Title: Grad Student Seminar with Jacob Barnett

Speakers: Jacob Barnett
Date: April 17, 2023 - 2:00 PM

URL: https://pirsa.org/23040152

Abstract: Jacob Barnett, Perimeter Institute

Locality and Exceptional Points in Pseudo-Hermitian Physics

This talk discusses the role of non-Hermitian operators in fundamental and effective theories of physics. An implicit assumption of the tensor product model of locality is that the inner product factorizes with the tensor product. Quasi-Hermitian quantum frameworks can be used to lift this assumption while preserving the reality of spectra and unitarity. After characterizing local observable algebras and expectation values, I will examine Bell's inequality and its generalizations, the nonlocal games, in the setting of quasi-Hermitian theories. Pseudo-Hermitian operators characterize systems with time-reversal symmetry. These operators exhibit rich perturbative and symmetry-breaking properties that are unparalleled in the Hermitian regime. I will convey some geometric and topological aspects of these features, with emphasis placed on non-interacting many-body systems.

Locality and Exceptional Points in Pseudo-Hermitian Physics

Jacob L. Barnett

April 17th, 2022

UNIVERSITY OF WATERLOO

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Outline

1. Introduction:
   - Hermitian quantum theory.
   - Motivating non-Hermitian operators.
   - Qubit Example.
2. Non-Hermitian Novelties:
   - Geometry, perturbations, and topology $\leftrightarrow$ Barnett, J. L., and Y. N. Joglekar. arXiv:2302.13204 (2023).
   - New types of locality $\leftrightarrow$ Barnett, J. L. *J. Phys. A* 54.29 (2021): 295307.
3. Future Work

Quantum Theory: Hermiticity

- An **operator**, O, is a linear map on a Hilbert space[1], $\mathcal{H}$.
- The **adjoint** of an operator, $O \mapsto O^\dagger$, satisfies $\langle\psi|O\phi\rangle = \langle O^\dagger\psi|\phi\rangle \ \forall \psi, \phi \in \mathcal{H}$.
- **Observables** are *Hermitian* operators, so $O = O^\dagger$.
- Average of observable in $\psi \in \mathcal{H}$ is $\langle\psi|O\psi\rangle$.

[1]Hilbert spaces are *finite-dimensional* in this talk.

Why do we assume observables are Hermitian?

- **Spectral decomposition** of Hermitian observables $\Rightarrow$
  1. **Measurement outcomes** are **real-valued** elements of the spectrum (e.g. eigenvalues).
  2. States define **probability measures**.

- Hermitian Hamiltonians generate **unitary** time evolution.

Five reasons for non-Hermiticity

- Reason 1:

  Hermiticity $\Rightarrow$ Real-valued measurement outcomes / Consistent probabilities

  $\Updownarrow$

  Quasi-Hermiticity

- O is **quasi-Hermitian** $\Leftrightarrow \eta O = O^\dagger \eta$ for some positive-definite **metric operator**, η.

Five reasons for non-Hermiticity

- Quasi-Hermitian means Hermitian with a new inner product.
- $\boxed{\langle \cdot | \cdot \rangle_{\text{phys}} := \langle \cdot | \eta \cdot \rangle}$.
- New kinds of **locality** and **time**: η could be **entangled** or **time-dependent**.

$$\begin{array}{c} \text{Quasi-Hermitian Theory} \Leftrightarrow \text{Hermitian Theory} \\ \cup\!\!\!\nmid \\ \text{Local Quasi-Hermitian Theory} \\ \cup\!\!\!\nmid \\ \text{Local Hermitian Theory} \end{array}$$

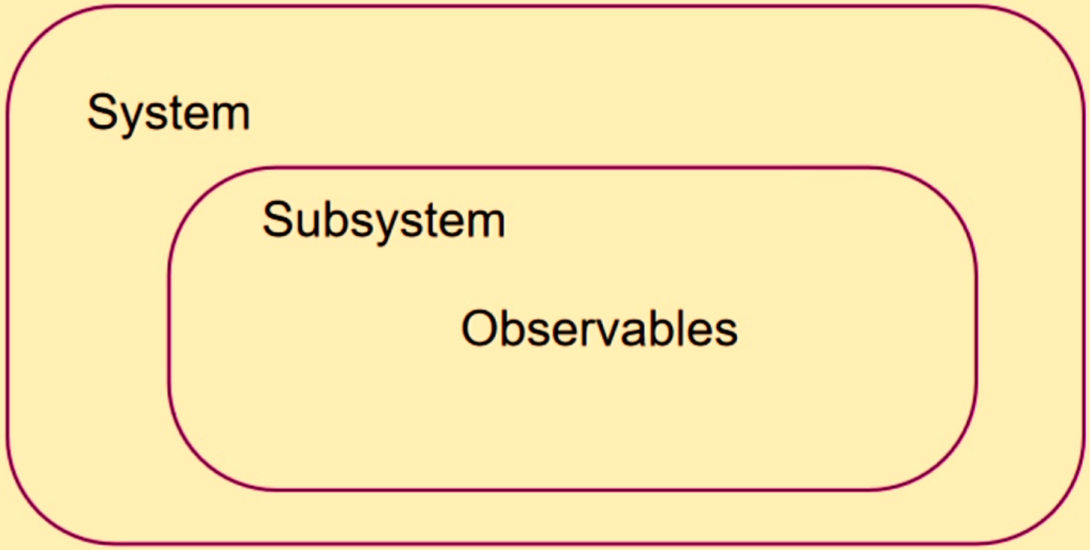
Five reasons for non-Hermiticity

Reason 2: Quantum Gravity?

A wavefunction description for a localized quantum particle in curved spacetimes T Rick Perche[4,1,2] and Jonas Neuser[3] Published 13 August 2021 • © 2021 IOP Publishing Ltd Classical and Quantum Gravity, Volume 38, Number 17	Article | Open Access | Published: 21 April 2022 **Curving the space by non-Hermiticity** Chenwei Lv, Ren Zhang, Zhengzheng Zhai & Qi Zhou *Nature Communications* **13**, Article number: 2184 (2022) | Cite this article
Open Access Einstein's quantum elevator: Hermitization of non-Hermitian Hamiltonians via a generalized vielbein formalism Chia-Yi Ju, Adam Miranowicz, Fabrizio Minganti, Chuan-Tsung Chan, Guang-Yin Chen, and Franco Nori Phys. Rev. Research **4**, 023070 – Published 25 April 2022	Strings from position-dependent noncommutativity Andreas Fring[1], Laure Gouba[2] and Frederik G Scholtz[2,3] Published 19 July 2010 • 2010 IOP Publishing Ltd Journal of Physics A: Mathematical and Theoretical, Volume 43, Number 34

Five reasons for non-Hermiticity

- Reason 3: Effective dynamics need not be unitary.



Essential Classes of Non-Hermitian Matrices

- Time-reversal is an **antilinear** symmetry, Θ, of the Hamiltonian, H:
  1. Θ is invertible
  2. $\Theta(\alpha |\psi\rangle + \beta |\phi\rangle) = \alpha^* \Theta |\psi\rangle + \beta^* \Theta |\phi\rangle$
  3. $\Theta H = H \Theta$.

E.P. Wigner. *Nachr. Ges. Wiss. Göttingen, Math.-Phys. Kl.* (1932): 546-559

Antilinear Symmetry

$\Uparrow$

Unbroken Antilinear Symmetry

$\Uparrow$

Quasi-Hermitian

$\Uparrow$

Hermitian

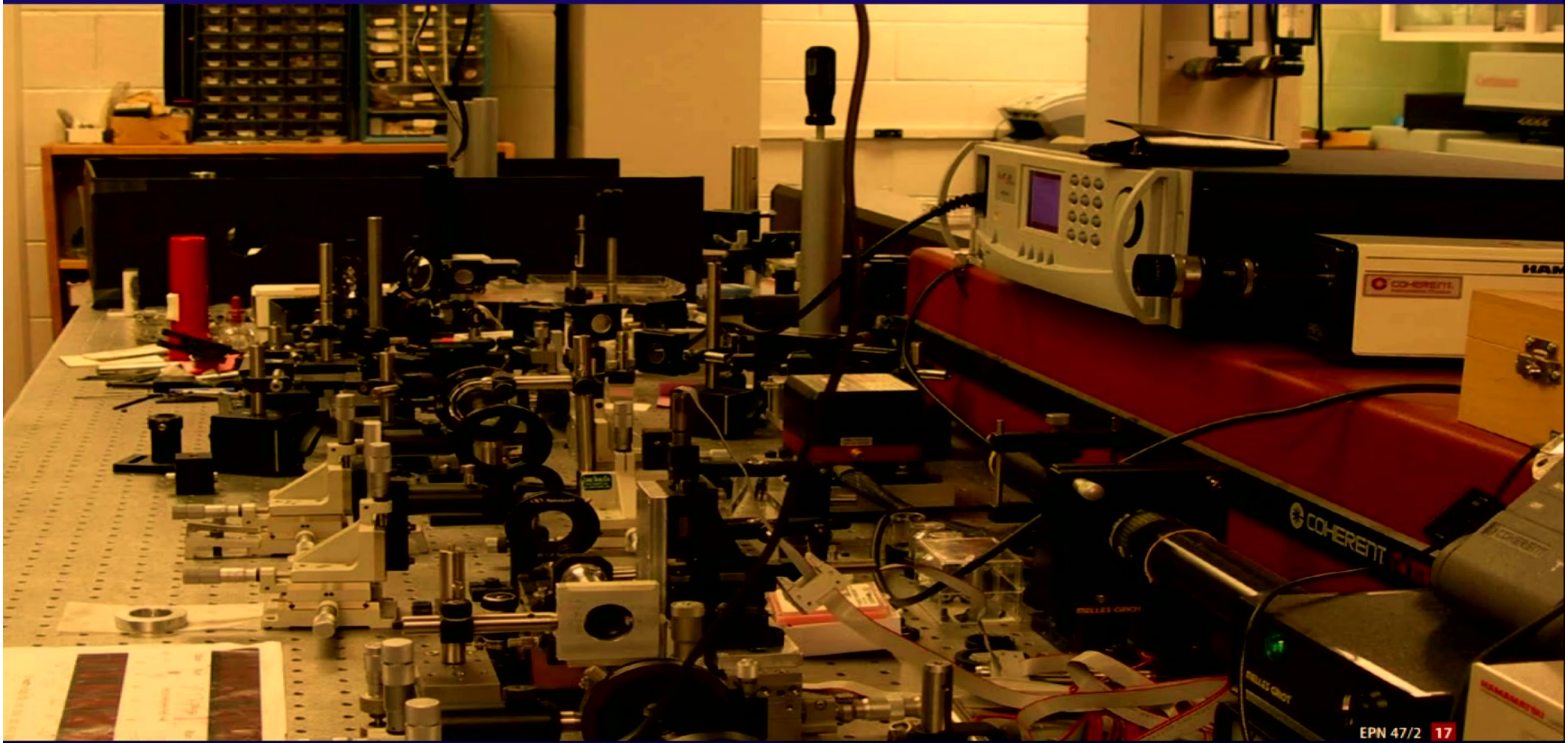
Five reasons for non-Hermiticity

- Reason 4: Non-Hermitian operators teach us about Hermitian ones!
- Ex: $H = A + \lambda B$, $\lambda \in \mathbb{R}$, $A = A^\dagger$, $B = B^\dagger$. Analytic continuation to $\lambda \in \mathbb{C}$ is non-Hermitian.
- Analytic continuation tells us about the original Hamiltonian.

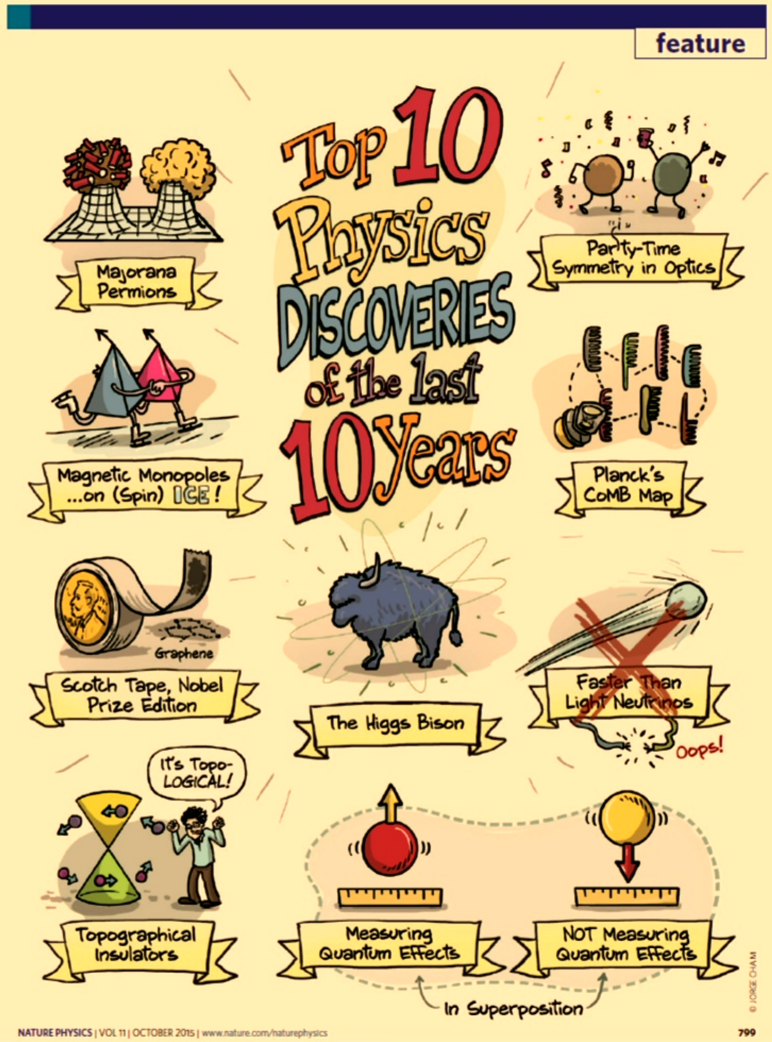
Five reasons for non-Hermiticity

- Reason 5: Schrödinger equations with non-Hermitian Hamiltonians appear outside quantum theory.
- Antilinear symmetry $\Leftrightarrow$ balanced loss and gain.

Non-Hermitian Optics



Cham J. *Nat. Phys.* 11.10 (2015): 799-799.



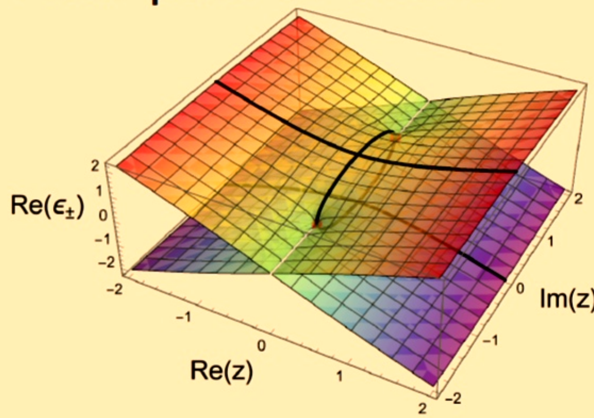
Qubit Example: Spectrum

$$H = \begin{pmatrix} z & 1 \\ 1 & -z \end{pmatrix}$$

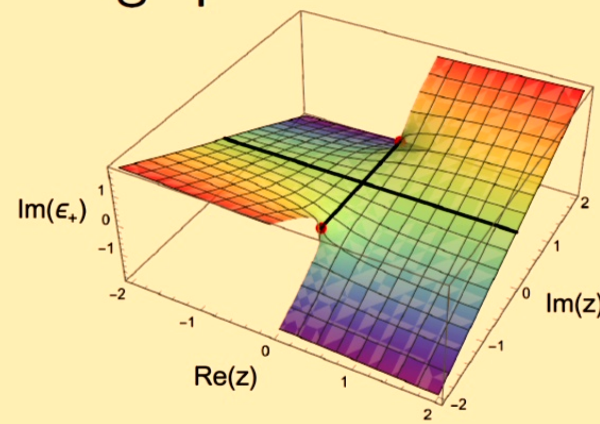
$$\epsilon_\pm = \pm\sqrt{1+z^2}$$

$$\epsilon_+ \approx 1 + \frac{z^2}{2} - \frac{z^4}{8} + \mathcal{O}(z^6)$$

Real part of eval.s

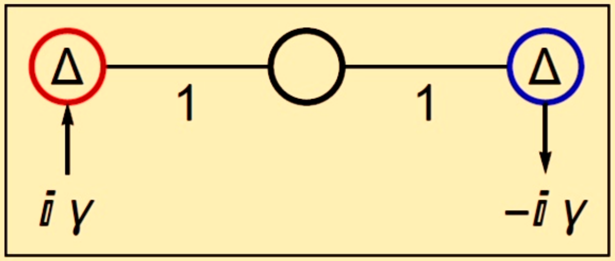


Imag. part of eval.s

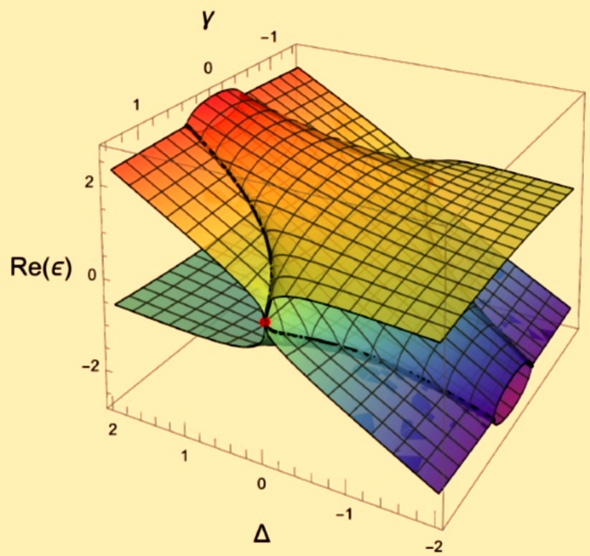


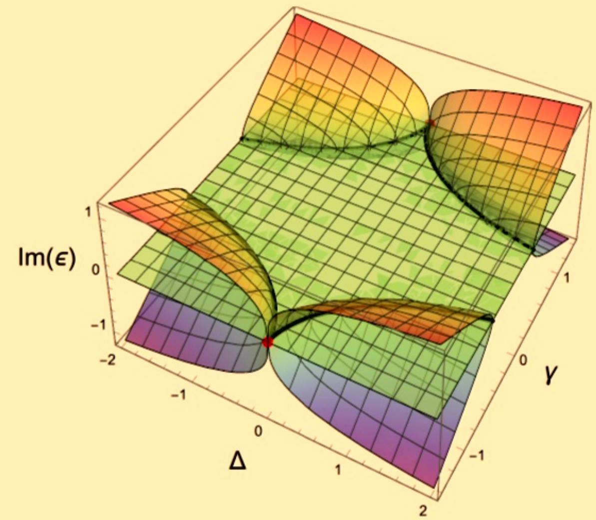
Radius of convergence dictated by branch points at $z = \pm i$.

3×3 model



$$H = \begin{pmatrix} \Delta + i\gamma & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & \Delta - i\gamma \end{pmatrix}$$



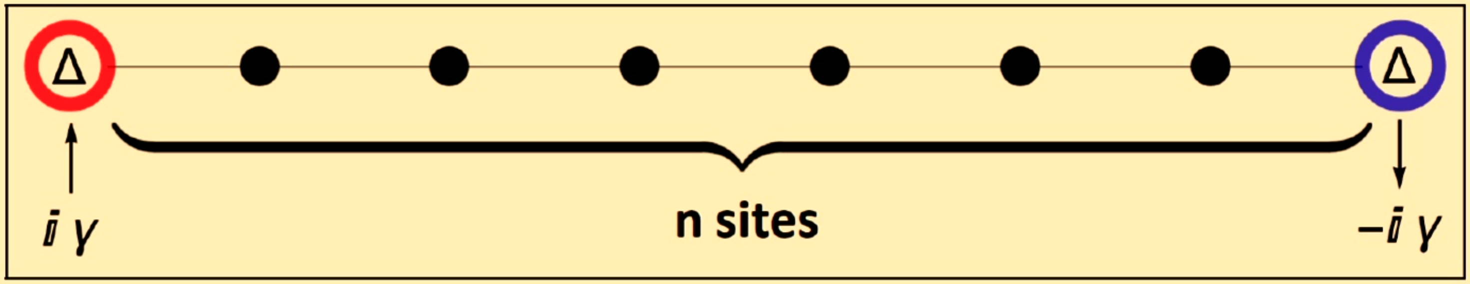


Features which generalize:

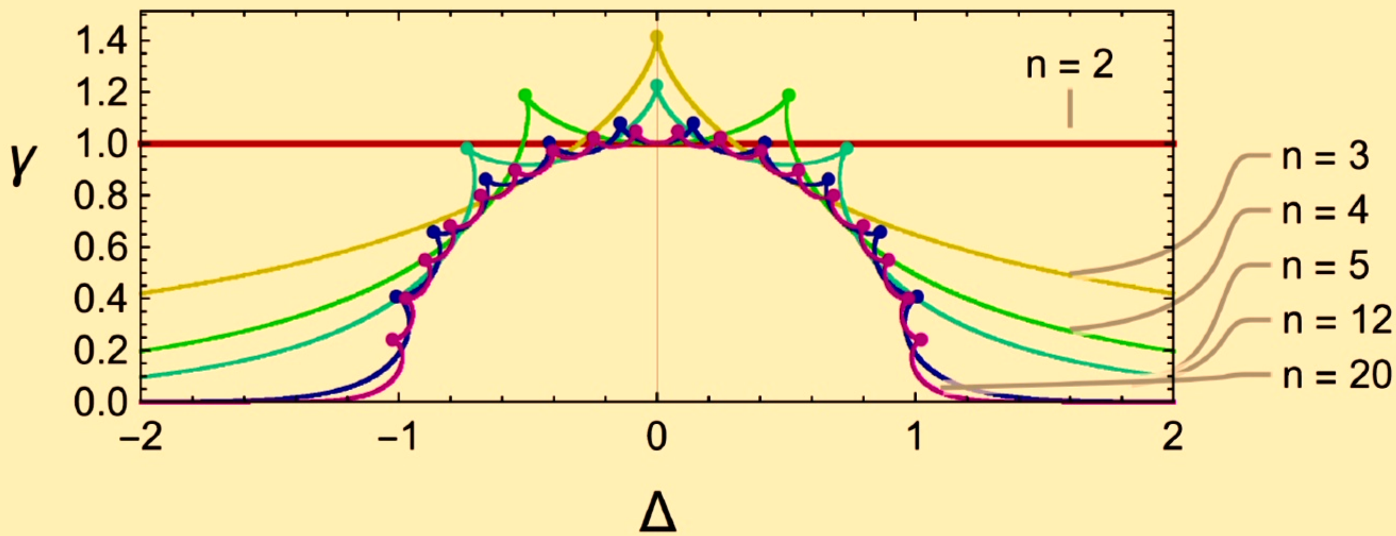
- EPs form an algebraic curve.
- Cusp singular points are higher order EPs.

Tight-Binding Model

Hamiltonian →

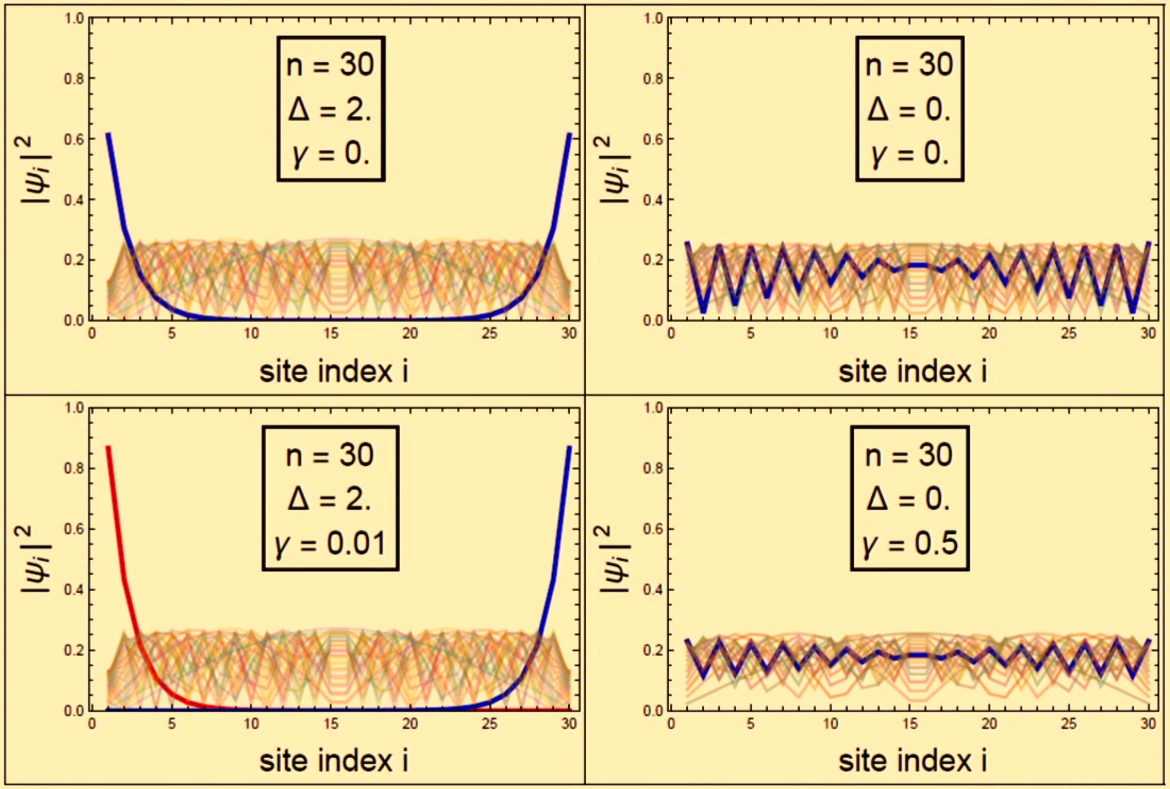


Exceptional Points →

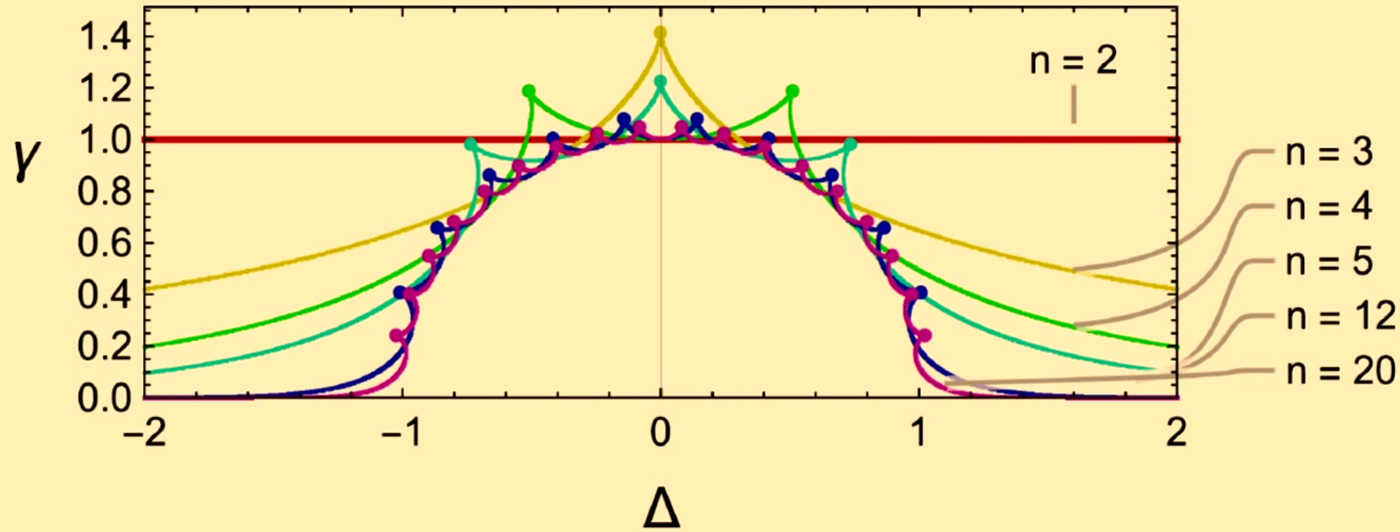


Symmetry Breaking and Edge States

- $\mathcal{PT}$-symmetry breaks outside unit disk due to **edge states**.



Tight-Binding Model Exceptional Points.



Features which generalize:

- EPs form an algebraic curve.
- Cusp singular points are higher order EPs.

Model-specific features:

- $\mathcal{PT}$ is unbroken inside the unit disk.
- $|\gamma| \sim \frac{1}{\Delta^{n-2}}$ as $\Delta \to \pm\infty$.

Locality

Universe

Local Subsystem $\mathfrak{A}_{\text{Jacob}}$

Local Subsystem $\mathfrak{A}_{\text{Jacobino}}$

Hermitian Tensor Product Model

- Space is a **finite** set of points, Σ.

$$\mathcal{H} = \bigotimes_{p\in\Sigma} \mathcal{H}_p$$

$$\mathfrak{A}_S = \mathcal{B}\left(\bigotimes_{p\in S} \mathcal{H}_p\right)$$

- The local algebras generate the entire algebra.

Tensor Product Local Quasi-Hermitian Observables

Schmidt decomposition over a spatial subsystem S with complement S':

$$\begin{array}{cccc} \eta = \sum_i & \zeta_{i,S} & \otimes & \zeta_{i,S'} \\ & \Cup & & \Cup \\ & \mathfrak{A}_S & & \mathfrak{A}_{S'} \end{array}$$

Quasi-Hermiticity generalizes the tensor product model because

- $\langle \psi_1 \otimes \phi_1 | \psi_2 \otimes \phi_2 \rangle_\eta \neq \langle \psi_1 | \phi_1 \rangle_M \langle \psi_2 | \phi_2 \rangle_{M'}$ for any M, M'.
- $\mathfrak{A}_S(\eta) \subsetneq \mathfrak{A}_S$, so $\mathfrak{A}_S(\eta)$ doesn't generate all operators.

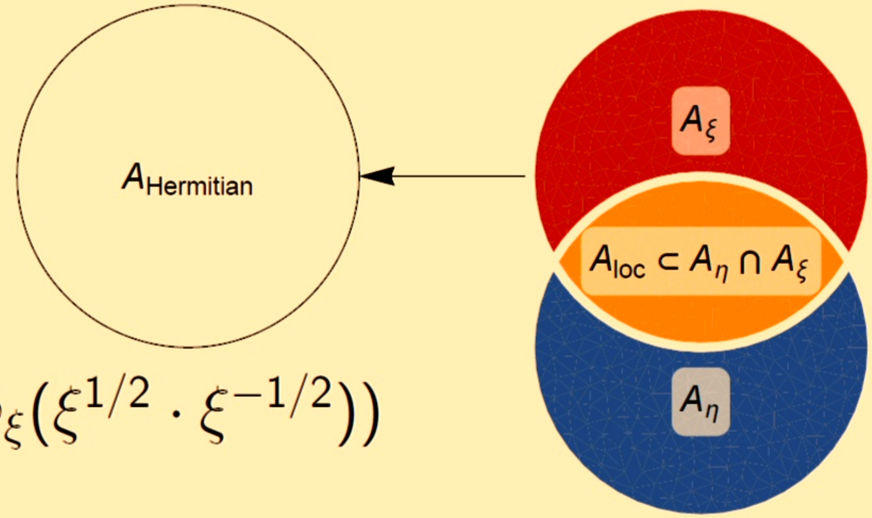
Characterization of Local Quasi-Hermitian Observables

$$\eta = \sum_i \begin{array}{ccc} \zeta_{i,S} & \otimes & \zeta_{i,S'} \\ \Cup & & \Cup \\ \mathfrak{A}_S & & \mathfrak{A}_{S'} \end{array}$$

- $\exists \xi \in \text{span}(\zeta_i)$ positive-definite such that $\xi O_S = O_S^\dagger \xi$.

Local Equivalence of Quasi-Hermitian and Hermitian Frameworks

$$
\begin{aligned}
&\varphi && : \mathfrak{A}(\eta) \to \mathbb{C} \\
&\downarrow && \text{(restriction)} \\
&\varphi|_{\mathfrak{A}_{\text{loc}}(\eta)} && : \mathfrak{A}_{\text{loc}}(\eta) \to \mathbb{C} \\
&\downarrow && \text{(Hahn-Banach)} \\
&\varphi_\xi && : \mathfrak{A}(\xi) \to \mathbb{C} \\
&\downarrow && \text{(local similarity, } \varphi_H(\cdot) = \varphi_\xi(\xi^{1/2} \cdot \xi^{-1/2})) \\
&\varphi_H && : \mathfrak{A} \to \mathbb{C}
\end{aligned}
$$



Summary

- Quasi-Hermitian quantum theory generalizes locality.
  1. Quasi-Hermitian local observable algebras are smaller than their Hermitian counterparts
  2. New notion of locality is "tame": Tsirelson's bound on Bell's inequality violation still holds!
- Perturbations of non-Hermitian operators $\leftrightarrow$ algebraic geometry.
- Higher order exceptional points $\sim$ cusp singularities.

Possible Future Work

- Pseudo-Hermitian random matrices.
- Ansatz for interacting non-Hermitian many-body problems (e.g. tensor networks).
- Second-quantized non-Hermiticity with pair creation and annihilation (e.g. Kitaev's quantum wire).
- Time: Tomita-Takesaki theory.
- Entanglement: inner product, entropy.

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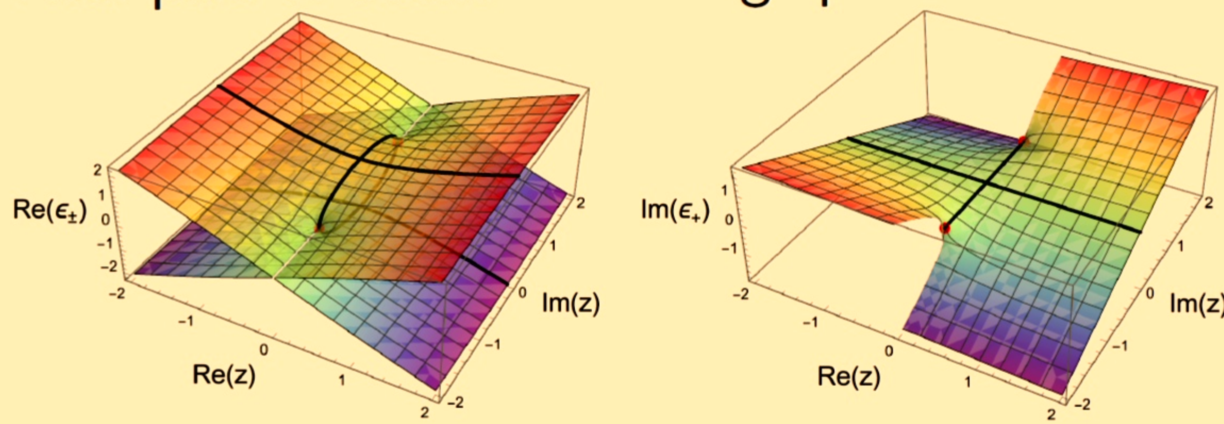

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