Title: Quantum Matter Lecture (230411)

Speakers: Ganapathy Baskaran

Collection: Quantum Matter (2022/2023)

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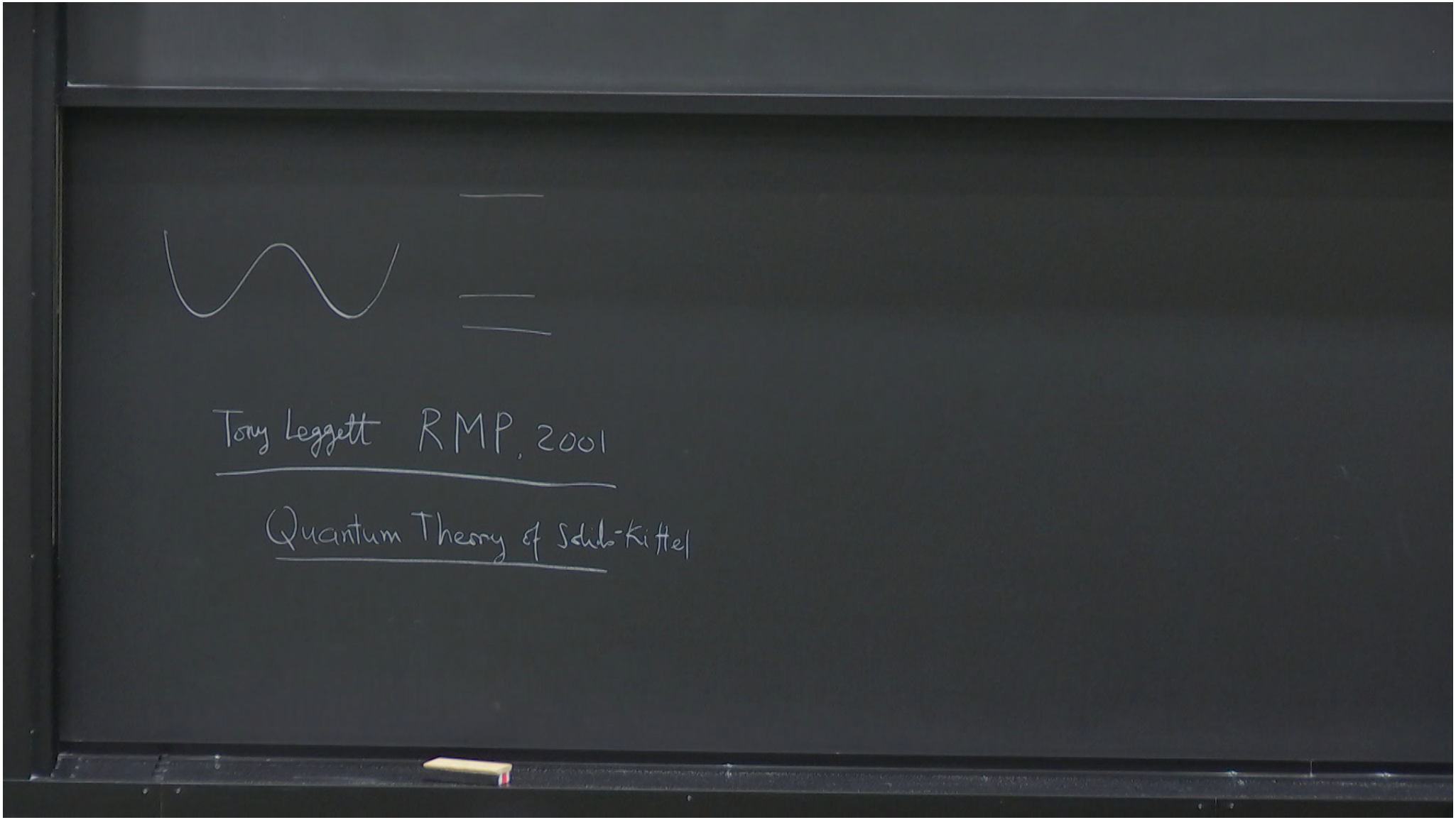

Tony Leggett RMP, 2001
Quantum Theory of Schub-Kittel

Adv. Q CMP

2020

Michel El Batanouny

Tony Leggett RMP, 2001

Quantum Theory of Schib-Kittel

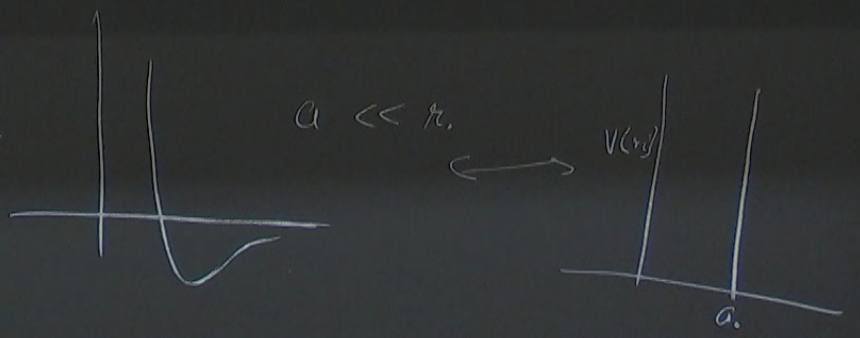
{ Daniel I. Khomskii

{ Basic Aspects of QTS

Bogoliubov Theory of Weakly Interacting Bose Gas

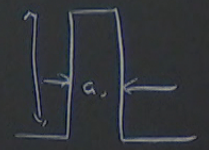
$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2M} + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

$$V(\vec{r}_{ij}) \sim V_0\, \delta(\vec{r}_{ij})$$



$$H = \sum \left(\frac{\hbar^2 k^2}{2M}\right) a_k^\dagger a_k + \left(\frac{V_0}{N}\right) \sum a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4}\, \delta(k_1 + k_2 - k_3 - k_4)$$

$$\frac{V_0}{\sqrt{a}}\, e^{-\left(\frac{r_{ij}}{a}\right)^2}$$

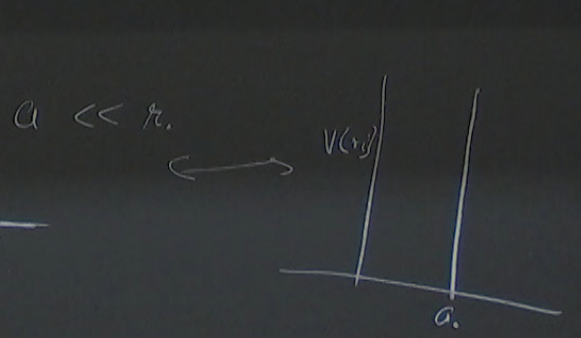


Bogoliubov Theory of Weakly Interacting Bose Gas

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2M} + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

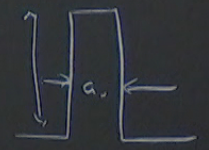
$$\phi(x) = \frac{1}{\sqrt{L}} e^{i\vec{k}\cdot\vec{x}}$$

$$V(\vec{r}_{ij}) \sim V_0\, \delta(\vec{r}_{ij})$$



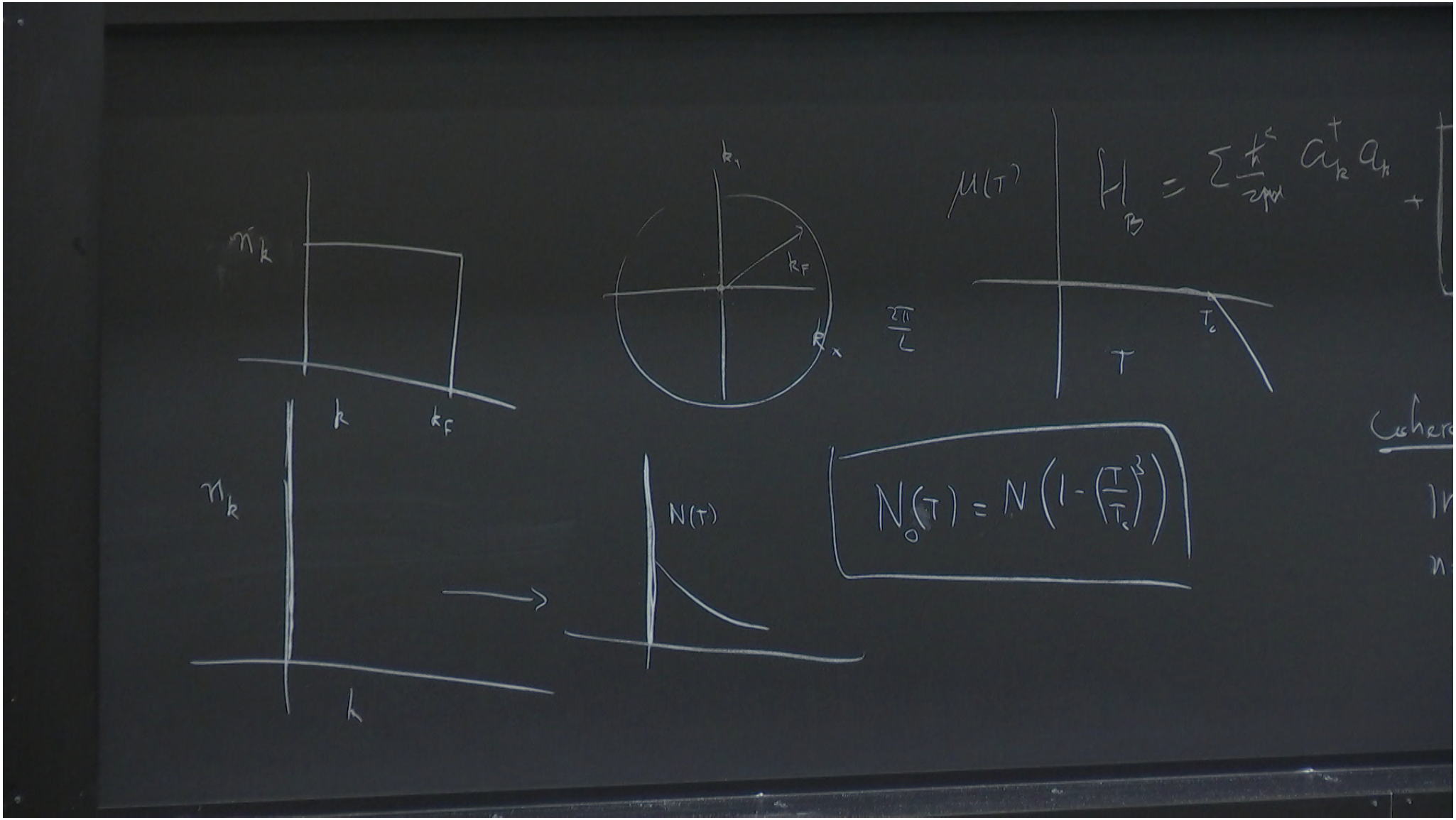
$$H = \sum \left(\frac{\hbar^2 k^2}{2M}\right) a_k^\dagger a_k + \left(\frac{V_0}{N}\right) \sum a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4}\, \delta(k_1 + k_2 - k_3 - k_4)$$

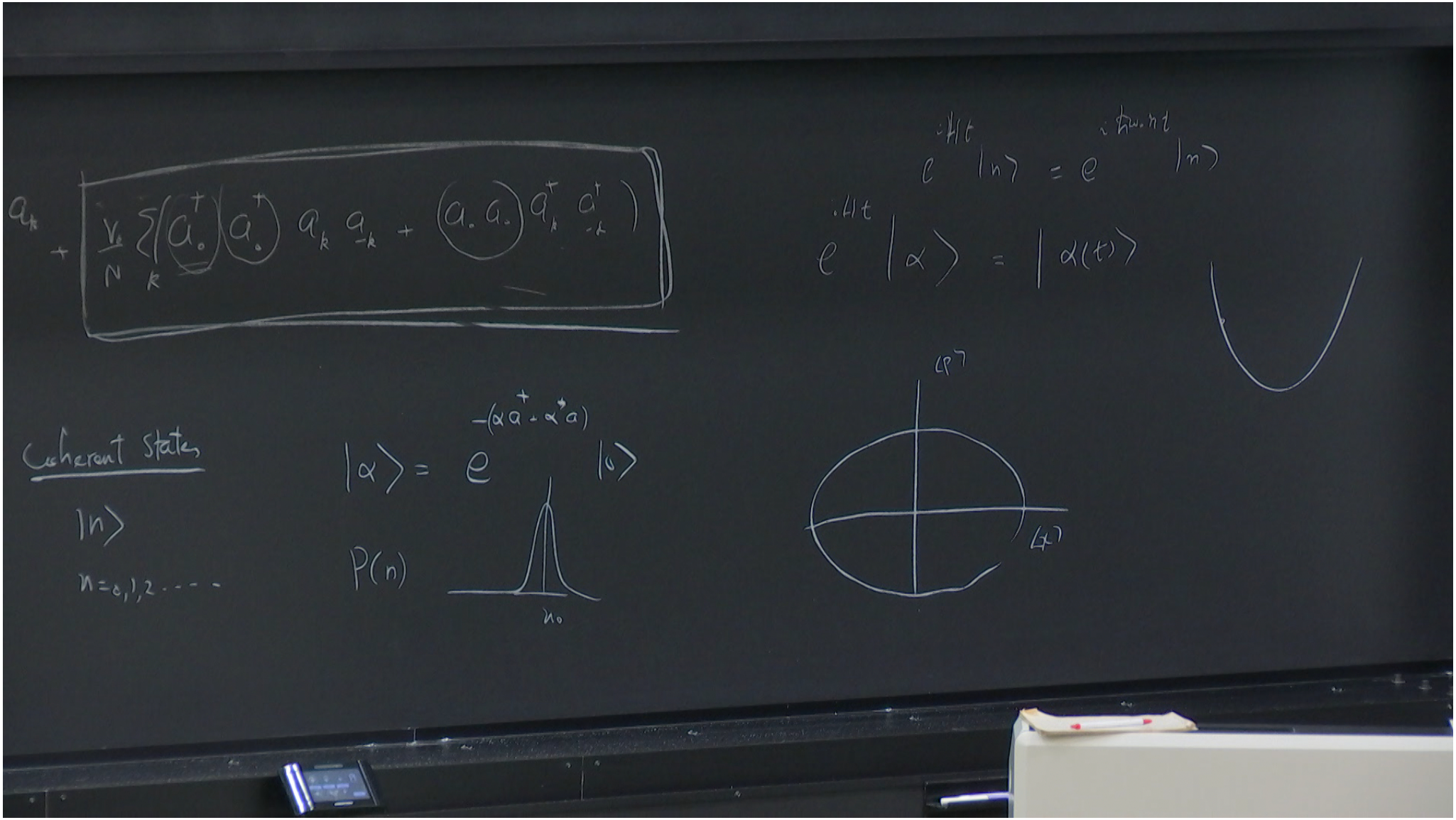
$$\frac{V_0}{\sqrt{L}}\, e^{-\left(\frac{r_{ij}}{a}\right)^2}$$

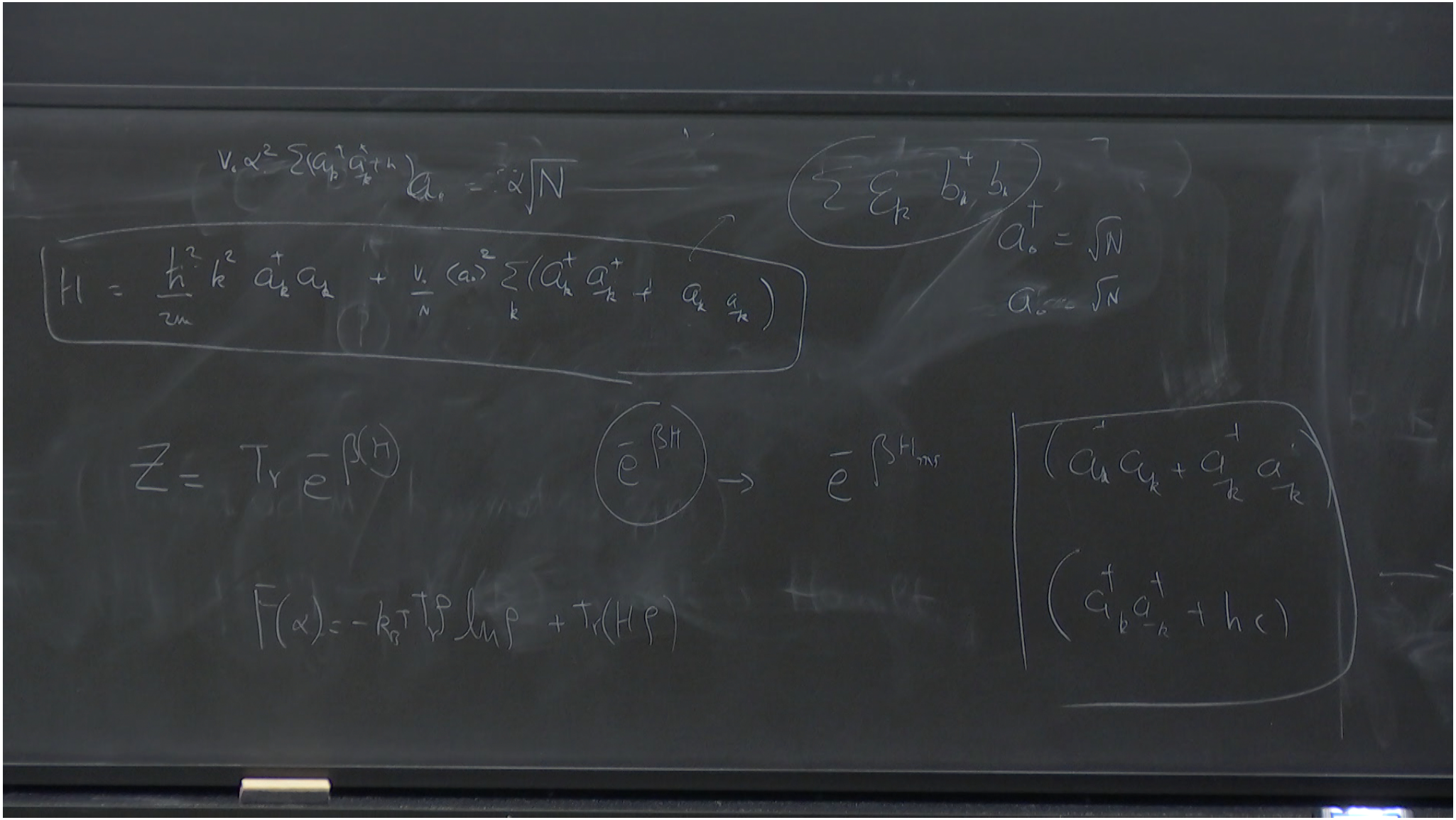


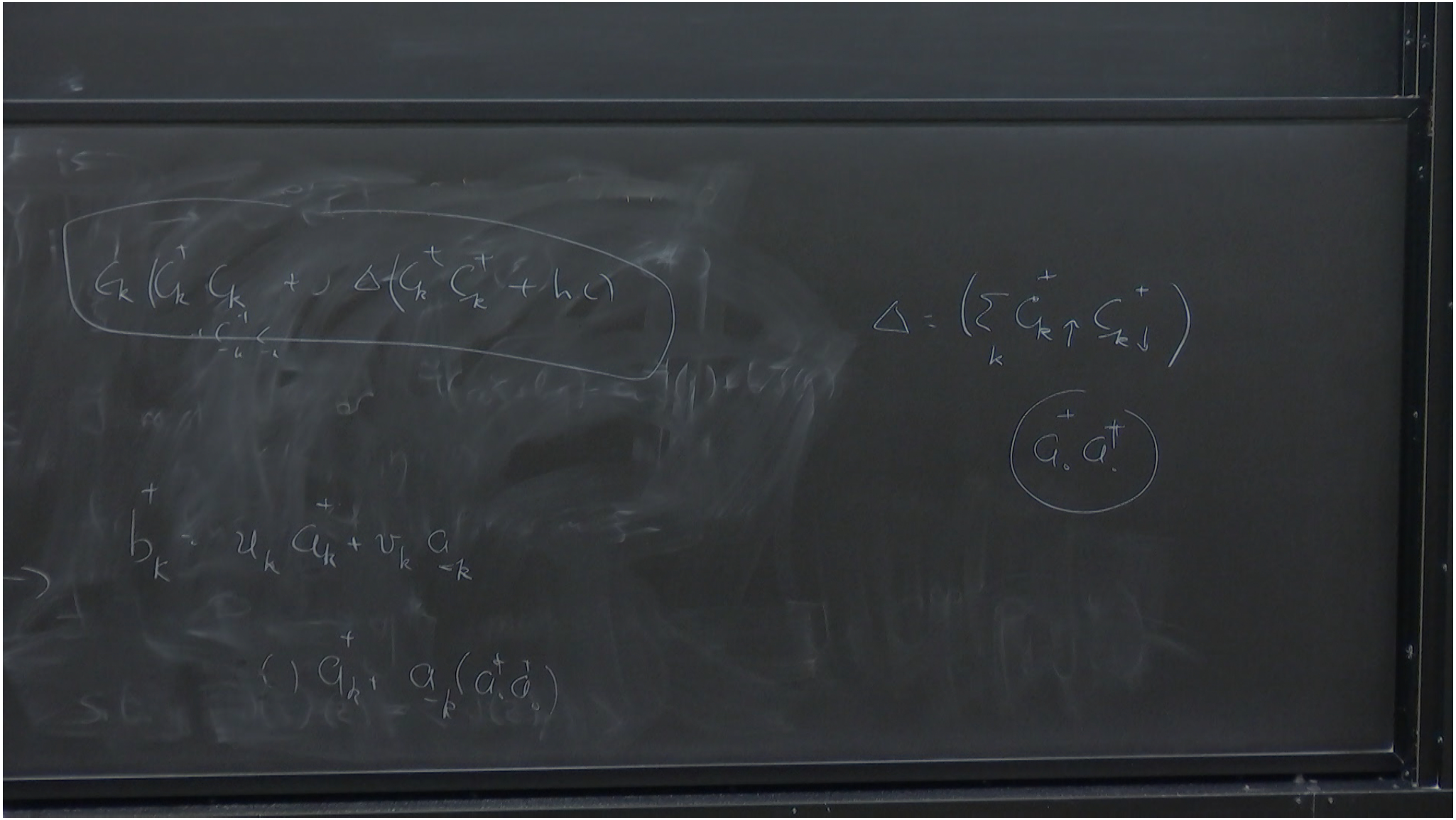
$$n_0 = N \qquad \boxed{a_0^\dagger a_0 = N} \qquad \left\{a_0, a_0^\dagger\right\} = 1 \qquad a_0 a_0^\dagger - a_0^\dagger a_0 = 1$$

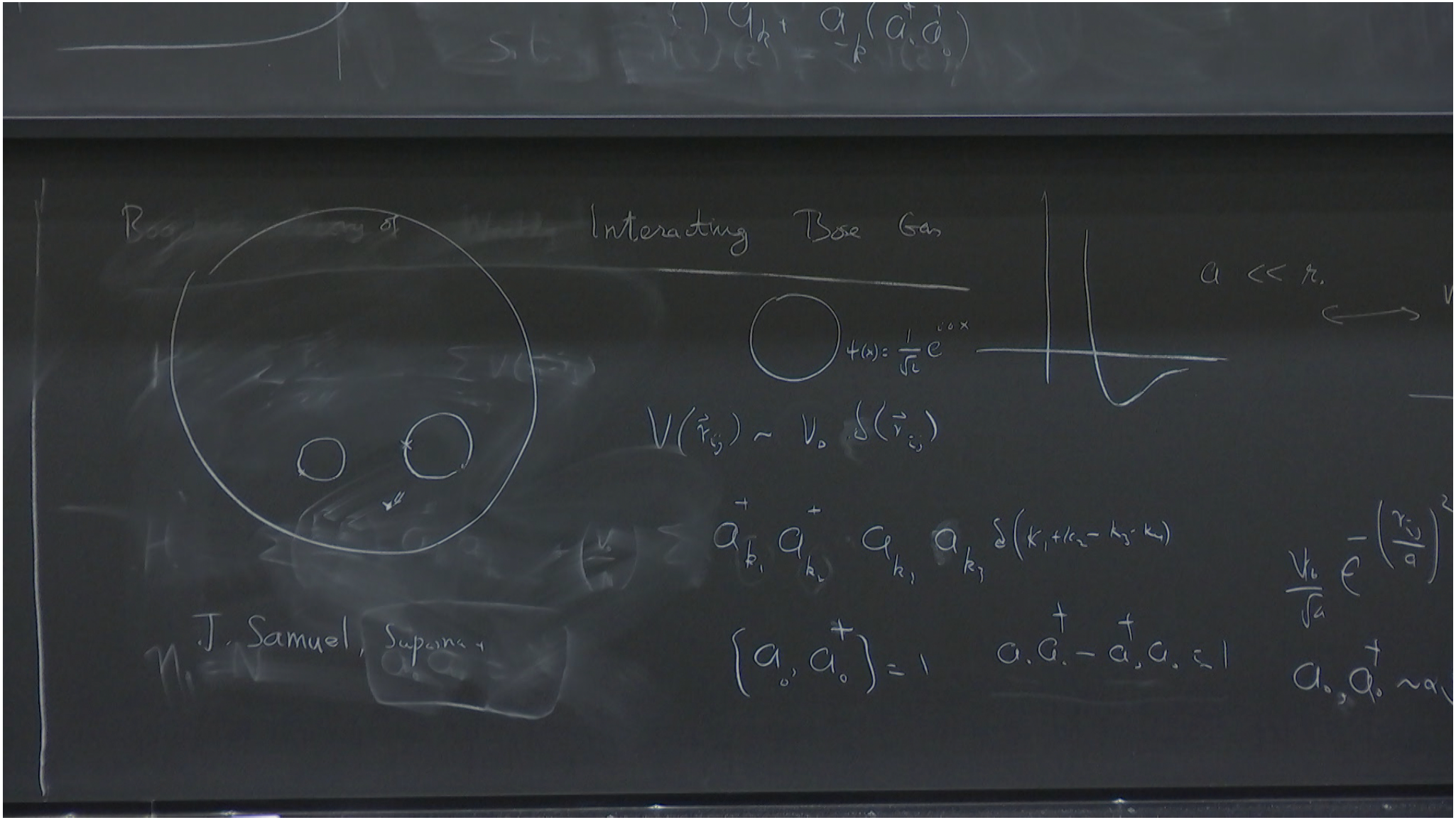
$$a_0, a_0^\dagger \sim a\sqrt{N}$$


n_k
k
k_F
k_y
k_x
$\frac{2\pi}{L}$
$\mu(T)$
T
T_c
$H_B = \sum \frac{\hbar^2}{2m} a_k^\dagger a_k$ +
n_k
k
$N(T)$
$N_0(T) = N\left(1 - \left(\frac{T}{T_c}\right)^3\right)$


Coherent States
|n⟩
n = 0, 1, 2 . . . .
P(n)
n₀
⟨p⟩
⟨x⟩


$V_0 \alpha^2 \sum (a_k^\dagger a_k^\dagger + h.c.) \sqrt{N} = \alpha \sqrt{N}$
$\sum \varepsilon_k b_k^\dagger b_k$
$H = \frac{\hbar^2}{2m} k^2 a_k^\dagger a_k + \frac{V_0}{N} (a_0)^2 \sum_k (a_k^\dagger a_{-k}^\dagger + a_k a_{-k})$
$a_0^\dagger = \sqrt{N}$
$a_0 = \sqrt{N}$
$Z = Tr\, e^{-\beta(H)}$
$e^{-\beta H} \rightarrow e^{-\beta H_{new}}$
$(a_k^\dagger a_k + a_{-k}^\dagger a_{-k})$
$(a_k^\dagger a_{-k}^\dagger + h.c.)$
$F(\alpha) = -k_B T Tr \rho \ln \rho + Tr(H\rho)$


$\Delta = \left(\sum_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)$
$a_\circ a^{\dagger}_\circ$
$b_k = u_k c_k + v_k c^{\dagger}_{-k}$
$a_k + a_{-k}(a^{\dagger}_\circ a_\circ)$


Interacting Bose Gas
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J. Samuel, Supern