Title: Quiver algebras and their representations for arbitrary quivers

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Series: Mathematical Physics

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Abstract: I will first review the construction of quiver Yangians for the quiver and superpotential from string theory on general toric Calabi-Yau threefolds; they serve as BPS algebras of these systems and their characters reproduce the unrefined BPS indices. I will then explain how to generalize the construction to arbitrary quivers and how to incorporate refined BPS indices. The entire construction allows for straightforward generalizations to trigonometric, elliptic, and generalized cohomologies. Time permitting, I will discuss some applications, such as on BPS counting and on Gauge/Bethe correspondence.

Zoom link: https://pitp.zoom.us/j/96812116687?pwd=RjlsSVhDaUVBREtUUnd5S04rQjhpZz09

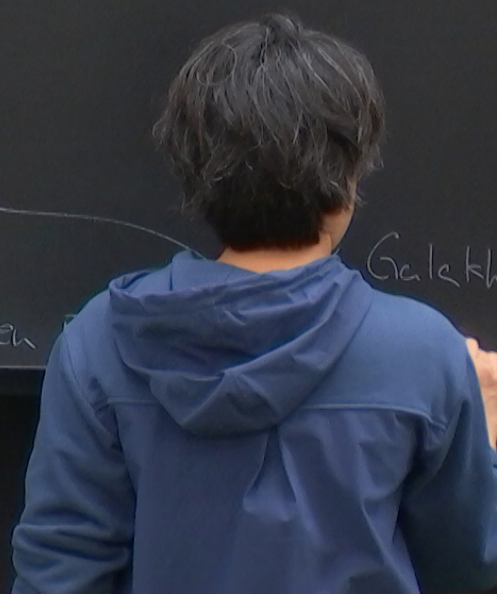
Quiver algebras and Representations for general quivers

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(cf. toric CY_3 quivers
2003.08908 w/ M. Yamazaki

(I) Quiver Yangians (for general quivers
(II) As BPS algebras for IIA sty on toric CY_3
(III) $\Rightarrow$ general quivers
(IV) extension / application

- toroidal elliptic
- Non-vacuum rep
  $\Rightarrow$ different chamber.



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- Gauge/Bethe correspondence
- toroidal elliptic
- Non-vacuum rep $\Rightarrow$ differed chamber.
- Open BPS et.

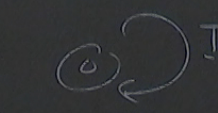
w/ Galakhov Yamazaki

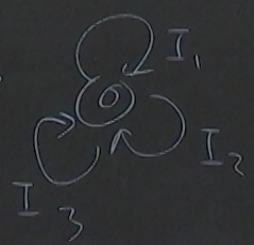
(I) Quiver Yangian

(I-1) Quiver $Q = (Q_0, Q_1)$

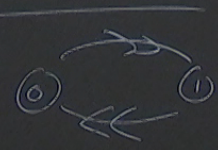
$Q_0 = \{\text{vertex } a\}$ $\quad a = 0, \ldots, |Q_0| - 1$

$Q_1 = \{\text{arrow } I^{a \to b}\}$ $\left(\{a \to b\} \equiv \{ I^{a \to b} \in Q_1 \}\right)$

eg.  Jordan



$\mathbb{C}^3$



resolved conifold



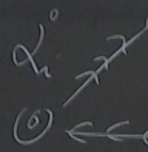
m-krone

I_i $i = 1, \ldots, m$

- knot-quiver correspondence

w/ Galakhov Yamazaki

application ... different chamber. Open BPS et.) Yamazaki.



$\mathbb{C}^3/\mathbb{Z}_3$

$W = \epsilon^{ijk}\, \alpha_i^0 \alpha_j^1 \alpha_k^2$

- symmetric $\forall\, a, b \in Q_0$

$$|a \to b| = |b \to a|$$

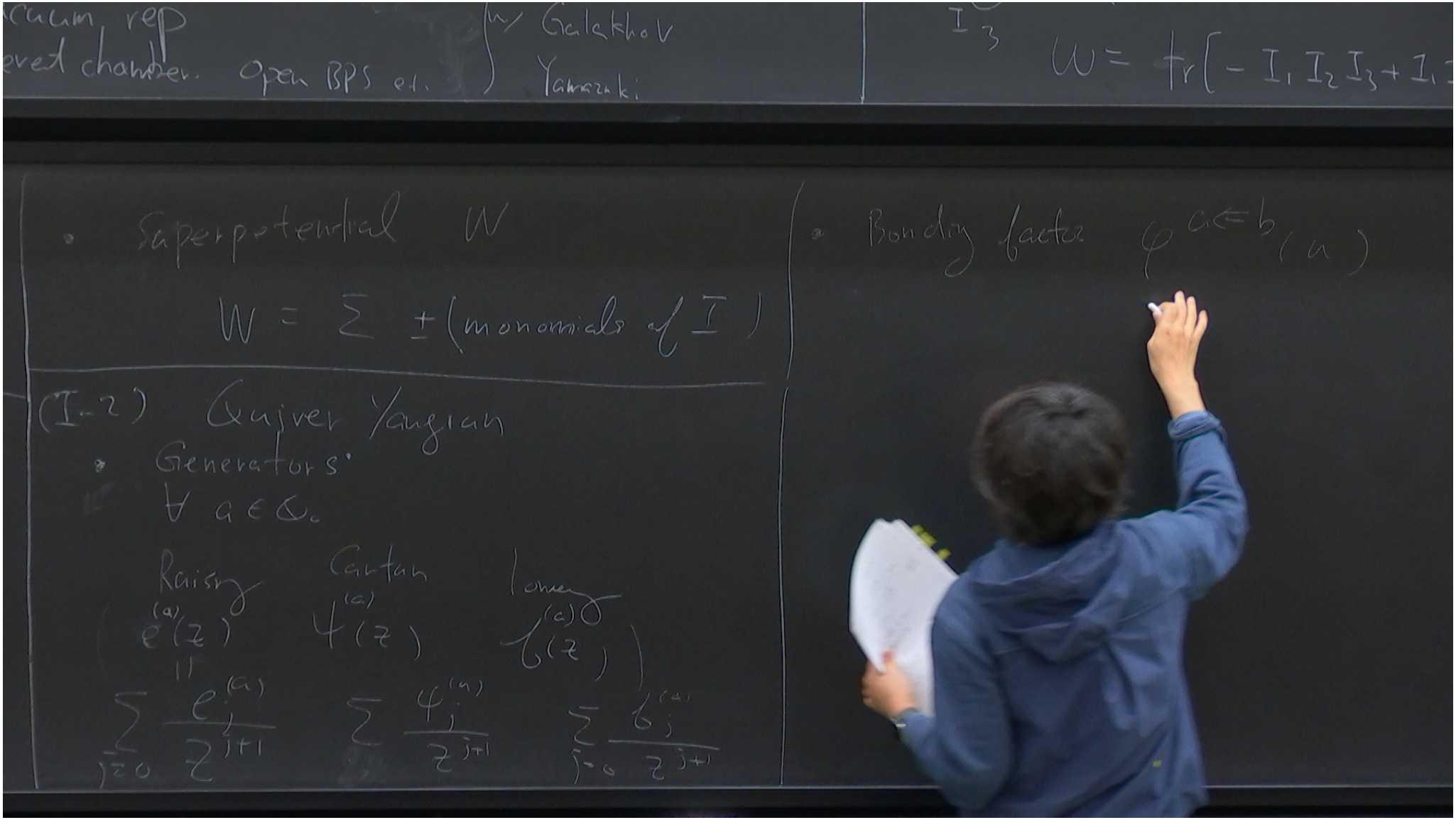
$$(\exists\, |\{a \to b\}|)$$

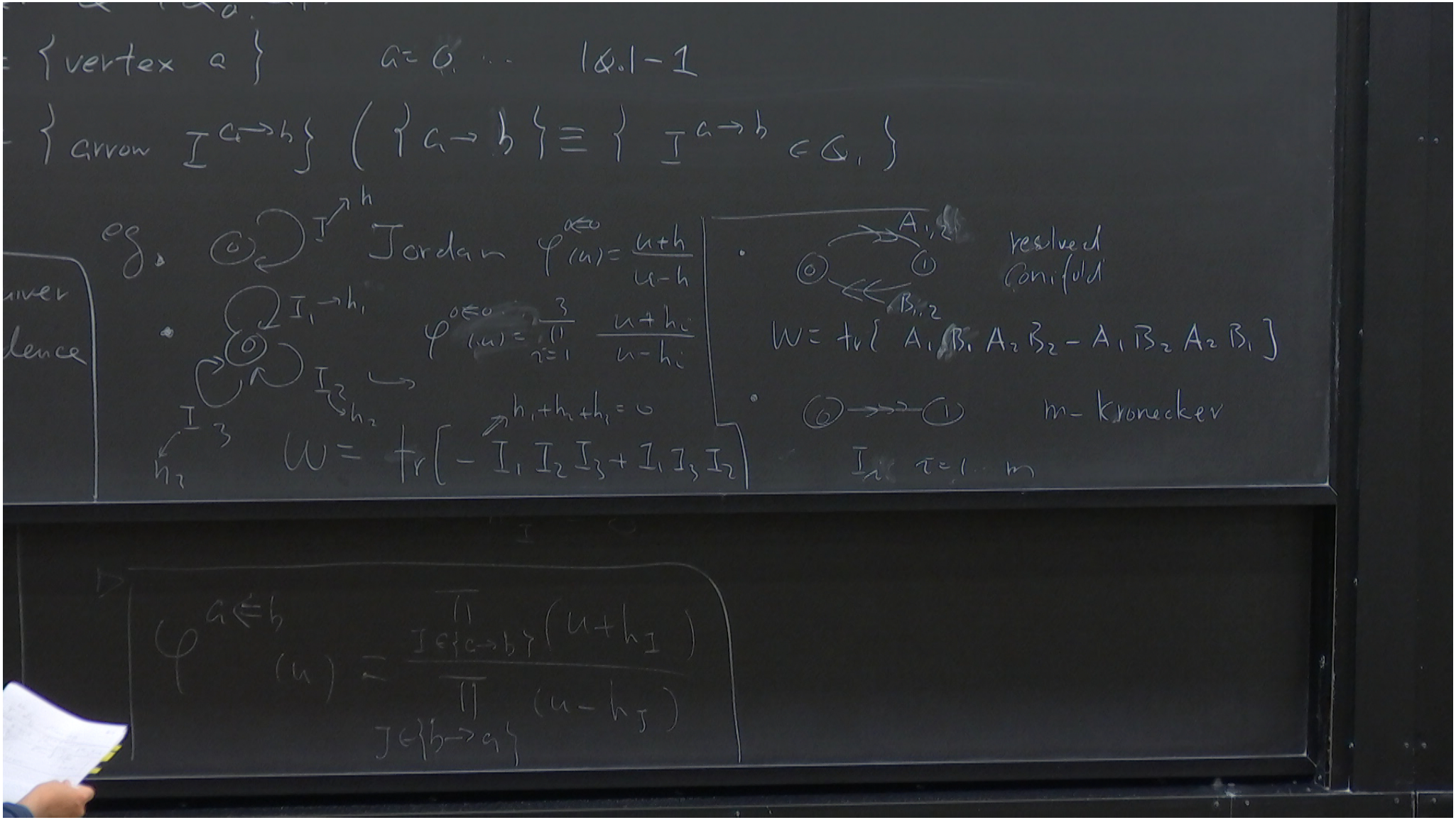
- chiral $\exists\, a, b \in Q_0$

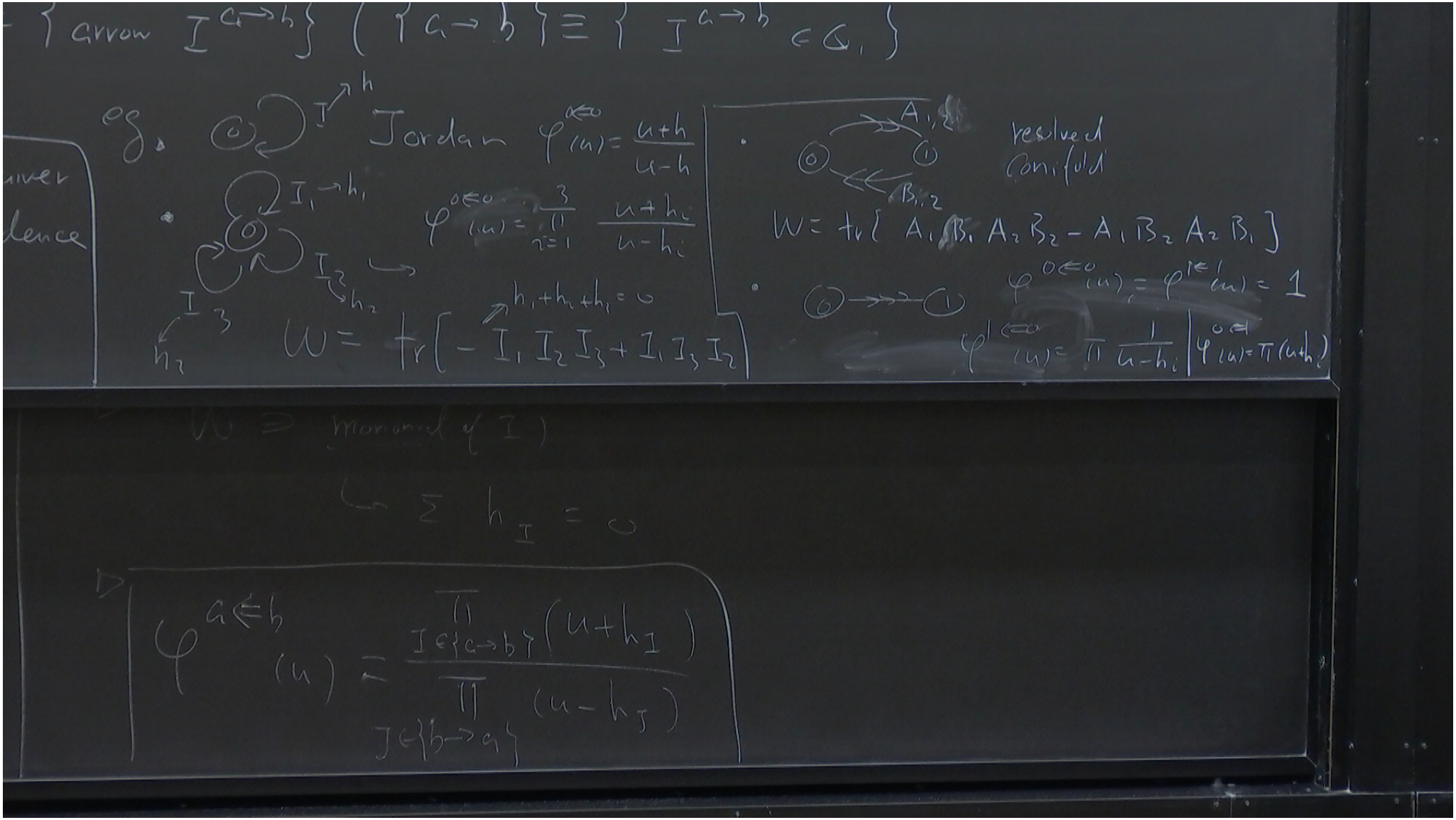
$$|a \to b| \neq |b \to a|$$

- Superpotential W

$W = \sum \pm$ (monomials of


cuum rep
ret chamber. open BPS et.
w/ Galakhov
Yamazaki
I_3
$W = \mathrm{tr}(-I_1 I_2 I_3 + I_1$
• Superpotential W
$W = \sum \pm$ (monomials of I)
• Bondy factor $\varphi^{a \in b}(u)$
(I-2) Quiver Yangian
• Generators
$\forall a \in Q_0$
Raising
Cartan
Lowering
$e_j^{(a)}$
$\psi_j^{(a)}$
$f_j^{(a)}$
$\sum_{j \geq 0} \frac{e_j^{(a)}}{z^{j+1}}$
$\sum \frac{\psi_j^{(a)}}{z^{j+1}}$
$\sum_{j \geq 0} \frac{f_j^{(a)}}{z^{j+1}}$


{vertex a}
a = 0, ... |Q_0| - 1
{arrow I^{a→b}} ({a→b} ≡ {I^{a→b} ∈ Q_1})
eg.
Jordan
resolved conifold
W = tr{A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1}
m - kronecker
W = tr(-I_1 I_2 I_3 + I_1 I_3 I_2)


{ arrow $I^{a\to b}$ } ({ $a\to b$ } ≡ { $I^{a\to b} \in Q_1$ }
eg.
Jordan $\varphi^{0\in 0}(u) = \frac{u+h}{u-h}$
$I_1 \to h_1$
$I_2 \to h_2$
$I_3 \to h_3$
$h_1+h_2+h_3 = 0$
$W = tr(-I_1 I_2 I_3 + I_1 I_3 I_2)$
$\varphi^{0\in 0}(u) = \prod_{i=1}^{3} \frac{u+h_i}{u-h_i}$
resolved conifold
$W = tr\{ A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1 \}$
$\varphi^{0\in 0}(u) = \varphi^{1\in 1}(u) = 1$
$\varphi^{1\in 0}(u) = \prod \frac{1}{u-h_i}$
$\varphi^{0\in 1}(u) = \prod (u+h_i)$
$W \supset$ monomial of I
$\hookrightarrow \sum h_I = 0$
$\varphi^{a\in b}(u) = \frac{\prod_{I\in\{a\to b\}} (u+h_I)}{\prod_{J\in\{b\to a\}} (u-h_J)}$

(I-3) Quadratic Relations

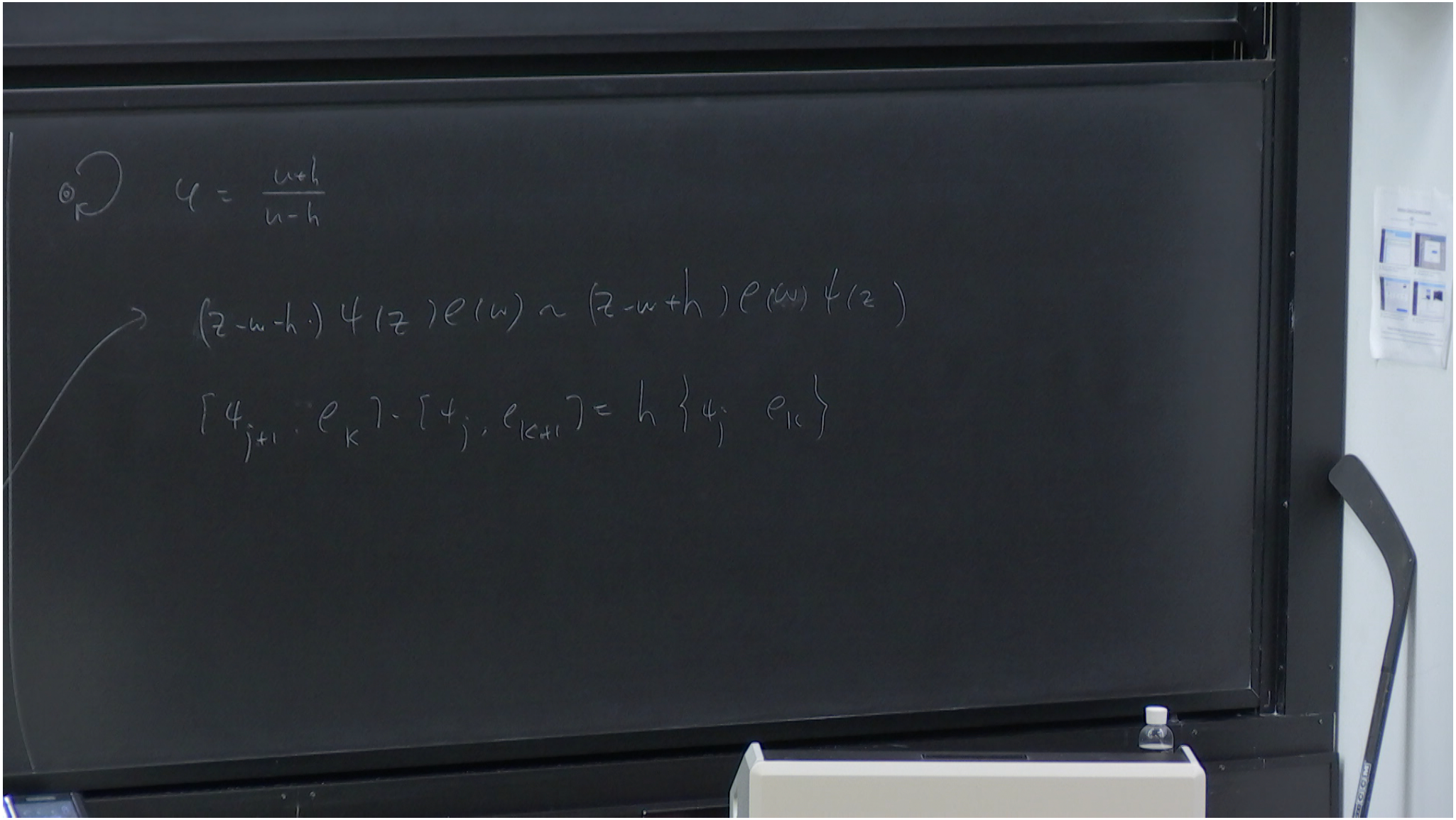
- $[\psi^{(a)}(z)\ \psi^{(b)}(w)] = 0$
- $[e^{(a)}(z),\ f^{(b)}(w)\}\sim -\delta^{(ab}\ \dfrac{\psi^{(a)}(z)-\psi^{(a)}(w)}{z-w}$ → all (Q, w)
- $\psi^{(a)}(z)\,e^{(b)}(w) \sim \varphi^{a\in b}(z-w)\,e^{(b)}(w)\,\psi^{(a)}(z)$

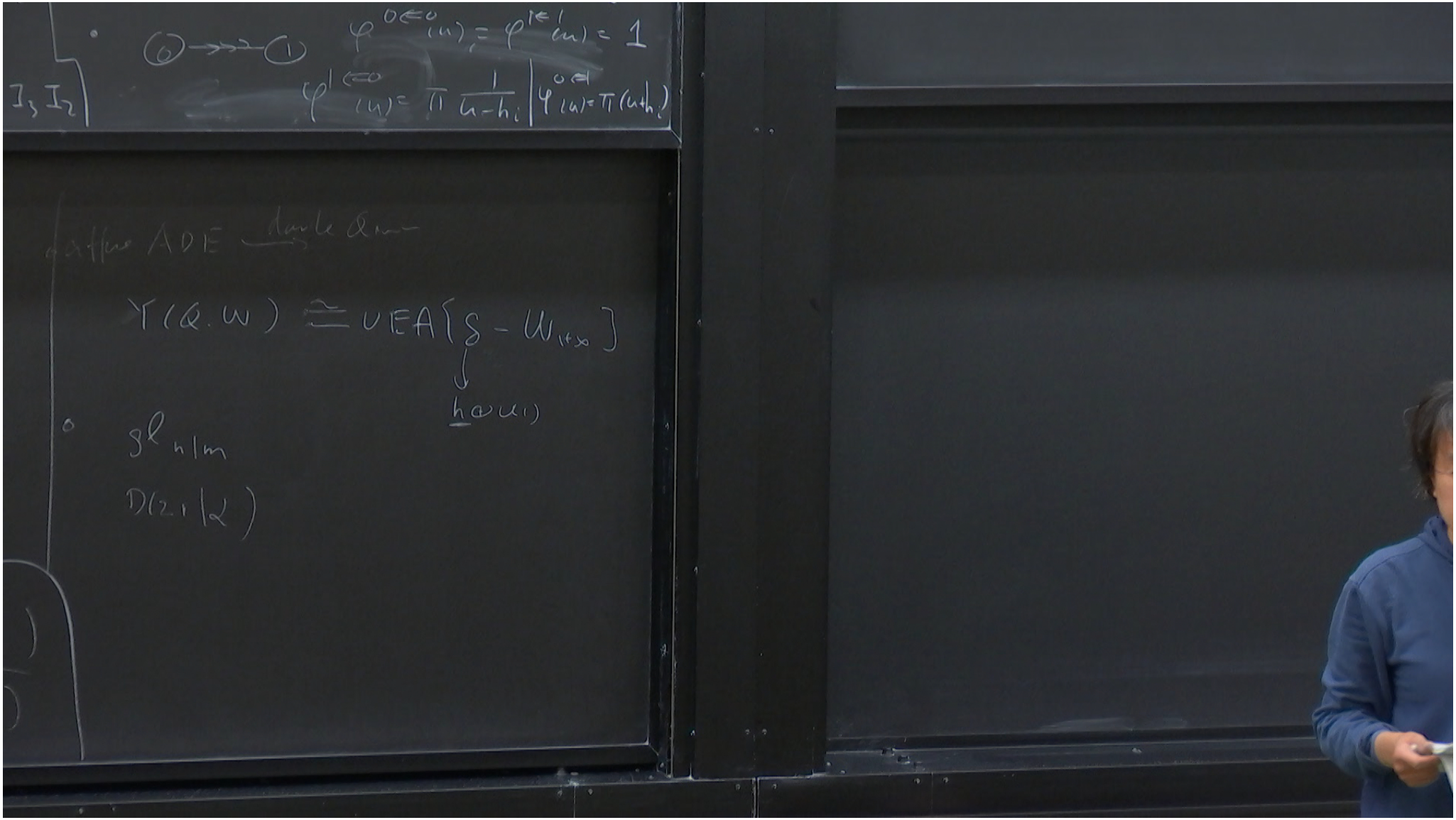
$\dfrac{N(z-w)}{D(z-w)}$

$\downarrow t$: t $\quad \varphi^{a\in b}(z-w)^{-1}\, f^{(b)}(w)$;

- $e^{(a)}(z)\,e^{(b)}(w) \sim \pm\, \varphi^{a\in b}(z-w)\cdot e^{(b)}(w)\,e^{(a)}(z)$

$t\,t$ $\quad t$ $\quad t$ $\quad ; \varphi^{a\in b}(z-w)^{-1}$ $\quad t$ $\quad t$


$q = \frac{w+h}{w-h}$
$(z-w-h)\,\psi(z)\,\varphi(w) \sim (z-w+h)\,\varphi(w)\,\psi(z)$
$[\psi_{j+1}, \varphi_k] - [\psi_j, \varphi_{k+1}] = h\{\psi_j\ \varphi_k\}$


affine ADE
$Y(Q,W) \cong \mathrm{UEA}[S - W_{1+\infty}]$
$\mathfrak{gl}_{n|m}$
$D(2,1|2)$

$$\varphi^{a\in\mathfrak{h}}(u) = \frac{\prod_{I\in\{c\to b\}}(u+h_I)}{\prod_{J\in\{b\to a\}}(u-h_J)}$$

Λ_1 = "unit" BPS state = $\square$

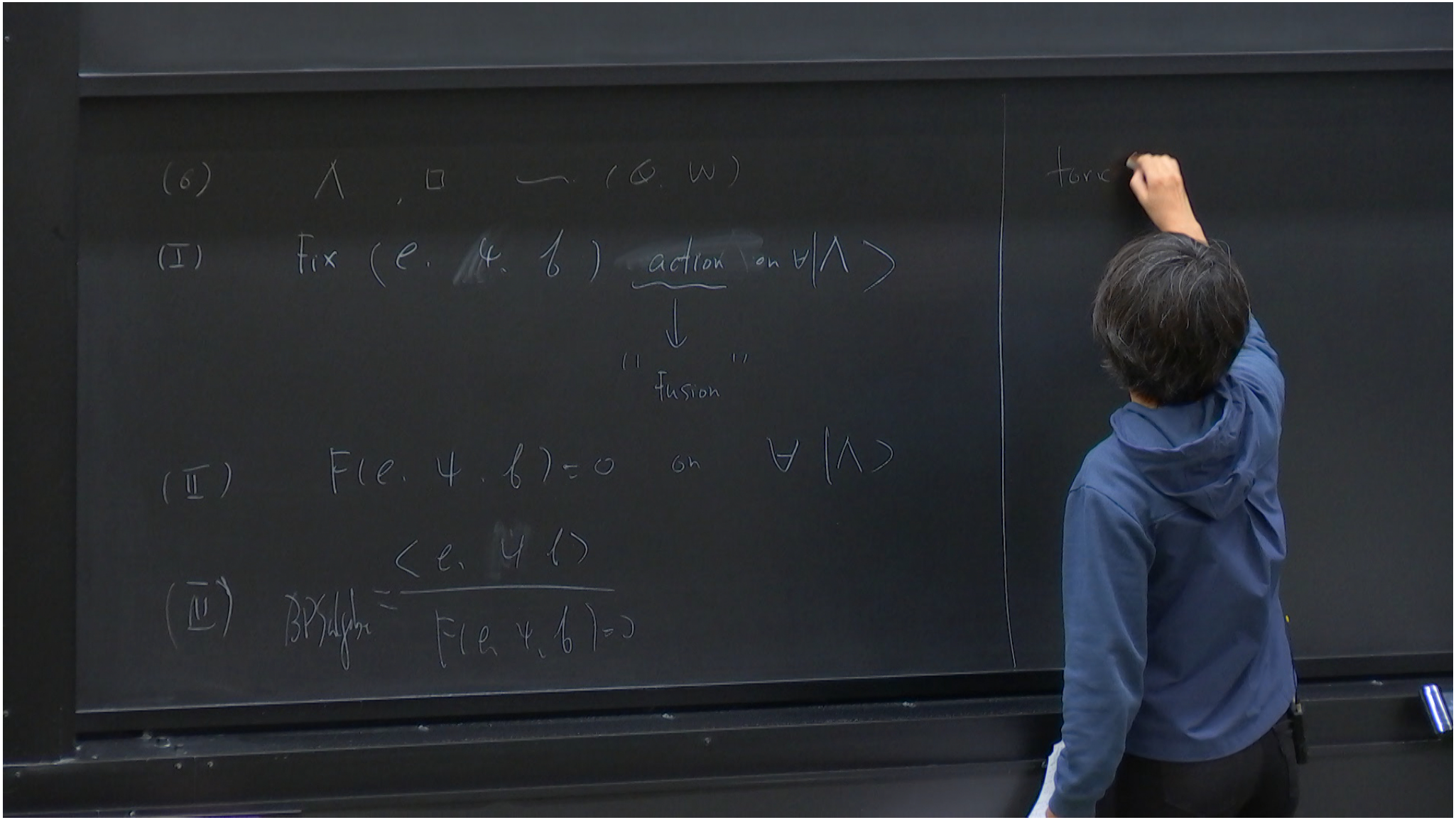
Λ_2 : arbitrary

$\square \otimes \Lambda \longrightarrow \Lambda + \square \in \mathcal{H}_{BPS}$

- $\hat{e}(z)$

$\hat{\psi}(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$

- $f: \mathcal{F}_\Lambda \to \mathcal{F}_{\Lambda-\square}$


(6) Λ . □ — (Φ W)
(I) Fix (e. 4. b) action on ∀|Λ>
" "
fusion
(II) F(e. 4. b) = 0 on ∀|Λ>
(III) BPS states = <e. 4 ()> / F(e, 4, b) = 0

- Higher order Relation
  (Serre relation
- Co-product

$\mathcal{N}=4$ $\mathcal{O}\mathcal{M}$

Q_0 $a \in Q_0 \longrightarrow U(N_a)$

Q_1 $I^{a\to b} \in Q_1 \longrightarrow \Phi_I$ $\overline{U(N_a) \times U(N_b)}$

$W = \mathrm{tr}\, \Sigma \left(\Phi_I \right) \longrightarrow$ F-term $\partial_I W = 0$

$\{ \Phi_I \}$ F-term, D-term

$\Phi_I \cdot \Phi_J = 0$ $t(I) = s(I)$

general quiver

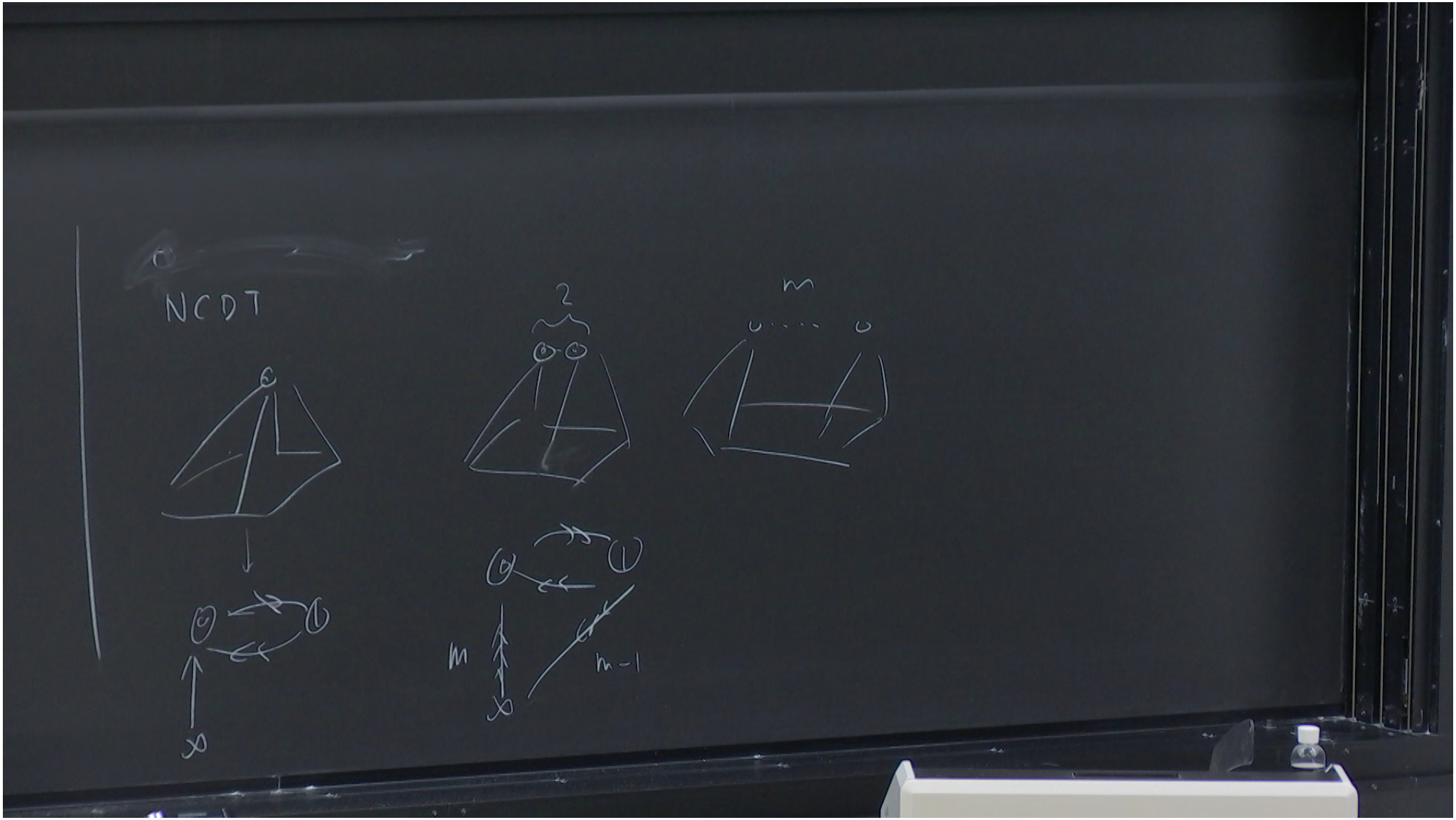
$$\mathcal{O} \quad |\emptyset\rangle$$

$$1 \quad |\square\rangle$$

$$d \to d+1$$

$$\mathcal{E}(z)\,|\Lambda\rangle = z^{-\left(\frac{\operatorname{Res} \bar{\Psi}^{(a)}_{\Lambda}(u)}{z - h(\square)}\right)^{1/2}} |\Lambda + \square\rangle$$

$$\bar{\Psi}^{(a)}_{\Lambda}(u) = \psi_a(u) \prod_{b \in Q_1} \prod_{\square \in \Lambda} \varphi^{a \in b}\left(u - h(\square)\right)$$


NCDT
2
m
m
m-1

$$\varphi^{a<b}(u) = \frac{\prod_{I\in\{a\to b\}}(u+h_I)}{\prod_{J\in\{b\to a\}}(u-h_J)}$$

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$\hat{\psi}(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$

$-\ b : \ell_{(z)}\Lambda \to \Lambda - \square$

$\mathcal{Q}-\mathcal{W}_{1+\infty}$ = DIM = $\mathcal{H}_1$

2D $(2,2)$ — sinhβz

3D $\mathcal{N}=2$ — $\Theta_q(z)$ → ellipt.

$$\varphi^{c\in\mathbb{Z}} = \frac{(u+h_J)}{(u-h_J)} \quad z$$