Title: Cosmology Lecture (230502)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

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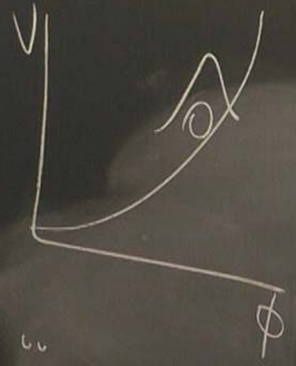

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$\ddot{x} + \omega^2 x = 0$$

$$v(t) = \frac{e^{\pm i\omega t}}{\sqrt{2\omega}}$$

$$\langle x^2 \rangle = |v|^2 = \frac{1}{2\omega}$$

$$x = v(t)\, a + v^*(t)\, a^{+}$$

$$[x, p] = i$$

$$\Rightarrow |v|^2 = \frac{1}{2\omega}$$

$$ds^2 = -(1+2A)\,dt^2 - 2a\,B_i\,dt\,dx^i + a^2\,\delta_{ij}\,dx^i dx^j$$

$$\phi = \phi_0 + \delta\phi$$

$$d\tau = d\eta = \frac{dt}{a}$$

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \nabla^2\delta\phi = 0$$

$$\eta = \int \frac{dt}{a} = \int \frac{da}{a^2 H} = \frac{1}{H}\int_{a_0}^{a} \frac{da}{a^2} = \frac{1}{a_0 H} - \frac{1}{aH}$$

$$\tau = \eta - \eta_{end} = \frac{1}{a_{end} H} - \frac{1}{aH} \simeq -\frac{1}{aH}$$

$$ds^2 = -(1+2A)\,dt^2 - 2a\,B_i\,dt\,dx^i + a^2\,\delta_{ij}\,dx^i dx^j$$

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$$|\tau| \approx \frac{1}{aH}$$

$$\tau = \eta - \eta_{end} = \frac{1}{a_{end} H} - \frac{1}{aH} \approx -\frac{1}{aH}$$

$$\frac{d^2\delta\phi}{d\tau^2} - \frac{2}{\tau}\frac{d\delta\phi}{d\tau} - \nabla^2\delta\phi = 0$$

$$\delta\phi = \frac{x}{a} \Rightarrow \frac{d^2x}{d\tau^2} + \left(k^2 - \frac{2}{\tau^2}\right)x = 0$$

$$v(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

$|k\tau| \gg 1$: inside horizon

$\Rightarrow$ matches harmonic oscillator

$|k\tau| \ll 1$ outside

$\Rightarrow v$ stops oscillating

quantum fluctuations

$\rightarrow$ classical fluctuations

$$\frac{d^2\delta\phi}{d\tau^2} - \frac{2}{\tau}\frac{d\delta\phi}{d\tau} - \nabla^2\delta\phi = 0$$

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$$v(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

$$\langle\delta\phi^2\rangle = \frac{\langle x^2\rangle}{a^2} = \frac{|v|^2}{a^2} = \frac{1}{2k^3\tau^2a^2} = \frac{H^2}{2k^3}$$

$|k\tau| \gg 1$: inside horizon

$\Rightarrow$ matches harmonic oscillator

$|k\tau| \ll 1$ outside

$\Rightarrow v$ stops oscillating

quantum fluctuations

$\rightarrow$ classical fluctuations

dx^j

$= \frac{dt}{a}$

$\frac{1}{aH}$

$$\int \frac{4\pi k^2 dk}{(2\pi)^3} \langle \delta\phi(k)^2 \rangle = \int \frac{dk}{k} \Delta^2(k)$$

$$\Delta^2(k) = \frac{k^3 \langle \delta\phi(k)^2 \rangle}{2\pi^2} = \left(\frac{H}{2\pi}\right)^2$$

scale invt. because $H \sim$ const.

$$ij\, dx^i\, dx^j$$

$$= d\eta = \frac{dt}{a}$$

$$|\tau| \approx \frac{1}{aH}$$

$$\int \frac{4\pi k^2 dk}{(2\pi)^3} \langle \delta\phi(k)^2 \rangle = \int \frac{dk}{k} \Delta^2(k)$$

$$\Delta^2(k) = \frac{k^3 \langle \delta\phi(k)^2 \rangle}{2\pi^2} = \left(\frac{H}{2\pi}\right)^2 \Bigg|_{aH=k}$$

scale invt. because $H \sim$ const

$$\varepsilon = \frac{d}{dt}\left(\frac{1}{H}\right)$$

$$\delta\phi \to \Phi,\ \Psi,\ \delta_m,\ \Theta_0,\ \ldots$$

$$\delta\phi \to \delta t_{end} \to \delta a$$

$$\delta t = \frac{\delta\phi}{\dot\phi}$$

$$\frac{\delta a}{a} = H\delta t = \frac{H}{\dot\phi}\,\delta\phi$$

$\delta\phi \to \Phi, \Psi, \delta_m, \Theta_0, \ldots$

gauge invt. $\zeta = -\Phi - HV \qquad N$

$\delta\phi \to \delta t_{end} \to \delta a$

$= -\frac{H}{\dot\phi}\delta\phi$

$\delta t = \frac{\delta\phi}{\dot\phi}$

$P_\zeta = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{2k^3} = \frac{2\pi G H^2}{\varepsilon k^3}\Big|_{aH=k}$

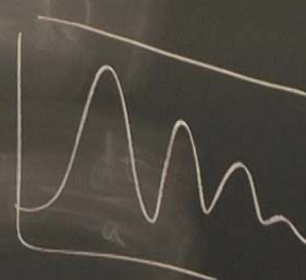
$\dot\phi^2 = \varepsilon \frac{H^2}{4\pi G}$

$$\frac{d \log P_\zeta}{d \log k} = n_s - 4 \quad \Rightarrow \quad n_s = 1 - 6\varepsilon + 2\eta$$

$$= -3 + 2\,\frac{d\log H}{d\log k} - \frac{d\log \varepsilon}{d\log k}$$

$$n_s \approx 0.96$$

$$4\varepsilon - 2\eta$$



$$\int \frac{4\pi k^2 dk}{(2\pi)^3}$$

$$\Delta^2($$

Predictions of inflation

1. nearly scale invt. spectrum $n_s \lesssim 1$

2. adiabatic perturbations

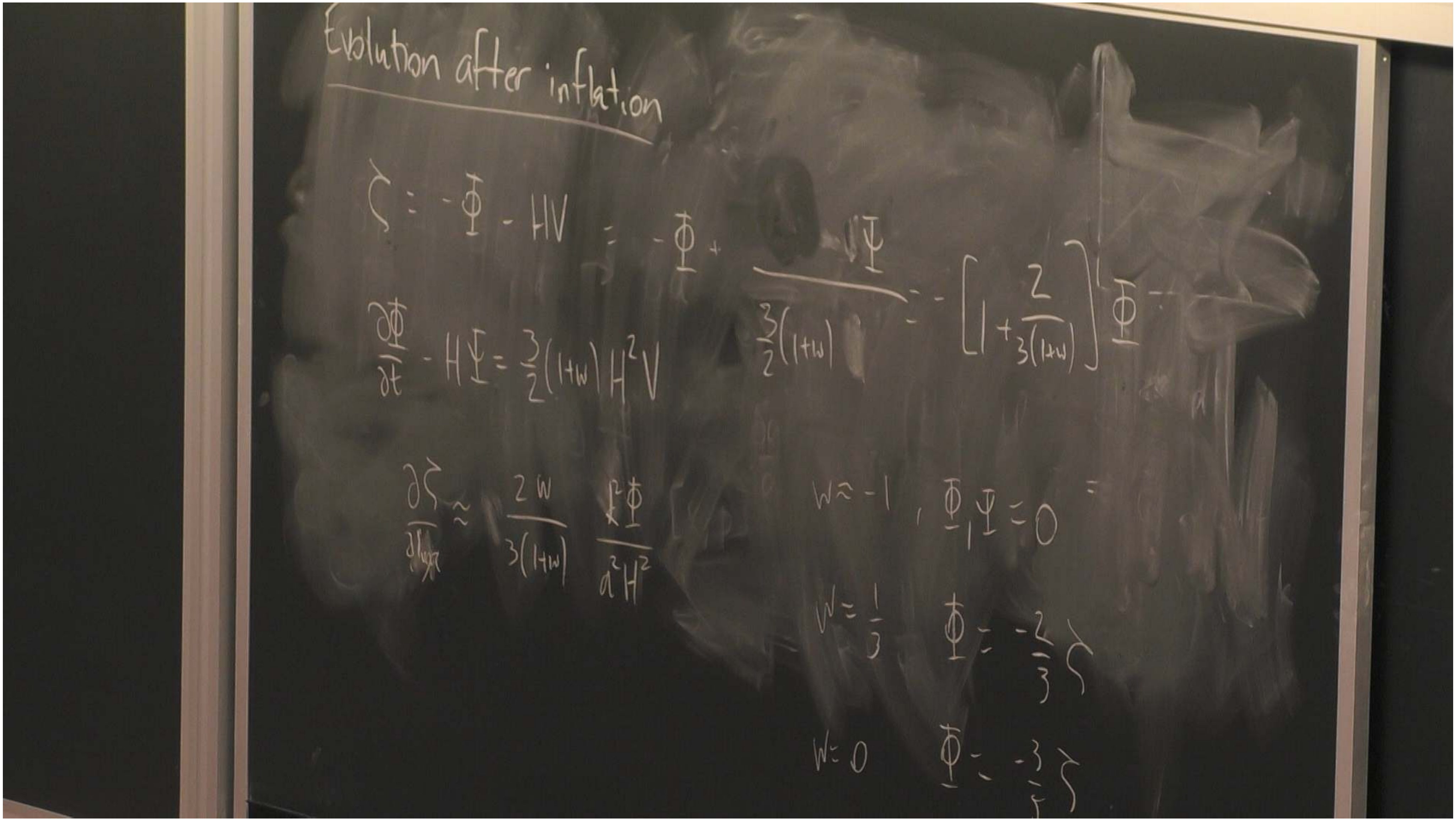
rad: $\rho \propto a^{-4} \Rightarrow \delta_r = \frac{\delta \rho}{\rho_r} = -4 \frac{\delta a}{a}$

m $\rho \propto a^{-3}$

$$\delta_m = \frac{\delta \rho_m}{\rho_m} = -3 \frac{\delta a}{a}$$

$$\boxed{\delta_{dm} = \delta_b = \frac{3}{4} \delta_\gamma = \frac{3}{4} \delta_\nu}$$

3. Gaussian fluctuations

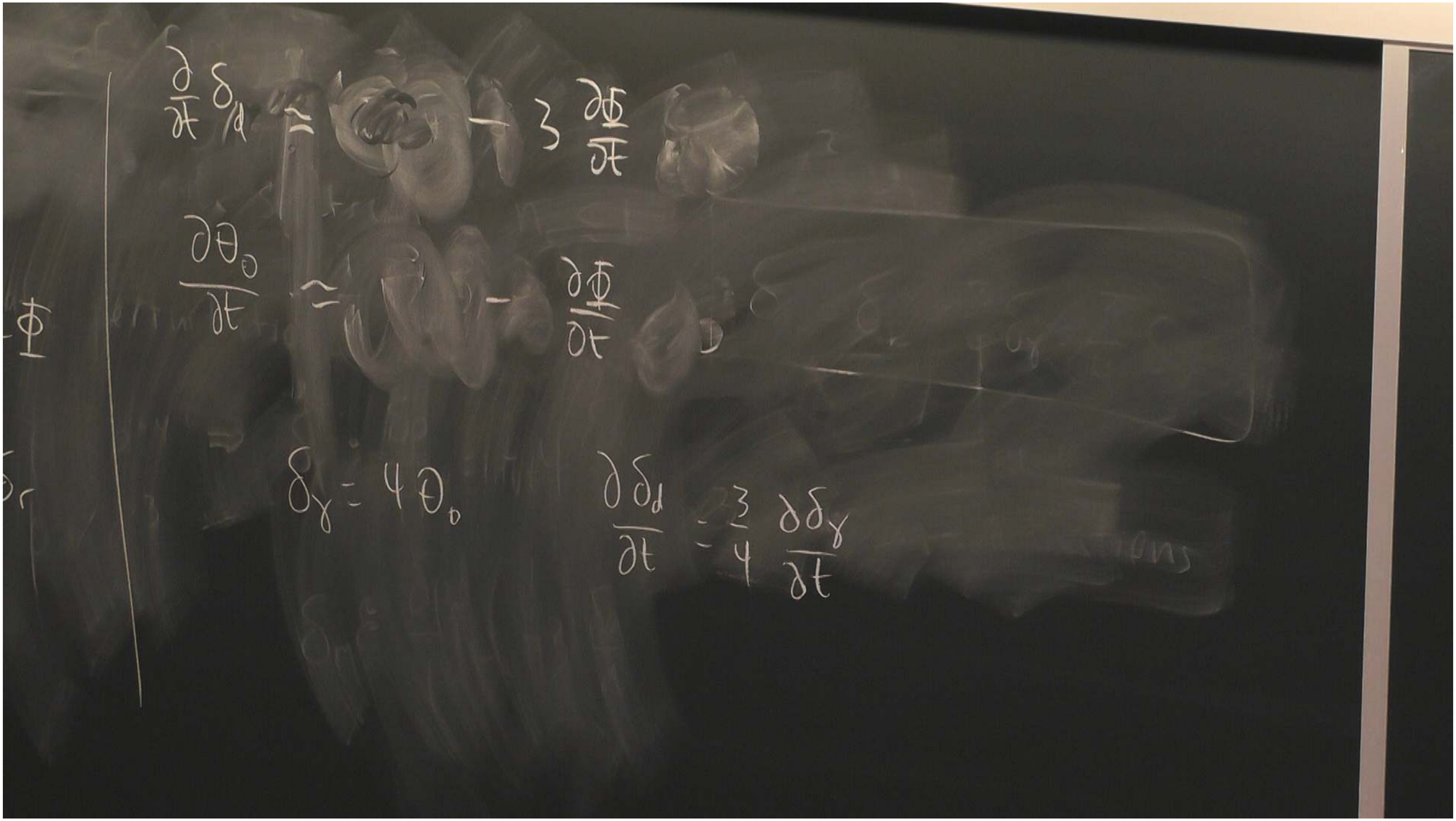

Evolution after inflation
$\zeta = -\Phi - HV = -\Phi + \frac{\dot{\Phi} - H\Psi}{\frac{3}{2}(1+w)} = -\left[1 + \frac{2}{3(1+w)}\right]\Phi$
$\frac{\partial \Phi}{\partial t} - H\Psi = \frac{3}{2}(1+w)H^2 V$
$\frac{\partial \zeta}{\partial \ln a} \approx \frac{2w}{3(1+w)} \frac{k^2 \Phi}{a^2 H^2}$
$w \approx -1, \quad \Phi, \Psi \approx 0$
$w = \frac{1}{3} \quad \Phi = -\frac{2}{3}\zeta$
$w = 0 \quad \Phi = -\frac{3}{5}\zeta$

$$\Phi_{MD}(k) = \frac{9}{10}\,\Phi_{RD}(k)$$

$$3H^2\Phi - \ldots = -4\pi G\,\delta\rho = -\frac{3}{2}H\,\delta_{tot} \Rightarrow \delta_{tot} = 2\Phi$$

$$\delta_{tot} = \frac{\rho_r\delta_r + \rho_m\delta_m + \ldots}{\rho_r + \rho_m + \ldots}$$

$$\delta_m = \frac{3}{4}\delta_r$$


$\frac{\partial}{\partial t}\delta_d \simeq$
$- 3 \frac{\partial \Phi}{\partial t}$
$\frac{\partial \Theta_0}{\partial t} \simeq$
$- \frac{\partial \Phi}{\partial t}$
$\delta_\gamma = 4\Theta_0$
$\frac{\partial \delta_d}{\partial t} = \frac{3}{4} \frac{\partial \delta_\gamma}{\partial t}$

$$\frac{\partial}{\partial t}\delta_d \simeq -3\frac{\partial \Phi}{\partial t}$$

$$\frac{\partial \Theta_0}{\partial t} \simeq -\frac{\partial \Phi}{\partial t}$$

$\ldots_{tot} = 2\Phi$

$\delta_m = \frac{3}{4}\delta_r$

$\delta_\gamma = 4\Theta_0$

$$\frac{\partial \delta_d}{\partial t} = \frac{3}{4}\frac{\partial \delta_\gamma}{\partial t}$$

$x'' + Ax' + Bx = 0$

$$\frac{d^2\delta\phi}{d\tau^2}$$

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