Title: The Higher Berry Phase and Matrix Product States

Speakers: Shuhei Ohyama

Series: Quantum Matter

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Abstract: The Berry phase, discovered by M.V. Berry in 1984, has been applied to the construction of various invariants in topological phase of matters. The Berry phase measures the non-triviality of a uniquely gapped system as a family and takes its value in $H^2({parameter space};Z)$.

In recent years, there have been several attempts to generalize it to higher-dimensional many-body lattice systems[1,2,3,4], called the "higher" Berry phase. In the case of spatial dimension d it is believed that the higher Berry phase takes its value in $H^{d+2}({parameter space};Z)$. However, in general dimensions, the definition of the higher Berry phase in lattice systems is not yet known.

In my talk, I'll explain about the way to extract the higher Berry phase in 1-dimensional systems by using the "higher inner product" of three matrix product states and how to construct the topological invariant which takes its value in $H^3({parameter space};Z)$. This talk is based on [3] and [4].

Refs:
[1] A. Kapustin and L. Spodyneiko Phys. Rev. B 101, 235130
[2] X. Wen, M. Qi, A. Beaudry, J. Moreno, M. J. Pflaum, D. Spiegel, A. Vishwanath and M. Hermele arXiv:2112.07748
[3] S. Ohyama, Y. Terashima and K. Shiozaki arXiv:2303.04252
[4] S. Ohyama and S. Ryu arXiv:2304.05356

Zoom link: https://pitp.zoom.us/j/93720709850?pwd=RTliMDNMRWo2V2k1MnBKUjlRMjBqZz09

The Higher Berry Phase
& Matrix Product States
Shuhei Ohyama from YITP
Based on Kyoto Univ.

- arXiv:2303.04252
  with Y. Terashima & K. Shiozaki
- arXiv:2304.05356
  with S. Ryu

Office #367 ~ May 29

§1. Motivation

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ITP

oto Univ.

. Shiozaki

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§1. Motivation

Kitaev's conjecture.

spatial dim

a. g.s. of a unique gapped Ham.

Let $\mathcal{M}_d := \{$ the moduli of d-dim invertible states $\}$

Then $\{\mathcal{M}_d\}$ is a Ω-spectrum., $\Omega \mathcal{M}_{d+1} \sim \mathcal{M}_d$

Many people (e.g. Gaiotto-Johnson-Freyd)

checked this conjecture in various context.

I'm interested in an approach with lattice systems.

Ex

a family of d-dim invertible states

$\ldots n_\mu \ldots$, where n_μ is the

associated with $\{u_{d}\}$

For example $\exists$ top phase

classified by $H^{d+2}(X;\mathbb{Z})$

In the case of $d=0$ (QM)

this non-triviality is measured

by the Berry phase $\in H^2(X;\mathbb{Z})$

———— // ———— $d>0$

————————————

the higher Berry phase.

M)
ured.
(X;$\mathbb{Z}$)

In lattice system.
[Kapustin–Spodyneiko]
[Kapustin–Sopenko] gave a def of
the higher Berry curvature $\in H^{d+2}(X;\mathbb{R})$
but no def of the higher Berry phase is known.

$$H^{d+2}(X;\mathbb{Z}) \cong \mathbb{Z}^{\oplus k} \oplus \bigoplus_{\lambda} \mathbb{Z}/p_e\mathbb{Z}$$

$M)$

ured.

$(X;\mathbb{Z})$

In lattice system.

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the higher Berry curvature. $\in H^{d+2}(X;\mathbb{R})$

but no def of the higher Berry phase is known.

d=0

$X = \mathbb{RP}^2$.

d=1

$X = \mathbb{RP}^2 \times S^1,\ L(3;1) \times S^1 \longrightarrow \mathbb{Z}/2,\ \mathbb{Z}/3$

$H^{d+2}(X;\mathbb{Z}) \cong \mathbb{Z}^{\oplus k} \oplus \bigoplus \mathbb{Z}/p_i\mathbb{Z}$

$H^3(X;\mathbb{Z})$

Overview & Main idea.

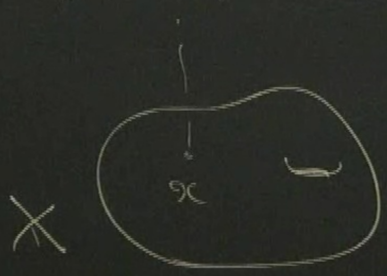
We propose a def. of the Berry phase
for $d=1$ by using the matrix product
state representation.

0 d

State representation.

<u>0 d</u>

$\mathbb{C}\ |\psi(x)\rangle$



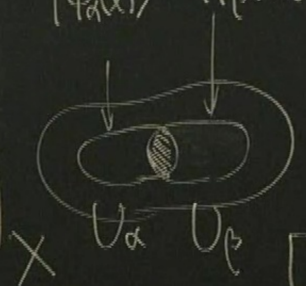
$\rightsquigarrow$ cpx line bdl $\mathcal{L}$ on X

(g.s. line bdl)

$\cap$

$H^2(X;\mathbb{Z})$

<u>How about the lin?</u>



$C_{\alpha\beta}(x) := \langle\psi_\alpha(x)|\psi_\beta(x)\rangle \in U(1)$

transition function of $\mathcal{L}$.

$\rightsquigarrow [C_{\alpha\beta}] \in H^1(X;\underline{U(1)})$

...tation.

...e inv?

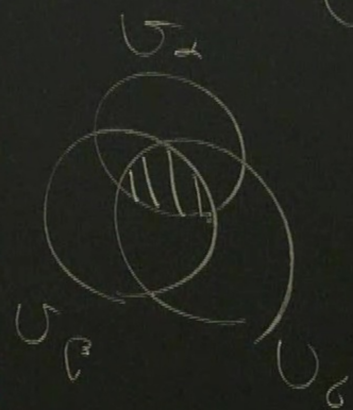
$c_{\alpha\beta}(x) := \langle \psi_\alpha(x) | \psi_\beta(x) \rangle$
$\in U(1)$

transition function
of $\mathcal{L}$.

$\rightsquigarrow [c_{\alpha\beta}] \in H^1(X; \underline{U(1)})$
$\cong H^2(X; \mathbb{Z})$

X

How about the inv.?



$C_{\alpha\beta\gamma} :=$ "triple inner prod." on $U_{\alpha\beta\gamma}$

Goal

$H^3(X; \mathbb{Z})$

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...ssociated with $\{\mu_d\}$

example $\exists$ top phase

...ssified by $H^{d+2}(X;\mathbb{Z})$

the case of $d=0$ (QM)

its non-triviality is measured by the Berry phase $\in H^2(X;\mathbb{Z})$

$d>0$

the higher Berry phase.

$\cong \tilde{h}^d_\mu(X)$, where $\tilde{h}^d_\mu$ is the cohomology theory

In lattice system
[Kapustin-Spodyneiko]
[Kapustin-Sopenko] gave a def of

the higher Berry curvature $\in H^{d+2}(X;\mathbb{R})$

but no def of the higher Berry phase is known.

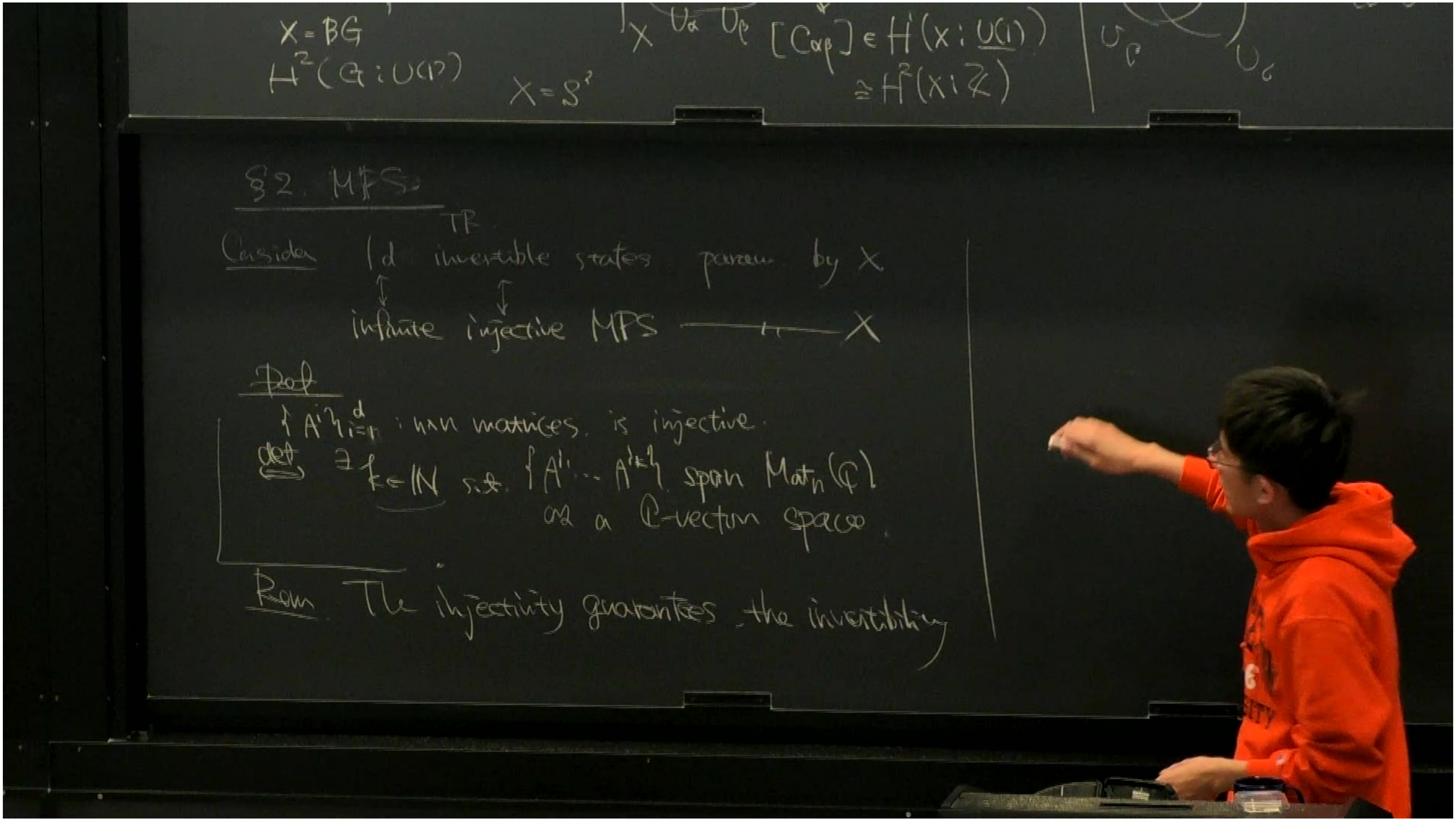
$\mathbb{K}^{\otimes} \otimes \mathbb{R}$

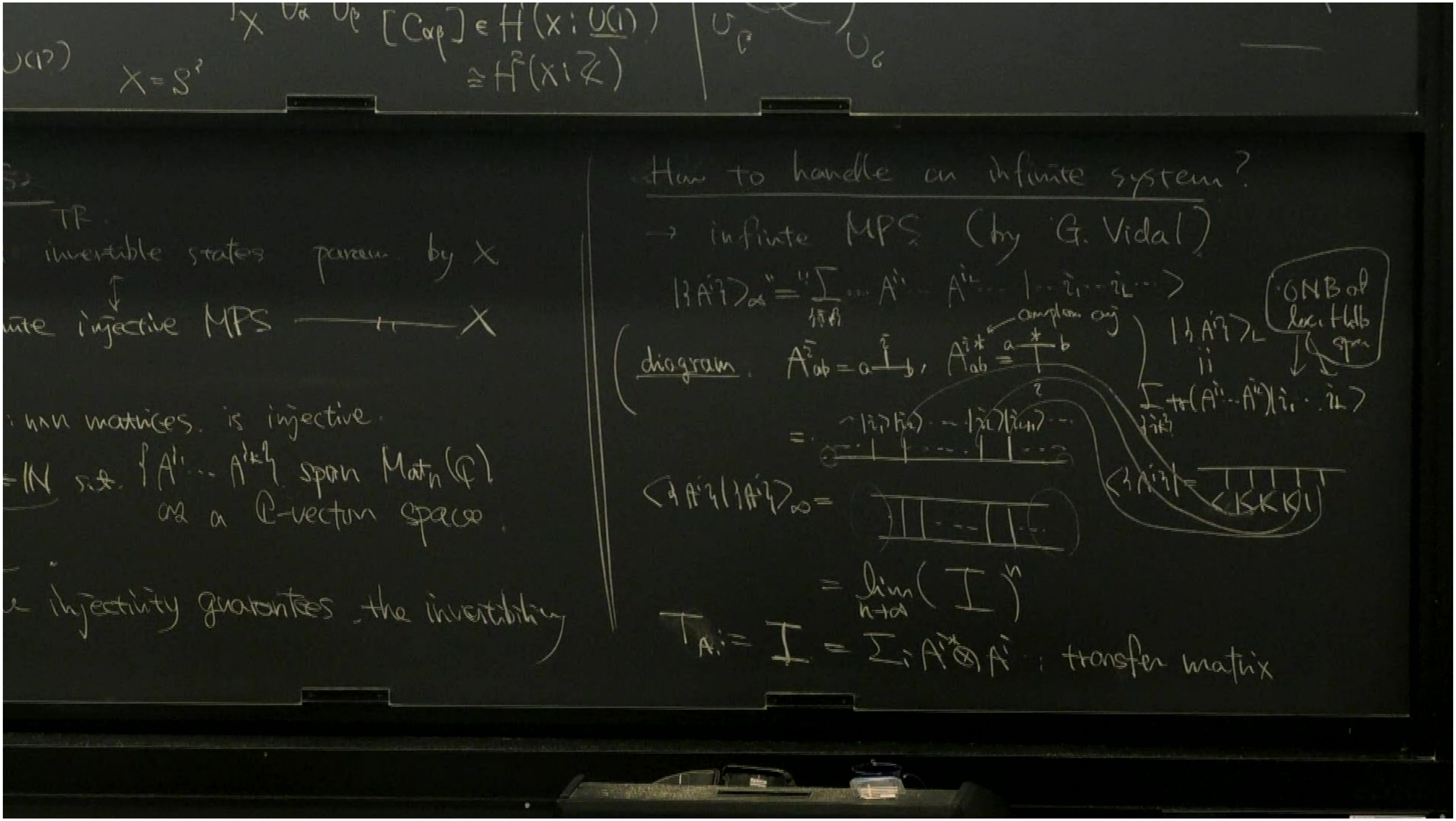
$I\Omega^{Spin}_3(X) \cong H^0(X;\mathbb{Z}/2) \oplus H^1(X;\mathbb{Z}/2) \oplus H^2(X;\mathbb{Z})$

$d=0$
$X = \mathbb{RP}^2$.
$d=1$
$X = \mathbb{RP}^2 \times S^1$, $L(3;1) \times S^1$ $\longrightarrow$ $\mathbb{Z}/2$, $\mathbb{Z}/3$

$H^{d+2}(X;\mathbb{Z}) \cong \mathbb{Z}^{\oplus k} \oplus \bigoplus \mathbb{Z}/p_i\mathbb{Z}$

$H^3(X;\mathbb{Z})$


X = BG
H²(G : U(1))
X = S²
[C_αβ] ∈ H¹(X : U(1)) ≅ H²(X : ℤ)
§2. MPS
Consider {d TP invertible states param. by X
infinite injective MPS —— X
Def
{A^i}_{i=1}^d : n×n matrices is injective
def ⟺ ∃ k ∈ ℕ s.t. {A^{i_1} ⋯ A^{i_k}} span Mat_n(ℂ)
as a ℂ-vector space.
Rem The injectivity guarantees the invertibility



How to handle an infinite system?
→ infinite MPS (by G. Vidal)
diagram
transfer matrix
ONB of loc. Hilb space
invertible states param. by X
injective MPS
n×n matrices is injective
span $Mat_n(\mathbb{C})$
as a $\mathbb{C}$-vector space
injectivity guarantees the invertibility
$T_{A_i} := \sum_i A^{i*} \otimes A^i$: transfer matrix

T_A acts on $\mathrm{Mat}_n(\mathbb{C})$
from left / right side :

$\forall M \in \mathrm{Mat}_n(\mathbb{C})$

$$\begin{cases} T_A \cdot M := \sum_i A^i M A^{i\dagger} \Leftrightarrow I \cdot) =] \\ M \cdot T_A := \sum_i A^{i\dagger} M A^i \Leftrightarrow (\cdot I = (I \end{cases}$$

$r_A :=$ the spectral radius of T_A w.r.t. the first action

cur

the l

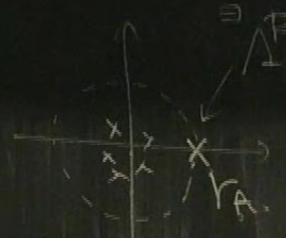
T_A acts on $\mathrm{Mat}_n(\mathbb{C})$
from left / righ side :

$\forall M \in \mathrm{Mat}_n(\mathbb{C})$

$$\begin{cases} T_A \cdot M := \sum_i A^i M A^{i\dagger} \Leftrightarrow \mathrm{I} \cdot \supset = \mathrm{I}\!\supset \\ M \cdot T_A := \sum_i A^{i\dagger} M A^i \Leftrightarrow \subset \cdot \mathrm{I} = \subset\!\mathrm{I} \end{cases}$$

$r_A :=$ the spectral radius of T_A w.r.t. the first action

$l_A :=$ ____________________ the second action.



$\Rightarrow$ I•) = ⊐

$\Rightarrow$ (• I = (I

of T_A w.r.t. the first action

the second action.

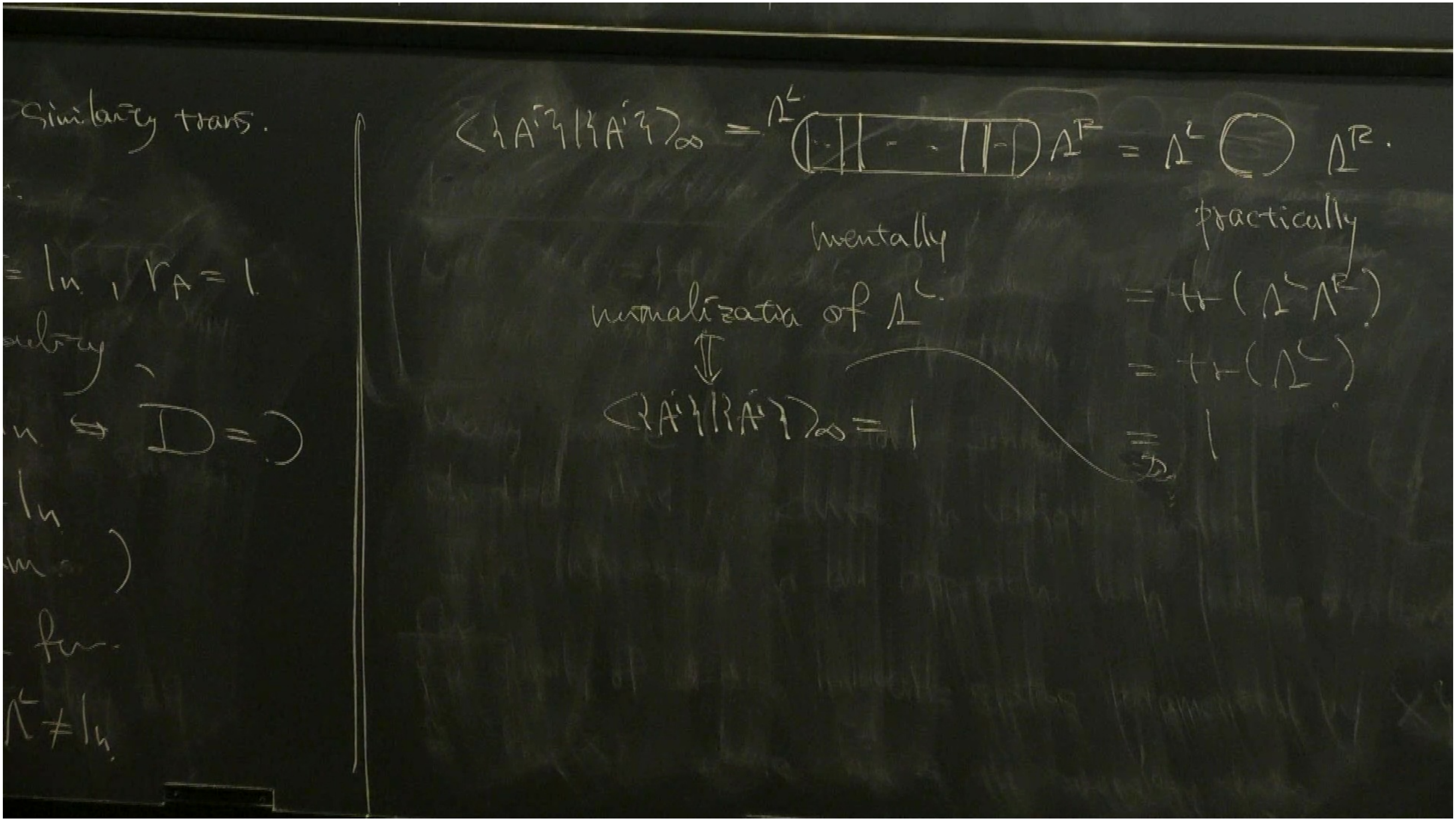
Thm (Sanz-Perez-Garcia-Wolf-Cirac)

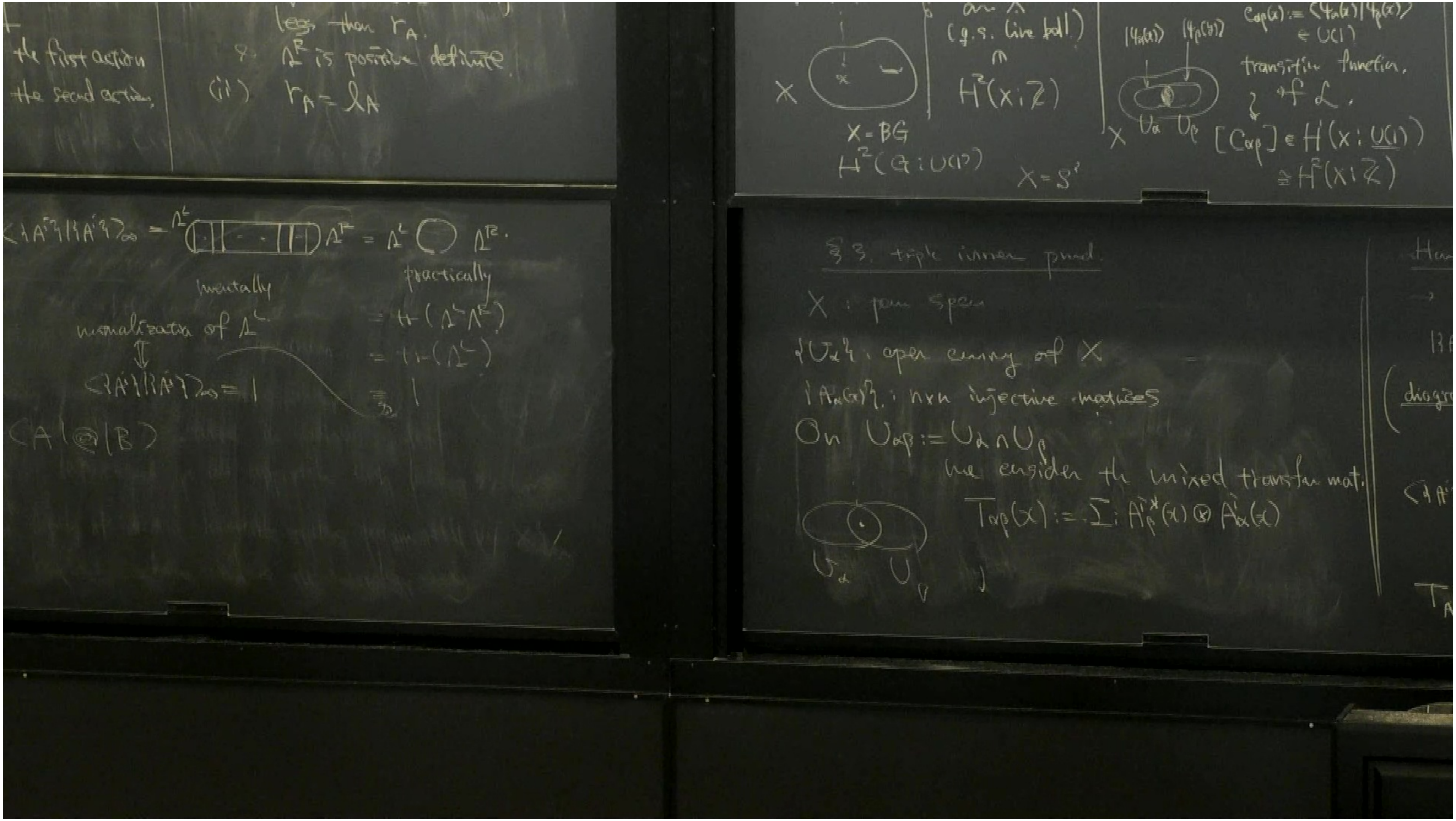
$\{A^i\}$: $n \times n$ injective matrices

(i) $\exists! \Lambda^R$: right eigenvector with eigenvalue r_A

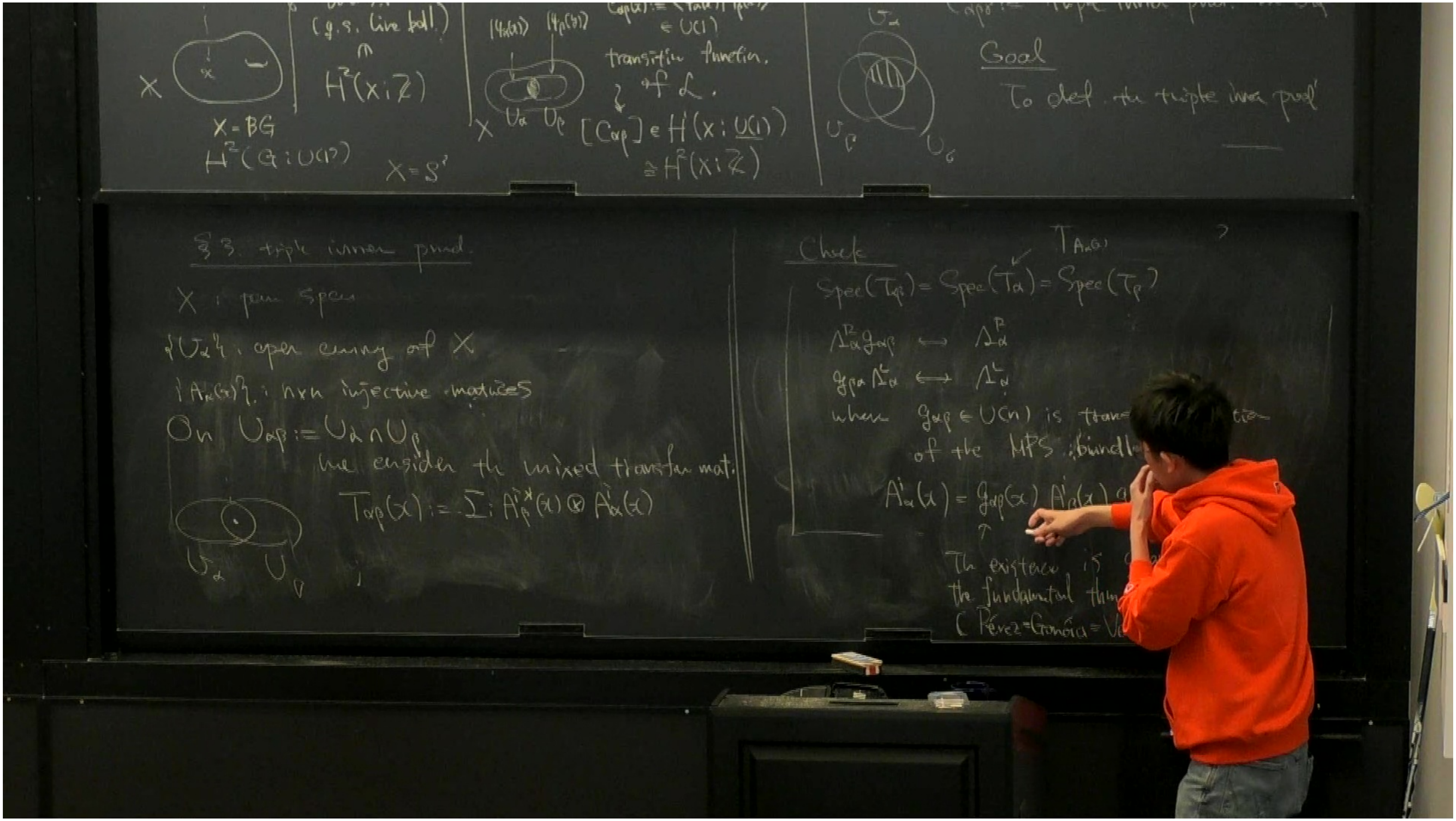
(i.e. $T_A \cdot \Lambda^R = r_A \Lambda^R$)

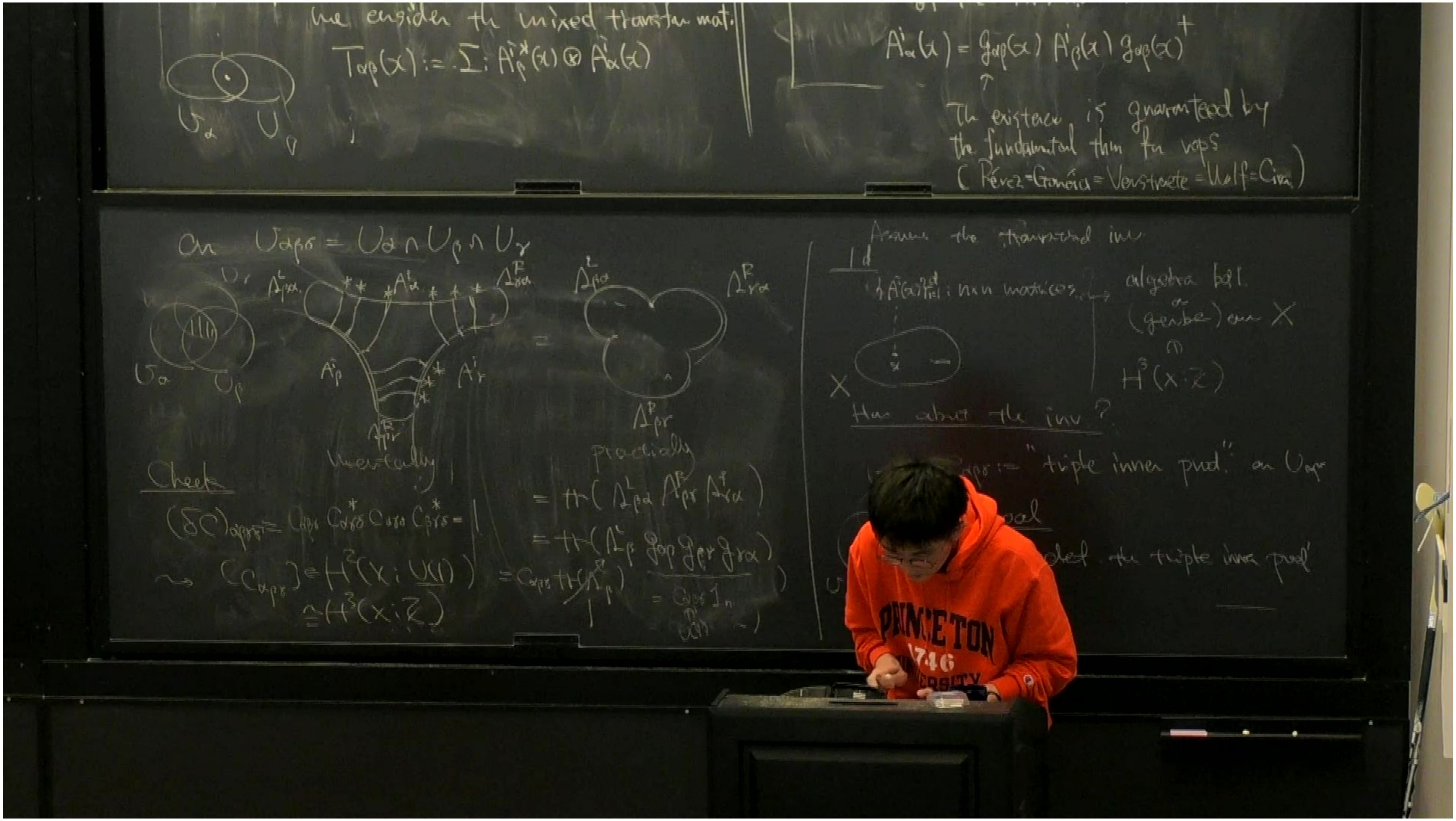
& the absolute values of the other eigenvalues are strictly less than r_A.


Similarity trans.
= 1n , rA = 1
D =)
⟨{A^i}|{A^i}⟩∞ = Λ^L Λ^R = Λ^L Λ^R.
mentally
practically
normalization of Λ^L.
⟨{A^i}|{A^i}⟩∞ = 1
= tr(Λ^L Λ^R)
= tr(Λ^L)
= 1
Λ^L ≠ 1n



less than r_A
the first action
the second action
Λ^R is positive definite
(ii) $r_A = \lambda_A$
mentally
practically
normalization of Λ^L
$= \mathrm{tr}(\Lambda^L \Lambda^R)$
$= \mathrm{tr}(\Lambda^L)$
$\langle A | \otimes | B \rangle$
(g.s. line bdl)
$H^2(X;\mathbb{Z})$
$X = BG$
$H^2(G; U(1))$
$X = S^2$
$C_{\alpha\beta}(x) := \langle \psi_\alpha(x) | \psi_\beta(x) \rangle \in U(1)$
transition function of $\mathcal{L}$.
U_α U_β
$[C_{\alpha\beta}] \in H^1(X; \underline{U(1)}) \cong H^2(X;\mathbb{Z})$
§3. triple inner prod.
X : top. space
$\{U_\alpha\}$: open covering of X
$\{A_\alpha(x)\}$: $n \times n$ injective matrices
On $U_{\alpha\beta} := U_\alpha \cap U_\beta$
we consider the mixed transfer mat.
$T_{\alpha\beta}(x) := \sum_i A_\beta^{i*}(x) \otimes A_\alpha^i(x)$


Goal
To def. the triple inner prod
X = BG
X = S'
transition function
§3. triple inner prod.
X : pure spec
{U_α} : open covering of X
{A_α(x)} : n×n injective matrices
On U_αβ := U_α ∩ U_β
we consider the mixed transfer mat.
T_αβ(x) := Σ_i A_β^i*(x) ⊗ A_α^i(x)
Check
Spec(T_αβ) = Spec(T_α) = Spec(T_β)?
where g_αβ ∈ U(n) is
of the MPS bundle
The existence is
the fundamental
(Pérez-García


we consider the mixed transfer mat.
The existence is guaranteed by the fundamental thm for mps
(Pérez-García - Verstraete - Wolf - Cirac)
Check
vertically
practically
algebra bdl
(gerbe) over X
How about the inv?
"triple inner prod."
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