Title: Quantization of causal diamonds in 2+1 dimensional gravity

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Series: Quantum Gravity

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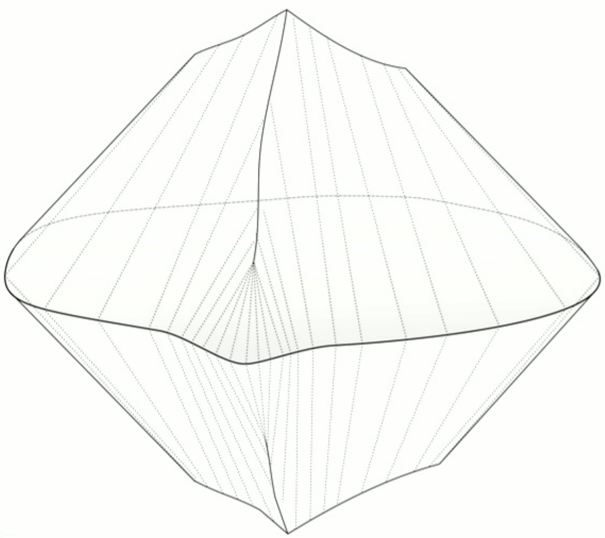
Abstract: We develop the reduced phase space quantization of causal diamonds in $2+1$ dimensional gravity with a nonpositive cosmological constant. The system is defined as the domain of dependence of a spacelike topological disk with fixed (induced) boundary metric. By solving the constraints in a constant-mean-curvature time gauge and removing all the spatial gauge redundancy, we find that the phase space is the cotangent bundle of $Diff^+(S^1)/PSL(2, \mathbb{R})$, i.e., the group of orientation-preserving diffeomorphisms of the circle modulo the projective special linear subgroup. Classically, the states correspond to causal diamonds embedded in AdS_3 (or $Mink_3$ if $\Lambda = 0$), with a fixed corner length, that have the topological disk as a Cauchy surface. Because this phase space does not admit a global system of coordinates, a generalization of the standard canonical (coordinate) quantization is required --- in particular, since the configuration space is a homogeneous space for a Lie group, we apply Isham's group-theoretic quantization scheme. The Hilbert space of the associated quantum theory carries an irreducible unitary representation of the BMS_3 group, and can be realized by wavefunctions on a coadjoint orbit of Virasoro with labels in irreducible unitary representations of the corresponding little group. A surprising result is that the twist of the diamond boundary loop is quantized in terms of the ratio of the Planck length to the corner length.

Zoom link: https://pitp.zoom.us/j/94369372201?pwd=NWNsYno3RmZIWUx0LytWZ09PVDVVQT09

Quantization of causal diamonds in *2+1* dimensional gravity



Rodrigo Andrade e Silva

Perimeter Institute
April 27, 2023

In collaboration with **Ted Jacobson**

Challenges of canonical quantum gravity

- Non-linear constraints/phase space
- Absence of local observables
- Perturbatively non-renormalizable in d>3
- Problem of time

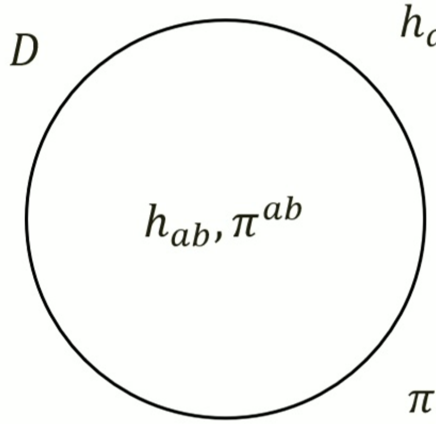
It is worth to explore the non-perturbative quantization of gravity in simplified settings

The system: causal diamonds

2+1 dimensional Einstein-Hilbert gravity with $\Lambda \leq 0$

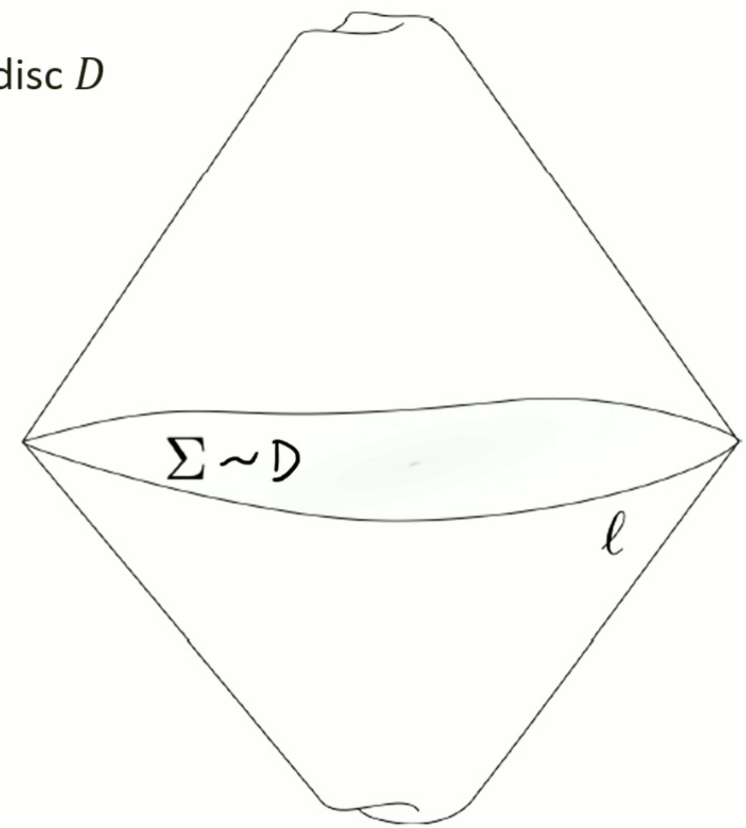
Spacetime: domain of dependence of a topological disc D

Dirichlet condition for the induced boundary metric
(corresponds to fixing the total boundary length ℓ)

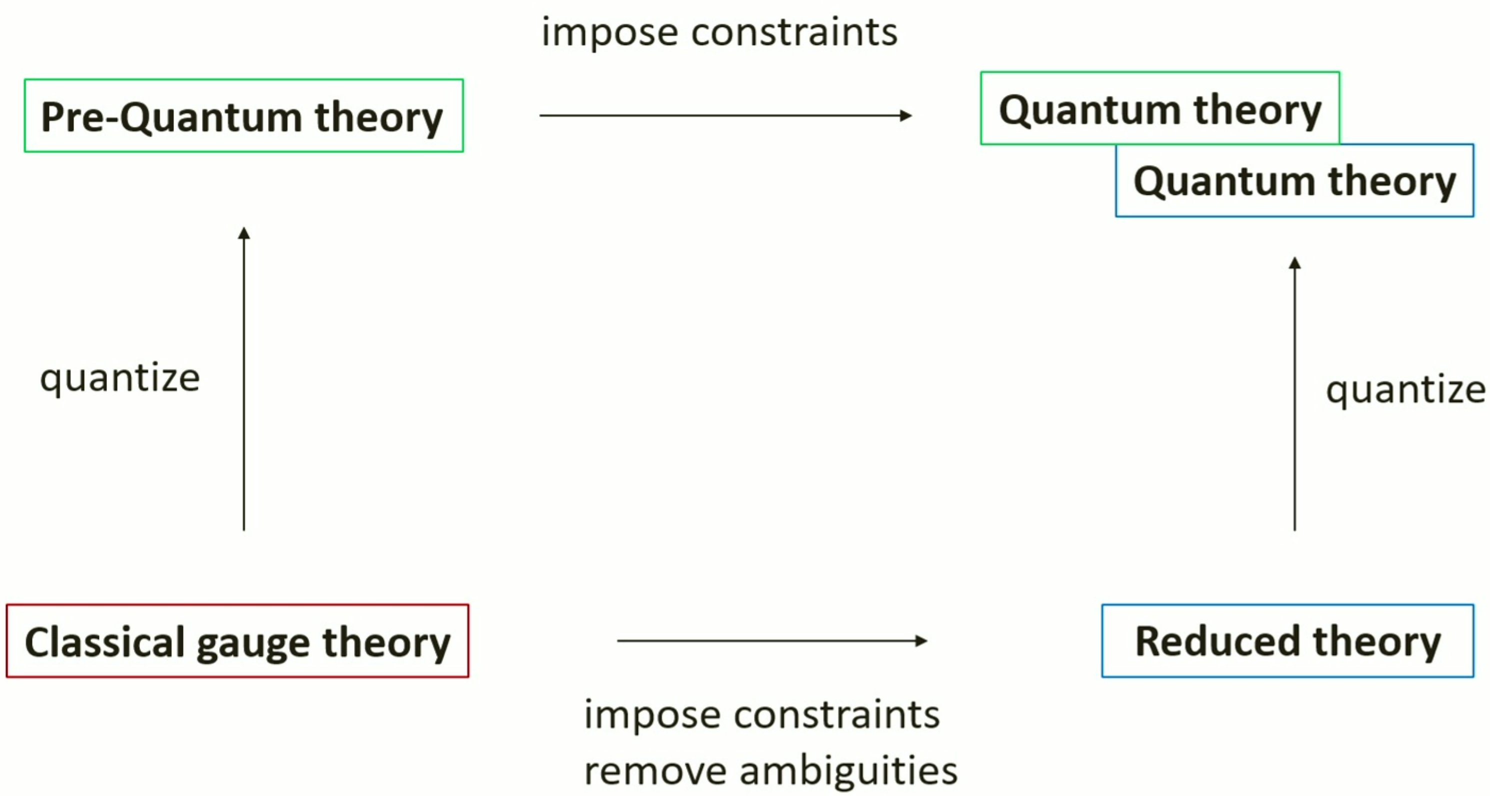


$$h_{ab}\Big|_{\partial D} = \gamma$$

$$\pi^{ab} = \sqrt{\det h}\left(K^{ab} - K h^{ab}\right)$$



Two approaches in gauge theory



Motivation

- Similar approaches have been considered for closed spacetimes. This leads to a finite-dimensional reduced phase space. *[Moncrief, Fischer, Carlip, Witten, …]*

- The causal diamonds are natural system to consider if we wish to understand quantum gravity in a quasi-local setting.

- The low-dimensionality gives us great control of the problem, without making it too trivial (the phase space is infinite-dimensional due to “boundary gravitons”).

Contents

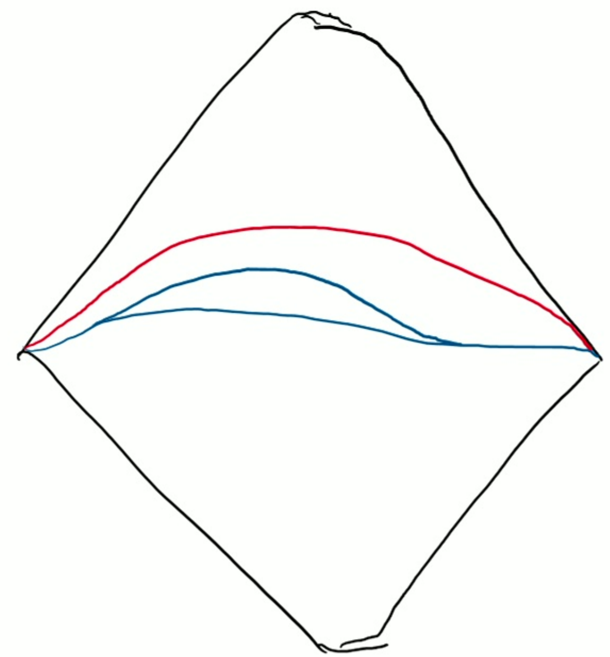
- Constant-mean-curvature (CMC) time
- Solving constraints and removing gauge
- Reduced Hamiltonian
- Canonical quantization, Isham's method
- Representation theory, spin/twist quantization

Constant-mean-curvature time

In GR, there is gauge associated with both time and space diffeomorphisms.

When there are corners/boundaries, we need to be particularly careful to say something is gauge or not.

The condition of fixing the induced metric on the corner, $h_{ab}|_\partial = \gamma$, implies that *all* refoliations are in fact gauge.



Constant-mean-curvature time

The diamond can be nicely foliated by CMC slices ($\Lambda \leq 0$ *ensures this*).

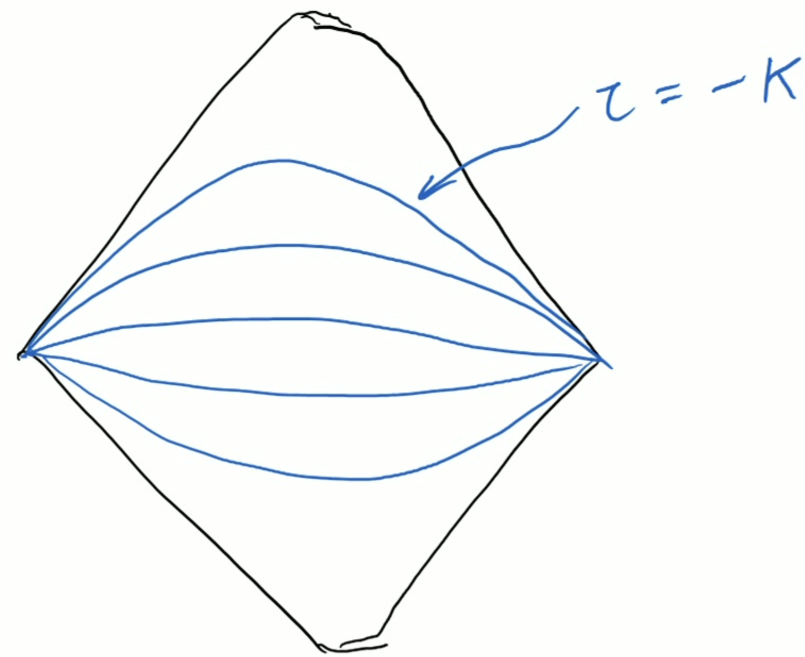
CMC = Constant-mean-curvature surface

$$K = h_{ab}K^{ab} = \text{constant}$$

where K^{ab} is the extrinsic curvature.

This provides a convenient "gauge-fixing" for time, the *York time* $\tau = -K$

[York 72]



Constant-mean-curvature time

Writing K^{ab} as

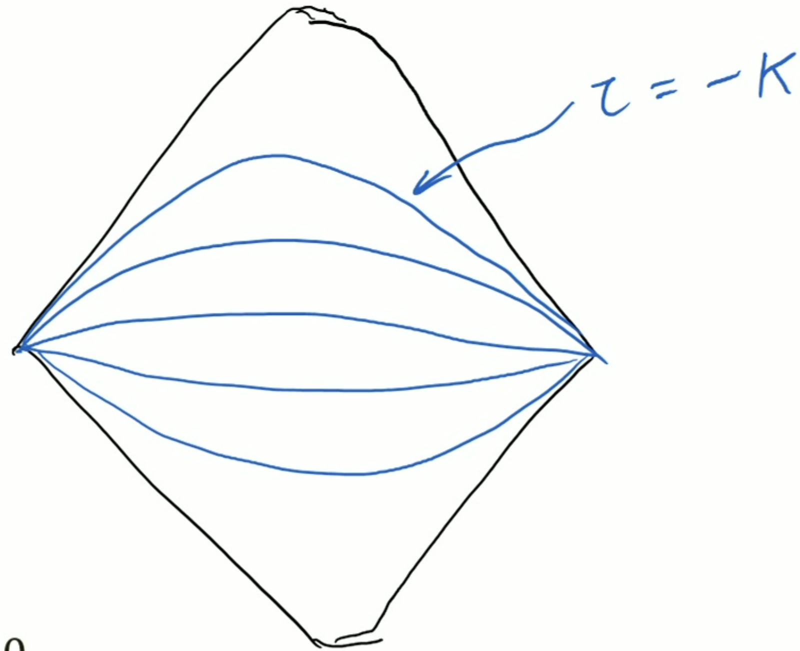
$$K^{ab} = \sigma^{ab} + \frac{1}{2} K h^{ab}$$

where σ^{ab} is the trace-free part of K^{ab},
the initial value constraints become

Momentum constraint: $\nabla_a^{(h)} \sigma^{ab} = 0$

Hamiltonian constraint: $-R_{(h)} + \sigma^{ab}\sigma_{ab} - \chi = 0$

where $\chi = -2\Lambda + \frac{1}{2}\tau^2$



Solving the constraints — *Lichnerowicz method*

[Lichnerowicz 44, Moncrief 89]

Start with "*seed data*" (h_{ab}, σ^{ab}) satisfying

momentum constraint $\nabla_a \sigma^{ab} = 0$

boundary condition $h|_{\partial D} = \gamma$

(But not necessarily the Hamiltonian constraint)

Apply Weyl transformation $(h_{ab}, \sigma^{ab}) \longrightarrow (\tilde{h}_{ab}, \tilde{\sigma}^{ab}) := (e^{\phi} h_{ab}, e^{-2\phi} \sigma^{ab})$

Momentum constraint $\nabla_a \sigma^{ab} = 0 \quad \Leftrightarrow \quad \widetilde{\nabla}_a \tilde{\sigma}^{ab} = 0$

Boundary condition $\tilde{h}|_{\partial D} = \gamma \quad \Leftrightarrow \quad \phi|_{\partial D} = 0$

Hamiltonian constraint $\nabla^2 \phi - R_{(h)} + e^{-\phi} \sigma^{ab} \sigma_{ab} - e^{\phi} \chi = 0$

(Lichnerowicz equation in 2+1)

Solving the constraints — *Existence and uniquiness*

The non-positive cosmological constant implies

$$\chi = -2\Lambda + \frac{1}{2}\tau^2 \geq 0$$

which ensures that the Lichnerowicz equation always has *one and only one* solution.

Each family of Weyl-transformed "*seed data*" leads to a *unique* ***valid initial data***.

Any valid data corresponds to an equivalence class $\longrightarrow$ $[(h_{ab}, \sigma^{ab}) \sim (e^{\lambda} h_{ab}, e^{-2\lambda}\sigma^{ab})]$

$$\lambda\Big|_{\partial D} = 0$$

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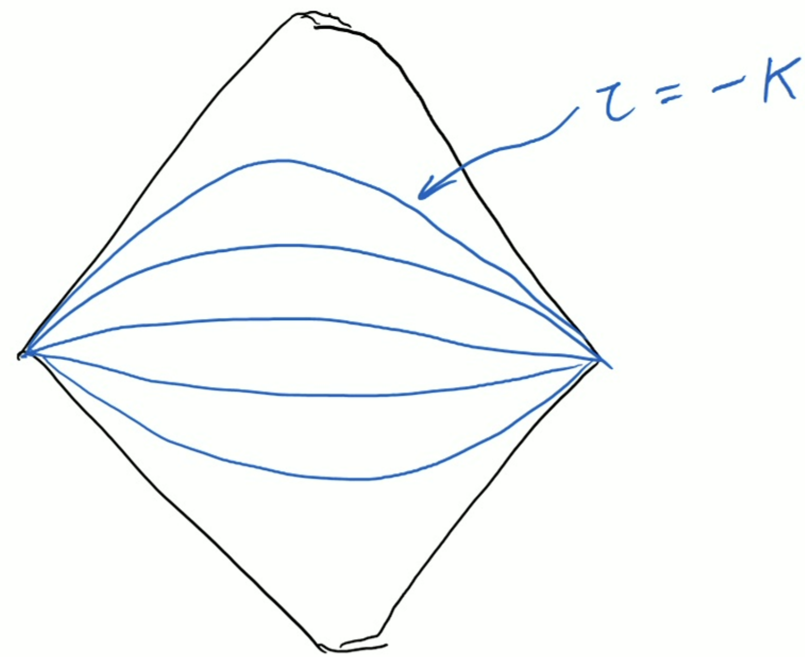

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$$\lambda\Big|_{\partial D} = 0$$

Reduced phase space — *Removing gauge*

<u>*Remove gauge*</u>: Boundary-trivial spatial diffeomorphisms $\Psi : D \to D$

Reduced phase space $=$ *Space of physically inequivalent (classical) states*

$$\left[\left(h_{ab}, \sigma^{ab}\right) \sim \left(\Psi_* e^{\lambda} h_{ab}, \Psi_* e^{-2\lambda} \sigma^{ab}\right)\right]$$

with $\Psi|_{\partial D} = I$
and $\lambda|_{\partial D} = 0$

This can be identified with the ***cotangent bundle of the space of "conformal geometries" of the disc***,

$$\tilde{\mathcal{P}} = T^* \mathcal{Q}$$

$$\left[h_{ab} \sim \Psi_* e^{\lambda} h_{ab}\right]$$

with $\Psi|_{\partial D} = I$
and $\lambda|_{\partial D} = 0$

Reduced phase space — *Determining* $\mathcal{Q}$

Note that $\mathrm{Diff}^+(S^1)$, the *group of orientation-preserving diffeos on* $S^1 \sim \partial D$, acts on $\mathcal{Q}$.

Given $\psi \in \mathrm{Diff}^+(S^1)$, let $\varphi \in C^\infty(S^1, \mathbb{R})$ be such that the boundary metric is preserved

$$\psi_* e^\varphi \gamma = \gamma$$

Now extend this transformation arbitrarily into the disc,

$$(\Psi, \Phi) \in \mathrm{Diff}^+(D) \times C^\infty(D, \mathbb{R}) \ , \qquad (\Psi, \Phi)\Big|_{\partial D} = (\psi, \varphi)$$

The natural action of ψ on $[h] \in \mathcal{Q}$ is

$$\psi[h] \coloneqq [\Psi_* e^\Phi h]$$

Reduced phase space — *Determining* $\mathcal{Q}$

This action is ***transitive*** because all discs are conformally equivalent (under conformal transformations that act non-trivially at the boundary)

Therefore $\mathcal{Q} = \mathrm{Diff}^+(S^1)/H$, for some little group H.

To determine H we can look at a particular point of $\mathcal{Q}$, e.g., the class of equivalence of the *unit round disc*

$$[dr^2 + r^2 d\theta^2]$$

The group of *conformal isometries* of the unit disc is $\mathrm{PSL}(2, \mathbb{R})$, so

$$\boxed{\mathcal{Q} = \mathrm{Diff}^+(S^1)/\mathrm{PSL}(2, \mathbb{R})}$$

Reduced phase space — *Structure of* $\tilde{\mathcal{P}}$

The reduced phase space is thus

$$\tilde{\mathcal{P}} = T^*[\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]$$

The symplectic form is the natural one (associated with the cotangent bundle structure).

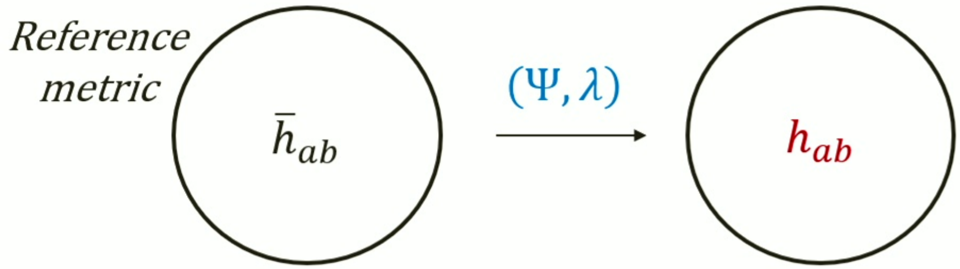
There is a non-trivial symplectomorphism to the reduced phase space of pure asymp AdS_3,

$$\mathcal{Q} \times \mathcal{Q} = [\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})] \times [\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]$$

[Maloney, Witten 10]
[Scarinci, Krasnov 13]

Reduced phase space — *Alternative approach*

Change from the *ADM coordinates* (h_{ab}, σ^{ab}) to "*conformal coordinates*" $(\Psi, \lambda, \bar{\sigma}^{ab})$



$$h_{ab} = \Psi_* e^{\lambda} \bar{h}_{ab}$$

$$\sigma^{ab} = \Psi_* e^{-2\lambda} \bar{\sigma}^{ab}$$

- Possible to carry the reduction process explicitly by quotienting over degenerate directions of the ADM (pre)-symplectic form.

- Induces a natural "coordinalization" for the reduced phase space.

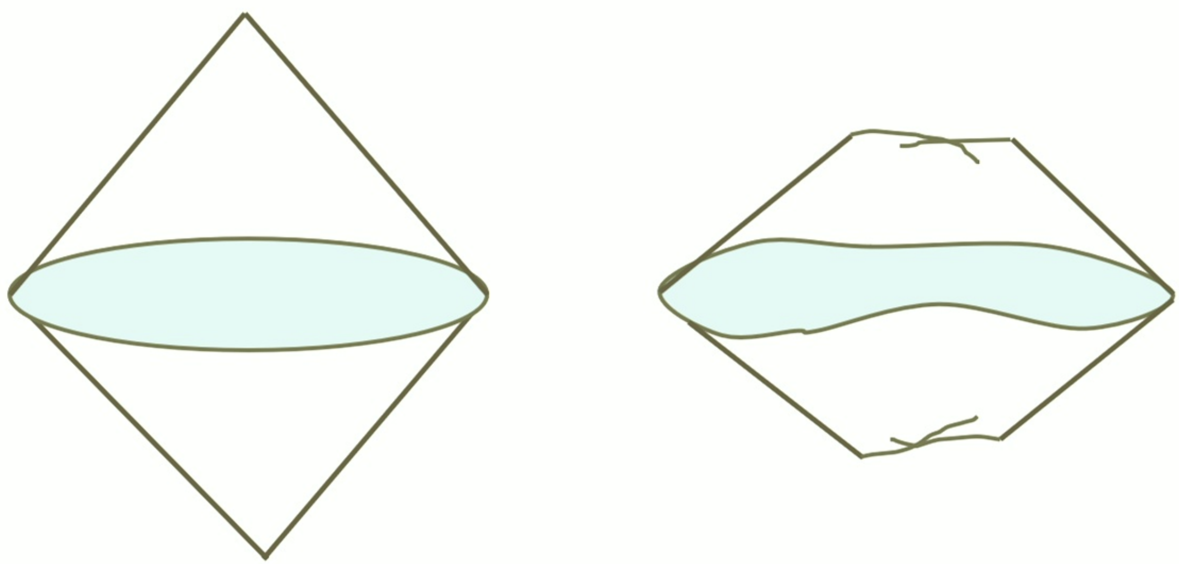
- Useful for interpreting the physical meaning of quantum observables.

Physical interpretation of the states

What are all those states? There are ***no local degrees of freedom*** in *2+1* dimensions.

Since the diamond is locally AdS, it embeds into *global* AdS_3 spacetime.

Because the diamond is finite, ***its shape is observable***.

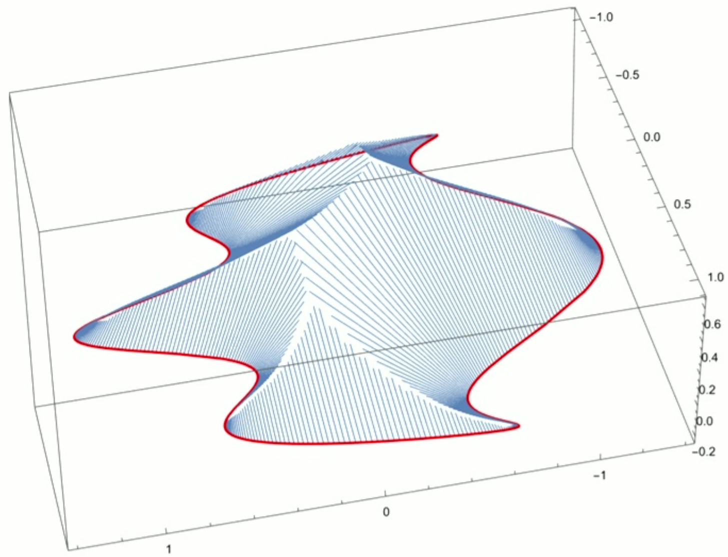


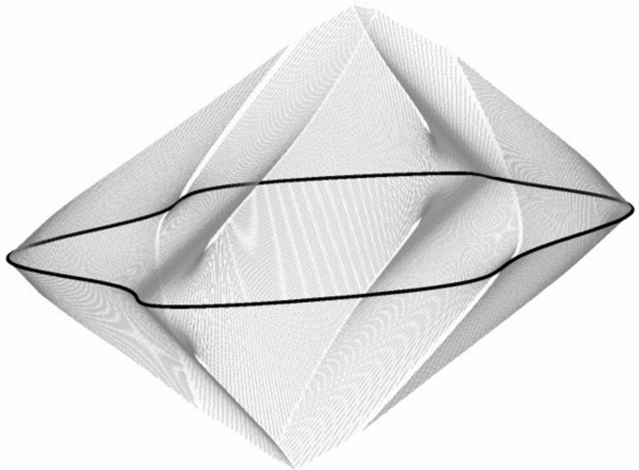
Physical interpretation of the states

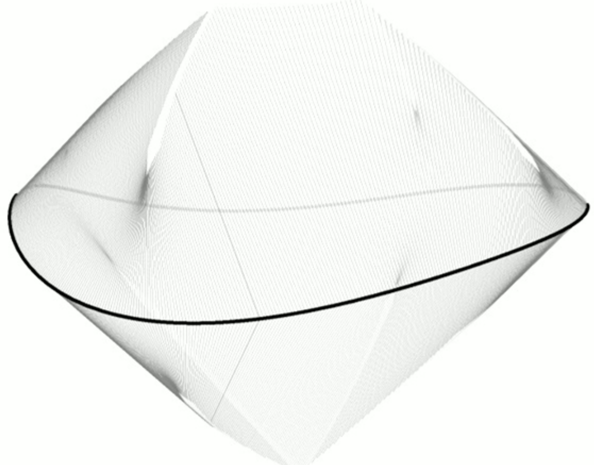
We can think of the phase space $\tilde{\mathcal{P}} = T^*[\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]$ as:

"A space of loops, with fixed length, that can be embedded into AdS_3 (as the boundary of a spacelike disc)."

Some more pictures…







Produced with *Mathematica*

(Assuming Minkowski)

The CMC Hamiltonian

We now identify the Hamiltonian $\widetilde{H}$ that generates (CMC) time evolution on the reduced phase space.

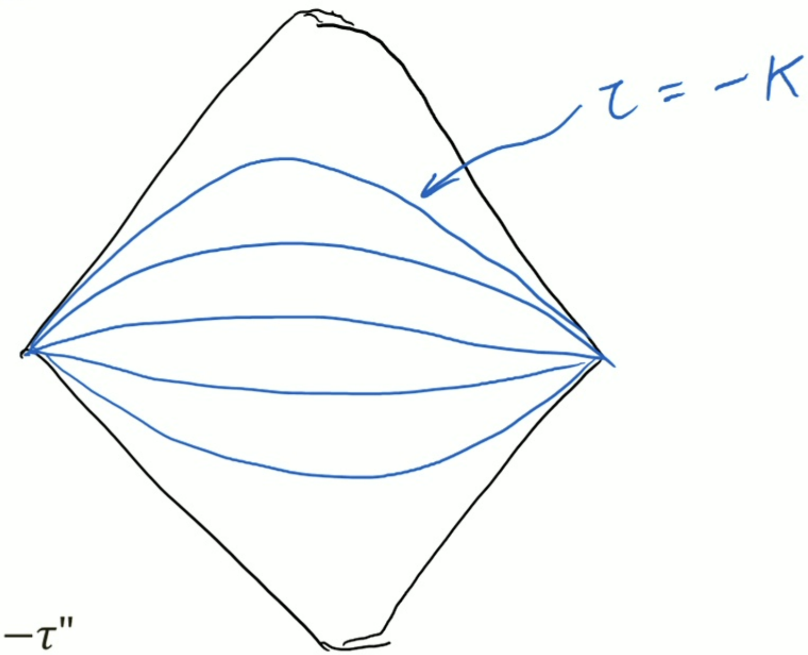
Write the action in the form

$$S = \int_{\gamma} dt\left(\pi^{ab}\dot{h}_{ab} - 0\right) = \int_{\widetilde{\gamma}} d\tau\left(p\dot{q} - \widetilde{H}\right)$$

and read $\widetilde{H}$.

We obtain,

$$\widetilde{H} = \int_{D} d^2x\, e^{\lambda} = \text{"}\mathit{area\ of\ the\ CMC\ with}\ K = -\tau\text{"}$$



[York 72]

Reintroducing physical scales

This Hamiltonian is complicated. There are regimes where it simplifies.

We must reintroduce the physical scales:

Corner length, ℓ

AdS length, $\ell_{AdS} := \frac{1}{\sqrt{-\Lambda}}$

Planck length, $\ell_P := \hbar G$

Quantization

Now we wish to quantize our reduced phase space, $\tilde{\mathcal{P}} = T^*[\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]$

Since it is not trivial (no natural global coordinates), we must be careful

Example: A particle on the half-line

Phase space $T^*\mathbb{R}^+ \sim \mathbb{R}^+ \times \mathbb{R}$, with canonical coordinates (x, p)

If x and p can be represented as self-adjoint operators satisfying $[x, p] = i$, then p can be exponentiated to a generator of spatial translations

$$e^{-iap}|x\rangle = |x + a\rangle$$

Since a can be arbitrarily negative, this is an improper quantization.

Isham's quantization scheme

"Find a transitive group G of symplectic symmetries of the phase space, and then construct the quantum theory based on unitary irreducible (projective) representations of G."

Each generator ξ_i of the group is associated with a Hamiltonian charge Q_i.

The Poisson algebra of these charges is homomorphic to the algebra of G.

Transitiveness implies that this set of charges is complete (i.e., any observable can be locally expressed in terms of them).

Quantization proceeds by finding unitary irreducible representations of this algebra

$$\{Q_i, Q_j\} = c_{ij}^k Q_k \quad \longrightarrow \quad \frac{1}{i\hbar}[\hat{Q}_i, \hat{Q}_j] = c_{ij}^k \hat{Q}_k$$

Action on the configuration space

The configuration space $Q = \text{Diff}^+(S^1)/\text{PSL}(2,\mathbb{R})$ is a homogeneous space for $\text{Diff}^+(S^1)$

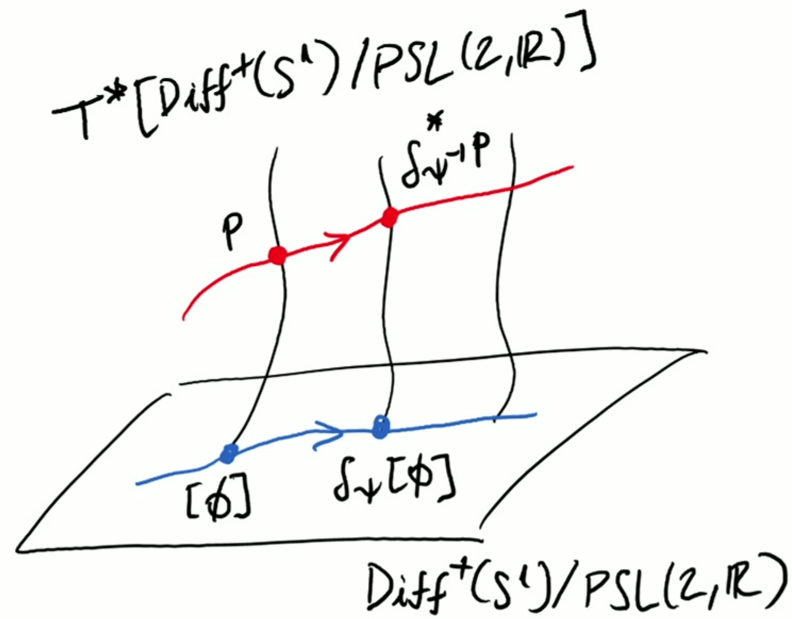
$$\delta_\psi[\phi] \coloneqq \psi[\phi] \coloneqq [\psi \circ \phi]$$

Naturally, this can be lifted to a (symplectic) action on the cotangent bundle

$$\tilde{\delta}_\psi(p) \coloneqq \delta^*_{\psi^{-1}} p$$

But this does not act transitively on the phase space – it only acts "horizontally".

We need also some sort of "vertical" action.



Momentum translations

There is a natural way to define vertical transformations, given a group K acting on the configuration space Q

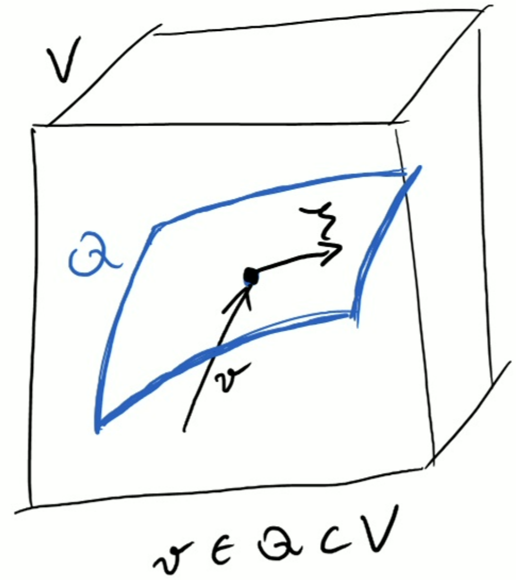
"Find a representation of K on a vector space V such that at least one orbit in V is homeomorphic to Q"

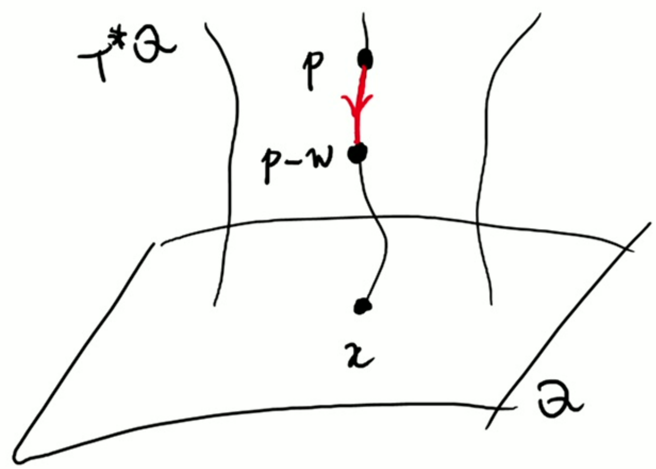
Any $w \in V^*$ can be restricted to $Q \subset V$ to define a 1-form field on Q.

Then the momentum translation is defined by

$$\zeta_w(p) \coloneqq p - w$$

where $p \in T^*Q$.





Canonical group for the diamond

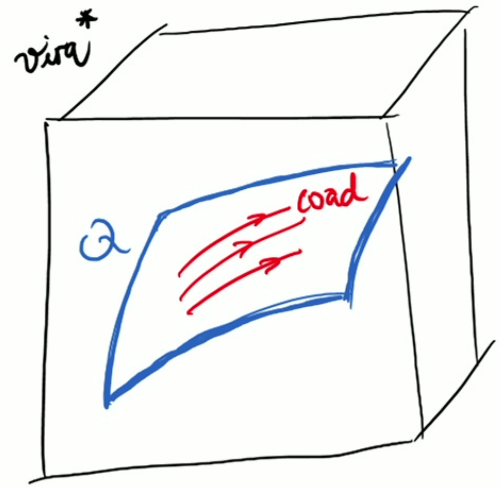
We could not find any representation of $K = \mathrm{Diff}^+(S^1)$ which had an orbit isomorphic to $\mathcal{Q} = \mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})$.

It turns out that the *coadjoint representation* of the *Virasoro group* Vira (extension of $\mathrm{Diff}^+(S^1)$ by $\mathbb{R}$) on its dual Lie algebra $\mathfrak{vira}^* \sim \mathfrak{diff}^* \oplus_S \mathbb{R}$, does have an orbit isomorphic to $\mathcal{Q}$.

Thus, taking $K = \mathrm{Vira}$ and $V = \mathfrak{vira}^*$, we can have a transitive group of symplectomorphisms of $\tilde{\mathcal{P}} = T^*[\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]$ defined by

$$G \coloneqq (\mathfrak{vira}^*)^* \rtimes \mathrm{Vira}$$

$$\Gamma_{(w,\tilde{\psi})}(p) \coloneqq \mathrm{coad}^*_{\tilde{\psi}^{-1}} p - w$$



where $w \in (\mathfrak{vira}^*)^* \sim \mathfrak{vira}$, $\tilde{\psi} \in \mathrm{Vira}$ and p is a cotangent vector on $\mathcal{Q} \subset \mathfrak{vira}^*$

Canonical algebra for the diamond

The algebra of G is $\mathfrak{vira}^c \oplus_S \mathfrak{vira}$. Recall that $\mathfrak{vira} \sim \mathbb{R} \oplus_S \mathfrak{diff}(S^1) \sim \mathbb{R} \oplus_S \mathrm{Vect}(S^1)$

It is convenient to consider a Fourier basis

Q-translations	$L_n = \left(0, e^{in\theta}\partial_\theta\right)$	$R = (0, \tilde{c})$	*central elements*
Momentum transl.	$K_n = \left(e^{in\theta}\partial_\theta, 0\right)$	$T = (\tilde{c}, 0)$	

which gives

$$[L_n, L_m] = i(n-m)L_{n+m} - 4\pi i n^3 \delta_{n+m,0} R$$

$$[K_n, L_m] = i(n-m)K_{n+m} - 4\pi i n^3 \delta_{n+m,0} T$$

$$[K_n, K_m] = 0$$

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We could not find any representation of $K = \mathrm{Diff}^+(S^1)$ which had an orbit isomorphic to $\mathcal{Q} = \mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})$.

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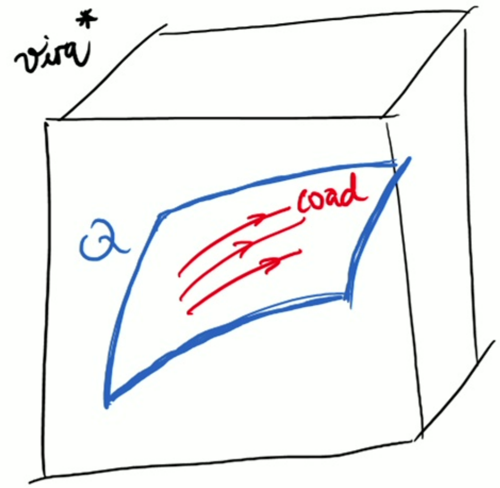

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central elements

Q-translations $\quad L_n = \left(0, e^{in\theta}\partial_\theta\right) \quad R = (0, \tilde{c})$

Momentum transl. $\quad K_n = \left(e^{in\theta}\partial_\theta, 0\right) \quad T = (\tilde{c}, 0)$

which gives

$$[L_n, L_m] = i(n-m)L_{n+m} - 4\pi i n^3 \delta_{n+m,0} R$$

$$[K_n, L_m] = i(n-m)K_{n+m} - 4\pi i n^3 \delta_{n+m,0} T$$

$$[K_n, K_m] = 0$$

Canonical charges

We can evaluate the canonical charges generated on the phase space. In this Fourier basis,

$$L_n \mapsto P_n \qquad\qquad K_n \mapsto Q_n$$

with central charges $R \mapsto 0$ and $T \mapsto 1$.

Their Poisson algebra is

$$\{P_n, P_m\} = i(n-m)P_{n+m}$$

$$\{Q_n, P_m\} = i(n-m)Q_{n+m} - 4\pi i n^3 \delta_{n+m,0}$$

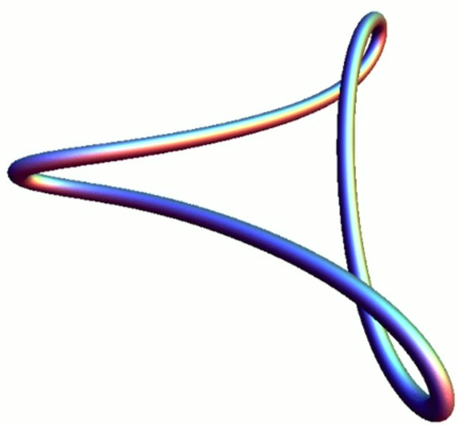
$$\{Q_n, Q_m\} = 0$$

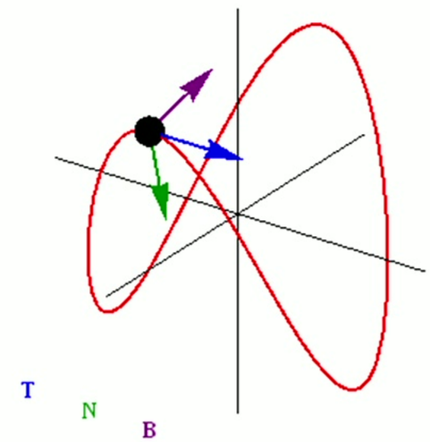
This corresponds to the BMS_3 algebra (symmetries of *2+1* asymptotically flat spacetimes at null infinity). *[Barnich, Compere 07; Oblak 17]*

Twist of a loop

The ***twist*** $\mathcal{T}$ is the integrated torsion along a loop

(The torsion of a curve gives how its adapted frame rotates around it)





Euclidean example of twisted loop. (Lorentzian is similar.)

Spin/Twist

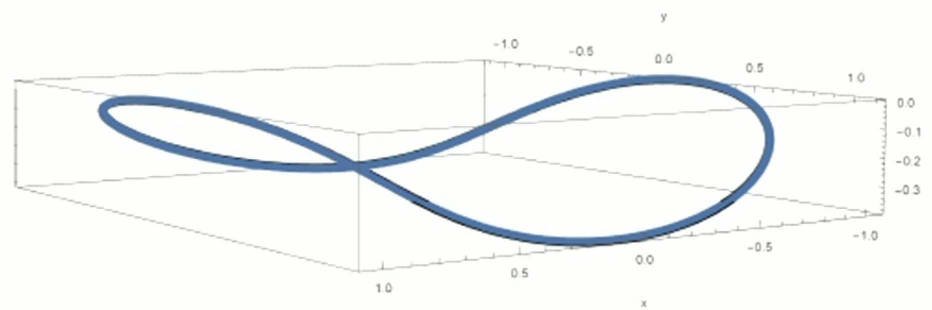
The charge P_0 can be interpreted as the ***spin*** of the diamond.

It corresponds to the $SO(2)$ subgroup of $\mathrm{Diff}^+(S^1) \subset \mathrm{Vira}$

It coincides it the ADM generator of diffeos that act as isometries of the corner

We can also show that P_0 is proportional to the ***twist*** $\mathcal{T}$ of the diamond corner

$$P_0 = \frac{\ell}{16\pi^2 G}\mathcal{T}$$



Quantum diamonds

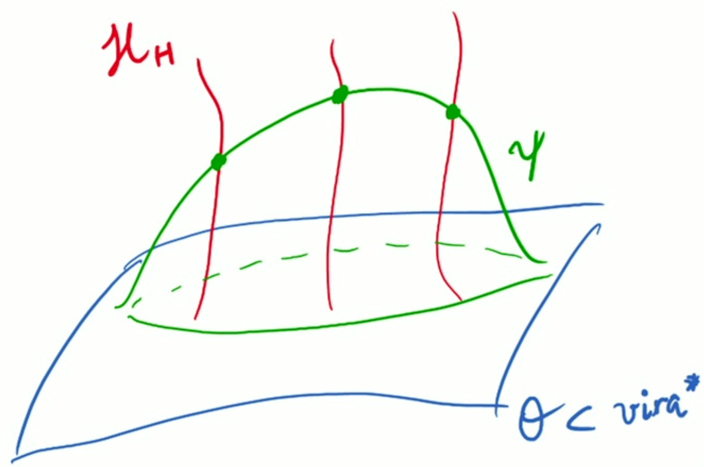
The quantum theory is based on some unitary irreducible (projective) representation of the canonical group $G = (\mathfrak{vira}^*)^* \rtimes \mathrm{Vira}$

Since G is a semi-direct product of the form *abelian* $\rtimes$ *group*, we "can" apply Mackey's theory of induced representations. *[Oblak 16]*

A representation is given by "wavefunctions" on a coadjoint orbit of Vira with labels on some unitary irrep of the corresponding little group H.

If the orbit is chosen as Q, the little group is

$$H = \text{"}\mathrm{Vira}/[\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]\text{"} = \mathbb{R} \times \mathrm{PSL}(2,\mathbb{R})$$



This is compatible with the Casimir T being represented as 1.
Imposing $R = 0$ picks the trivial irrep for $\mathbb{R}$.

Quantum diamonds

Equivalently, the Hilbert space $\mathcal{H}$ carries an irreducible representation of the algebra

$$[P_n, P_m] = \hbar(m-n)P_{n+m}$$

$$[Q_n, P_m] = \hbar(m-n)Q_{n+m} + 4\pi\hbar n^3 \delta_{n+m,0}$$

$$[Q_n, Q_m] = 0$$

satisfying the "reality" conditions $P_n^\dagger = P_{-n}$ and $Q_n^\dagger = Q_{-n}$

Spin (twist) quantization

Note that the P_n's and Q_n's act as ladder operators for the spin P_0

$$[P_0, P_n] = n\hbar P_n \qquad [P_0, Q_n] = n\hbar Q_n$$

Since these operators are represented irreducibly,

$$Spectrum(P_0) = \{(s+n)\hbar,\ \forall\, n \in \mathbb{Z}\}$$

where s is some real number. We can take $s \in [0,1)$.

Assuming that *time-reversal symmetry* is realized in the quantum theory, $s = 0$ or ${}^1/_2$

Therefore, *the twist of the diamond corner loop is quantized* as

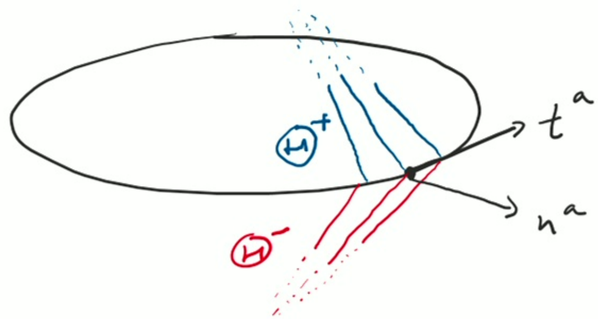
$$\mathcal{T} = \frac{16\pi^2 \ell_P}{\ell}(s+n)\,, \quad n \in \mathbb{Z}$$

What are those Q charges?

Unlike the P charges, the geometrical meaning of the Q charges remains a mystery.

Speculation: The Q charges may be some linear combination of the following set of "*corner conformal-deforming*" Hamiltonian charges (on-shell)

$$H(\chi, \zeta^+, \zeta^-) \doteq 2 \int ds \big(\chi t^a n^b K_{ab} + \zeta^+ \Theta^+ - \zeta^- \Theta^-\big)$$



Associated with diffeos χt^a along the corner. These are precisely the P charges.

Associated with flows $\zeta^\pm$ along the future/past horizon null generators. ($\Theta^\pm$ are the respective expansion parameters.)

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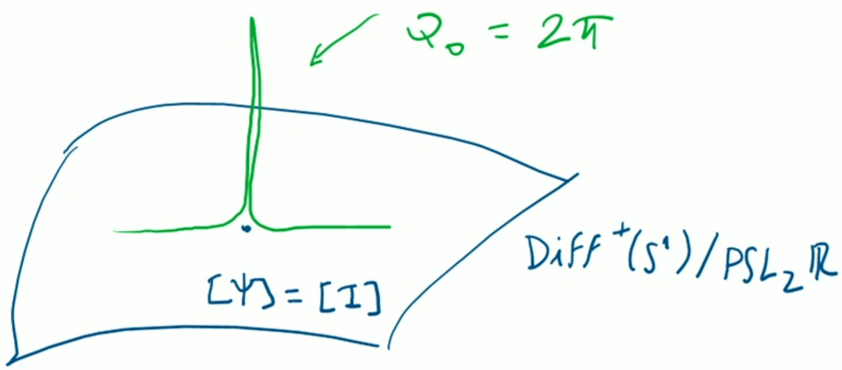
We know their formula in the abstract phase space variables, and we also know that they must depend only on the conformal class of the spatial metric.

Moreover, both classically *and quantum-mechanically*, we know that $-\infty < Q_0 \leq 2\pi$. The value 2π is attained only by a (non-normalizable) wave-function concentrated at the conformal class of the symmetric disc.



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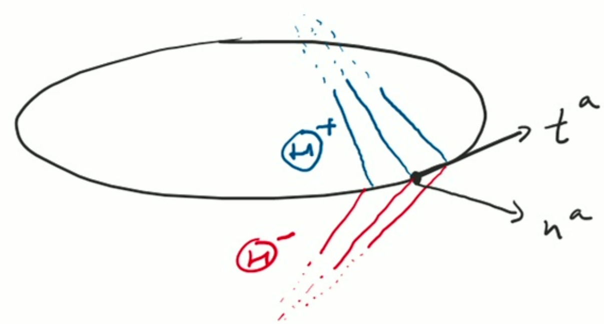

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Summary

- We considered *2+1* pure gravity, with $\Lambda \leq 0$, in the domain of dependence of topological discs with fixed induced corner metric (*causal diamonds*).
- Using a constant-mean-curvature gauge for time, we solved the constraints and eliminated all the gauge ambiguities. We found the reduced phase space $\tilde{\mathcal{P}} = T^*[\mathrm{Diff}^+(S^1)/\mathrm{PSL}(2,\mathbb{R})]$;
- The CMC Hamiltonian was given by the area of the CMC with $K = -\tau$, a complicated function on the reduced phase space. It becomes "free" in a neighborhood of the symmetric diamond.
- We applied Isham's group-theoretic method to quantize the system. The canonical group was $(\mathfrak{vira}^*)^* \rtimes \mathrm{Vira} \sim \mathrm{BMS}_3$. Mackey's theory gives representations carried by "wavefunctions" on coadjoint orbits of Virasoro, with labels in unitary irreps of the corresponding little group.
- We found that the spin is related to the twist of the diamond corner loop, which is quantized in integer or half-integer multiples of $16\pi^2\ell_P/\ell$.

Open questions

- What is the nature of a "*quantum causal diamond*"? What are "shapes" given that $[Q, P] \neq 0$?
- What is the *quantum dynamics* of the diamond? Can the Hamiltonian be (perturbatively) quantized in certain regimes?
- Why has the geometrical meaning of the Q charges been so elusive, given that the P charges have very simple interpretations?
- Can the causal diamond be seen as a "*subsystem*" of a larger quantum spacetime? Can the *twist quantization* be promoted to a general statement about loops in AdS_3? Can we obtain a finite entropy by fixing certain parameters (like P_0 and $\widetilde{H}$, or P_0 and Q_0)?

Thank You!