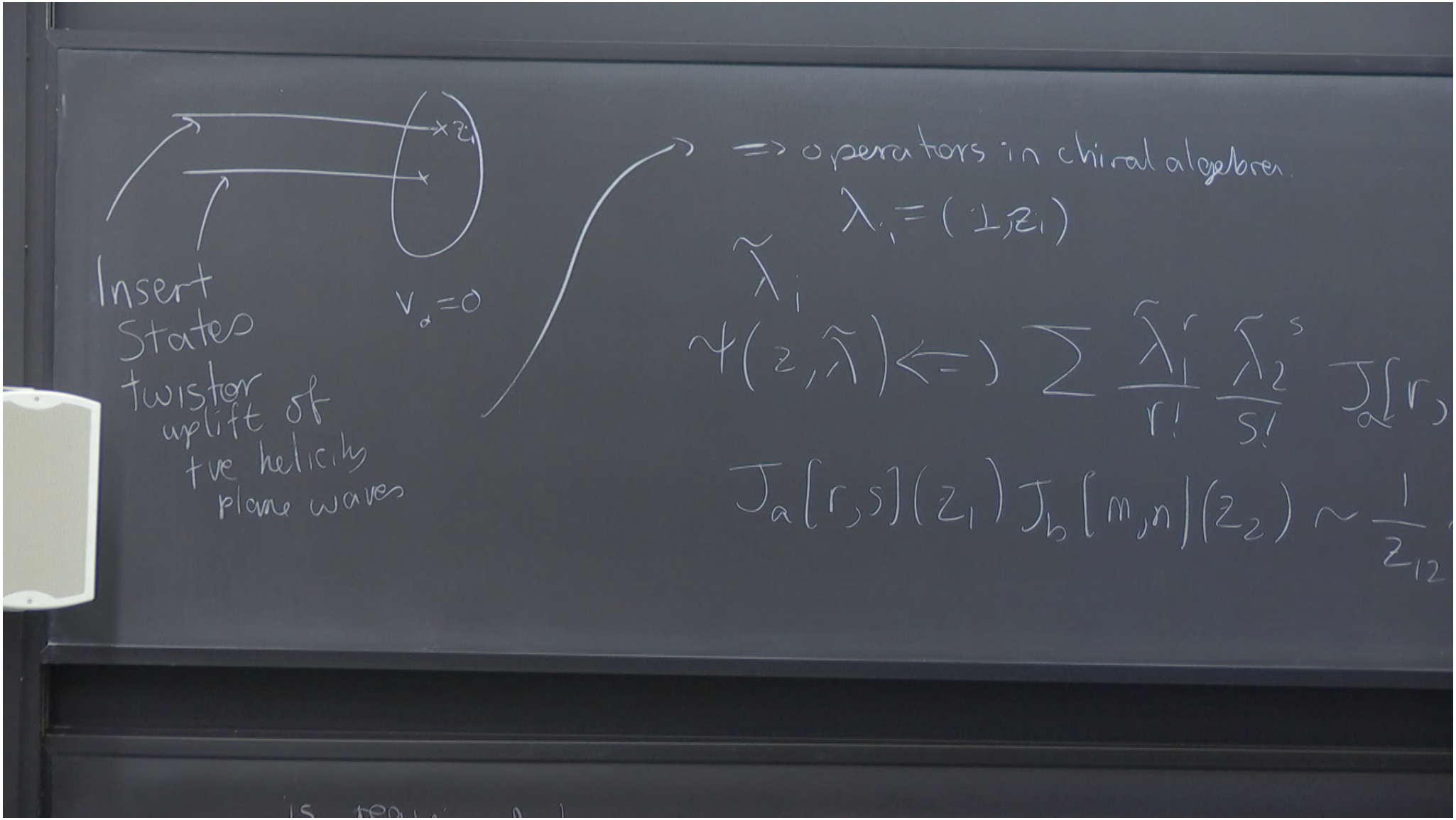
Title: Mathematical Physics Lecture (230505)

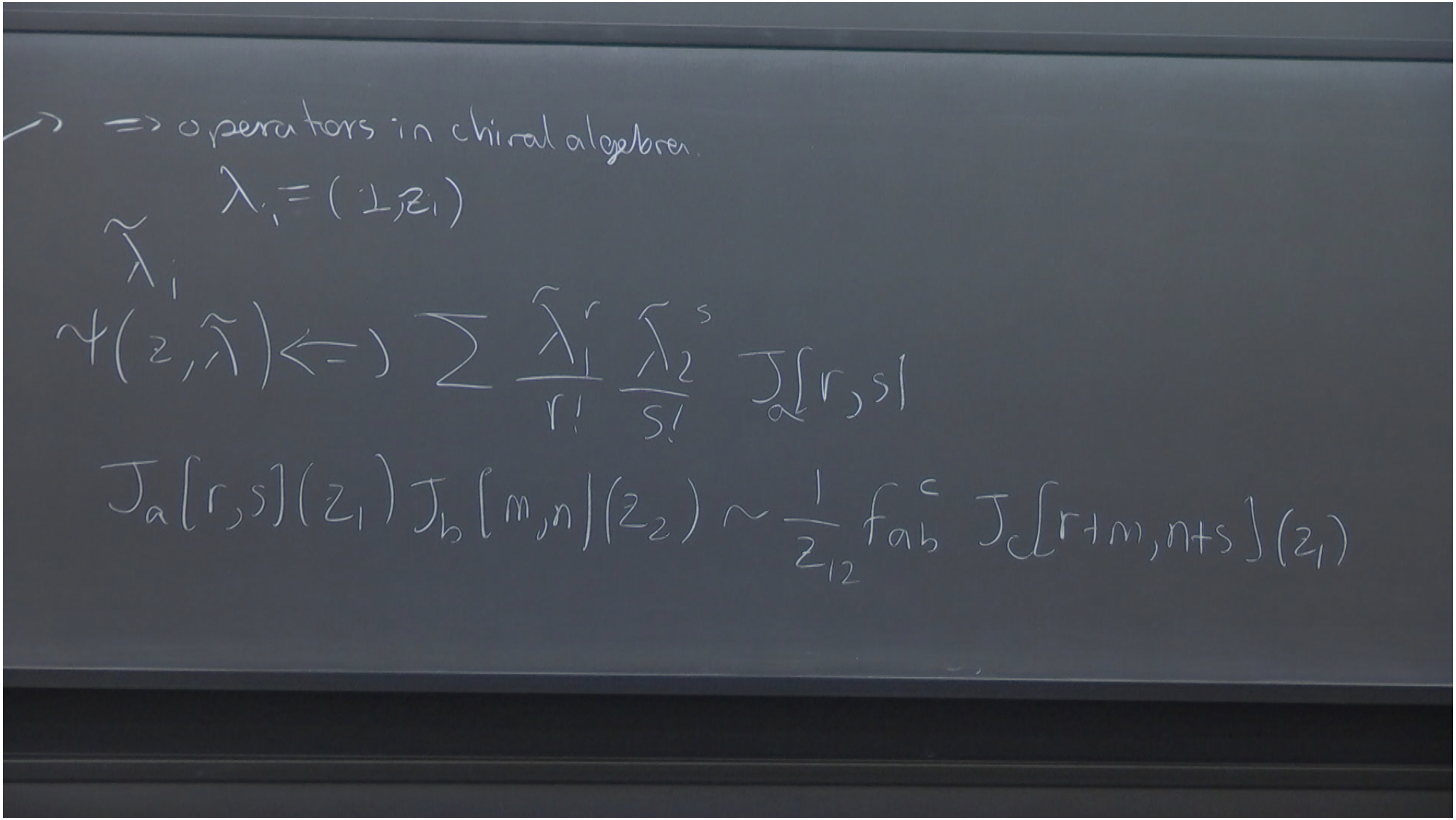
Speakers: Kevin Costello

Collection: Mathematical Physics - Elective (2022/2023)

Date: May 05, 2023 - 11:30 AM

URL: https://pirsa.org/23050012


Insert
States
twistor
uplift of
+ve helicity
plane waves
$V_\alpha = 0$
z_i
$\Rightarrow$ operators in chiral algebra
$\lambda_i = (1, z_i)$
$\tilde{\lambda}_i$
$\psi(z, \tilde{\lambda}) \Leftarrow \sum \frac{\tilde{\lambda}_1^r}{r!} \frac{\tilde{\lambda}_2^s}{s!} J_a[r,$
$J_a[r,s](z_1) J_b[m,n](z_2) \sim \frac{1}{z_{12}}$


=> operators in chiral algebras.
$\lambda_i = (1, z_i)$
$\tilde{\lambda}_i$
$\psi(z, \tilde{\lambda}) \leftrightarrow \sum \frac{\tilde{\lambda}_1^r}{r!} \frac{\tilde{\lambda}_2^s}{s!} J_a[r,s]$
$J_a[r,s](z_1) J_b[m,n](z_2) \sim \frac{1}{z_{12}} f_{ab}^c J_c[r+m, n+s](z_1)$

What is the gauge theory meaning?

1) $J_a[\tilde{\lambda}](z)$ is a state with momentum

$$P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \qquad \lambda = (1, z)$$

(*) says that in any tree level form factor 2 external states with momenta p^1, p^2

is required by gauge invariance

If $J_a[\tilde{\lambda}] = \sum \frac{\tilde{\lambda}_1^r \lambda_2^s}{r!\,s!} J_a[r,s]$

OPE is

$$(*) \quad J_a[\tilde{\lambda}^1](z_1) \cdot J_b[\tilde{\lambda}^2](z_2) \sim \frac{1}{z_{12}} J_c[\tilde{\lambda}^1 + \tilde{\lambda}^2](z_1)$$

For gravity, what is the algebra?

Lagrangian:

$$h \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2))$$

$$g \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(-6))$$

$$\int g(\bar{\partial} h + \tfrac{1}{2}\{h, h\})$$

$$\{a, b\} = \varepsilon^{\alpha\beta} \partial_{V_\alpha} a \, \partial_{V_\beta} b$$

$$\delta h = \bar{\partial}\chi + \{\chi, h\}$$

Most general defect that couples to h is

$$\sum_{r,s} \frac{1}{r!}\frac{1}{s!} \int_{v_i=0} (\partial_{v_1}^r \partial_{v_2}^s h)(z)\, W[r,s](z)\, dz$$

$W[r,s]$ currents in the 2d CFT.

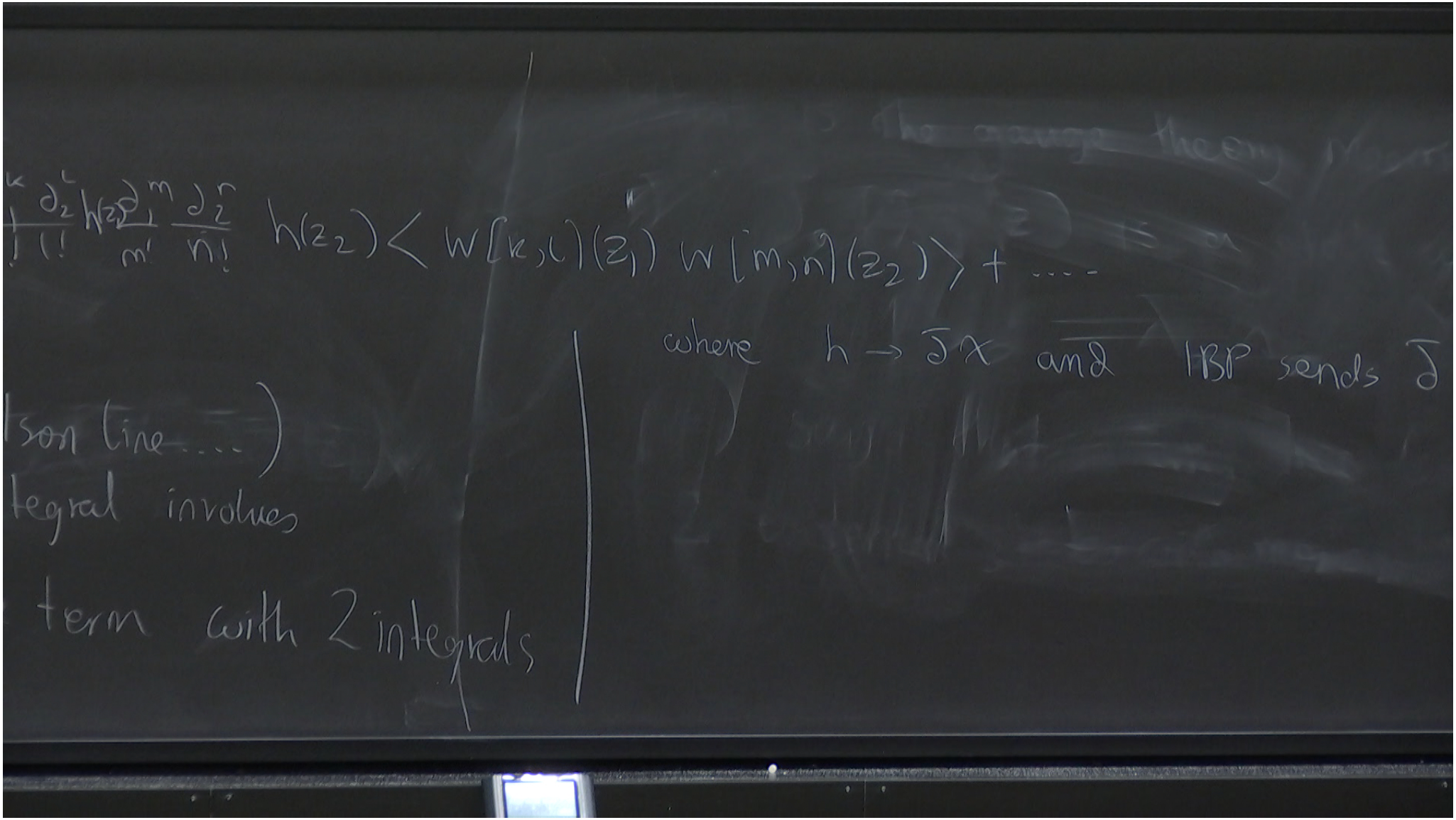
The expression

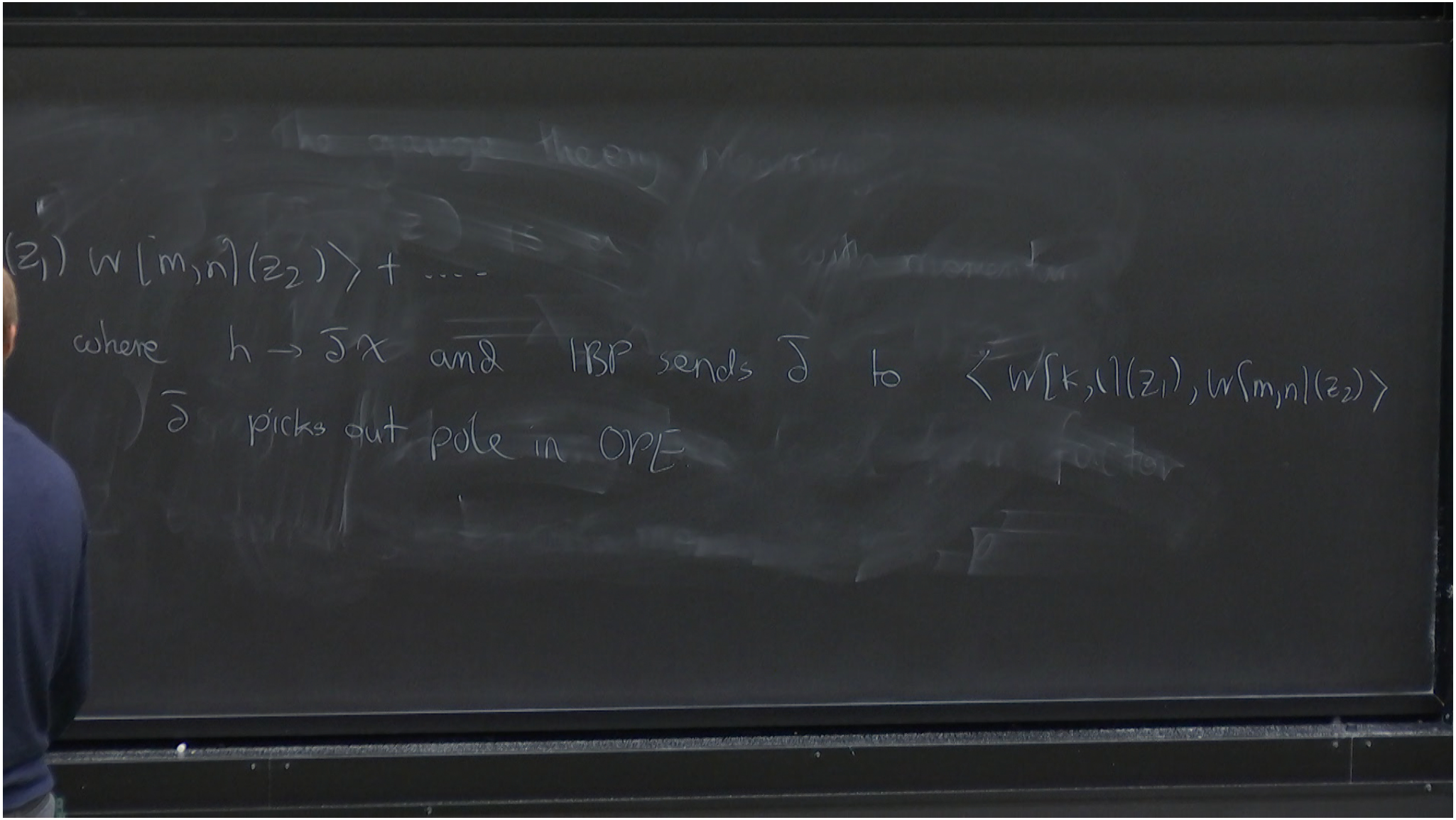
$$\sum_{r,s} \int \frac{\partial_1^r \partial_2^s}{r!\, s!} h(z_2) \langle w[r,s](z) \rangle + \sum_{\substack{k,l \\ m,n}} \int \frac{\partial_1^k \partial_2^l}{k!\, l!} h(z_2) \frac{\partial_1^m}{m!} \frac{\partial_2^n}{n!} h(z_2) \langle$$

must be gauge invariant (like Wilson line....)

The term in gauge variation with one integral involves

$\{\chi, h\}(z)$ must be cancelled by the term with $2 \ldots$

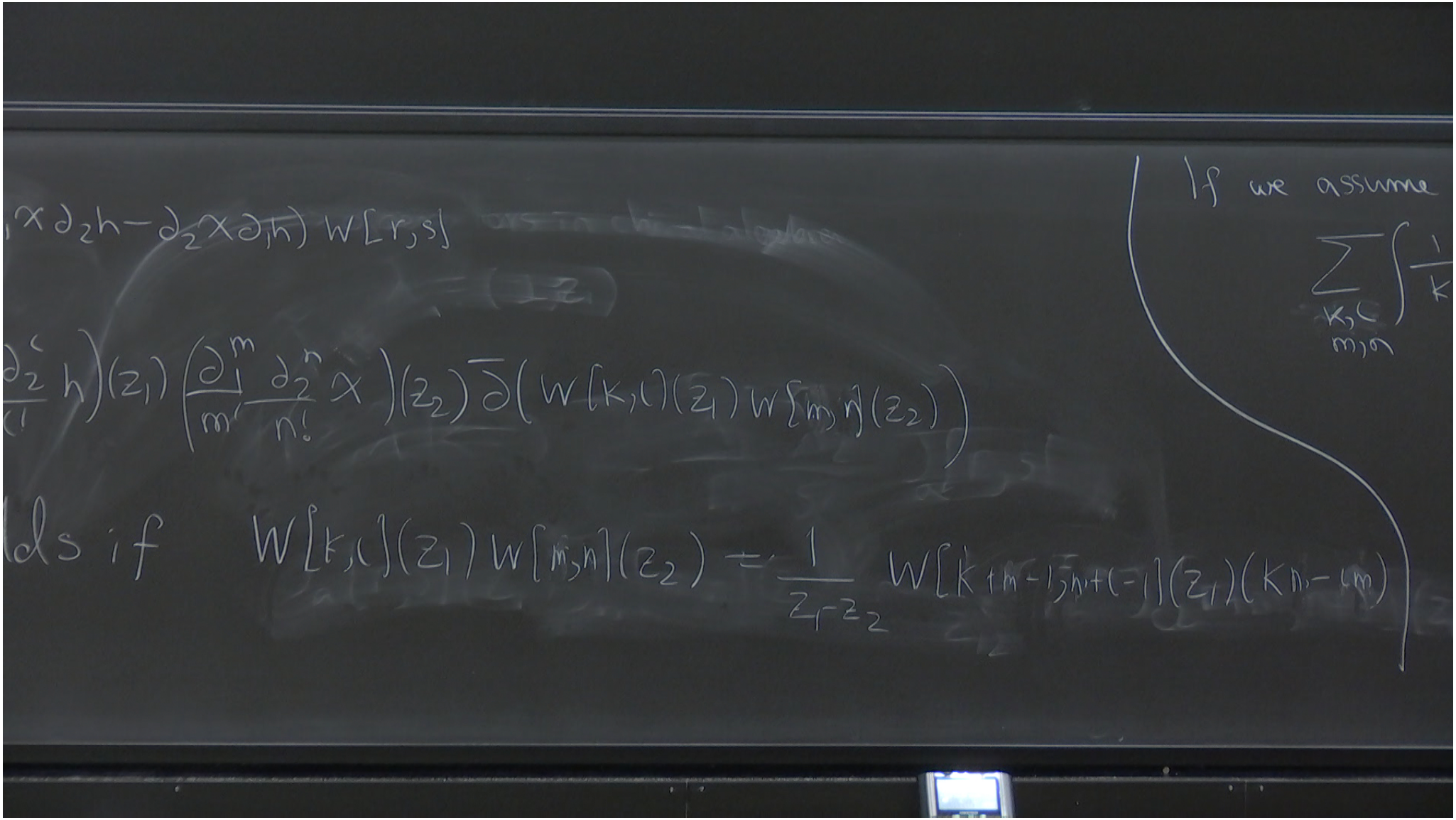

$\frac{\partial_2^l}{l!} h(z_2) \frac{\partial_1^m}{m!} \frac{\partial_2^n}{n!} h(z_2) \langle W[k,l](z_1)\, W[m,n](z_2) \rangle + \ldots$
where $h \to \partial x$ and IBP sends ∂
Ison line ...)
tegral involves
term with 2 integrals


$(z_1)\, W[m,n](z_2)\rangle + \cdots$
where $h \to \bar{\partial} x$ and IBP sends $\bar{J}$ to $\langle W[k,l](z_1), W[m,n](z_2)\rangle$
$\bar{\partial}$ picks out pole in OPE

$$\sum_{r,s} \int \frac{\partial_1^r}{r!} \frac{\partial_2^s}{s!} (\partial_1 x \partial_2 h - \partial_2 x \partial_1 h)\, W[r,s]$$

$$= \sum_{k,\ell,m,n} \iint \left(\frac{\partial_1^k}{k!} \frac{\partial_2^\ell}{\ell!} h \right)(z_1) \left(\frac{\partial_1^m}{m!} \frac{\partial_2^n}{n!} x \right)(z_2)\, \bar{\partial}\left(W[k,\ell](z_1)\, W[r,s](z_2) \right)$$

Claim This holds if $W[k,\ell](z_1)\, W[r,s](z_2) = \frac{1}{z_1 - z_2} W[k+$


$\partial_1 x \partial_2 h - \partial_2 x \partial_1 h) W[r,s]$
$\left(\frac{\partial_2^c}{c!} h\right)(z_1) \left(\frac{\partial_1^m}{m!} \frac{\partial_2^n}{n!} x\right)(z_2) \bar{\partial}\left(W[k,l](z_1) W[m,n](z_2)\right)$
$W[k,l](z_1) W[m,n](z_2) = \frac{1}{z_1 - z_2} W[k+m-1, n+l-1](z_1)(kn - lm)$
If we assume
$\sum_{\substack{k,l \\ m,n}} \int \frac{1}{k}$

If we assume the OPE, RHS is

$$\sum_{\substack{k,\ell \\ m,n}} \int \frac{1}{k!}\frac{1}{\ell!}\frac{1}{m!}\frac{1}{n!}\left(\partial_1^k \partial_2^\ell h\, \partial_1^m \partial_2^n \chi\right)(z)\,(k,\ell-(m,n))$$

$$= \text{LHS}$$

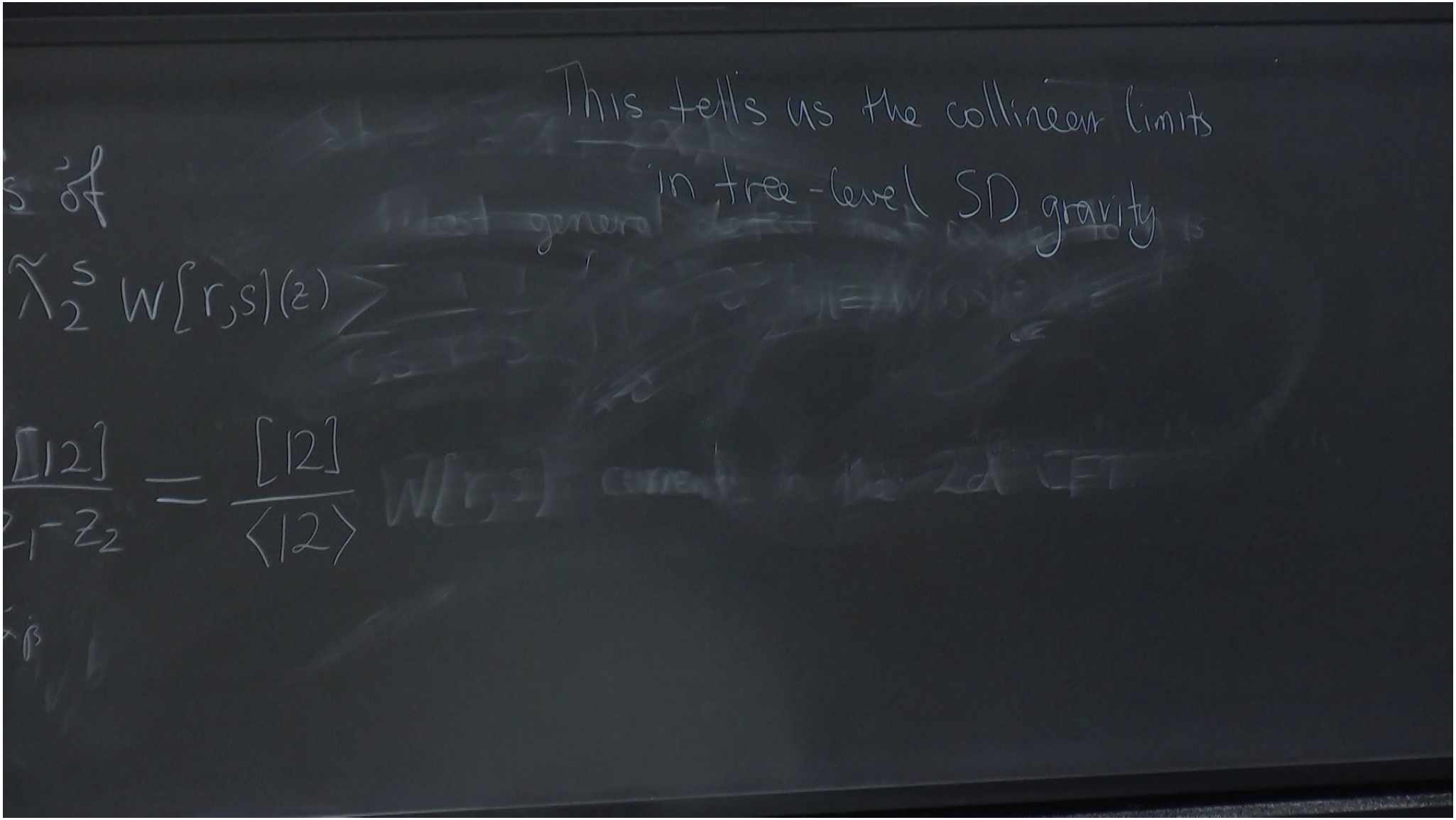
$$\ldots + (-1)^{\ldots}(z_1)(k_n - (m_n))$$

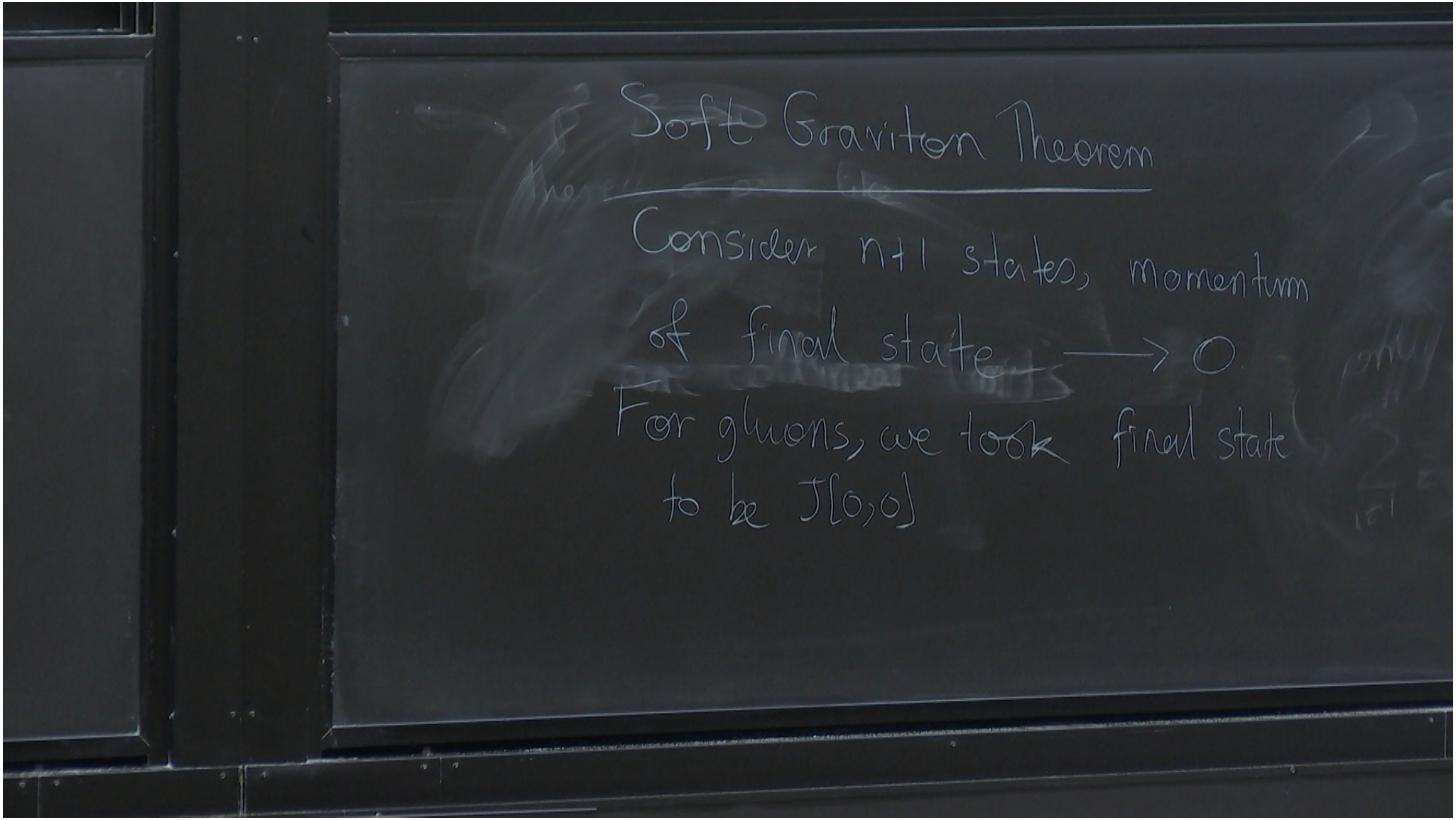
If we rewrite in terms of

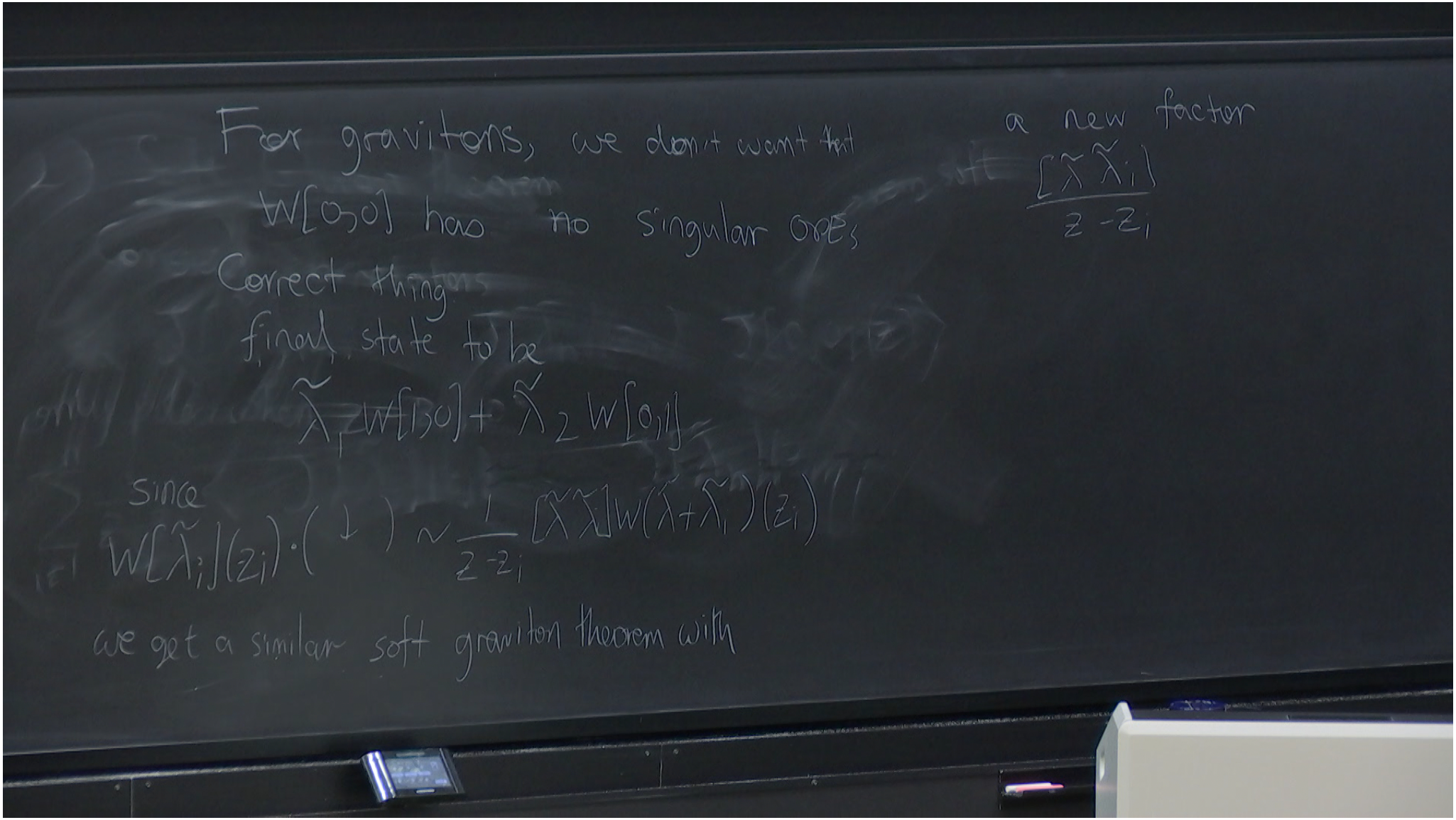
$$W[\tilde{\lambda}](z) = \sum \frac{1}{r!s!} \tilde{\lambda}_1^r \tilde{\lambda}_2^s W[r,s](z)$$

$$W[\tilde{\lambda}^1](z_1)\, W[\tilde{\lambda}^1](z_2) = \frac{[12]}{z_1 - z_2} = \frac{[12]}{\langle 12 \rangle}$$

$$[12] = \tilde{\lambda}^1_\alpha \tilde{\lambda}^2_\beta \varepsilon^{\alpha\beta}$$


This tells us the collinear limits
in tree-level SD gravity
$\lambda_2^s W[r,s](z)$
$\frac{[12]}{z_1 - z_2} = \frac{[12]}{\langle 12 \rangle}$


Soft Graviton Theorem
Consider n+1 states, momentum
of final state → 0
For gluons, we took final state
to be J[0,0]


For gravitons, we don't want that
W[0,0] has no singular OPEs
Correct thing
final state to be
$\tilde{\lambda}_1 W[1,0] + \tilde{\lambda}_2 W[0,1]$
since
$W[\tilde{\lambda}_i](z_i) \cdot (\downarrow) \sim \frac{1}{z - z_i} [\tilde{\lambda}\tilde{\lambda}] W(\tilde{\lambda} + \tilde{\lambda}_i)(z_i)$
we get a similar soft graviton theorem with
a new factor
$\frac{[\tilde{\lambda}\tilde{\lambda}_i]}{z - z_i}$