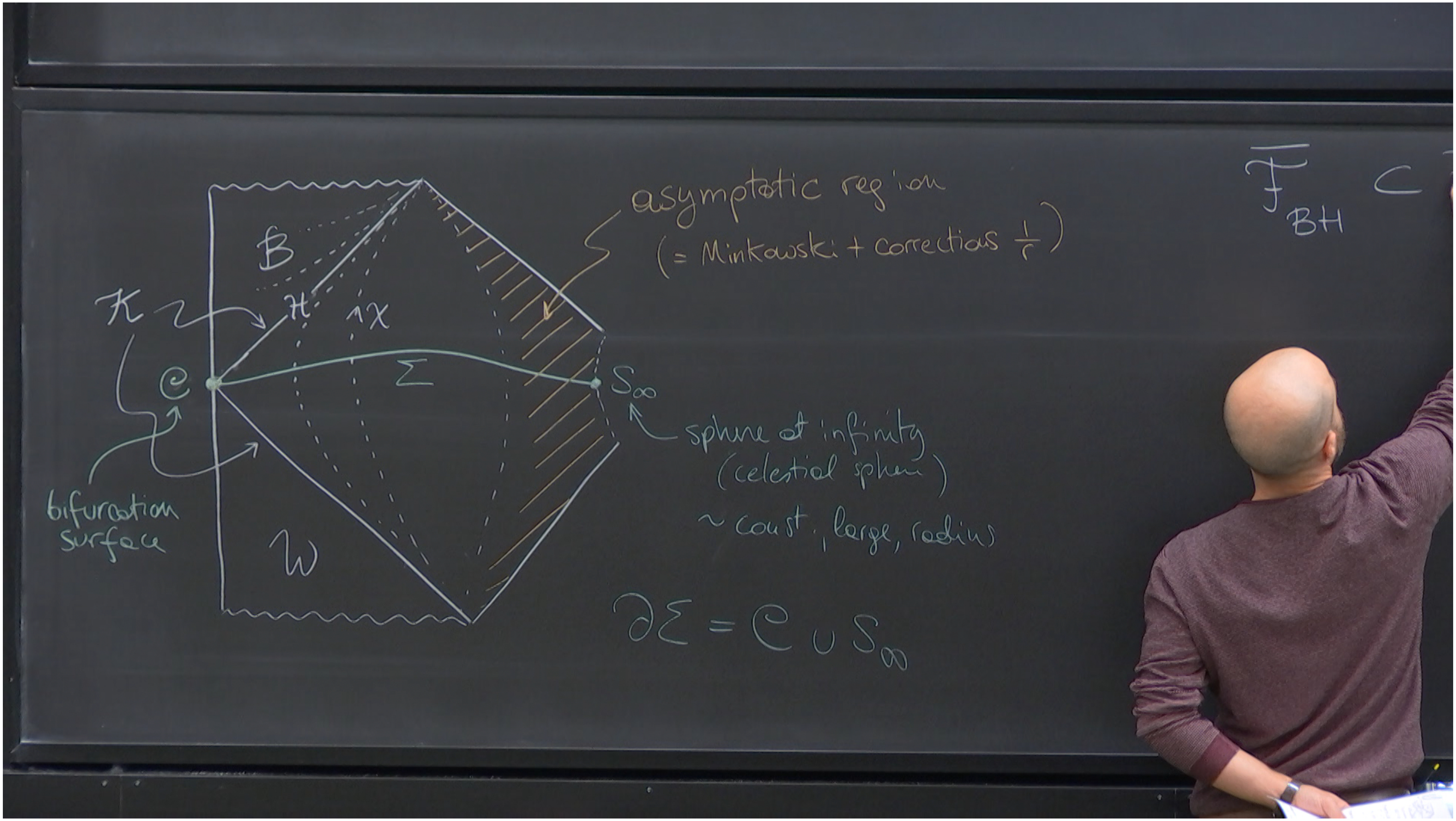
Title: Quantum Gravity Lecture (230501)

Speakers: Aldo Riello

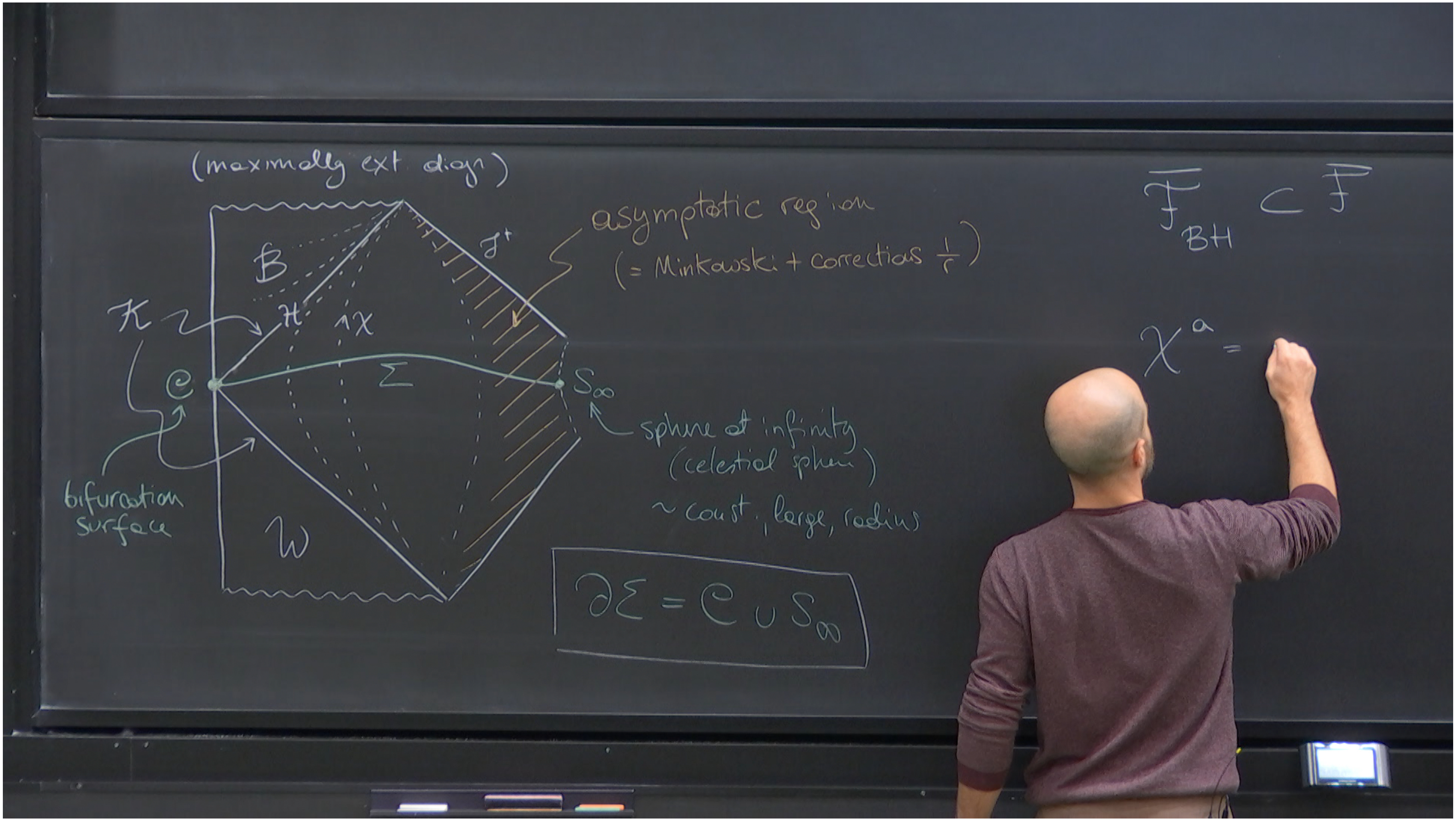
Collection: Quantum Gravity (2022/2023)

Date: May 01, 2023 - 2:00 PM

URL: https://pirsa.org/23050006



asymptotic region
(= Minkowski + corrections $\frac{1}{r}$)
sphere at infinity
(celestial sphere)
~ const. large radius
bifurcation surface
$\partial \Sigma = \mathcal{C} \cup S_\infty$
$\mathcal{F}_{BH}$


(maximally ext. diagr)
asymptotic region
(= Minkowski + corrections $\frac{1}{r}$)
$\mathcal{B}$
$\mathcal{J}^+$
$\mathcal{K}$
$\mathcal{H}$
χ
$\mathcal{C}$
Σ
S_∞
sphere at infinity
(celestial sphere)
~ const, large, radius
bifurcation surface
$\mathcal{W}$
$\partial\Sigma = \mathcal{C} \cup S_\infty$
$\mathcal{F}_{BH} \subset \mathcal{F}$
$\chi^a =$

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$$\chi^a = t^a + \omega_H \phi^a$$

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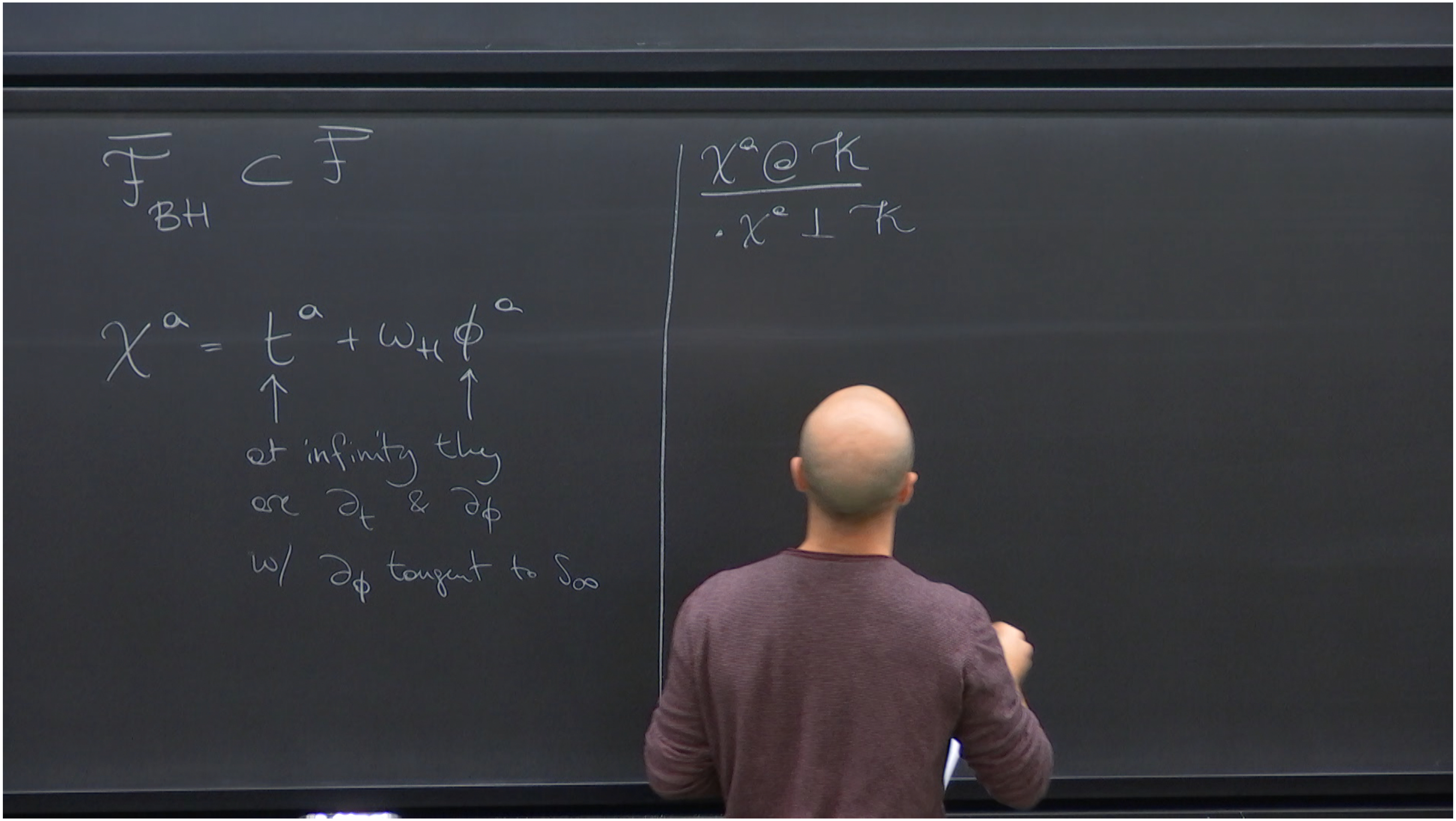
at infinity they are ∂_t & ∂_ϕ


$\mathcal{F}_{BH} \subset \mathcal{F}$
$\chi^a = t^a + \omega_H \phi^a$
at infinity they
are ∂_t & ∂_ϕ
w/ ∂_ϕ tangent to S_∞
χ^a @ $\mathcal{K}$
· $\chi^a \perp \mathcal{K}$

$\underline{\chi^a \text{ @ } \mathcal{K}}$

surface gravity, $\underline{\text{thm}}$: κ const on $\mathcal{H}$

- $\chi^a \perp \mathcal{K}$, $\chi^a \parallel \mathcal{K}$
- $\nabla_a(\chi\cdot\chi) = -2\kappa\, \chi_a \rightarrow \chi^b \nabla_b \chi^a = \kappa \chi^a$
- @ $\mathcal{C}$: $\chi^a = 0$, $\nabla_a \chi_b = \kappa\, \varepsilon_{ab}$

χ^a @ $\mathcal{K}$

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χ is a killing (isometry) & therefore it depends on $g_{ab} \in \overline{\mathcal{F}}_{BH}$

$\Rightarrow \delta\chi^a \neq 0$

$\underline{\chi^a \,@\, \mathcal{K}}$

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χ is a killing (isometry) & therefore it depends on $g_{ab} \in \overline{\mathcal{F}}_{BH}$

$\Rightarrow \delta\chi^a \neq 0$ $\quad$ <u>Field dependent!</u>

$\omega/\ \partial_\phi$ tangent to S_∞

$$\boxed{\partial\Sigma = \mathcal{C} \cup S_\infty}$$

Adapt Ham flow eq.

$d\xi = 0$: $\quad \mathring{i}_{\rho(\xi)}\Omega_\Sigma = -dQ_\Sigma(\xi) - \underbrace{\int_\Sigma i_\xi \underline{E}}_{\text{not here}} + \underbrace{\int_{\partial\Sigma} i_\xi \underline{\Theta}}_{\text{not here}}$

$\rho(\xi) = \int L_\xi g_{ab} \frac{\delta}{\delta g_{ab}} \qquad \Omega^{(ab)(a'b')}(g)\, dg_{ab} \wedge dg_{a'b'}$

$\Rightarrow$ $\underline{\underline{\text{no}}}$ d acting on ξ

$= d(Q_\Sigma(\xi)) - Q_\Sigma(d\xi)$

$\rightsquigarrow$ generalizes to $d\xi \neq 0$ as follows: $\quad \mathring{i}_{\rho(\xi)}\Omega_\Sigma = -(dQ_\Sigma)(\xi) - \int_\Sigma i_\xi \underline{E} + \int_{\partial\Sigma} i_\xi \underline{\Theta}$

or: plug field-dependence of ξ AFTER taking variations

Let's work on $\mathcal{F}_{BH}$ & plug in $\xi = \chi$!

1) $\underline{E} \approx 0$ on $\mathcal{F}_{BH}$

2) $\rho(\xi) = \int_\Sigma (\mathcal{L}_\xi g_{ab}) \frac{\delta}{\delta g_{ab}} \overset{(\xi=\chi)}{=} 0 \Rightarrow i_{\rho(\chi)} \Omega_\Sigma = 0$

Komar charge

3) $Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi) = \int_\Sigma (\underline{C}_\Sigma)_\alpha \xi^\alpha + \int_{\partial\Sigma} \frac{1}{2} j^{ab}(\xi)\, \underline{\epsilon}_{ab} \approx q_{\partial\Sigma}(\xi)$

constraints ≈ 0

Komar current

4) $\int_{\partial\Sigma} i_\chi \underline{\Theta} = \int_{\mathcal{H}} i_\chi \underline{\Theta} + \int_{S_\infty} i_\chi \underline{\Theta} = \underset{\chi|_{\mathcal{H}}=0}{0} + \int_{S_\infty} i_t \underline{\Theta} + \omega_H \int_{S_\infty} i_{\phi_0} \underline{\Theta}$

Let's work on $\mathcal{F}_{BH}$ & plug in $\xi = \chi$!

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2) $\rho(\xi) = \int_\Sigma \left(\mathcal{L}_\xi g_{ab}\right) \frac{\delta}{\delta g_{ab}} \overset{(\xi=\chi)}{=} 0 \Rightarrow i_{\rho(\chi)} \Omega_\Sigma = 0$

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Komar current

zero b.c. $\phi \parallel S_\infty$

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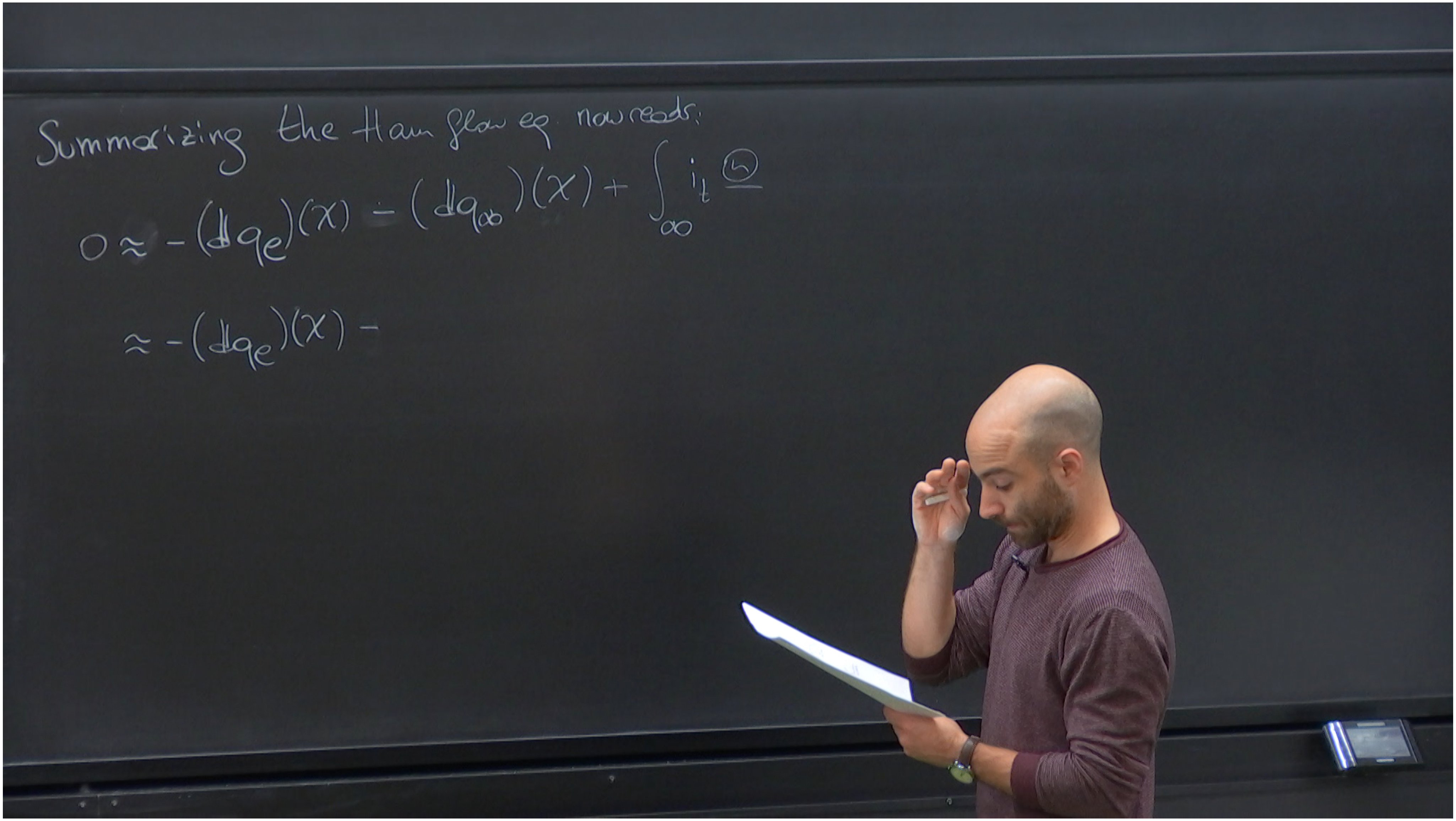
Summarizing the Ham flow eq. now reads:
$0 \approx -(dq_e)(x) - (dq_\infty)(x) + \int_\infty i_t \, \omega$
$\approx -(dq_e)(x) -$

Summarizing the Ham flow eq. now reads:

$$0 \approx -(dq_e)(\chi) - (dq_\infty)(\chi) + \int_\infty i_t \underline{\Omega}$$

$$\approx -(dq_e)(\chi) - (dq_\infty)(t^e) - \omega_H (dq_\infty)(\phi^e) + \int i_t \underline{\Omega}$$

<u>not varied</u> even though it depends on which sol. I'm looking at.

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$\boxed{\text{Fixed}}$ "boundary condition" or "asymptotic background"

<u>Obs</u>: @ infinity <u>all</u> $g_{ab} \to \eta_{ab}$ + corrections

and thus t^e & ϕ^e are fixed ("anchored" to background) $\to dt^e = d\phi^e = 0$

(not ω_H!)

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$$\approx -(dq_e)(X) - (dq_\infty)(t^a) - \omega_H (dq_\infty)(\phi^a) + \int_{\partial\Sigma} i_t \underline{\omega}$$

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and thus t^a & ϕ^a are fixed ("anchored" to background) $\to dt^a = d\phi^a = 0$

(not ω_H: $d\omega_H \neq 0$)

$\left(\text{not } \omega_H : \not{d}\,\omega_H \neq 0\right)$

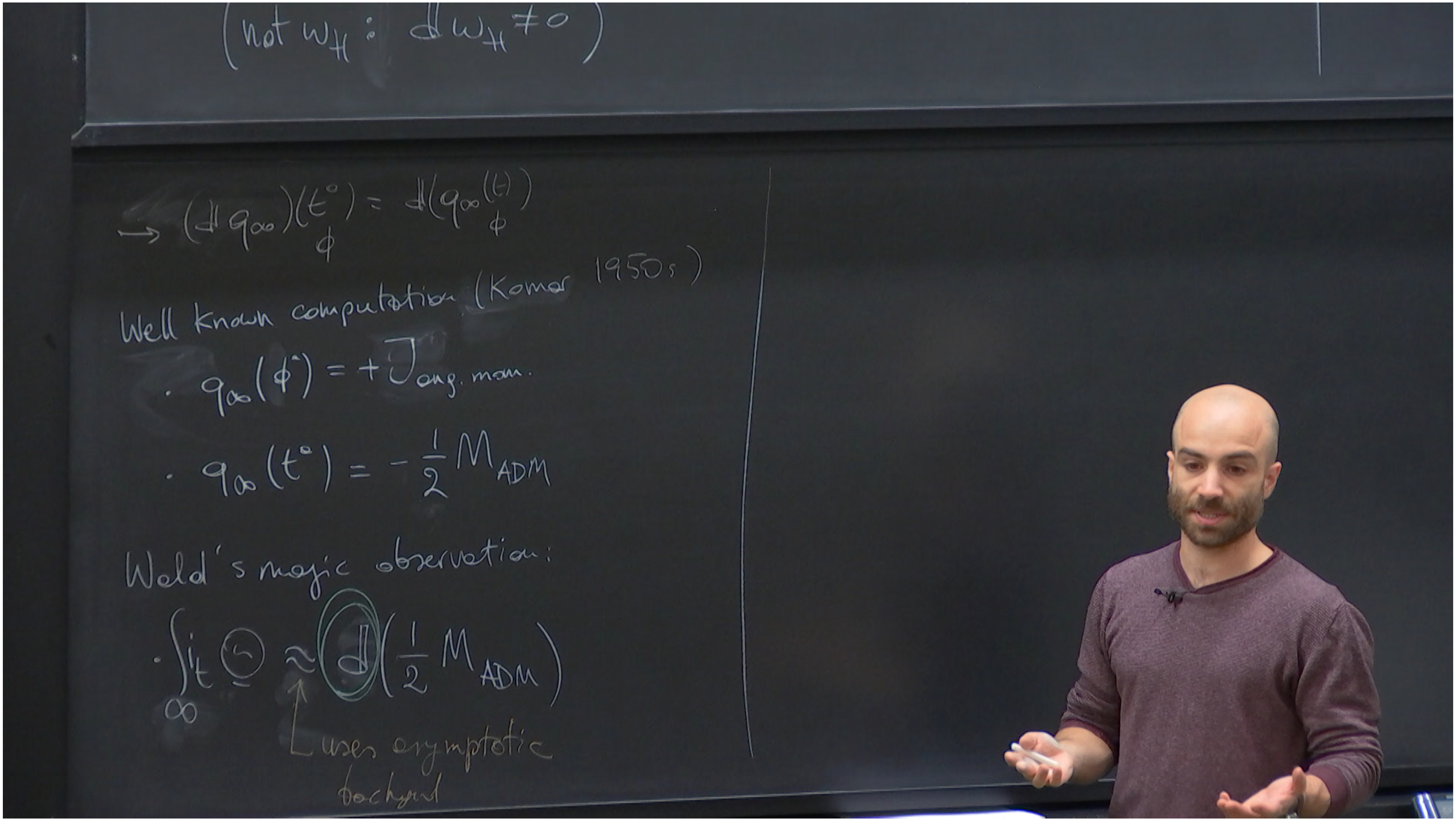
$\to (\not{d}\, q_\infty)(t^\circ_\phi) = \not{d}(q_\infty(t_\phi))$

Well known computation (Komar 1950s)

- $q_\infty(\phi^\circ) = + J_{\text{ang. mom.}}$
- $q_\infty(t^\circ) = -\frac{1}{2} M_{ADM}$

Wald's magic observation:

- $\int_\infty i_t \Omega \approx \not{d}\left(\frac{1}{2} M_{ADM}\right)$



(not ω_H : $d\omega_H \neq 0$)
$\rightarrow (d q_\infty)(t^\circ_\phi) = d(q_\infty(t^\circ_\phi))$
Well known computation (Komar 1950s)
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· $\int_\infty i_t ☺ \approx d\left(\frac{1}{2} M_{ADM}\right)$
uses asymptotic background

(not $\omega_{\mathcal{H}}$: $d\omega_{\mathcal{H}} \neq 0$)

$\to (d\, q_\infty)(t^\circ_\phi) = d(q_\infty(t_\phi))$

Well known computation (Komar 1950s)

- $q_\infty(\phi^\circ) = + J_{\text{ang. mom.}}$
- $q_\infty(t^\circ) = -\frac{1}{2} M_{ADM}$, $M_{ADM} \geq 0$ & $=0$ iff Mink.

Wald's magic observation:

- $\int_\infty i_t \underline{\Theta} \approx d\left(\frac{1}{2} M_{ADM}\right)$

uses asymptotic background

Putting everything together:

$$\underbrace{(dq_e)(X)}_{\substack{\text{geom. quantity} \\ \text{def @ horizon}}} \hat{\approx} \underbrace{-\omega_H \, d J_{\text{sym}} + d M_{ADM}}_{\substack{\text{ang. mom \& energy as} \\ \text{defined from infinity}}}$$

$(\mathrm{d}q_e)(\chi) = ?$

$$q_e(\xi) = \frac{1}{4}\int_e (\nabla_a \xi_b - \nabla_b \xi_a)\,\underline{\epsilon}^{ab}$$

$$= \frac{1}{2}\int_e \partial_{[a}(g_{b]c}\,\xi^c)\,\underline{\epsilon}^{ab}$$

need index up to plug in $\xi^a = \chi^a$ after variation!

<u>Warm up</u>. $q_e(\chi) = \frac{1}{4}\int_e (\nabla_a\chi_b - \nabla_b\chi_a)\,\underline{\epsilon}^{ab} = \frac{1}{2}\int_e \kappa\, \underbrace{\varepsilon^{ab}\,\underline{\epsilon}_{ab}}_{2\underline{\epsilon}_e} = \kappa\,\mathrm{Area}(e)$

vol. element of e

$\mathrm{d}q_e(\xi)$

Field dependent

$$\not{d} q_e(\xi) = \frac{1}{2}\int_e \partial_{[e}(\not{d} g_{b]c}\,\xi^c)\,\underline{\epsilon}^{ab} + \partial_{[e}(g_{b]c}\,\xi^c)\,\not{d}\underline{\epsilon}^{ab}$$

$$(\not{d} q_e)(\chi) = \frac{1}{2}\int_e \partial_{[e}(\not{d} g_{b]c}\,\chi^c)\,\underline{\epsilon}^{ab} + \partial_{[e}(g_{b]c}\,\chi^c)\,\not{d}\underline{\epsilon}^{ab}$$

$$\Big\downarrow \chi|_e = 0 \qquad\qquad \Big\downarrow (*)$$

$$= \frac{1}{2}\int_e (\partial_{[e}\chi^c)\,\not{d} g_{b]c}\,\underline{\epsilon}^{ab} + \nabla_{[e}\chi_{b]}\,\not{d}\underline{\epsilon}^{ab}$$

$$=$$

$\neq 0$ Field dependent!

$$d q_e(\xi) = \frac{1}{2}\int_e \partial_{[e}\left(d g_{b]c}\,\xi^c\right)\epsilon^{ab} + \partial_{[e}\left(g_{b]c}\,\xi^c\right) d\epsilon^{ab}$$

$$(d q_e)(\chi) = \frac{1}{2}\int_e \partial_{[e}\left(d g_{b]c}\,\chi^c\right)\epsilon^{ab} + \partial_{[e}\left(g_{b]c}\,\chi^c\right) d\epsilon^{ab}$$

$\uparrow (*)$

$\chi|_e = 0 \downarrow$

$$= \frac{1}{2}\int_e \left(\partial_{[e}\chi^c\right) d g_{b]c}\,\epsilon^{ab} + \nabla_{[e}\chi_{b]}\, d\epsilon^{ab}$$

$\Gamma\ \chi|_e = 0 \downarrow$ $\quad\downarrow \nabla_a \chi_b = \kappa\,\varepsilon_{ab}$

$$= \frac{1}{2}\int_e \nabla_{[e}\chi^c\, d g_{b]c}\,\epsilon^{ab} + \kappa\,\varepsilon_{ab}\, d\epsilon^{ab}$$

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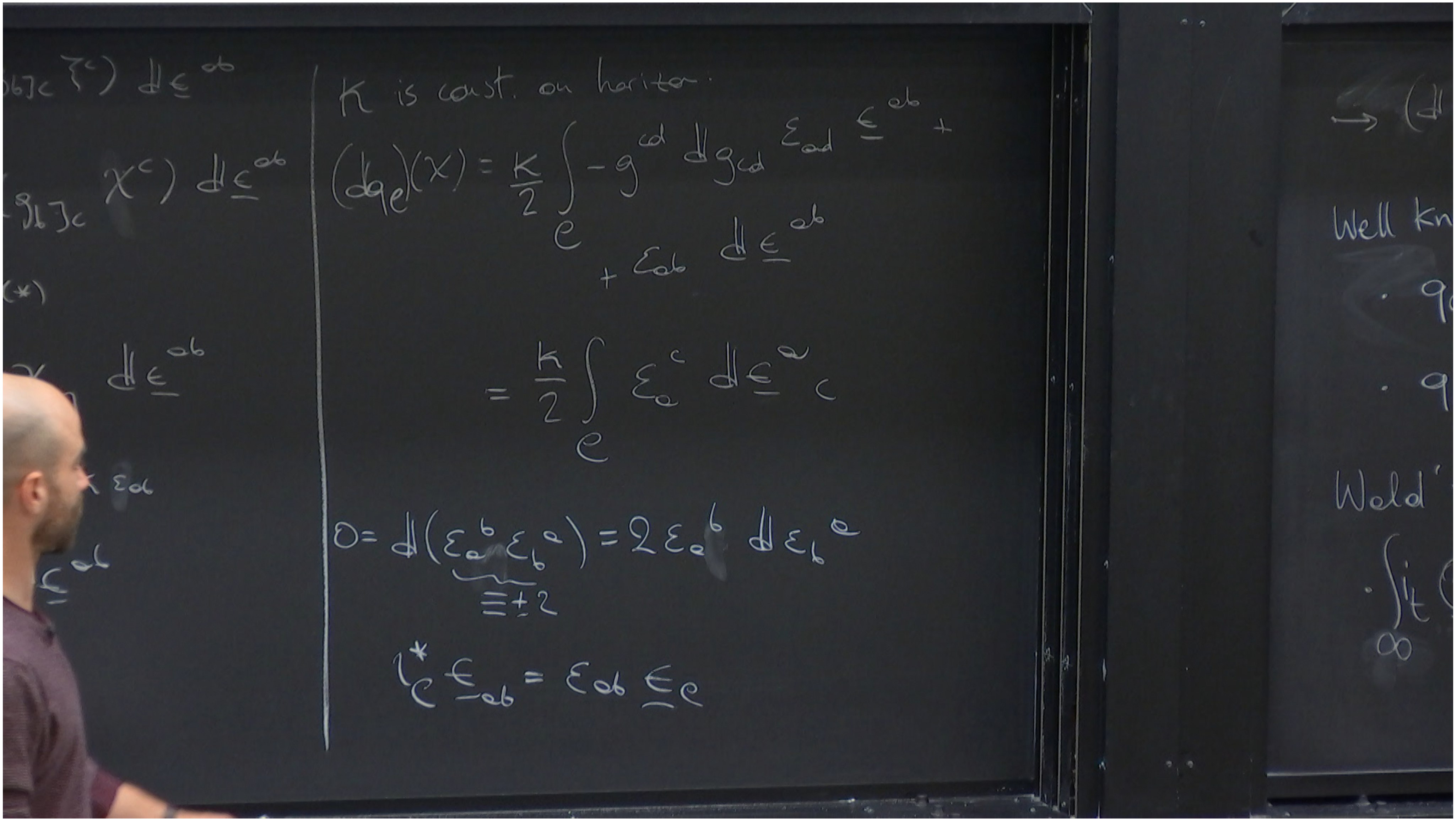
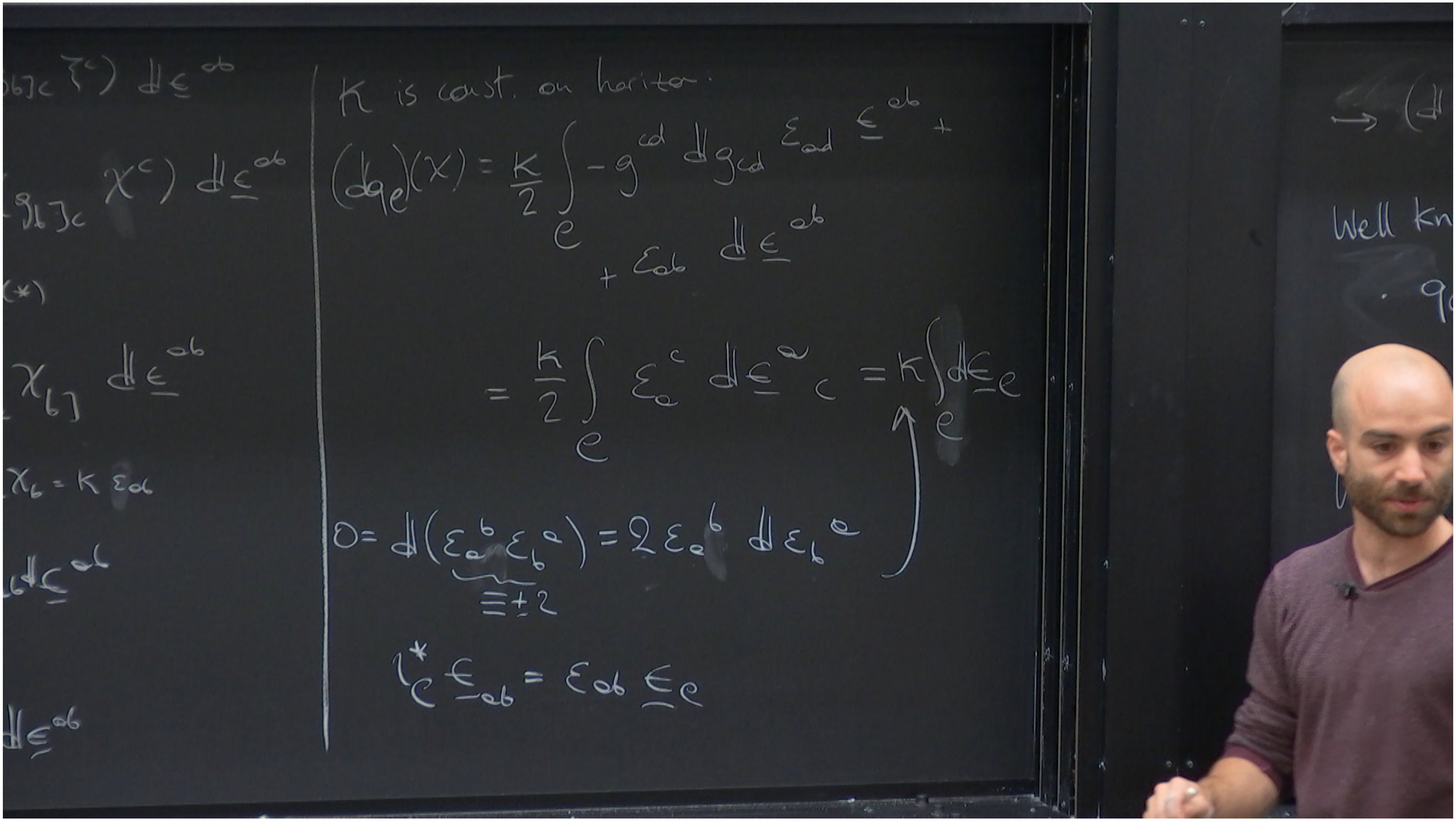

κ is const. on horizon:
$(\mathrm{d}q_e)(\chi) = \frac{\kappa}{2}\int_e -g^{cd}\,\mathrm{d}g_{cd}\,\varepsilon_{ad}\,\epsilon^{ab} +$
$+\ \varepsilon_{ab}\,\mathrm{d}\epsilon^{ab}$
$= \frac{\kappa}{2}\int_e \varepsilon_a{}^c\,\mathrm{d}\epsilon^a{}_c$
$0 = \mathrm{d}(\underbrace{\varepsilon_a{}^b\varepsilon_b{}^a}_{\equiv \pm 2}) = 2\varepsilon_a{}^b\,\mathrm{d}\varepsilon_b{}^a$
$\iota_e^*\epsilon_{ab} = \varepsilon_{ab}\,\epsilon_e$
Well
Wald


K is const. on horizon.
$(\mathrm{d}q_e)(\chi) = \frac{K}{2}\int_e -g^{cd}\,\mathrm{d}g_{cd}\,\varepsilon_{ab}\,\epsilon^{ab} + \varepsilon_{ab}\,\mathrm{d}\epsilon^{ab}$
$= \frac{K}{2}\int_e \varepsilon_a{}^c\,\mathrm{d}\epsilon^a{}_c = K\int_e \mathrm{d}\epsilon_e$
$0 = \mathrm{d}(\underbrace{\varepsilon_a{}^b\varepsilon_b{}^a}_{\equiv \pm 2}) = 2\varepsilon_a{}^b\,\mathrm{d}\varepsilon_b{}^a$
$\iota_e^*\epsilon_{ab} = \varepsilon_{ab}\,\epsilon_e$
Well kn

κ is const. on horizon:

$$(\delta q_e)(\chi) = \frac{\kappa}{2}\int_e -g^{cd}\,\delta g_{cd}\,\varepsilon_{ab}\,\underline{\epsilon}^{ab} + \varepsilon_{ab}\,\delta\underline{\epsilon}^{ab}$$

$$= \frac{\kappa}{2}\int_e \varepsilon_a{}^c\,\delta\underline{\epsilon}^a{}_c = \kappa\int_e \delta\underline{\epsilon}_e$$

index structure!

$$0 = \delta(\underbrace{\varepsilon_a{}^b\,\varepsilon_b{}^a}_{\equiv \pm 2}) = 2\,\varepsilon_a{}^b\,\delta\varepsilon_b{}^a$$

$$\iota_e^*\,\underline{\epsilon}_{ab} = \varepsilon_{ab}\,\underline{\epsilon}_e$$

(50s)

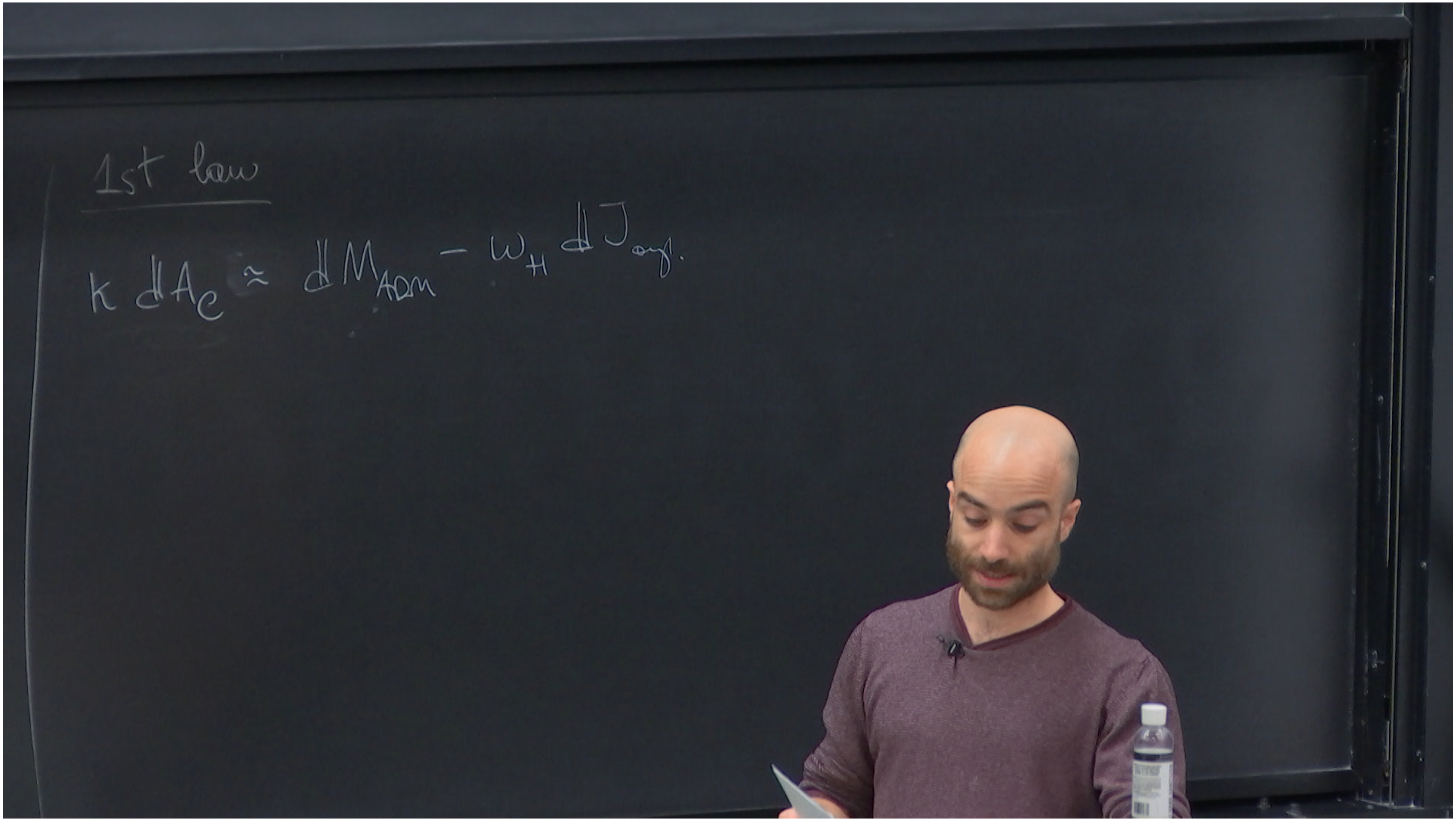
$M_{ADM} \geq 0$
& $= 0$ iff
Mink.

Putting everything together:

$$\underbrace{(dq_e)(X)}_{\substack{\text{geom. quantity} \\ \text{def @ horizon}}} \approx \underbrace{-\omega_H \, dJ_{asym} + dM_{ADM}}_{\substack{\text{ang. mom \& energy as} \\ \text{defined from \underline{infinity}}}}$$

$$\|$$

$$\kappa \int_e d\epsilon_e = \kappa \, d\mathrm{Area}(e)$$


1st law
$\kappa \, dA_C \simeq dM_{ADM} - \omega_H \, dJ_{ang.}$