

Title: The B-RNS-GSS formalism in heterotic supergravity backgrounds

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Series: Quantum Fields and Strings

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Abstract: A new approach for the first quantization of superstrings, called B-RNS-GSS formalism, is being constructed. It consists of quantizing embeddings of super surfaces into superspaces. As in the classical theory of super-embeddings, it has twistor-like variables. In this talk, besides motivating the need for such a formalism, I will review the work done in hep-th: 2211.06899, where the heterotic supergravity equations of motion were derived from BRST nilpotency.

Zoom link: <https://pitp.zoom.us/j/98656433153?pwd=NjlpcUtITDAwWlgycUtUZUVsZ3QrQT09>

$$\partial \bar{\psi}^{\alpha} + \partial \lambda^{\alpha} \omega_{\alpha}$$

$$\partial (c_{\beta} + \frac{1}{2} \gamma_{\beta}^{\alpha})$$

$$+ \partial \bar{c}_{\beta} + \partial (\bar{c}_{\beta})$$

B-RNS-GSS : Flat Background

$$Q_{B-RNS-GSS} = \int \gamma \psi^{\alpha} E_{\alpha} + cT - (\gamma^2 - \partial c c) / \gamma$$

$$\bar{Q} = \int \bar{c}T + \partial \bar{c} \bar{c} \bar{b}$$

$$- \frac{1}{2} \gamma^{\alpha} + \lambda^{\alpha} (d_{\alpha} + e^{-\phi} \gamma^{\alpha} \Sigma_{\alpha}) + \gamma^2 E^{\alpha} \omega_{\alpha} - \frac{L}{2} \frac{\lambda^{\alpha} \gamma^{\alpha} \lambda^{\beta} \psi_{\alpha} \psi_{\beta}}{\gamma}$$

$$d_{\alpha} = p_{\alpha} + \frac{1}{2} \partial \lambda^{\alpha} (\gamma \theta)_{\alpha} - \frac{1}{8} (\partial \theta \gamma^{\alpha} \theta) (\gamma \theta)_{\alpha}$$

$$E^{\alpha} = \partial \theta^{\alpha}, E^{\beta} = \partial \lambda^{\beta} - \frac{L}{2} (\partial \theta \gamma^{\alpha} \theta)$$

Twisted B-RNS-GSS:  $p_{\alpha} \lambda^{\alpha} \psi^{\alpha} \omega_{\alpha}$

Ghost #	0	+1	0	-1
Conformal weight	+1	0	+1/2	+1

- RNS
- GS
- PS

B-RNS-GSS

$\frac{\alpha}{\beta} \frac{\psi}{\chi}$   
 SS:  $P_\alpha \Lambda^\alpha \psi^\alpha w_\alpha$   
 $0 \quad +1 \quad 0 \quad -1$   
 $+1 \quad 0 \quad +\frac{1}{2} \quad +1$

- RNS.
- GS.
- PS.

B-RNS-GSS:

$\psi^\alpha, \chi^\alpha, \beta, \gamma, c, b, \Lambda^\alpha, w_\alpha, P_\alpha, \Theta^\alpha,$

$\gamma, c, b, \Lambda^\alpha, \omega_\alpha, P_\alpha, \Theta^\alpha,$

MFS

$$\beta \sim \alpha \zeta e^{-\phi}$$

$$\gamma \sim \zeta e^{\phi}$$

$$\psi^{\pm} \sim e^{\alpha \cdot H} \quad \begin{matrix} \text{ev.} \\ +1/2 \end{matrix}$$

$$\Sigma_{\alpha} \sim e^{-\alpha \cdot H} \quad \begin{matrix} \text{ev.} \\ +5/8 \end{matrix}$$

c) 6

$$E^a \omega_a = \frac{1}{2} \frac{\Lambda^\alpha \gamma_\alpha^\beta \Lambda^\beta \psi_a}{\gamma}$$

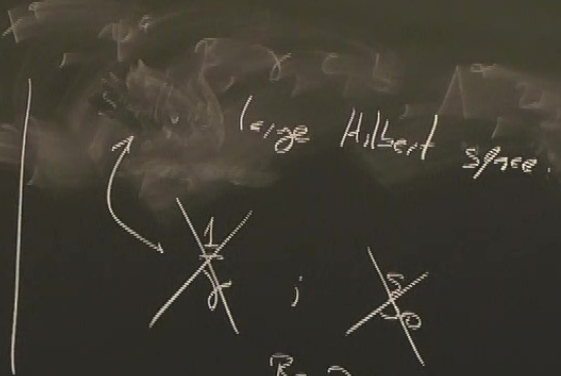
Twisted B-RNS-GS:  $P_\alpha \Lambda^\alpha \psi^a \omega_a$

Ghost #	0	+1	0	-1
Conformal weight	+1	0	+1/2	+1

~~X~~, ~~z~~, ~~\phi~~, ~~R~~

$\gamma, \psi \in \mathcal{H}$

$$Q_{BRST} = Q_{RNS} + \int \zeta$$



large Hilbert space.

$\beta, \psi \in \mathcal{H}$   
 $\gamma, \psi \in \mathcal{H}$

$$\gamma, \zeta \in \mathbb{P}$$

large Hilbert space

~~$\frac{1}{2}$~~  ;  ~~$\frac{5}{2}$~~   
 $\beta \nu \partial \zeta \in \mathbb{P}$   
 $\gamma, \zeta \in \mathbb{P}$

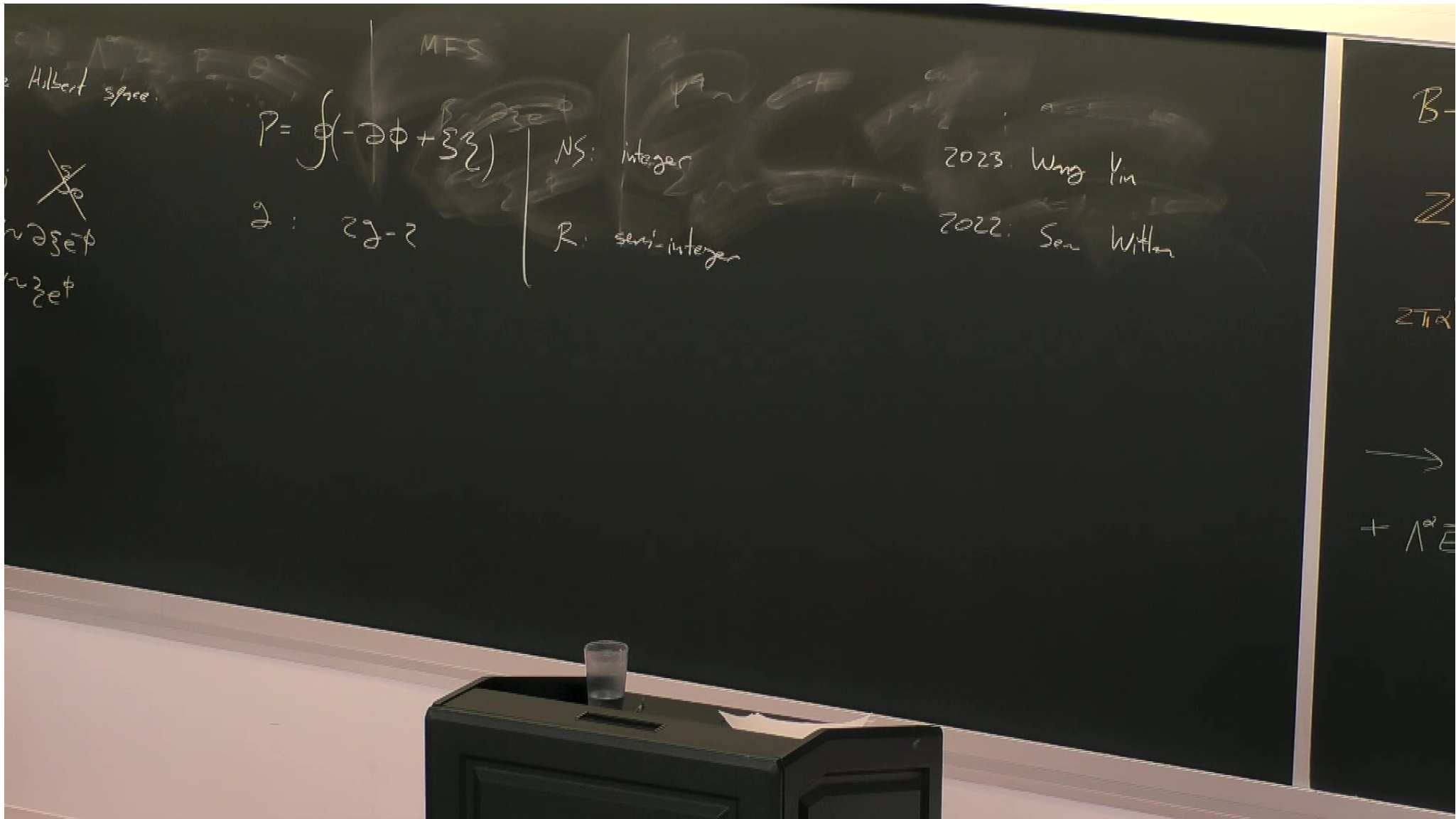
MFS

$$P = \int \left( -\partial \phi + \sum_{\zeta \in \mathbb{P}} \zeta \right)$$

$$Q : \quad 2\zeta - \zeta$$

NS: integer

R: semi-integer



$$V_{\text{Gluino}} = \frac{1}{2} \int d^2z e^{-\phi_{12}} \sum_{\alpha} W^{\alpha i} \bar{\psi}_i \bar{\psi}_i$$

$$\alpha = 1, \dots, 16$$

$$\beta = 1, \dots, 16$$

Type II

$$V_{RR} = \int d^2z e^{-\phi_{12}} e^{-\phi_{12}} \sum_{\alpha} \sum_{\beta} F^{\alpha\beta}$$

$$\phi F = 0$$

$$\begin{matrix} \Lambda^{\alpha} & \psi^{\alpha} & w_{\alpha} \\ +1 & 0 & -1 \\ 0 & +\frac{1}{2} & +1 \end{matrix}$$

B-Field



$$\int d^2z e^{-\phi_L} \sum_{\alpha} W^{\alpha} \bar{\psi}_L \bar{\psi}_L$$

$$\int d^2z e^{-\phi_L} \sum_{\alpha} \sum_{\beta} F_{\alpha\beta}^{\gamma} \psi_{\alpha}^{\gamma}$$

$\alpha = 1, \dots, 16$   
 $\beta = 1, \dots, 16$

$\phi = 0$

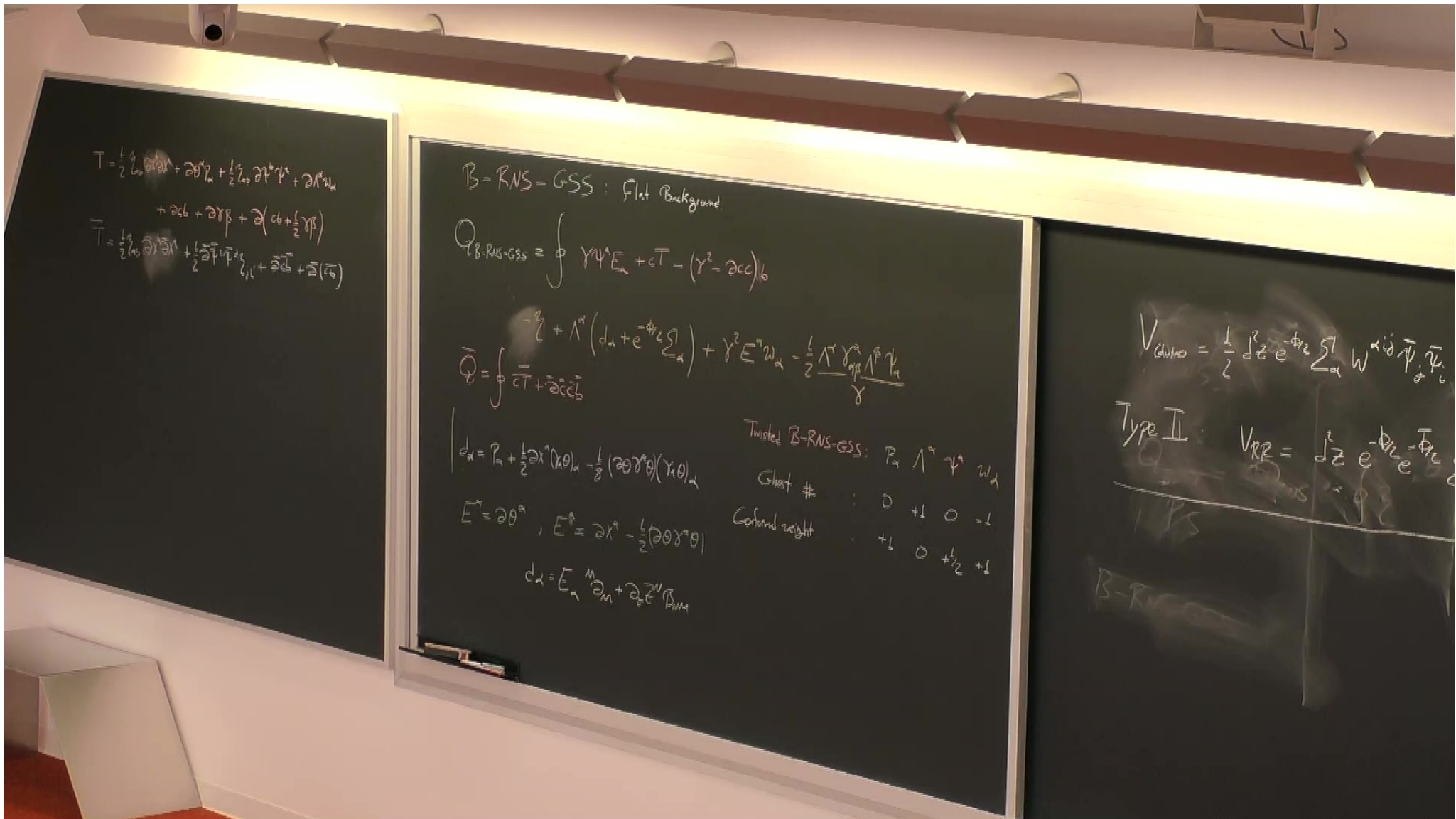
GS.

MFS

$\psi^{\alpha}, \lambda^{\alpha}, \omega_{\alpha}, \gamma, \beta, \epsilon, \zeta, R_{\alpha}$

$$2\pi\alpha' L = \frac{1}{2} \int d\tau d\sigma \left( \dot{X}^{\mu} \dot{X}_{\mu} + \dot{\psi}^{\alpha} \dot{\psi}_{\alpha} \right) + \int d\tau d\sigma \left( \mathcal{H}_{\text{het}} + \frac{1}{2} \partial \bar{\psi}^{\alpha} \psi_{\alpha} \right)$$

$d_{\alpha}$



$$T = \frac{1}{2} \int d^4x (\partial_\mu \psi^\dagger \partial^\mu \psi + \partial_\mu \psi^\dagger \gamma^\mu \psi + \partial^\mu \psi^\dagger \gamma_\mu \psi) + \partial c \bar{b} + \partial \bar{c} b + \partial (c \bar{b} + \bar{c} b)$$

$$\bar{T} = \frac{1}{2} \int d^4x (\partial_\mu \bar{\psi} \partial^\mu \psi + \partial_\mu \bar{\psi} \gamma^\mu \psi + \partial^\mu \bar{\psi} \gamma_\mu \psi) + \partial \bar{c} b + \partial c \bar{b}$$

B-RNS-GSS : Flat Background

$$Q_{B-RNS-GSS} = \int \gamma \psi^\dagger E_\alpha + c \bar{T} - (\gamma^2 - \partial c \bar{c}) b$$

$$Q = \int \bar{c} T + \partial \bar{c} \bar{c} b$$

$$- \frac{1}{2} \int \Lambda^\alpha (d_\alpha + e^{-\phi} \gamma_\alpha) + \gamma^2 E^\alpha \omega_\alpha - \frac{1}{2} \frac{\Lambda^\alpha \gamma_{\alpha\beta} \Lambda^\beta \psi_\alpha}{\gamma}$$

$$d_\alpha = p_\alpha + \frac{1}{2} \partial x^\mu (\gamma_\mu)_\alpha - \frac{1}{8} (\partial \theta \gamma^\mu \theta) (\gamma_\mu)_\alpha$$

$$E^\alpha = \partial \theta^\alpha, E^\beta = \partial x^\beta - \frac{1}{2} (\partial \theta \gamma^\beta \theta)$$

$$d_\alpha = E_\alpha^\mu \partial_\mu + \partial_\nu E^\mu \Gamma_{\alpha\mu}^\nu$$

Twisted B-RNS-GSS:  $P_\alpha \Lambda^\alpha \gamma^\alpha \omega_\alpha$

Ghost #	0	+1	0	-1
Conformal weight	+1	0	+1/2	+1

$$V_{GMS} = \frac{1}{2} \int d^2z e^{-\phi(z)} \sum_\alpha W^{\alpha\beta} \bar{\psi}_\alpha \psi_\beta$$

Type II:  $V_{RR} = \int d^2z e^{-\phi(z)} \bar{\psi}_R \psi_R$

B-RNS

$$V_{RR} = \int d^2z e^{-\phi_L - \bar{\phi}_L} \sum_{\alpha} \sum_{\beta} W^{\alpha\beta} \bar{\psi}_i \bar{\psi}_j$$

$$V_{RR} = \int d^2z e^{-\phi_L - \bar{\phi}_L} \sum_{\alpha} \sum_{\beta} F^{\alpha\beta}$$

$\alpha = 1, \dots, 16$   
 $\beta = 1, \dots, 16$

$\phi = 0$

GS.

MFS

$$2\pi\alpha' L = \frac{1}{2} E^{\mu\nu} \dot{X}^{\mu} \dot{X}^{\nu} + \frac{1}{2} \partial_{\mu} X^{\nu} \partial_{\nu} X^{\mu} + \frac{1}{2} \partial_{\mu} \psi^i \partial_{\nu} \bar{\psi}^j \gamma^{\mu\nu}$$

$$d_{\alpha} = 0$$

$$\{d_{\alpha}, d_{\beta}\} = E^{\mu\nu} \gamma_{\mu\nu\alpha\beta}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E^{\mu\nu} \gamma_{\mu\nu} = 0$$

GS.

MFS

$\psi^a, \lambda^a, \omega_a, \gamma, \beta, c, b, P_2$

$$\begin{aligned}
 \mathcal{Z}\pi^\alpha L = & \frac{1}{2} E^a \bar{E}^b + \mathcal{P}_{Het} + \frac{1}{2} \partial \bar{\psi}^i \psi^j \partial z_{ji} \\
 & + \bar{E}^\alpha d_\alpha
 \end{aligned}
 \quad \Rightarrow \quad E^a E_a = 0$$

1986. Siegel, Yin

B-RNS

$$Z^M = \bar{Z}^M$$

$$\mathcal{Z}\pi^\alpha L = \frac{1}{2} E^a \bar{E}^b$$

$$\begin{aligned}
 & \rightarrow \dots + \\
 & + \lambda^a \bar{E}^b (H_{b\alpha c}
 \end{aligned}$$

$$\left( \int d\sigma \lambda^\alpha d_\alpha \right)^2 = 0 \quad \text{W}$$

$$d_\alpha = E_{\alpha\beta}^M \partial_{M\alpha} - \partial_{\alpha\beta}^M \bar{B}_{M\alpha} + \frac{L}{2} A_{\alpha\beta}^i \partial_{\alpha\beta}^i + \Omega_{\alpha\beta}^{\gamma\delta} \sum_{\gamma} \bar{p}^{\gamma} + \frac{L}{2} \bar{E}_{\alpha\beta}^{\gamma\delta} \sum_{\delta} L_{\delta\gamma}$$

GS.

MFS

$\psi^\alpha, \lambda^\alpha, \omega_\alpha, \gamma, \beta, \epsilon, \zeta, P_\alpha$

$$2\pi\alpha' L = \frac{1}{2} \dot{E}^\alpha \dot{E}_\alpha + \mathcal{P}_{\text{Hof}} + \frac{1}{2} \partial_\alpha \bar{\psi} \psi + \partial_\alpha \bar{\lambda} \lambda + \dot{E}^\alpha d_\alpha$$

$P_\alpha$	$\lambda^\alpha$	$\psi^\alpha$	$\omega_\alpha$
0	+1	0	-1
+1	0	+1/2	+1

B-Field

GS.

MFS

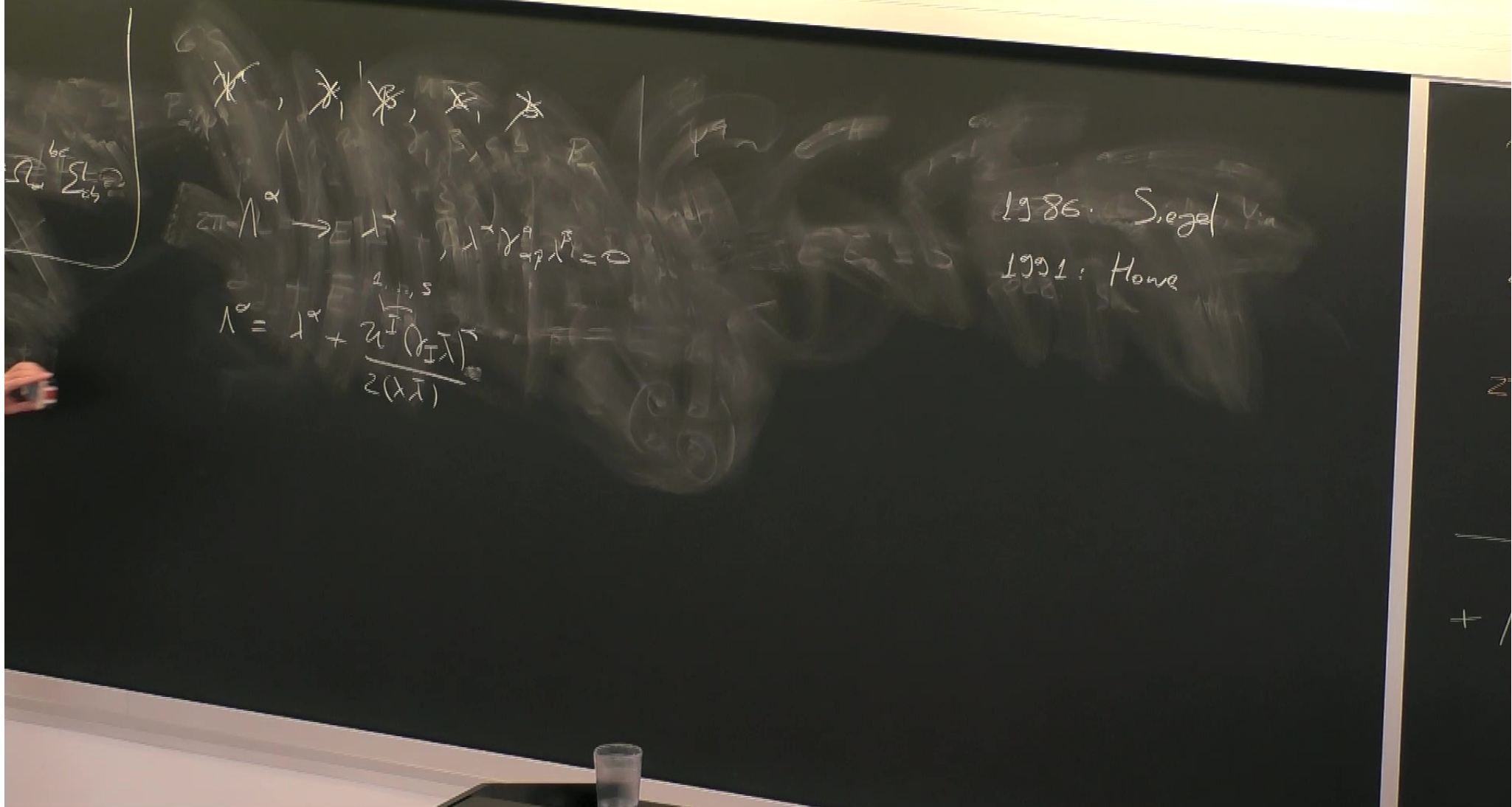
$\omega^a, \Lambda^a, \omega_a, \gamma, \beta, \epsilon, \zeta, P_s$

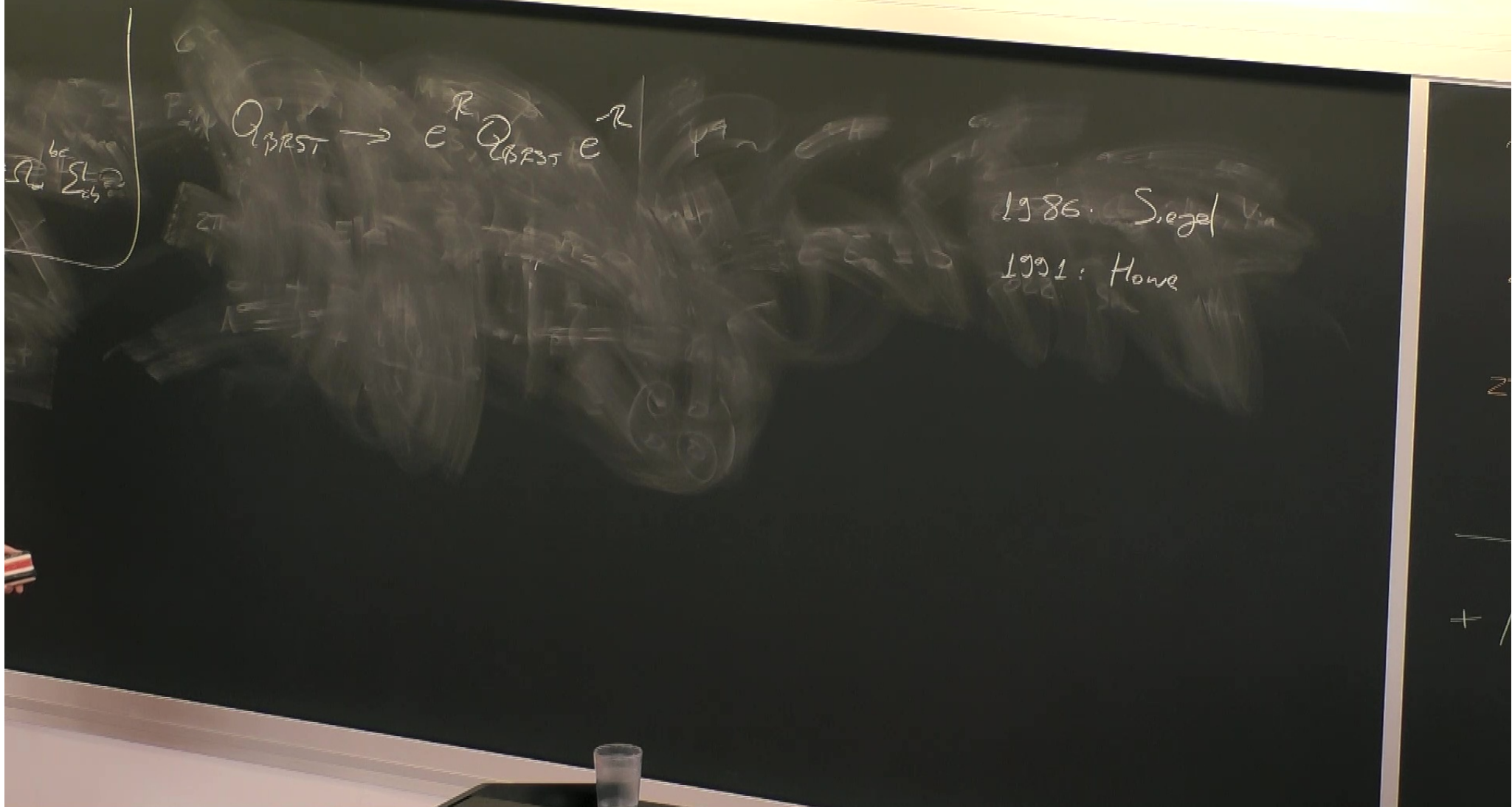
$$L = \frac{1}{2} E^a \bar{E}^b + B_{Het} + \frac{1}{2} \partial \bar{\psi} \psi + \frac{1}{2} \partial \bar{\psi} \psi + \bar{E}^a \omega_a$$

$$E^a E_a = 0$$

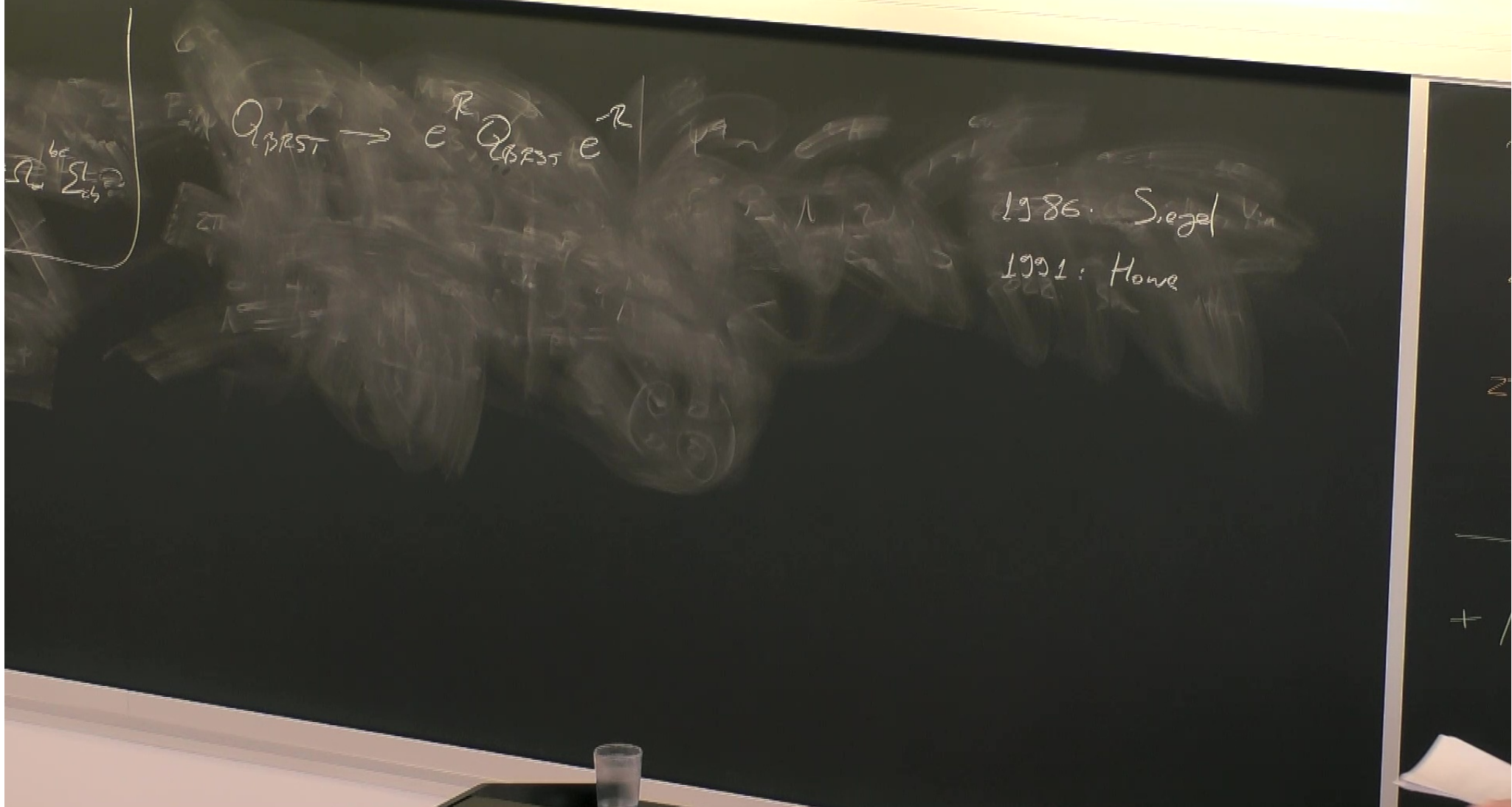
1986: Siegel

1991: Howe









bc  
SL<sub>2</sub>  
2g

$$Q_{p^2 RST} \rightarrow e^R Q_{p^2 RST} e^{-R}$$

1986: Siegel Yiu  
1991: Howe

$bc$   
 $\sum_{i=1}^n$

$$Q_{p,q,r,s,t} \xrightarrow{R} e^R Q_{p,q,r,s,t} e^{-R}$$

$SO(1,6)$      $U(5)$

$$\tilde{\psi}^a = (\psi^I, \psi_{\bar{I}}) ; \quad \tilde{\psi}^I = \gamma \psi^I ; \quad \tilde{\psi}_{\bar{I}} = \gamma^2 \psi_{\bar{I}}$$

$$i \left( \frac{\lambda^a - \gamma^2 \lambda^{\bar{a}}}{2(\lambda^I - \lambda_{\bar{I}})} \right) = \tilde{g}^a ; \quad \tilde{\psi}_{\bar{I}} = \gamma^{-1} \psi_{\bar{I}}$$

1986: Siegel, Yin

1991: Howe

$\int_{\mathcal{L}} \sum_{\mathcal{L}} \dots$

$$e^{-R} Q e^R = \int \left( \lambda^{\alpha} d_{\alpha} + \tilde{\gamma} b + \psi I u_{\Gamma} - \tilde{z} \right)$$

$$Q_{BH} = \int \lambda^{\alpha} d_{\alpha}$$

1986: Siegel  
1991: Howe

$$Q_b = T$$

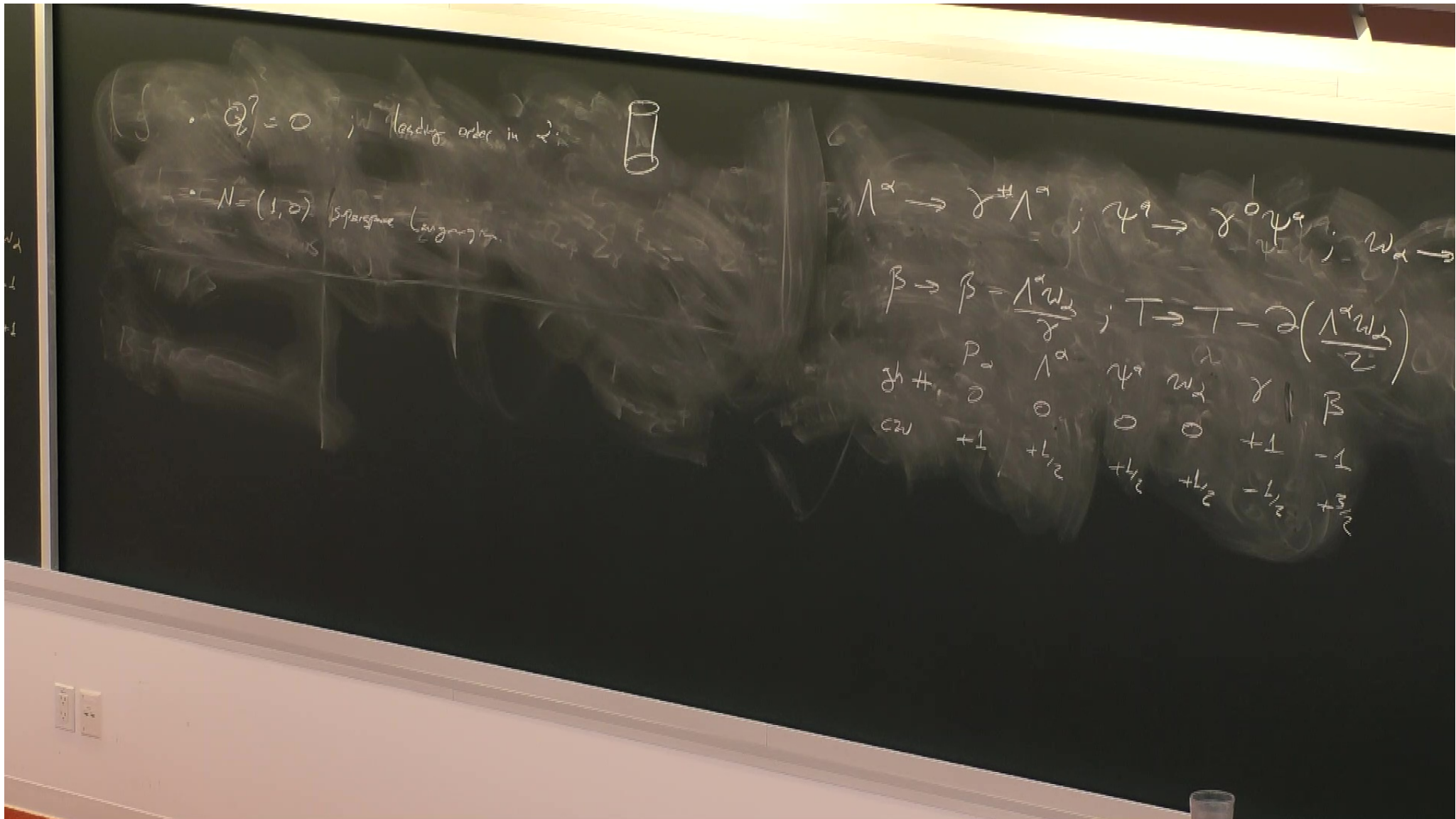
$$b = S^{\frac{1}{2}} \bar{\lambda}_d + \frac{\bar{\lambda}_d G}{\lambda T} = \frac{\bar{\lambda}_d \rho H^{\beta}}{(\lambda T)^2} \pm \dots = \mathcal{O}((\lambda T)^{-4})$$


$$\Omega_{ab} = \sum_{\gamma} \rho_{\gamma} + \frac{1}{2} \Omega_{ab} = \sum_{\gamma} \rho_{\gamma}$$

$$\Lambda^{\alpha} \rightarrow \gamma^{\#} \Lambda^{\alpha} ; \psi^{\alpha} \rightarrow \gamma^{\circ} \psi^{\alpha} ; \omega_{\alpha} \rightarrow \Lambda^{-1} \omega_{\alpha}$$

$$\beta \rightarrow \beta = \frac{\Lambda^{\alpha} \omega_{\alpha}}{\gamma} ; T \rightarrow T - 2 \left( \frac{\Lambda^{\alpha} \omega_{\alpha}}{2} \right)$$

	$P_{\alpha}$	$\Lambda^{\alpha}$	$\psi^{\alpha}$	$\omega_{\alpha}$	$\gamma$	$\beta$
gh #	0	0	0	0	+1	-1
cw	+1	+1/2	+1/2	+1/2	-1/2	+3/2



$Q = 0$  ; leading order in  $\epsilon$  

$N = (1, 0)$  Spherically Symmetric Lagrangian

$$\Lambda^\alpha \rightarrow \gamma^\# \Lambda^\alpha ; \psi^\alpha \rightarrow \gamma^0 \psi^\alpha ; w_\alpha \rightarrow w_\alpha$$

$$\beta \rightarrow \beta - \frac{\Lambda^\alpha w_\alpha}{\gamma} ; T \rightarrow T - 2 \left( \frac{\Lambda^\alpha w_\alpha}{2} \right)$$

$\gamma^\#$	$P_0$	$\Lambda^\alpha$	$\psi^\alpha$	$w_\alpha$	$\gamma$	$\beta$
$c_{21}$	0	0	0	0	+1	-1
	+1	+1/2	+1/2	+1/2	-1/2	+3/2

$\int \mathcal{L} = T$   
 $\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = p_i$   
 $\frac{\partial \mathcal{L}}{\partial q_i} = -V_i$   
 $\frac{\partial \mathcal{L}}{\partial t} = -\dot{V}_i$   
 $\frac{\partial \mathcal{L}}{\partial t} = -\dot{V}_i$

$\int \mathcal{L} = 0$  ; leading order in  $\dot{q}_i$   
 $N = (1, 0)$  ; prepare Lagrangian  
 $Ad_{S^3} \Rightarrow S^3$   
 $= \text{Multi-loop}$



$\Lambda^\alpha \rightarrow \gamma^\# \Lambda^\alpha$  ;  $\psi^a \rightarrow \gamma^0 \psi^a$  ;  $w_\alpha \rightarrow \Lambda^{-1} w_\alpha$   
 $\beta \Rightarrow \beta = \frac{\Lambda^\alpha w_\alpha}{\gamma}$  ;  $T \Rightarrow T - 2 \left( \frac{\Lambda^\alpha w_\alpha}{\gamma} \right)$   

$\gamma^\#$	$\Lambda^\alpha$	$\psi^a$	$w_\alpha$	$\gamma$	$\beta$
$\partial$	$0$	$0$	$0$	$+1$	$-1$
$\partial w$	$+1$	$+1/2$	$+1/2$	$+1/2$	$+3/2$