Title: Mathematical Physics Lecture (230501)

Speakers: Kevin Costello

Collection: Mathematical Physics - Elective (2022/2023)

Date: May 01, 2023 - 11:30 AM

URL: https://pirsa.org/23050010

Topics

1) Connections with celestial holography
2) Loop level quantities on twistor space
3) CSW rules for more complicated tree-level amplitudes

In

In SDYM, we can compute the form factor
for tr B^2 as a hol. Wilson line

Celestial holography:

We can recast this as a computation in a 2d CF

Reminder: The formula for a Wilson line

$$P\mathrm{Exp}\int_{\mathbb{R}} A$$

Is BAD!

Not the fundamental

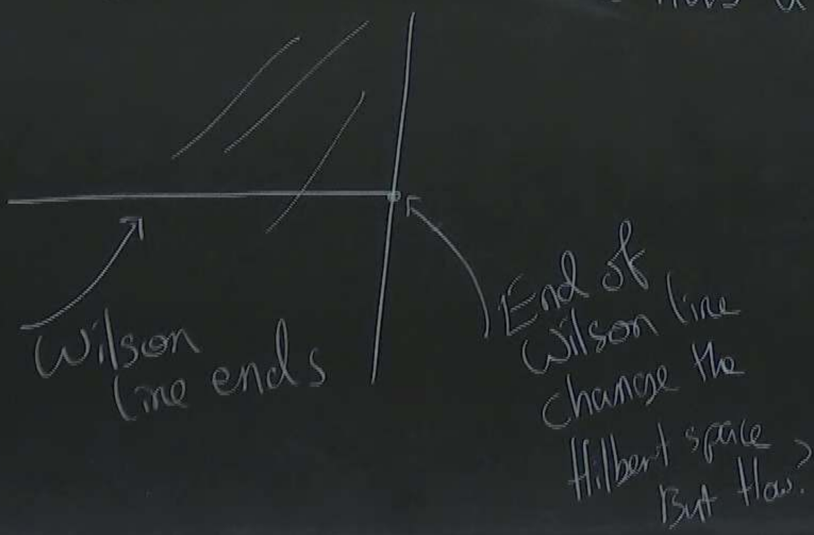
definition.

E.g

space-tim

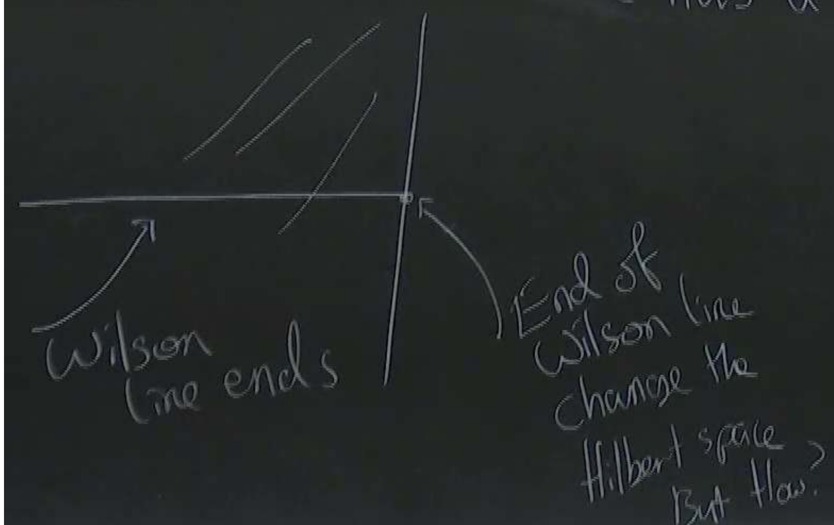
definition.

E.g space-time has a boundary:



definition

E.g space-time has a boundary:



Better.

Choose a rep. R of G.

A Wilson line is a QM system on the line, with Hilbert space R and Hamiltonian, A in this rep.

(Component of A pointing along line)

Evolution of a state

$\psi \in R$ from 0 to T

<u>Is</u> $P\mathrm{Exp}\int_0^T A_t\,dt$

Evolution of a state
$\psi \in R$ from 0 to T
is $\quad P\mathrm{Exp}\int_0^T A_t\,dt$

$\psi(t=0)$

χ gauge param.

$$\delta\psi = \chi(t=0)\,\psi$$

$$\frac{\partial\psi}{\partial t} = A_t\cdot\psi$$

$$\left[\frac{\partial}{\partial t}, \chi\right] = 0$$

$\chi(t=0)\psi$

$A_t \cdot \psi$

$] = 0$

<u>Ex</u>

$G = su(n)$

$$R = \bigoplus_{i=0}^{n} \Lambda^i F$$

This comes from a Lagrangian system

$\psi_i \in F$

$\bar{\psi}^i \in \bar{F}$

$$\int_{\mathbb{R}} \psi(\partial_t + A_t)\bar{\psi}$$

$\psi, \bar{\psi}$ transform under gauge symmetry in the natural way, this is gauge invariant.

Hilbert space of $\psi, \bar{\psi}$ system is gen. by $|\emptyset\rangle$ with

$\bar{\psi}^i |\emptyset\rangle = 0$

States are

$\psi_{i_1} \cdots \psi_{i_k} |\emptyset\rangle$

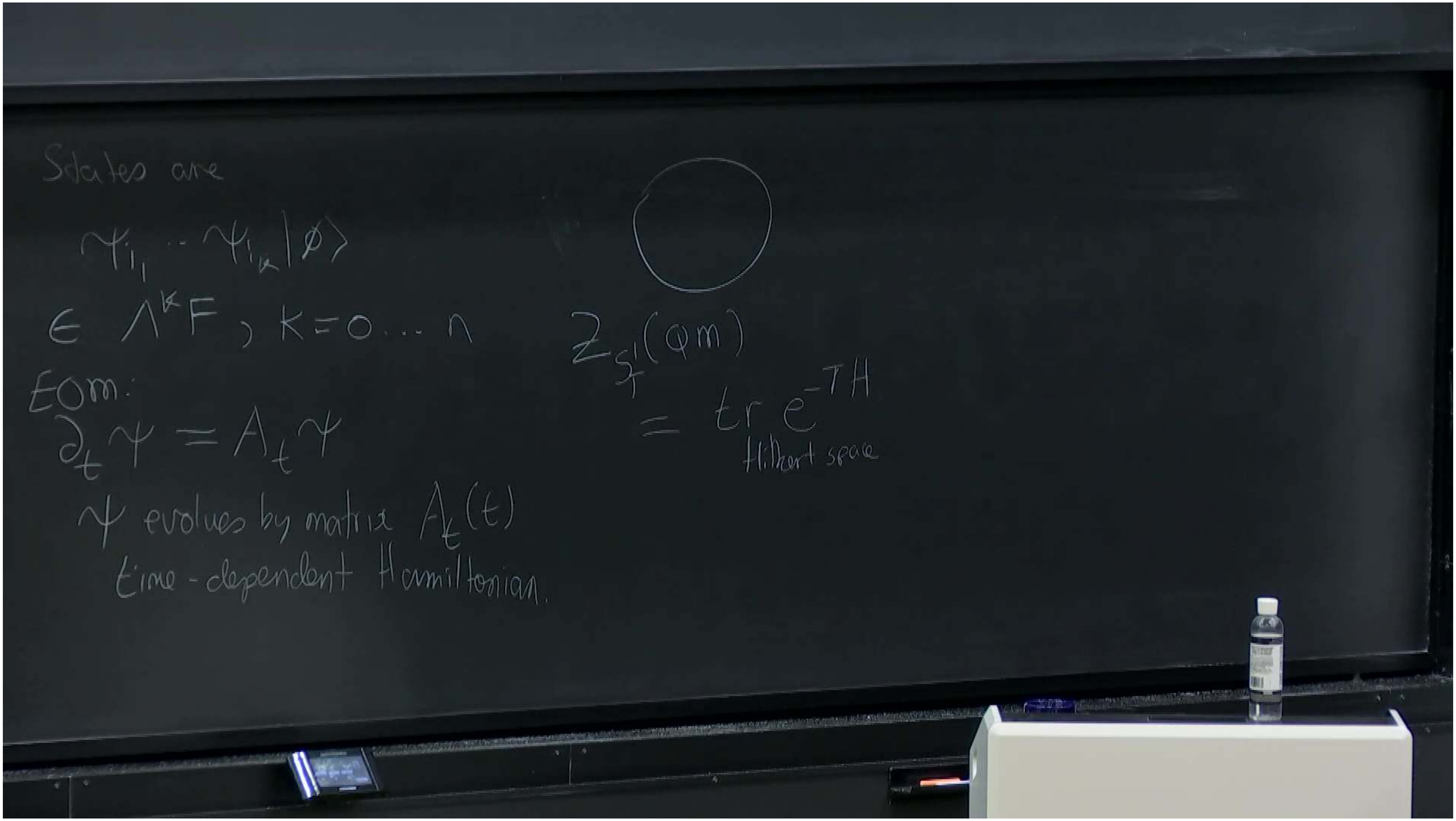
$\in \Lambda^k F$, $k = 0 \ldots n$

EOM:

$\partial_t \psi = A_t \psi$

ψ evolves by matrix $A_t(t)$

time-dependent Hamiltonian.


States are
$\psi_{i_1}\cdots\psi_{i_k}|\phi\rangle$
$\in \Lambda^k F$, $k=0\dots n$
EOM:
$\partial_t\psi = A_t\psi$
ψ evolves by matrix $A_t(t)$
time-dependent Hamiltonian.
$Z_{S^1_T}(QM)$
$= \mathrm{tr}\, e^{-TH}$
Hilbert space

$P\mathrm{Exp}\int_0^T A_t\,dt \quad R_{t=0} \longrightarrow R_{t=T} \qquad = \chi(t=T)$

$\psi \in R_{t=0}$

We need, for χ a gauge transformation,

$\left(P\mathrm{Exp}\int_0^T A_t\,dt\right)\chi(t=0)\,\psi$

$P\,Exp \int_0^T A_t\,dt$ $\quad R_{t=0} \xrightarrow{\text{formula}} R_{t=T}$ $\quad = \hat{X}(t$

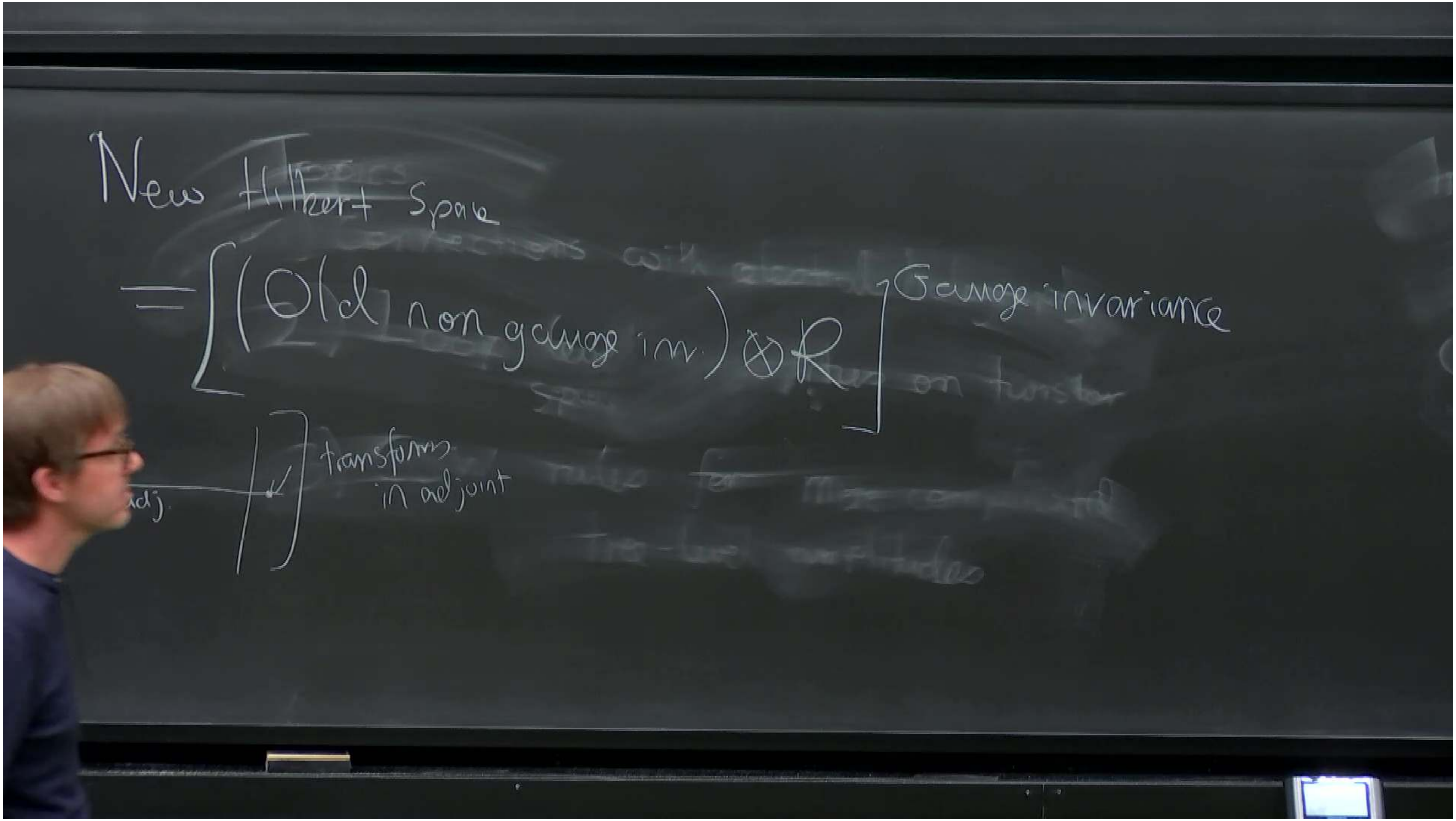
$\Psi \in R_{t=0}$

We need, for X a gauge transformation,

$\left(P\,Exp \int_0^T A_t\,dt\right) X(t=0)\,\Psi$

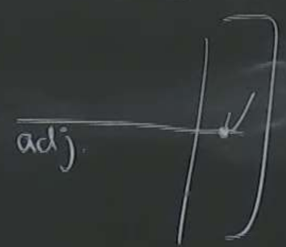
$$= \chi(t=T)\, P\mathrm{Exp}\left(\int_0^T A_t\, dt\right) \psi(t=0)$$

This is true by standard argument


New Hilbert Space
= [(Old non gauge inv.) ⊗ R] Gauge invariance
adj.
transforms in adjoint

New Hilbert Space

$$= \Big[\big(\text{Old non gauge inv.} \big) \otimes R \Big]_{\text{Gauge invariance}}$$



transforms in adjoint

What about 2d Systems?

On twistor space (a patch, $= \mathbb{C}^3$)

$A \in \Omega^{0,1}(\mathbb{C}^3, \mathfrak{g})$

$B \in \Omega^{3,1}(\mathbb{C}^3, \mathfrak{g})$

$$\int B F^{0,2}(A)$$

Hol. Wilson line is roughly

$$\int_{z_1 \ldots z_n \in \mathbb{C}} \frac{A(z_1) \cdots A(z_n)}{z_{12} \cdots z_{n1}}$$

Want to realize this by
Coupling to a 2d chiral system.

2d chiral CFT:

- States (= operators) $\mathcal{O}_i \in$ vector space V
- $\mathcal{O}_i$ has dimension (a.k.a. spin) d_i
- Operator product expansion

$$\mathcal{O}_i(0)\,\mathcal{O}_j(z) \sim \sum_{\substack{k,\\ d_i+d_j-d_k>0}} \frac{1}{z^{d_i+d_j-d_k}}\,\mathcal{O}_k\, C_{ij}^k$$

In any computation,

$\mathcal{O}_i(0)\,\mathcal{O}_j(z)$ can be replaced by

OPE plus regular terms

Suppose we have a 2d CFT, C

and we couple to holomorphic BF

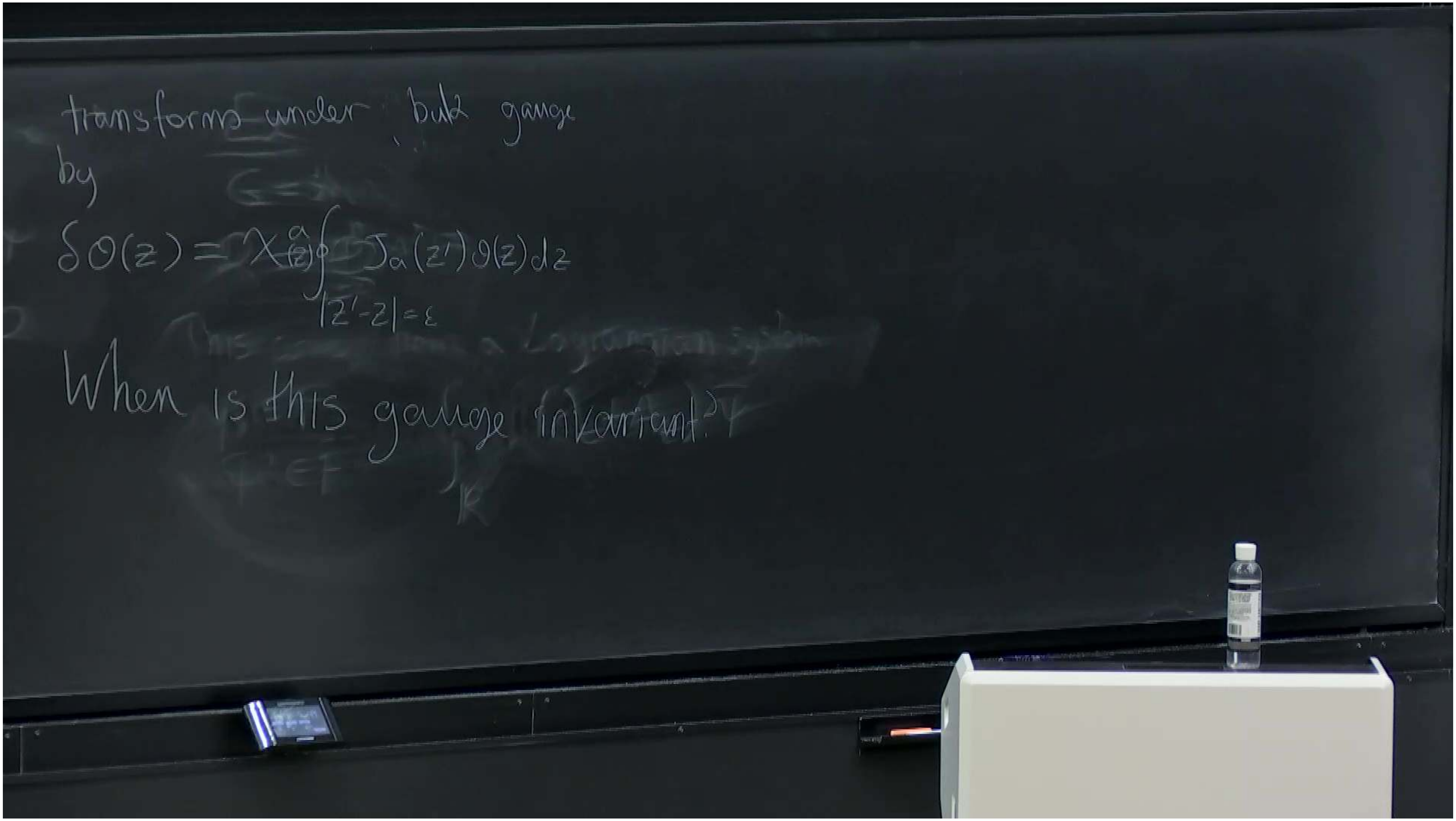
theory by saying:

1) $A^a_{\bar{z}}$ couples to operators

$J_a(z)$ of spin 1

2) Any operator $\mathcal{O}(z)$

transforms under


transforms under bulk gauge
by
$\delta O(z) = \chi^a(z) \oint_{|z'-z|=\varepsilon} J_a(z') O(z) \, dz$
When is this gauge invariant?

If we $\int$ out CFT degrees of freedom

we are left with

$$\sum_{n\geq 0} \int_{\mathbb{C}^n} A(z_1)\cdots A(z_n) \langle J(z_1)\cdots J(z_n)\rangle dz_1\cdots dz_n$$

1st 2 terms:

$n=1$ $\quad \int_{\mathbb{C}} A(z)\langle J(z)\rangle \qquad A \to A + \bar\partial X + [X, A]$

this becomes

$$\int \bar{\partial} \chi \langle J \rangle + \int [\chi, A] \langle J \rangle$$

$$\shortparallel$$

$$-\int \chi \langle \bar{\partial} J \rangle = 0$$ as we have a chiral CFT.

$$\int [\chi, A] \langle J(z) \rangle$$ must cancel ω. $n=2$ case:

$$\iint A(z_1) A(z_2) \langle J(z_1) J(z_2) \rangle$$

gauge variation gives

$$\iint \bar{\partial} \chi(z_1) A(z_2) \langle J(z_1) J(z_2) \rangle + \dots$$

IBP:

$$\iint \chi(z_1) A(z_2) \langle \bar{\partial} J(z_1) J(z_2) \rangle = \int [\chi, A](z) \langle J \rangle$$

Conclude:

$$\bar{\partial}\, J^a(z_1)\, J^b(z_2) = \delta_{z_1=z_2} f^{ab}_{\ \ c} J^c(z_1)$$

So we have OPE:

$$\boxed{2\pi i\, J^a(z_1)\, J^b(z_2) \sim \frac{1}{z_1-z_2} f^{ab}_{\ \ c} J^c(z_1)}$$

WZW

or Kac-Moody at level 0

Conclude:

$$\bar{\partial}\, J^a(z_1)\, J^b(z_2) = \delta_{z_1=z_2}\, f^{ab}_{\ \ c}\, J^c(z_1)$$

So we have OPE:

$$\boxed{2\pi i\, J^a(z_1)\, J^b(z_2) \sim \frac{1}{z_1-z_2}\, f^{ab}_{\ \ c}\, J^c(z_1)}$$

Structure constants of gauge algebra

WZW or Kac-Moody at level 0

Conclude:

$$\bar\partial\, J^a_{(z_1)} J^b_{(z_2)} = \delta_{z_1=z_2} f^{ab}_{\ c} J^c(z_1)$$

So we have OPE:

$$\boxed{2\pi i\, J^a(z_1)\, J^b(z_2) \sim \frac{1}{z_1-z_2} f^{ab}_{\ c} J^c(z_1)} \left(+ \frac{K}{(z_{12})^2}\, \mathrm{Id} \right)$$

level K

WZW
or Kac-Moody at level 0

Structure constants of gauge algebra

Conclude:

$$\bar{\partial}\, J^a(z_1)\, J^b(z_2) = \delta_{z_1=z_2}\, f^{ab}_c\, J^c(z_1)$$

So we have OPE:

$$\boxed{2\pi i\, J^a(z_1)\, J^b(z_2) \sim \frac{1}{z_1-z_2} f^{ab}_c\, J^c(z_1)} \left(+ \frac{K}{(z_{12})^2} \mathrm{Id} \right)$$

level K

Structure constants of gauge algebra

WZW or Kac-Moody at level 0

2d chiral

A, J