Title: Tutorial 4A: Navigator examples. Bootstrapping N=1 Super-Ising

Speakers: Ning Su

Collection: Mini-Course of Numerical Conformal Bootstrap

Date: April 27, 2023 - 1:30 PM

URL: https://pirsa.org/23040146

IsingOE_nvg.nb - Wolfram Mathematica 11.1

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```
Get["./Scripts/config.m"];
Get[ReconfigCmd@"[Local.PackageDirectory]/Bootstrapper.m"];

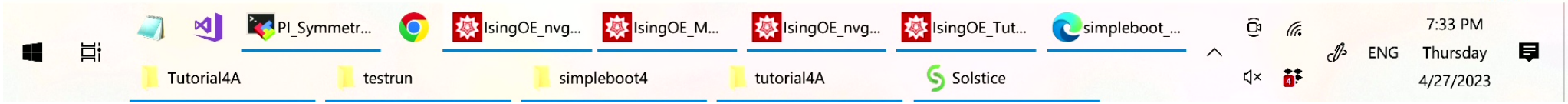
Bootstapper packages Loaded. Version : 4.0
MMA Precision set to 200.

Cluster$SetConfig["[Cluster.ProjectDirectory]","Proj_IsingOE_nvg"];  (* folder for the current project (inside the workspace) *)
CheckDirectory[Cluster$GetConfig["[Cluster.ProjectDirectory]"]];  (* create the folder if it doesn't exist *)
```

Input : bootstrap condition

$$V_{\text{even}} = \begin{pmatrix} \begin{pmatrix} F_{-,\Delta,\prime}^{11,11} & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & F_{-,\Delta,\prime}^{22,22} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2} F_{-,\Delta,\prime}^{11,22} \\ \frac{1}{2} F_{-,\Delta,\prime}^{11,22} & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2} F_{+,\Delta,\prime}^{11,22} \end{pmatrix} \end{pmatrix}, \qquad V_{\text{odd}} = \begin{pmatrix} 0 \\ 0 \\ F_{-,\Delta,\prime}^{12,12} \\ (-1)^{\prime} F_{-,\Delta,\prime}^{21,12} \\ -(-1)^{\prime} F_{-,\Delta,\prime}^{21,12} \end{pmatrix}$$

120%



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```
"VBlock$Identity"→{Fp[sig,sig,sig,sig,0]
                   Fp[eps,eps,eps,eps,0]
                   0
                   Fp[sig,sig,eps,eps,0]
                   Hp[sig,sig,eps,eps,0]}, (* crossing vector for V_identity=(1 1)·V_even(Δ=0, ℓ=0)·(1 1) *)

"VBlock$Deriv"→{"odd"
                "odd"
                "odd"
                "odd"
                "even"}, (* indicate the 5 compenents are even/odd under u<->v exchange *)

"VBlock$External"→{"sig","eps"} (* the name of external operators as String *)
};
```

Block specifications

```
(* conformal block specifications : spacetime dimension, derivative order, pole keeping order, r_ order, spins *)
blockconfobj={"dim"→3,"Λmax"→11,"κ"→12,"rN"→48,"lset"→Range[0,20]~Join~{49,52}};

dim = 3;
Lambda = 27;
AutoCB3$BlockSetting["DSD"][dim, Lambda] (* some presetting in simpleboot *)
```

120%

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```
GFFnormalization},

(*
BC$normalization=-AutoCB3$Vector[CrossVec[op[op,"E",1,1],2AutoCB3$CorOP2VarName["esp"],{√2,0},0]+
CrossVec[op[op,"E",1,1],2AutoCB3$CorOP2VarName["sig"],{0,√2},0]+
CrossVec[op[op,"O",1,1],AutoCB3$CorOP2VarName["sig"]+AutoCB3$CorOP2VarName["esp"],{1},0]];
*)

BC$normalization=AutoCB3$SigmaNavigatorNormalizer[GapConfiguration];  (* α.V_identity=1 *)
BC$objective=AutoCB3$Vector[CrossVec["identity"]];
BC$condition=AutoCB3$Condition[1,CrossVec["V$theta"],0,{0}]~Join~  (* α.V_θ>0 *)
AutoCB3$Condition[GapConfiguration];  (* the rest positivity conditions *)

sdp=SDPData[BC$objective,BC$normalization,BC$condition];

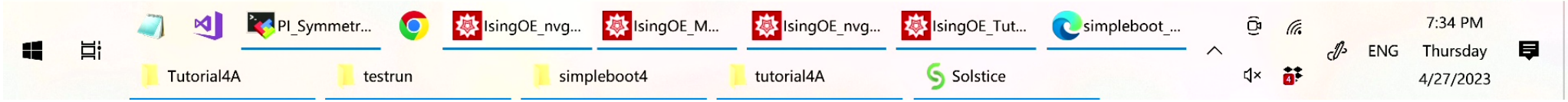
sdp
];
```

~Join~ : see the note

```
(* save the template as default template for this project *)
AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];
```

Generate SDP

120%



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```
GFFnormalization},

(*
BC$normalization=-AutoCB3$Vector[CrossVec[op[op,"E",1,1],2AutoCB3$CorOP2VarName["esp"],{√2,0},0]+
 CrossVec[op[op,"E",1,1],2AutoCB3$CorOP2VarName["sig"],{0,√2},0]+
 CrossVec[op[op,"O",1,1],AutoCB3$CorOP2VarName["sig"]+AutoCB3$CorOP2VarName["esp"],{1},0]];
*)

BC$normalization=AutoCB3$SigmaNavigatorNormalizer[GapConfiguration];  (* α.V_identity=1 *)
BC$objective=AutoCB3$Vector[CrossVec["identity"]];
BC$condition=AutoCB3$Condition[1,CrossVec["V$theta"],0,{0}]~Join~  (* α.V_θ>0 *)
AutoCB3$Condition[GapConfiguration];  (* the rest positivity conditions *)

sdp=SDPData[BC$objective,BC$normalization,BC$condition];

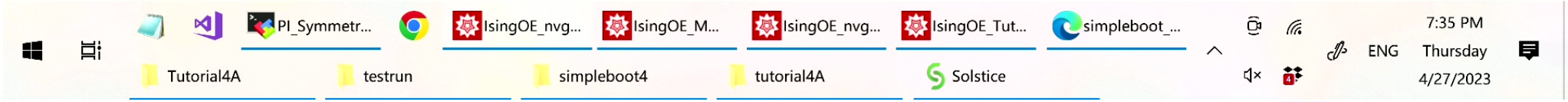
sdp
];
```

~Join~ : see the note

```
(* save the template as default template for this project *)
AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];
```

Generate SDP

120%



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```
sdp=SDPData[BC$objective,BC$normalization,BC$condition];

sdp
];
```

In[23]:=
```
2 AutoCB3$CorOP2VarName["eps"]
```

Out[23]=
```
2 ExtOp$eps
```

```
BC$normalization = -AutoCB3$Vector[CrossVec[op[op, "E", 1, 1], 2 AutoCB3$CorOP2VarName["esp"], {√2, 0}, 0] +
    CrossVec[op[op, "E", 1, 1], 2 AutoCB3$CorOP2VarName["sig"], {0, √2}, 0] +
    CrossVec[op[op, "O", 1, 1], AutoCB3$CorOP2VarName["sig"] + AutoCB3$CorOP2VarName["esp"], {1}, 0]];
```

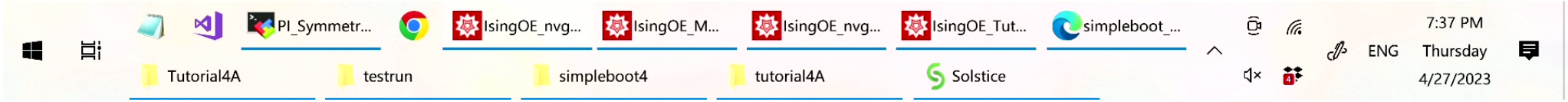
~Join~ : see the note

```
(* save the template as default template for this project *)
AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];
```

Generate SDP

This function generate a SDP. It will be used by the Delaunay search scanner.

130%



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Output:

SDP filename

load packages

In[17]:=
```
Get[ReconfigCmd@"[Local.PackageDirectory]/Indicator1.7.m"];
```

In[18]:=
```
(* For navigator runs, we use a modified SDPB based on SDPB2.4.0 *)
Cluster$SetConfig["[sdpb.script]",
"mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdpb2.4.0_midck --procsPerNode=$phys_cores_per_node "];

Cluster$SetConfig["[AutoCB3.sdp2input_mod.script]",
"mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdp2input_mod_2.4.0"];
```

In[20]:=
```
Cluster$SetConfig["[AutoCB3.FixPrefactor]",True];
```

Navigator run

130%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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paramFilename: SDP filename. Automatic : AutoCB3$GenerateSDP will choose a proper name automatically.

Output:
SDP filename

load packages

In[17]:=
```
Get[ReconfigCmd@"[Local.PackageDirectory]/Indicator1.7.m"];
```

In[18]:=
```
(* For navigator runs, we use a modified SDPB based on SDPB2.4.0 *)
Cluster$SetConfig["[sdpb.script]",
"mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdpb2.4.0_midck --procsPerNode=$phys_cores_per_node "];

Cluster$SetConfig["[AutoCB3.sdp2input_mod.script]",
"mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdp2input_mod_2.4.0"];
```

In[20]:=
```
Cluster$SetConfig["[AutoCB3.FixPrefactor]",True];
```

130%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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load packages

In[24]:=
```
Cluster$Configuration
```

Out[24]=
```
{[Cluster.LoginServer] → 10.10.21.98, [Cluster.Account] → nsu2,
 [Cluster.WorkspaceDirectory] → /gpfs/nsu2/PITutorial/Tutorial1A, [Cluster.PackageDirectory] → /gpfs/nsu2/simpleboot4_package,
 [Local.PackageDirectory] → C:\Users\shinn\OneDrive\Bootstrap_Projects\BootstrapperV2_Latest_Packages\simpleboot4,
 [AutoCB3.scalar_blocks_mod.script] →
  mpirun --bind-to none -n 1 /gpfs/nsu2/bootstrap_bin/scalar_blocks_mod --num-threads $phys_cores_per_node,
 [AutoCB3.sdp2input_mod.script] → mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdp2input_mod_2.4.0,
 [AutoCB3.sdpdd] → mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdpdd --procsPerNode=$phys_cores_per_node ,
 [sdpb.script] →
  mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdpb2.4.0_midck --procsPerNode=$phys_cores_per_node ,
 [DynamicSDPB.script] → mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/dynamical_sdp_RV1B,
 [ClusterModel] → srun, [SDPB.Version] → sdpb_El, [QUADRATICNET.SCRIPT] →
  /home/nsu2/packages/stack/stack-2.5.1-linux-x86_64/stack exec -- /home/nsu2/packages/quadratic-net-old/quadratic-net-exe
    --pvm2sdpExecutable /home/nsu2/packages/quadratic-net-old/quadratic-net-master/scripts/pvm2sdp.sh
    --sdpbExecutable /home/nsu2/packages/quadratic-net-old/quadratic-net-master/scripts/sdpb.sh --workDir tmp,
 [QUADRATICNET.SWITCH] → False, [UsePackage.scalar_block] → False, [UsePackage.sdp2input] → False,
 [Debug.TimingStatistics] → True, [UsePackage.AutoCB3] → True, [AutoCB3.no-fake-poles] → True,
 [AutoCB3.sdp2input_mod] → True, [AutoCB3.scalar_blocks_mod.cmdprec] → 120,
 [AutoCB3.scalar_blocks_mod.precision] → 1024, [AutoCB3.sdp2input_mod.precision] → 1024,
```

130%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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paramFilename: SDP filename. Automatic : AutoCB3$GenerateSDP will choose a proper name automatically.

Output:
SDP filename

load packages

In[17]:=
```
Get[ReconfigCmd@"[Local.PackageDirectory]/Indicator1.7.m"];
```

In[18]:=
```
(* For navigator runs, we use a modified SDPB based on SDPB2.4.0 *)
Cluster$SetConfig["[sdpb.script]",
"mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdpb2.4.0_midck --procsPerNode=$phys_cores_per_node "];
Cluster$SetConfig["[AutoCB3.sdp2input_mod.script]",
"mpirun -n $phys_cores_per_node /gpfs/nsu2/bootstrap_bin/sdp2input_mod_2.4.0"];
```

In[20]:=
```
Cluster$SetConfig["[AutoCB3.FixPrefactor]",True];
```

130%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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Navigator run

In[21]:=

```
BFGSoptions={
"Algorithm"→ConstrBFGS, (* MyBFGS, or ConstrBFGS . Default : MyBFGS *)
"BoundingBox"→SetPrec@{
{0.515,0.523},
{1.38,1.45}
}, (* Default : Automatic *)
"FeasibilityQ"→False,

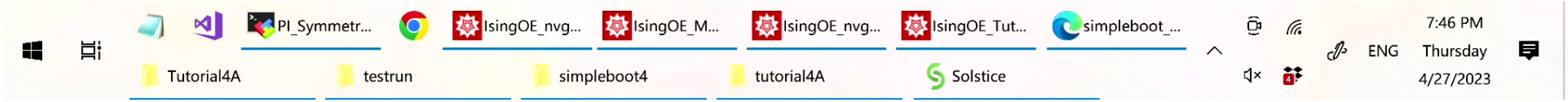
"PrecisionGoal.g"→10^-20, (* Default : 10^-10 *)
"PrecisionGoal.s"→10^-20, (* Default : 10^-10 *)
"PrecisionGoal.f"→10^-20, (* Default : 10^-10 *)
"InitInvHessian"→Automatic[0.02], (* Default : Automatic *)

"ConstrBFGS.OptimizeDirection"→ {1,"max"}, (* {1,"max"} or {2,"min"}. Default: {1,"max"} *)

"ConstrBFGS.InfeasibilityTolerence"→{"Relative",2}, (* {"Absolute",10^-20} or {"Relative",2}. Default: {"Relative",2} *)
"LineSearch.SafeInterval"→{1/30, 9/10}, (* Default : {1/10, 9/10} *)

"SDPD.Derivative.Infinitesimal"→10^-20   (* Default : 10^-20 *)
```

130%



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```
        {0.515,0.523},
        {1.38,1.45}
        }, (* Default : Automatic *)
        "FeasibilityQ"→False,

        "PrecisionGoal.g"→10^-20, (* Default : 10^-10 *)
        "PrecisionGoal.s"→10^-20, (* Default : 10^-10 *)
        "PrecisionGoal.f"→10^-20, (* Default : 10^-10 *)
        "InitInvHessian"→Automatic[0.02], (* Default : Automatic *)

        "ConstrBFGS.OptimizeDirection"→ {1,"max"}, (* {1,"max"} or {2,"min"}. Default: {1,"max"} *)

        "ConstrBFGS.InfeasibilityTolerence"→{"Relative",2}, (* {"Absolute",10^-20} or {"Relative",2}. Default: {"Relative",2} *)
        "LineSearch.SafeInterval"→{1/30, 9/10}, (* Default : {1/10, 9/10} *)

        "SDPD.Derivative.Infinitesimal"→10^-20   (* Default : 10^-20 *)
        };

    SSH$UploadCurrentNotebook[] (* save and upload current notebook as package (.m) to the cluster *)
```

130%

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```
        {0.515,0.523},
        {1.38,1.45}
        }, (* Default : Automatic *)
        "FeasibilityQ"→False,

        "PrecisionGoal.g"→10^-20, (* Default : 10^-10 *)
        "PrecisionGoal.s"→10^-20, (* Default : 10^-10 *)
        "PrecisionGoal.f"→10^-20, (* Default : 10^-10 *)
        "InitInvHessian"→Automatic[0.02], (* Default : Automatic *)

        "ConstrBFGS.OptimizeDirection"→ {1,"max"}, (* {1,"max"} or {2,"min"}. Default: {1,"max"} *)

        "ConstrBFGS.InfeasibilityTolerence"→{"Relative",2}, (* {"Absolute",10^-20} or {"Relative",2}. Default: {"Relative",2} *)
        "LineSearch.SafeInterval"→{1/30, 9/10}, (* Default : {1/10, 9/10} *)

        "SDPD.Derivative.Infinitesimal"→10^-20   (* Default : 10^-20 *)
        };

      SSH$UploadCurrentNotebook[] (* save and upload current notebook as package (.m) to the cluster *)
```

130%

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```
ClusterAsyn$Evaluate[
 AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];

 initpt = {0.52, 1.44} // SetPrec;

 SB$NavigatorRun[BFGSoptions, SDP$IsingOE, initpt,
  "--saveMiddleCheckpointMuThreshold=1e-8 --maxIterations=1000 --dualityGapThreshold=1e-25 --primalErrorThreshold=1e-15
    --dualErrorThreshold=1e-15 --precision=765 --initialMatrixScalePrimal=1e+20 --initialMatrixScaleDual=1e+20
    --maxComplementarity=1e+70  --writeSolution=\"x,y,X,Y\"", 1000, False]
]
```

```
408976
```

```
ClusterAsyn$JobOutput["408976"]
(* Check the job output. Input : job ID *)
ClusterAsyn$JobOutput["408581"]
SB$LoadProject[];
ObjGet[SB$Proj$calculated, 2, "LastCheckpoint"]
```

```
0.520030122632_1.43997505299_Apr21_18h13m42s.sdp
```

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65	33	1.5e+25	1.35e+28	-0.0105	1.00	+1.43e-211	+2.81e-185	+0.305	0.538	0.543	0.300
66	33	9.3e+24	9.40e+27	-0.00998	1.00	+1.31e-211	+4.62e-185	+0.140	0.591	0.538	0.300
67	34	5.6e+24	6.00e+27	-0.00961	1.00	+1.31e-211	+3.20e-185	+0.0645	0.561	0.501	0.300
68	34	3.5e+24	3.88e+27	-0.00935	1.00	+1.43e-211	+8.92e-185	+0.0322	0.549	0.484	0.300
69	35	2.2e+24	2.50e+27	-0.00914	1.00	+1.31e-211	+4.07e-185	+0.0166	0.536	0.531	0.300
70	35	1.4e+24	1.63e+27	-0.00894	1.00	+1.31e-211	+5.38e-185	+0.00778	0.549	0.647	0.300
71	36	8.0e+23	1.03e+27	-0.00880	1.00	+1.19e-211	+8.78e-185	+0.00274	0.829	0.646	0.300
72	36	3.5e+23	4.39e+26	-0.00875	1.00	+1.31e-211	+3.41e-183	+0.000972	0.866	0.655	0.300
73	37	1.5e+23	1.79e+26	-0.00875	1.00	+1.43e-211	+4.16e-183	+0.000335	0.949	0.661	0.300
74	37	5.7e+22	6.52e+25	-0.00874	1.00	+1.31e-211	+6.35e-183	+0.000114	0.991	0.666	0.300
75	38	2.2e+22	2.34e+25	-0.00874	1.00	+1.54e-211	+6.66e-183	+3.80e-05	1.00	0.670	0.300
76	38	8.4e+21	8.78e+24	-0.00875	1.00	+1.31e-211	+1.73e-183	+1.25e-05	0.992	0.672	0.300
77	39	3.4e+21	3.46e+24	-0.00876	1.00	+1.78e-211	+9.98e-184	+4.10e-06	0.997	0.673	0.300
78	39	1.3e+21	1.37e+24	-0.00878	1.00	+1.66e-211	+1.31e-183	+1.34e-06	1.00	0.674	0.300
79	40	5.3e+20	5.48e+23	-0.00879	1.00	+1.54e-211	+2.40e-183	+4.38e-07	1.00	0.674	0.300
80	40	2.1e+20	2.20e+23	-0.00881	1.00	+1.43e-211	+1.53e-183	+1.43e-07	0.985	0.675	0.300
81	41	8.6e+19	8.99e+22	-0.00881	1.00	+1.54e-211	+1.25e-183	+4.64e-08	0.989	0.675	0.300
82	41	3.4e+19	3.63e+22	-0.00880	1.00	+1.31e-211	+7.90e-185	+1.51e-08	0.978	0.676	0.300
83	42	1.4e+19	1.50e+22	-0.00876	1.00	+1.54e-211	+1.01e-184	+4.88e-09	0.966	0.677	0.300
84	42	5.5e+18	6.25e+21	-0.00870	1.00	+1.31e-211	+3.10e-184	+1.58e-09	0.951	0.677	0.300
85	43	2.2e+18	2.63e+21	-0.00864	1.00	+1.66e-211	+6.52e-184	+5.09e-10	0.945	0.675	0.300
86	43	8.8e+17	1.10e+21	-0.00857	1.00	+1.31e-211	+2.70e-184	+1.65e-10	0.894	0.672	0.300

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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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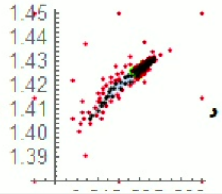
28	{0.520869794409, 1.43201042929}	$-6.43574682959\times10^{-11}$	$\{0.000014148, -1.06535\times10^{-6}\}$
29	{0.520892152519, 1.43231830004}	$-1.75922323397\times10^{-11}$	$\{8.4288\times10^{-6}, -2.88681\times10^{-7}\}$
30	{0.520896156846, 1.43240052676}	$-1.08776345829\times10^{-12}$	$\{6.61742\times10^{-6}, -4.18259\times10^{-8}\}$
31	{0.520896358818, 1.43241093095}	$-4.17849119208\times10^{-15}$	$\{6.31751\times10^{-6}, -1.07564\times10^{-9}\}$
32	{0.520896359500, 1.43241117127}	$-1.07015377365\times10^{-18}$	$\{6.30963\times10^{-6}, -9.14048\times10^{-12}\}$
33	{0.520896359501, 1.43241117329}	$-2.10770894055\times10^{-24}$	$\{6.30956\times10^{-6}, -2.05805\times10^{-15}\}$
34	{0.520896359501, 1.43241117329}	$-1.64652840466\times10^{-26}$	$\{6.30956\times10^{-6}, 1.04104\times10^{-19}\}$
35	{0.520896359501, 1.43241117329}	$-1.36213138885\times10^{-28}$	$\{6.30956\times10^{-6}, 7.18911\times10^{-21}\}$

```
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 1, "LastCheckpoint"]

(* automatic stall recovery and save middle checkpoint *)
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 3, "LastCheckpoint"]

ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 10, "LastCheckpoint"]

ListPlot[IndicatorHistory[[All, 1]], Joined → True, PlotMarkers → {Automatic, Tiny}, AspectRatio → 1,
  PlotStyle → {Lighter@Red}] //
```




```
Show[#, , ListPlot[IndicatorHistory[[-1 ;; -1, 1]], PlotMarkers → {⋆, Scaled[0.05]}, AspectRatio → 1,
```

130%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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8:01 PM ENG Thursday 4/27/2023

IsingOE_nvg.nb * - Wolfram Mathematica 11.1

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26	{0.520772395715, 1.43095692221}	$1.92472196496 \times 10^{-10}$	$\{0.0000432394, -4.96977 \times 10^{-6}\}$
27	{0.520813488151, 1.43144820702}	$-1.83905667917 \times 10^{-10}$	$\{0.0000219229, -2.13631 \times 10^{-6}\}$
28	{0.520869794409, 1.43201042929}	$-6.43574682959 \times 10^{-11}$	$\{0.000014148, -1.06535 \times 10^{-6}\}$
29	{0.520892152519, 1.43231830004}	$-1.75922323397 \times 10^{-11}$	$\{8.4288 \times 10^{-6}, -2.88681 \times 10^{-7}\}$
30	{0.520896156846, 1.43240052676}	$-1.08776345829 \times 10^{-12}$	$\{6.61742 \times 10^{-6}, -4.18259 \times 10^{-8}\}$
31	{0.520896358818, 1.43241093095}	$-4.17849119208 \times 10^{-15}$	$\{6.31751 \times 10^{-6}, -1.07564 \times 10^{-9}\}$
32	{0.520896359500, 1.43241117127}	$-1.07015377365 \times 10^{-18}$	$\{6.30963 \times 10^{-6}, -9.14048 \times 10^{-12}\}$
33	{0.520896359501, 1.43241117329}	$-2.10770894055 \times 10^{-24}$	$\{6.30956 \times 10^{-6}, -2.05805 \times 10^{-15}\}$
34	{0.520896359501, 1.43241117329}	$-1.64652840466 \times 10^{-26}$	$\{6.30956 \times 10^{-6}, 1.04104 \times 10^{-19}\}$
35	{0.520896359501, 1.43241117329}	$-1.36213138885 \times 10^{-28}$	$\{6.30956 \times 10^{-6}, 7.18911 \times 10^{-21}\}$

```
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 1, "LastCheckpoint"]
(* automatic stall recovery and save middle checkpoint *)
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 3, "LastCheckpoint"]
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 10, "LastCheckpoint"]
ListPlot[IndicatorHistory[[All, 1]], Joined → True, PlotMarkers → {Automatic, Tiny}, AspectRatio → 1,
  PlotStyle → {Lighter@Red}] //
```

130%

IsingOE_nvg.nb * - Wolfram Mathematica 11.1

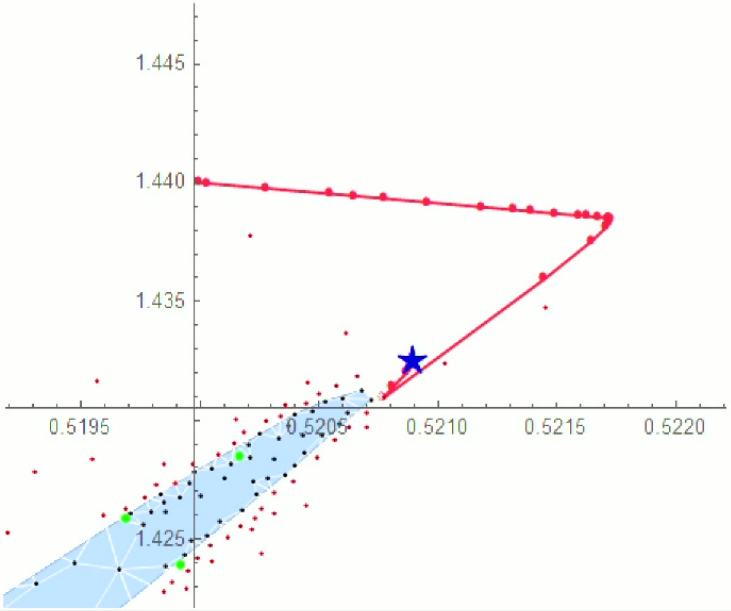
File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

```
 PlotStyle → {Blue}],
PlotRange → Transpose@{{0.5192419365518848, 1.418705864820847}, {0.5221499751744999, 1.446028503257329}}] &
```

N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.

N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.

PlotMarkers: Invalid size Scaled[0.05]. A PlotMarker must be of the form {marker, size}.



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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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ENG

IsingOE_nvg.nb * - Wolfram Mathematica 11.1

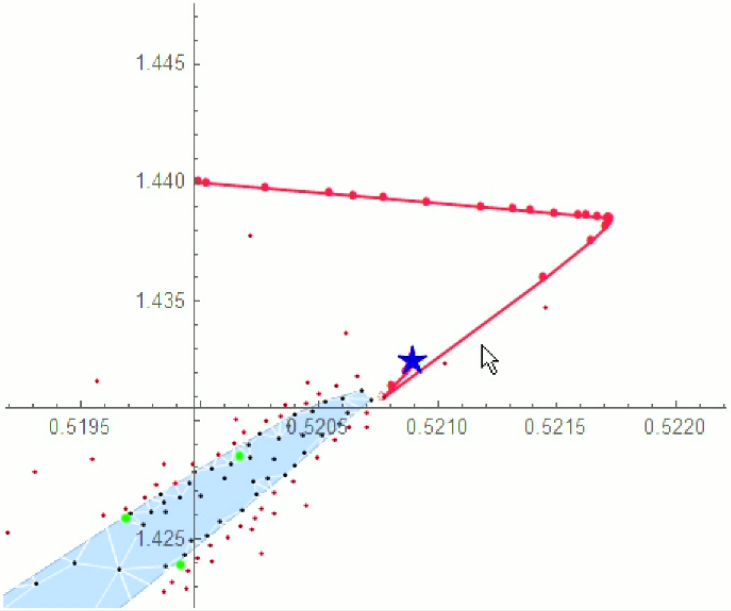
File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

```
 PlotStyle → {Blue}],
PlotRange → Transpose@{{0.5192419365518848, 1.418705864820847}, {0.5221499751744999, 1.446028503257329}}] &
```

N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.

N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.

PlotMarkers: Invalid size Scaled[0.05]. A PlotMarker must be of the form {marker, size}.



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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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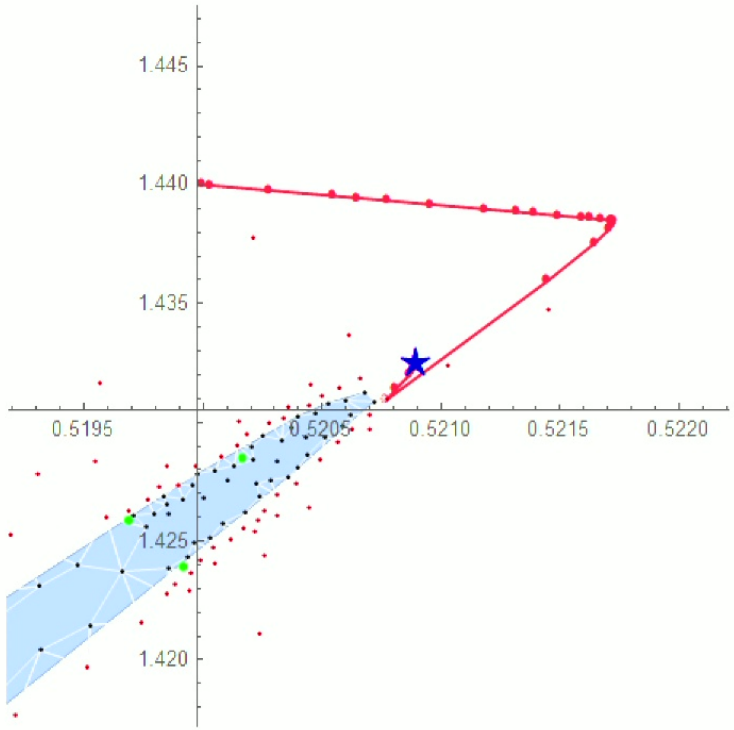
8:06 PM Thursday 4/27/2023 ENG

IsingOE_nvg.nb * - Wolfram Mathematica 11.1

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

PlotMarkers: Invalid size Scaled[0.05]. A PlotMarker must be of the form {marker, size}.



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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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ENG

IsingOE_nvg.nb * - Wolfram Mathematica 11.1

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```
  "PrecisionGoal.g"→10^-20, (* Default : 10^-10 *)
  "PrecisionGoal.s"→10^-20, (* Default : 10^-10 *)
  "PrecisionGoal.f"→10^-20, (* Default : 10^-10 *)
  "InitInvHessian"→Automatic[0.02], (* Default : Automatic *)

  "ConstrBFGS.OptimizeDirection"→ {1,"max"}, (* {1,"max"} or {2,"min"}. Default: {1,"max"} *)

  "ConstrBFGS.InfeasibilityTolerence"→{"Relative",2}, (* {"Absolute",10^-20} or {"Relative",2}. Default: {"Relative",2} *)
  "LineSearch.SafeInterval"→{1/30, 9/10}, (* Default : {1/10, 9/10} *)

  "SDPD.Derivative.Infinitesimal"→10^-20   (* Default : 10^-20 *)
  };

SSH$UploadCurrentNotebook[] (* save and upload current notebook as package (.m) to the cluster *)


ClusterAsyn$Evaluate[
 AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];

 initpt = {0.52, 1.44} // SetPrec;
```

130%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... simpleboot_...

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IsingOE_nvg.nb * - Wolfram Mathematica 11.1

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```
    --dualErrorThreshold=1e-15 --precision=765 --initialMatrixScalePrimal=1e+20 --initialMatrixScaleDual=1e+20
    --maxComplementarity=1e+70  --writeSolution=\"x,y,X,Y\"", 1000, False]
]

408976

ClusterAsyn$JobOutput["408976"]

(* Check the job output. Input : job ID *)
ClusterAsyn$JobOutput["408581"]

SB$LoadProject[];
ObjGet[SB$Proj$calculated, 2, "LastCheckpoint"]

0.520030122632_1.43997505299_Apr21_18h13m42s.sdp

(* Check the SDPB output. Input : SDP file name *)
ClusterAsyn$SDPBOutput["0.520000000000_1.44000000000_Apr21_18h11m18s.sdp"]

ClusterAsyn$SDPBOutput["0.520030122632_1.43997505299_Apr21_18h13m42s.sdp"]

SSH$DownloadFile@SB$Proj$FileName; (* download current project data from the cluster *)
SB$LoadProject[]; (* load current project data *)


SB$RawPlot[]
```

1.440

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IsingOE_nvg.nb - Wolfram Mathematica 11.1

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```
spectrumdata // AutoCB3$PrintSpectrum[#, True] &
```

V$theta	L=0	{0.000[0.3666,4.620]}
op[op, 0, 1, 1]	L=0	{3.000[0.2846], 5.856[0.04586]}
op[op, E, 1, 1]	L=0	{3.000[0.002278,2.848], 3.977[0.04829,1.505], 9.380[0.0003717,8.465×10^7]}
op[op, E, 1, 1]	L=2	{11.000[1061.,3114.], 11.092[-44.16,4281.], 12.456[6.139,876.1], 13.071[12.93,176.4], 14.010[2.162,393.6], 18.142[0.04335,0.00009873]}
op[op, E, 1, 1]	L=4	{13.000[-0.001371,1106.], 13.031[239.3,1273.], 13.895[1.010, 0.0001704], 15.551[-2.050,1152.], 16.253[-3.459,1534.], 17.294[-16.50,7245.]}
op[op, E, 1, 1]	L=6	{15.000[-8.957,5227.], 15.025[106.6,0.3243], 16.900[-32.08,14090.], 17.405[0.8320,0.001895], 19.966[-141.9,62320.]}
op[op, E, 1, 1]	L=8	{17.058[-27.81,12780.], 19.183[0.8362,0.001904], 22.923[-1071.,470300.]}
op[op, E, 1, 1]	L=10	{19.004[-435.1,191100.], 19.493[2.357,0.005367]}
op[op, E, 1, 1]	L=12	{21.000[11.34,0.02582]}
op[op, E, 1, 1]	L=14	{23.000[6.224,0.01417]}
op[op, E, 1, 1]	L=16	{}
op[op, E, 1, 1]	L=18	{}
op[op, E, 1, 1]	L=20	{}
op[op, E, 1, 1]	L=52	{}
op[op, 0, 1, -1]	L=1	{10.000[2.022], 18.924[0.2192]}

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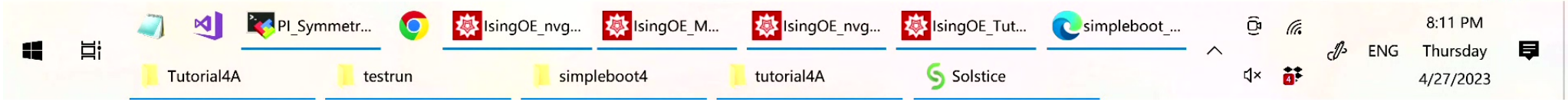

lecture1.nb - Wolfram Mathematica 11.1

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Text

More complicated CFTs : need more correlators

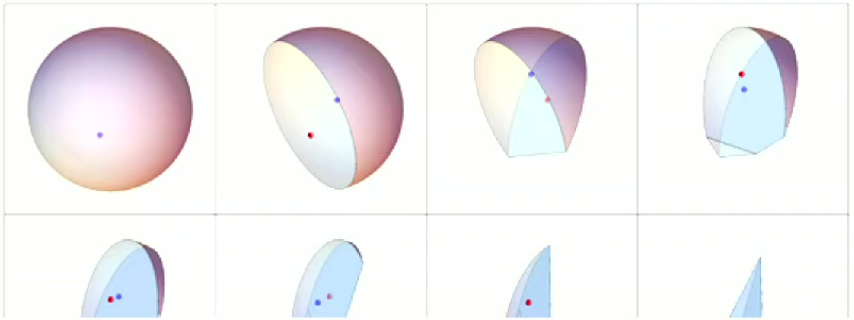
More powerful computational method ⟶ explore bigger set of constraints ⟶ access to more CFTs and data

Slide 14 of 39

Cutting surface algorithm

Scan $\left\{ \frac{\lambda_{\phi\phi t}}{\lambda_{\phi\phi s}}, \frac{\lambda_{ttt}}{\lambda_{\phi\phi s}}, \frac{\lambda_{sss}}{\lambda_{\phi\phi s}} \right\}$: cutting surface algorithm : cost linear to number of OPE coeff

⟶ success in O(2), O(3), GNY



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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

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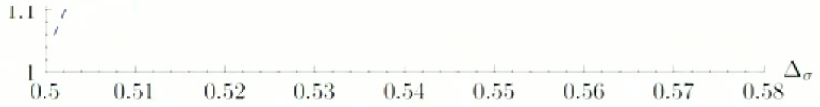
Text



Reference : Weizmann Lectures on the Numerical Conformal Bootstrap. Shai M. Chester

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Bootstrap equations for scalars

Consider four different scalar $\phi_1,\ \phi_2,\ \phi_3,\ \phi_4$, the conformal partial wave decomposition is

$$\langle \overline{\phi_1(x_1)\,\phi_2}\,(x_2)\,\overline{\phi_3(x_3)\,\phi_4}\,(x_4)\rangle = \frac{1}{x_{12}^{\Delta_1-\Delta_2}\,x_{34}^{\Delta_3-\Delta_4}}\left(\frac{x_{24}}{x_{14}}\right)^{\Delta_{12}}\left(\frac{x_{14}}{x_{13}}\right)^{\Delta_{34}}\sum_{\mathcal{O}}\lambda_{ij\mathcal{O}}\,\lambda_{kl\mathcal{O}}\,g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(u,\,v)\ ,\quad \Delta_{ij}=\Delta_i-\Delta_j\ .$$

Crossing : $\langle \overline{\phi_1\,\phi_2}\,\overline{\phi_3\,\phi_4}\rangle_{u,v} - \langle \overline{\phi_3\,\phi_2}\,\overline{\phi_1\,\phi_4}\rangle_{u,v} = 0$. Under $1 \leftrightarrow 3$ exchange, $u \leftrightarrow v$, $z \leftrightarrow 1-z,\ \bar{z} \leftrightarrow 1-\bar{z}$

$$\sum_{\mathcal{O}}\lambda_{12\mathcal{O}}\,\lambda_{34\mathcal{O}}\,F_{\pm,\Delta,\ell}^{12,\,34} \mp \lambda_{32\mathcal{O}}\,\lambda_{14\mathcal{O}}\,F_{\pm,\Delta,\ell}^{32,\,14} = 0 \text{ where } F_{\pm,\mathcal{O}}^{12;34}(u,\,v) = v^{\frac{\Delta_2-\Delta_3}{2}}\,g_{\mathcal{O}}^{\Delta_{12},\Delta_{34}}(u,\,v) \pm u^{\frac{\Delta_2-\Delta_3}{2}}\,g_{\mathcal{O}}^{\Delta_{12},\Delta_{34}}(v,\,u)$$

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Text

SDPB in optimality mode : bound OPE coefficient

$$\sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\phi\phi\mathcal{O}}{}^2 F_{\Delta,\ell}(u,v) = 0 \quad \longrightarrow \quad \lambda_{\mathcal{O}_0}{}^2 F_{\Delta_0,\ell_0}(u,v) = -F_{0,0}(u,v) - \sum_{\mathcal{O}} \lambda_{\mathcal{O}}{}^2 F_{\Delta,\ell}(u,v)$$

Find a linear functional α such that

$\alpha\left(F_{\Delta_0,\ell_0}(u,v)\right) = 1$ (1)

$\alpha\left(F_{\Delta,\ell}(u,v)\right) \geq 0$ for $\Delta > \Delta_{\text{min}}$ (2)

If a α exist, we find an inequality

$$\lambda_{\mathcal{O}_0}{}^2 = -\alpha\left(F_{0,0}(u,v)\right) - \sum_{\mathcal{O}} \lambda_{\mathcal{O}}{}^2\, \alpha\left(F_{\Delta,\ell}(u,v)\right) \leq -\alpha\left(F_{0,0}(u,v)\right)$$

To the most restrictive bound, we need α that minimizing $-\alpha\left(F_{0,0}(u,v)\right)$ subject to (1), (2)

Such that α should satisfy $\alpha\left(F_{\Delta,\ell}(u,v)\right) = 0$ for Δ, ℓ of each $\mathcal{O}$ in the $\sum_{\mathcal{O}\in\phi\times\phi}$ ...

Zeros in $\alpha\left(F_{\Delta,\ell}(u,v)\right) \longrightarrow$ physical spectrum Δ (**Extremal Functional Method**, EFM)

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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

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IsingOE_nvg.nb - Wolfram Mathematica 11.1

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```
spectrumdata // AutoCB3$PrintSpectrum[#, True] &
```

Operator	L	Spectrum
V$theta	L=0	$\{0.000^{\{0.3666,4.620\}}\}$
op[op, 0, 1, 1]	L=0	$\{3.000^{\{0.2846\}}, 5.856^{\{0.04586\}}\}$
op[op, E, 1, 1]	L=0	$\{3.000^{\{0.002278,2.848\}}, 3.977^{\{0.04829,1.505\}}, 9.380^{\{0.0003717,8.465\times10^{-7}\}}\}$
op[op, E, 1, 1]	L=2	$\{11.000^{\{1061.,3114.\}}, 11.092^{\{-44.16,4281.\}}, 12.456^{\{6.139,876.1\}}, 13.071^{\{12.93,176.4\}}, 14.010^{\{2.162,393.6\}}, 18.142^{\{0.04335,0.00009873\}}\}$
op[op, E, 1, 1]	L=4	$\{13.000^{\{-0.001371,1106.\}}, 13.031^{\{239.3,1273.\}}, 13.895^{\{1.010,-0.0001704\}}, 15.551^{\{-2.050,1152.\}}, 16.253^{\{-3.459,1534.\}}, 17.294^{\{-16.50,7245.\}}\}$
op[op, E, 1, 1]	L=6	$\{15.000^{\{-8.957,5227.\}}, 15.025^{\{106.6,0.3243\}}, 16.900^{\{-32.08,14090.\}}, 17.405^{\{0.8320,0.001895\}}, 19.966^{\{-141.9,62320.\}}\}$
op[op, E, 1, 1]	L=8	$\{17.058^{\{-27.81,12780.\}}, 19.183^{\{0.8362,0.001904\}}, 22.923^{\{-1071.,470300.\}}\}$
op[op, E, 1, 1]	L=10	$\{19.004^{\{-435.1,191100.\}}, 19.493^{\{2.357,0.005367\}}\}$
op[op, E, 1, 1]	L=12	$\{21.000^{\{11.34,0.02582\}}\}$
op[op, E, 1, 1]	L=14	$\{23.000^{\{6.224,0.01417\}}\}$
op[op, E, 1, 1]	L=16	{}
op[op, E, 1, 1]	L=18	{}
op[op, E, 1, 1]	L=20	{}
op[op, E, 1, 1]	L=52	{}
op[op, 0, 1, -1]	L=1	$\{10.000^{\{2.022\}}, 18.924^{\{0.2192\}}\}$

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IsingOE_MMA.nb - Wolfram Mathematica 11.1

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```
Cluster$SetConfig["[Cluster.ProjectDirectory]","Proj_IsingOE_MMA"];  (* folder for the current project (inside the workspace) *)
CheckDirectory[Cluster$GetConfig["[Cluster.ProjectDirectory]"]];  (* create the folder if it doesn't exist *)
```

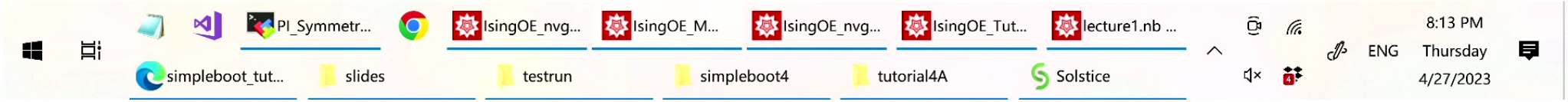
Input : bootstrap condition

$$V_{\text{even}} = \begin{pmatrix} \begin{pmatrix} F^{11,11}_{-,\Delta,\ell} & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & F^{22,22}_{-,\Delta,\ell} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2} F^{11,22}_{-,\Delta,\ell} \\ \frac{1}{2} F^{11,22}_{-,\Delta,\ell} & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2} F^{11,22}_{+,\Delta,\ell} \\ \frac{1}{2} F^{11,22}_{+,\Delta,\ell} & 0 \end{pmatrix} \end{pmatrix}, \qquad V_{\text{odd}} = \begin{pmatrix} 0 \\ 0 \\ F^{12,12}_{-,\Delta,\ell} \\ (-1)^{\ell} F^{21,12}_{-,\Delta,\ell} \\ -(-1)^{\ell} F^{21,12}_{-,\Delta,\ell} \end{pmatrix}$$

$$V_{\text{identity}} = (1 \quad 1) \cdot V_{\text{even}}(\Delta = 0,\ \ell = 0) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$V_{\theta} = V_{\text{even}}(\Delta = \Delta_{\epsilon},\ \ell = 0) + V_{\text{odd}}(\Delta = \Delta_{\sigma},\ \ell = 0) \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

120%




IsingOE_Tutorial1A_Exercise4.nb - Wolfram Mathematica 11.1
File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
SDP$IsingOE[point_List,filename_:Automatic]:=Module[
{Δσ,Δε},
{Δσ,Δε}=SetPrec[point];
AutoCB3$GenerateSDP[{Δσ,Δε},{},filename]
];
AutoCB3$GenerateSDP[externalOperatorList, userDefinedVariables, paramFilename]
AutoCB3$GenerateSDP call C++ programs to generate blocks (if the block data is not generated) and generate an actual SDP based on the template and user-defined variables
Input:
externalOperatorList: the values of the external operators. The order should match with "VBlock$External"
userDefinedVariables: user-defined variable in the template. Format : {"variable1"→value1, "variable2"→value2, ...}
paramFilename: SDP filename. Automatic : AutoCB3$GenerateSDP will choose a proper name automatically.
Output:
SDP filename
Delaunay Search
SSH$UploadCurrentNotebook[] (* save and upload current notebook as package (.m) to the cluster *)
initpts = GeneratePointsInRectangular[{0.53, 0.60}, {1.1, 2.2}, 3, 3] // SetPrec;
initpts // ListPlot
... N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.
F[sig,sig,eps,eps]
0
100%
120%
PI_Symmetr...
IsingOE_nvg...
IsingOE_M...
IsingOE_nvg...
IsingOE_Tut...
lecture1.nb ...
simpleboot_tut...
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IsingOE_Tutorial1A_Exercise4.nb - Wolfram Mathematica 11.1

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```
Table[
 ClusterAsyn$Evaluate[
  AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];

  initpts = GeneratePointsInRectangular[{0.53, 0.60}, {1.1, 2.2}, 3, 3] // SetPrec;

  SB$FeasibilityScanner[
   SDP$IsingOE, (* function that generates a SDP *)
   initpts, (* initial points to scan *)
   "--maxIterations=1000 --dualityGapThreshold=1e-50 --primalErrorThreshold=1e-60 --dualErrorThreshold=1e-10 --precision=765
     --initialMatrixScalePrimal=1e+20 --initialMatrixScaleDual=1e+20 --maxComplementarity=1e+70 --findPrimalFeasible --findDualFeasible
     --detectPrimalFeasibleJump --detectDualFeasibleJump ", (* SDPB parameters *)

    {"Delaunay"}, (* scan method *)
    200, (* maximal points to scan *)
    False (* initial checkpoint. False means no initial checkpoint. *)
  ]
 ]
 , 2]
{408482, 408483}

ClusterAsyn$JobOutput["408478"] (* check job output *)
SSH$DownloadFile@SB$Proj$FileName; (* download current project data from the cluster *)
SB$LoadProject[]; (* load current project data *)

ClusterAsyn$SDPBOutput["0.530000000000_2.19989000000_Apr19_19h45m13s.sdp"]
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated$primal, 1, "LastCheckpoint"]
```

100%

120%

||1 F[sig,sig,eps,eps] 0 ||

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

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IsingOE_Tutorial1A_Exercise4.nb - Wolfram Mathematica 11.1

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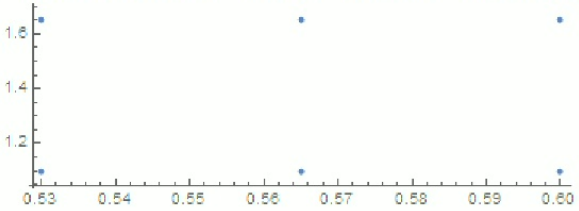


```
(* turn of the hotstart *)
ThetaSampler$ΔScanner$SingleRun$FindHotStartData[Δlist_List]:={False,False};

Table[
 ClusterAsyn$Evaluate[
  AutoCB3$SaveSDPTemplate@SDPTemplate$IsingOE[];

  initpts = GeneratePointsInRectangular[{0.53, 0.60}, {1.1, 2.2}, 3, 3] // SetPrec;

  SB$FeasibilityScanner[
   SDP$IsingOE, (* function that generates a SDP *)
   initpts, (* initial points to scan *)
   "--maxIterations=1000 --dualityGapThreshold=1e-50 --primalErrorThreshold=1e-60 --dualErrorThreshold=1e-10 --precision=765
     --initialMatrixScalePrimal=1e+20 --initialMatrixScaleDual=1e+20 --maxComplementarity=1e+70 --findPrimalFeasible --findDualFeasible
     --detectPrimalFeasibleJump --detectDualFeasibleJump ", (* SDPB parameters *)

    {"Delaunay"}, (* scan method *)
    200, (* maximal points to scan *)
    False (* initial checkpoint. False means no initial checkpoint. *)
   ]
  ]
  , 2]
```

100%

IsingOE_Tutorial1A_Exercise4.nb - Wolfram Mathematica 11.1

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149	98	0.18	+1.27e-44	+0.00	1.27e-44	+1.84e-46	+7.36e-35	+0.000874	0.477	0.435	0.300
150	99	0.13	+6.65e-45	+0.00	6.65e-45	+9.63e-47	+3.85e-35	+0.000494	0.477	0.435	0.300
151	99	0.087	+3.48e-45	+0.00	3.48e-45	+5.03e-47	+2.01e-35	+0.000279	0.477	0.435	0.300
152	100	0.061	+1.82e-45	+0.00	1.82e-45	+2.63e-47	+1.05e-35	+0.000158	0.478	0.434	0.300
153	101	0.042	+9.49e-46	+0.00	9.49e-46	+1.37e-47	+5.49e-36	+8.93e-05	0.478	0.434	0.300
154	101	0.029	+4.96e-46	+0.00	4.96e-46	+7.17e-48	+2.87e-36	+5.06e-05	0.478	0.434	0.300
155	102	0.020	+2.59e-46	+0.00	2.59e-46	+3.75e-48	+1.50e-36	+2.86e-05	0.478	0.434	0.300
156	102	0.014	+1.35e-46	+0.00	1.35e-46	+1.96e-48	+7.82e-37	+1.62e-05	0.478	0.433	0.300
157	103	0.0098	+7.07e-47	+0.00	7.07e-47	+1.02e-48	+4.08e-37	+9.18e-06	0.478	0.433	0.300
158	104	0.0068	+3.69e-47	+0.00	3.69e-47	+5.34e-49	+2.13e-37	+5.21e-06	0.478	0.433	0.300
159	104	0.0047	+1.93e-47	+0.00	1.93e-47	+2.79e-49	+1.11e-37	+2.95e-06	0.478	0.433	0.300
160	105	0.0033	+1.01e-47	+0.00	1.01e-47	+1.45e-49	+5.81e-38	+1.67e-06	0.478	0.433	0.300
161	106	0.0023	+5.25e-48	+0.00	5.25e-48	+7.59e-50	+3.03e-38	+9.50e-07	0.478	0.432	0.300
162	106	0.0016	+2.74e-48	+0.00	2.74e-48	+3.96e-50	+1.58e-38	+5.39e-07	0.478	0.432	0.300
163	107	0.0011	+1.43e-48	+0.00	1.43e-48	+2.07e-50	+8.26e-39	+3.06e-07	0.478	0.432	0.300
164	108	0.00076	+7.46e-49	+0.00	7.46e-49	+1.08e-50	+4.31e-39	+1.74e-07	0.478	0.432	0.300
165	108	0.00053	+3.89e-49	+0.00	3.89e-49	+5.63e-51	+2.25e-39	+9.88e-08	0.478	0.432	0.300
166	109	0.00037	+2.03e-49	+0.00	2.03e-49	+2.93e-51	+1.17e-39	+5.61e-08	0.478	0.431	0.300
167	110	0.00026	+1.06e-49	+0.00	1.06e-49	+1.53e-51	+6.12e-40	+3.19e-08	0.479	0.431	0.300
168	110	0.00018	+5.52e-50	+0.00	5.52e-50	+7.98e-52	+3.19e-40	+1.82e-08	0.479	0.431	0.300
169	111	0.00012	+2.88e-50	+0.00	2.88e-50	+4.16e-52	+1.66e-40	+1.03e-08	0.479	0.431	0.300
170	112	8.6e-05	+1.50e-50	+0.00	1.50e-50	+2.17e-52	+8.67e-41	+5.88e-09	0.479	0.431	0.300
171	112	6.0e-05	+7.83e-51	+0.00	7.83e-51	+1.13e-52	+4.52e-41	+3.35e-09	0.479	0.431	0.300
172	113	4.2e-05	+4.08e-51	+0.00	4.08e-51	+5.90e-53	+2.36e-41	+1.91e-09	0.479	0.430	0.300
173	114	2.9e-05	+2.13e-51	+0.00	2.13e-51	+3.07e-53	+1.23e-41	+1.09e-09	0.479	0.430	0.300
174	114	2.0e-05	+1.11e-51	+0.00	1.11e-51	+1.60e-53	+6.40e-42	+6.19e-10	0.479	0.430	0.300
175	115	1.4e-05	+5.77e-52	+0.00	5.77e-52	+8.34e-54	+3.33e-42	+3.53e-10	0.479	0.430	0.300
176	116	9.8e-06	+3.01e-52	+0.00	3.01e-52	+4.35e-54	+1.74e-42	+2.01e-10	0.479	0.430	0.300
177	116	6.8e-06	+1.57e-52	+0.00	1.57e-52	+2.26e-54	+9.05e-43	+1.15e-10	0.479	0.430	0.300

-----found dual feasible solution-----

primalObjective -

100%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

simpleboot_tut... slides testrun simpleboot4 tutorial4A Solstice

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IsingOE_Tutorial1A_Exercise4.nb - Wolfram Mathematica 11.1

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157	103	0.0098	+7.07e-47	+0.00	7.07e-47	+1.02e-48	+4.08e-37	+9.18e-06	0.478	0.433	0.300
158	104	0.0068	+3.69e-47	+0.00	3.69e-47	+5.34e-49	+2.13e-37	+5.21e-06	0.478	0.433	0.300
159	104	0.0047	+1.93e-47	+0.00	1.93e-47	+2.79e-49	+1.11e-37	+2.95e-06	0.478	0.433	0.300
160	105	0.0033	+1.01e-47	+0.00	1.01e-47	+1.45e-49	+5.81e-38	+1.67e-06	0.478	0.433	0.300
161	106	0.0023	+5.25e-48	+0.00	5.25e-48	+7.59e-50	+3.03e-38	+9.50e-07	0.478	0.432	0.300
162	106	0.0016	+2.74e-48	+0.00	2.74e-48	+3.96e-50	+1.58e-38	+5.39e-07	0.478	0.432	0.300
163	107	0.0011	+1.43e-48	+0.00	1.43e-48	+2.07e-50	+8.26e-39	+3.06e-07	0.478	0.432	0.300
164	108	0.00076	+7.46e-49	+0.00	7.46e-49	+1.08e-50	+4.31e-39	+1.74e-07	0.478	0.432	0.300
165	108	0.00053	+3.89e-49	+0.00	3.89e-49	+5.63e-51	+2.25e-39	+9.88e-08	0.478	0.432	0.300
166	109	0.00037	+2.03e-49	+0.00	2.03e-49	+2.93e-51	+1.17e-39	+5.61e-08	0.478	0.431	0.300
167	110	0.00026	+1.06e-49	+0.00	1.06e-49	+1.53e-51	+6.12e-40	+3.19e-08	0.479	0.431	0.300
168	110	0.00018	+5.52e-50	+0.00	5.52e-50	+7.98e-52	+3.19e-40	+1.82e-08	0.479	0.431	0.300
169	111	0.00012	+2.88e-50	+0.00	2.88e-50	+4.16e-52	+1.66e-40	+1.03e-08	0.479	0.431	0.300
170	112	8.6e-05	+1.50e-50	+0.00	1.50e-50	+2.17e-52	+8.67e-41	+5.88e-09	0.479	0.431	0.300
171	112	6.0e-05	+7.83e-51	+0.00	7.83e-51	+1.13e-52	+4.52e-41	+3.35e-09	0.479	0.431	0.300
172	113	4.2e-05	+4.08e-51	+0.00	4.08e-51	+5.90e-53	+2.36e-41	+1.91e-09	0.479	0.430	0.300
173	114	2.9e-05	+2.13e-51	+0.00	2.13e-51	+3.07e-53	+1.23e-41	+1.09e-09	0.479	0.430	0.300
174	114	2.0e-05	+1.11e-51	+0.00	1.11e-51	+1.60e-53	+6.40e-42	+6.19e-10	0.479	0.430	0.300
175	115	1.4e-05	+5.77e-52	+0.00	5.77e-52	+8.34e-54	+3.33e-42	+3.53e-10	0.479	0.430	0.300
176	116	9.8e-06	+3.01e-52	+0.00	3.01e-52	+4.35e-54	+1.74e-42	+2.01e-10	0.479	0.430	0.300
177	116	6.8e-06	+1.57e-52	+0.00	1.57e-52	+2.26e-54	+9.05e-43	+1.15e-10	0.479	0.430	0.300

```
-----found dual feasible solution-----
primalObjective =
  8.1634433626464130953717540374373656481820913840595147474337350047810204046653879270630178348984912004583345439458993963798773059625403951148319573158130522099762
   3622497878023612773052712010761265642574688907010476797845942169824585 69e-53
dualObjective =
  0.00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
   0000000000000000000000000000000000000000000000000000000000000000000000
dualityGap =
  8.1634433626464130953717540374373656481820913840595147474337350047810204046653879270630178348984912004583345439458993963798773059625403951148319573158130522099762
```

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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ... ENG 8:15 PM Thursday 4/27/2023

simpleboot_tut... slides testrun simpleboot4 tutorial4A Solstice

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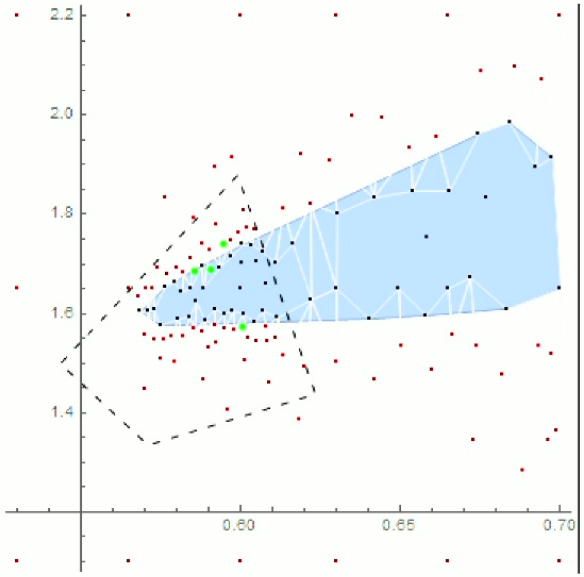
```
SB$DelaunayPlot[] (* plot the scan result *)
```




```
ThetaSampler$ΔScanner$Plot[]
```

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PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

simpleboot_tut... slides testrun simpleboot4 tutorial4A Solstice

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$V_{identity} = (1 \quad 1) \cdot V_{even}(\Delta = 0, l = 0) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$V_\theta = V_{even}(\Delta = \Delta_\epsilon, l = 0) + V_{odd}(\Delta = \Delta_\sigma, l = 0) \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Crossing vectors

Channel name :

op[op,
"E", (* rep name *)
1 , (* always 1 in this tutorial *)
1 (* 1: spin even; -1 : spin odd *)
]

```
(* data for crossing vectors. see ArXiv:1406.4858 *)

crossvecobj={"VBlock"→{

op[op,"E",1,1]→{{{F[sig,sig,sig,sig] 0},{0 0}},{{0 0},{0 F[eps,eps,eps,eps]}},{{0 0},{0 0}}},{{0 1/2 F[sig,sig,eps,eps]},{1/2 F[sig,sig,eps,eps] 0}}},  (* crossing vector V_even for Z_2 even, spin even channel *)
```

120%

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$$V_{\text{even}} = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & F^{22,22}_{-,\Delta,l} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2} F^{11,22}_{-,\Delta,l} \\ \frac{1}{2} F^{11,22}_{-,\Delta,l} & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2} F^{11,22}_{+,\Delta,l} \\ \frac{1}{2} F^{11,22}_{+,\Delta,l} & 0 \end{pmatrix} \end{pmatrix}, \qquad V_{\text{odd}} = \begin{pmatrix} 0 \\ 0 \\ F^{12,12}_{-,\Delta,l} \\ (-1)^l F^{21,12}_{-,\Delta,l} \\ -(-1)^l F^{21,12}_{-,\Delta,l} \end{pmatrix}$$

$$V_{\text{identity}} = (1 \quad 1) \cdot V_{\text{even}}(\Delta = 0,\ l = 0) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

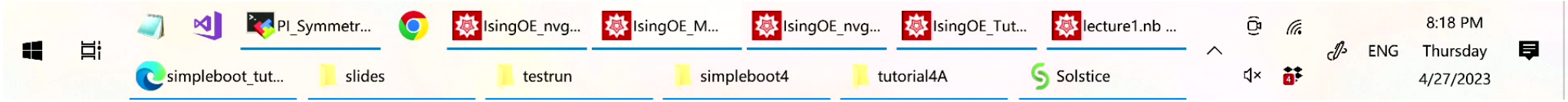
$$V_{\theta} = V_{\text{even}}(\Delta = \Delta_{\epsilon},\ l = 0) + V_{\text{odd}}(\Delta = \Delta_{\sigma},\ l = 0) \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Crossing vectors

Channel name :

```
op[op,
"E", (* rep name *)
1 , (* always 1 in this tutorial *)
1 (* 1: spin even; -1 : spin odd *)
]
```

120%



IsingOE_MMA.nb - Wolfram Mathematica 11.1

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```
sdp
];

{op[op, "O", 1, 1], 3, 0}

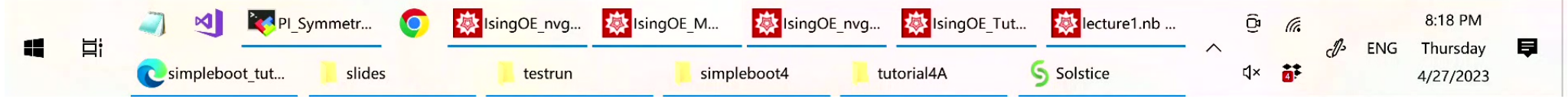
(* 0.68629^(a+x) / ((-1/2+a+x) (1/2+a+x) (1+a+x) (3/2+a+x) (5/2+a+x) (3+a+x) (7/2+a+x) (9/2+a+x) (5+a+x) (7+a+x) (9+a+x) (11+a+x)) *)

PrefactorDR[op[op, "E", 1, 1], 0, x]

DampedRational[
 0.6862915010152396095864902063224153714426249969844154145865620960741401723031436891968997253788674181198892301527016238006013115531941022988470280232005133486942186725799552579536003959907777435154282 4^x,
 {-11 - x, -9 - x, -7 - x, -5 - x, -9/2 - x, -7/2 - x, -3 - x, -5/2 - x, -3/2 - x, -1 - x, -1/2 - x, 1/2 - x},
 0.6862915010152396095864902063224153714426249969844154145865620960741401723031436891968997253788674181198892301527016238006013115531941022988470280232005133486942186725799552579536003959907777435154282 4, x]

op[op, "E", 1, 1] → {{{F[sig, sig, sig, sig], 0}, {0, 0}}, {{0, 0}, {0, F[eps, eps, eps, eps]}}, {{0, 0}, {0, 0}}, {{0, 1/2 F[sig, sig, eps, eps]}, {1/2 F[sig, sig, eps, eps], 0}}}
```

120%



IsingOE_MMA.nb - Wolfram Mathematica 11.1

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$$\begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{1}{2}\,\text{CBPoly\$Hold}[\text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \Delta] \\ \frac{1}{2}\,\text{CBPoly\$Hold}[\text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \Delta] & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{1}{2}\,\text{CBPoly\$Hold}[\text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \Delta] \\ \frac{1}{2}\,\text{CBPoly\$Hold}[\text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \Delta] & 0 \end{pmatrix}$$

```
CrossVec["V$theta"] // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} \text{CBPoly\$Hold}[\text{Fixed}, \text{FS}[\text{sig}, \text{sig}, \text{sig}, \text{sig}], \ell, \text{m}, \text{n}, \text{ExtOp\$eps}] & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & \text{CBPoly\$Hold}[\text{Fixed}, \text{FS}[\text{eps}, \text{eps}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$eps}] \end{pmatrix} \\ \begin{pmatrix} \text{CBPoly\$Hold}[\text{Fixed}, \text{F}[\text{sig}, \text{eps}, \text{sig}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$sig}] & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \text{CBPoly\$Hold}[\text{Fixed}, \text{F}[\text{eps}, \text{sig}, \text{sig}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$sig}] & \frac{1}{2}\,\text{CBPoly\$Hold}[\text{Fixed}, \text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$eps}] \\ \frac{1}{2}\,\text{CBPoly\$Hold}[\text{Fixed}, \text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$eps}] & 0 \end{pmatrix}$$

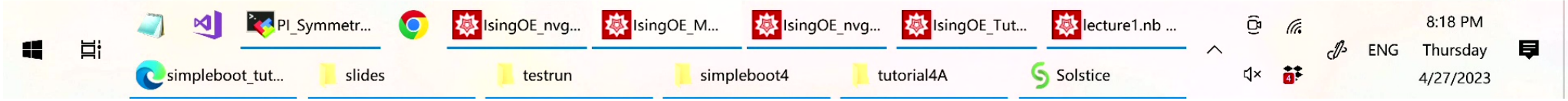
$$\begin{pmatrix} -\text{CBPoly\$Hold}[\text{Fixed}, \text{F}[\text{eps}, \text{sig}, \text{sig}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$sig}] & \frac{1}{2}\,\text{CBPoly\$Hold}[\text{Fixed}, \text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$eps}] \\ \frac{1}{2}\,\text{CBPoly\$Hold}[\text{Fixed}, \text{FS}[\text{sig}, \text{sig}, \text{eps}, \text{eps}], \ell, \text{m}, \text{n}, \text{ExtOp\$eps}] & 0 \end{pmatrix}$$

FS["FSsigsigepseps", ℓ, m, n, x] : $\partial_z^m \partial_{\bar{z}}^n F^{\sigma\sigma\epsilon\epsilon}_{\Delta=\ell+x+d-2,\ell}$

P["FSsigsigsigsig", ℓ, m, n, shift] : $\partial_z^m \partial_{\bar{z}}^n P^{\sigma\sigma\epsilon\epsilon}_{\Delta=\ell+x+\text{shift}+d-2,\ell}$

P["FSsigsigsigsig", 0, 1, 0, 2] : $\partial_z^m \partial_{\bar{z}}^n P^{\sigma\sigma\epsilon\epsilon}_{\Delta=x+3,0}$

120%



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$$\left\{\left\{0, \frac{1}{2} P[\text{FSsigsigepseps}, 0, 7, 3, 2.]\right\}, \left\{\frac{1}{2} P[\text{FSsigsigepseps}, 0, 7, 3, 2.], 0\right\}\right\},$$

$$\left\{\left\{0, \frac{1}{2} P[\text{FSsigsigepseps}, 0, 8, 2, 2.]\right\}, \left\{\frac{1}{2} P[\text{FSsigsigepseps}, 0, 8, 2, 2.], 0\right\}\right\},$$

$$\left\{\left\{0, \frac{1}{2} P[\text{FSsigsigepseps}, 0, 9, 1, 2.]\right\}, \left\{\frac{1}{2} P[\text{FSsigsigepseps}, 0, 9, 1, 2.], 0\right\}\right\},$$

$$\left\{\left\{0, \frac{1}{2} P[\text{FSsigsigepseps}, 0, 10, 0, 2.]\right\}, \left\{\frac{1}{2} P[\text{FSsigsigepseps}, 0, 10, 0, 2.], 0\right\}\right\}\}$$

```
BootstrapCondition[AutoCB3$DerivConfig, PrefactorDR[op[op, "E", 1, 1], 0, 3],
   CrossVec["V$theta"], 3, {0}][[1, 2]];
Cases[%, FS["FSsigsigepseps", ___], Infinity] /. -1.`200. → -1
```

```
{FS[FSsigsigepseps, 0, 1, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 1, 0, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 2, 1, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 2, 1, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 3, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 3, 0, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 3, 2, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 3, 2, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 4, 1, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 4, 1, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 5, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 5, 0, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 4, 3, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 4, 3, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 5, 2, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 5, 2, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 6, 1, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 6, 1, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 7, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 7, 0, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 5, 4, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 5, 4, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 6, 3, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 6, 3, -1 + ExtOp$eps],
 FS[FSsigsigepseps, 0, 7, 2, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 7, 2, -1 + ExtOp$eps],
```

120%

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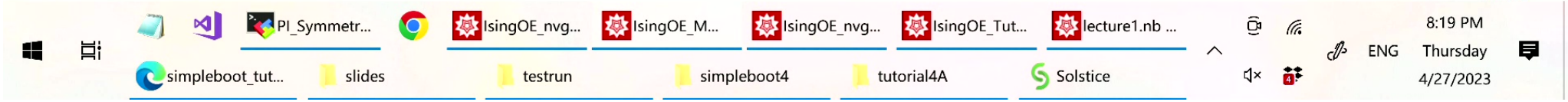
```
FS[FSsigsigepseps, 0, 5, 1, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 5, 1, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 6, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 6, 0, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 4, 4, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 4, 4, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 5, 3, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 5, 3, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 6, 2, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 6, 2, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 7, 1, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 7, 1, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 8, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 8, 0, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 5, 5, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 5, 5, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 6, 4, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 6, 4, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 7, 3, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 7, 3, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 8, 2, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 8, 2, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 9, 1, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 9, 1, -1 + ExtOp$eps],
FS[FSsigsigepseps, 0, 10, 0, -1 + ExtOp$eps], FS[FSsigsigepseps, 0, 10, 0, -1 + ExtOp$eps]}
```

Use MMA to evaluate a polynomial

```
SSH2$UploadCurrentNotebook[]


ClusterAsyn$Evaluate[
 AutoCB3$TemplateExpressionEvaluate[
  {0.518, 1.412} // SetPrec, {"Var1" → 10, "Var2" → 20},
  P["FSsigsigepseps", 0, 2, 1, Var1]
```

120%



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80 640 3 628 800

```
FS["FSsigsigepseps", 0, 8, 2, -1 + ExtOp$eps]

ClusterAsyn$Evaluate[
 templateExpr = Sum[1/m! 1/n! FS["FSsigsigepseps", 0, Max[m, n], Min[m, n], Var1] (z - 1/2)^m (zb - 1/2)^m, {m, 0, 11, 2}, {n, 0, 11 - m, 2}];
 AutoCB3$TemplateExpressionEvaluate[
  {0.518, 1.412} // SetPrec, {"Var1" → 10, "Var2" → 20},
  templateExpr
 ]
]
```

409282

```
ClusterAsyn$JobOutput["409282"] // ClusterAsyn$JobOutput$ToExpression // N
```

$$893.615 - 3169.52\,z + 5360.3\,z^2 - 4802.9\,z^3 + 3477.26\,z^4 - 1354.05\,z^5 + 556.886\,z^6 - 91.2394\,z^7 + 23.7868\,z^8 - 0.651277\,z^9 + 0.130255\,z^{10} -$$
$$3169.52\,zb + 17\,059.6\,z\,zb - 35\,849.9\,z^2\,zb + 42\,726.4\,z^3\,zb - 34\,588.3\,z^4\,zb + 16\,881.9\,z^5\,zb - 7321.3\,z^6\,zb + 1467.65\,z^7\,zb -$$
$$386.45\,z^8\,zb + 13.0255\,z^9\,zb - 2.60511\,z^{10}\,zb + 5360.3\,zb^2 - 35\,849.9\,z\,zb^2 + 99\,939.5\,z^2\,zb^2 - 154\,629.\,z^3\,zb^2 + 145\,970.\,z^4\,zb^2 -$$
$$89\,491.4\,z^5\,zb^2 + 41\,743.3\,z^6\,zb^2 - 10\,351.7\,z^7\,zb^2 + 2763.76\,z^8\,zb^2 - 117.23\,z^9\,zb^2 + 23.446\,z^{10}\,zb^2 - 4802.9\,zb^3 + 42\,726.4\,z\,zb^3 -$$
$$154\,629.\,z^2\,zb^3 + 297\,712.\,z^3\,zb^3 - 343\,778.\,z^4\,zb^3 + 262\,469.\,z^5\,zb^3 - 135\,469.\,z^6\,zb^3 + 41\,875.6\,z^7\,zb^3 - 11\,406.7\,z^8\,zb^3 + 625.225\,z^9\,zb^3 -$$
$$125.045\,z^{10}\,zb^3 + 3477.26\,zb^4 - 34\,588.3\,z\,zb^4 + 145\,970.\,z^2\,zb^4 - 343\,778.\,z^3\,zb^4 + 499\,976.\,z^4\,zb^4 - 465\,673.\,z^5\,zb^4 + 276\,486.\,z^6\,zb^4 -$$
$$106\,565.\,z^7\,zb^4 + 29\,923.6\,z^8\,zb^4 - 2188.29\,z^9\,zb^4 + 437.658\,z^{10}\,zb^4 - 1354.05\,zb^5 + 16\,881.9\,z\,zb^5 - 89\,491.4\,z^2\,zb^5 + 262\,469.\,z^3\,zb^5 -$$
$$465\,673.\,z^4\,zb^5 + 518\,053.\,z^5\,zb^5 - 370\,380.\,z^6\,zb^5 + 175\,755.\,z^7\,zb^5 - 51\,816.7\,z^8\,zb^5 + 5251.89\,z^9\,zb^5 - 1050.38\,z^{10}\,zb^5 + 556.886\,zb^6 -$$

120%

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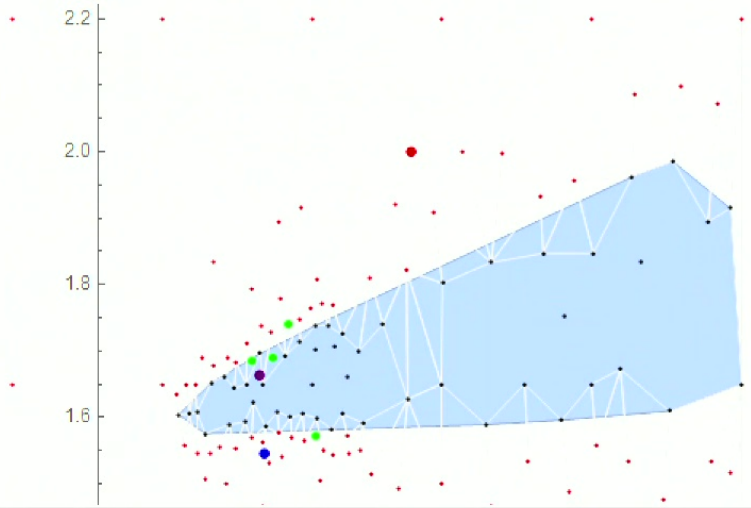
```
ClusterLoginNode$Evaluate@DeleteDirectory[ReconfigCmd@"[Cluster.ProjectDirectory]", DeleteContents → True]
```

```
//
```

```
Show[#,
  ListPlot[List /@ {{0.6228254476530574, 1.9993151417172765`}, {0.5875470892301244, 1.6629719549936235`},
    {0.5888405172482473, 1.5456119773339352`}}, PlotStyle → {Red, Purple, Blue}], ImageSize → 400] &
```

… N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.



120%

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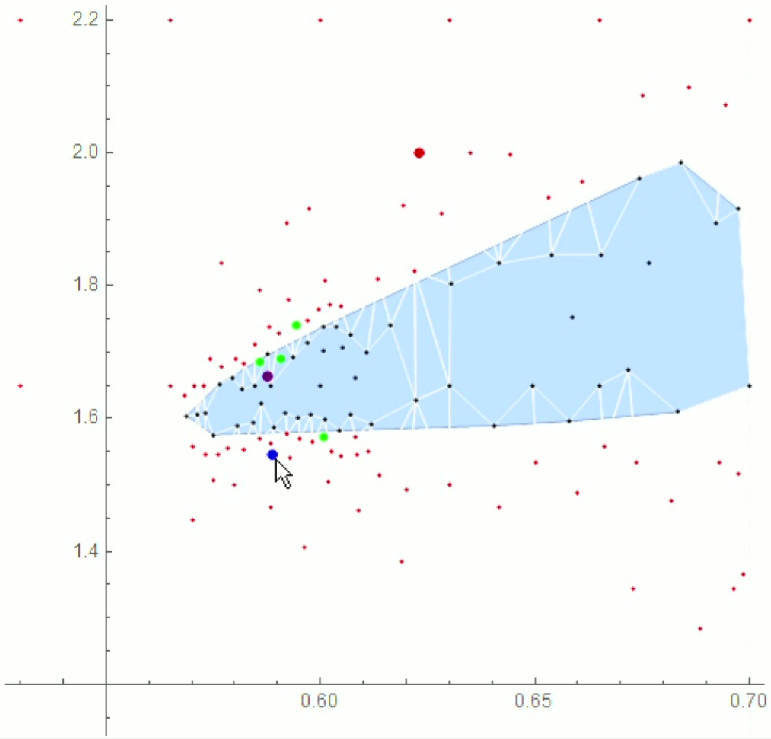
```
Show[#,
  ListPlot[List /@ {{0.6228254476530574, 1.9993151417172765`}, {0.5875470892301244, 1.6629719549936235`},
    {0.5888405172482473, 1.5456119773339352`}}, PlotStyle → {Red, Purple, Blue}], ImageSize → 400] &
```

··· N: Requested precision 16 is smaller than $MinPrecision. Using $MinPrecision instead.



120%

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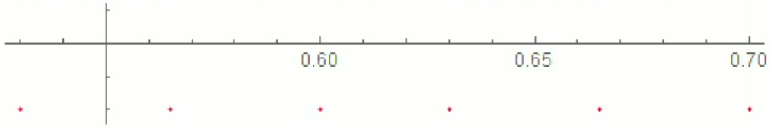


```
ClusterAsyn$Evaluate[
 AutoCB3$SaveSDPDataTemplate@SDPTemplate$IsingOE[];

 ObjSet[BFGSoptions, "Algorithm"] = "None";

 allpts = {{0.6228254476530574, 1.9993151417172765`}, {0.5875470892301244, 1.6629719549936235`},
    {0.5888405172482473, 1.5456119773339352`}} // SetPrec;


 SB$NavigatorRun[BFGSoptions, SDP$IsingOE, initpt,
  "--saveMiddleCheckpointMuThreshold=1e-8 --maxIterations=1000 --dualityGapThreshold=1e-25 --primalErrorThreshold=1e-15
    --dualErrorThreshold=1e-15 --precision=765 --initialMatrixScalePrimal=1e+20 --initialMatrixScaleDual=1e+20
    --maxComplementarity=1e+70  --writeSolution=\"x,y,X,Y\"", 1000, False]
]
409568

ClusterAsyn$JobOutput["409568"]
SSH$DownloadFile@SB$Proj$FileName;
SB$LoadProject[];
```

120%

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```
 SB$NavigatorRun[BFGSoptions, SDP$IsingOE, initpt,
  "--saveMiddleCheckpointMuThreshold=1e-8 --maxIterations=1000 --dualityGapThreshold=1e-25 --primalErrorThreshold=1e-15
    --dualErrorThreshold=1e-15 --precision=765 --initialMatrixScalePrimal=1e+20 --initialMatrixScaleDual=1e+20
    --maxComplementarity=1e+70  --writeSolution=\"x,y,X,Y\"", 1000, False]
]
```

409568

```
ClusterAsyn$JobOutput["409568"]
SSH$DownloadFile@SB$Proj$FileName;
SB$LoadProject[];

ObjGet[SB$Proj$calculated, All, "DeltaList"] // N
```

{{0.622825, 1.99932}, {0.622825, 1.99932}, {0.587547, 1.66297}}

Red point : {0.6228254476530574,1.9993151417172765`}

```
SSH$UploadCurrentNotebook[]


ClusterAsyn$JobOutput["409285"]
SSH$DownloadFile@SB$Proj$FileName;
SB$LoadProject[];
```

120%

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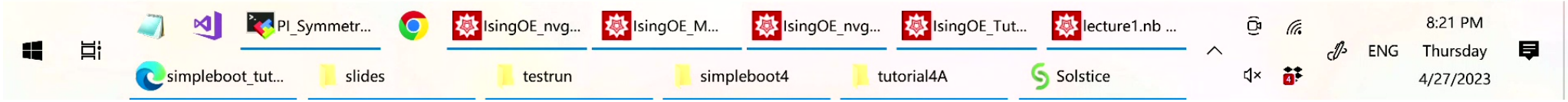
```
  1.412`}], {Var1 ï¢ 10, Var2 ï¢ 20}, P[FSsigsigepseps, 0, 2, 1, Var1]]]. Start evaluating...
Bootstapper packages Loaded. Version : 4.0
MMA Precision set to 200.
[RemoteExecuteReturnBegin]QXV0b0NCMyRUZW1wbGF0ZUV4cHJlc3Npb25FdmFsdWF0ZVt7MC41MTgwMDAwMDAwMDAwMDAwMTU5ODcyMTE1NTQ2MDIyNTQx:
   ODIxMDAyOTYwMjA1MDc4MTI1YDIwMC4sIDEuNDExOTk5OTk5OTk5OTIxODQwMjk5MDY2Mzg4OTc5NTU0MTc2MzMwNTY2NDA2MjVgMjAwLn0sIHsiVmFyM:
   SIgLT4gMTAsICJWYXIyIiAtPiAyMH0sIFBbIkZTc2lnc2lnZXBzZXBzIiwgMCwgMiwgMSwgVmFyMV1d[RemoteExecuteReturnEnd]
SSH$Evaluate result:
AutoCB3$TemplateExpressionEvaluate[{0.518000000000000015987211554602254182100296020507812 5`200.,
  1.411999999999999921840299066388979554176330566406 25`200.}, {Var1 ï¢ 10, Var2 ï¢ 20}, P[FSsigsigepseps, 0, 2, 1, Var1]]

ClusterAsyn$Evaluate[
 BC$normalization$template = AutoCB3$SigmaNavigatorNormalizer[GapConfiguration];
 BC$normalization = AutoCB3$TemplateExpressionEvaluate[
   {0.6228254476530574, 1.9993151417172765`} // SetPrec, {},
   BC$normalization$template
  ];

 functionalAlpha =
  SDPB$ReadOut$Alpha[FullPath$SDPBFiles$WithSuffix["0.622825447653_1.99931514172_Apr21_21h51m00s.out", ".out"],
   BC$normalization];

 SaveExpression["functionalAlpha.txt", functionalAlpha, True];
]
```

120%



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```
BC$normalization = AutoCB$TemplateExpressionEvaluate[
  {0.6228254476530574, 1.9993151417172765`} // SetPrec, {},
  BC$normalization$template
 ];

functionalAlpha =
 SDPB$ReadOut$alpha[FullPath$SDPBFiles$WithSuffix["0.622825447653_1.99931514172_Apr21_21h51m00s.out", ".out"],
  BC$normalization];

SaveExpression["functionalAlpha.txt", functionalAlpha, True];
]
```

```
409293
```

```
ClusterAsyn$JobOutput["409293"]
```

```
Fri Apr 21 22:28:46 EDT 2023
CPU Information:
     64  AMD EPYC 7532 32-Core Processor
/gpfs/nsu2/simpleboot4_tutorial1A
Project director:
Proj_IsingOE_MMA
running MMA file:
/gpfs/nsu2/simpleboot4_package/boot.m
with parameter:
```

120%

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```
Length@AutoCB3$DerivConfig[[1, 1]]
```

```
21
```

```
AutoCB3$DerivConfig[[1, 1]]
```

```
{{1, 0}, {2, 1}, {3, 0}, {3, 2}, {4, 1}, {5, 0}, {4, 3}, {5, 2}, {6, 1}, {7, 0},
 {5, 4}, {6, 3}, {7, 2}, {8, 1}, {9, 0}, {6, 5}, {7, 4}, {8, 3}, {9, 2}, {10, 1}, {11, 0}}
```

```
functionalAlpha = LoadExpression["functionalAlpha.txt"];
```

```
functionalAlpha5norms = Norm /@ Partition[LoadExpression["functionalAlpha.txt"], Length@AutoCB3$DerivConfig[[1, 1]]] // N
% // Length
```

$$\{0.00484697,\ 5.3426\times10^{-65},\ 1.87259\times10^{-31},\ 2.06867\times10^{-31},\ 2.24855\times10^{-31}\}$$

```
5
```

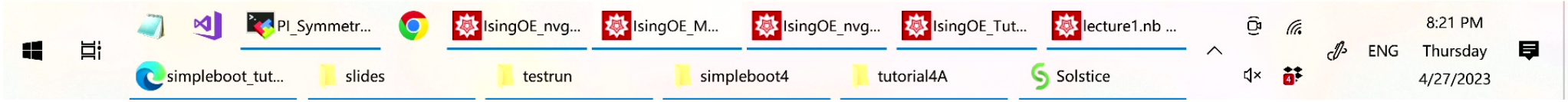
```
Cases[#, _F | _H, Infinity][[1]] & /@ Transpose[ObjGet[crossvecobj, "VBlock"][[All, 2]]]
```

```
{F[sig, sig, sig, sig], F[eps, eps, eps, eps], F[sig, eps, sig, eps], F[sig, sig, eps, eps], H[sig, sig, eps, eps]}
```

```
{Cases[#, _F | _H, Infinity][[1]] & /@ Transpose[ObjGet[crossvecobj, "VBlock"][[All, 2]]], functionalAlpha5norms} // Transpose //
 MatrixForm
```

F[sig, sig, sig, sig]	0.00484697
F[eps, eps, eps, eps]	5.3426×10^{-65}
F[sig, eps, sig, eps]	1.87259×10^{-31}
F[sig, sig, eps, eps]	2.06867×10^{-31}

120%



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```
functionalAlpha = LoadExpression["functionalAlpha.txt"];

functionalAlpha5norms = Norm /@ Partition[LoadExpression["functionalAlpha.txt"], Length@AutoCB3$DerivConfig[[1, 1]]] // N
% // Length
```

$$\{0.00484697,\ 5.3426\times10^{-65},\ 1.87259\times10^{-31},\ 2.06867\times10^{-31},\ 2.24855\times10^{-31}\}$$

```
5

Cases[#, _F | _H, Infinity][[1]] & /@ Transpose[ObjGet[crossvecobj, "VBlock"][[All, 2]]]
```

$$\{F[sig, sig, sig, sig],\ F[eps, eps, eps, eps],\ F[sig, eps, sig, eps],\ F[sig, sig, eps, eps],\ H[sig, sig, eps, eps]\}$$

```
{Cases[#, _F | _H, Infinity][[1]] & /@ Transpose[ObjGet[crossvecobj, "VBlock"][[All, 2]]], functionalAlpha5norms} // Transpose //
 MatrixForm
```

$$\begin{pmatrix} F[sig, sig, sig, sig] & 0.00484697 \\ F[eps, eps, eps, eps] & 5.3426\times10^{-65} \\ F[sig, eps, sig, eps] & 1.87259\times10^{-31} \\ F[sig, sig, eps, eps] & 2.06867\times10^{-31} \\ H[sig, sig, eps, eps] & 2.24855\times10^{-31} \end{pmatrix}$$

check all points

```
computeAlpha[ptsindex_]:=Module[
```

120%

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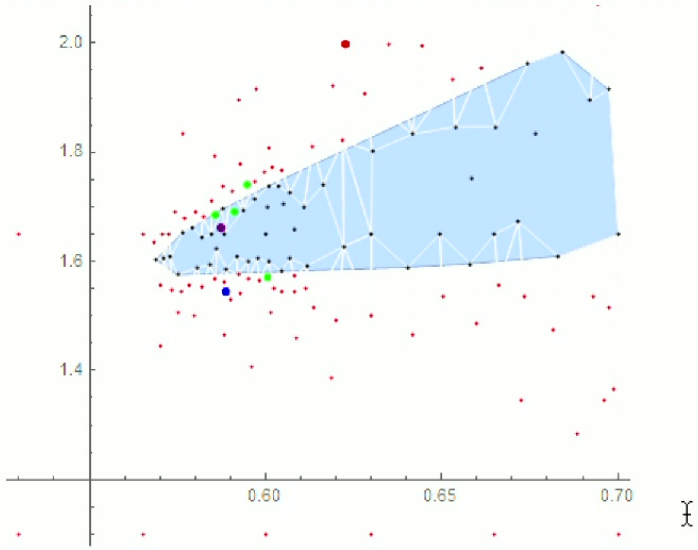


```
Print["Norm of αᵢ :"];
(Norm /@ Partition[#, componentLength]) & /@ alphadata[[All, 1]] // N // Prepend[#, corlist] & // Transpose //
  Prepend[#, {"correlator", "red", "purple", "blue"}] & // MatrixForm
```

Norm of α_i :

correlator	red	purple	blue
F[sig, sig, sig, sig]	0.00484697	3.4594×10^{-6}	0.00414189
F[eps, eps, eps, eps]	5.3426×10^{-65}	5.64305×10^{-76}	0.0000342222
F[sig, eps, sig, eps]	1.87259×10^{-31}	1.75155×10^{-37}	0.00408897
F[sig, sig, eps, eps]	2.06867×10^{-31}	3.96601×10^{-37}	0.00166957
H[sig, sig, eps, eps]	2.24855×10^{-31}	4.72323×10^{-37}	0.00234527

110%

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```
eyJJc2luZ09FLm0iLCBIb2xkW1NCJExvYWRQcm9qZWN0W107IChSdW5TaGVsbFtTdHJpbmdKb2luWyJzY2FuY2VsICIsICMxXV0gJiApIC9AIFNCJFByb2okY2FsY3VsYXRpbmdbW0F
   sbCwyXV07IF19 evaluate
SSH$Evaluate: filename=IsingOE.m, expr=Hold[SB$LoadProject[]
  ; (RunShell[scancel  <> #1] &)/@SB$Proj$calculating ã  All, 2 ã  ;]. Start evaluating...
Bootstapper packages Loaded. Version : 4
MMA Precision set to 200.
[RemoteExecuteReturnBegin]TnVsbA==[RemoteExecuteReturnEnd]
SSH$Evaluate result:
Null
```

Manually add points for Delaunay scanner to scan

```
(* ClusterLoginNode$Evaluate: execute MMA command on the login node of the cluster *)
ClusterLoginNode$Evaluate[
 (* Cluster$AddPoints : add a list of point for the scanner to compute *)
 Cluster$AddPoints[
  {{0.5177103695412963, 1.4055967885780287`}, {0.5174195656793055, 1.4110164420007305`}, {0.5184664595824726, 1.4188071937958642`},
    {0.5188735849892598, 1.4079678869504608`}} // SetPrec]
]
```

Manually add points for Delaunay scanner to scan

```
ClusterLoginNode$Evaluate[
 (* Add a test region in the Delaunay scanner. The scanner will only search for new points inside the test region. *)
 SB$LockProject[];
 ObjSet[SB$Proj, "method"] =
```

110%

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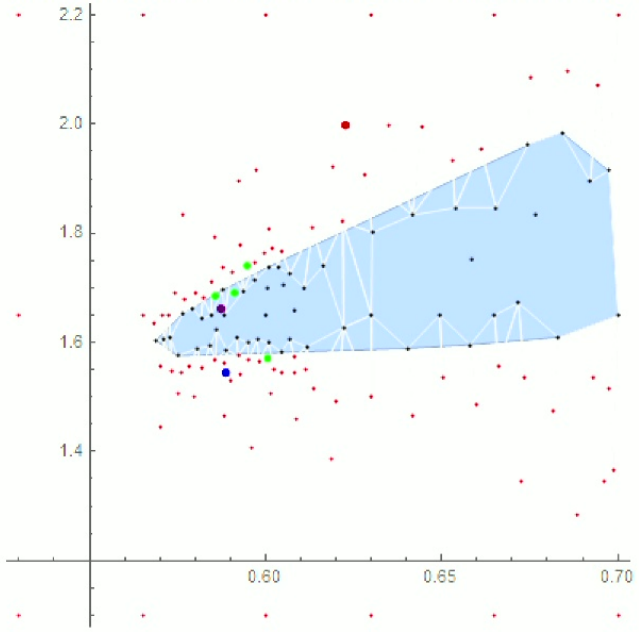


```
Print["Norm of αᵢ :"];
(Norm /@ Partition[#, componentLength]) & /@ alphadata[[All, 1]] // N // Prepend[#, corlist] & // Transpose //
  Prepend[#, {"correlator", "red", "purple", "blue"}] & // MatrixForm
```

Norm of α_i :

correlator	red	purple	blue
F[sig, sig, sig, sig]	0.00484697	3.4594×10^{-6}	0.00414189
F[eps, eps, eps, eps]	5.3426×10^{-65}	5.64305×10^{-76}	0.0000342222
F[sig, eps, sig, eps]	1.87259×10^{-31}	1.75155×10^{-37}	0.00408897

110%

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```
Print["Norm of αᵢ :"];
(Norm /@ Partition[#, componentLength]) & /@ alphadata[[All, 1]] // N //
    Prepend[#, corlist] & // Transpose //
  Prepend[#, {"correlator", "red", "purple", "blue"}] & // MatrixForm
```

Norm of α_i :

correlator	red	purple	blue
F[sig, sig, sig, sig]	0.00484697	3.4594×10^{-6}	0.00414189
F[eps, eps, eps, eps]	5.3426×10^{-65}	5.64305×10^{-76}	0.0000342222
F[sig, eps, sig, eps]	1.87259×10^{-31}	1.75155×10^{-37}	0.00408897
F[sig, sig, eps, eps]	2.06867×10^{-31}	3.96601×10^{-37}	0.00166957
H[sig, sig, eps, eps]	2.24855×10^{-31}	4.72323×10^{-37}	0.00234527

```
Print["αᵢ.nᵢ :"];
Module[{alpha, normalization},
      alpha = Partition[#, componentLength] & /@ alphadata[[All, 1]];
      normalization = Partition[#, componentLength] & /@ alphadata[[All, 2]];
```

160%

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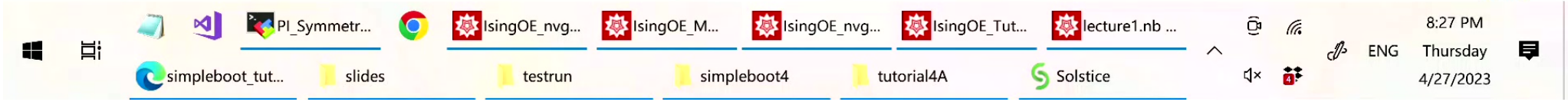
```
MapIndexed[{#2, NumberForm[#1[[1, 1 ;; -1]], 12], NumberForm[#1[[2, 1]], 12], ToString[N@#1[[2, 2]], StandardForm]} &,
   IndicatorHistory] // Prepend[#, {"Step", "Point", "Indicator", "Gradient"}] & // TableForm
```

Step	Point	Indicator	Gradient
1	{0.587547089230, 1.66297195499, 0.500000000000}	0.000903703269651	{0.0192532, -0.0257968, -0.0225902}
2	{0.587514358865, 1.66343629659, 0.503614430987}	0.000811354657274	{0.0185984, -0.0246409, -0.0216663}
3	{0.585785581552, 1.67600275310, 0.601831555264}	$-3.24647392005 \times 10^{-8}$	{-0.000112904, 0.0000381183, 0.0000206571}
4	{0.585773068110, 1.67602885145, 0.602037592311}	$-2.56497065441 \times 10^{-8}$	{-0.000116312, 0.0000394669, 0.0000217481}
5	{0.585728267829, 1.67611200276, 0.602697052167}	$-1.12882024191 \times 10^{-9}$	{-0.000128651, 0.000044308, 0.0000255819}
6	{0.585723282269, 1.67610621986, 0.602656294797}	$-1.78352229260 \times 10^{-9}$	{-0.000128255, 0.0000441412, 0.0000254208}
7	{0.584451999852, 1.67484937080, 0.593918984366}	$-3.36135187239 \times 10^{-8}$	{-0.0000899377, 0.0000267225, 5.6684×10^{-6}}
8	{0.583816564562, 1.67424924843, 0.589768079832}	$-1.10457776896 \times 10^{-8}$	{-0.0000934467, 0.0000272949, 4.16346×10^{-6}}
9	{0.583473658507, 1.67377366631, 0.586381029791}	$-9.34330598894 \times 10^{-10}$	{-0.0000926284, 0.0000243443, -3.80153×10^{-6}}
10	{0.583463024035, 1.67378920279, 0.586509036681}	$4.91256789335 \times 10^{-11}$	{-0.0000933617, 0.0000250465, -2.37196×10^{-6}}
11	{0.583463136341, 1.67377916446, 0.586435736396}	$-9.71030029548 \times 10^{-12}$	{-0.0000931097, 0.0000247162, -3.12619×10^{-6}}
12	{0.583462898620, 1.67377516931, 0.586411917646}	$-8.66386654396 \times 10^{-12}$	{-0.0000930292, 0.0000246072, -3.37732×10^{-6}}
13	{0.583462183849, 1.67376183773, 0.586348537481}	$-2.11023703063 \times 10^{-11}$	{-0.0000928688, 0.0000241485, -4.56746×10^{-6}}
14	{0.583461018570, 1.67375055578, 0.586326565753}	$-7.74073201242 \times 10^{-11}$	{-0.000092793, 0.0000239236, -5.15133×10^{-6}}
15	{0.583439180816, 1.67354846376, 0.586136847989}	$-1.35056743612 \times 10^{-9}$	{-0.0000915817, 0.0000223488, -8.68841×10^{-6}}
16	{0.583348648506, 1.67275825560, 0.585667294875}	$-4.06770311771 \times 10^{-9}$	{-0.0000893595, 0.0000200331, -0.0000134933}
17	{0.583128393521, 1.67060295100, 0.584710458937}	$-1.34860335710 \times 10^{-8}$	{-0.0000906109, 0.0000197414, -0.0000151045}
18	{0.581077629765, 1.64552706420, 0.574082104048}	0.0000154611924361	{-0.000556628, -0.00127699, -0.00276154}
19	{0.582075253938, 1.65772559793, 0.579252423511}	$2.80162496740 \times 10^{-7}$	{-0.000194018, -0.0000592876, -0.000251219}
20	{0.582239114648, 1.65972921858, 0.580101653350}	$6.21849328333 \times 10^{-8}$	{-0.00010077, -8.61256×10^{-6}, -0.0000835327}
21	{0.582359178497, 1.66119730942, 0.580723900155}	$3.46117767844 \times 10^{-9}$	{-0.0000814809, 6.02074×10^{-7}, -0.0000535966}

120%



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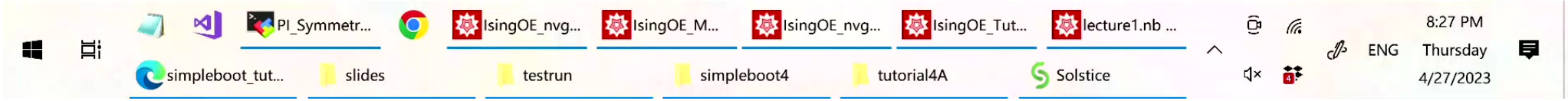
30	{0.568350690412, 1.58784308391, 0.653413184381}	$-7.65397580394 \times 10^{-8}$	{-0.0000484939, 0.0000169486, 0.0000142529}
31	{0.563513783468, 1.54874532355, 0.684064213193}	0.000124529309230	{0.0343234, -0.0224379, -0.00852744}
32	{0.566504885258, 1.57292304339, 0.665109879730}	$-5.75981182027 \times 10^{-8}$	{-0.0000258769, 2.63865×10^{-6}, 0.0000119803}
33	{0.565459527995, 1.57130259334, 0.665910650944}	$-1.90509039111 \times 10^{-8}$	{-0.0000509356, 0.0000143987, 0.0000198924}
34	{0.564254512584, 1.57188263028, 0.661528341942}	$-2.42181186379 \times 10^{-8}$	{-0.0000671816, 0.0000244341, 0.0000216703}
35	{0.559475704398, 1.56875234375, 0.648295032236}	$-3.82907636088 \times 10^{-8}$	{-0.0000515396, 0.0000179698, 0.0000138447}
36	{0.553874365760, 1.56444861249, 0.632735157333}	$5.55854290290 \times 10^{-8}$	{-0.0000299784, 2.6082×10^{-7}, -4.34823×10^{-6}}
37	{0.555335475504, 1.56436192208, 0.638847996080}	$4.47820631627 \times 10^{-11}$	{-6.62464×10^{-6}, -7.1405×10^{-6}, -5.51847×10^{-6}}
38	{0.554393777000, 1.55803695399, 0.652087278143}	$2.44548272034 \times 10^{-7}$	{-0.00028859, 0.000101987, 0.0000767211}
39	{0.555266931845, 1.56390154495, 0.639811647222}	$-1.41921674430 \times 10^{-9}$	{-8.59468×10^{-6}, -6.13629×10^{-6}, -4.79369×10^{-6}}
40	{0.554748734061, 1.56044264592, 0.646832323234}	$7.30237205412 \times 10^{-8}$	{-0.00012635, 0.000044517, 0.0000323074}
41	{0.555179927859, 1.56332080530, 0.640990399452}	$-3.49890923681 \times 10^{-10}$	{-0.0000338055, 6.23202×10^{-6}, 4.15667×10^{-6}}
42	{0.555138954249, 1.56330029694, 0.640642476052}	$9.37026858784 \times 10^{-11}$	{-0.0000184122, -1.34479×10^{-6}, -1.02717×10^{-6}}
43	{0.555141906826, 1.56325018155, 0.640776646105}	$8.82116605543 \times 10^{-13}$	{-0.0000212108, 5.54486×10^{-8}, 5.95442×10^{-8}}
44	{0.555141937824, 1.56325913463, 0.640760704740}	$2.57528145611 \times 10^{-16}$	{-0.0000210478, -2.72602×10^{-8}, -1.52319×10^{-8}}
45	{0.555141937427, 1.56326078309, 0.640758886267}	$-6.22699038268 \times 10^{-16}$	{-0.0000211003, -1.98187×10^{-9}, -1.11634×10^{-9}}
46	{0.555141937395, 1.56326090752, 0.640758751082}	$7.58810206628 \times 10^{-20}$	{-0.0000211044, 4.52171×10^{-12}, 3.01357×10^{-12}}

```
feasiblepts = Select[IndicatorHistory, #[[2, 1]] < 0 &][[All, 1, 1 ;; 2]];

(* automatic stall recovery and save middle checkpoint *)
ClusterAsyn$SDPBOutput@ObjGet[SB$Proj$calculated, 3, "LastCheckpoint"]

ListPlot[IndicatorHistory[[All, 1, 1 ;; 2]], Joined → True, PlotMarkers → {Automatic, Tiny}, AspectRatio → 1,
  PlotStyle → {Lighter@Red}] //
```

120%



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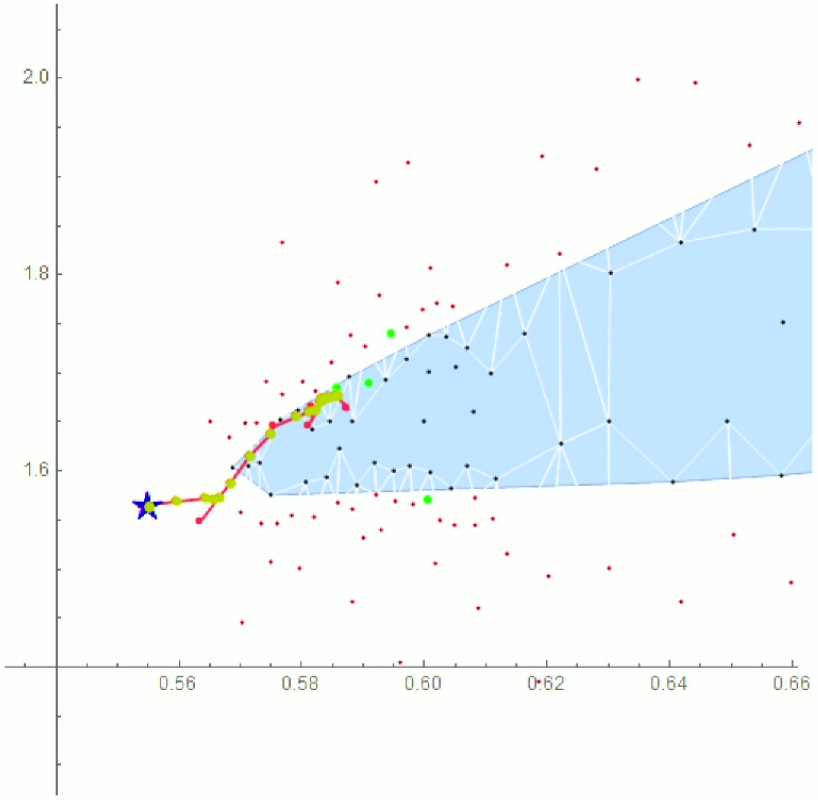
··· General: Further output of N::precsm will be suppressed during this calculation.



check instability

120%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

simpleboot_tut... slides testrun simpleboot4 tutorial4A Solstice

ENG 8:27 PM Thursday 4/27/2023

IsingOE_MMA.nb * - Wolfram Mathematica 11.1

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```
Print["Norm of αᵢ :"];
(Norm /@ Partition[#, componentLength]) & /@ alphadata[[All, 1]] // N //
    Prepend[#, corlist] & // Transpose //
  Prepend[#, {"correlator", "red", "purple", "blue"}] & // MatrixForm
```

Norm of α_i :

correlator	red	purple	blue
F[sig, sig, sig, sig]	0.00484697	3.4594×10^{-6}	0.00414189
F[eps, eps, eps, eps]	5.3426×10^{-65}	5.64305×10^{-76}	0.0000342222
F[sig, eps, sig, eps]	1.87259×10^{-31}	1.75155×10^{-37}	0.00408897
F[sig, sig, eps, eps]	2.06867×10^{-31}	3.96601×10^{-37}	0.00166957
H[sig, sig, eps, eps]	2.24855×10^{-31}	4.72323×10^{-37}	0.00234527

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Print["αᵢ.nᵢ :"];
Module[{alpha, normalization},
      alpha = Partition[#, componentLength] & /@ alphadata[[All, 1]];
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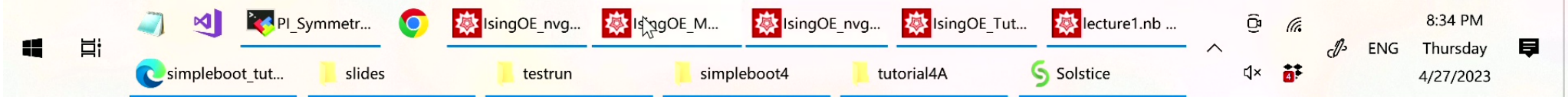
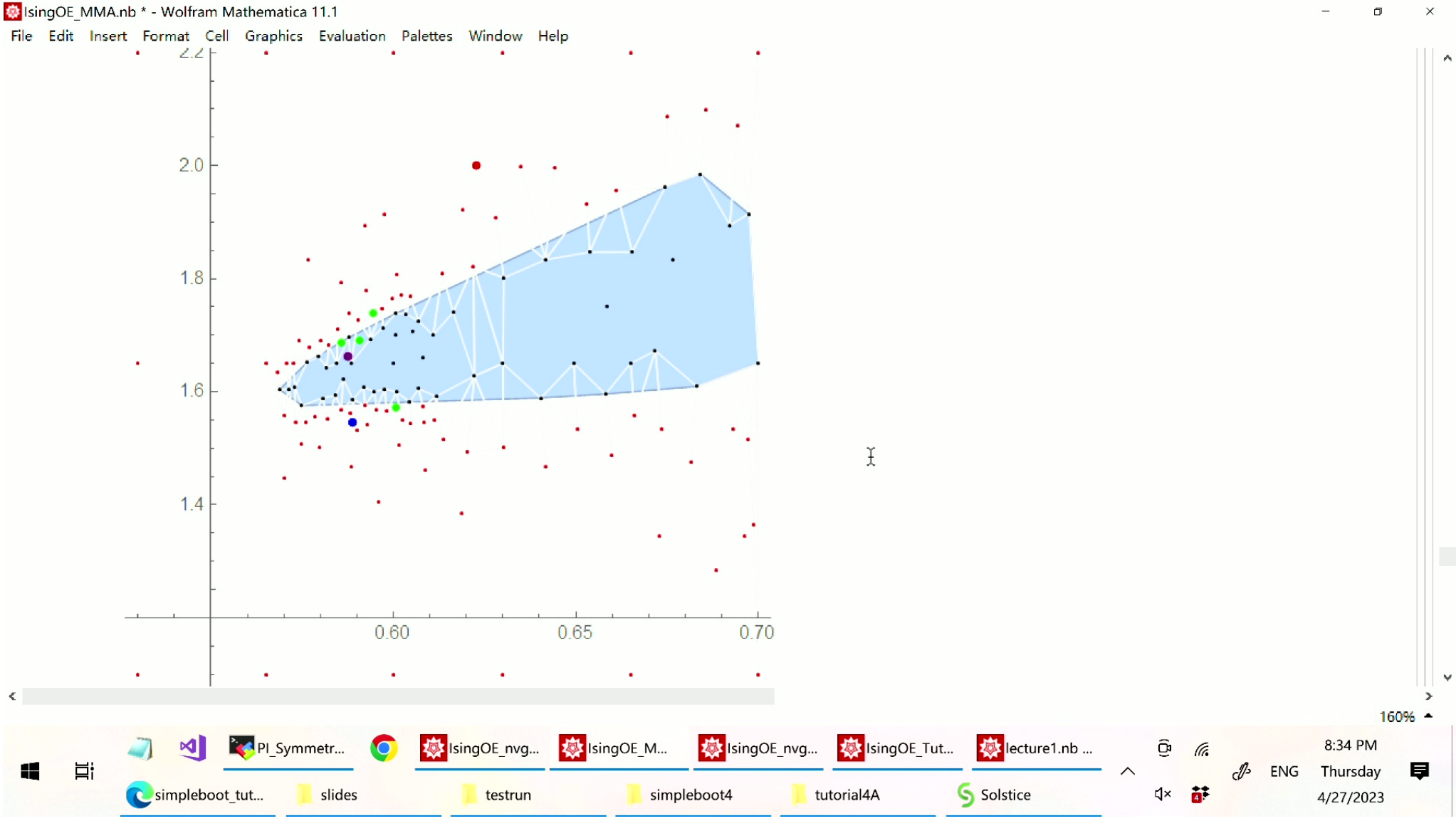
```

160%




IsingOE_MMA.nb * - Wolfram Mathematica 11.1
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2.0
1.8
1.6
1.4
0.60
0.65
0.70
160%
PI_Symmetr...
IsingOE_nvg...
IsingOE_M...
IsingOE_nvg...
IsingOE_Tut...
lecture1.nb ...
simpleboot_tut...
slides
testrun
simpleboot4
tutorial4A
Solstice
ENG
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4/27/2023

IsingOE_MMA.nb * - Wolfram Mathematica 11.1

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```
Print["Norm of αᵢ :"];
(Norm /@ Partition[#, componentLength]) & /@ alphadata[[All, 1]] // N //
    Prepend[#, corlist] & // Transpose //
  Prepend[#, {"correlator", "red", "purple", "blue"}] & // MatrixForm
```

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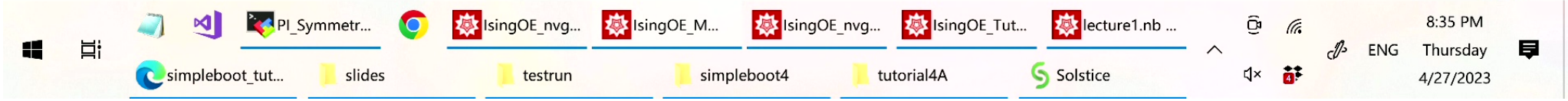
F[eps, eps, eps, eps]	5.3426×10^{-65}	5.64305×10^{-76}	0.0000342222
F[sig, eps, sig, eps]	1.87259×10^{-31}	1.75155×10^{-37}	0.00408897
F[sig, sig, eps, eps]	2.06867×10^{-31}	3.96601×10^{-37}	0.00166957
H[sig, sig, eps, eps]	2.24855×10^{-31}	4.72323×10^{-37}	0.00234527

```
Print["αᵢ.nᵢ :"];
Module[{alpha, normalization},
      alpha = Partition[#, componentLength] & /@ alphadata[[All, 1]];
      normalization = Partition[#, componentLength] & /@ alphadata[[All, 2]];
      Table[alpha[[pt, cor]].normalization[[pt, cor]], {pt, 3}, {cor, 5}]
     ] // N // Prepend[#, corlist] & // Transpose //
```

160%



IsingOE_MMA.nb * - Wolfram Mathematica 11.1

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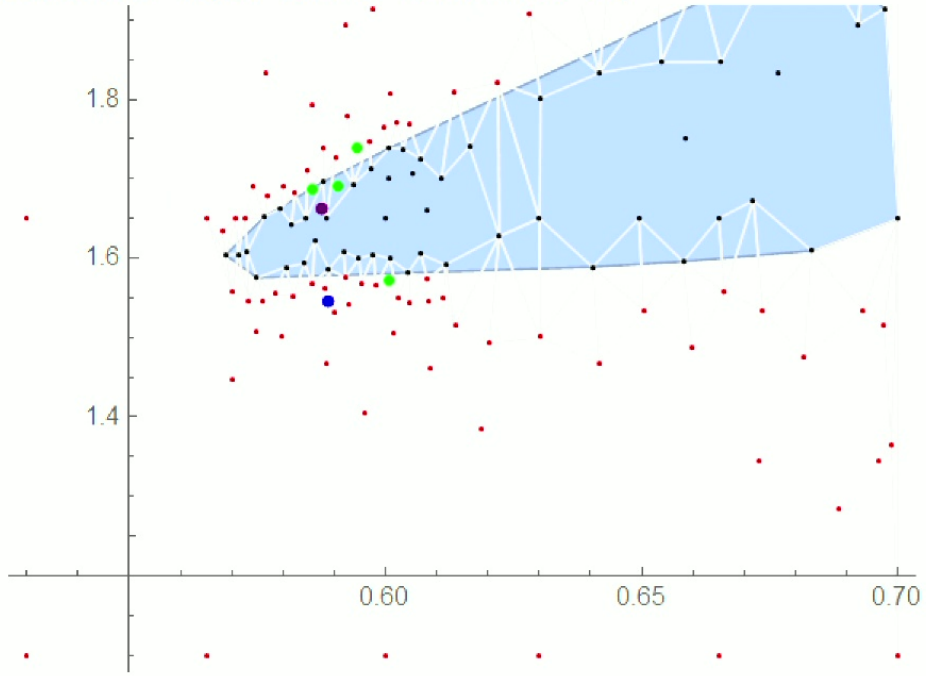


```
Print["Norm of αᵢ :"];
(Norm /@ Partition[#, componentLength]) & /@ alphadata[[All, 1]] // N //
    Prepend[#, corlist] & // Transpose //
  Prepend[#, {"correlator", "red", "purple", "blue"}] & // MatrixForm
```

160%

PI_Symmetr... IsingOE_nvg... IsingOE_M... IsingOE_nvg... IsingOE_Tut... lecture1.nb ...

simpleboot_tut... slides testrun simpleboot4 tutorial4A Solstice

8:36 PM ENG Thursday 4/27/2023