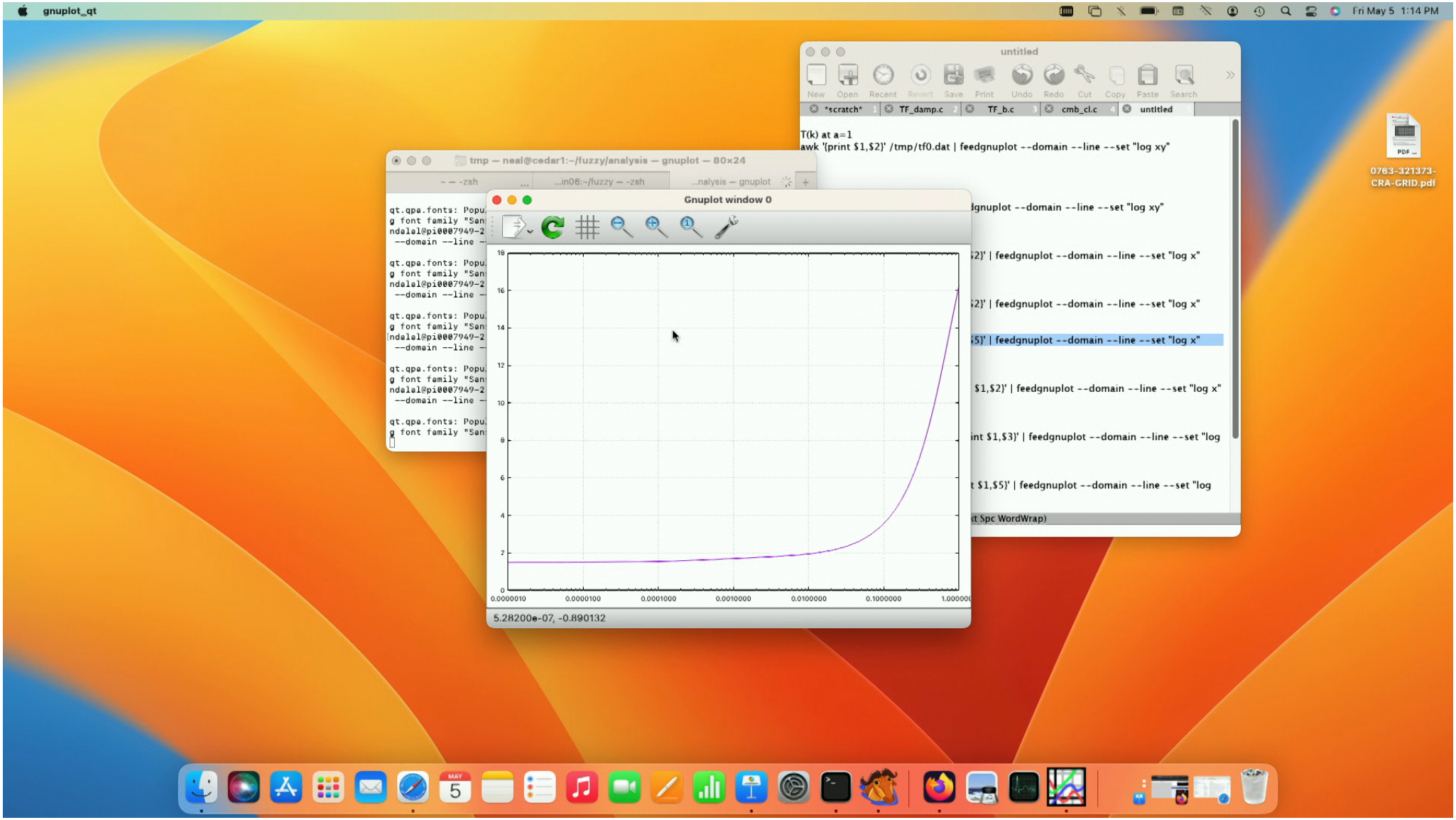

Transfer functions
$\Phi(k,a) = \Phi_{init}(k)\, T_\Phi(k,a)$
h/Mpc
$aH = \frac{h\ 100\ km/s/Mpc}{3\cdot 10^5\ km/s}$
$\sim 3\cdot 10^{-4}\ h/Mpc$

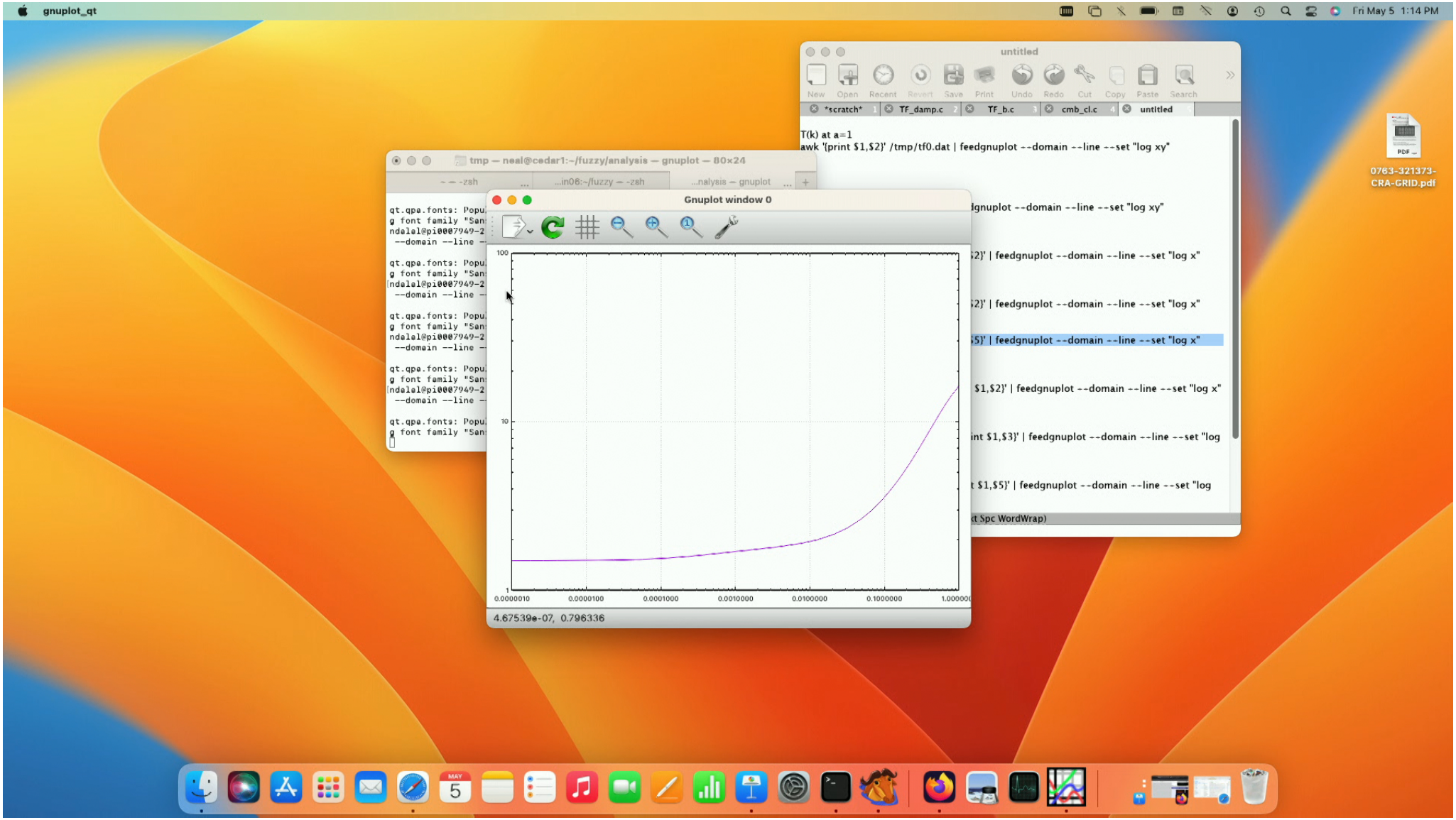

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T(k) at a=1
'{print $1,$2}' /tmp/tf0.dat | feedgnuplot --domain --line --set "log xy"
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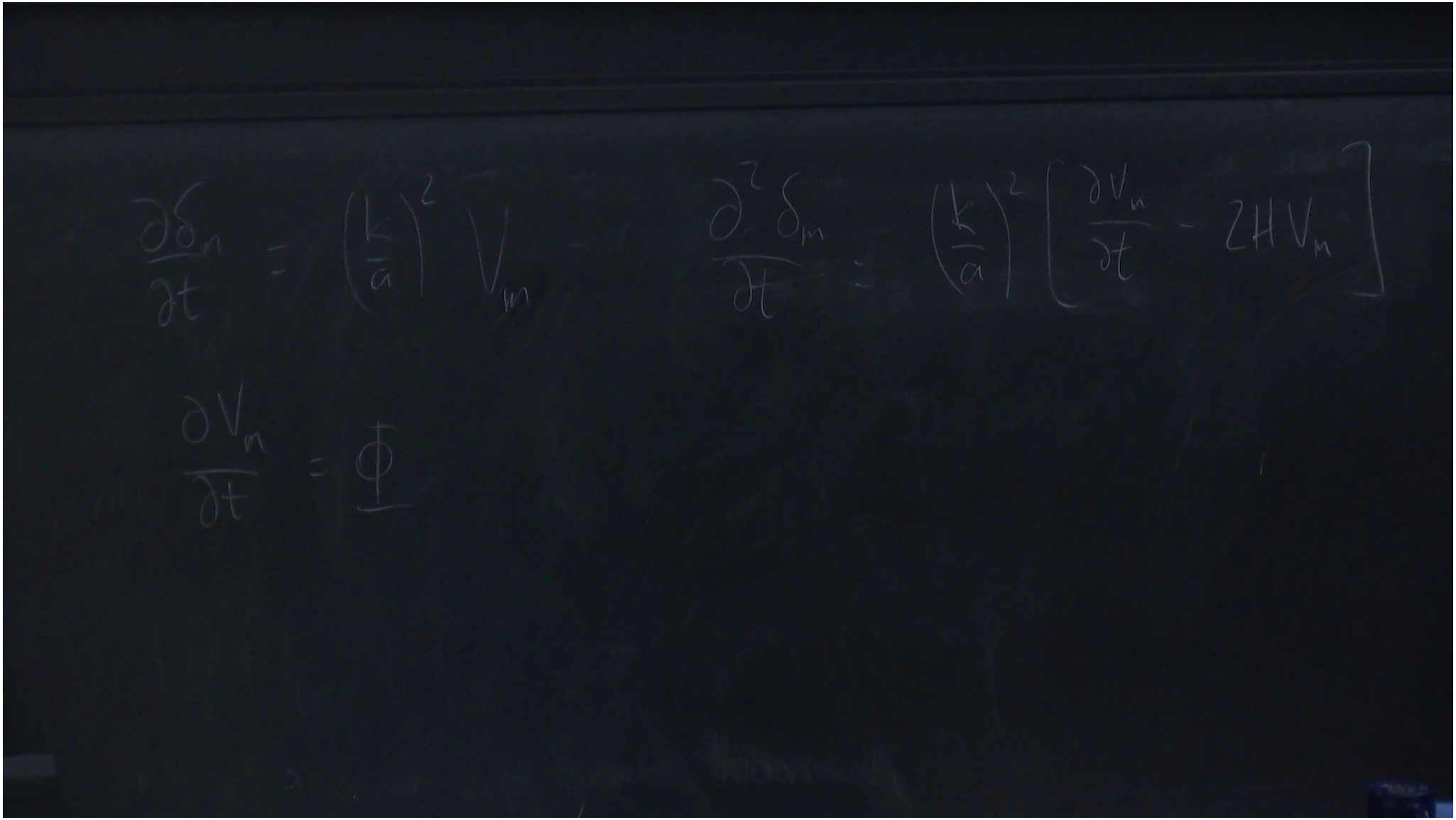

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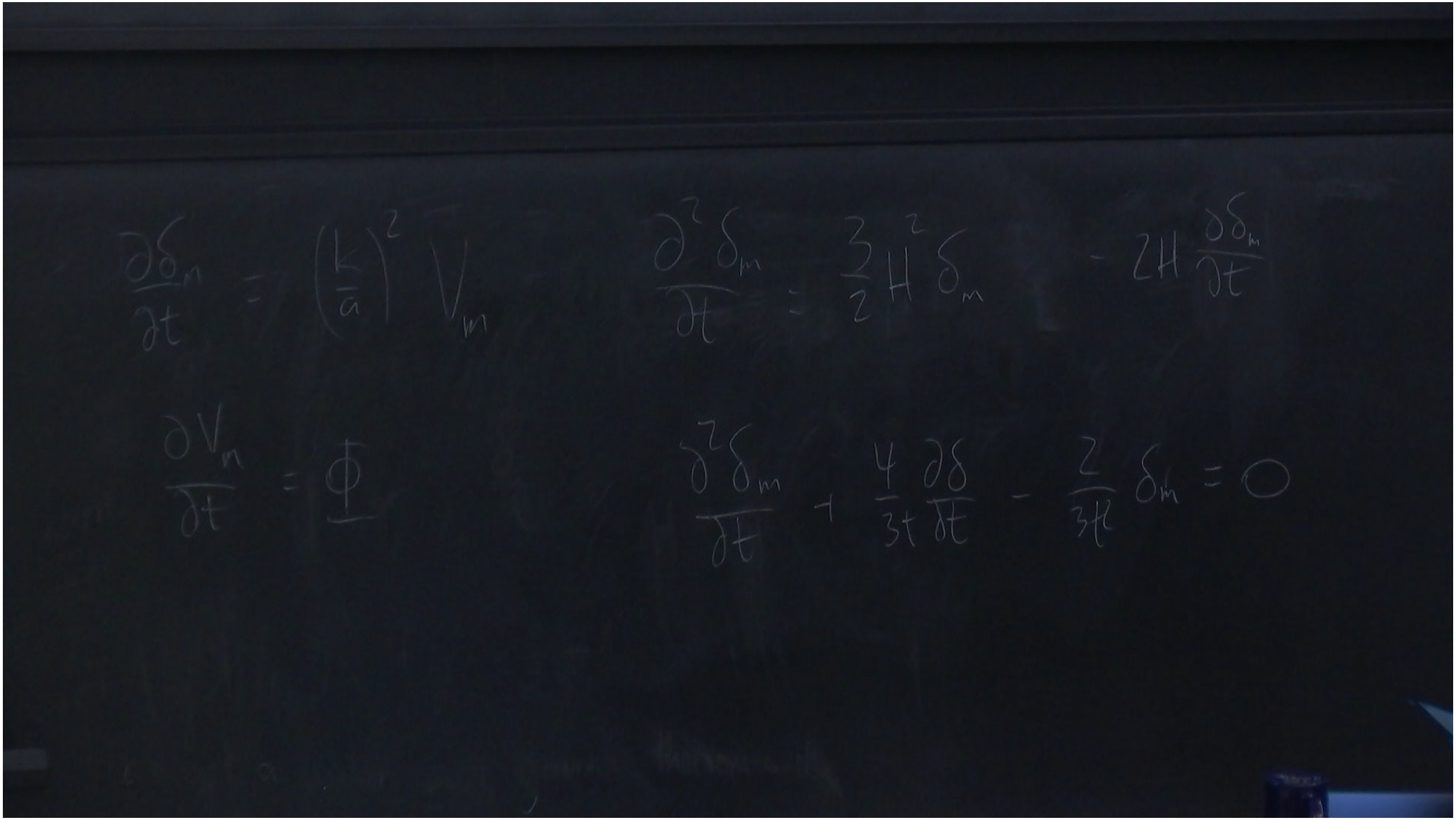

$\frac{\partial \delta_m}{\partial t} = \left(\frac{k}{a}\right)^2 V_m$
$\frac{\partial^2 \delta_m}{\partial t} = \left(\frac{k}{a}\right)^2 \left[\frac{\partial V_m}{\partial t} - 2HV_m\right]$
$\frac{\partial V_m}{\partial t} = \Phi$

$$\frac{\partial \delta_m}{\partial t} = \left(\frac{k}{a}\right)^2 V_m$$

$$\frac{\partial V_m}{\partial t} = \Phi$$

$$\frac{\partial^2 \delta_m}{\partial t} = \frac{3}{2} H^2 \delta_m - 2H \frac{\partial \delta_m}{\partial t}$$

$$\frac{\partial^2 \delta_m}{\partial t} + \frac{4}{3t} \frac{\partial \delta}{\partial t} - \frac{2}{3t^2} \delta_m = 0$$


$\frac{\partial \delta_m}{\partial t} = \left(\frac{k}{a}\right)^2 V_m$
$\frac{\partial^2 \delta_m}{\partial t} = \frac{3}{2} H^2 \delta_m - 2H \frac{\partial \delta_m}{\partial t}$
$\frac{\partial V_m}{\partial t} = \Phi$
$\frac{\partial^2 \delta_m}{\partial t} + \frac{4}{3t} \frac{\partial \delta}{\partial t} - \frac{2}{3t^2} \delta_m = 0$

$$\frac{\partial^2 \delta_m}{\partial t^2} = \frac{3}{2} H^2 \delta_m - 2H \frac{\partial \delta_m}{\partial t}$$

$$\delta \propto t^n$$

$$\delta \propto t^{2/3} \propto a$$

$$t^{-1}$$

$$\frac{\partial^2 \delta_m}{\partial t^2} + \frac{4}{3t} \frac{\partial \delta}{\partial t} - \frac{2}{3t^2} \delta_m = 0$$

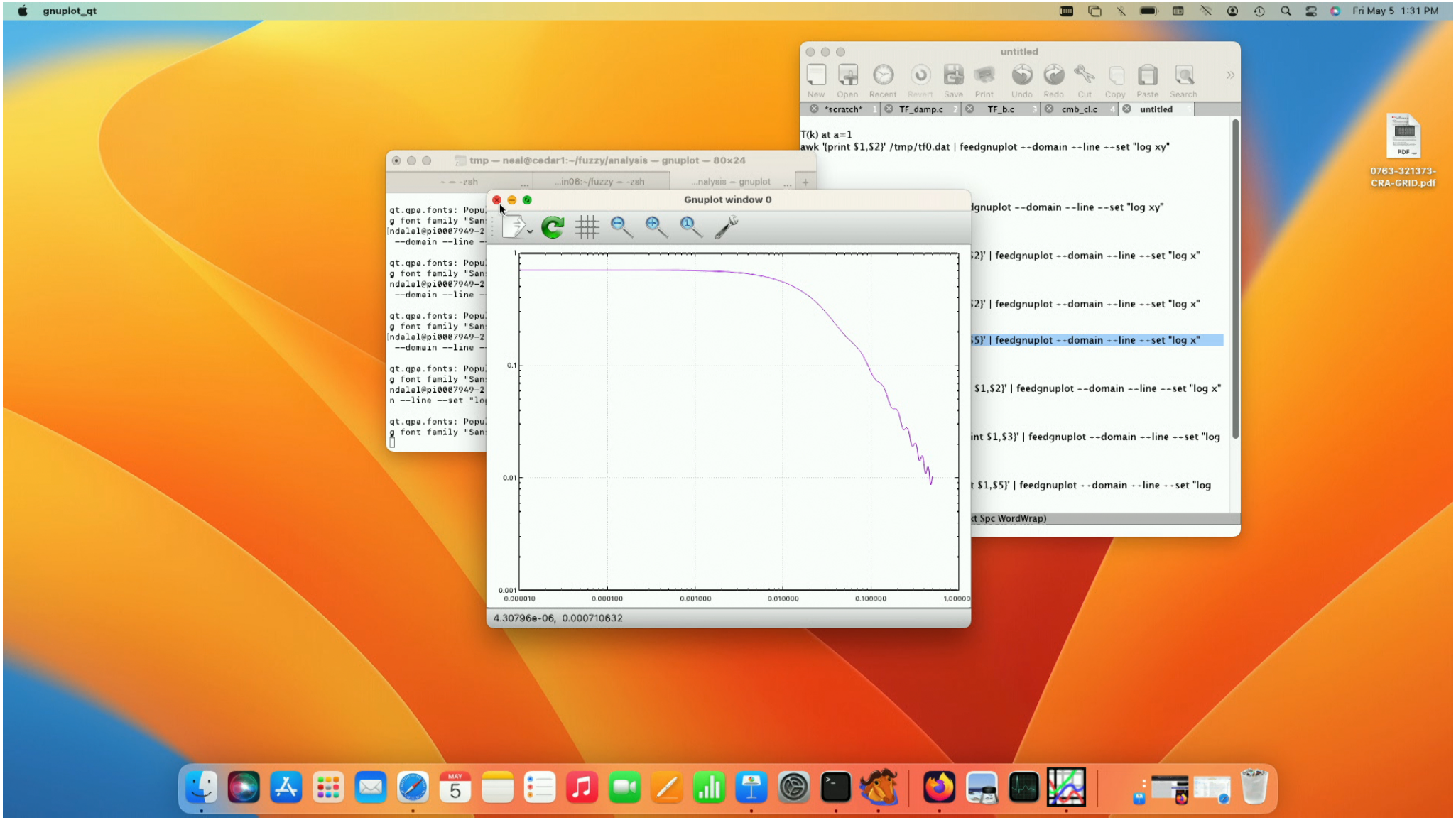
$$n(n-1) + \frac{4n}{3} + \frac{2}{3} = 0 = \left(n - \frac{2}{3}\right)(n+1) = 0$$

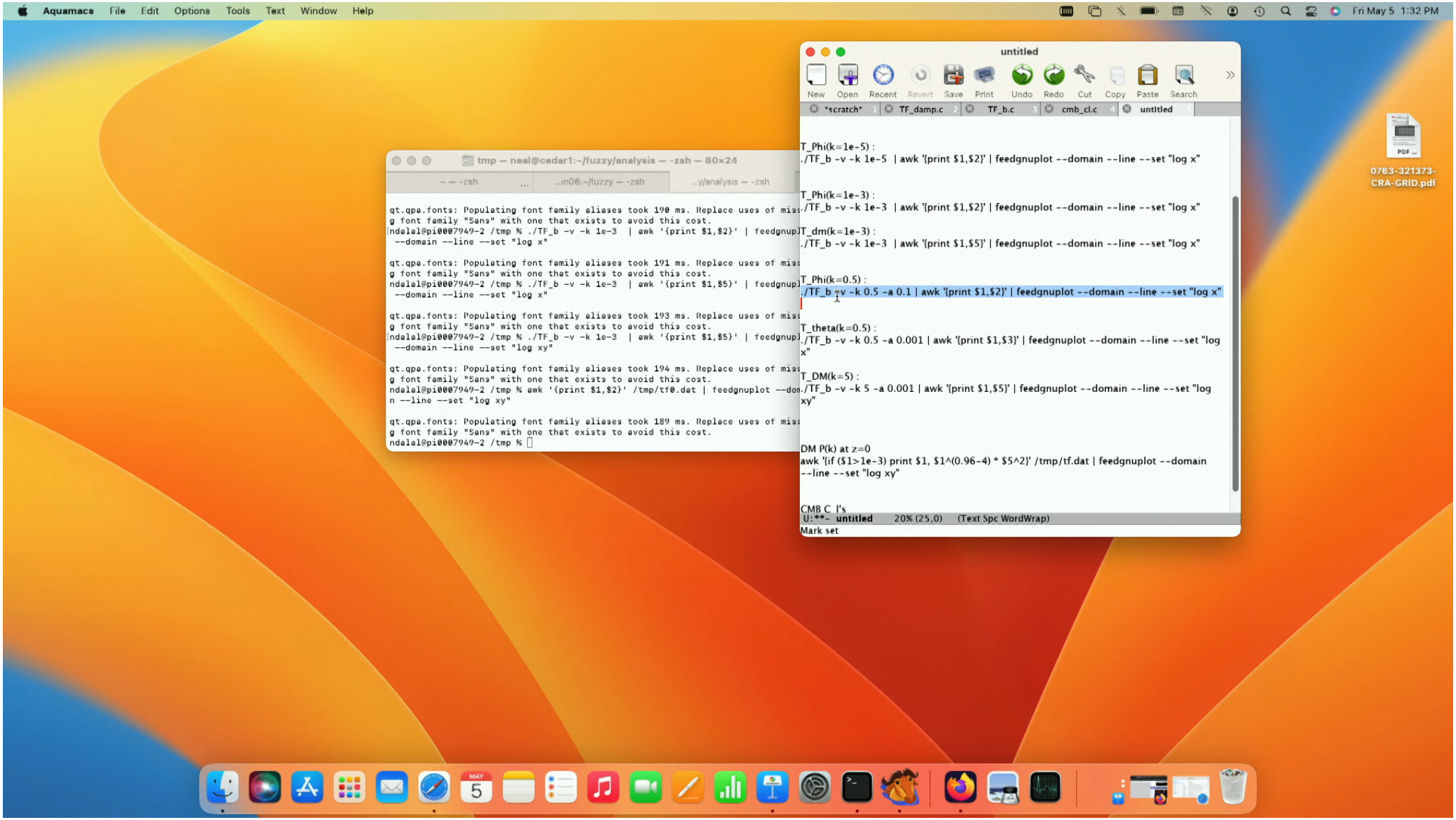
$$T_m(k, a) = T_m(k, a=1) \times D(a)$$

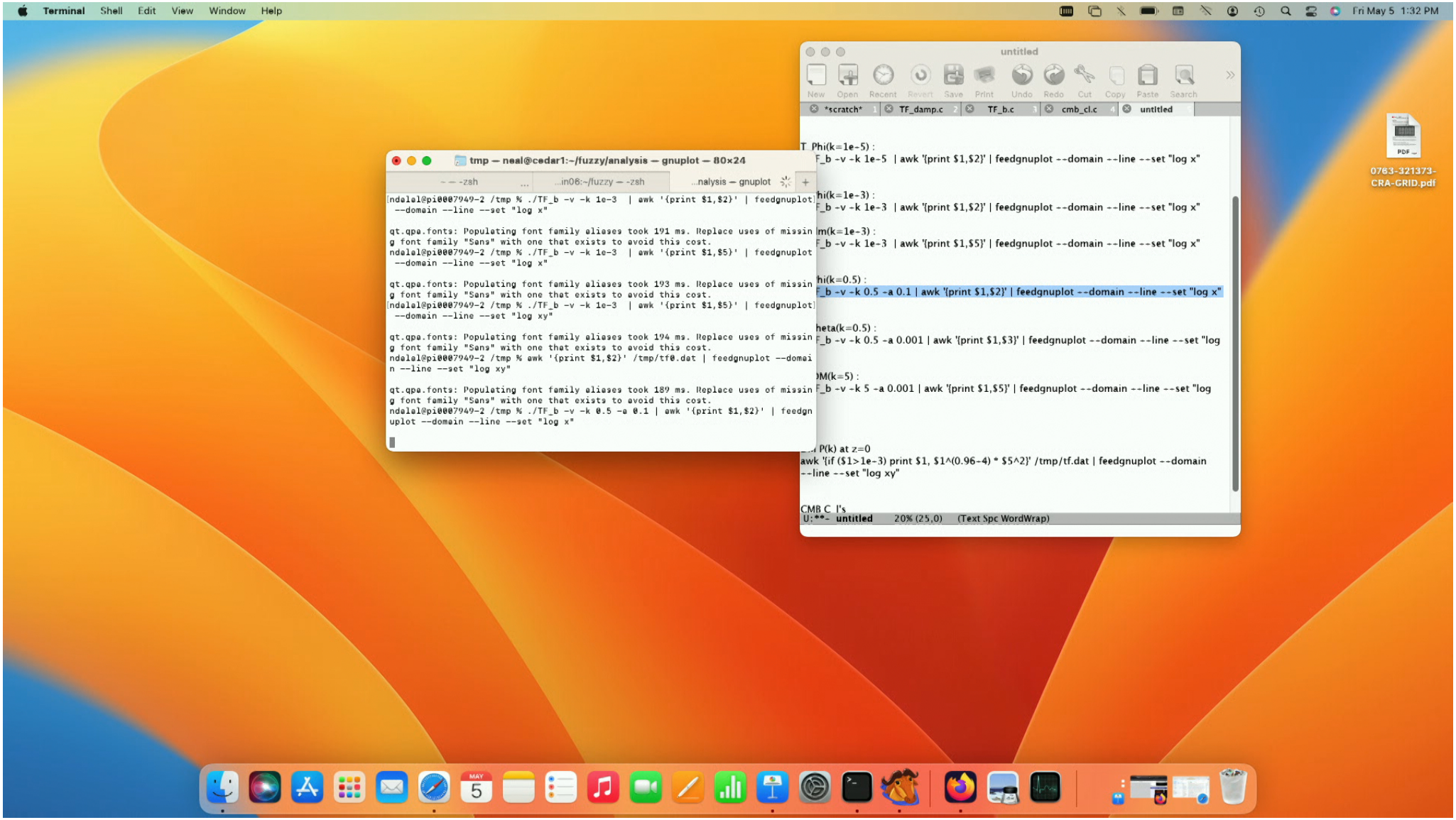
linear growth factor

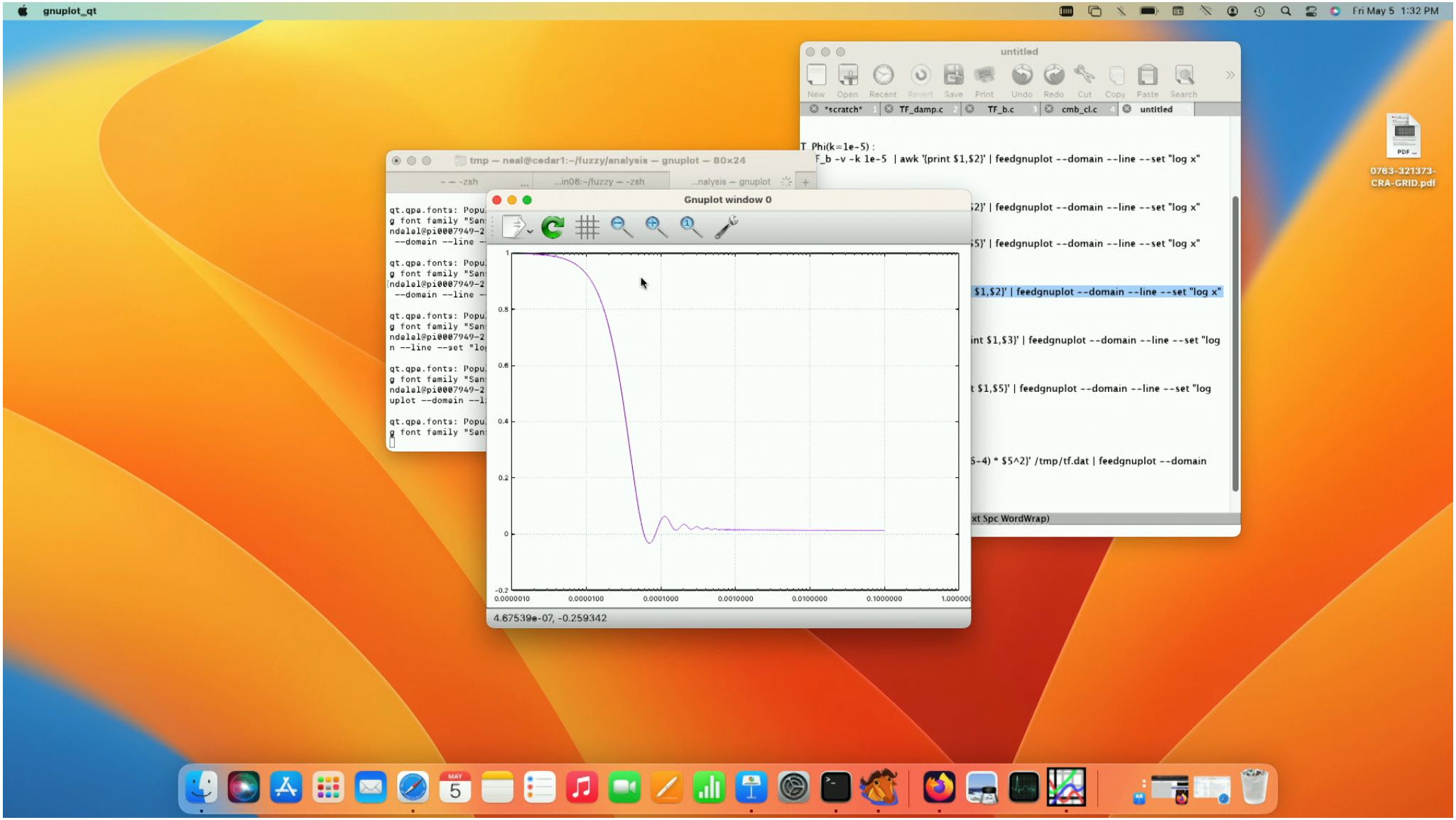
$$\frac{\partial^2 \delta}{\partial t} + \frac{4}{3t} \frac{\partial \delta}{\partial t} = 0$$

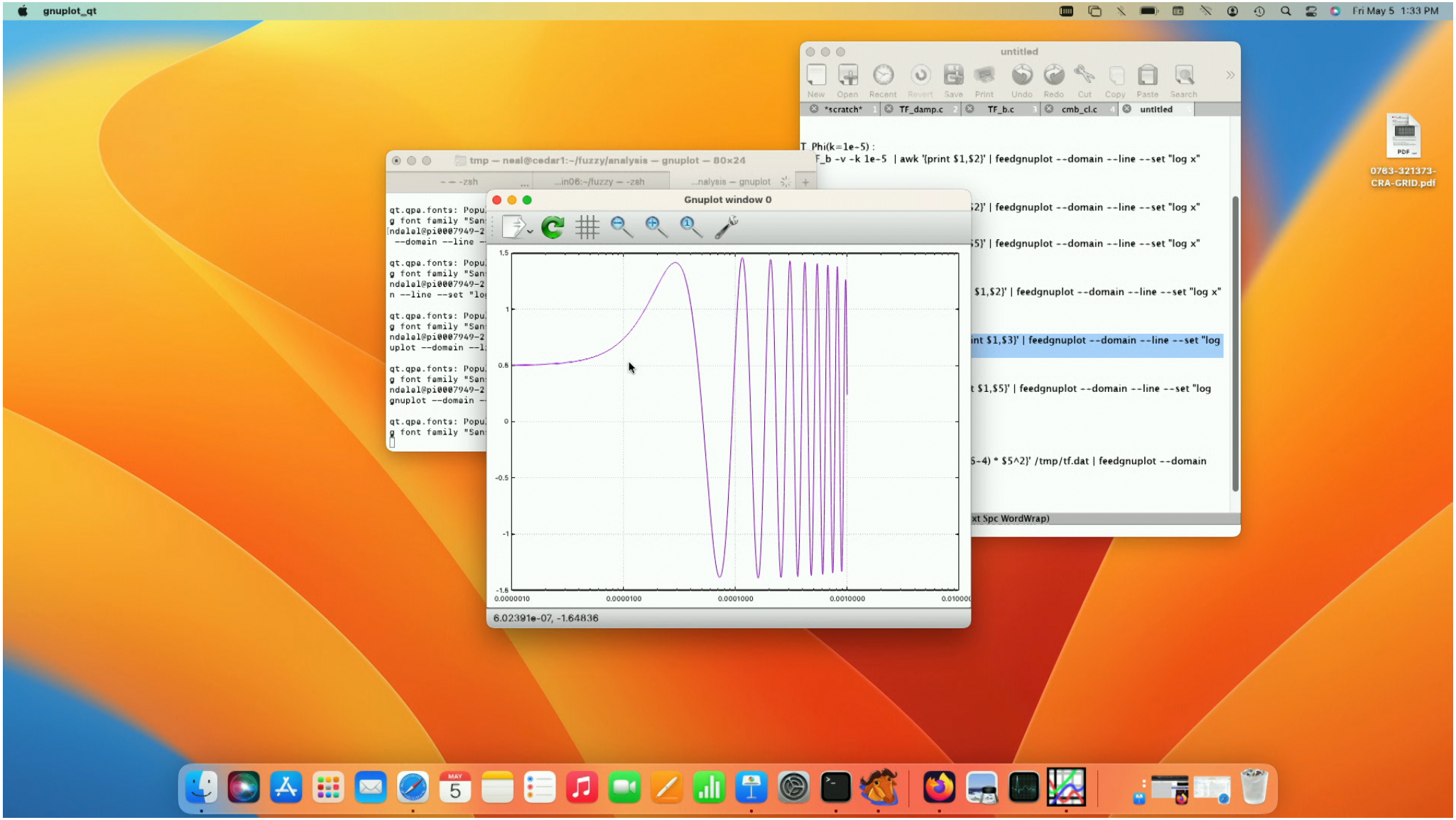
$$\delta_1 - \delta_b$$

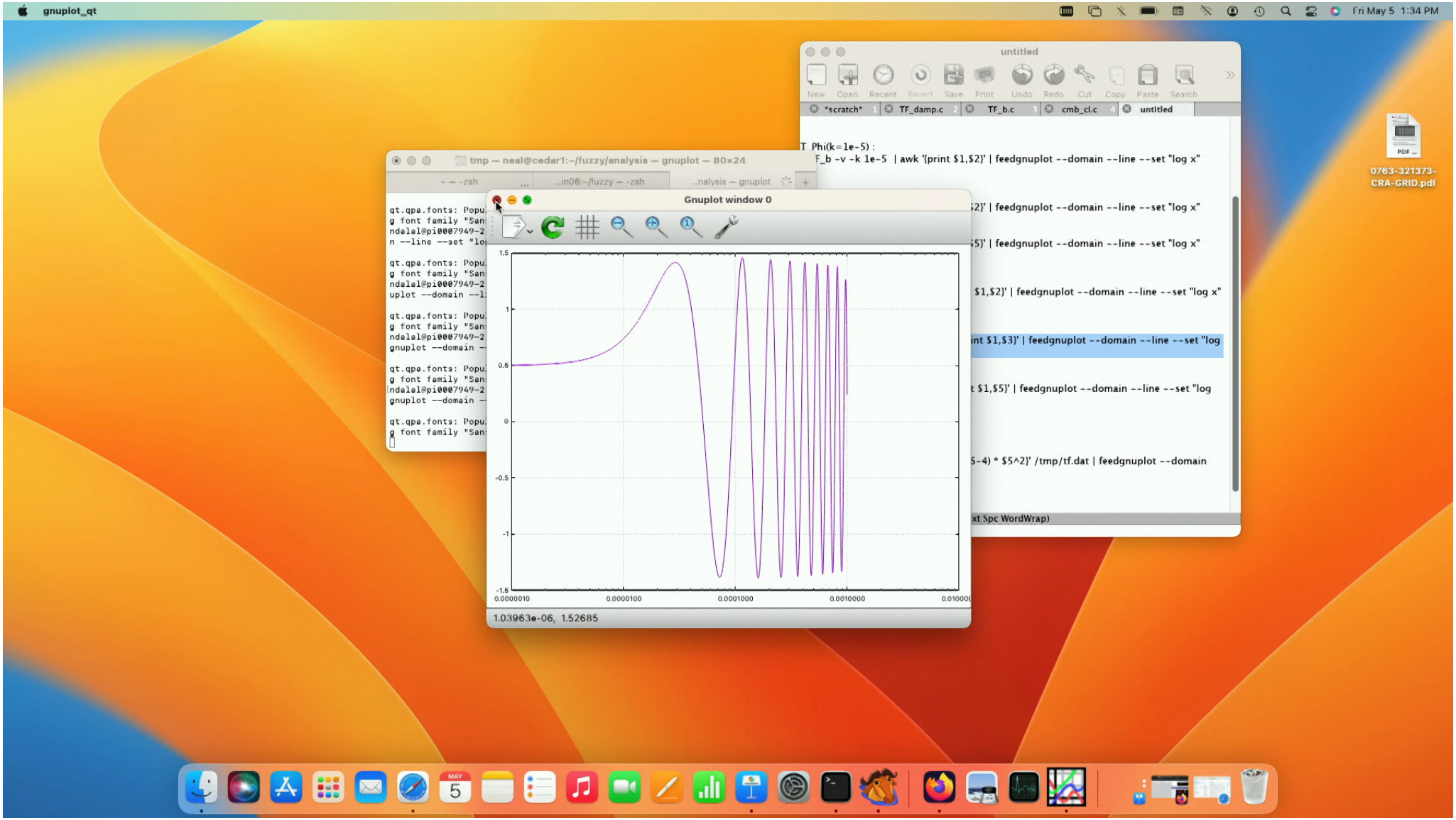

gnuplot_qt
Fri May 5 1:31 PM
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T(k) at a=1
awk '{print $1,$2}' /tmp/tf0.dat | feedgnuplot --domain --line --set "log xy"
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Gnuplot window 0
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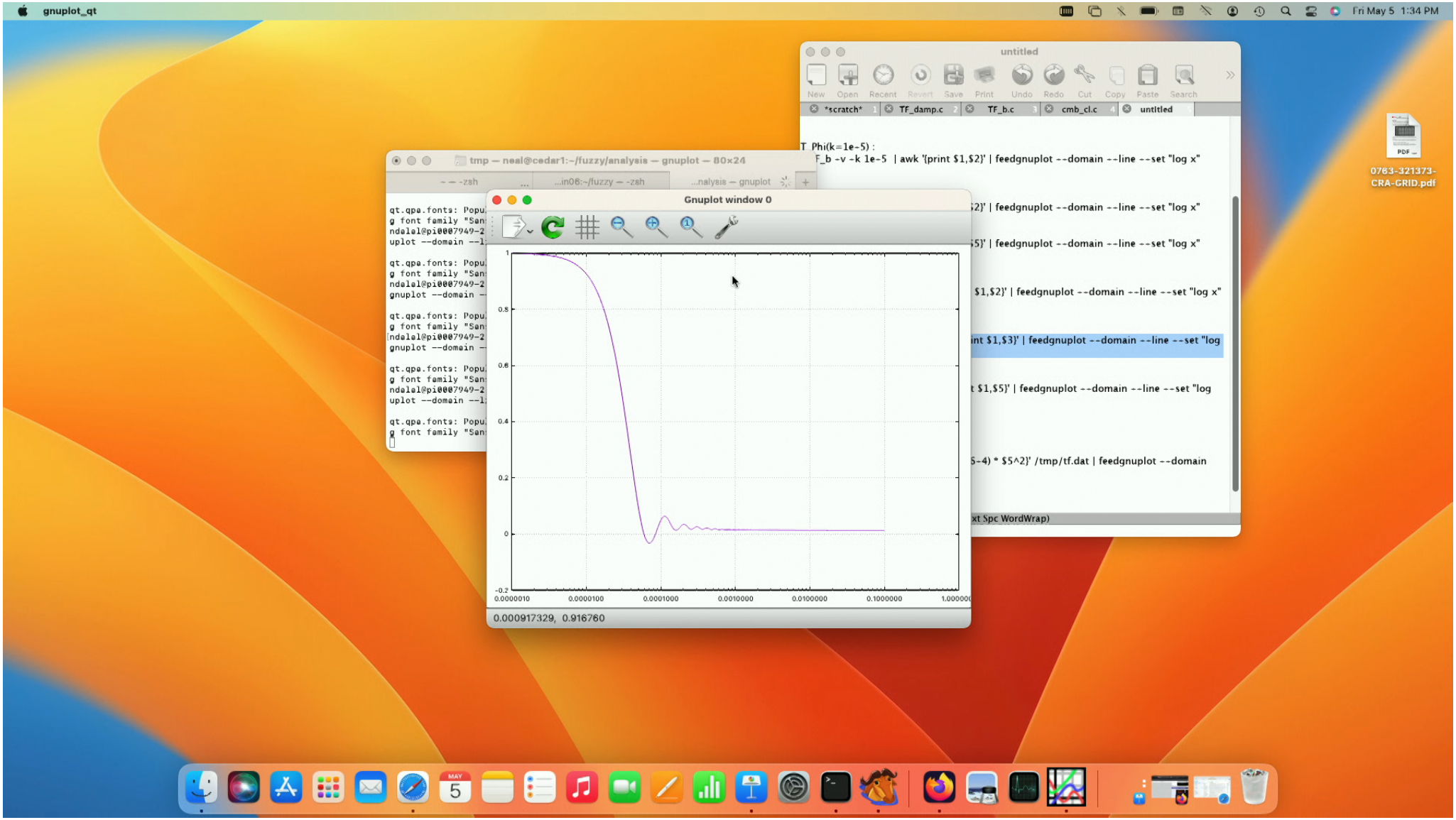

Aquamacs File Edit Options Tools Text Window Help
Fri May 5 1:32 PM
tmp — neal@cedar1:~/fuzzy/analysis — -zsh — 80×24
qt.qpa.fonts: Populating font family aliases took 190 ms. Replace uses of missing font family "Sans" with one that exists to avoid this cost.
ndalal@pi0007949-2 /tmp % ./TF_b -v -k 1e-3 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
qt.qpa.fonts: Populating font family aliases took 191 ms. Replace uses of missing font family "Sans" with one that exists to avoid this cost.
ndalal@pi0007949-2 /tmp % ./TF_b -v -k 1e-3 | awk '{print $1,$5}' | feedgnuplot --domain --line --set "log x"
qt.qpa.fonts: Populating font family aliases took 193 ms. Replace uses of missing font family "Sans" with one that exists to avoid this cost.
ndalal@pi0007949-2 /tmp % ./TF_b -v -k 1e-3 | awk '{print $1,$5}' | feedgnuplot --domain --line --set "log xy"
qt.qpa.fonts: Populating font family aliases took 194 ms. Replace uses of missing font family "Sans" with one that exists to avoid this cost.
ndalal@pi0007949-2 /tmp % awk '{print $1,$2}' /tmp/tf0.dat | feedgnuplot --domain --line --set "log xy"
qt.qpa.fonts: Populating font family aliases took 189 ms. Replace uses of missing font family "Sans" with one that exists to avoid this cost.
ndalal@pi0007949-2 /tmp %
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T_Phi(k=1e-5) :
./TF_b -v -k 1e-5 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
T_Phi(k=1e-3) :
./TF_b -v -k 1e-3 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
T_dm(k=1e-3) :
./TF_b -v -k 1e-3 | awk '{print $1,$5}' | feedgnuplot --domain --line --set "log x"
T_Phi(k=0.5) :
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T_theta(k=0.5) :
./TF_b -v -k 0.5 -a 0.001 | awk '{print $1,$3}' | feedgnuplot --domain --line --set "log x"
T_DM(k=5) :
./TF_b -v -k 5 -a 0.001 | awk '{print $1,$5}' | feedgnuplot --domain --line --set "log xy"
DM P(k) at z=0
awk '{if ($1>1e-3) print $1, $1^(0.96-4) * $5^2}' /tmp/tf.dat | feedgnuplot --domain --line --set "log xy"
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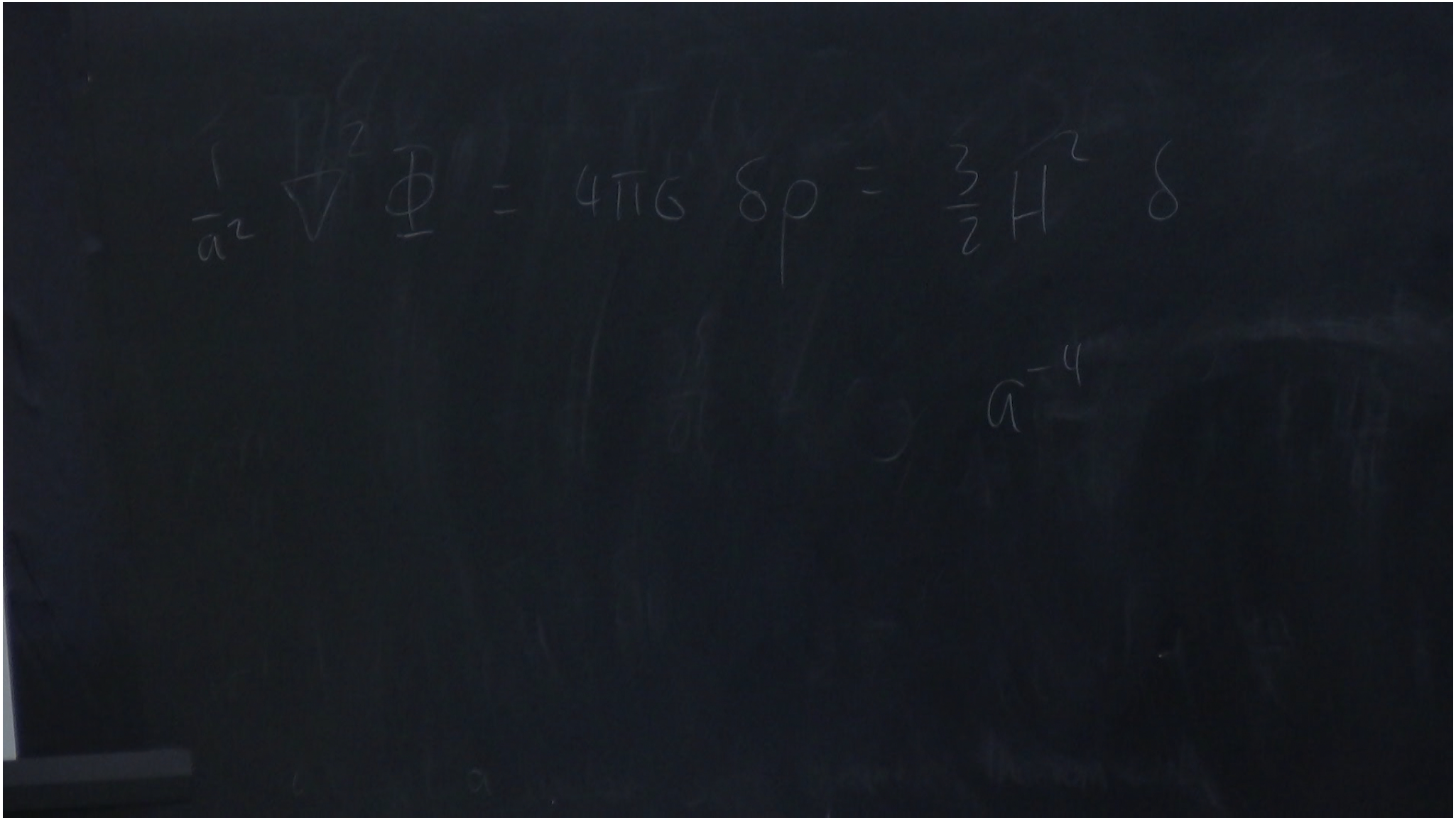

Terminal Shell Edit View Window Help
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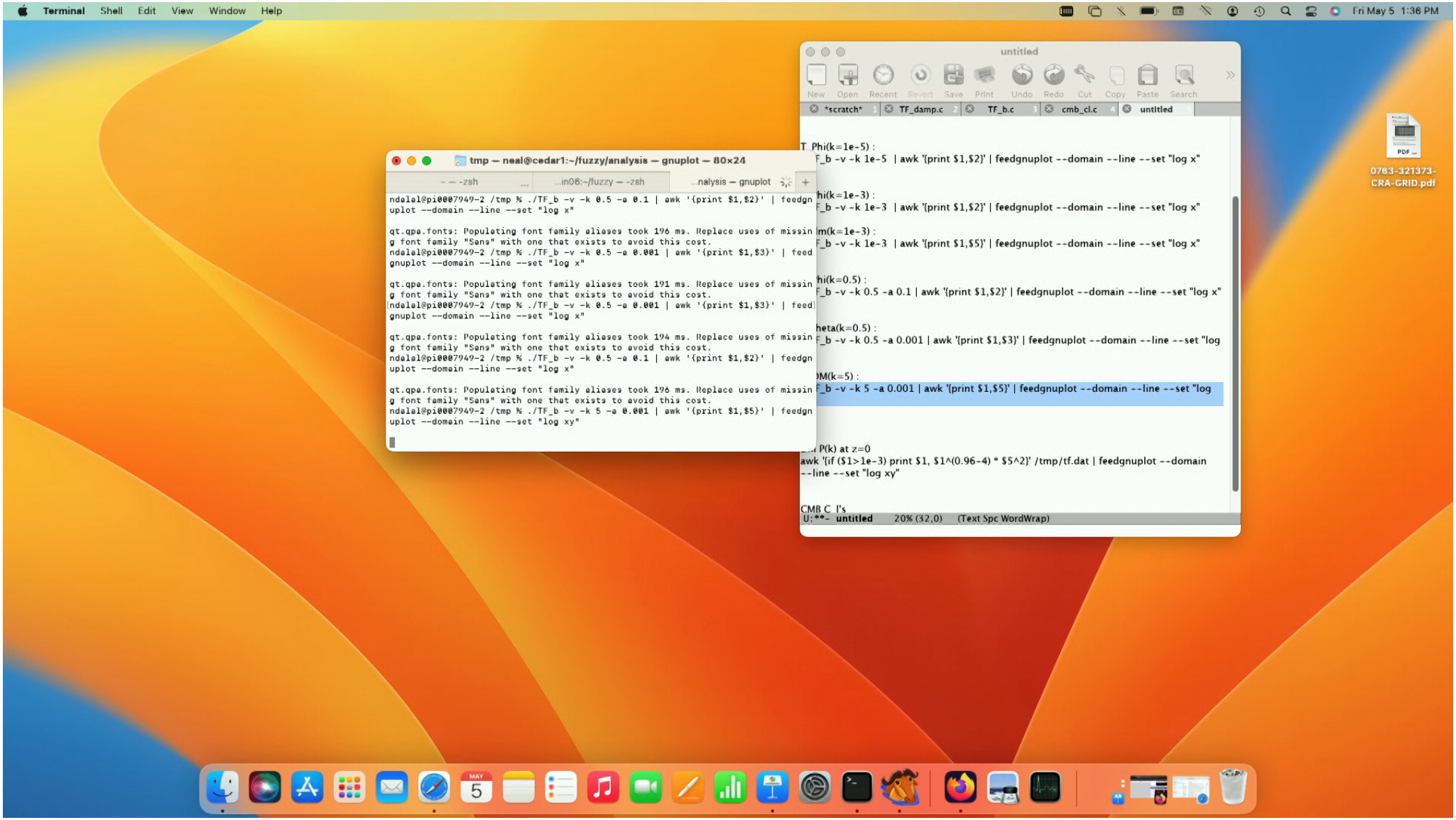

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T_Phi(k=1e-5) :
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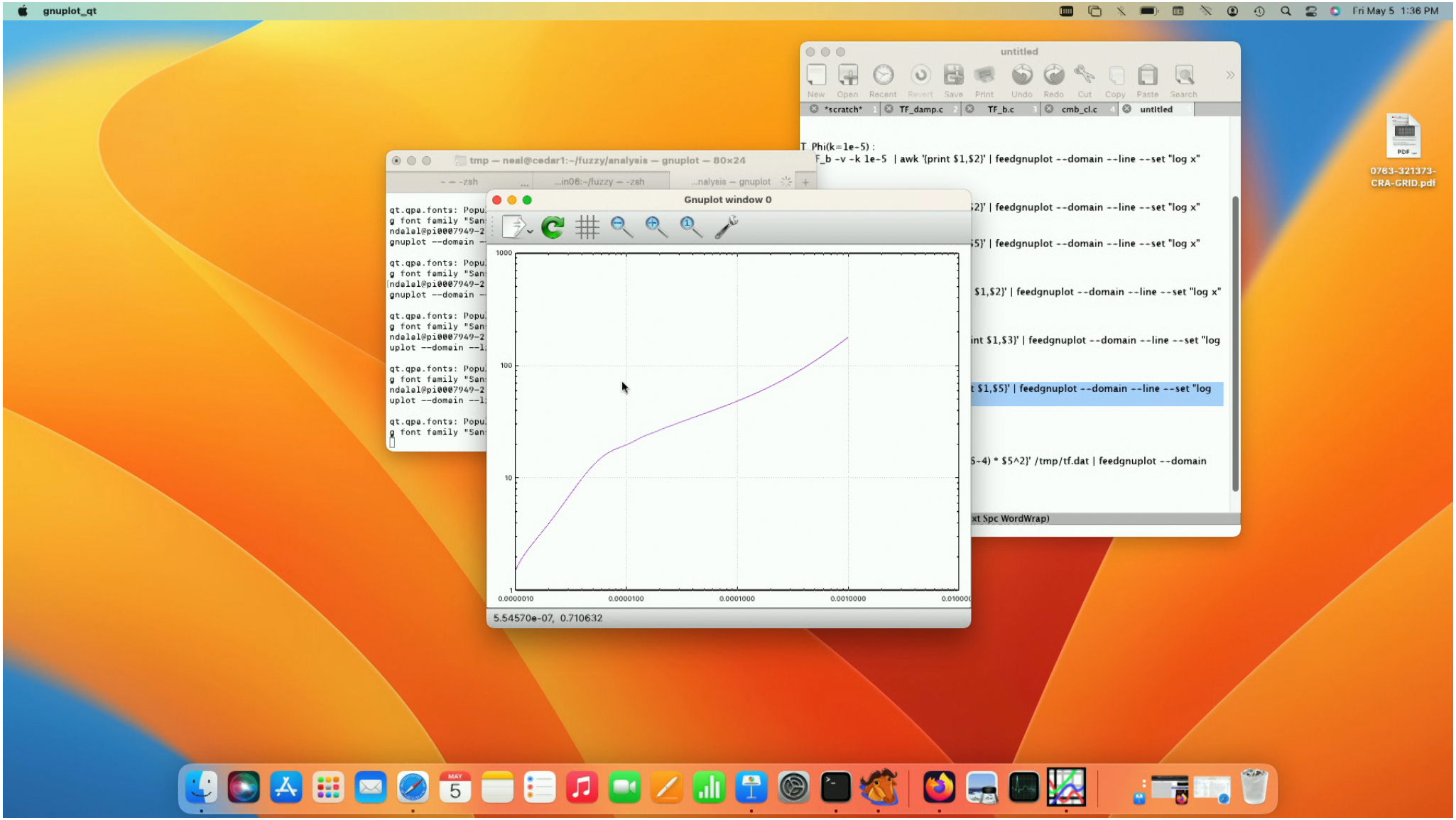

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F_b -v -k 1e-5 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
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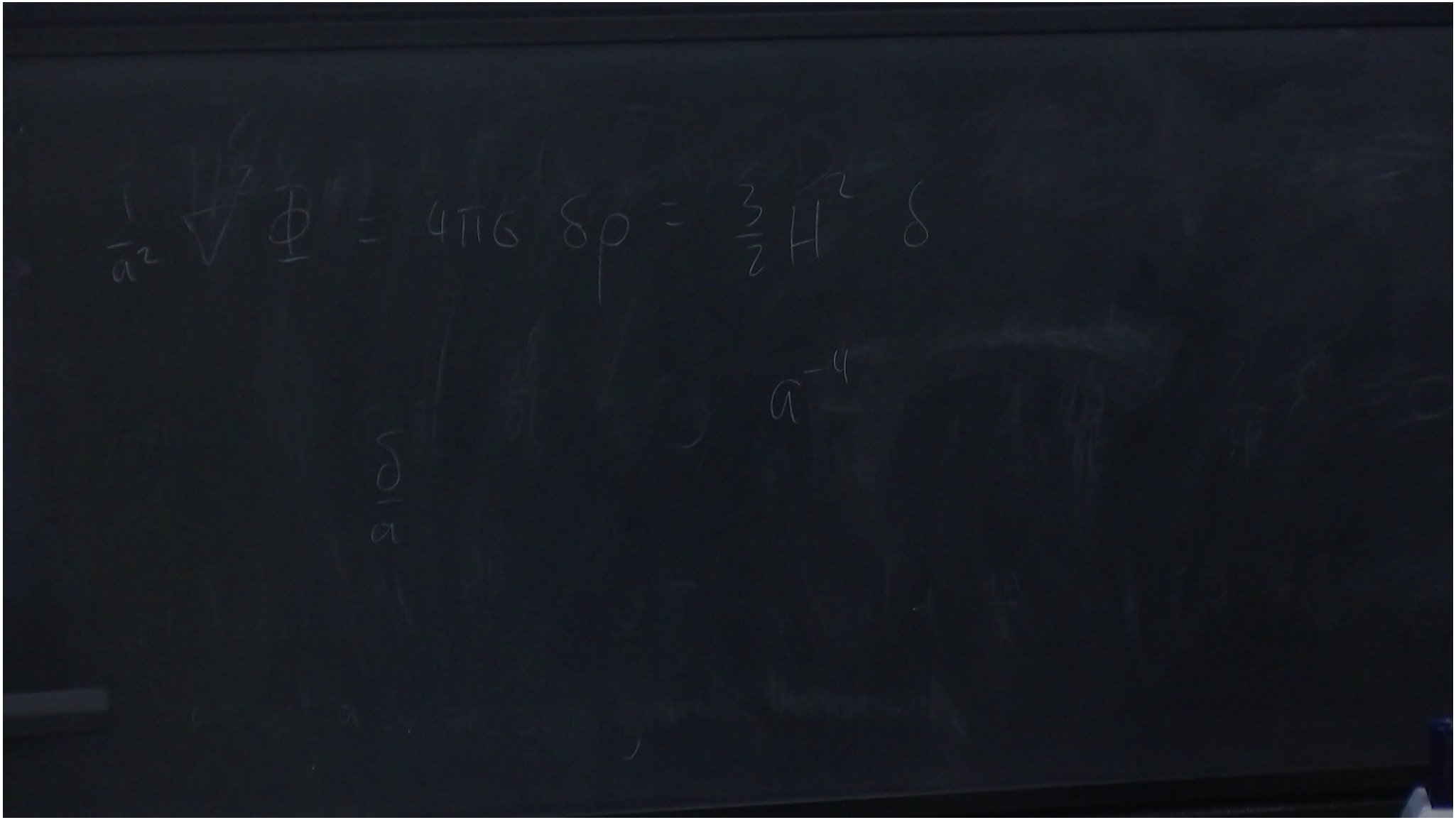

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T_Phi(k=1e-5) :
F_b -v -k 1e-5 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
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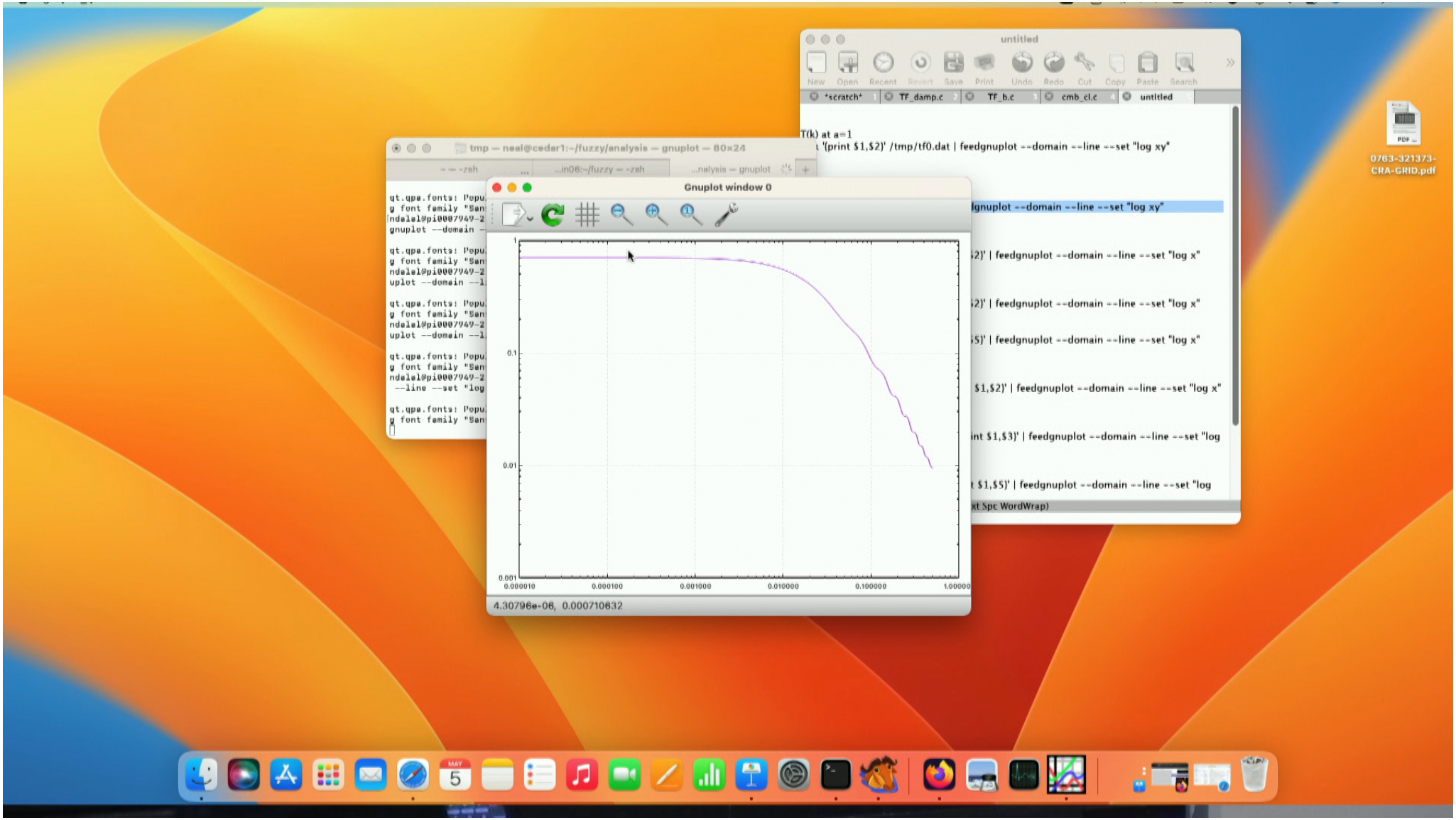

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T_Phi(k=1e-5) :
F_b -v -k 1e-5 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
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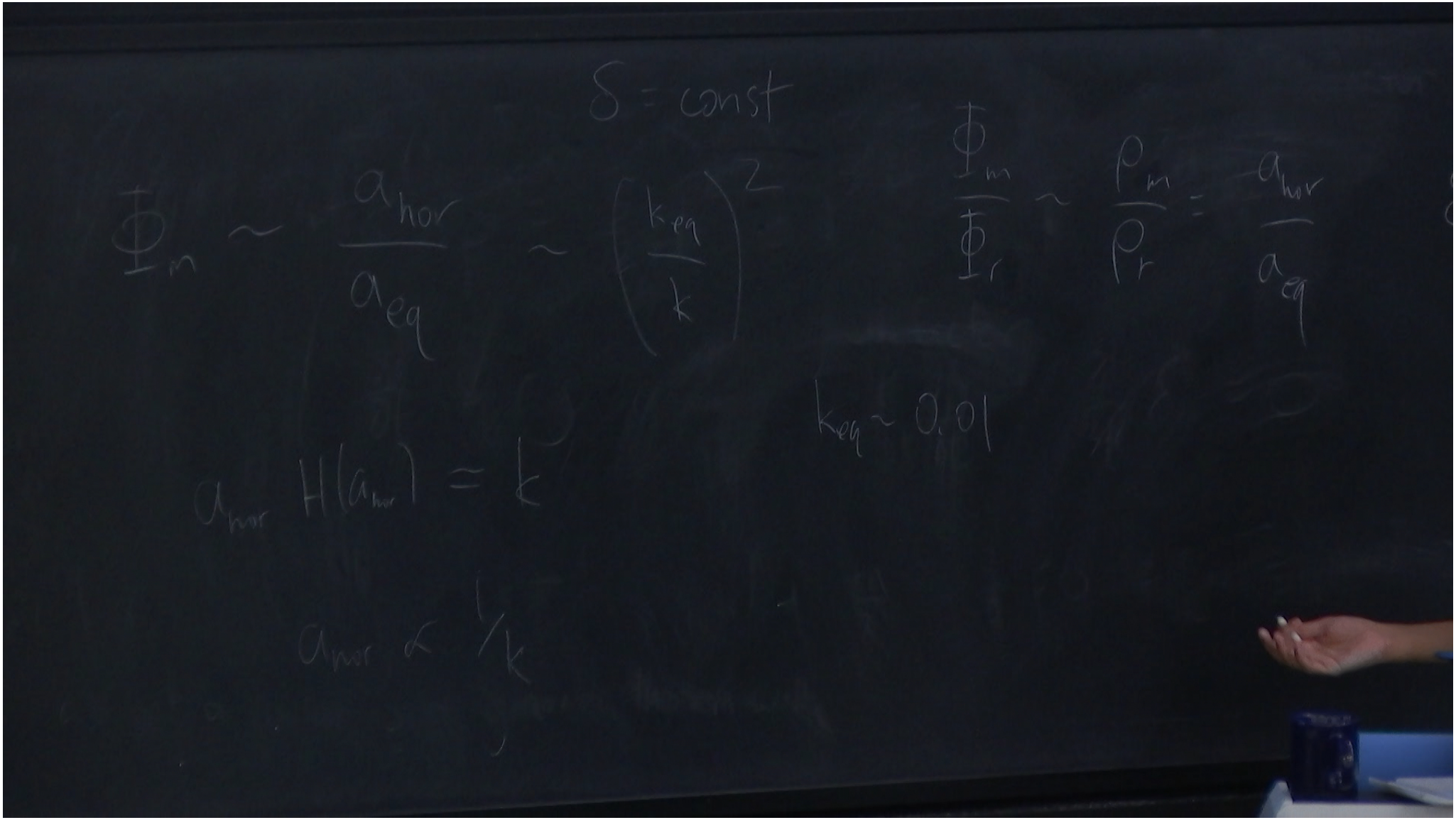

$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\,\delta\rho = \frac{3}{2}H^2\,\delta$
a^{-4}

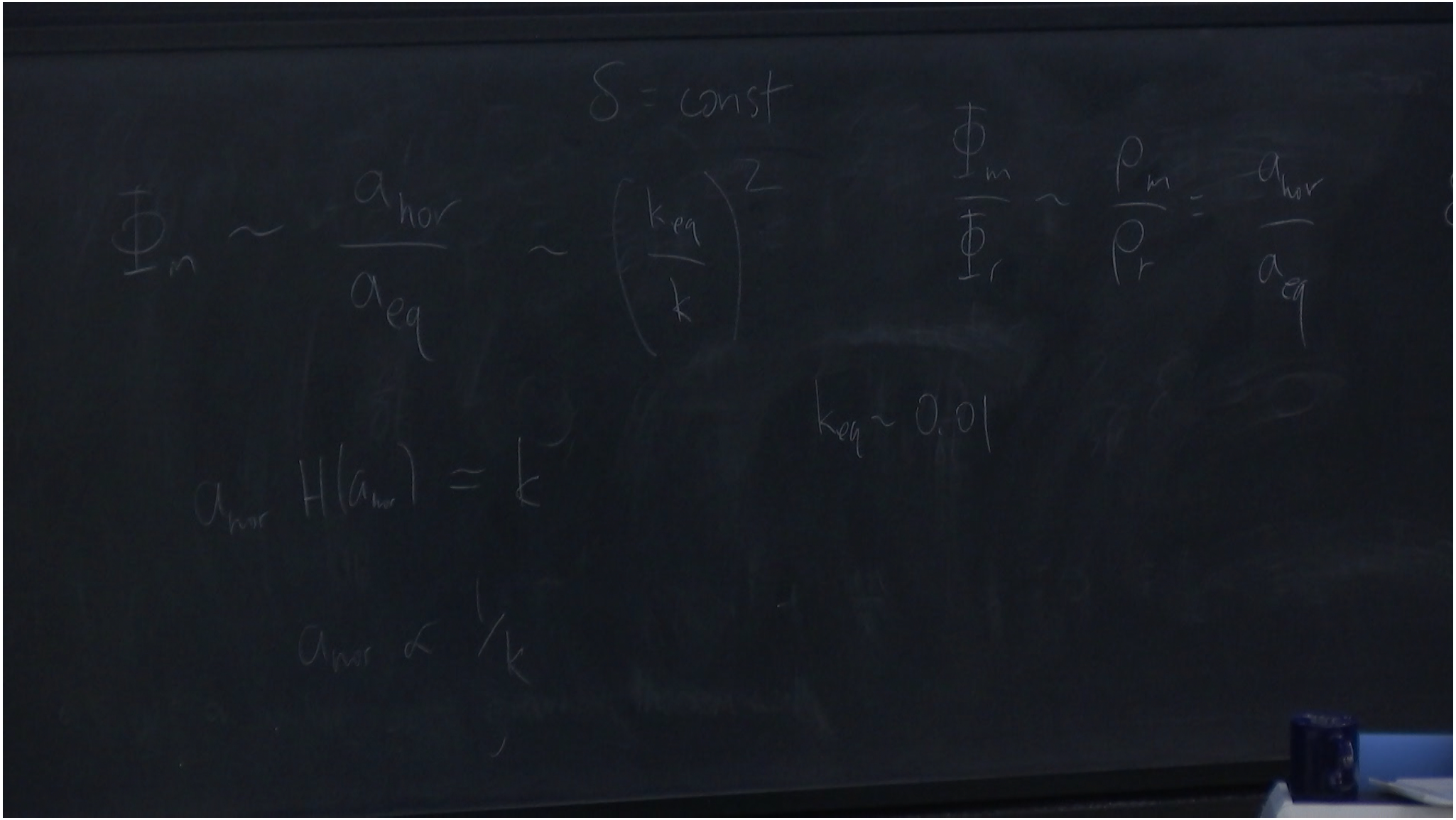

Terminal Shell Edit View Window Help
Fri May 5 1:36 PM
tmp — neal@cedar1:~/fuzzy/analysis — gnuplot — 80×24
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0763-321373-CRA-GRID.pdf

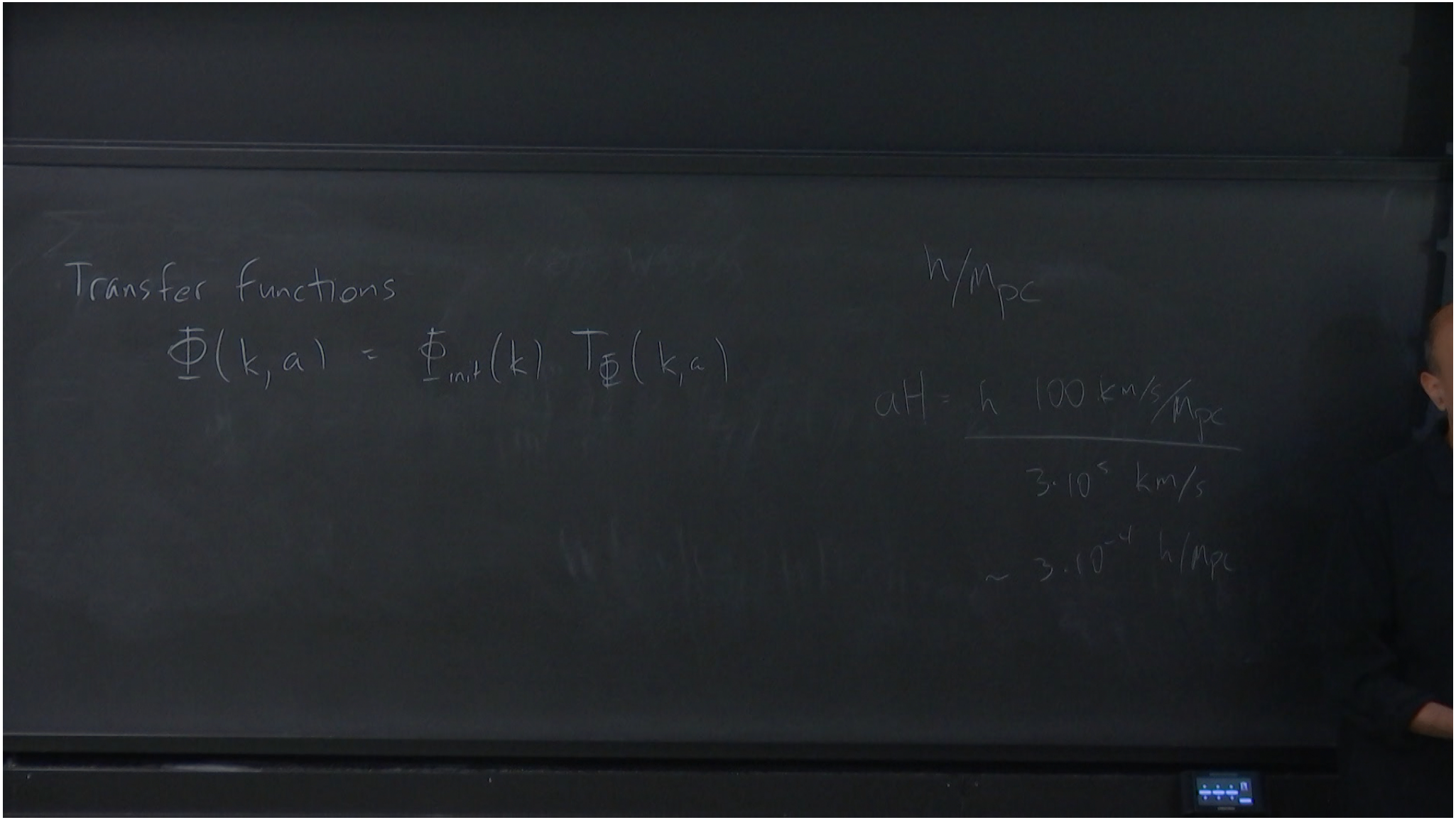

gnuplot_qt
Fri May 5 1:36 PM
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T_Phi(k=1e-5) :
F_b -v -k 1e-5 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
$1,$2}' | feedgnuplot --domain --line --set "log x"
int $1,$3}' | feedgnuplot --domain --line --set "log
t $1,$5}' | feedgnuplot --domain --line --set "log
5-4) * $5^2}' /tmp/tf.dat | feedgnuplot --domain
xt Spc WordWrap)
0763-321373-CRA-GRID.pdf
tmp — neal@cedar1:~/fuzzy/analysis — gnuplot — 80×24
Gnuplot window 0
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0.0010000
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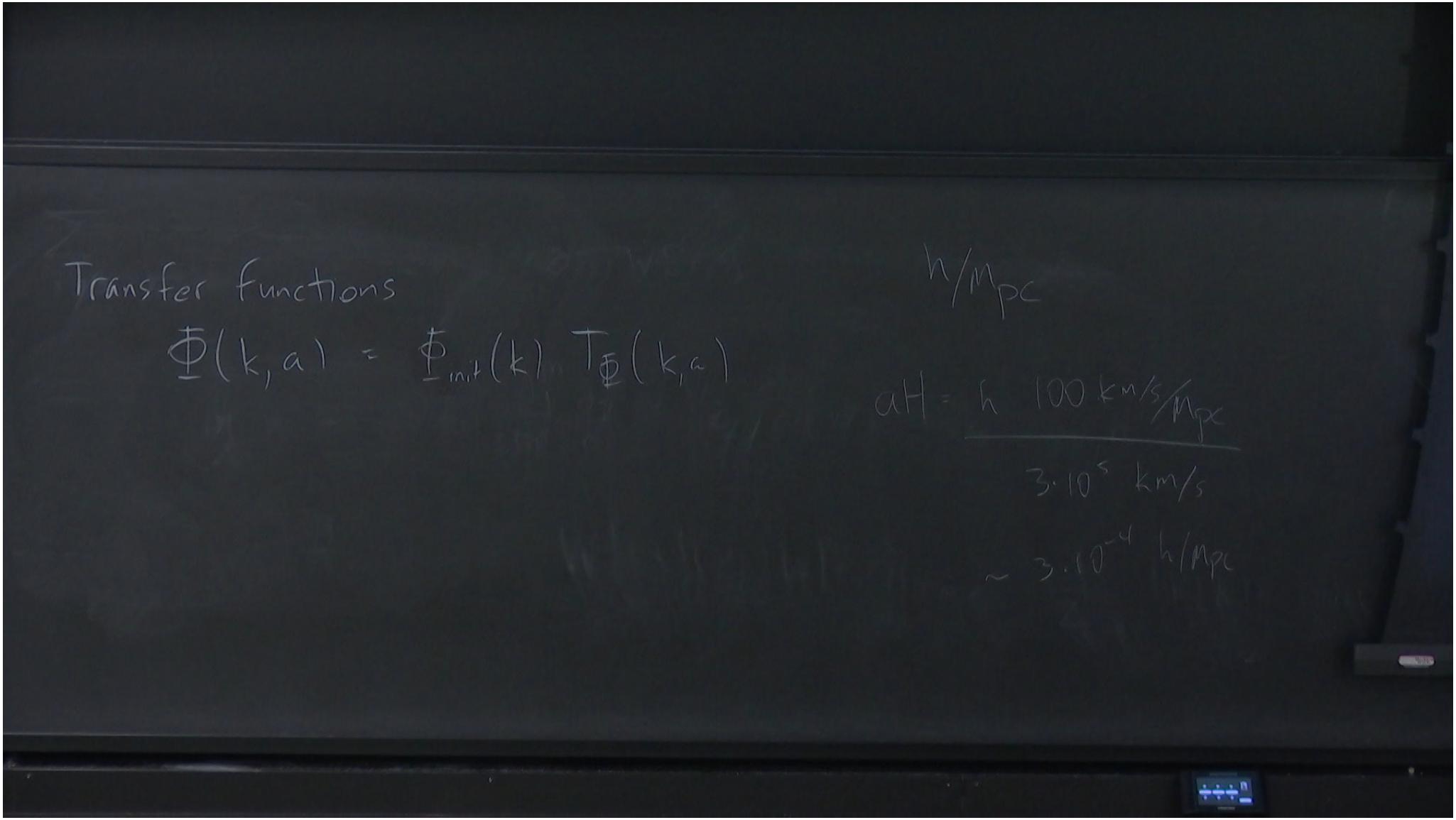

$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\,\delta\rho = \frac{3}{2}H^2\,\delta$
a^{-4}
$\frac{\delta}{a}$

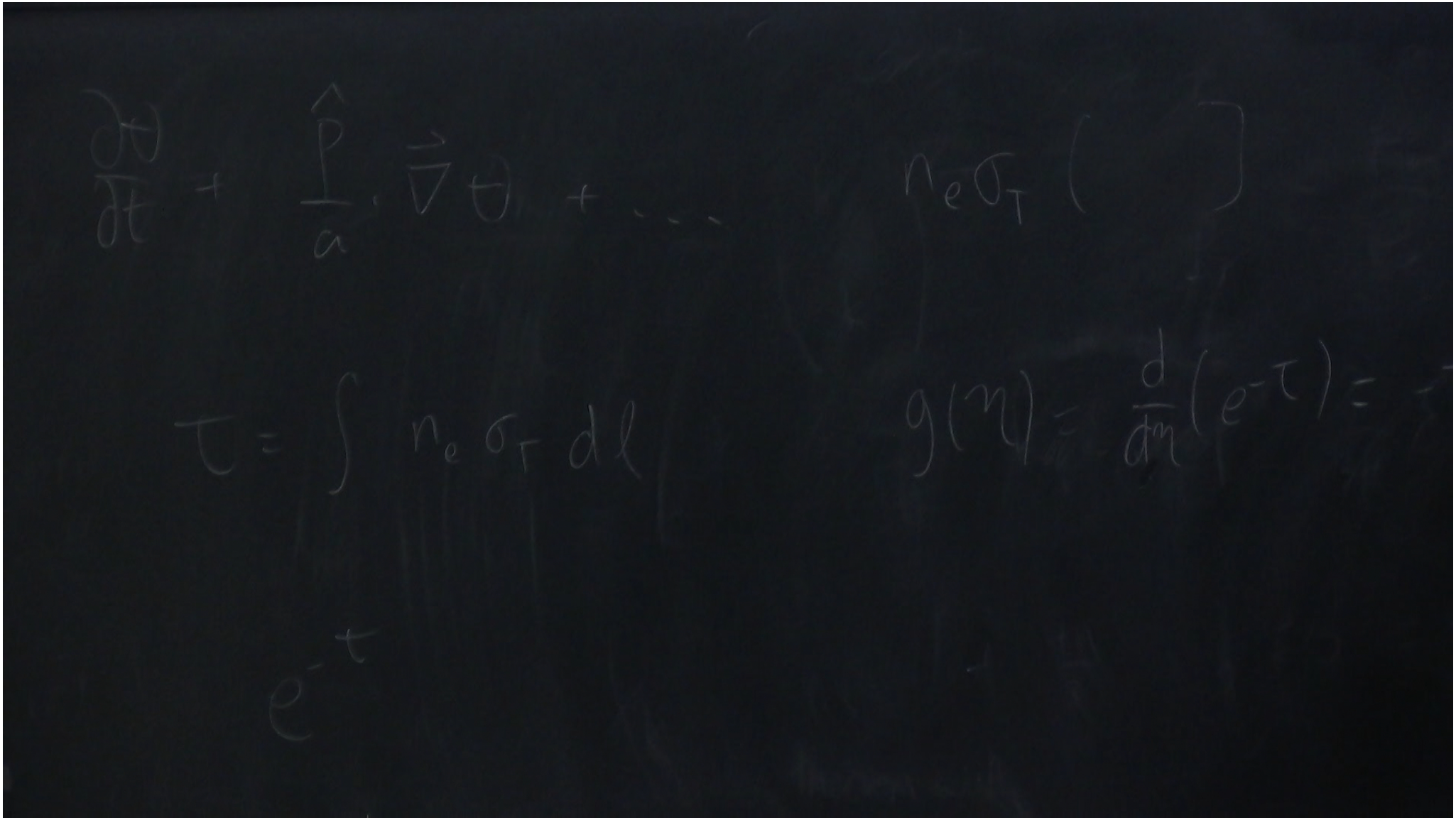

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untitled
T(k) at a=1
'{print $1,$2}' /tmp/tf0.dat | feedgnuplot --domain --line --set "log xy"
tmp — neal@cedar1:~/fuzzy/analysis — gnuplot — 80×24
Gnuplot window 0
1
0.1
0.01
0.001
0.000010
0.000100
0.001000
0.010000
0.100000
1.000000
4.30796e-06, 0.000710632
0763-321373-CRA-GRID.pdf

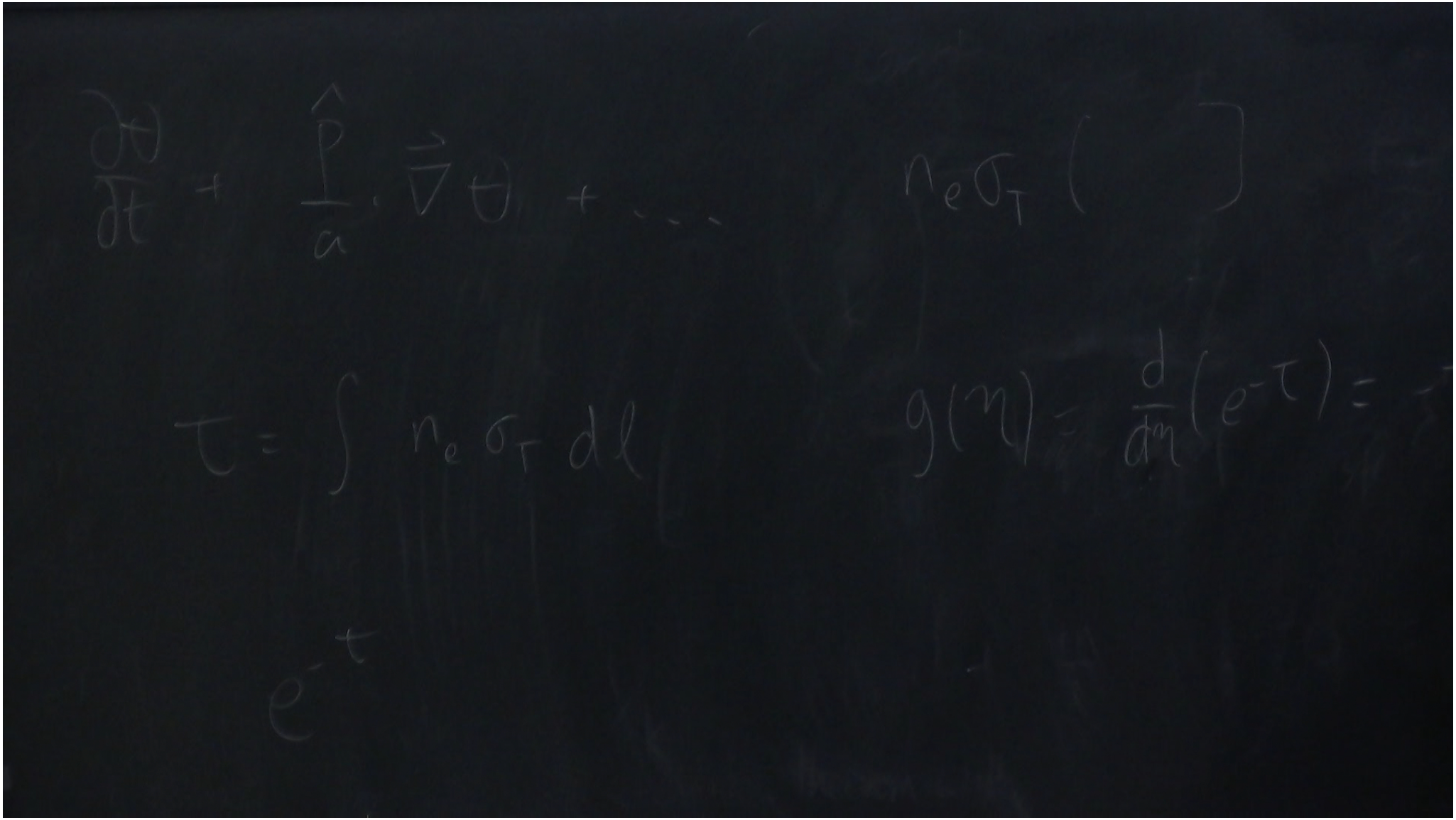

δ = const
$\frac{\Phi_m}{\Phi_r} \sim \frac{\rho_m}{\rho_r} = \frac{a_{hor}}{a_{eq}}$
$\Phi_m \sim \frac{a_{hor}}{a_{eq}} \sim \left(\frac{k_{eq}}{k}\right)^2$
$k_{eq} \sim 0.01$
$a_{hor} H(a_{hor}) = k$
$a_{hor} \propto \frac{1}{k}$

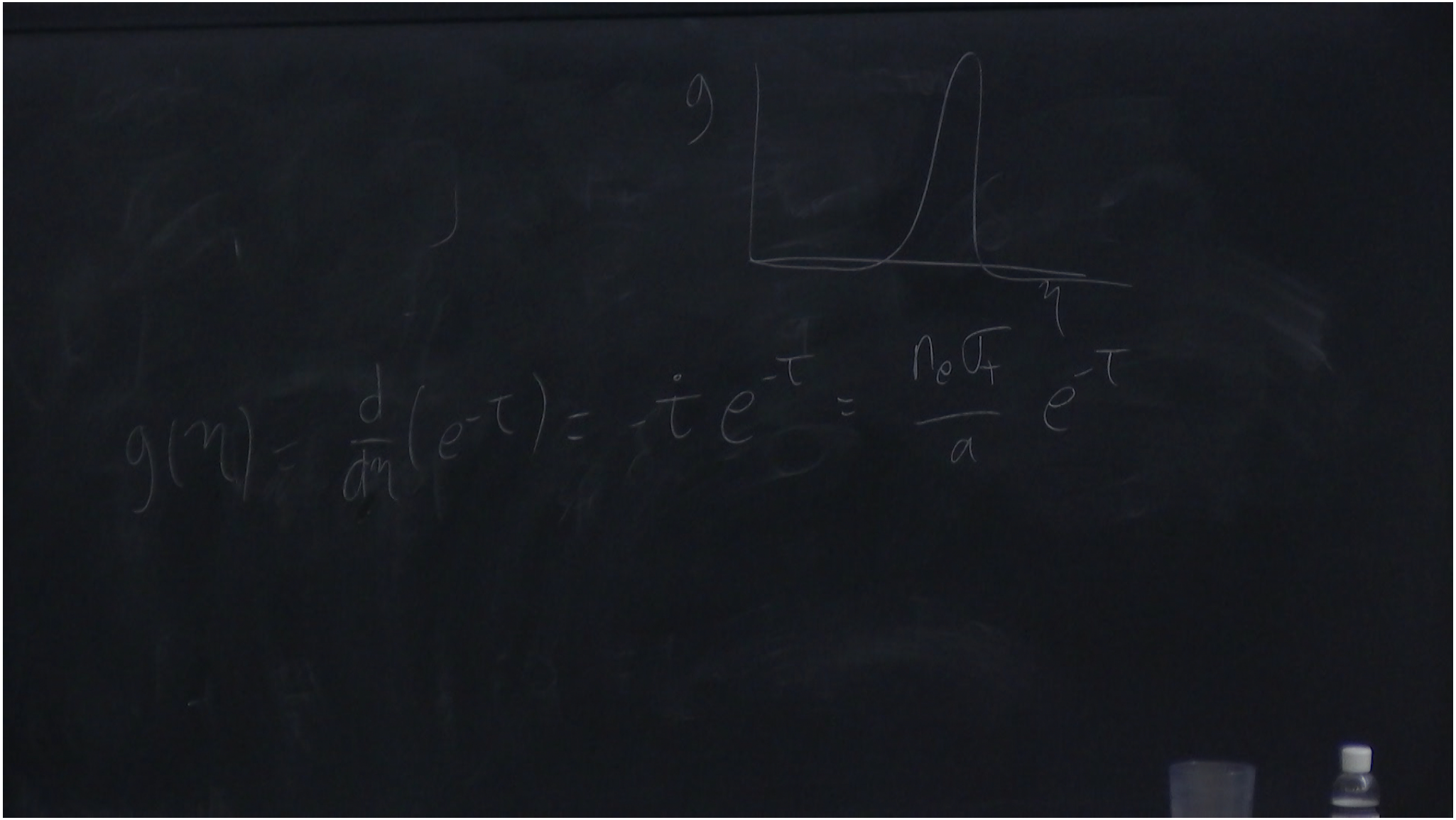

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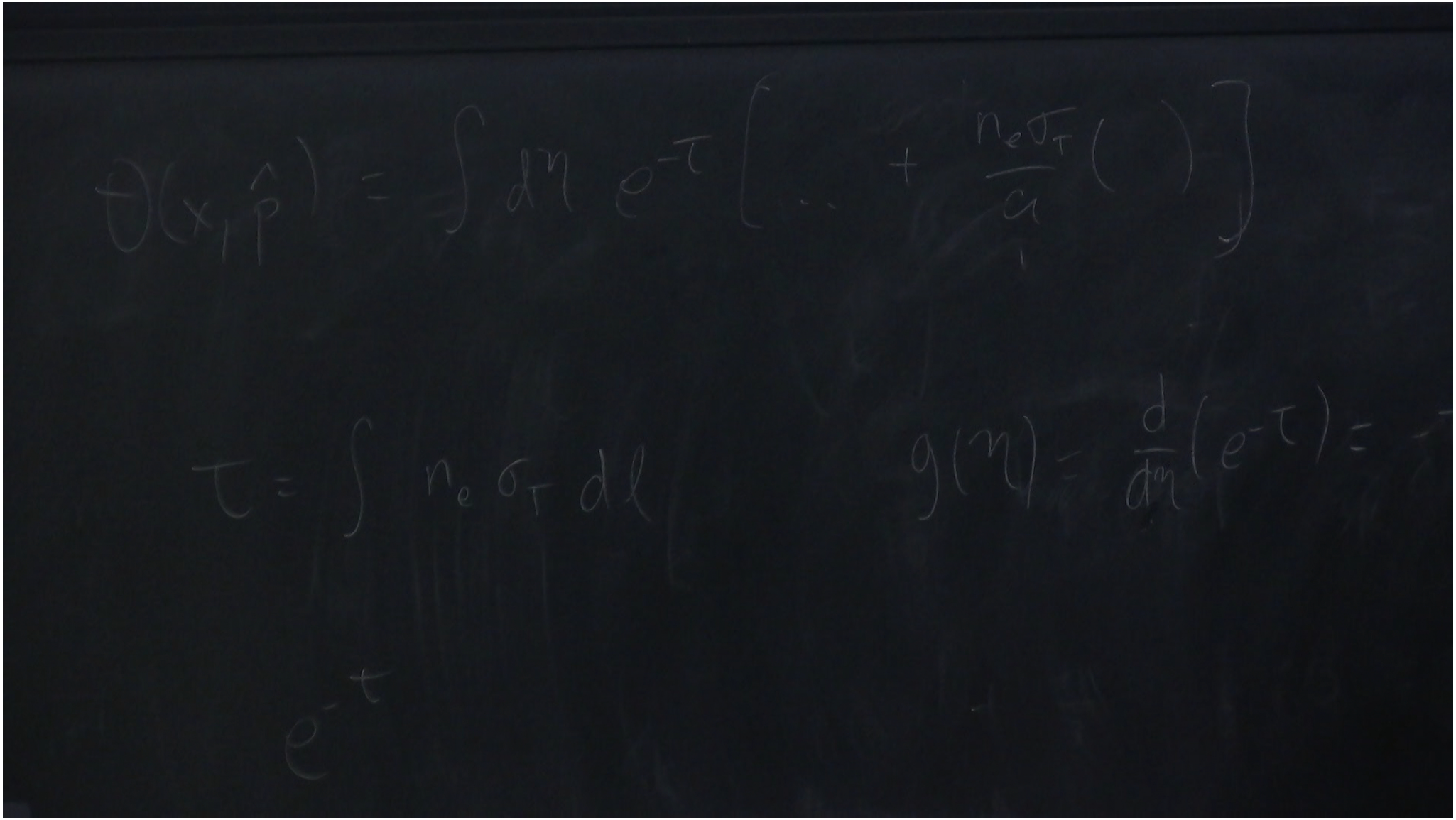

Transfer functions
$\Phi(k,a) = \Phi_{init}(k)\, T_\Phi(k,a)$
h/Mpc
$aH = \frac{h\ 100\ km/s/Mpc}{3\cdot 10^5\ km/s}$
$\sim 3\cdot 10^{-4}\ h/Mpc$

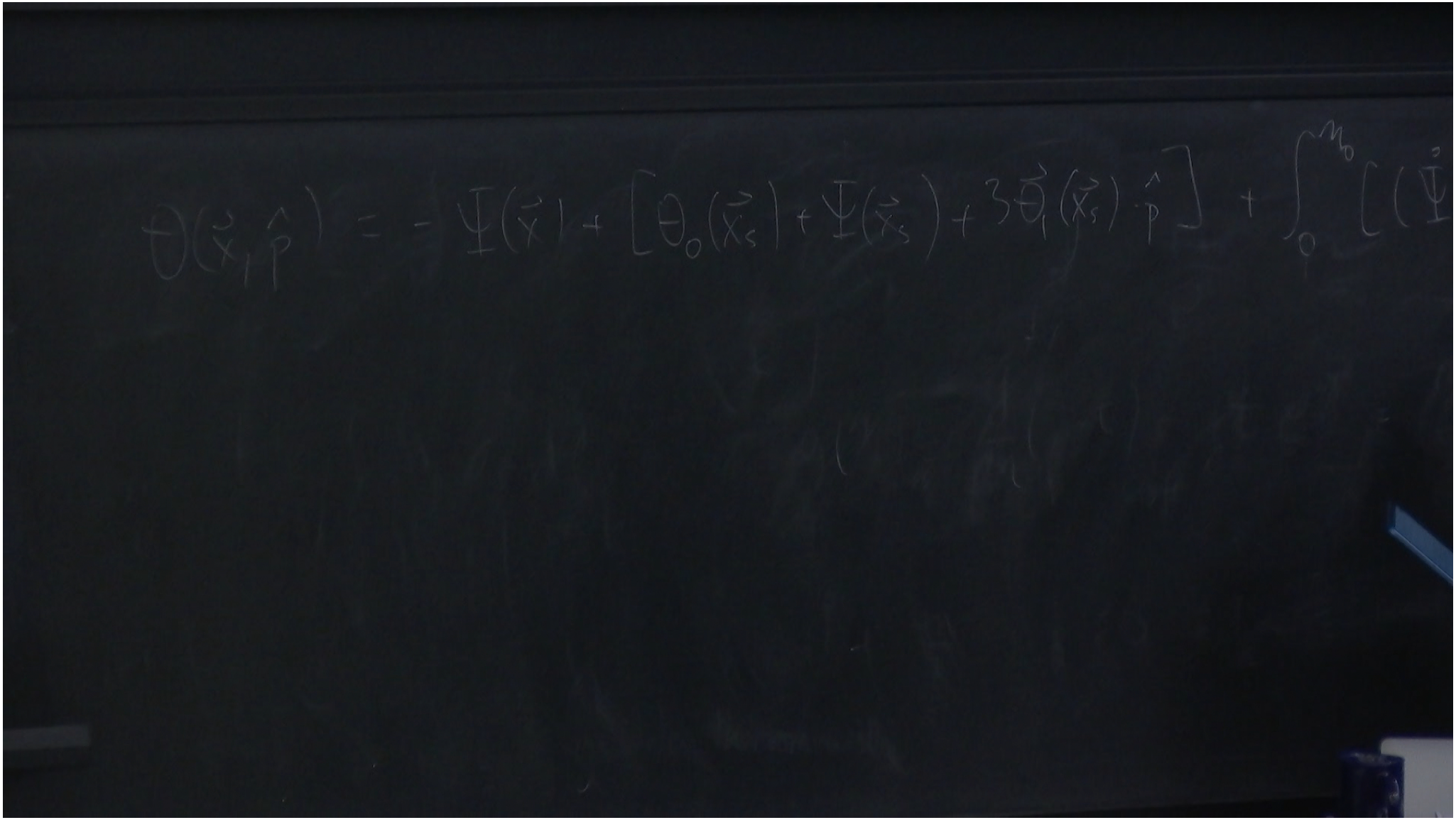

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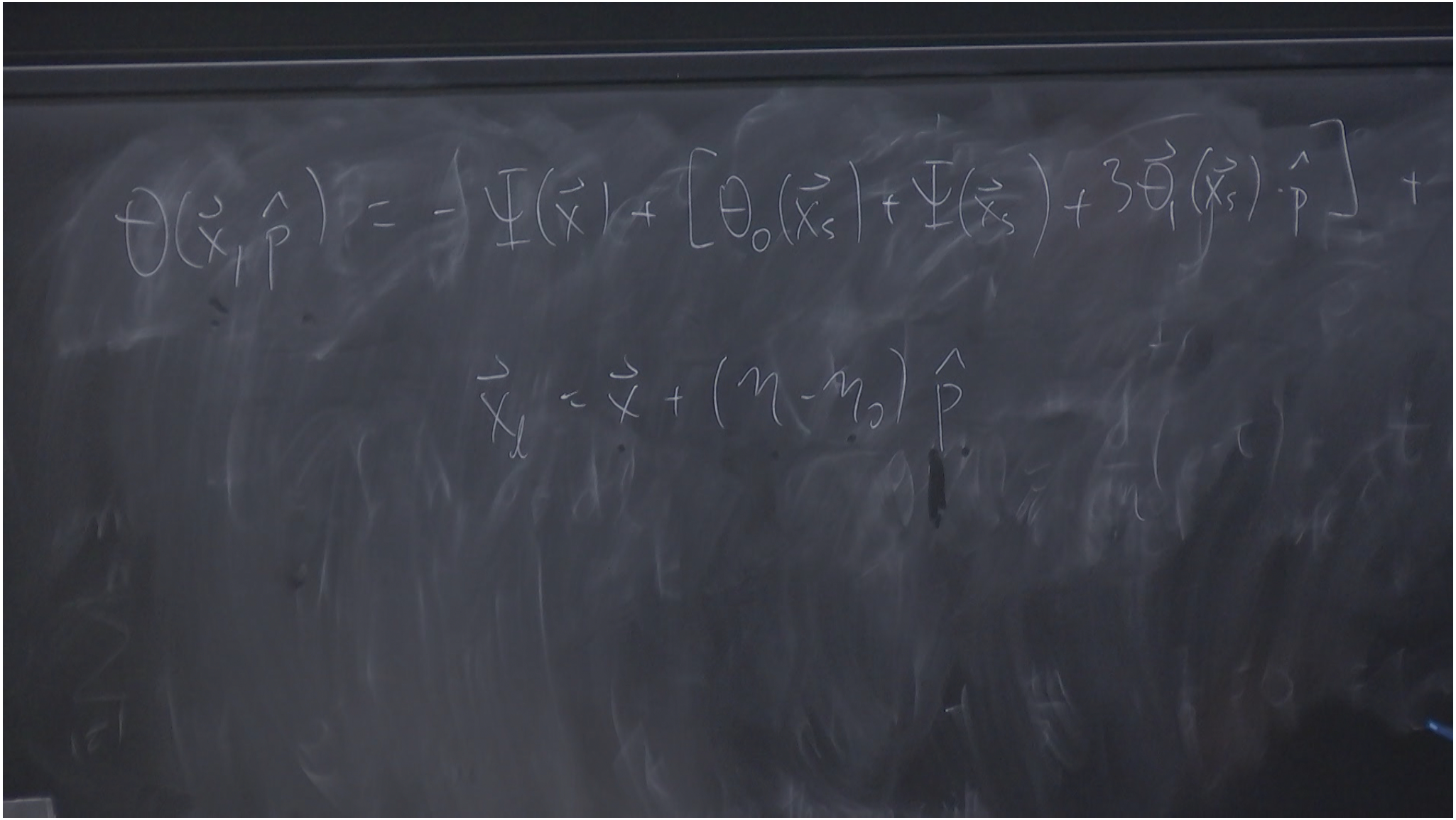

$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}}{a} \cdot \vec{\nabla} \Theta + \dots$
$n_e \sigma_T \left(\right)$
$\tau = \int n_e \sigma_T \, dl$
$g(\eta) = \frac{d}{d\eta}(e^{-\tau}) =$
e^{-t}

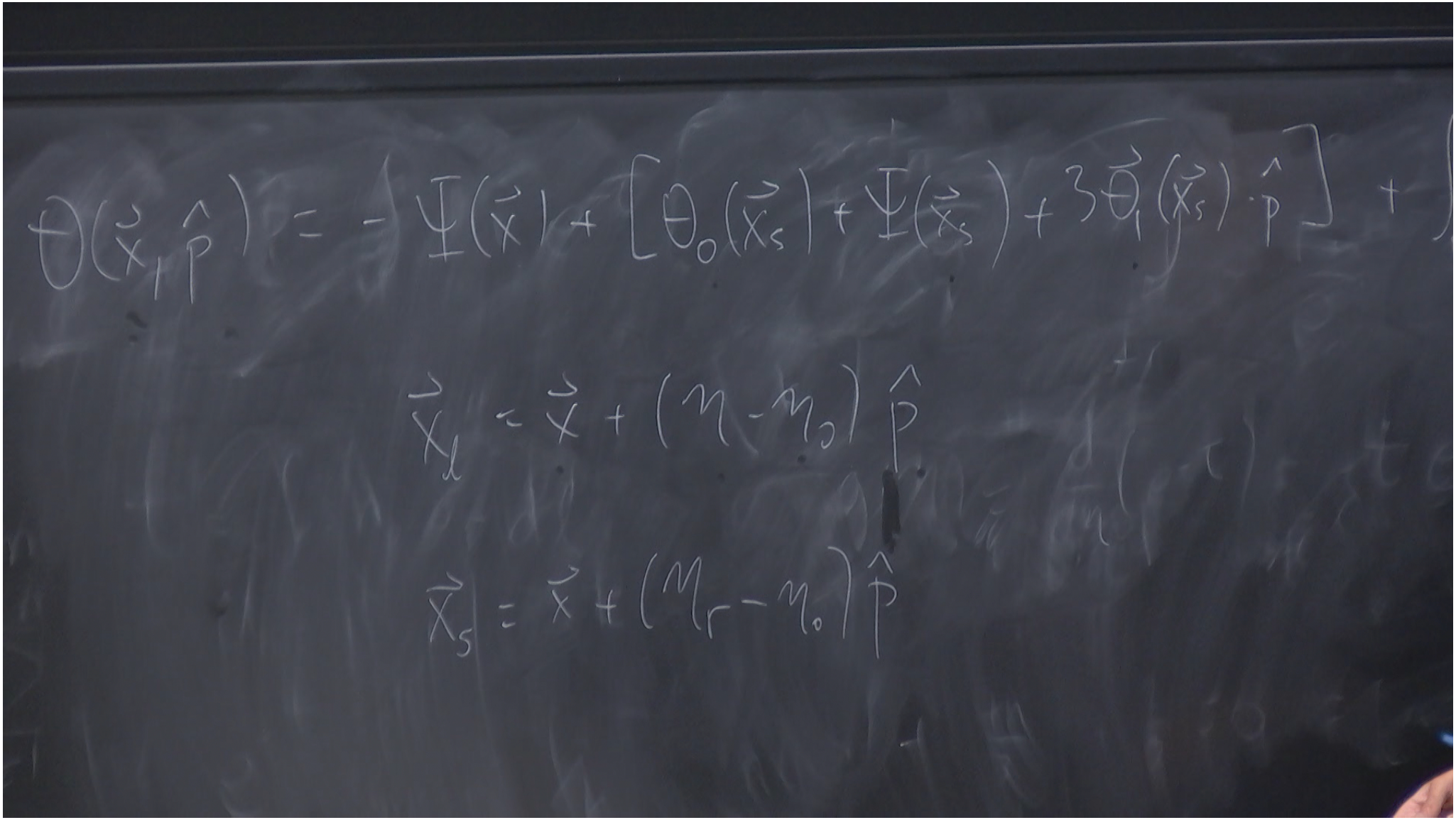

$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}}{a} \cdot \vec{\nabla} \Theta + \dots$
$n_e \sigma_T \left(\right)$
$\tau = \int n_e \sigma_T \, dl$
$g(\eta) = \frac{d}{d\eta}(e^{-\tau}) =$
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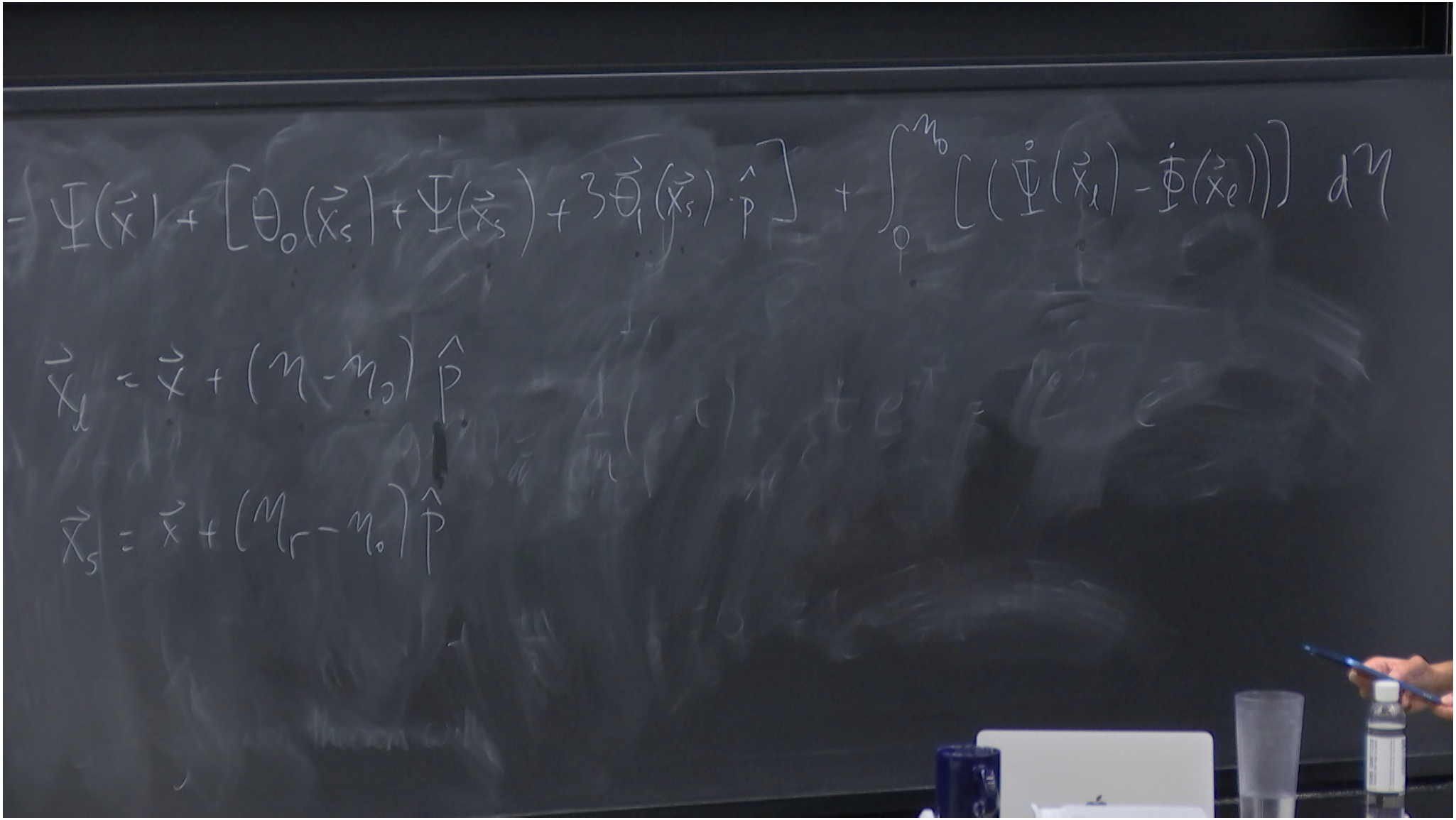

g
η
$g(\eta) = \frac{d}{d\eta}(e^{-\tau}) = \dot{\tau} e^{-\tau} = \frac{n_e \sigma_T}{a} e^{-\tau}$

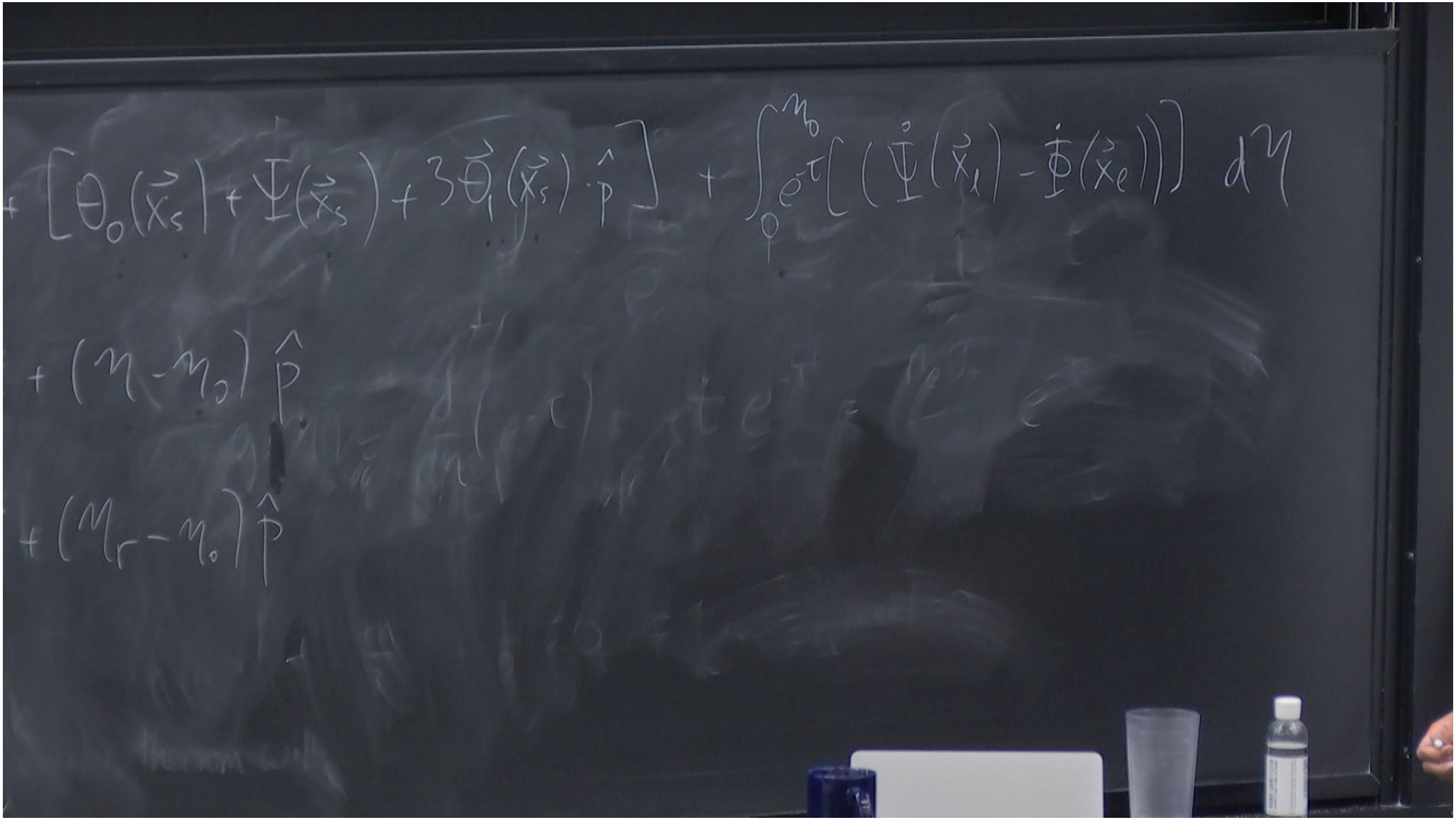

$\theta(x, \hat{p}') = \int d\eta\, e^{-\tau} \left[\ldots + \frac{n_e \sigma_T}{a} (\quad) \right]$
$\tau = \int n_e \sigma_T \, dl$
$g(\eta) = \frac{d}{d\eta}(e^{-\tau}) =$
e^{-t}

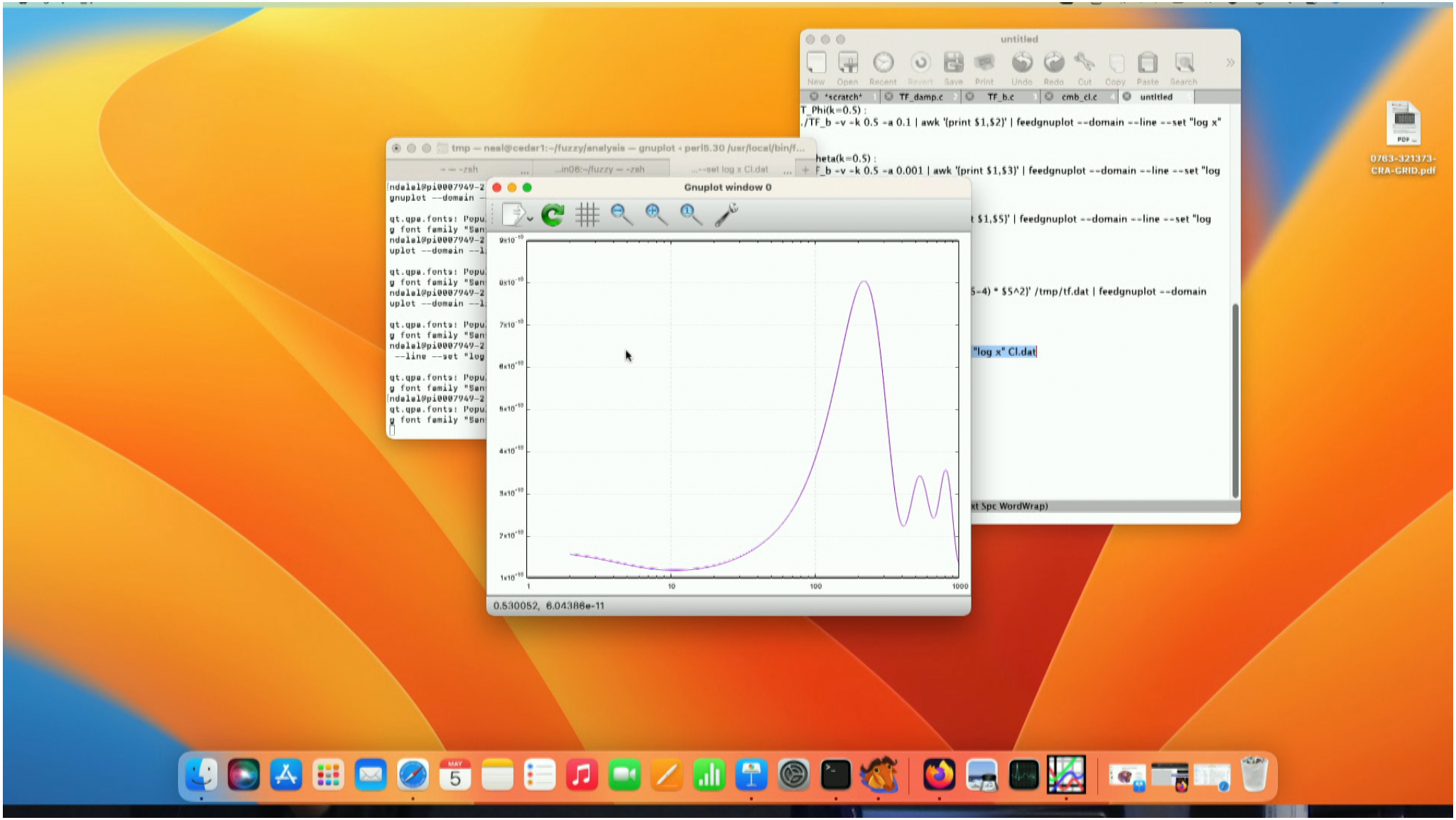

$\Theta(\vec{x}, \hat{p}) = -\Psi(\vec{x}) + [\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s) + 3\vec{v}_1(\vec{x}_s)\cdot\hat{p}] + \int_0^{\eta_0}$


$\Theta(\vec{x}, \hat{p}) = -\Psi(\vec{x}) + [\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s) + 3\vec{\Theta}_1(\vec{x}_s)\cdot\hat{p}] +$
$\vec{x}_s = \vec{x} + (\eta - \eta_0)\hat{p}$


$\Theta(\vec{x}, \hat{p}) = -\Psi(\vec{x}) + [\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s) + 3\vec{\Theta}_1(\vec{x}_s)\cdot\hat{p}] +$
$\vec{x}_d = \vec{x} + (\eta - \eta_0)\hat{p}$
$\vec{x}_s = \vec{x} + (\eta_r - \eta_0)\hat{p}$


$-\Psi(\vec{x}) + [\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s) + 3\vec{\Theta}_1(\vec{x}_s)\cdot\hat{p}] + \int_0^{\eta_0} [(\dot{\Psi}(\vec{x}_t) - \dot{\Phi}(\vec{x}_e))]\, d\eta$
$\vec{x}_t = \vec{x} + (\eta - \eta_0)\hat{p}$
$\vec{x}_s = \vec{x} + (\eta_r - \eta_0)\hat{p}$


$+\left[\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s) + 3\vec{\Theta}_1(\vec{x}_s)\cdot\hat{p}\right] + \int_0^{\eta_0} e^{-\tau}\left[\left(\dot{\Psi}(\vec{x}_l) - \dot{\Phi}(\vec{x}_e)\right)\right] d\eta$
$+ (\eta - \eta_0)\hat{p}$
$+ (\eta_r - \eta_0)\hat{p}$


untitled
New
Open
Recent
Revert
Save
Print
Undo
Redo
Cut
Copy
Paste
Search
scratch
TF_damp.c
TF_b.c
cmb_cl.c
untitled
T_Phi(k=0.5) :
./TF_b -v -k 0.5 -a 0.1 | awk '{print $1,$2}' | feedgnuplot --domain --line --set "log x"
heta(k=0.5) :
F_b -v -k 0.5 -a 0.001 | awk '{print $1,$3}' | feedgnuplot --domain --line --set "log
t $1,$5}' | feedgnuplot --domain --line --set "log
5-4) * $5^2}' /tmp/tf.dat | feedgnuplot --domain
"log x" Cl.dat
xt Spc WordWrap)
Gnuplot window 0
0.530052, 6.04386e-11
0763-321373-CRA-GRID.pdf