Title: Mathematical Physics Lecture (230503)

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Collection: Mathematical Physics - Elective (2022/2023)

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Last time:

Replace the hol. Wilson line
by a 2d chiral CFT

Parke - Taylor
= Correlators in 2d CFT

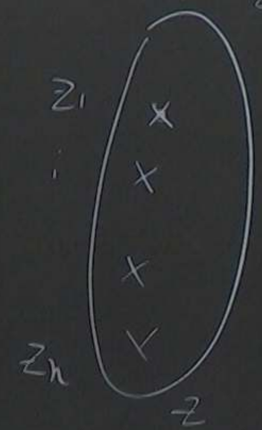
The most general 2d System = Celestial chiral algebra

Cartoon

4d states $\psi(\lambda,\tilde{\lambda}) \Longleftrightarrow \delta_{z=z_0}\, e^{v\tilde{\lambda}}$ $\lambda=(1,z_0)$

on twistor space

2d CFT on z-plane in the presence of states:



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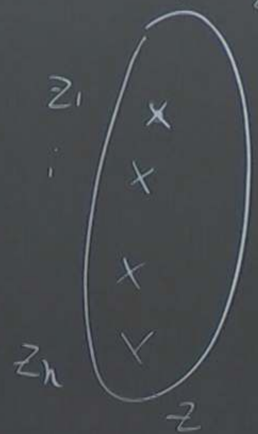

$\psi_i(\lambda_i, \tilde{\lambda}_i) \Longrightarrow$ a local operator

$\mathcal{O}_{\psi_i(\lambda_i)}(z_i)$

- Stay in 6d,
correlation functions
of $\mathcal{O}_{\tilde{\psi}_i(\tilde{\lambda}_i)}(z_i)$

CFT Correlators = Form factors

- Go to 4d, $\int$ out degrees of freedom on $\mathcal{C}$

$\Longrightarrow$ get a 4d local operator in the presence of these states.

4d Scattering process with a local operator ("form factor" of SDYM)

Correlators in 2d CFT

The most general 2d System = Celestial chiral algebra

z_n z $\mathcal{O}_{\tilde{\psi}_i(\lambda_i)}(z_i)$

- Go to 4d, $\int$ out degrees of freedom on $\mathbb{C}$
$\Rightarrow$ get a 4d local operator in the presence of these states.
4d Scattering process with a local operator ("form factor" of SDYM)

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- Stay in 6d, correlation functions of $\mathcal{O}_{\psi_i(\tilde{\lambda}_i)}(z_i)$

CFT Correlators = Form factors

Let's compute Parke-Taylor again

2 -ve helicity states $\Longleftrightarrow$ $\mathcal{B} \in \Omega^{3,1}(\mathbb{PT}) \otimes \mathfrak{g}$

Let's try to build a 2d chiral algebra

couples to both $\mathcal{B}$ and $\mathcal{A}$:

$$\int \mathcal{A}^a_{\bar{z}} J_a \, dz + \int \mathcal{B}^a_{\bar{z}} \tilde{J}_a \, dz$$

Last time

- Gauge invariance $\Rightarrow$

$$J_a(z_1)\, J_b(z_2) \sim f_{ab}^{\ c}\, J_c(z_1) \frac{1}{z_1 - z_2}$$

Including B we also have

$$J_a(z_1)\, \tilde{J}_b(z_2) \sim f_{ab}^{\ c}\, \tilde{J}_c(z_1) \frac{1}{z_1 - z_2}$$

$$\tilde{J}_a(z_1)\, \tilde{J}_b(z_2) \sim 0$$

Slightly tricky point

(this gives
$\langle J(z_1) J(z_2) \rangle = \frac{1}{z_{12}^2}$)

2d chiral algebra is
not a unitarity CFT.

E.g. in a unitary CFT
we always have

$$J_a(z_1) J_b(z_2) \sim \frac{1}{z_1 - z_2} J_c f_{ab}^c + \frac{K}{(z_1-z_2)^2} \mathbb{1}$$

$K > 0$

(this gives
$\langle J(z_1) J(z_2) \rangle = \frac{1}{z_{12}^2}$)

The OPEs are well-defined but, there may be more than one way to define correlation functions compatible with OPEs.

Fact

$\exists$ a unique correlator which is non-zero only with 2 $\tilde{J}$'s

$\langle \tilde{J}_a(z_1) \tilde{J}_b(z_2) \rangle = z_{12}^2 \delta_{ab}$

PT formula is

$$\langle 1^- 2^- 3^+ \cdots n^+ \rangle = \sum_{\sigma \in S_n} \frac{\langle 12 \rangle^4}{\langle \sigma_1 \sigma_2 \rangle \cdots \langle \sigma_n \sigma_1 \rangle} \, tr(t_{\sigma_1} \cdots t_{\sigma_n})$$

eg.

$$\langle 1^- 2^- 3^+ \rangle = \langle 12 \rangle^4 \left(\frac{tr(t_1 t_2 t_3)}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} + \frac{tr(t_2 t_1 t_3)}{\langle 21 \rangle \langle 13 \rangle \langle 31 \rangle} \right)$$

Chiral algebra

$A \in \Omega^{0,1}$

$B \in \Omega^{3,1}$

B has extra $dz\, dv_1\, dv_2$
is of $\dim^n$ -2

J $\dim^n$ 1

$\tilde{J}$ " -1

PT formula is

$\langle 1^- 2^- 3^+ \ \ n^+ \rangle = \sum$

eg.

$\langle 1^- 2^- 3^+ \rangle =$ $\langle 12 \rangle$

Ω point fn.

$$\ldots) J_c(z_3)\rangle \underset{z_3\to z_1}{\sim} \frac{1}{z_{31}} f_{ca}^{d} \langle \tilde{J}_d(z_1)\tilde{J}_b(z_2) = \frac{z_{12}^2}{z_{31}} f_{cab}$$

plane in the presence of state

local operator

Residue of pole → 2 point fn.

$$\langle \tilde{J}_a(z_1)\, \tilde{J}_b(z_2)\, J_c(z_3) \rangle \underset{z_3 \to z_1}{\sim} \frac{1}{z_{31}} f_c^{d} \ldots$$

$f_{ca}^{d}\, \tilde{J}_d$

In CFT, what is

$$\langle \tilde{J}_a(z_1)\, \tilde{J}_b(z_2)\, J_c(z_3) \rangle ?$$

Pole at z_{13} is

$$\tilde{J}_a(z_1)\, J_c(z_3) = \frac{1}{z_{31}} f_{ca}^{\ d}\, \tilde{J}_d$$

Pole at z_{23}

$$\tilde{J}_b(z_2)\, J_c(z_3) = \frac{1}{z_{32}} f_{cb}^{\ d}\, \tilde{J}_d$$

Resid

$\langle \tilde{J}$

$\langle \tilde{J}$

pole → 2 point fn.

$$\tilde{J}_b(z_2)\, J_c(z_3)\rangle \underset{z_3 \to z_1}{\sim} \frac{1}{z_{31}} f_{ca}^{\ d} \langle \tilde{J}_d(z_3)\, \tilde{J}_b(z_2) = \frac{z_{23}^2}{z_{31}} f_{cab}$$

Residue of pole → 2 point fn.

$$\langle \tilde{J}_a(z_1)\,\tilde{J}_b(z_2)\,J_c(z_3)\rangle \underset{z_1\to z_3}{\sim} \frac{1}{z_{31}} f_{ca}^{\;\;d}\langle \tilde{J}_d(z_3)\,\tilde{J}_b(z_2) \rangle =$$

$$\langle \tilde{J}_a(z_1)\,\tilde{J}_b(z_2)\,J_c(z_3)\rangle \underset{z_2\to z_3}{\sim} \frac{z_{13}}{z_{23}} f_{cba}$$

Residue of pole ⇒ 2 point fn.

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Claim: The unique global fn with these poles is

$$z_{12}^3\left(\frac{1}{z_{23}z_{31}} f_{abc}\right)$$

Residue of pole = 2 point fn.

$$\langle \tilde{J}_a(z_1)\tilde{J}_b(z_2)J_c(z_3)\rangle \underset{z_1\to z_3}{\sim} \frac{1}{z_{31}} f_{ca}^{\ d}\langle \tilde{J}_d(z_3)\tilde{J}_b(z$$

$$\langle \tilde{J}_a(z_1)\tilde{J}_b(z_2)J_c(z_3)\rangle \underset{z_2\to z_3}{\sim} \frac{z_{13}^2}{z_{23}} f_{cba}$$

Claim: The unique global fn with these poles is

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Twistor geometry (or spin/dimension)

$$\Rightarrow \langle \ldots J(z) \ldots \rangle \sim \frac{1}{z^2} \quad \text{as } z \to \infty$$

$$\langle \ldots \tilde{J} \ldots \rangle \sim z^2 \qquad z \to \infty$$

(this gives

$\langle J(z_1)\, J$

The OPE

but, the

than o

correl

compa

Residue of pole → 2 point fn.

$$\langle \tilde{J}_a(z_1)\, \tilde{J}_b(z_2)\, J_c(z_3)\rangle \underset{z_1 \to z_3}{\sim} \frac{1}{z_{31}} f_{ca}{}^{d} \langle \tilde{J}_d(z_3)\, \tilde{J}_b(z_2)\rangle =$$

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Some algebra ⇒ this is 3pt PT formula

Twistor geometry (or spin/dimension)

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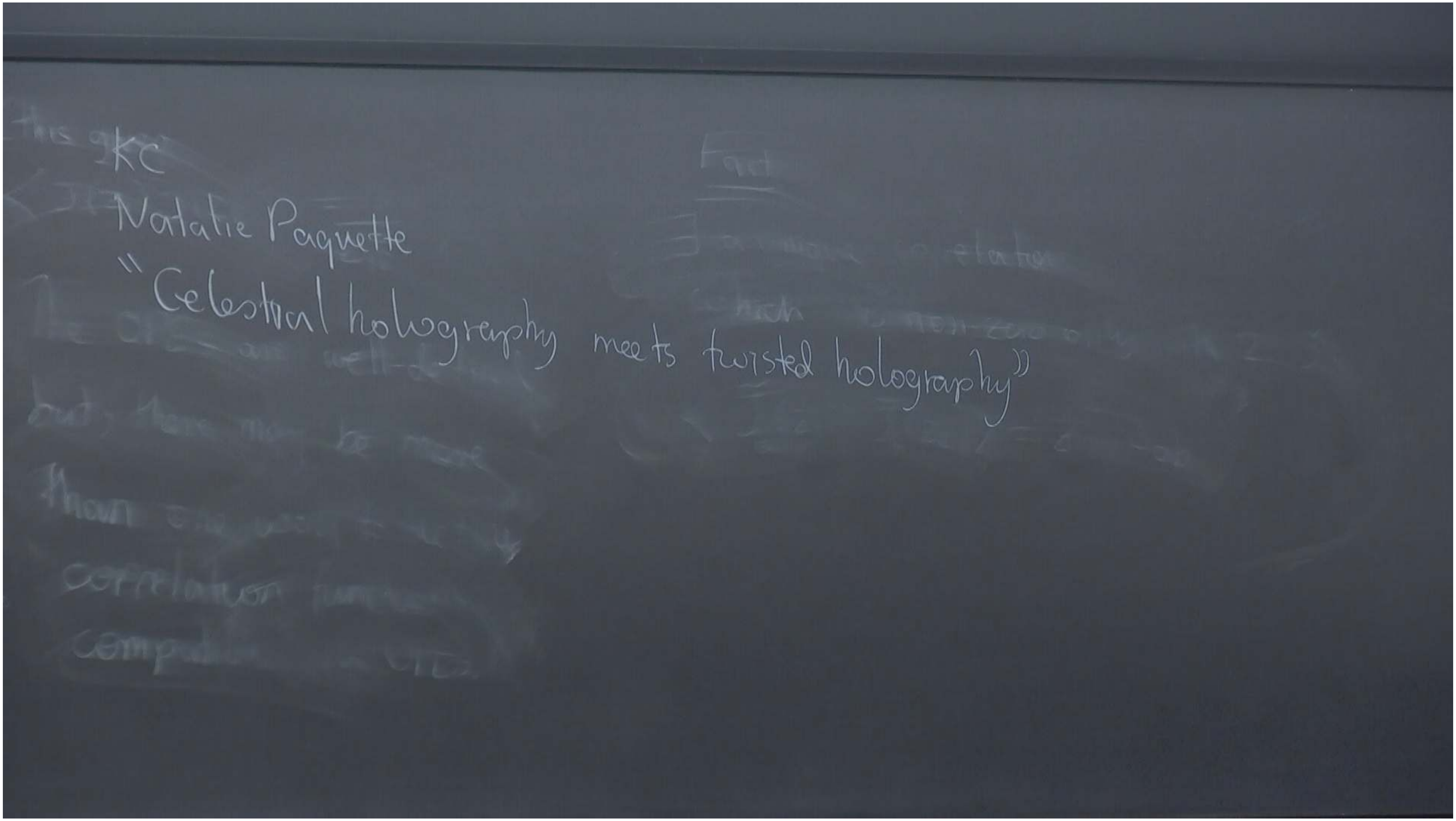
KC

Natalie Paquette

"Celestial holography


KC
Natalie Paquette
"Celestial holography meets twisted holography"

Exercise n-point function

$= $ Parke-Taylor n-point function.

What about more general couplings?

E.g. couple $\partial_{v_1} A$ to an operator?

The most general way of coupling a 2d chiral CFT to the theory on $\mathbb{PT}$ is:

$$\sum_{n,m \geq 0} \frac{1}{n!\,m!} \left(\int_{z} \partial_{v_1}^{n} \partial_{v_2}^{m} A^{a} J_{a}[n,m] + \int \partial_{v_1}^{n} \partial_{v_2}^{m} B^{a} \tilde{J}_{a}[n,m] \right)$$

Gauge invariance ⟹ the OPEs

$$J_a(z_1)[n,m]\, J_b(z_2)[r,s] \sim \frac{1}{z_{12}} J_c[n+r, m+s] f_{ab}^{\ c}$$

$$J_a(z_1)[n,m]\, \tilde{J}_b(z_2)[r,s] \sim \frac{1}{z_{12}} \tilde{J}_c[n+r, m+s] f_{ab}^{\ c}$$

$$\tilde{J}[\ ]\, \tilde{J}[\ ] \sim 0$$

This is the celestial chiral alg. for SDYM

A state of +ve (-ve) helicity maps to an element of the algebra by

+ve $\displaystyle\sum \frac{\tilde{\lambda}_1^r \tilde{\lambda}_2^s}{r!\, s!} J[r,s]$

-ve $\quad$ " $\quad \tilde{J}$

Gauge invariance $\Rightarrow$ the OPEs

$$J_a(z_1)[n,m]\, J_b(z_2)[r,s] \sim \frac{1}{z_{12}} J_c[n+r, m+s] f_{ab}^{c}$$

$$J_a(z_1)[n,m]\, \tilde{J}_b(z_2)[r,s] \sim \frac{1}{z_{12}} \tilde{J}_c[n+r, m+s] f_{ab}^{c}$$

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This

A

of

+ve

-ve

– Local operators in 4d

= Ways of defining correlation functions compatible w. OPE

– Form factors for a local operator = Correlators

Claim The uni

SD YM

YM $\quad \mathrm{tr}\, F_- \wedge F_-(z_2)$

$\frac{1}{\langle ij \rangle} \quad \frac{1}{z_i - z_j}$ ✓ plane in the

Claim $\frac{1}{[ij]}$ do not have a chiral algebra

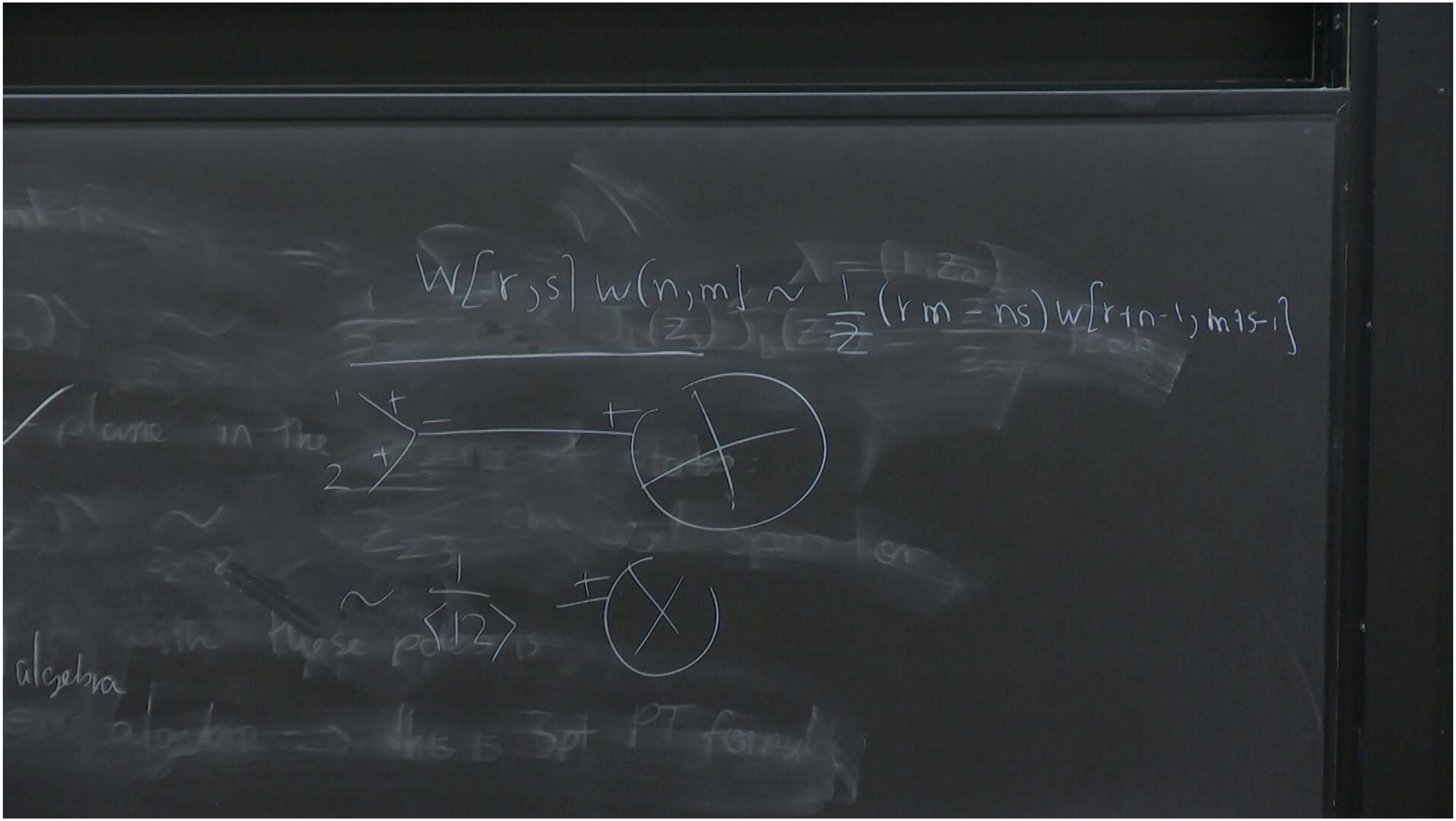
with these poles is

$\frac{f_{abc}}{\bar{z}_i - \bar{z}_j}$

algebra $\Rightarrow$ this is 3pt PT formula

$$W[r,s]\,W(n,m] \sim \frac{1}{z}(rm - ns)\,W[r+n-1, m+s-1]$$

algebra


W[r,s] w(n,m) ~ 1/z (rm - ns) w[r+n-1, m+s-1]
~ 1/⟨12⟩
algebra