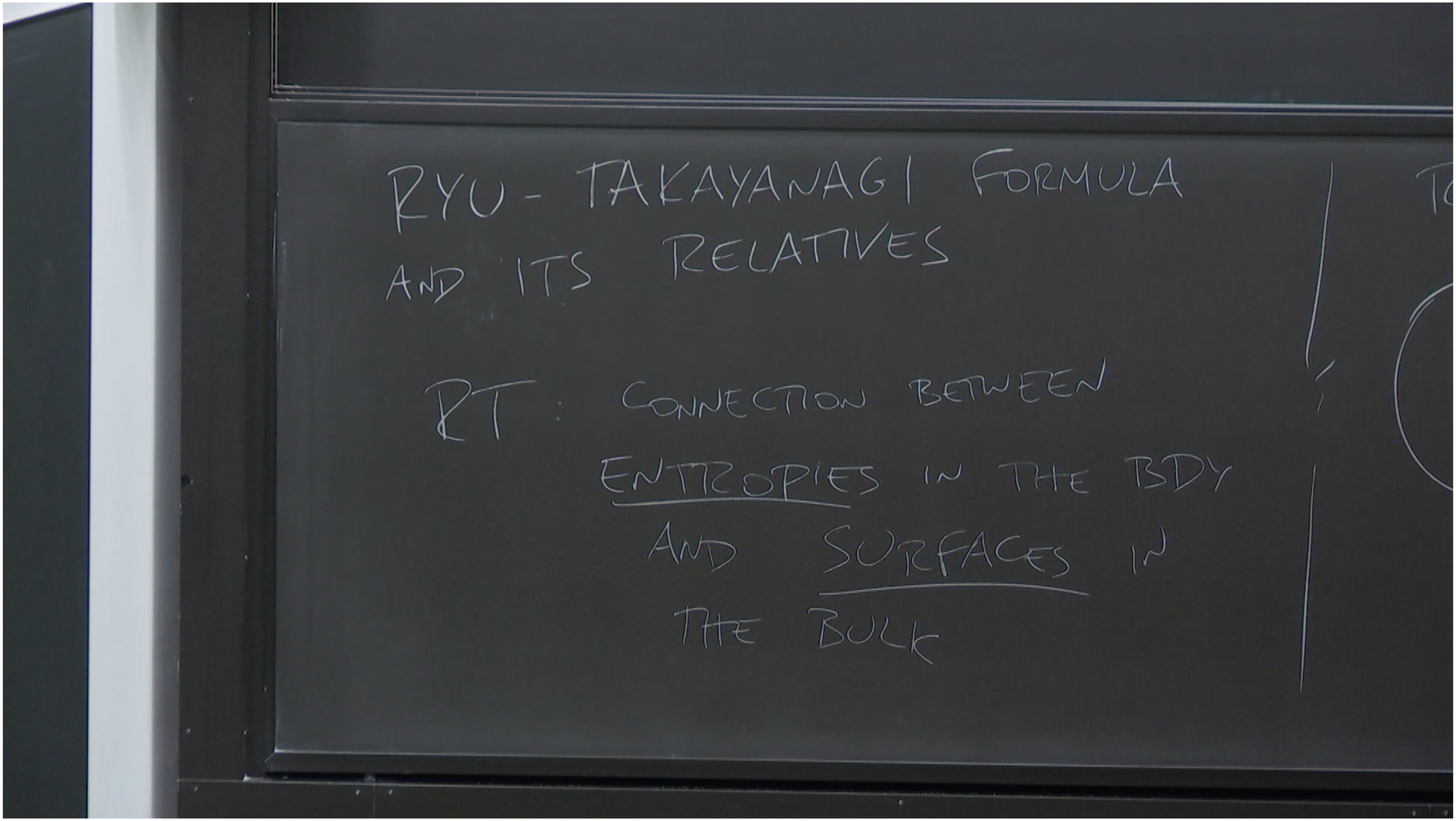
Title: AdS/CFT Lecture (230503)

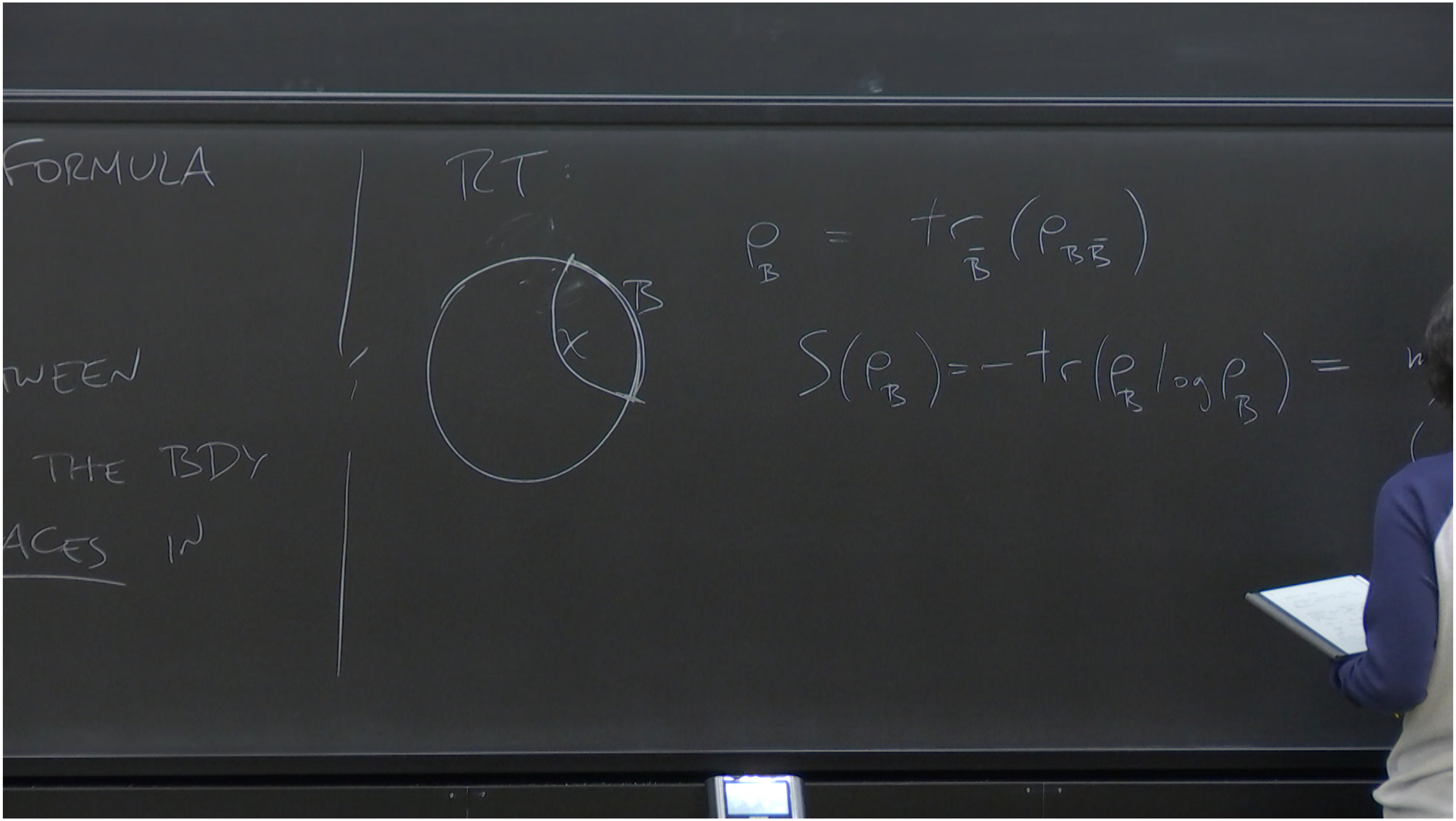
Speakers: David Kubiznak

Collection: AdS/CFT (2022/2023)

Date: May 03, 2023 - 9:00 AM

URL: https://pirsa.org/23050008


RYU - TAKAYANAGI FORMULA
AND ITS RELATIVES
RT : CONNECTION BETWEEN
ENTROPIES IN THE BDY
AND SURFACES IN
THE BULK


FORMULA
TWEEN
THE BDY
ACES IN
RT:
B
x
$\rho_B = \mathrm{tr}_{\bar{B}}(\rho_{B\bar{B}})$
$S(\rho_B) = -\mathrm{tr}(\rho_B \log \rho_B) =$

$$\mathrm{tr}_{\bar{B}}\left(\rho_{B\bar{B}}\right)$$

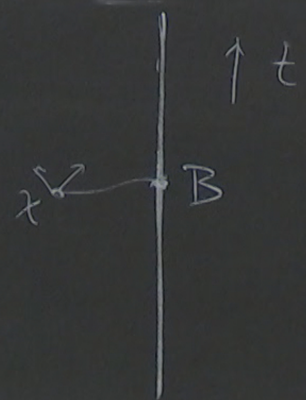
$$\rho_B\Big) = -\mathrm{tr}\left(\rho_B \log \rho_B\right) = \min_{x}\left(\frac{A(x)}{4G_N}\right)$$

x IS HOMOLOGOUS TO $B \iff \exists b$ SUCH THAT $\partial b = x \cup B$.

LATER IMPROVEMENTS:

1. HRT (COVARIANT EXTREMAL SURFACE)

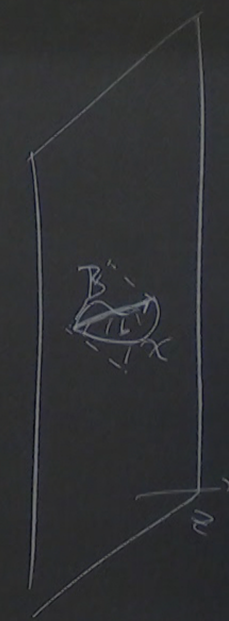
↳ TIME-DEPENDENT



$$S(\rho_B) = \min_{\chi} \, \mathrm{ext}\left(\frac{A(\chi)}{4G_N}\right)$$

↳ χ is EXTREMAL

$$\frac{\delta}{\delta\chi}\left(\frac{A(\chi)}{4G_N}\right) = 0$$



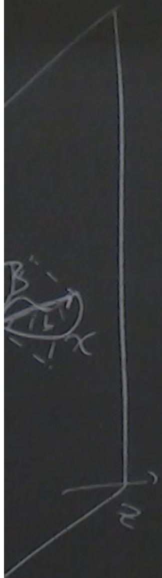
2. QUANTUM CONTRIBUTIONS:

2a. FAULKNER - LEWKOWYCZ - MALDACENA

$$S(\rho_B) = \min_x \operatorname{ext}\left(\frac{A(x)}{4G_N}\right) + S_{bulk}(b)$$

2b. ENGELHARDT - WALL (QUANTUM EXTREMAL SURFACE)

$$S(\rho_B) = \min_x \operatorname{ext}\left(\underbrace{\frac{A(x)}{4G_N} + S_{bulk}(b)}_{S_{gen}(x)}\right)$$



2. QUANTUM CONTRIBUTIONS.

2a. FAULKNER - LEWKOWYCZ - MALDACENA

$$S(\rho_B) = \min_x \mathrm{ext}\left(\frac{A(x)}{4G_N}\right) + S_{bulk}(b)$$

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$$S(\rho_B) = \min_x \mathrm{ext}\left(\underbrace{\frac{A(x)}{4G_N} + S_{bulk}(b)}_{S_{gen}(x)}\right)$$

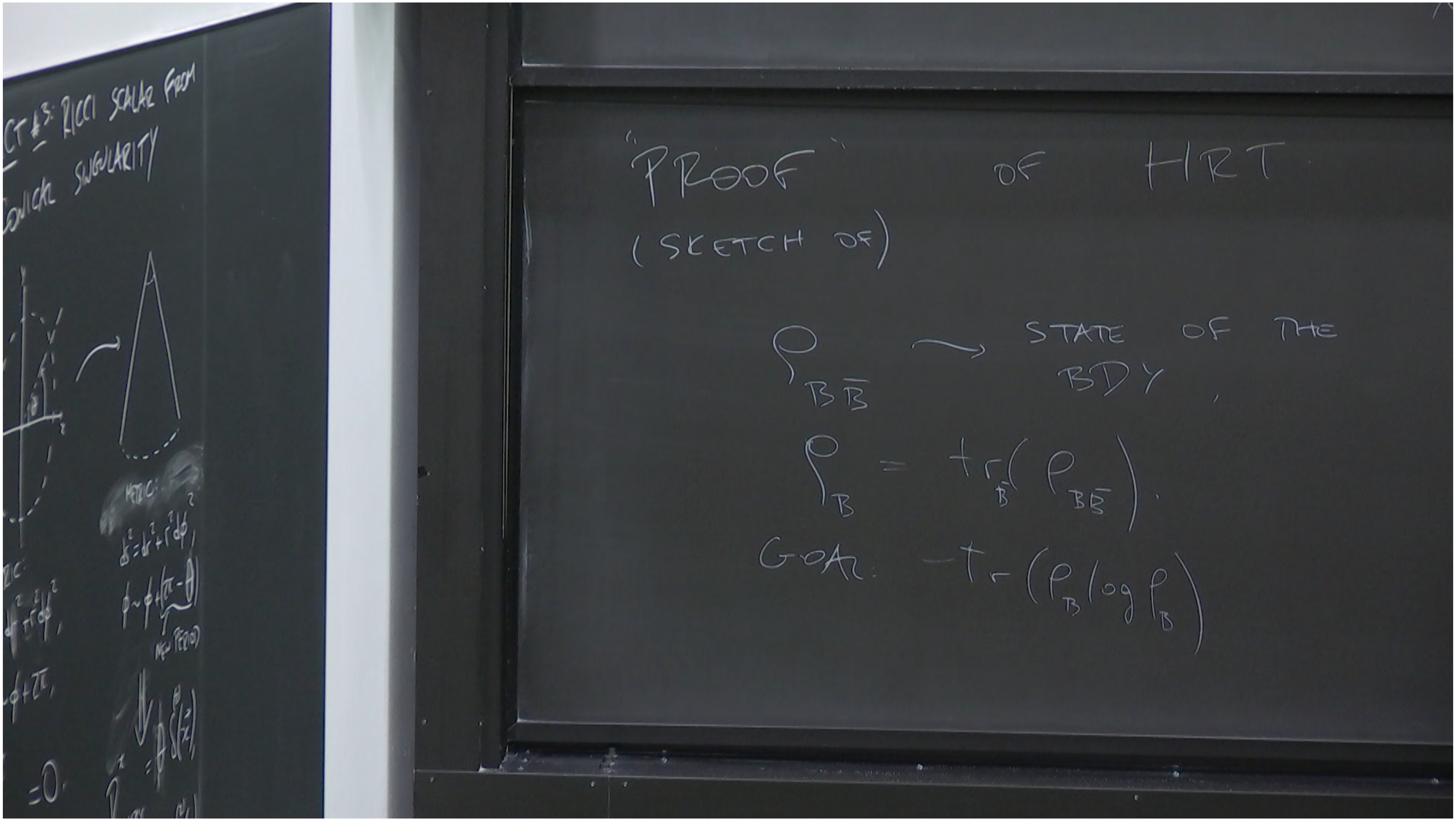
2. QUANTUM CORRECTIONS

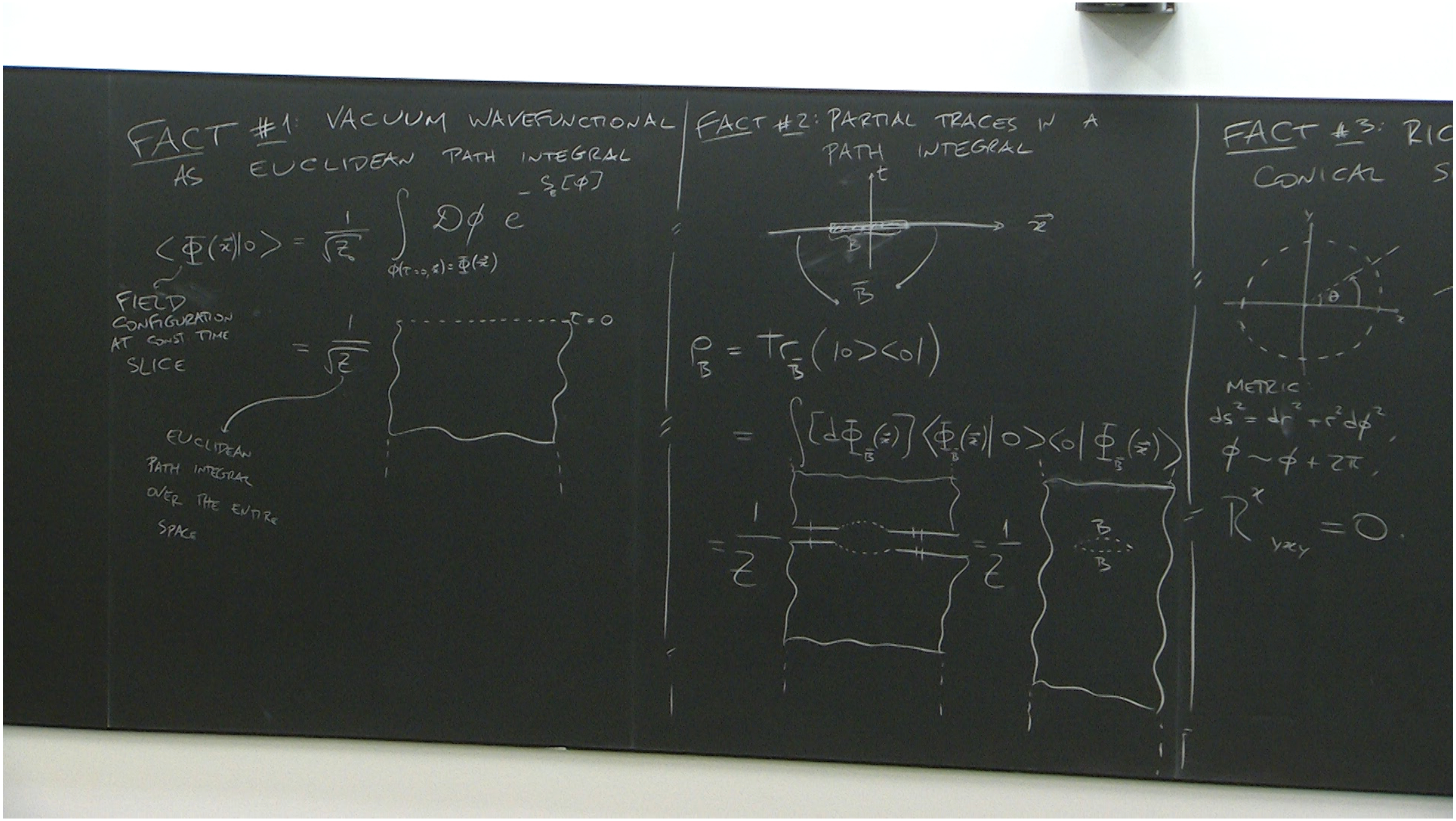
2a. FAULKNER - LEWKOWYCZ - MALDACENA

$$S(\rho_B) = \min_x \operatorname{ext}\left(\frac{A(x)}{4G_N}\right) + S_{bulk}(b)$$

2b. ENGELHARDT - WALL (QUANTUM EXTREMAL SURFACE)

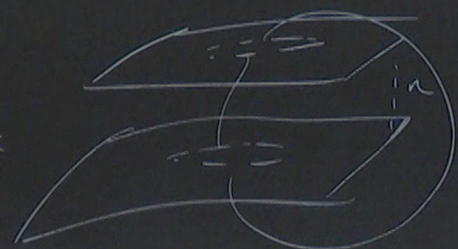
$$S(\rho_B) = \min_x \operatorname{ext}\left(\underbrace{\frac{A(x)}{4G_N} + S_{bulk}(b)}_{S_{gen}(x)}\right)$$


"PROOF" OF HRT
(SKETCH OF)
ρ_{BB̄} ~ STATE OF THE BDY,
ρ_B = tr_{B̄}(ρ_{BB̄}).
GOAL: −Tr(ρ_B log ρ_B)
RICCI SCALAR FROM
CONICAL SINGULARITY
METRIC:
NEW PERIOD


FACT #1: VACUUM WAVEFUNCTIONAL AS EUCLIDEAN PATH INTEGRAL
$\langle \Phi(\vec{x}) | 0 \rangle = \frac{1}{\sqrt{Z}} \int_{\phi(\tau=0,\vec{x}) = \Phi(\vec{x})} \mathcal{D}\phi \, e^{-S_E[\phi]}$
FIELD CONFIGURATION AT CONST TIME SLICE
$= \frac{1}{\sqrt{Z}}$
$\tau = 0$
EUCLIDEAN PATH INTEGRAL OVER THE ENTIRE SPACE
FACT #2: PARTIAL TRACES IN A PATH INTEGRAL
t
$\vec{x}$
B
$\bar{B}$
$\rho_B = \mathrm{Tr}_{\bar{B}} \left(|0\rangle\langle 0| \right)$
$= \int [d\Phi_{\bar{B}}(\vec{x})] \langle \Phi_{\bar{B}}(\vec{x}) | 0 \rangle \langle 0 | \Phi_{\bar{B}}(\vec{x}) \rangle$
$= \frac{1}{Z}$
$= \frac{1}{Z}$
B
B
FACT #3: RIC
CONICAL S
y
x
θ
METRIC
$ds^2 = dr^2 + r^2 d\phi^2$,
$\phi \sim \phi + 2\pi$,
$R^x{}_{yxy} = 0$.

$$\mathrm{tr}\, \rho_B^n = \frac{Z[n,B]}{Z_1^n}$$

$$Z[n,B] = \int_{\mathcal{M}_n(B)} \mathcal{D}\phi \, e^{-S_E[\phi]}$$

$\mathcal{M}_n(B) =$ 

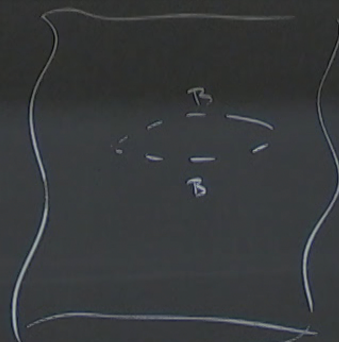
AdS/CFT THEN SAYS:

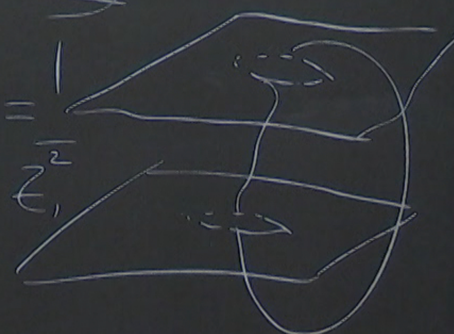
$$Z[n,B] = Z_{AdS}[n,B]$$

GRAVITY PATH INTEGRAL WITH CONFORMAL BDY $\mathcal{M}_n(B)$.

$S_{gen}(X)$

FOR INTEGER n.

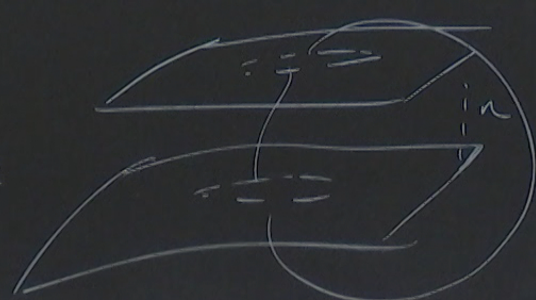
$\rho_B = \frac{1}{Z_1}$ 

$tr(\rho_B^2) = \sum_{jk} (\rho_B)_{jk} (\rho_B)_{kj} = \frac{1}{Z_1^2}$ 

$$\mathrm{tr}\,\rho_B^n = \frac{Z[n,B]}{Z_1^n}$$

$$Z[n,B] = \int_{\mathcal{M}_n(B)} \mathcal{D}\phi \; e^{-S_E[\phi]}$$

$$\mathcal{M}_n(B) =$$



AdS/CFT THEN

$$Z[n,B] = Z_{\mathrm{Ad}}$$

GRAVITY PA
WITH CON
BDY $\mathcal{M}$

ASSUMPTION 1: SEMICLASSICAL LIMIT:

$$Z_{AdS}[n, B] \simeq \max \left(e^{-S_{grav}[\mathcal{M}_n(B); g]} \right)$$

CLASSICAL ON-SHELL ACTION ON A GIVEN BULK GEOMETRY

ASSUMPTION 2:

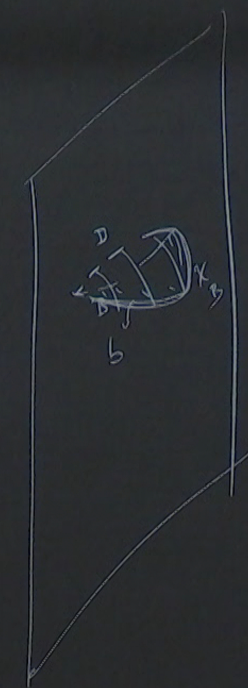
BULK SOLUTION IS ALSO SYMMETRIC UNDER CYCLIC PERMUTATIONS.

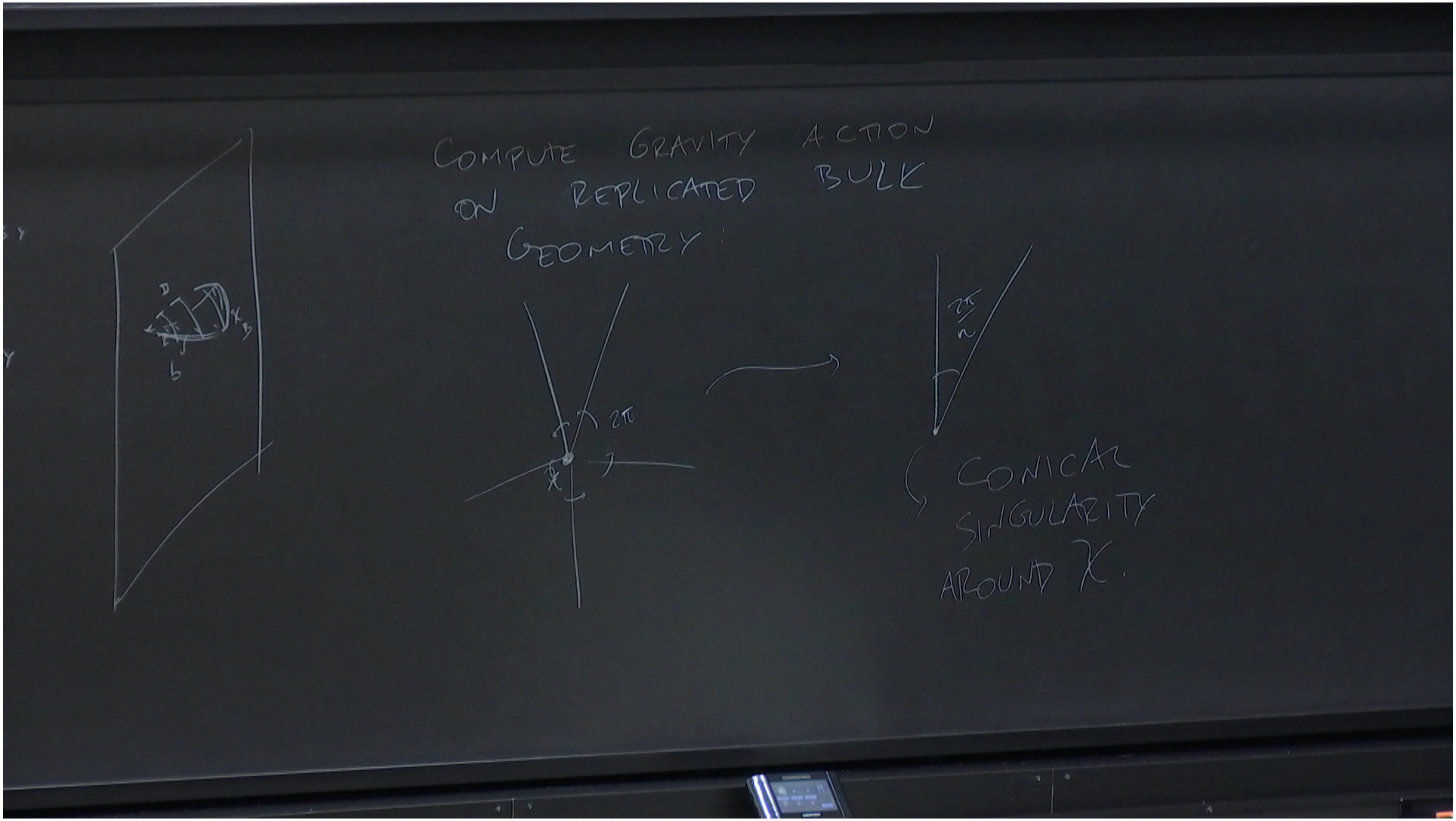
FROM ASSUMPTION 2:

BULK GEOMETRY DETERMINED BY

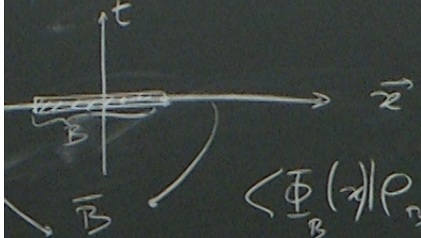
$g_1^{(n)} \longrightarrow$ BULK METRIC ON ONE COPY

$\chi_1^{(n)} \longrightarrow$ EXTENSION OF ∂B INTO THE BULK




COMPUTE GRAVITY ACTION
ON REPLICATED BULK
GEOMETRY:
2π
2π/n
CONICAL
SINGULARITY
AROUND X.

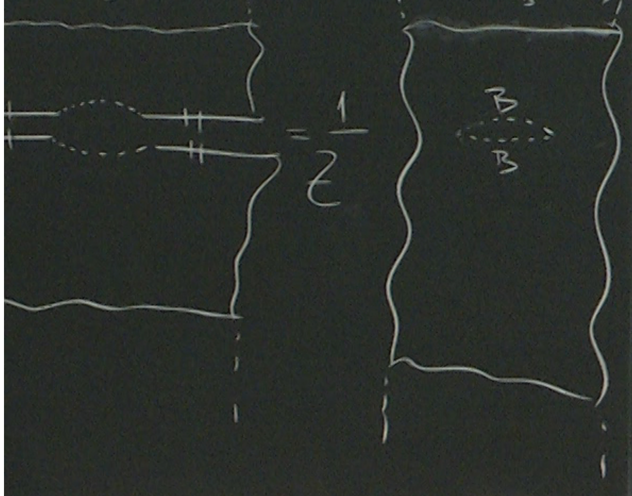
: PARTIAL TRACES IN A PATH INTEGRAL



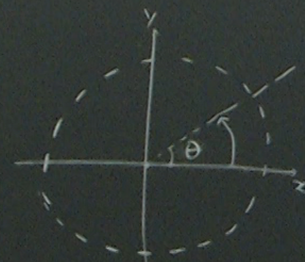
$$\langle \Phi_B(\vec{x}) | \rho_B | \Phi'_B(\vec{x}) \rangle =$$

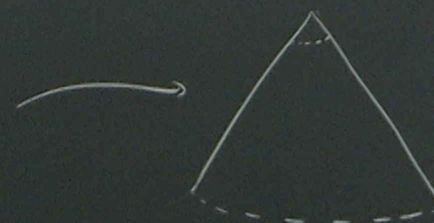
$$(|0\rangle\langle 0|)$$

$$[d\Phi_{\bar{B}}(\vec{x})] \langle \Phi_{\bar{B}}(\vec{x}) | 0 \rangle \langle 0 | \Phi_{\bar{B}}(\vec{x}) \rangle$$



FACT #3: RICCI SCALAR FROM CONICAL SINGULARITY





METRIC:

$$ds^2 = dr^2 + r^2 d\phi^2,$$

$$\phi \sim \phi + 2\pi,$$

$$R^x{}_{yxy} = 0.$$

METRIC:

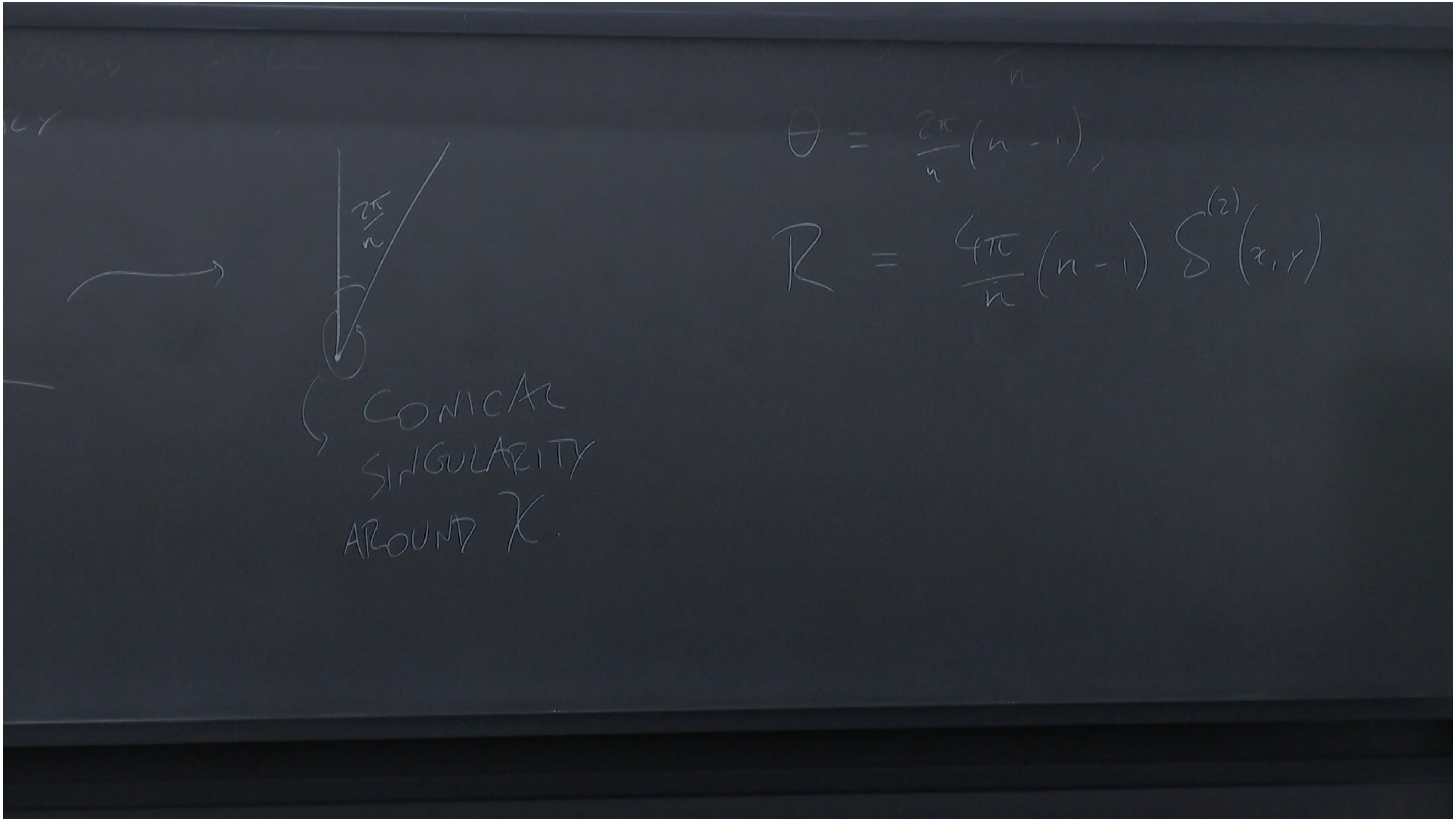
$$ds^2 = dr^2 + r^2 d\phi^2,$$

$$\phi \sim \phi + \underbrace{(2\pi - \theta)}_{\text{NEW PERIOD}}$$

$$\Downarrow$$

$$R^x{}_{yxy} = \theta\, \delta^{(2)}(\vec{x}),$$

$$R = 2\theta\, \delta^{(2)}(\vec{x})$$


$\theta = \frac{2\pi}{n}(n-1)$,
$R = \frac{4\pi}{n}(n-1)\,\delta^{(2)}(x,y)$
$\frac{2\pi}{n}$
CONICAL
SINGULARITY
AROUND X.

$$S_{grav} = -\frac{1}{16\pi G}\int d^d x \sqrt{g}\left(R - 2\Lambda\right) \Rightarrow$$

$$S_{grav}\left[M_n(B), g^{(n)}\right] = n \times S_{grav}\left[g_1^{(n)}\right] + \cancel{n} \times \frac{1}{4\,\cancel{16\pi} G} \frac{\cancel{4\pi}}{\cancel{n}}(n-1)A(x)$$

$$= n S_{grav}\left[g_1^{(n)}\right] + (n-1)\frac{A(x)}{4G_N}$$