Title: Quantum Gravity Lecture (230504)

Speakers: Aldo Riello

Collection: Quantum Gravity (2022/2023)

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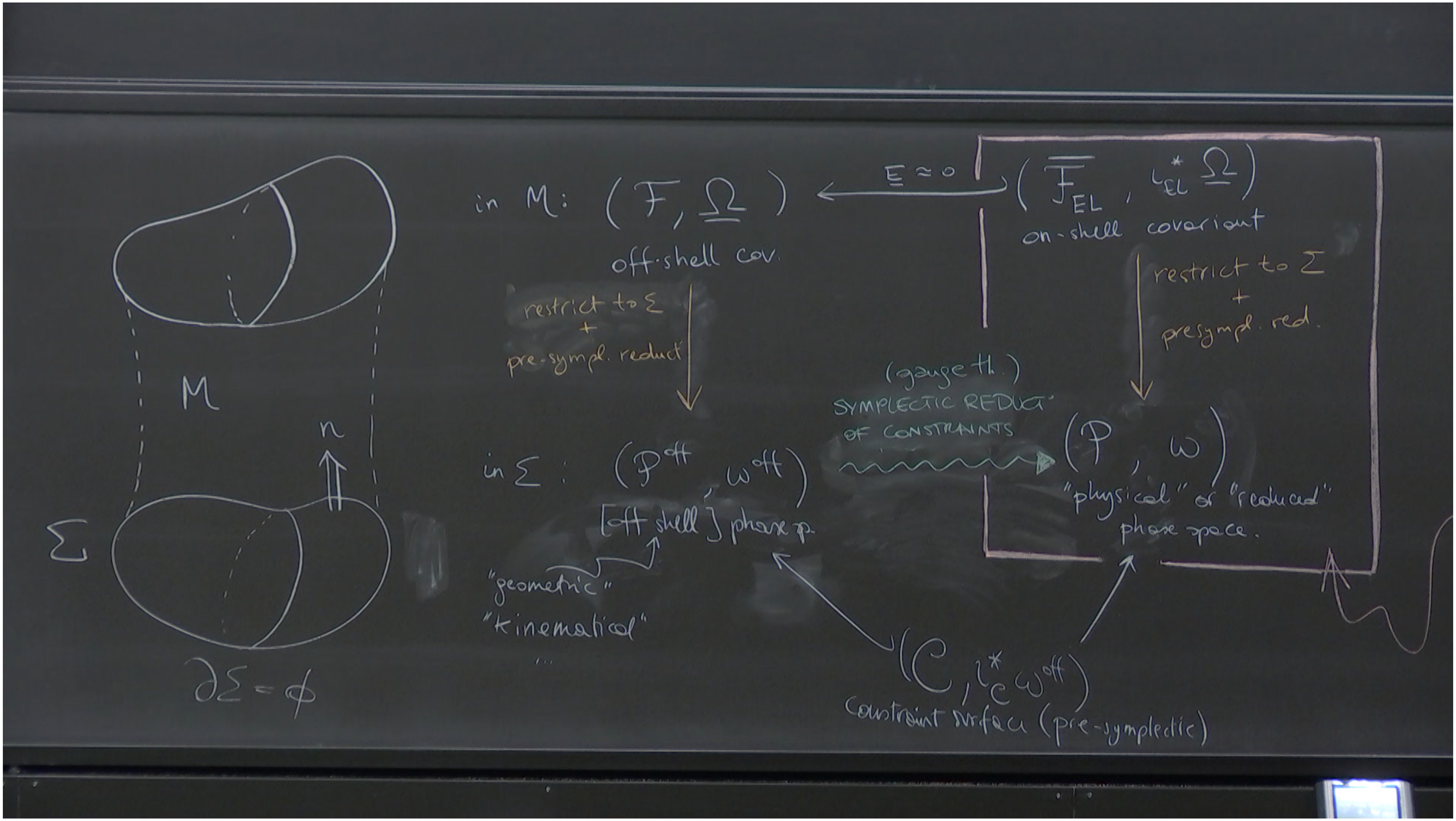
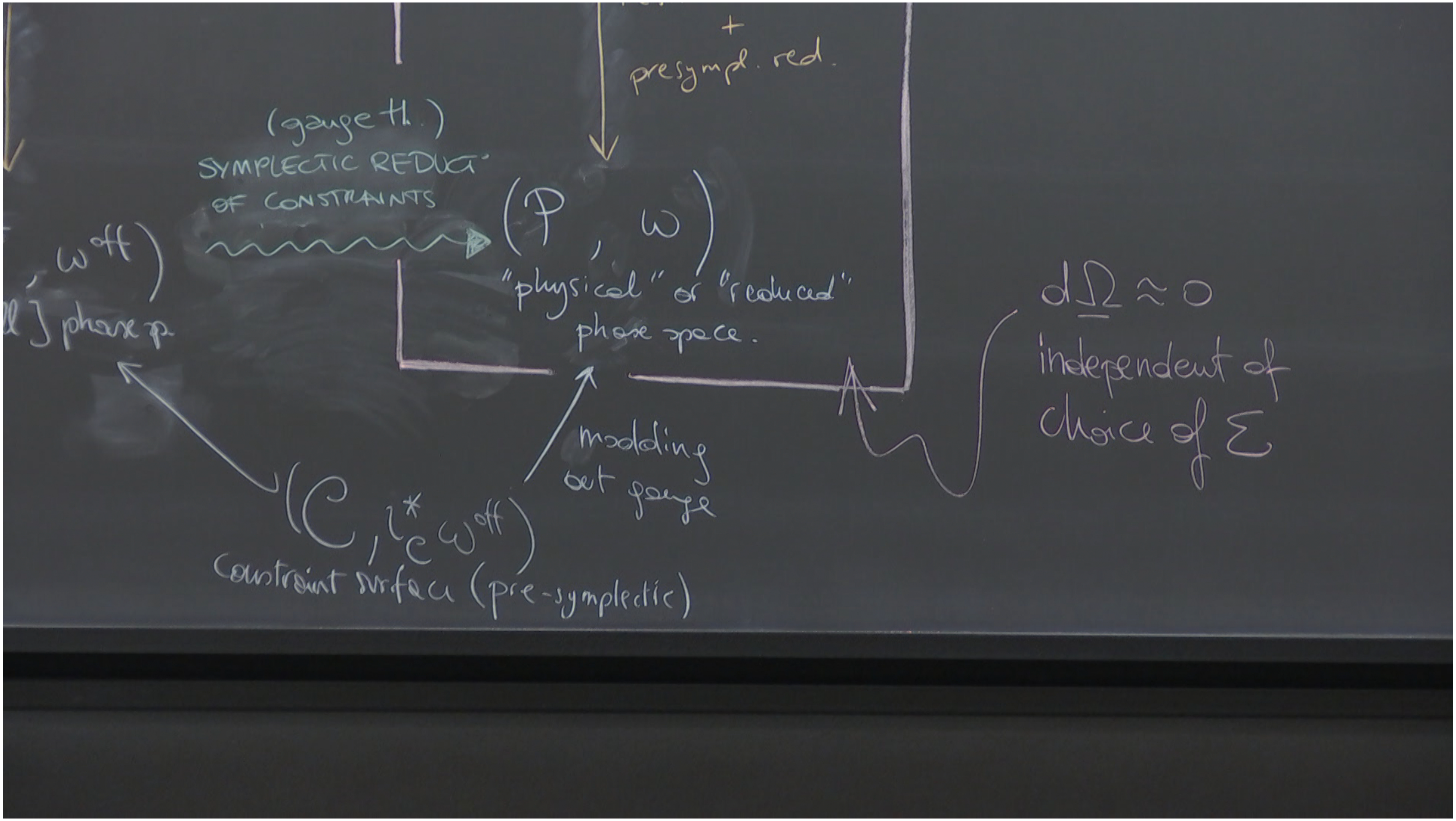

in M: $(\mathcal{F}, \Omega)$
off-shell cov.
$E \approx 0$
$(\mathcal{F}_{EL}, \iota^*_{EL}\Omega)$
on-shell covariant
restrict to Σ + pre-sympl. reduct
restrict to Σ + presympl. red.
M
n
Σ
in Σ: $(\mathcal{P}^{off}, \omega^{off})$
[off shell] phase p.
"geometric"
"kinematical"
...
$\partial\Sigma = \emptyset$
(gauge th.)
SYMPLECTIC REDUCT
OF CONSTRAINTS
$(\mathcal{P}, \omega)$
"physical" or "reduced" phase space.
$(\mathcal{C}, \iota^*_{\mathcal{C}}\omega^{off})$
constraint surface (pre-symplectic)


+
presympl. red.
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$, \omega^{\text{off}})$
l] phase sp.
$(\mathcal{C}, \iota_{\mathcal{C}}^{*}\omega^{\text{off}})$
constraint surface (pre-symplectic)
modding
out gauge
$d\underline{\Omega} \approx 0$
independent of
choice of Σ

Recall: <u>constraint</u> = the "bulk part" of the Noether current projected on Σ, it must vanish on-shell.

Configurations that do <u>not</u> satisfy the constraints are <u>not</u> viable initial conditions for the dynamic

$d\underline{\Omega} \approx 0$

independent of choice of Σ

$$\mathring{I}_{\rho(\xi)}\,\omega^{\text{off}}_{\Sigma} = -\,\mathbb{d}\,Q_{\Sigma}(\xi) \approx 0$$

Recall : time transl. scalar field.

$$\mathring{i}_{\rho(X)}\,\Omega_\Sigma = -d\!\!\!/\int_\Sigma \tfrac{1}{2}\dot{\phi}^2 X + \int_\Sigma \ddot{\phi}\, d\!\!\!/\phi$$

Σ in Hamilt. EoM

$$\int_\Sigma d\!\!\!/\dot{\phi} \wedge d\!\!\!/\phi$$

Recall [GR]

$$\Theta_\Sigma = \frac{1}{2}\int \sqrt{h}\, n_a \left(\nabla_b\, d\!\!\!/ g^{ab} - \nabla^a d\!\!\!/ g\right)$$

$$J^a(\xi) = \left(G^{ab} + \Lambda g^{ab}\right)\xi_b + \nabla_b\left(\text{Komar curr of } \xi\right)^b$$

$$\mathring{i}_{\rho(\xi)}\,\Omega_\Sigma = -d\!\!\!/\int_\Sigma \sqrt{h}\, n_a J^a(\xi) + \int_\Sigma i_\xi E^{ab}\, d\!\!\!/ g_{ab} + \int_{\partial\Sigma} i_\xi \Theta$$

Canonical GR

3+1 decomposition of Σ

- h_{ij} : the induced metric on Σ
- n^a : the normal to Σ $\to (N, N^i)$ lapse & shift
- $\pi^{ij} = \sqrt{h}\left(K^{ij} - K h^{ij}\right)$

K_{ij} = extrinsic curvature of Σ

$\left[= \frac{1}{2} \mathcal{L}_n h_{ab} \sim \dot{h}_{ab} \right]$

- $\omega^{\text{eff}} = \int_\Sigma d\pi^{ij} \wedge dh_{ij} + \cancel{(\text{corner terms})}$

no n^a! (similar to A_0 in YM)

of $\xi)^b$

$dg_{ab} + \int_{\partial\Sigma} i_\xi$ ⊙

(N, N^i) lapse & shift

of Σ

$\dot{h}_{ab}$]

terms)

similar to A_0 in YM)

Symmetries

$$\underset{\substack{\uparrow \\ TM}}{\xi} = \underset{\substack{\uparrow \\ T\Sigma}}{X} + \lambda\, n$$

$T\Sigma$: intrinsic notion to Σ

n: normal to Σ : depends on g_{ab} & cannot be "projected" on the canonical ph. sp.

Constraints

$$C^{(\xi)}_a \xi^a = \left[n_a J^a(\xi) \right]_{bulk}$$

$$= \underbrace{n_a n_b \left(G^{ab} + \Lambda g^{ab}\right)}_{H \text{ (Hamilt. constr)}} \lambda + \underbrace{n_a h^i_b \left(G^{ab} + \Lambda g^{ab}\right)}_{V \text{ (vector constr)}} X_i$$

After 3+1 split of Einstein's eqs:
(via Gauss-Codazzi-Mainardi eqs)

$$\begin{cases} V_i = \partial_j \pi^j{}_i & \text{(looks like Gauss)} \\ H = \dfrac{1}{2\sqrt{h}}\left(\pi_{kl}\pi^{kl} - \dfrac{1}{2}(\pi^k{}_k)^2\right) - \dfrac{1}{2}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right) \end{cases}$$

<u>Note</u>: $\hat{H}\Psi = 0$ is the Wheeler-DeWitt eq.

$^{(3)}R - 2\Lambda\Big)$

eq.

Reduction

① 3d diffeos (vector constr)

(*) projects nicely onto $\mathcal{P}^{\text{off}}$

and reads:

$$\mathbb{i}_{e(X)}\, \omega^{\text{off}} = -\mathbb{d}\int_\Sigma V_i X^i \equiv -\mathbb{d}\, V(X)$$

(h and K are 3d tensor)

$\longrightarrow$ Everything as in YM!

$$\{V(X), V(Y)\}^{\text{off}} = V([X,Y])$$

impor

important: $[X,Y]$ is the diff-3 Lie algebra bracket (does not depend on h nor K)

$= X^i \partial_i Y^j - Y^i \partial_i X^j$

off

$X^i \equiv - d V(X)$

)

$([X,Y])$

② [Lee Wald '91]

the action of normal diffeos does not project down to Diff.

reason: $\rho(\lambda n)\, h \sim \dot{h} \sim K \sim \Pi$ ✓

$\rho(\lambda_n)\, \Pi \sim \underbrace{\dot{\Pi} \sim \ddot{h}}_{\text{eom relate this quantity to } (\Pi, h)} \leftarrow \times$ not a phase space variable.

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$V(X)$

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$\rho(\lambda_n)\, \Pi \sim \underbrace{\dot{\Pi} \sim \ddot{h}}$ ← ✗ not a phase space variable.

eom relate this quantity to (Π, h)

$$J^a(\xi) = (G^{ab} + \Lambda g^{ab})\xi_b + \nabla_b(\text{Komar curr of } \xi)^b$$

$$\mathring{I}_{\rho(\xi)}\Omega_\Sigma = -d\int_\Sigma \sqrt{h}\, n_a J^a(\xi) + \int_\Sigma i_\xi E^{ab}\, dg_{ab} + \int_{\partial\Sigma} i_\xi \Theta \quad (*)$$

$\cdot\ \omega^{\text{off}} = \int$

(ADM) Σ

After 3+1 split of Einstein's eqs:
(via Gauss-Codazzi-Mainardi eqs)

$$\begin{cases} V_i = \partial_j \pi^j{}_i & \text{(looks like Gauss)} \\ H = \dfrac{1}{2\sqrt{h}}\left(\pi_{kl}\pi^{kl} - \dfrac{1}{2}(\pi^k{}_k)^2\right) - \dfrac{1}{2}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right) \end{cases}$$

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<u>Reduction</u>

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and reads:

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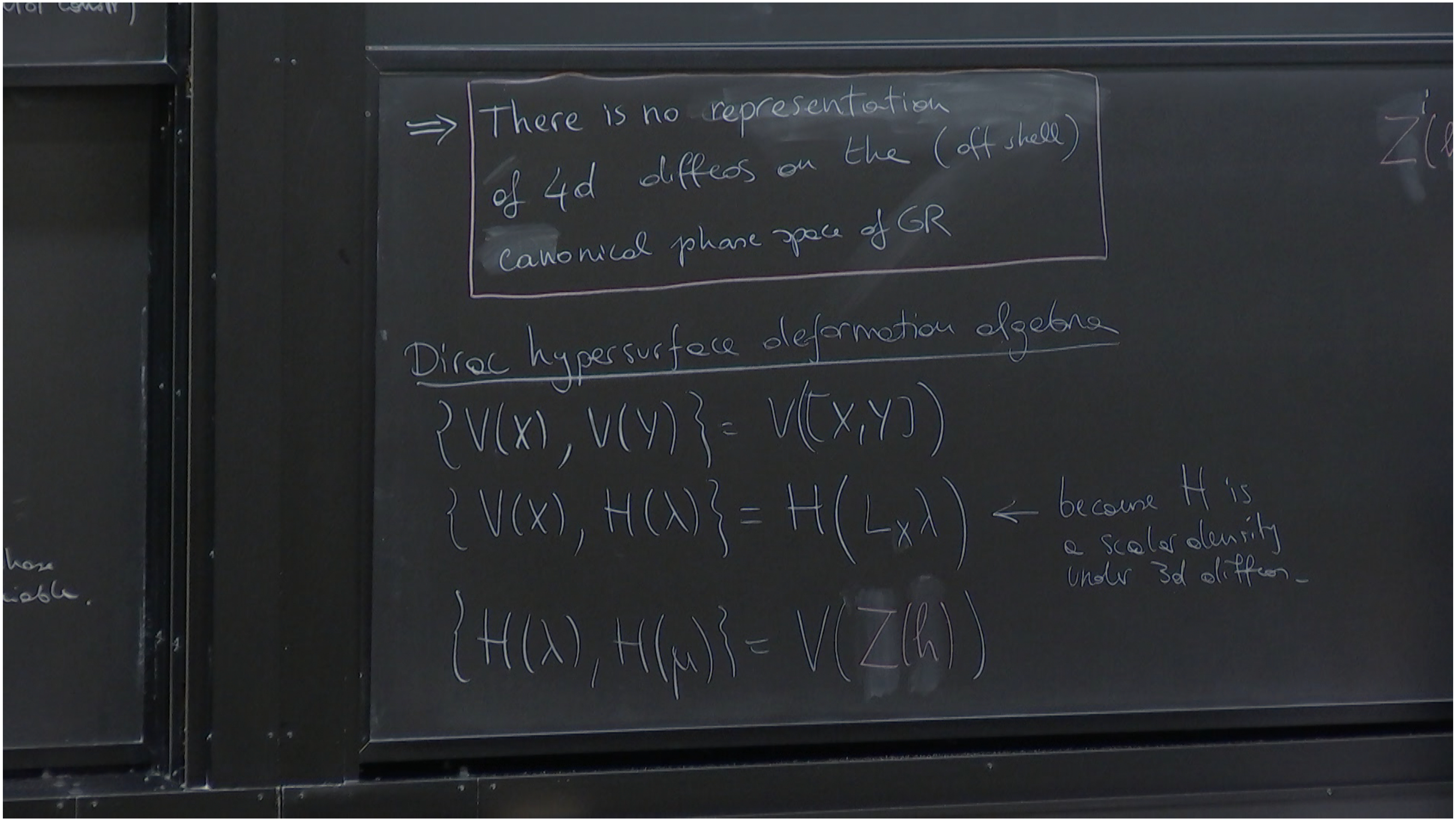
$\{ V(X)$



⇒ There is no representation of 4d diffeos on the (off shell) canonical phase space of GR
Dirac hypersurface deformation algebra
$\{V(X), V(Y)\} = V([X,Y])$
$\{V(X), H(\lambda)\} = H(L_X \lambda)$ ← because H is a scalar density under 3d diffeos.
$\{H(\lambda), H(\mu)\} = V(Z(h))$

$$Z^i(h) = h^{ij}(\lambda \partial_j \mu - \mu \partial_j \lambda)$$

depends on the configuration we are at!

"Algebra" of constraints closes

(Hamilt. flows of constr. tangent to $\mathcal{C} \hookrightarrow P^{\text{off}}$)

$\rightarrow$ can still define $\mathcal{P} = \mathcal{C} / (\text{flow of constraints})$

t is
density
diffeo-

Thm (Kuchař + Hojman + Teitelboim)

$\mathcal{P}^{\text{off}} = (\pi, h)$, modulo techn. hyp.

$\exists!$ $V(h,\pi)$, $H(h,\pi)$ such that they satisfy the DHSD Algebra.

Moreover, this is GR (w/ dynamics & 4d diffeo invariance implemented)