Title: AdS/CFT Lecture (230501)

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Collection: AdS/CFT (2022/2023)

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HYDRODYNAMICS $\equiv$ EFT DESCRIBING LONG RANGE ($\vec{q} \to 0$), LOW ENERGY ($\omega \to$
FLUCTUATIONS

LOCAL EQ.

"$\lambda_{\text{FLUCTUATIONS}} \gg \lambda_{\text{MFP}}$"

DYNAMICS: $\boxed{\nabla_\mu T^{\mu\nu} = 0}$

$\underbrace{T(x), u^\mu(x)}_{d\text{-UNKNOWNS}}$

... d-EOM

$T), \eta, \xi, \ldots)$

$$T, u^\mu) = T^{\mu\nu}\big(\rho(T), P(T), \underbrace{\eta, \xi, \ldots}_{\text{TRANSPORT COEFFS.}}\big)$$

(GOD GIVEN)

$$= T^{\mu\nu}(T, u^\mu) = T^{\mu\nu}\Big(\rho(T), P(T), \underbrace{\eta, \xi, \ldots}_{\text{TRANSPORT COEFFS. (GOD GIVEN)}}\Big)$$

STITUTIVE
ELATIONS

VE EXPANSION FOR $T^{\mu\nu}$:

$$= \underbrace{(\rho+P)u^\mu u^\nu + P g^{\mu\nu}}_{\text{0TH-ORDER}} + \underbrace{\tau^{\mu\nu}}_{\text{1st}} + O\left(\frac{\lambda_{MFP}}{\lambda_{FLUCT}}\right)^2$$

α - UNKNOWNS

IN LANDAU'S GAUGE ($T^{\mu\nu} u_\nu = -\rho u^\mu$)

SPORT COEFFS. $\tau^{\mu\nu} = -2\eta\sigma^{\mu\nu} - 2\zeta\theta P^{\mu\nu}$

VEN)

SHEAR

BULK ($\zeta = 0$ FOR CFT)

0TH-ORDER 1ST

WE WANT TO CALCULATE η/s:

STEP I: USE HYDRO DESCRIPTION (MACROSCOPIC PHYSICS)

$$\underbrace{\delta \tau^{xy}}_{\text{MACROSCOPIC}} \overset{TS}{=} -i\omega\eta\, \underbrace{h^{0}_{xy}}_{\substack{\text{FICTICIOUS GRAV. FIELD} \\ \text{MANAGING THE CHANGE} \\ \text{OF } \tau^{xy}}}$$

<u>STEP 2:</u> TO RELATE THIS TO MICROSCOPIC PHYSICS,
WE USE <u>LINEAR RESPONSE THEORY</u>

ASSUME SYSTEM IN EQUILIBRIUM (ρ_0)
PERTURBED BY EXTERNAL SOURCE ϕ_0

$$\delta S = \int d^d x \, O(x)\, \phi_0(x)$$

$$\Rightarrow \boxed{\delta \langle O(k) \rangle = - G_R^{OO}(k)\, \phi_0(k)}$$

FOR OS

$$\boxed{\delta\langle \tau^{xy}\rangle = -G_R(k)\, h^0_{xy}(k)} \quad (2)$$

(1) TO (2) GIVES

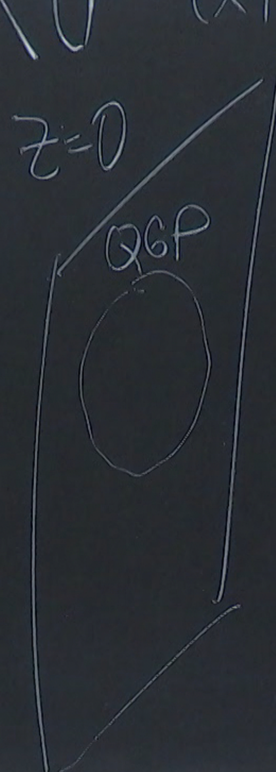
$$\boxed{\eta = \frac{1}{i\omega} G_R(k)}$$

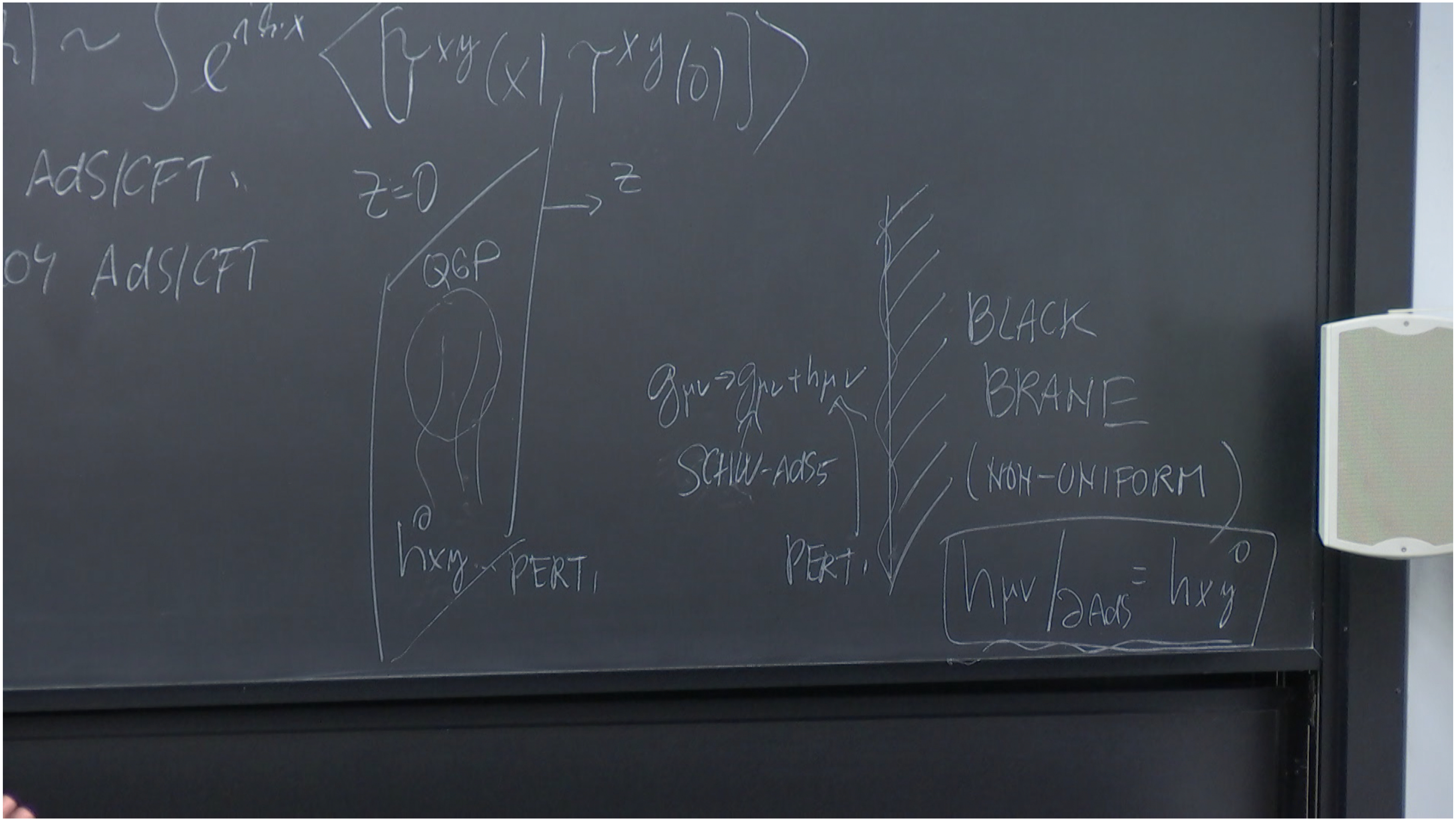
KUBO FORMULA

WHERE $G(k) \sim \int e^{ik\cdot x} \langle [T^{xy}(x), T^{xy}(0)] \rangle$

SO FAR NO AdS/CFT.

STEP 3: EMPLOY AdS/CFT




AdS/CFT
AdS/CFT
z=0
z
QGP
PERT.
SCHW-AdS5
PERT.
BLACK
BRANE
(NON-UNIFORM)

WE NEED TO CALCULATE

$$\langle \gamma^{xy} \gamma^{xy} \rangle \ldots \text{ 2PT FUNCTION.}$$

- WE SOLVE EOM FOR PERT $h_{\mu\nu}$.
- EXPRESS THIS IN TERMS OF THE BOUNDARY $\mathring{h}_{xy}$
- PLUG TO $S(h_{\mu\nu}) \to S[\mathring{h}_{xy}] = W[\mathring{h}_{xy}]$

ON.

$$\langle \tau^{xy}\,\tau^{xy}\rangle \simeq -\frac{\delta^2 S[h^0_{xy}]}{\delta h^0_{xy}\,\delta h^0_{xy}}$$

h^0_{xy}

$h^0_{xy}]$

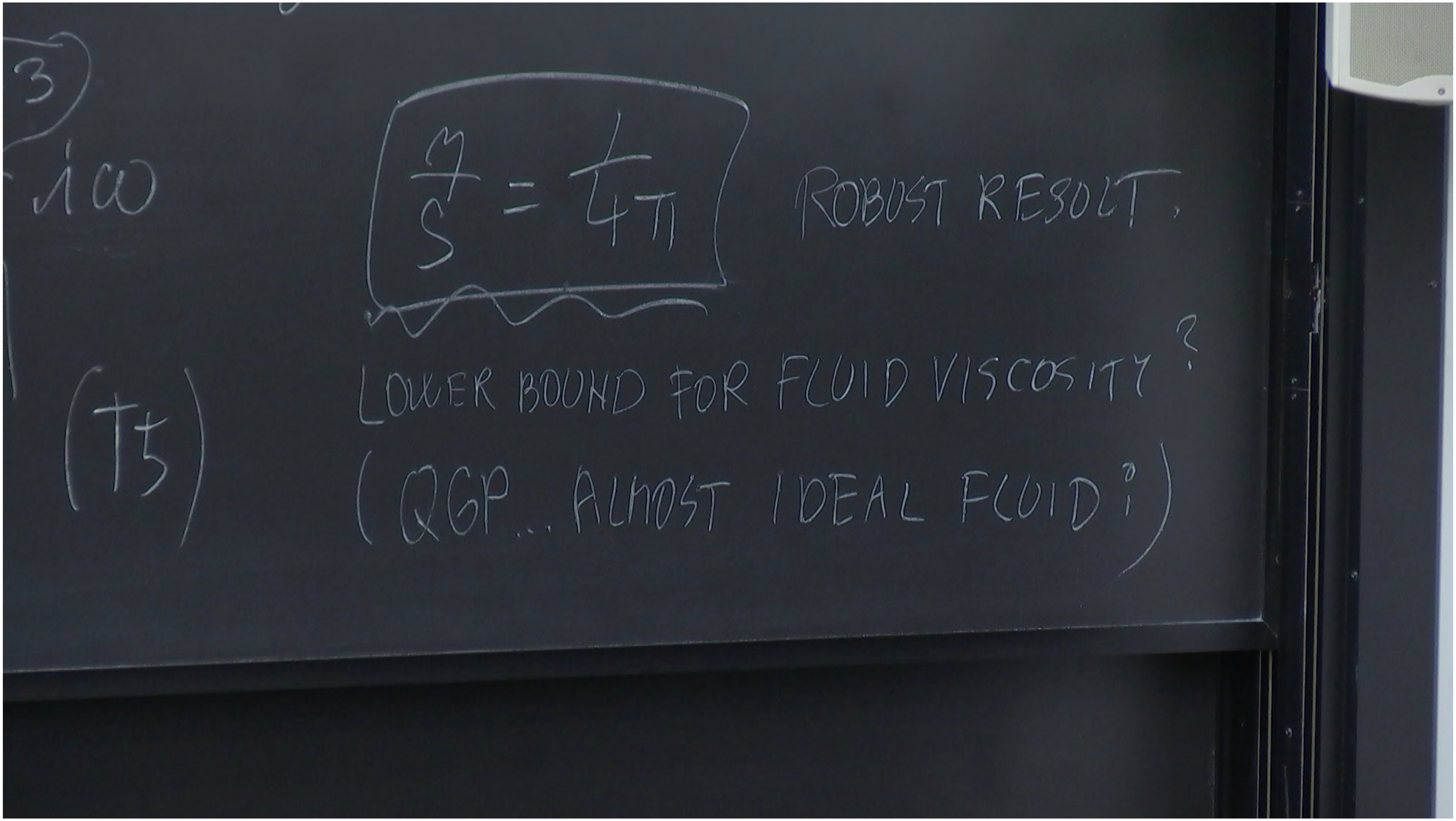
• RESULT OF CALC.

$$G(k) = \frac{\pi N_c^2 T^3}{8}\, i\omega$$

η

$$S = \frac{\pi^2}{2} N_c^2 T^3 \quad (+5)$$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$


3
iω
η/s = 1/4π
ROBUST RESULT,
LOWER BOUND FOR FLUID VISCOSITY ?
(T5)
(QGP... ALMOST IDEAL FLUID ?)

HOLOGRAPHIC ENTANGLEMENT ENTROPY

- QM: • VON-NEUMANN ENTROPY

$$S_{VN} = -\mathrm{Tr}(\rho \log \rho) \begin{cases} 0 & \text{PURE STATE} \\ \text{MAXIMAL} & \text{FOR EQ. PROB. DISTR.} \end{cases}$$

- $$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

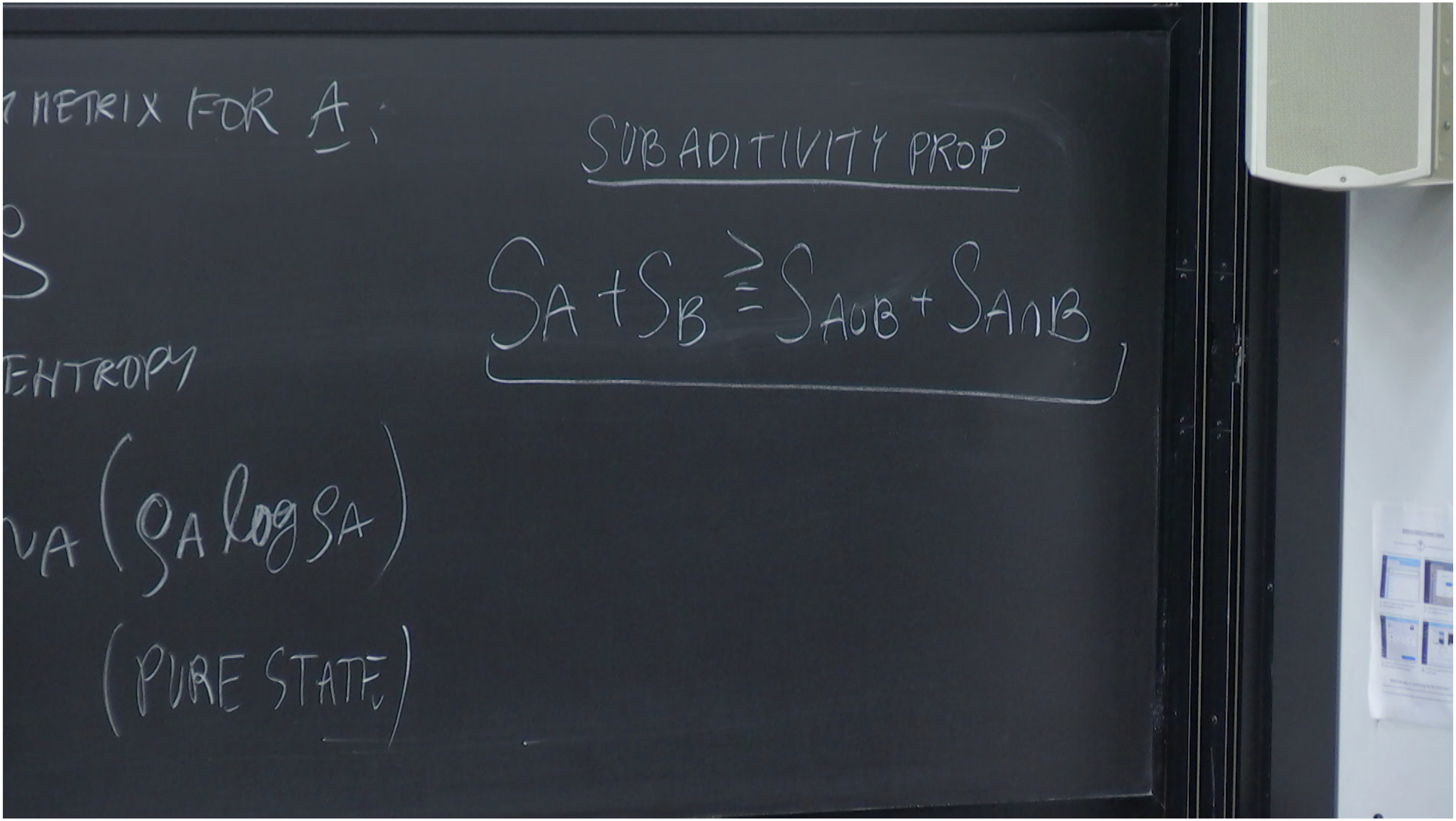
- REDUCED DENSITY MATRIX FOR $\underline{A}$:

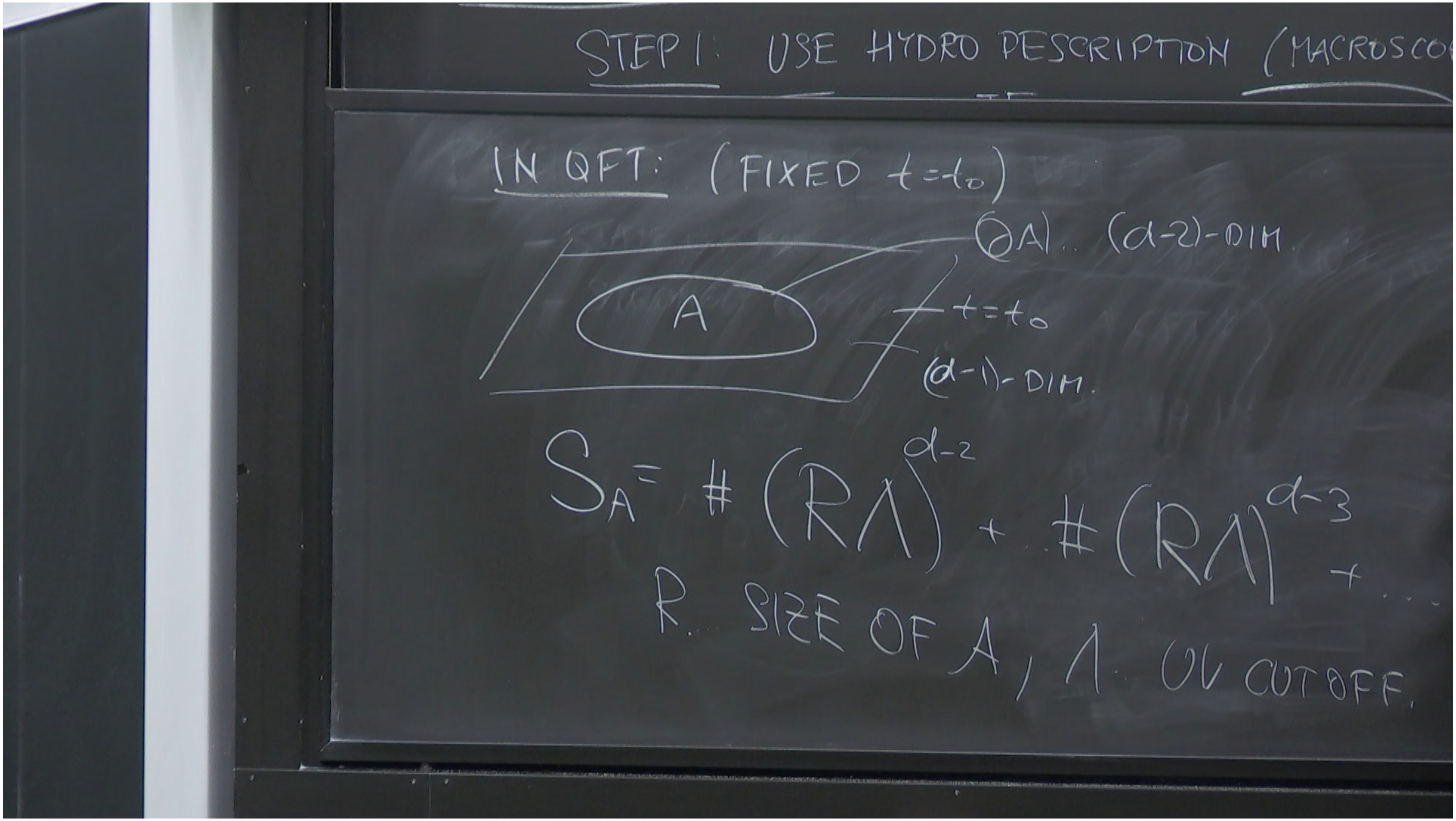
$$\rho_A = \mathrm{Tr}_B\, \rho$$

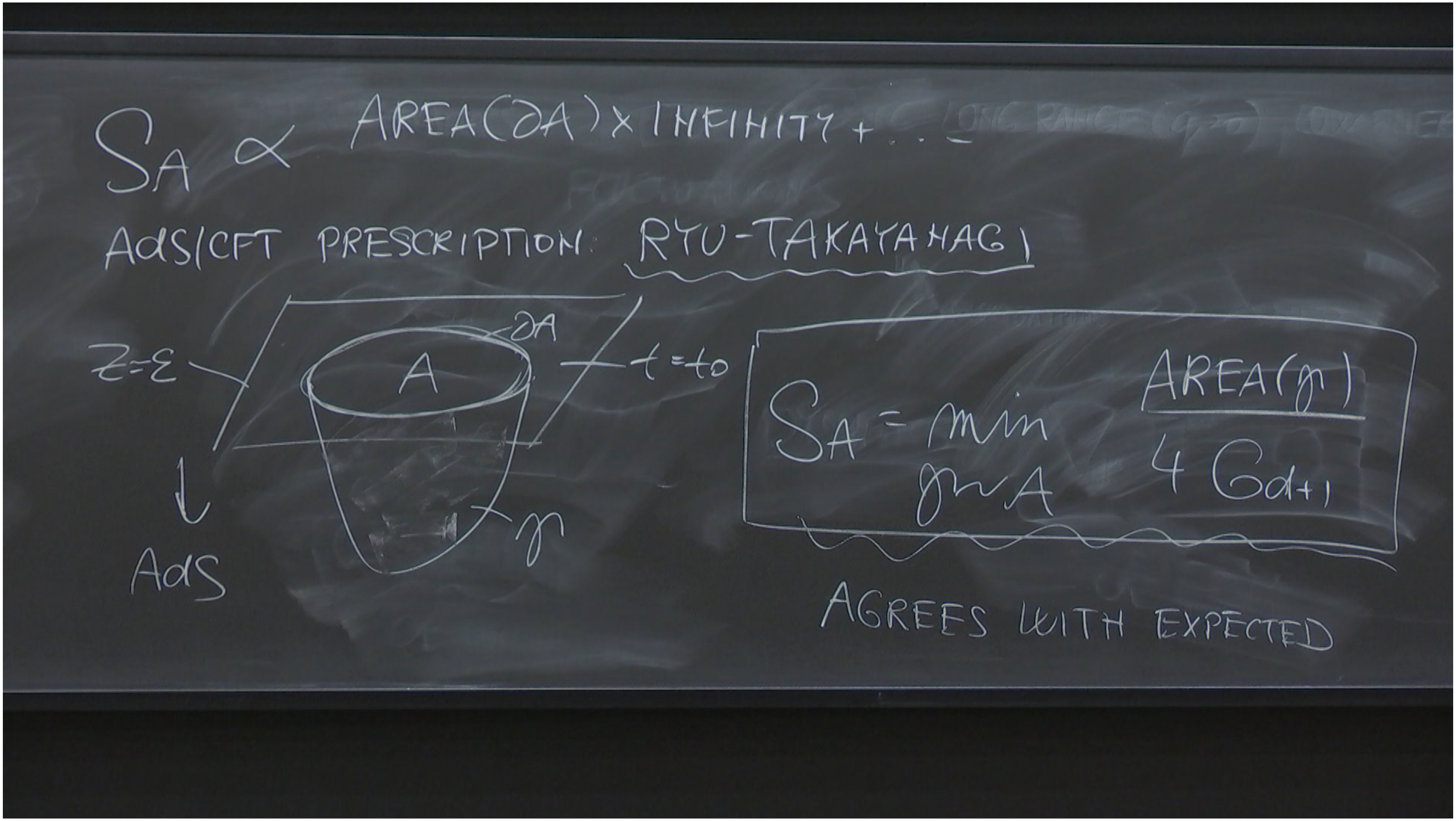
- ENTANGLEMENT ENTROPY

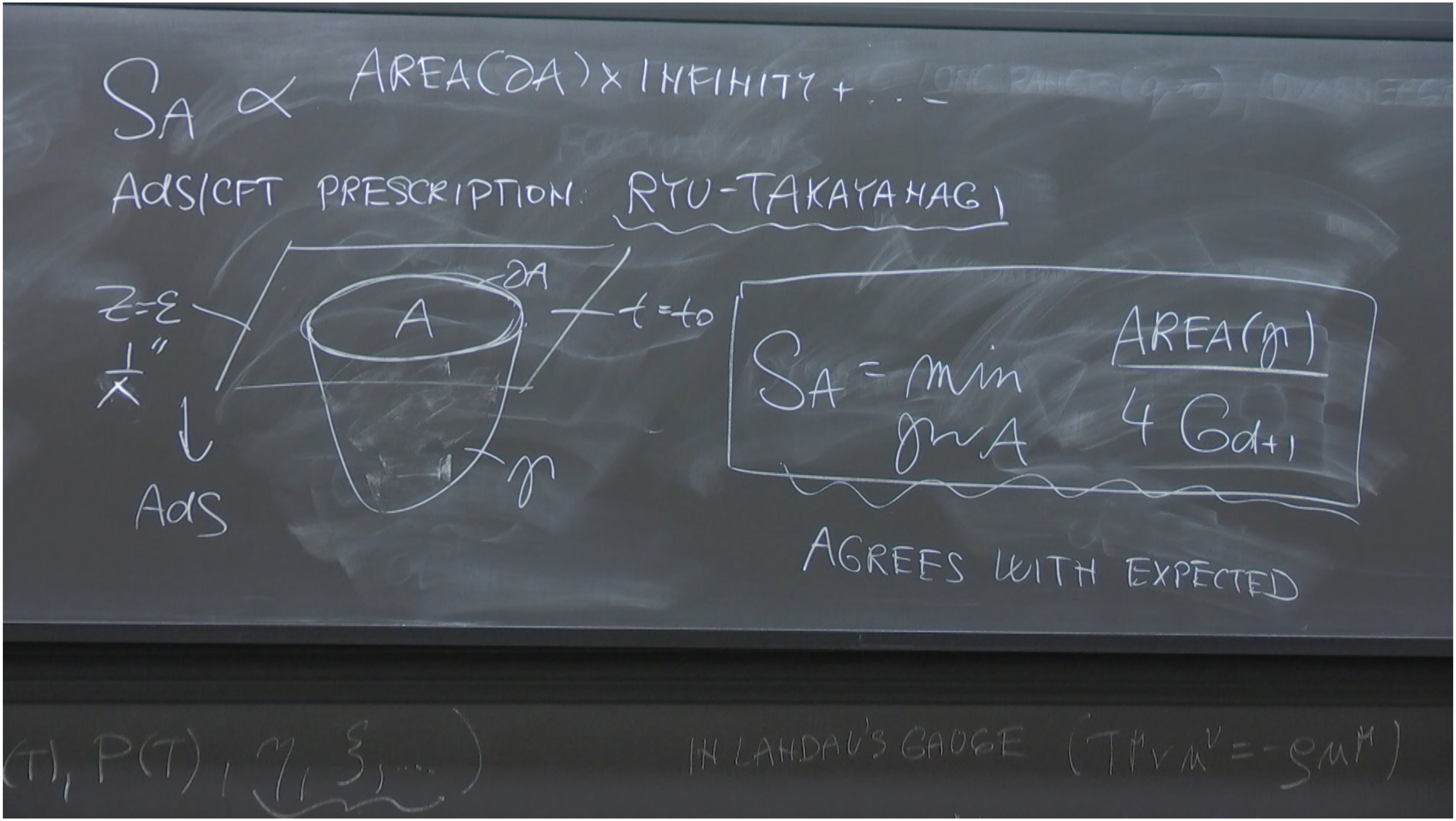
$$S_A = -\mathrm{Tr}_A\left(\rho_A \log \rho_A\right)$$

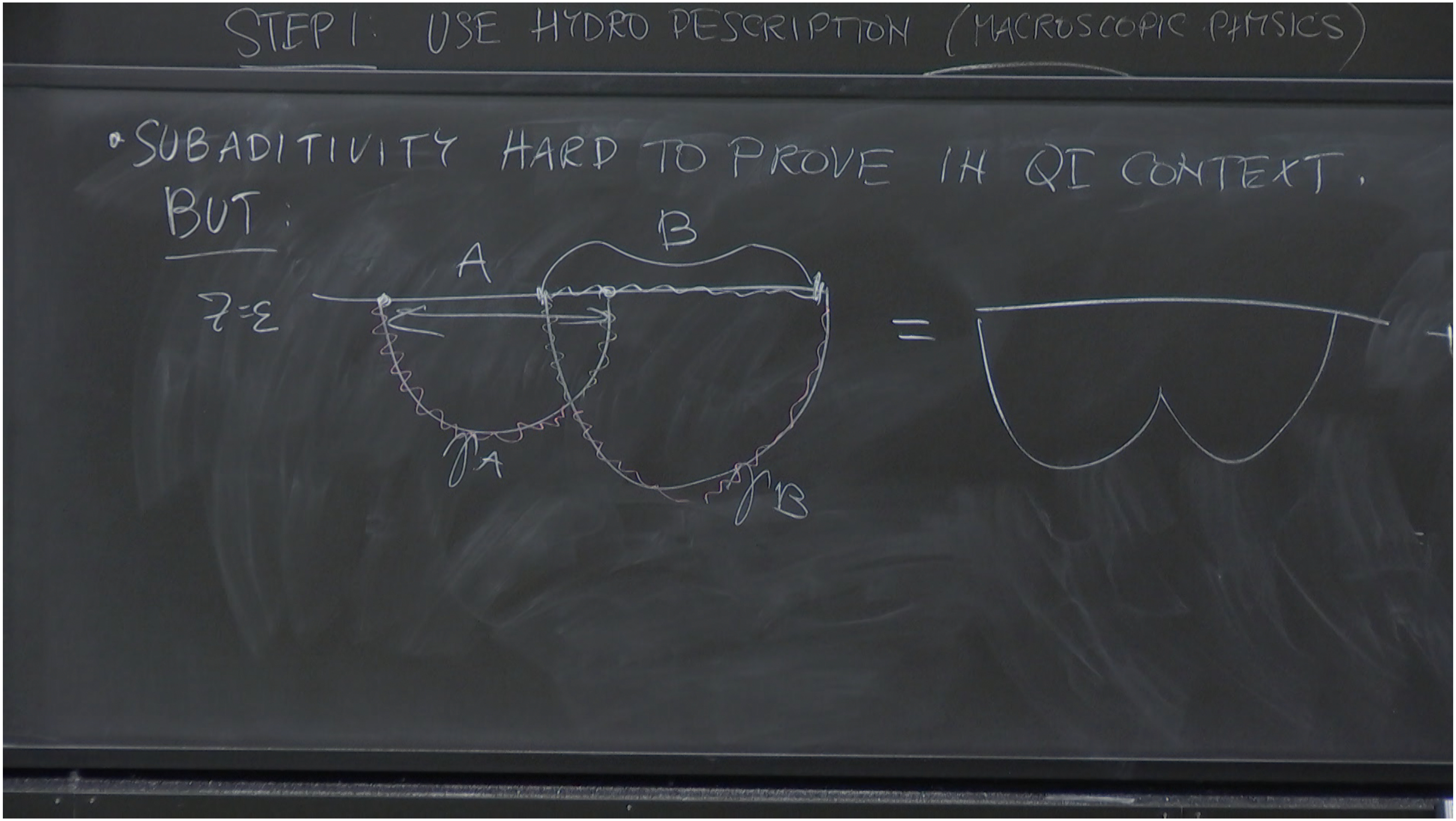
- $S_A = S_B$ (PURE STATE)

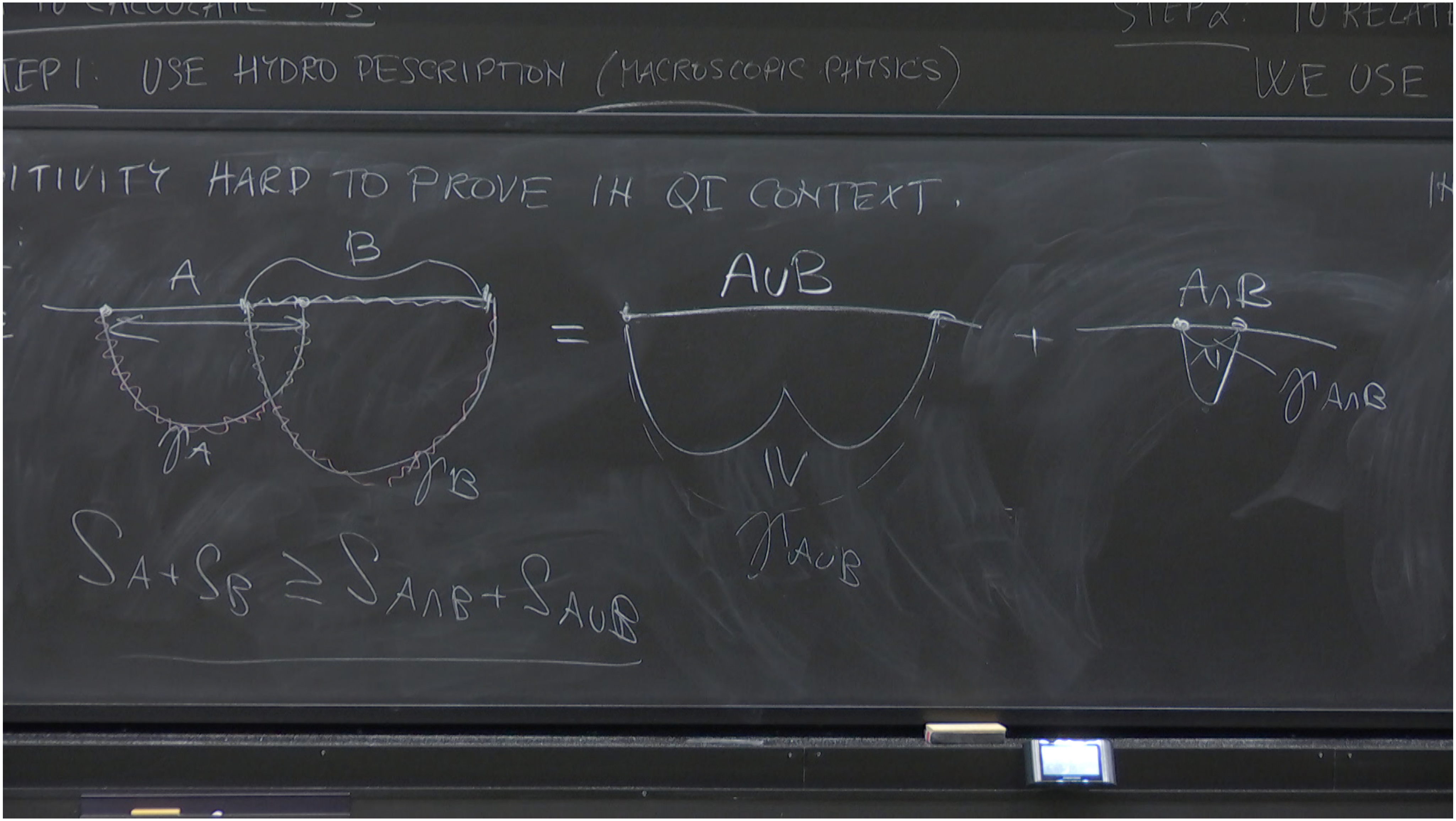

METRIX FOR A ;
ENTROPY
A (ρA log ρA)
(PURE STATE)
SUBADITIVITY PROP
SA + SB ≥ SA∪B + SA∩B


STEP 1: USE HYDRO DESCRIPTION (MACROSCO
IN QFT: (FIXED t = t0)
∂A .. (d-2)-DIM.
A
t = t0
(d-1)-DIM.
S_A = # (RΛ)^{d-2} + # (RΛ)^{d-3} + ...
R .. SIZE OF A, Λ .. UV CUTOFF.


$S_A \propto$ AREA(∂A) × INFINITY + ...
AdS/CFT PRESCRIPTION: RYU-TAKAYANAGI
$Z = \varepsilon$
∂A
A
$t = t_0$
AdS
γ
$S_A = \min_{\gamma \sim A} \frac{\text{AREA}(\gamma)}{4 G_{d+1}}$
AGREES WITH EXPECTED


S_A ∝ AREA(∂A) x INFINITY + ...
AdS/CFT PRESCRIPTION: RYU-TAKAYANAGI
Z=ε
1/λ "
AdS
∂A
A
t=t0
γ
S_A = min_{γ~A} AREA(γ) / 4 G_{d+1}
AGREES WITH EXPECTED
(T), P(T), η, ξ, ...)
IN LANDAU'S GAUGE (T^{μν} u_ν = -ρ u^μ)


STEP I: USE HYDRO DESCRIPTION (MACROSCOPIC PHYSICS)
• SUBADITIVITY HARD TO PROVE IN QI CONTEXT.
BUT:
A
B
z = ε
γ_A
γ_B
=


TEP 1: USE HYDRO DESCRIPTION (MACROSCOPIC PHYSICS)
STEP 2: TO RELATE
WE USE
ITIVITY HARD TO PROVE IN QI CONTEXT.
A
B
γ_A
γ_B
$A \cup B$
$\gamma_{A \cup B}$
$A \cap B$
$\gamma_{A \cap B}$
$S_A + S_B \geq S_{A \cap B} + S_{A \cup B}$